

Table of contents:

Nuclear moments: *what and why?*

magnetic moments

quadrupole moments

definitions, properties, nuclear structure information to deduce

Nuclear moments: How?

basic principles

angular distribution of radiation

orientation of the nuclear spins

formalisms, methods, experimental results

perturbation of spin-orientation by electromagnetic fields

the Zeeman interaction

the quadrupole interaction

the combined interaction methods to measure nuclear moments

methods to measure nuclear moments

time differential methods

time integrated methods

Moments of ground states

selected examples of experiments, methods, results

Moments of isomeric states

Basic principles to measure nuclear moments

The nuclear moments can be measured by investigating their interaction with the electromagnetic fields in their immediate environment

→ the magnetic moment interacts with magnetic fields (static or dynamic)

measure the magnetic interaction frequency $\nu_B = g\mu_N B/h$

→ the quadrupole moment interacts with the gradient of an electric field ($\partial^2 V / \partial z \partial z$)

measure the quadrupole interaction frequency $\nu_Q = eQV_{zz}/h$

The nuclear properties (detected radiation, lifetime, spin, production method, ...) determine which method can/needs to be used to measure it's nuclear moments

→ many methods have been developed (more than 25 for radioactive nuclei !)

distinguish two main categories, depending on the origin of the electromagnetic field

The electromagnetic fields can be induced by two means:

→ **By the electron cloud in free atoms** (atoms or ions in a beam)

measure the hyperfine structure of atoms/ions via **laser spectroscopy methods**

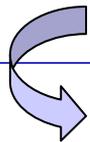
(lecture R. Neugart, Les Houches, 2002)

→ **By implantation in a solid** : crystal-induced electromagnetic fields occur and also external magnetic fields are applied.

The interaction of these fields with the nuclear spin causes a **perturbation of the nuclear spin-orientation**, which is reflected in **the angular distribution of the radiation**.

measure the perturbation of the angular distribution of the radioactive decay

by **hyperfine interaction methods**



I will concentrate on these types of experiments !

Requirements:

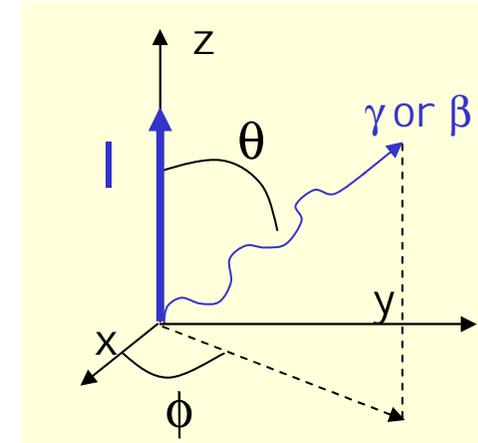
1. The **lifetime of the nuclear state** is longer than the interaction time, such that the interaction can cause a significant perturbation of the nuclear spin orientation before the nucleus decays.
$$(\tau > 1/\omega_i)$$
2. The **ensemble of nuclear spins needs to be oriented** (anisotropic distribution) in order to allow measuring a change in the anisotropy induced by the applied interactions.

Angular distribution of radiation

The direction in which the radiation, emitted by a radioactive nucleus, is emitted, depends on the direction of its nuclear spin.

This is formally described by the 'angular distribution' function:

$$W(\theta, \phi, t) = \sum_{k,n} \frac{\sqrt{4\pi}}{2k+1} A_k B_k^n(t) Y_k^n(\theta, \phi)$$



A_k = radiation parameter → properties of detected radiation

B_k^n = orientation tensor → describes the orientation of the spin ensemble
 unperturbed: $B_k^n(t=0)$ (at time of implantation)

perturbed: $B_k^n(t) = \sum_{k',n'} G_{kk'}^{nn'}(t) B_k^n(t=0)$

Perturbation factor

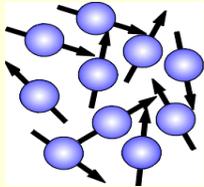
Y_k^n = spherical tensors

Measuring the angular distribution, for γ -decay or β -decaying nuclei, allows to deduce information about the interaction of their nuclear spins with their environment → nuclear moments

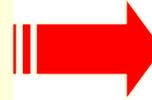
Orientation of an ensemble of nuclear spins

