Different $SO(10)$ Paths to Fermion Masses and Mixings

Based on: G.Altarelli, G.B. JHEP 1103:133,2011

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Outline

1. Masses and mixings
   - Flavour structures
   - Neutrino sector
2. GUT
   - SU(5)
   - SO(10)
3. A possible SO(10) strategy
   - A class of models
   - Compared fit

Beyond the Standard Model

Standard Model

$$\left(\overline{\psi}^i_u Y^{ij}_u \psi^j_u + \overline{\psi}^i_d Y^{ij}_d \psi^j_d + \overline{\psi}^i_e Y^{ij}_e \psi^j_e\right) \frac{v}{\sqrt{2}} \quad (1)$$

$Y^{ij}_{u,d,l}$ are completely **free parameters** → masses and mixings are not theoretically motivated
Neutrino are massless

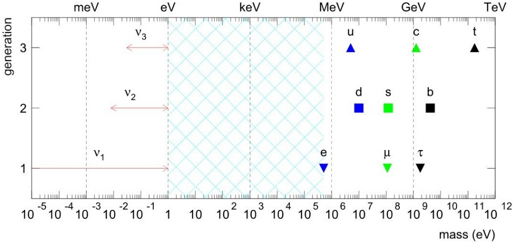
FLAvour **SY**mmetry → a more foundamental theory explaining these patterns

Grand Unified Theory

$$\overline{\psi}^{c\,i}_{\nu\,L} g^{ij}_{\nu} \psi^j_{\nu\,L} \frac{v^2}{2\Lambda} \quad (2)$$

Neutrino masses point to a lepton number violating scale close to M_{GUT} where lepton number is naturally violated !!!

The observed flavour structures (May '11)



$$|V_{\rm CKM}| \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix} \qquad |U_{\rm PMNS}| \sim \begin{pmatrix} 0.8 & 0.5 & < 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

De Gouvea '08

Possible patterns for leptons

The observed lepton mixings can be well reproduced with special and easy structures, including:

Tri-Bimaximal mixing

$$V_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In charged leptons diagonal basis

$$m_{TBM} = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix}$$

Usually $O(\theta_C^2)$ corrections from charged leptons

Bimaximal mixing

$$V_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In charged leptons diagonal basis

$$m_{BM} = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_0 & f_2 - f_0 \\ f_1 & f_2 - f_0 & f_0 \end{pmatrix}$$

Complementarity: $\theta_C + \theta_{12} \sim \pi/4$

$\rightarrow O(\theta_C)$ corrections

Descrete symmetries

m_{TB} and m_{BM} are symmetric under

$$m_{TB} = A_{23}^T m_{TB} A_{23}, \qquad m_{TB} = S_{TB}^T m_{TB} S_{TB}, \qquad m_l = T_{TB}^T m_l T_{TB}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_{TB} = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T_{TB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$m_{BM} = A_{23}^T m_{BM} A_{23}, \qquad m_{BM} = S_{BM}^T m_{BM} S_{BM}, \qquad m_l = T_{BM}^T m_l T_{BM}$$

$$S_{BM} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad T_{BM} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}$$

If TB or BM are relevants the neutrino sector points to a **discrete flavour symmetry**, such as S_3, A_4, S_4, T', ...

Then we have to **spontaneously break** the discrete group in the subgroups A_{23}, S in the neutrino sector and T in the lepton sector

Flavour in SU(5)

Matter content

$$d^c, e, \nu_e \supset \bar{5} \qquad u, u^c, d, e^c \supset 10 \qquad \nu_e^c \supset 1 \tag{3}$$

Mass terms: minimal version

$$\bar{5}\,10\,\bar{5}_H + 10\,10\,5_H \tag{4}$$

next to minimal

$$\bar{5}\,10\,\overline{45}_H + 10\,10\,45_H \tag{5}$$

Flavour relations at M_{GUT}

$$M_5^e = M_5^{d\,T} \qquad M_{45}^e = -3M_{45}^{d\,T} \tag{6}$$

How to obtain TB in SU(5)?

In general different discrete group representations for $\bar{5}$, 10 and 1

For example: $(SU(5), A_4) = (\bar{5}, 3), (10_1, 1''), (10_2, 1'), (10_3, 1), (1, 3)$
Altarelli, Feruglio, Hagedron '08

Flavour in SO(10)

All the SM particle + ν_R for each family in **one irreducible representation**

$$16 \supset \overline{5} + 10 + 1 \tag{7}$$

Mass term in SO(10)

$$16 \times 16 = 10 + 126 + 120 \tag{8}$$

(10 and 126 symmetric, 120 antisymmetric)

$$W_Y \; = \; h\,\psi\psi 10_H + f\,\psi\psi\overline{126}_H + h'\,\psi\psi 120_H \tag{9}$$

