SUSY adjoint $SU(5)$ grand unified model with S_4 flavor symmetry

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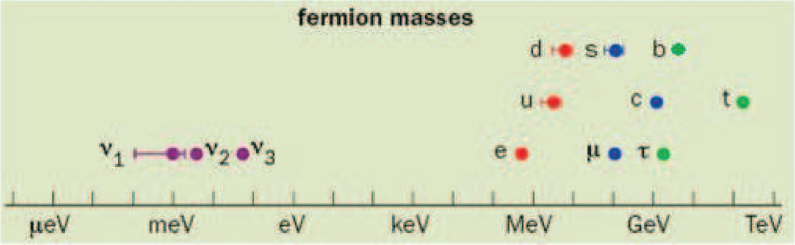
Outline

1. Observations
2. Tri-bimaximal mixing and S_4
3. SUSY adjoint $SU(5)$ model with S_4 flavor symmetry
4. Phenomenological implications of the model
5. Summary

SM flavor puzzle

- Experimental observation on fermion masses



- Experimental data on CKM matrix

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

with

$$\lambda = 0.2257^{+0.0009}_{-0.0010},\ A = 0.814^{+0.021}_{-0.022}, \bar{\rho} = 0.135^{+0.031}_{-0.016}, \eta = 0.349^{+0.015}_{-0.017}$$

- Neutrino oscillation parameters

parameter	best fit $\pm 1\sigma$	2σ	3σ
$\Delta m^2_{21}\,[10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
$\Delta m^2_{31}\,[10^{-3}\mathrm{eV}^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28-2.64$ $-(2.17-2.54)$	$2.18-2.73$ $-(2.08-2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	$\lesssim 0.027$ $\lesssim 0.031$	$\lesssim 0.035$ $\lesssim 0.039$

Schwetz,Tortola and Valle, New J.Phys.13:063004,2011,arXiv:1103.0734

- The current data within 2σ is well approximated by the so-called tri-bimaximal mixing

$$U_{\rm TB}=\begin{pmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

$\sin^2\theta_{12,\rm TB}=\frac{1}{3}$, $\sin^2\theta_{23,\rm TB}=\frac{1}{2}$ and $\sin^2\theta_{13,\rm TB}=0$

What is the origin of fermion mass hierarchy and flavor mixing?$\Longrightarrow$ We have to go beyond the SM

Lessons from the TB mixing

- The light neutrino mass matrix in the charged lepton diagonal basis, $M^{TB}_{\nu}=U^{*}_{TB}\mathrm{diag}(m_{\nu_1},m_{\nu_2},m_{\nu_3})U^{\dagger}_{TB}$

$$M^{TB}_{\nu}=\frac{m_{\nu_1}}{6}\begin{pmatrix}4 & -2 & -2\\ -2 & 1 & 1\\ -2 & 1 & 1\end{pmatrix}+\frac{m_{\nu_2}}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}+\frac{m_{\nu_3}}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix}$$

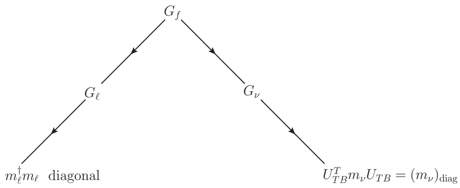
- Symmetry of M^{TB}_{ν}, i.e., $G^{T}_{\nu}M^{TB}_{\nu}G_{\nu}=M^{TB}_{\nu}$

$$G_{\nu_1}=\frac{1}{3}\begin{pmatrix}1 & -2 & -2\\ -2 & -2 & 1\\ -2 & 1 & -2\end{pmatrix},G_{\nu_2}=\frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix},G_{\nu_3}=-\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

The symmetry of light neutrino mass matrix is **Klein-Four** K_4 symmetry, $G^2_{\nu_i}=1$ and $G_{\nu_i}G_{\nu_j}=G_{\nu_j}G_{\nu_i}=G_{\nu_k}$ with $i\neq j\neq k$.

- Where does Klein-Four symmetry come from? $\Longrightarrow$ a flavor symmetry at high energy, which should have 3-dimensional irreducible representation.