Definitions

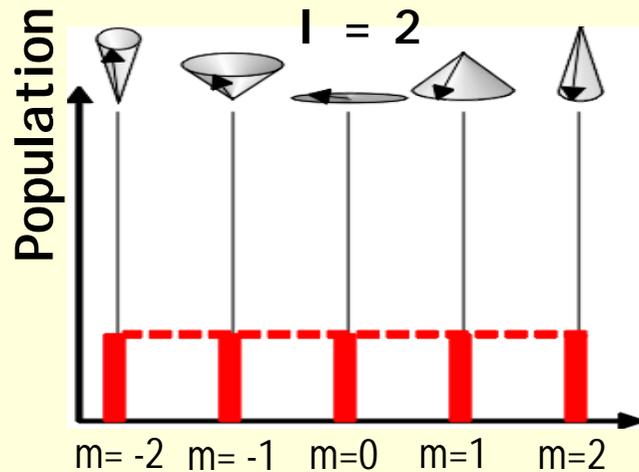
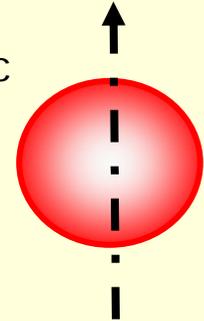
An ensemble of spins is said to be **isotropic**, if all spins are pointing in random directions.



Classical :
spherical symmetric distribution
of the nuclear spins

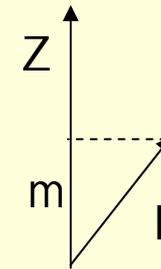


Spherical symmetric
radiation pattern

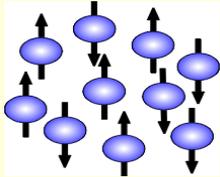


Quantum mechanical:
All spin projected states (m-quantum states)
are equally populated:

$$p(m) \text{ equal } \forall m$$



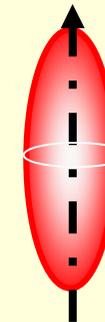
An ensemble of spins is said to be **aligned**,
if all spins are preferentially along a certain direction (**axial symmetry**)



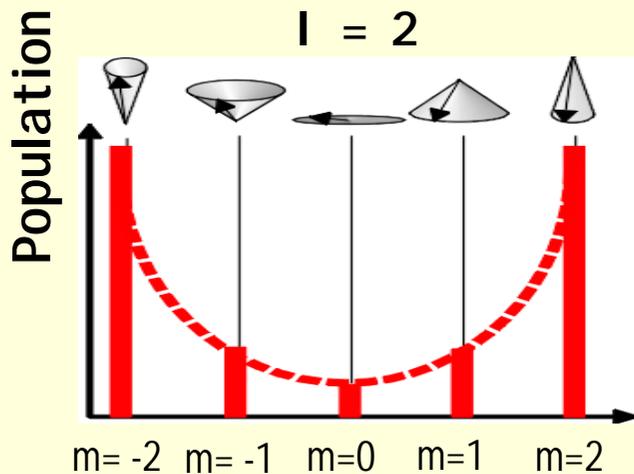
Classical :
equal amount of spins are
pointing up and down



Aligned radiation pattern
(up/down symmetry
along the axial symmetry axis)



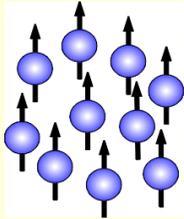
$N(0^\circ) = N(180^\circ)$
and
no dependence on ϕ
Measure $N(0)/N(90)$



Quantum mechanical:
Equal amount of spins with +/-m :

$p(m) = p(-m)$
+ axial symmetry around Z

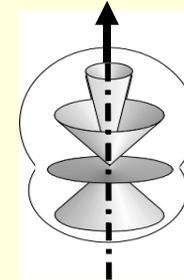
An ensemble of spins is said to be **polarized**,
 if all spins are preferentially along a certain direction (**axial symmetry**)
 and if the UP/DOWN symmetry is broken



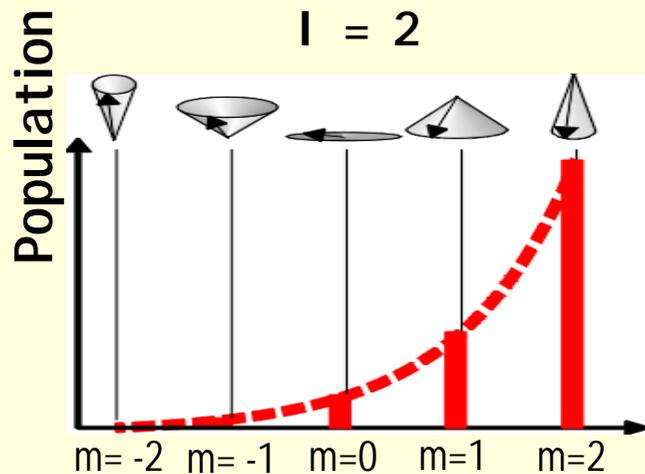
Classical :
 all spin point in the same
 preferential direction



Polarized radiation pattern
 (up/down symmetry is broken)



