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A_4-based neutrino masses with Majoron decaying dark matter

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Outline

A_4-based neutrino masses with Majoron decaying dark matter

1. Spontaneous breaking of letpon number and the Majoron (Schechter and Valle)
2. The model
3. Results
4. Conclusions

1-2-3 Model (Schechter/Valle)

J. Schechter and J.W.F. Valle, Phys. Rev. D 25, 774 (1982).

1-2-3 model: $SU(2)$ singlet $\phi(l=-2)$, doublet $h(l=0)$ and triplet $T(l=-2)$

$$\langle\phi\rangle=\sigma,\ \langle h\rangle=v,\ \langle T\rangle=u$$

$$\mathcal{L}_{\rm mass}=-\frac{1}{2}\rho^T i\sigma_2 M\rho+\ \text{h.c.}. \qquad M=\begin{pmatrix} M_{II} & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \epsilon=O\left(\frac{m_D}{M_R}\right)$$

$$U^T M U=\begin{pmatrix} m & 0 \\ 0 & m_R \end{pmatrix}=\text{real positive, diagonal,}$$

$$\rho=U\nu.$$

Ansatz: $U=(\exp iH)V,$

$$H=\begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix},\quad V=\begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}$$

gives...

1-2-3 Model (Schechter/Valle)

$$S = -im_D^* \left(M_R^*\right)^{-1}$$

$$V_1^T \left(-m_D M_R^{-1} m_D^T + M_{II}\right) V_1 = m = \text{real positive, diagonal,}$$
$$V_2^T \left(M_R + \frac{1}{2}(M_R^*)^{-1} m_D^\dagger m_D + \frac{1}{2} m_D^T m_D^* (M_R^*)^{-1}\right) V_2 = m_R = \text{real positive, diagonal}$$

The transformation matrix U is then

$$U = \begin{pmatrix} U_a & U_b \\ U_c & U_d \end{pmatrix}$$
$$= \begin{pmatrix} \left(1 - \frac{1}{2} m_D^* (M_R^*)^{-1} M_R^{-1} m_D^T\right) V_1 & m_D^* (M_R^*)^{-1} V_2 \\ -M_R^{-1} m_D^T V_1 & \left(1 - \frac{1}{2} M_R^{-1} m_D^T m_D^* (M_R^*)^{-1}\right) V_2 \end{pmatrix}$$
$$+ O(\epsilon^2).$$

1-2-3 Model (Schechter/Valle)

Yukawa couplings:

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\rho^T i\sigma_2 \begin{pmatrix} M_{II}\frac{T}{u} & m_D\frac{h}{v} \\ m_D^T\frac{h}{v} & M_R\frac{\phi}{\sigma} \end{pmatrix} \rho + \text{ h.c.}$$

Gauge Transformations

Example: Hyper-charge, Triplet

$$\begin{cases} \delta(T_r) = -2\alpha_Y(T_i) \\ \delta(T_i) = 2\alpha_Y(T_r) \end{cases}$$

Invariance of Higgs potential

hyper-charge
$$\frac{\partial V}{\partial \alpha_Y} = 2\frac{\partial V}{\partial T_r}T_i - 2\frac{\partial V}{\partial T_i}T_r + \frac{\partial V}{\partial h_r}h_i - \frac{\partial V}{\partial h_i}h_r = 0$$

lepton number
$$\frac{\partial V}{\partial \alpha_l} = \frac{\partial V}{\partial \phi_r}\phi_i - \frac{\partial V}{\partial \phi_i}\phi_r + \frac{\partial V}{\partial T_r^0}T_i^0 - \frac{\partial V}{\partial T_i^0}T_r^0 = 0$$

1-2-3 Model (Schechter/Valle)

Taking vevs with no spont. CP break and diff. with respect to ϕ_i

$$\left\langle \frac{\partial V}{\partial \phi_r} \right\rangle - \langle \phi_r \rangle \left\langle \frac{\partial^2 V}{\partial \phi_i^2} \right\rangle - \langle T_r^0 \rangle \left\langle \frac{\partial^2 V}{\partial \phi_i \partial T_i^0} \right\rangle = 0$$