The minimal group producing TB mixing



- Minimal means the order of the group is minimal.
- The minimal symmetry of the charged lepton sector at low energy, $G_\ell = Z_3$
  Generator:
  $$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad \text{or} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$
- The flavor symmetry group should contain both the above $G_\ell = Z_3$ and $G_\nu = K_4$ symmetries as subgroup, **the minimal group is S_4.** (C.S.Lam,Phys.Rev.Lett.101:121602,2008)

Discrete group S_4

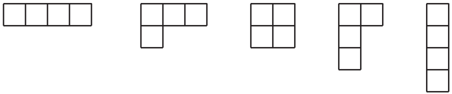
- Geometrically S_4 is the symmetry group of cube.

- S_4 is the permutation group of 4 objects, it has 4!=24 elements. S_4 can be generated by two basic permutations S and T given by $S=(1234)$ and $T=(123)$, which obey the relation

$$S^4=T^3=1, ST^2S=T$$

- S_4 has 30 subgroups, its elements belong to 5 conjugate classes.

Representation of S_4



1_1 3_1 2 3_2 1_2

- S_4 has 5 irreducible representations: 1_1, 1_2, 2, 3_1 and 3_2

1_1 $\quad S=1 \quad T=1$

2 $\quad S=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad T=\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$

3_1 $\quad S=\frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix} \quad T=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

1_2 and 3_2 representations can be obtained from 1_1 and 3_1 by only shifting the sign of S.

- We have more alternatives to produce tri-bimaximal mixing. The double representation can be used to extend the flavor symmetry to the quark sector, $2+1$ assignment for quarks.

Adjoint $SU(5)$ GUT

- Matter fields: $\bar{\mathbf{5}}+\mathbf{10}$

$$\bar{\mathbf{5}}\sim\begin{pmatrix} d_r^C \\ d_b^C \\ d_g^C \\ e^- \\ -\nu_e \end{pmatrix}_L, \qquad \mathbf{10}\sim\begin{pmatrix} 0 & u_g^C & -u_b^C & -u_r & -d_r \\ -u_g^C & 0 & u_r^C & -u_b & -d_b \\ u_b^C & -u_r^C & 0 & -u_g & -d_g \\ u_r & u_b & u_g & 0 & -e^C \\ d_r & d_b & d_g & e^C & 0 \end{pmatrix}_L$$

- Higgs sector: H_{24}, H_5 and $H_{\bar{5}}$ break $SU(5)$ gauge symmetry down to standard model symmetry and subsequently into the residual $SU(3)_c\times U(1)_{em}$. H_{45} and $H_{\overline{45}}$ are introduced to avoid the wrong relation $M_d^T=M_\ell$.

- Generating neutrino masses:
  1) Type-I see-saw: introducing three right-handed neutrinos ν_i^c (Hagedorn et al, JHEP 1006:048,2010)
  2) Type-II see-saw: H_{15} Higgs.
  3)Type-I +Type III see-saw: introducing three adjoint fields A_i,
  $24=(8,1,0)\oplus(1,3,0)\oplus(3,2,-5/6)\oplus(\bar{3},2,5/6)\oplus(1,1,0)$

Structure of the model

- The flavor symmetry is $S_4 \times Z_3 \times Z_4$

Fields	T_3	$(T_2, T_1)^T$	F	A	H_5	H_{45}	$H_{\overline{5}}$	$H_{\overline{45}}$	H_{24}	χ	φ	ζ	ϕ	η	Δ	ξ
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{\overline{5}}$	$\mathbf{24}$	$\mathbf{5}$	$\mathbf{45}$	$\mathbf{\overline{5}}$	$\mathbf{\overline{45}}$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1_1}$	$\mathbf{2}$	$\mathbf{3_1}$	$\mathbf{3_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{3_1}$	$\mathbf{2}$	$\mathbf{1_2}$	$\mathbf{3_1}$	$\mathbf{2}$	$\mathbf{3_1}$	$\mathbf{1_1}$
Z_3	1	ω	1	1	1	1	ω	1	1	1	1	**1**	ω^2	ω^2	ω	ω
Z_4	1	i	$-i$	i	1	-1	1	-1	1	-1	-1	-1	i	i	i	i
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