$N(0^\circ) \neq N(180^\circ)$
 and
 no dependence on ϕ



Quantum mechanical:
 Different amount of spins with +/-m :

$p(m) \neq p(-m)$
 + axial symmetry around Z

Methods to Orient Nuclear Spins

Direct methods → spin-orientation given by the production process

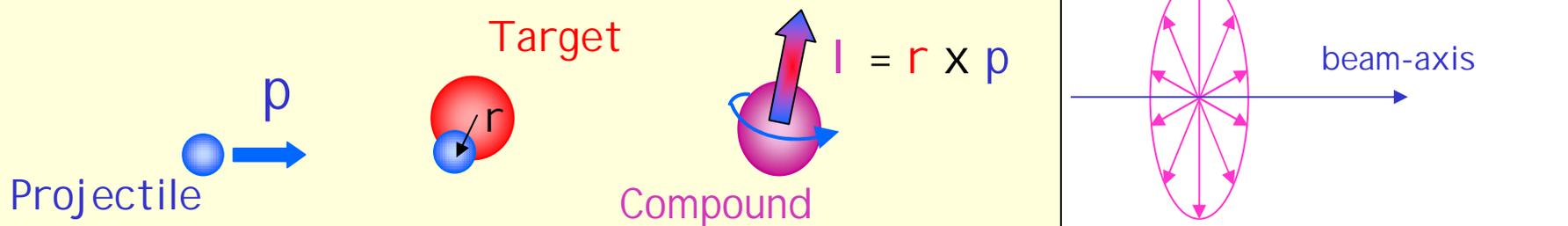
- the nuclear reaction produces spin-oriented reaction products
- the spin-orientation is maintained until implantation into a host material

examples: fusion-evaporation
projectile fragmentation
coulomb excitation

....

Fusion evaporation: nearly complete **alignment**

Principle: transfer of momentum from projectile to target



Fusion evaporation: pro's and contra's

Experiments:

(1) in-beam (mostly)

- production + recoil into the target → **very high orientation !**
only neutron deficient nuclei

not always target = host possible !

- with recoil-implantation → avoid air gaps between target and host foil
(loss of orientation due to hyperfine interaction between randomly oriented electron spin and nuclear spin during the flight in vacuum)

Problems for experiments on exotic nuclei:

- **in-beam is no longer possible** (need selection)
- need a selection method that maintains orientation !

(2) after recoil mass separation (in-flight)

inverse kinematics + recoil separation !

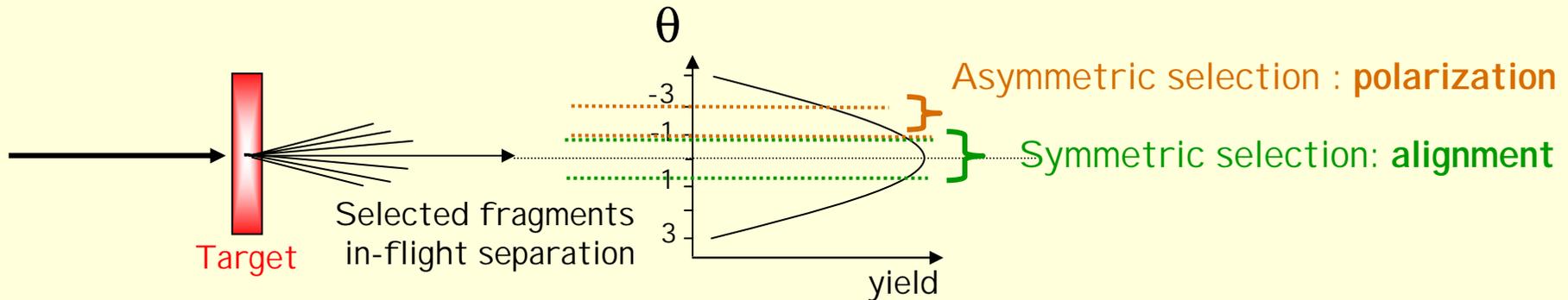
→ select isomers in noble-gas charge state