At the minimum: $$\sigma \left\langle \frac{\partial^2 V}{\partial \phi_i^2} \right\rangle + u \left\langle \frac{\partial^2 V}{\partial \phi_i \partial T_i^0} \right\rangle = 0.$$

$$\sigma \left\langle \frac{\partial^2 V}{\partial T_i \partial \phi_i} \right\rangle + u \left\langle \frac{\partial^2 V}{\partial (T_i^0)^2} \right\rangle = 0$$

$$v \left\langle \frac{\partial^2 V}{\partial (h_i^0)^2} \right\rangle + 2u \left\langle \frac{\partial^2 V}{\partial h_i^0 \partial T_i^0} \right\rangle = 0$$

$$v \left\langle \frac{\partial^2 V}{\partial h_i^0 \partial T_i^0} \right\rangle + 2u \left\langle \frac{\partial^2 V}{\partial (T_i^0)^2} \right\rangle = 0$$

$$\sigma \left\langle \frac{\partial^2 V}{\partial \phi_i \partial h_i^0} \right\rangle + u \left\langle \frac{\partial^2 V}{\partial T_i^0 \partial h_i} \right\rangle = 0$$

1-2-3 Model (Schechter/Valle)

$$M_{\text{Higgs}} = \begin{pmatrix} 1 & \frac{v}{2\sigma} & -\frac{v}{2u} \\ \frac{v}{2\sigma} & \frac{v^2}{4\sigma^2} & -\frac{v^2}{4\sigma u} \\ -\frac{v}{2u} & -\frac{v^2}{4\sigma u} & \frac{v^2}{4u^2} \end{pmatrix} a \qquad a = \left\langle \frac{\partial^2 V}{\partial (h_i^0)^2} \right\rangle .$$

One 0 eigenvalue with double multiplicity

Majoron

$$J = N\left(-2vu^2 h_i^0 + \sigma(v^2+4u^2)\phi_i + uv^2 T_i^0\right)$$

$$\phi^* = -iN\sigma(v^2+4u^2)J + \ldots$$

$$h^0 = -2iNvu^2 J + \ldots$$

$$T^0 = iNuv^2 J + \ldots$$

1-2-3 Model (Schechter/Valle)

Coupling of the Majoron with neutrinos

$$\mathcal{L}_{\nu J} = -\frac{1}{2}\nu^T i\sigma_2 \left(\frac{T^0}{u} U_a^T M_{II} U_a + \frac{h^0}{v}(U_a^T m_D U_c + U_c^T m_D^T U_a) + \frac{\phi^*}{\sigma} U_c^T M_R U_c \right) \nu + \text{ h.c.}$$

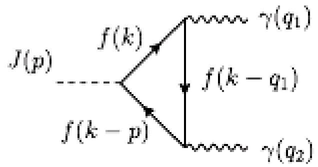
$$\mathcal{L}_{\nu J} = -\frac{1}{2} N J v^2 \nu^T i\sigma_2 V_1^T \left(M_{II} - m_D M_R^{-1} m_D^T \right) V_1 \nu + \text{ h.c.} + O(\epsilon^3)$$

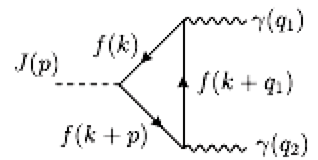
Coupling of the Majoron with photons

$$g_{J\gamma\gamma}^f \equiv \frac{N_f \alpha^2 g_{Jff} Q_f^2 X_f}{8\pi m_f},$$

$$X_f = -2m_f^2 C_0(0,0,m_J^2,m_f^2,m_f^2,m_f^2) \simeq 1 + m_J^2/(12 m_f^2)$$

$$g_{Jff} = -\frac{2u_\Delta^2}{v^2 u_\sigma} m_f(-2T_3^f).$$





A_4 neutrino masses w/ spont. break. lepton num.