- We assign $\begin{pmatrix} T_2 \\ T_1 \end{pmatrix} \sim 2$ and $T_3 \sim 1_1$ to account for the heavy top mass. If we assign $\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \sim 2$, we obtain wrong relation $m_e \sim m_\mu$ and $m_d \sim m_s$.

$$(TF)_{3_1} \sim \begin{pmatrix} T_1F_2 + T_2F_3 \\ T_1F_3 + T_2F_1 \\ T_1F_1 + T_2F_2 \end{pmatrix}, \quad (TF)_{\mathbf{3_2}} \sim \begin{pmatrix} T_1F_2 - T_2F_3 \\ T_1F_3 - T_2F_1 \\ T_1F_1 - T_2F_2 \end{pmatrix}$$

Spontaneous breaking of flavor symmetry

Fields	χ^0	φ^0	ϕ^0	ρ^0	Δ^0
S_4	3_2	2	3_1	1_1	3_1
Z_3	1	1	ω^2	ω^2	ω
Z_4	1	1	-1	-1	-1
$U(1)_R$	2	2	2	2	2

- Using the driving field method with the help of supersymmetry (Altarelli and Feruglio, Nucl.Phys.B 741, 215 (2006))

$$\langle\chi\rangle=\begin{pmatrix} v_\chi \\ v_\chi \\ v_\chi \end{pmatrix},\quad \langle\varphi\rangle=\begin{pmatrix} v_\varphi \\ v_\varphi \end{pmatrix},\quad \langle\zeta\rangle=0\Rightarrow \text{breaks } S_4 \text{ to Klein four}$$

$$\langle\phi\rangle=\begin{pmatrix} 0 \\ v_\phi \\ 0 \end{pmatrix},\quad \langle\Delta\rangle=\begin{pmatrix} v_\Delta \\ 0 \\ 0 \end{pmatrix},\quad \langle\eta\rangle=\begin{pmatrix} 0 \\ v_\eta \end{pmatrix},\quad \langle\xi\rangle=v_\xi$$

- All the VEVs are of the same order of magnitude which is determined by the observed fermion mass hierarchies and flavor mixings

$$\varepsilon\equiv\frac{v_\chi}{\Lambda}\sim\frac{v_\varphi}{\Lambda}\sim\frac{v_\phi}{\Lambda}\sim\frac{v_\eta}{\Lambda}\sim\frac{v_\Delta}{\Lambda}\sim\frac{v_\xi}{\Lambda}\sim\lambda_c^2$$

Neutrino

The LO superpotential is

$$w_\nu = y_\nu (FA)_{1_1} H_5 + \lambda_1 (AA)_{3_1}\chi + \lambda_2 (AA)_2 \varphi$$

then Dirac and Majorana neutrino mass matrices are

$$M^D_{\rho_3} = \frac{1}{2} y_\nu v_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad M^D_{\rho_0} = \frac{\sqrt{15}}{10} y_\nu v_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M^M_{\rho_3} = \begin{pmatrix} 2\lambda_1 v_\chi & -\lambda_1 v_\chi + \lambda_2 v_\varphi & -\lambda_1 v_\chi + \lambda_2 v_\varphi \\ -\lambda_1 v_\chi + \lambda_2 v_\varphi & 2\lambda_1 v_\chi + \lambda_2 v_\varphi & -\lambda_1 v_\chi \\ -\lambda_1 v_\chi + \lambda_2 v_\varphi & -\lambda_1 v_\chi & 2\lambda_1 v_\chi + \lambda_2 v_\varphi \end{pmatrix}$$

$$M^M_{\rho_0} = M^M_{\rho_3}$$

Light neutrino mass matrix is given by see-saw formula

$$\begin{aligned} M_\nu &= -(M^D_{\rho_3})^T (M^M_{\rho_3})^{-1} M^D_{\rho_3} - (M^D_{\rho_0})^T (M^M_{\rho_0})^{-1} M^D_{\rho_0} \\ &= \begin{pmatrix} \frac{-a-b}{5b(3a-b)} & \frac{-a+b}{5b(3a-b)} & \frac{-a+b}{5b(3a-b)} \\ \frac{-a+b}{5b(3a-b)} & \frac{-3a^2-4ab+b^2}{5b(9a^2-b^2)} & \frac{-3a^2+2ab-b^2}{5b(9a^2-b^2)} \\ \frac{-a+b}{5b(3a-b)} & \frac{-3a^2+2ab-b^2}{5b(9a^2-b^2)} & \frac{-3a^2-4ab+b^2}{5b(9a^2-b^2)} \end{pmatrix} y^2_\nu v^2_5 \end{aligned}$$