proof of principle : E. Dafni and M. Satteson, Phys. Rev. C38 (1988) 2949

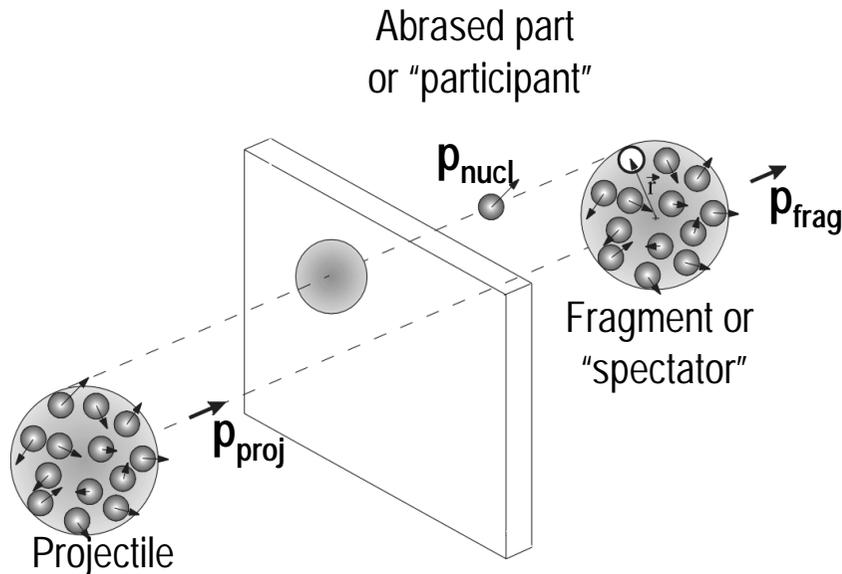
(only) 15% of initial alignment is maintained !

Projectile fragmentation: rather low orientation (5-30%)

Principle: transfer of momentum via nucleon removal in the projectile



the participant spectator model: peripheral collisions



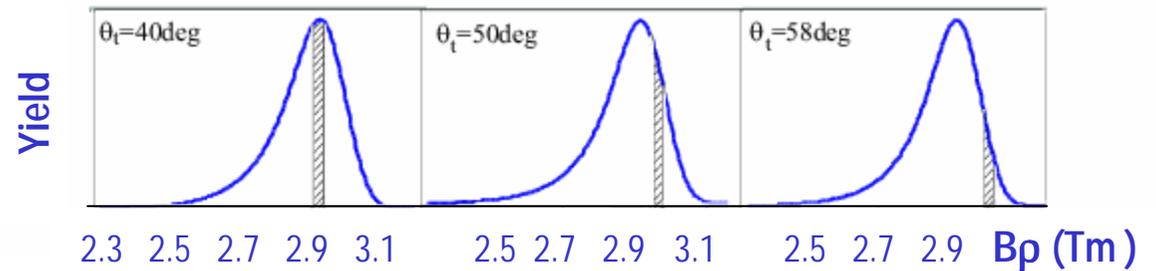
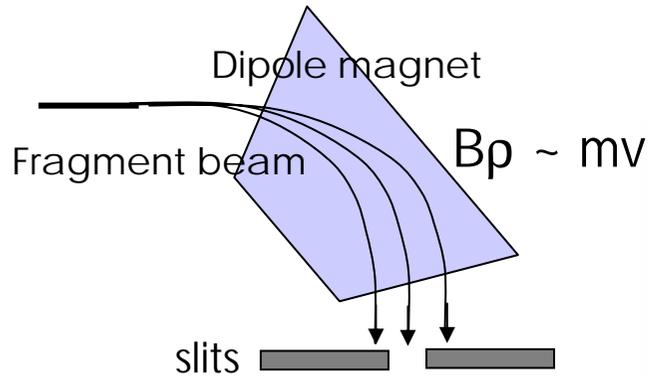
Spin-orientation from projectile fragmentation: still under investigation !

It depends on

- the energy of the projectile beam
- the Z of the target , the thickness of the target
- the Z (and N ?) of the selected fragment
- the selected energy of the fragment beam (longitudinal momentum)
- the selected angle of the fragment beam (transverse momentum)

Projectile fragmentation: dependence on fragment energy

A fragment separator has a certain **momentum acceptance**
→ selection of the fragment momentum (energy) by slits



Examples of fragment separators:

LI SE at GANIL

A1200 at MSU

RIPS at RIKEN

FRS at GSI

GANIL
GRAND ACCELERATEUR NATIONAL D'IONS LOURDS

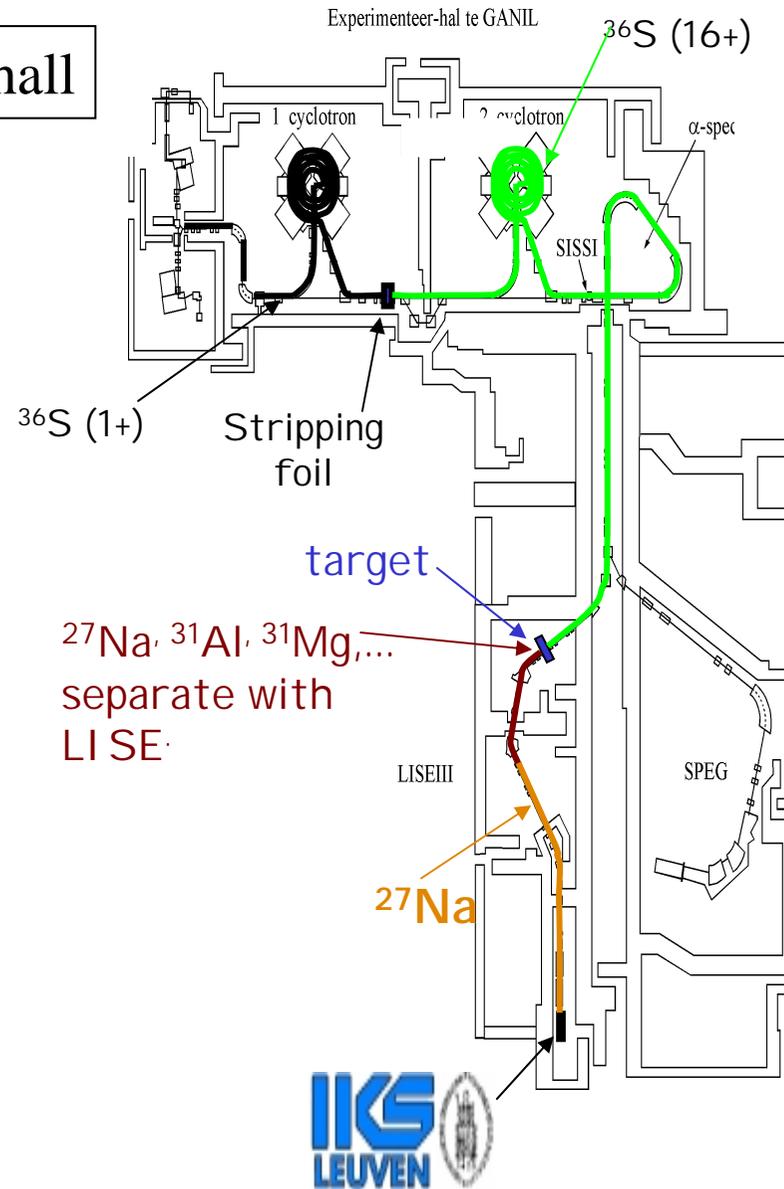


Projectile fragmentation: dependence on fragment energy

Experiment @ GANIL:

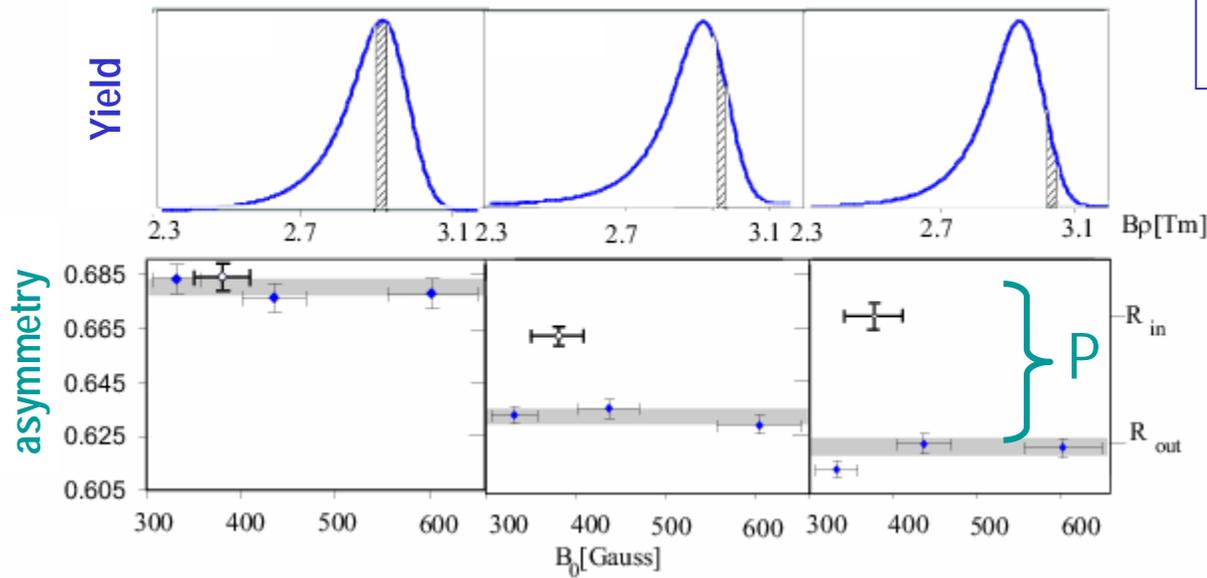
^{36}S (77 MeV/A) + ^9Be

Experimental hall

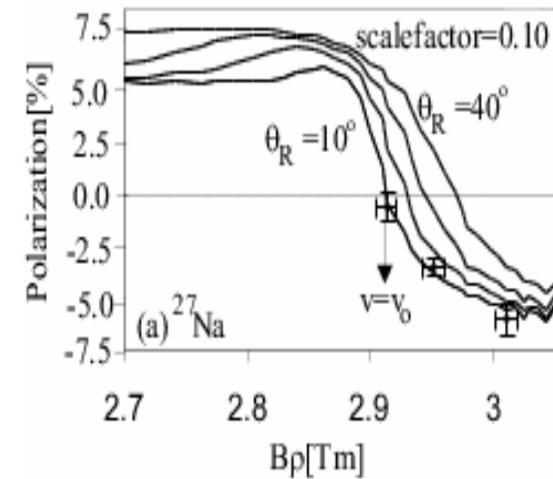


Projectile fragmentation: dependence on fragment energy

Experiment @ GANIL:
 ^{36}S (77 MeV/A) + ^9Be



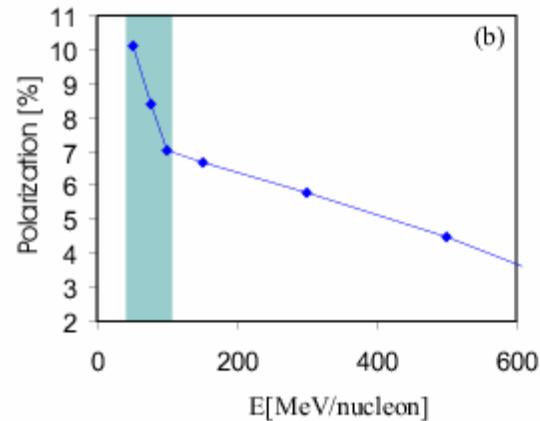
Polarization is maximal in wing of momentum distribution (lowest yield) (for reaction on a low-Z target).
 Model : H. Okuno, K. Asahi et al., PL B335 (1994) 29



Projectile fragmentation: dependence on beam energy

Prediction by the kinematical model:

strong reduction of polarization with increasing beam energy
(in combination with strong forward peaked yield)



Experimental results:

At GANIL energies (50-80 MeV/A)

A = 10 - 15% P = 3 - 10%

High GSI energies (> 500 MeV)

A = 15-30% P < 1%

Ref.: PLB 393 (97) 36

PRC 66 (2003) 054601

Z.Phys. A350 (94) 215

PRC 57(98)2205

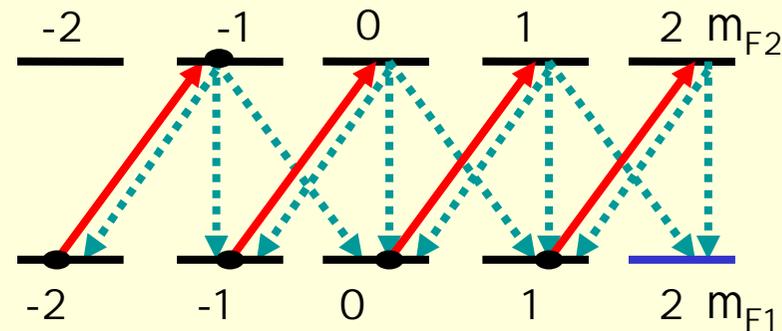
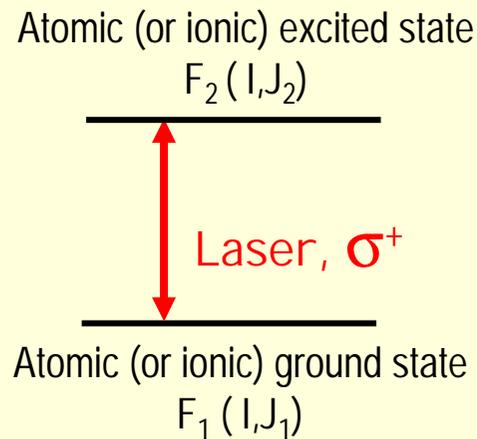
Methods to Orient Nuclear Spins

Indirect methods → spin-orientation produced after the production reaction
(a requirement for ISOL-beams !)

examples: optical pumping with a collinear laser beam
tilted foils polarization
low temperature orientation ($T \sim 20$ mK, $B \sim 10$ T)

Optical pumping: interaction of a radioactive beam and a laser beam
(dedicated set-up: COLLAPS – ISOLDE) only one worldwide ?