$$-\mathcal{L}_L = h_1 \overline{L}_1 (\nu_R \Phi)'_1 + h_2 \overline{L}_2 (\nu_R \Phi)_1 + h_3 \overline{L}_3 (\nu_R \Phi)''_1 + \lambda L_1^T C \Delta L_2 + \lambda L_2^T C \Delta L_1 + \lambda' L_3^T C \Delta L_3 + M_R \left(\overline{S_L} \nu_R\right)_1 + h \left(S_L^T C S_L\right)'_1 \sigma + \text{h.c.}$$

Table 1: Lepton multiplet structure ($Q = T_3 + Y/2$)

	L_1	L_2	L_3	l_{Ri}	ν_{iR}	Φ_i	Δ	σ	S_{iL}
$SU(2)$	2	2	2	1	1	2	3	1	1
$U(1)_Y$	-1	-1	-1	-2	0	-1	2	0	0
A_4	$1'$	1	$1''$	3	3	3	$1''$	$1''$	3
L	1	1	1	1	1	0	-2	-2	1

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}, \quad m_D = v\,\mathrm{diag}(h_1, h_2, h_3)\, U, \quad U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega \\ 1 & 1 & 1 \\ 1 & \omega & \omega^2 \end{pmatrix}$$

Inverse seesaw

$$\mathcal{M}_\nu^{\mathrm{I}} = m_D M^{T^{-1}} \mu M^{-1} m_D^T = \frac{h v^2 u_\sigma}{M_R^2} \begin{pmatrix} h_1^2 & 0 & 0 \\ 0 & 0 & h_2 h_3 \\ 0 & h_2 h_3 & 0 \end{pmatrix} \qquad \mathcal{M}_\nu^{\mathrm{II}} = 2 u_\Delta \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & \lambda' \end{pmatrix}$$

A_4 neutrino masses w/ spont. break. lepton num.

Invariant Higgs Potential

$$V = V(\Phi) + V(\Phi, \Delta, \sigma) + \dots$$

$$V(\Phi) = m_\Phi^2 \left(\Phi^\dagger\Phi\right)_1 + \lambda_1 \left(\Phi^\dagger\Phi\right)_1 \left(\Phi^\dagger\Phi\right)_1 + \lambda_2 \left(\Phi^\dagger\Phi\right)_{1'} \left(\Phi^\dagger\Phi\right)_{1''}$$
$$+ \lambda_3 \left(\Phi^\dagger\Phi\right)_{3s} \cdot \left(\Phi^\dagger\Phi\right)_{3s} + \lambda_4 \left(\Phi^\dagger\Phi\right)_{3s} \cdot \left(\Phi^\dagger\Phi\right)_{3a} + \lambda_5 \left(\Phi^\dagger\Phi\right)_{3a} \cdot \left(\Phi^\dagger\Phi\right)_{3a}$$

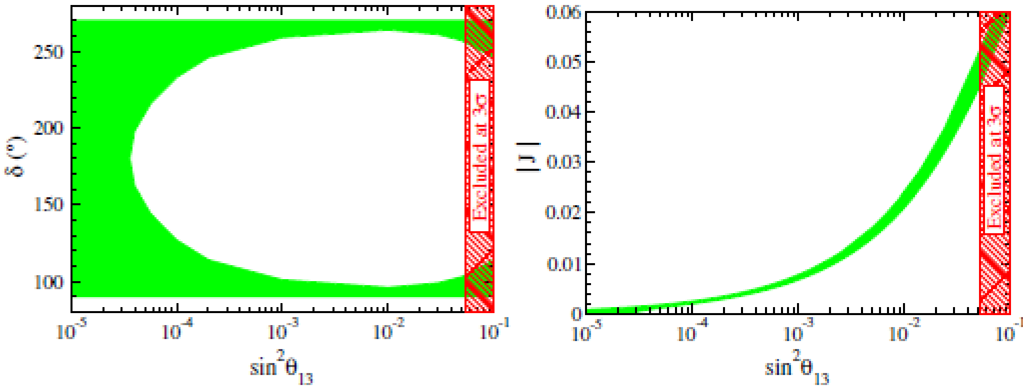
$$V(\Phi, \Delta, \sigma) = \left(M_\Delta^2 + \rho|\sigma|^2\right) \mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_\sigma|\sigma|^4 + \left[m_\sigma^2 + \xi \left(\Phi^\dagger\Phi\right)_1\right] |\sigma|^2 - (\delta\Phi^T\Delta\Phi\sigma^* + \text{h.c.})$$