with $a = \lambda_1 v_\chi$ and $b = \lambda_2 v_\varphi$

- TB mixing is automatically guaranteed, $U^T_{TB} M_\nu U_{TB} = \mathrm{diag}(m_1, m_2, m_3)$

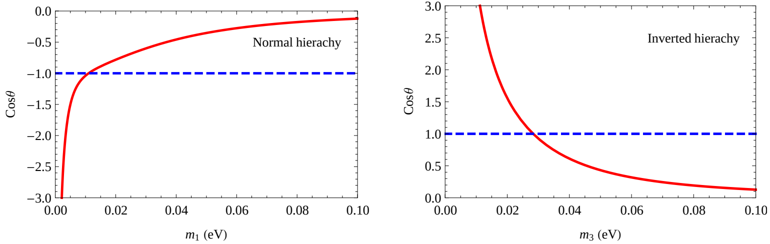
$$m_1 = \frac{1}{b-3a}, \quad m_2 = \frac{-1}{2b}, \quad m_3 = \frac{-1}{3a+b}$$

in unit of $2y_\nu^2 v_5^2/5$, this implies the neutrino mass sum rule

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$

- Neutrino mass spectrum can be normal hierarchy or inverted hierarchy

$$m_1 \geq 0.011\text{eV}, \quad \text{for normal hierarchy}$$
$$m_3 \geq 0.028\text{eV}, \quad \text{for inverted hierarchy}$$



Up quarks

$$
\begin{aligned}
w_u &= y_t T_3 T_3 H_5 + \sum_{i=1}^{4} \frac{y_{ci}}{\Lambda^2} TT\,\mathcal{O}_i^{(1)} H_5 + \frac{y_{ut_1}}{\Lambda^2} TT_3(\phi\chi)_2 H_5 + \frac{y_{ut_2}}{\Lambda^2} TT_3(\eta\varphi)_2 H_5 \\
&\quad + \frac{y_{ut_3}}{\Lambda^2} TT_3 \eta\zeta H_5 + \frac{y_{ct}}{\Lambda} TT_3 \eta H_{45}
\end{aligned}
$$

with

$$
\mathcal{O}^{(1)} = \{(\phi\phi)_{1_1}, (\phi\phi)_2, (\eta\eta)_{1_1}, (\eta\eta)_2\}
$$

$$
M_u = \begin{pmatrix} 0 & 0 & \varepsilon^2 v_5 \\ 0 & \varepsilon^2 v_5 & \varepsilon v_{45} \\ \varepsilon^2 v_5 & -\varepsilon v_{45} & v_5 \end{pmatrix}
$$

Mass hierarchy among up quarks is produced

$$
m_u : m_c : m_t \approx \varepsilon^4 : \varepsilon^2 : 1 \approx \lambda_c^8 : \lambda_c^4 : 1
$$

Down quarks and charged leptons

$$\begin{aligned} w_d &= \frac{y_b}{\Lambda}T_3F\phi H_{\overline{5}}+\frac{y_{s_1}}{\Lambda^2}(TF)_{3_1}(\Delta\Delta)_{3_1}H_{\overline{45}}+\frac{y_{s_2}}{\Lambda^2}(TF)_{3_1}\Delta\xi H_{\overline{45}} \\ &+\sum_{i=1}^{9}\frac{y_{d_i}}{\Lambda^3}T_3F\,\mathcal{O}_i^{(2)}H_{\overline{5}}+\sum_{i=1}^{6}\frac{x_{d_i}}{\Lambda^3}T_3F\,\mathcal{O}_i^{(3)}H_{\overline{45}}+\sum_{i=1}^{7}\frac{z_{d_i}}{\Lambda^3}TF\,\mathcal{O}_i^{(4)}H_{\overline{5}}+\ldots \end{aligned}$$