Principle: polarize the atom (or ion) by resonant excitation with a polarized laser beam



Total atomic spin F gets polarized through pumping !
Consequence: **nuclear spin polarization**
(via the hyperfine interaction)

Formal description of the orientation of an ensemble of nuclear spins:

from	density matrix	$\rho_{mm'}$	(population of m-levels)
to	density tensor	ρ_k^n	(easy to rotate)
to	orientation tensor	B_k^n	(used in ang. Distr.)

The **density matrix** describes the **population** and **coherences** between m-quantum states of an ensemble of nuclear spins:

$$\rho_{mm} = \langle m | \rho | m \rangle = p(m) \quad \text{population of level } |I m\rangle$$

$$\rho_{mm'} = \langle m | \rho | m' \rangle \quad (m \neq m') \quad \text{coherence between level } m \text{ and } m'$$

In case of axial symmetry \rightarrow only $p(m)$ exists, non-diagonal terms are zero

The **density tensor for ensemble of spins (I)** is obtained by projection on the spherical tensor basis:

$$\begin{aligned} \rho_k^n &= \langle U_k^n | \rho \rangle \\ &= \sqrt{2k+1} \sum_{m,m'} (-1)^{l+m} \begin{pmatrix} I & I & k \\ -m & m' & n \end{pmatrix} \rho_{mm'} \end{aligned}$$

In case of axial symmetry \rightarrow only ρ_k^0 exists, terms with $n \neq 0$ are zero

The **orientation tensor** is defined as: $B_k^n = \sqrt{2l+1} \rho_k^{n*}$

Properties of alignment and polarization

An isotropic ensemble has $B_0 = 1$, all other components are zero

An aligned ensemble has B_k^0 components for $k=\text{even}$,
 $n \neq 0$ components are zero
odd tensors are zero

$B_2, B_4 \sim$ amount of alignment A

$$A = \sum_m \frac{[3m^2 - I(I+1)] p(m)}{I(I+1)} \quad (\text{integer } I)$$

A polarized ensemble has B_k^0 components for $k=\text{odd}$ and $k=\text{even}$!
 $n \neq 0$ components are zero

$B_1 \sim$ amount of polarization P

$$P = \sum_m \frac{m p(m)}{I}$$

Simplifications in the angular distribution function



Table of contents:

Nuclear moments: *what and why?*

magnetic moments

quadrupole moments

definitions, properties, nuclear structure information to deduce

Nuclear moments: How?

basic principles

angular distribution of radiation

orientation of the nuclear spins

formalisms, methods, experimental results

perturbation of spin-orientation by electromagnetic fields

the Zeeman interaction

the quadrupole interaction

the combined interaction methods to measure nuclear moments

methods to measure nuclear moments

time differential methods

time integrated methods

Moments of ground states

selected examples of experiments, methods, results

Moments of isomeric states

The angular distribution for γ - and β -decay: simplifications !

In most cases:

→ γ -detection: A_k only for k =even → detect alignment

$$W(\theta, \phi, t) = 1 + \frac{\sqrt{4\pi}}{5} \sum_n A_2 B_2^n(t) Y_2^n(\theta, \phi) + \dots$$

→ β -detection: A_k only for k =1 → detect polarization
+ detection along the polarization axis ($n=0$)

$$W(\theta, \phi, t) = 1 + A_1 B_1^0(t) \cos\theta$$

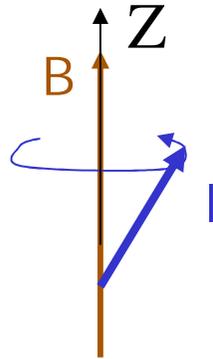
$$B_1^0(t) = \sum_{k,n} G_{1k}^{0n}(t) B_k^n(t=0)$$

Consequence: for study of isomers: need aligned beam !
 for study of ground states: need polarized beam !

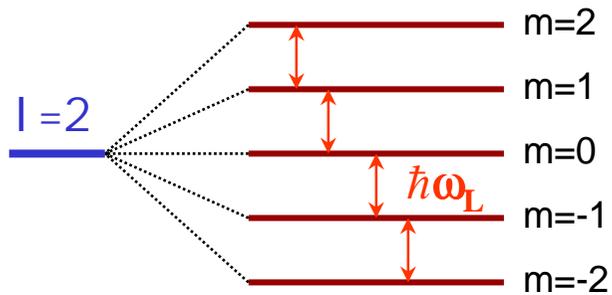
Static Magnetic and Quadrupole Interaction