$$\frac{\delta V}{\delta\Delta} = 0 \Rightarrow (M_\Delta^2 + \rho\, u_\sigma^2)\, u_\Delta - \delta v^2 u_\sigma = 0 \quad \Longrightarrow \quad \boxed{u_\Delta = \frac{\delta v^2 u_\sigma}{M_\Delta^2 + \rho u_\sigma^2} \simeq \frac{\delta v^2}{\rho u_\sigma}}$$

"vev-seesaw" relation: $\boxed{u_\Delta u_\sigma \sim v^2}$

$$\frac{\delta V}{\delta\sigma} = 0 \Rightarrow 2\lambda_\sigma u_\sigma^3 + \left(m_\sigma^2 + \xi v^2 + \rho u_\Delta^2\right) u_\sigma - 2\delta v^2 u_\Delta = 0 \quad \underset{u_\Delta,\, v \,<<\, u_\sigma}{\Longrightarrow} \quad \boxed{u_\sigma = \sqrt{-\frac{m_\sigma^2}{2\lambda_\sigma}}}$$

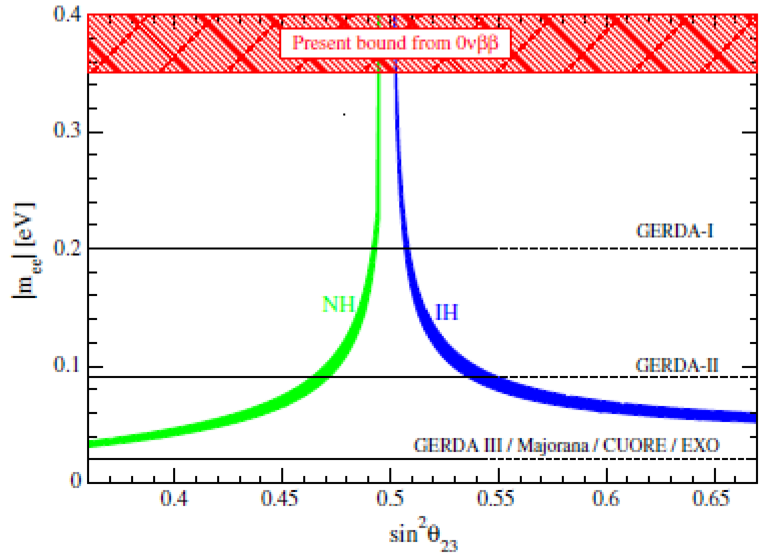
A_4 neutrino masses w/ spont. break. lepton num.



$$J = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta$$

$$m_1/m_3 = \tan^2\theta_{23}$$

$$|m_{ee}| = \left|c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{2i\alpha} + s_{13}^2 m_3 e^{2i\beta}\right|$$



Majoron decaying Dark-Matter

Present Majoron density: $n_J(t_0) = n_J(t_D) e^{-t_0/\tau}$

$$\frac{n_J(t_0)}{n_\gamma(t_0)} = \frac{g_{*s}(t_0)}{g_{*s}(t_D)} \frac{n_J(t_D)}{n_\gamma(t_D)} e^{-t_0/\tau}$$

$$\Omega_J h^2 = \beta \frac{m_J}{1.36 \text{ keV}} e^{-t_0/\tau}$$

Majorons can explain DM if

$$\Gamma_{J\nu\nu} < 1.3 \times 10^{-19} \text{ s}^{-1} \ , \ 0.12 \text{ keV} < \beta\, m_J < 0.17 \text{ keV}$$