where

$$\begin{aligned} \mathcal{O}^{(2)} &= \{\chi^2\phi,\chi^2\eta,\varphi\chi\phi,\varphi\chi\eta,\varphi^2\phi,\chi\phi\zeta,\chi\eta\zeta,\varphi\phi\zeta,\phi\zeta^2\} \\ \mathcal{O}^{(3)} &= \{\phi^3,\phi^2\eta,\phi\eta^2,\Delta^3,\Delta^2\xi,\Delta\xi^2\} \\ \mathcal{O}^{(4)} &= \{\phi^2\chi,\phi^2\varphi,\phi^2\zeta,\eta\phi\chi,\eta\phi\varphi,\eta\phi\zeta,\eta^2\chi\} \end{aligned}$$

The mass matrices for down quarks and charged leptons are

$$M_d=\begin{pmatrix} y^d_{11}\varepsilon^3v_{\overline{5}} & y^d_{12}\varepsilon^3v_{\overline{5}} & y^d_{13}\varepsilon^3v_{\overline{5}}+\mathbf{2}y^{d'}_{13}\varepsilon^3v_{\overline{45}} \\ y^d_{21}\varepsilon^3v_{\overline{5}} & \mathbf{2}y^d_{22}\varepsilon^2v_{\overline{45}}+y^{d'}_{22}\varepsilon^3v_{\overline{5}} & y^d_{23}\varepsilon^3v_{\overline{5}} \\ \mathbf{2}y^d_{22}\varepsilon^2v_{\overline{45}}+y^{d'}_{31}\varepsilon^3v_{\overline{5}} & y^d_{32}\varepsilon^3v_{\overline{5}} & y^d_{33}\varepsilon v_{\overline{5}} \end{pmatrix}$$

$$M_\ell=\begin{pmatrix} y^d_{11}\varepsilon^3v_{\overline{5}} & y^d_{21}\varepsilon^3v_{\overline{5}} & -\mathbf{6}y^d_{22}\varepsilon^2v_{\overline{45}}+y^{d'}_{31}\varepsilon^3v_{\overline{5}} \\ y^d_{12}\varepsilon^3v_{\overline{5}} & -\mathbf{6}y^d_{22}\varepsilon^2v_{\overline{45}}+y^{d'}_{22}\varepsilon^3v_{\overline{5}} & y^d_{32}\varepsilon^3v_{\overline{5}} \\ y^d_{13}\varepsilon^3v_{\overline{5}}-\mathbf{6}y^{d'}_{13}\varepsilon^3v_{\overline{45}} & y^d_{23}\varepsilon^3v_{\overline{5}} & y^d_{33}\varepsilon v_{\overline{5}} \end{pmatrix}$$

- Relating down quarks to charged leptons

$$m_\tau \simeq m_b\,, \quad m_\mu \simeq 3m_s$$

$$m_d : m_s : m_b \approx \lambda_c^4 : \lambda_c^2 : 1, \quad m_e : m_\mu : m_\tau \approx \lambda_c^4 : 3\lambda_c^2 : 1$$

- Predictions for CKM matrix: we choose $v_{\overline{45}} \sim \lambda_c v_{\overline{5}}$ in order to reproduce the Cabibbo mixing angle

$$V_{CKM} \simeq \begin{pmatrix} 1 & \mathcal{O}(\lambda_c) & \mathcal{O}(\lambda_c^3) \\ \mathcal{O}(\lambda_c) & 1 & \mathcal{O}(\lambda_c^2) \\ \mathcal{O}(\lambda_c^3) & \mathcal{O}(\lambda_c^2) & 1 \end{pmatrix}$$