To avoid large Higgs representations one can consider the above couplings as effective, ie coming from **higher order operators** as

$$10 \times 45 = 10 + 120 + 320 \tag{10}$$

$$16_H \times 16_H = 10 + 126 + 120 \tag{11}$$

Neutrino masses are naturally generated (ν_R is not a singlet)

See-saw 2 dominance

In a general **renormalizable** scheme, assuming two light Higgs doublets

$$Y_u = h + r_2 f + r_3 h' \qquad Y_e = r_1(h - 3f + c_e h') \tag{12}$$
$$Y_d = r_1(h + f + h') \qquad Y_{\nu^D} = h - 3r_2 f + c_\nu h' \tag{13}$$

In the $\overline{126}$ both the SM singlet (v_R) and the $SU(2)_L$ triplet (v_L) can get vev → type-1 +type-2 see-saw (Dutta, Mimura, Mohapatra '10)

$$m_\nu = f v_L - M_D \frac{1}{f v_R} M_D^T \simeq f v_L \qquad \begin{array}{l} v_L\text{: vev of **triplet** in } \overline{126}_H \\ v_R\text{: Majorana mass for } \nu_R \end{array} \tag{14}$$

assuming **type 2 see-saw dominance** (... Melfo '10)

In this way it possible to partially **disentangle neutrino and quark sectors**

Quarks, charged leptons and TB neutrinos

Taking f in the TB form, h dominantly 33 and hermitian matrices (see below) we get

- **quark sector**:
  - heavy third generation masses from Y
  - small first and second generations masses and small CKM from h_{ij}, f, h'
- **lepton sector**:
  - m_ν TB from f
  - corrections to TB from charged leptons mixing

$$h = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & Y \end{pmatrix}$$

$$f = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix}$$

$$h' = i \begin{pmatrix} 0 & \sigma_{12} & \sigma_{13} \\ -\sigma_{12} & 0 & \sigma_{23} \\ -\sigma_{13} & -\sigma_{23} & 0 \end{pmatrix}$$

We assume an underlaying **parity symmetry** (Dutta '04)

- hermitian matrices $\rightarrow$ less parameters
- the fit is still very good

TB, BM and general matrices

- Note that we can **always go to a basis where f is TB**

$$f_{TB} = V^*_{TB} f'_{diag} V^\dagger_{TB} = V^*_{TB} V^T f' V V^\dagger_{TB} \tag{15}$$

rotating the 16 of fermions

- In the same way also **f BM** or with other structures with three free parameters in m_ν can be obtained by a rotation
- TB, BM, ... correspond to the **same fit** analysis

This analysis is **general** for SO(10) with 10_H, 120_H and $\overline{126}_H$ and type 2 see-saw dominance

Fit results

We **fitted the model** on fermion masses and mixing angles evolved at the high scale M_{GUT}

Observable	Best fit value
$m_u[MeV]$	0.553
$m_c[MeV]$	210
$m_t[GeV]$	82.6
$m_d[MeV]$	1.15
$m_s[MeV]$	22.4
$m_b[GeV]$	1.08
$m_e[MeV]$	0.3585
$m_\mu[MeV]$	75.67
$m_\tau[GeV]$	1.292
V_{us}	0.224
V_{cb}	0.0351
V_{ub}	0.00320
$J \times 10^{-5}$	2.19
$\Delta m^2_{21} \times 10^{-5}[eV^2]$	7.65
$\Delta m^2_{32} \times 10^{-3}[eV^2]$	2.40
$sin^2\theta_{13}$	0.0126
$sin^2\theta_{12}$	0.305
$sin^2\theta_{23}$	0.499
χ^2 quark	0.0959
χ^2 charged fermions	0.0959
χ^2 neutrino	0.0316
χ^2 totale	0.127
χ^2/dof totale	0.127
d_{FT}	469777

Parameter	Best fit value
$h_{11}v_u[GeV]$	0.808
$h_{12}v_u[GeV]$	1.17
$h_{13}v_u[GeV]$	6.06
$h_{22}v_u[GeV]$	5.37
$h_{23}v_u[GeV]$	5.64
$Yv_u[GeV]$	85.0
$f_0v_u[GeV]$	-2.20
$f_1v_u[GeV]$	-0.276
$f_2v_u[GeV]$	-0.228
$\sigma_{12}v_u[GeV]$	-0.270
$\sigma_{13}v_u[GeV]$	2.27
$\sigma_{23}v_u[GeV]$	6.37
$r_1/\tan\beta$	0.0129
r_2	1.66
r_3	0.612
c_e	3.85
$v_L/v_u \times 10^{-9}$	0.0112

Comparation with other models

Comparing with other **realistic SO(10) models** without TB, on the same set of data