$$J \longrightarrow \nu\nu$$

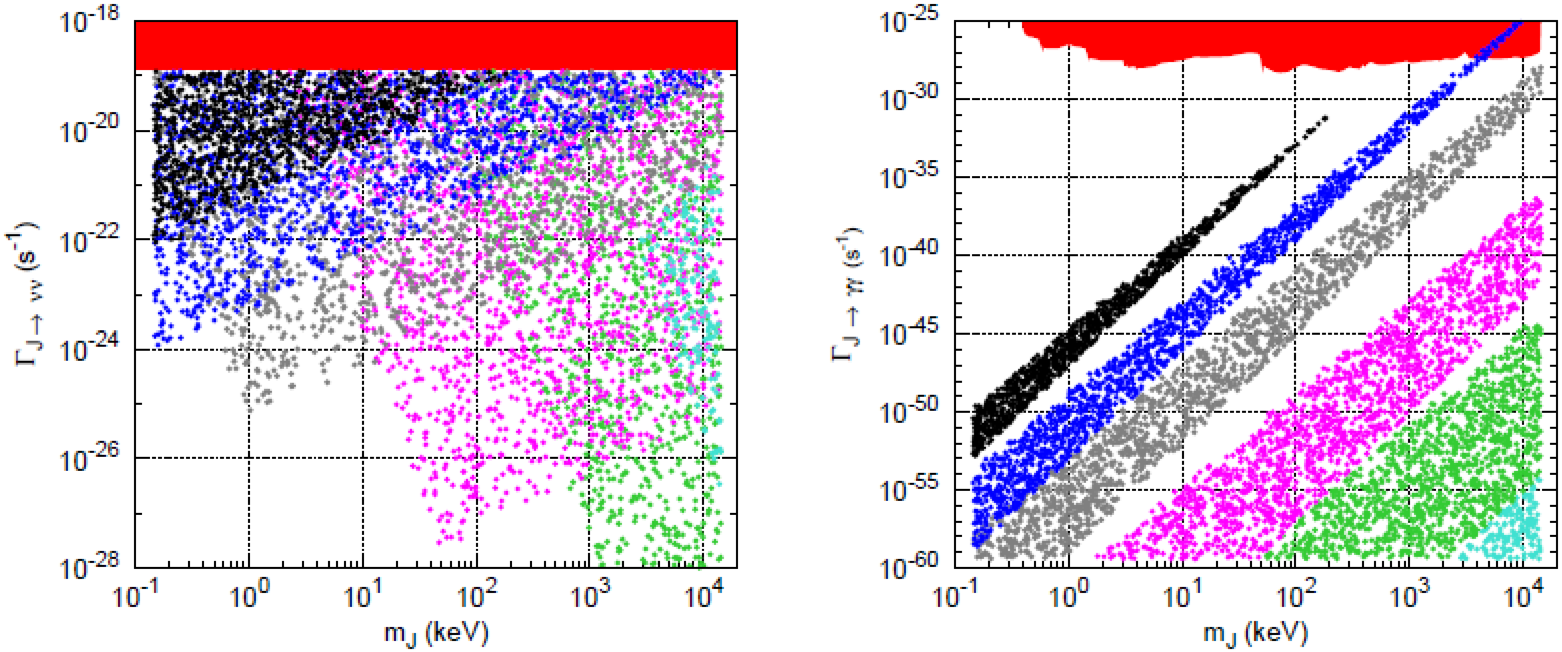
$$\Gamma_{J\nu\nu} = \frac{m_J}{32\pi} \frac{\sum_i (m_i^\nu)^2}{2u_\sigma^2}$$

$$J \longrightarrow \gamma\gamma$$

$$\Gamma_{J\gamma\gamma} = \frac{\alpha^2 m_J^3}{64\pi^3} \left| \sum_f N_f Q_f^2 \frac{2u_\Delta^2}{v^2 u_\sigma} (-2T_3^f) \frac{m_J^2}{12 m_f^2} \right|^2 ,$$

Majoron decaying Dark-Matter

$$\Gamma_{J\nu\nu} < 1.3 \times 10^{-19}\ \mathrm{s}^{-1}\ ,\ 0.12\ \mathrm{keV} < \beta\, m_J < 0.17\ \mathrm{keV}$$