- Lepton mixing parameters

$$\sin\theta_{13} \sim \frac{\lambda_c}{3\sqrt{2}} \simeq 2.97^\circ$$

$$|\sin^2\theta_{12} - \frac{1}{3}| \sim \frac{2}{9}\lambda_c$$

$$|\sin^2\theta_{23} - \frac{1}{2}| \sim \frac{\lambda_c^2}{36}$$

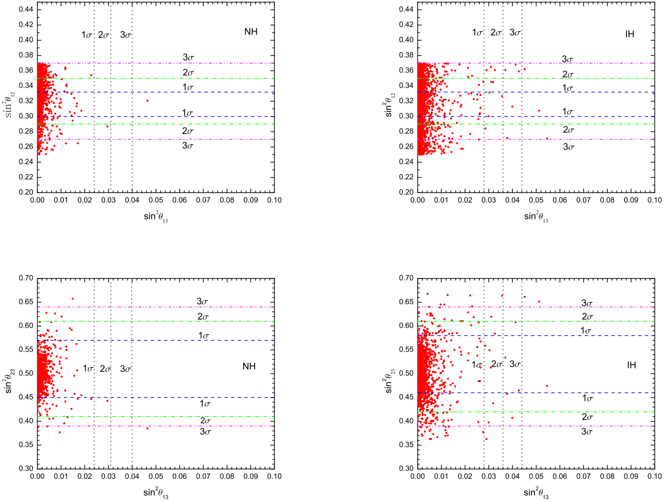
Next to leading order(NLO) corrections

- Leading order predictions are corrected by higher dimensional operators whose contributions are suppressed by additional powers of $1/\Lambda$
  ★ Higher dimensional operators in the driving superpotential w_v—- revising the vacuum alignment
  ★ Higher dimensional operators in the Yukawa superpotentials w_ν, w_u and w_d—-modifying the Yukawa couplings after the electroweak and flavor symmetry breaking

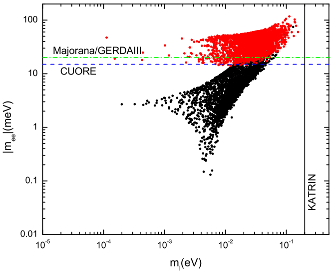
- The subleading corrections have been studied in details. All observables get a correction of relative order λ_c^2, and the leading order predictions are not spoiled.

Phenomenological consequences

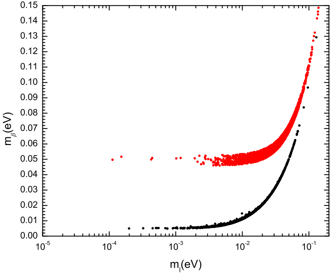
- Lepton mixing angles



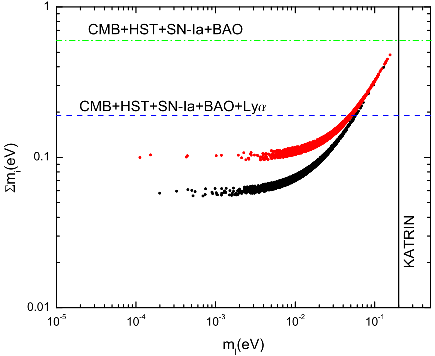
- Neutrinoless double beta decay: $|m_{ee}|=\frac{|m_1|}{3}\sqrt{2-\frac{|m_2|^2}{|m_1|^2}+2\frac{|m_2|^2}{|m_3|^2}}$



- Beta decay



- Sum of neutrino masses



- A SUSY SU(5) based on $S_4 \times Z_3 \times Z_4$ flavor symmetry is presented. Three generations of adjoint matter field are introduced, neutrino masses are generated via the combination of type-I and type-III see-saw mechanism.

- At leading order, S_4 symmetry is broken to Klein four symmetry in neutrino sector, and neutrino mass matrix is exactly diagonalized by tri-bimaximal mixing matrix, we obtain the neutrino mass sum rule $\frac{1}{m_1}+\frac{1}{m_2}=\frac{1}{m_3}$. The mixing of left-handed charged lepton results in corrections to the TB mixing, well-known leptonic mixing sum rules are produced, θ_{13} is predicted to be close to 3°.

- We assign $\begin{pmatrix} T_2 \\ T_1 \end{pmatrix} \sim 2$ and $T_3 \sim 1_1$. The observed hierarchies of quark masses and CKM mixing matrix are produced.

- The leading order predictions are stable under subleading corrections, and the phenomenological implications of the model are analyzed.

Thank you for your attention!