Model	d.o.f.	χ^2	χ^2/d.o.f.	d_{FT}	d_{Data}
DR [14]	4	0.41	0.10	7.0 10^3	1.3 10^3
ABB [16–18]	6	2.8	0.47	8.1 10^3	3.8 10^3
JLM [19]	4	2.9	0.74	9.4 10^3	3.8 10^3
BSV [33]	< 0	6.9	-	2.0 10^5	3.8 10^3
JK2 [38]	3	3.4	1.1	4.7 10^5	3.8 10^3
GK [40]	0	0.15	-	1.5 10^5	3.8 10^3
T-IID	1	0.13	0.13	4.7 10^5	3.8 10^3

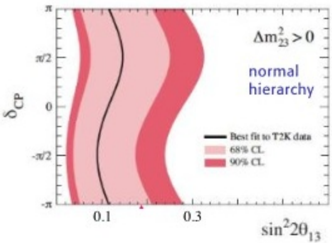
- DR: Dermisek, Raby '06
- ABB: Albright, Babu, Barr '01
- JLM: Ji, Li, Mohapatra '05
- BSV: Bajc, Senjanovic, Vissani '02
- JK2: Joshipura, Kodrani '09
- GK: Grimus, Kuhbock '06
- T-IID: this model

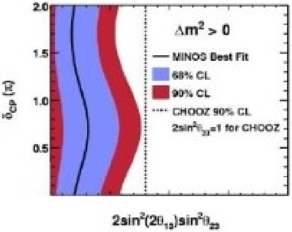
Degree of fine-tuning $\to d_{FT} = \sum \left| \frac{par_i}{err_i} \right|$

Excellent fit of this model but large fine-tuning

Why the fine-tuning? $f_1/f_0 \sim \sqrt{r}$ gives m_{Igen}/m_{IIgen} and δTB too big without any cancellation

Latest news (June '11)





T2K and **MINOS** announced strong hints for $\theta_{13} \neq 0$ in the neutrino sector

$$\text{T2K} \rightarrow 0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34) \quad \text{at } 90\% \text{ C.L.} \quad (16)$$

$$\text{MINOS} \rightarrow 0 < 2\sin^2 2\theta_{13} \sin^2\theta_{23} < 0.12(0.19) \quad \text{at } 90\% \text{ C.L.} \quad (17)$$

New analysis performed on these new data

New analysis

We fitted the model T-IID on the **new T2K data**

T-IID TB

Model	d.o.f.	χ^2	χ^2/d.o.f.	d_{FT}	d_{Data}
T-IID	1	0.13	0.13	$3.4\ 10^5$	$3.8\ 10^3$
T-IID (old data)	1	0.13	0.13	$4.7\ 10^5$	$3.8\ 10^3$

→ very good fit again

We considered also the same model with **f BM** with the new data

T-IID BM

Model	d.o.f.	χ^2	χ^2/d.o.f.	d_{FT}	d_{Data}
T-IID BM	1	0.13	0.13	$3.1\ 10^5$	$3.8\ 10^3$

→ consistency check (TB and BM differs by a 16 rotation → same fit)

Conclusions

- If TB (or BM ...) mixing is realized, the neutrino sector points to a **discrete flavour group**
- Quark sector is gerarchic and with no indication of such a discrete group
- Even if **Grand Unified Theories** connect the two sectors, it is possible to explain the two different patterns in a unified theory ($SU(5) \times A_4$ for example)
- The case of **SO(10) is more difficult** $\rightarrow$ every particle of each family must be in the same flavour group rapresentation
- SO(10) + **type-2 see-saw** offers a viable solution but more work has to be done (ig Dutta, Mimura, Mohapatra '09)

THANK YOU FOR THE ATTENTION

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New analysis results for T-IID TB

Observable	Best fit value
$m_u[MeV]$	0.551
$m_c[MeV]$	210
$m_t[GeV]$	82.5
$m_d[MeV]$	1.22
$m_s[MeV]$	21.4
$m_b[GeV]$	1.05
$m_e[MeV]$	0.3585
$m_\mu[MeV]$	75.67
$m_\tau[GeV]$	1.292
V_{us}	0.224
V_{cb}	0.0351
V_{ub}	0.00313
$J \times 10^{-5}$	2.34
$\Delta m^2_{21} \times 10^{-5}[eV^2]$	7.65
$\Delta m^2_{32} \times 10^{-3}[eV^2]$	2.40
$sin^2\theta_{13}$	0.0363
$sin^2\theta_{12}$	0.303
$sin^2\theta_{23}$	0.498
χ^2 quark	0.0921
χ^2 charged fermions	0.0921
χ^2 neutrino	0.0360
χ^2 totale	0.128
χ^2/dof totale	0.128
d_{FT}	344802