Left panel: $\Gamma_{J\nu\nu}$ as function of the Majoron mass for u_Δ =1 eV (turquoise), 100 eV (dark green), 10keV (magenta), 1MeV (grey), 10MeV (dark blue) and 100 MeV (black). Right panel: $\Gamma_{J\gamma\gamma}$ as function of the Majoron mass for the same values of the triplet vev as in the left panel. The upper orange shaded region is the excluded region from X-ray observations taken from Herder et al. arXiv 0906.1788 [astro-ph].

Conclusions

- The A_4 group as a flavour symmetry can accommodate the phenomenological neutrino data with an extended leptonic sector and with spontaneous breaking of letpon number.
- In this setup there is a goldstone boson, the Majoron, which although instable is very long lived and is a good candidate for Dark Matter.
- We exepct maximum CP-violation in the region for θ_{13} probed by the next generation experiments.
- The predicted 1-loop induced Majoron decay into photons can be tested through the search of cosmological X-ray, as it is the purpose of the Xenia experiment.

Majoron decaying Dark-Matter - 8

Representations of A_4 $\quad\quad n_1^2 + n_2^2 + n_3^2 + n_4^2 = 12$

$$\mathbf{1} : S = 1, T = 1$$

$$\mathbf{1}' : S = 1, T = \exp(2\pi i/3) = \omega \qquad 1 + \omega + \omega^2 = 0$$

$$\mathbf{1}'' : S = 1, T = \exp(4\pi i/3) = \omega^2$$

$\mathbf{3}$:

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

or

$$V = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

$$S' = VSV^\dagger = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad T' = VTV^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

Majoron decaying Dark-Matter - 9

Character table (Altarelli, hep-ph/0611117)

Class	$\chi 1$	$\chi 1'$	$\chi 1''$	$\chi 3$
C_1	1	1	1	3
C_2	1	ω	ω^2	0
C_3	1	ω^2	ω	0
C_4	1	1	1	-1

$$\chi(a \bigotimes b) = \chi(a) \cdot \chi(b) \longrightarrow \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1''} \oplus \mathbf{3} \oplus \mathbf{3}$$

$$(\mathbf{3} \otimes \mathbf{3})_1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$(\mathbf{3} \otimes \mathbf{3})_{1'} = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

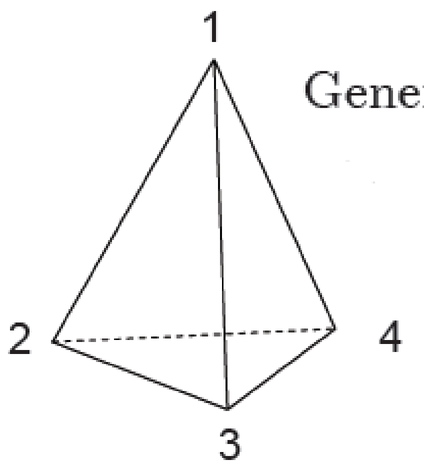
$$(\mathbf{3} \otimes \mathbf{3})_{1''} = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$(\mathbf{3} \otimes \mathbf{3})_3 = (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$(\mathbf{3} \otimes \mathbf{3})_3 = (a_3 b_2, a_1 b_3, a_2 b_1)$$

Majoron decaying Dark-Matter - 7

A_4 - Group of even permutation of 4 objects



Generated by $S = (4321)$ and $T = (2314)$: $S^2 = T^3 = (ST)^3 = 1$

A_4: Rotational invariance symmetry of the tetrahedron

S_4: Full invariance symmetry of the tetrahedron

4 Classes

$$C_1 : I = (1234)$$

$$C_2 : T = (2314), ST = (4132), TS = (3241), STS = (1423)$$

$$C_3 : T^2 = (3124), ST^2 = (4213), T^2S = (2431), TST = (1342)$$

$$C_4 : S = (4321), T^2ST = (3412), TST^2 = (2143)$$