Parameter	Best fit value
$h_{11}v_u[GeV]$	1.42
$h_{12}v_u[GeV]$	-0.723
$h_{13}v_u[GeV]$	-9.84
$h_{22}v_u[GeV]$	7.49
$h_{23}v_u[GeV]$	8.49
$Yv_u[GeV]$	82.9
$f_0v_u[GeV]$	-3.04
$f_1v_u[GeV]$	-0.603
$f_2v_u[GeV]$	0.0452
$\sigma_{12}v_u[GeV]$	1.43
$\sigma_{13}v_u[GeV]$	0.764
$\sigma_{23}v_u[GeV]$	9.88
$r_1/\tan\beta$	0.0125
r_2	1.55
r_3	0.751
c_e	3.08
$v_L/v_u \times 10^{-9}$	0.00909

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New analysis results for T-IID BM

Observable	Best fit value
$m_u[MeV]$	0.550
$m_c[MeV]$	210
$m_t[GeV]$	81.6
$m_d[MeV]$	1.26
$m_s[MeV]$	21.7
$m_b[GeV]$	1.10
$m_e[MeV]$	0.3585
$m_\mu[MeV]$	75.67
$m_\tau[GeV]$	1.292
V_{us}	0.224
V_{cb}	0.0351
V_{ub}	0.00318
$J \times 10^{-5}$	2.31
$\Delta m^2_{21} \times 10^{-5}[eV^2]$	7.65
$\Delta m^2_{32} \times 10^{-3}[eV^2]$	2.40
$sin^2\theta_{13}$	0.0417
$sin^2\theta_{12}$	0.305
$sin^2\theta_{23}$	0.493
χ^2 quark	0.107
χ^2 charged fermions	0.107
χ^2 neutrino	0.0186
χ^2 totale	0.126
χ^2/dof totale	0.126
d_{FT}	344802

Parameter	Best fit value
$h_{11}v_u[GeV]$	0.396
$h_{12}v_u[GeV]$	0.773
$h_{13}v_u[GeV]$	-1.67
$h_{22}v_u[GeV]$	5.99
$h_{23}v_u[GeV]$	7.19
$Yv_u[GeV]$	82.8
$f_0v_u[GeV]$	-2.19
$f_1v_u[GeV]$	-0.598
$f_2v_u[GeV]$	-0.164
$\sigma_{12}v_u[GeV]$	0.515
$\sigma_{13}v_u[GeV]$	1.76
$\sigma_{23}v_u[GeV]$	9.43
$r_1/\tan\beta$	0.0133
r_2	1.70
r_3	0.747
c_e	2.66
$v_L/v_u \times 10^{-9}$	-0.0117

$$h = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{12} & h_{22} & h_{23} \\ h_{13} & h_{23} & Y \end{pmatrix}$$

$$f = \begin{pmatrix} f_2 & f_1 & f_1 \\ f_1 & f_2 + f_0 & f_1 - f_0 \\ f_1 & f_1 - f_0 & f_2 + f_0 \end{pmatrix}$$

$$h' = i \begin{pmatrix} 0 & \sigma_{12} & \sigma_{13} \\ -\sigma_{12} & 0 & \sigma_{23} \\ -\sigma_{13} & -\sigma_{23} & 0 \end{pmatrix}$$

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Some analytic approximate relations for leptons

$$m_\tau \approx k[Y - 3(f_0 + f_2)] \qquad k = \frac{r_1 v_u}{\tan\beta} \tag{18}$$

$$m_\mu \approx k[h_{22} - 3(f_0 + f_2)] - m_\tau s_{23}^{e2} \tag{19}$$

$$s_{23}^e e^{i\phi_2^e} \approx \frac{k}{m_\tau}[h_{23} + 3(f_0 - f_1) + ic_e\sigma_{23}] \tag{20}$$

$$s_{13}^e e^{i(\delta^e + \phi_1^e + \phi_2^e)} \approx \frac{k}{m_\tau}[h_{13} - 3f_1 + ic_e\sigma_{13}] \tag{21}$$

$$s_{12}^e \approx \sqrt{\frac{k}{m_\mu}(h_{11} - 3f_2) - \frac{m_\tau}{m_\mu}s_{13}^{e2}} \tag{22}$$

$$(m_\mu s_{12}^e + m_\tau s_{13}^e s_{23}^e e^{i\delta^e})e^{i\phi_1^e} \approx k(h_{12} - 3f_1 + ic_e\sigma_{12}) \tag{23}$$

$$U_{12} \approx \frac{1}{\sqrt{3}}(1 - s_{12}^e e^{i\phi_1^e} - s_{13}^e e^{i(\delta^e + \phi_1^e + \phi_2^e)}) \tag{24}$$

$$U_{13} \approx \frac{1}{\sqrt{2}}(s_{12}^e - s_{13}^e e^{i(\delta^e + \phi_2^e)}) \tag{25}$$

$$U_{23} \approx \frac{-1}{\sqrt{2}}(1 + s_{23}^e e^{i\phi_2^e}) \tag{26}$$