Nucleus in static magnetic field B

$$H_B = -\bar{\mu} \cdot \bar{B}$$



$$E_m = -\hbar\omega_L m$$

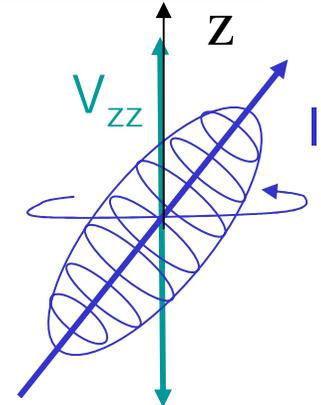


Zeeman splitting

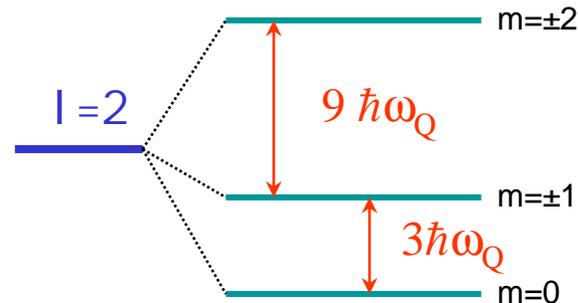
1 frequency ν_L
 Larmor frequency $\nu_L = g\mu_N B/h$
 Coupling constant: $\omega_L = 2\pi\nu_L$

Nucleus in electric field gradient V_{zz}

$$H_Q = \frac{\hbar}{\omega_Q} (3\bar{I}_z^2 - \bar{I}^2)$$



$$E_m = \hbar\omega_Q [(3m^2 - I(I+1))]$$



Quadrupole splitting

I (or $I - 1/2$) frequencies $\nu_Q(n) \sim \nu_0$
 Quadrupole frequency $\nu_0 = eQV_{zz}/h$
 Coupling constant: $\omega_Q = \nu_0 2\pi/4I(2I-1)$

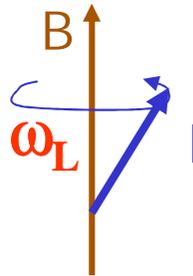
Experimental techniques to measure moments

Measure as a function of time (= time differential methods)

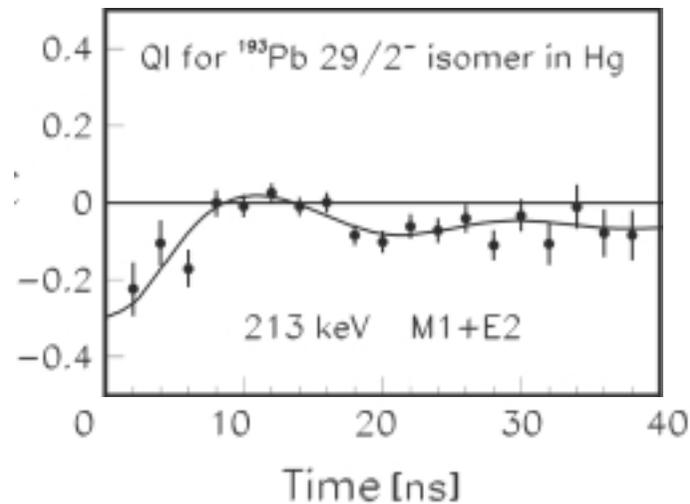
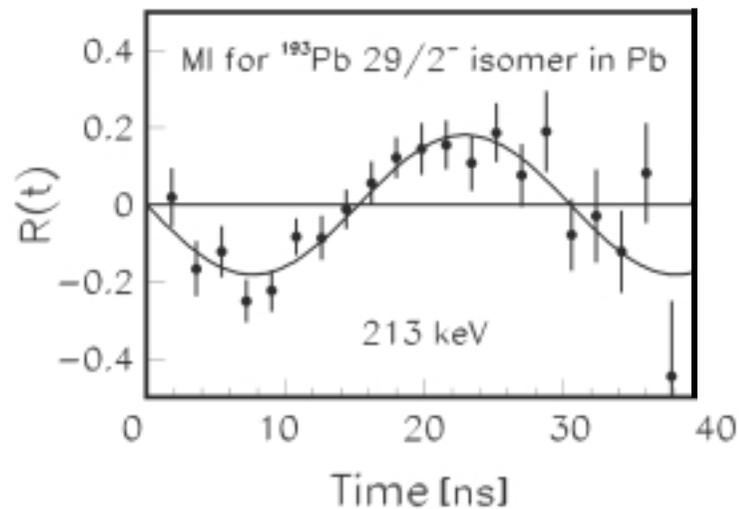
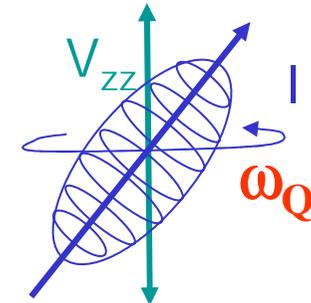
Measure the **change in anisotropy as a function time**

→ Time Differential Perturbed Angular Distribution (TDPAD) methods

Measure the **larmor precession** of an oriented ensemble submitted to a static field B

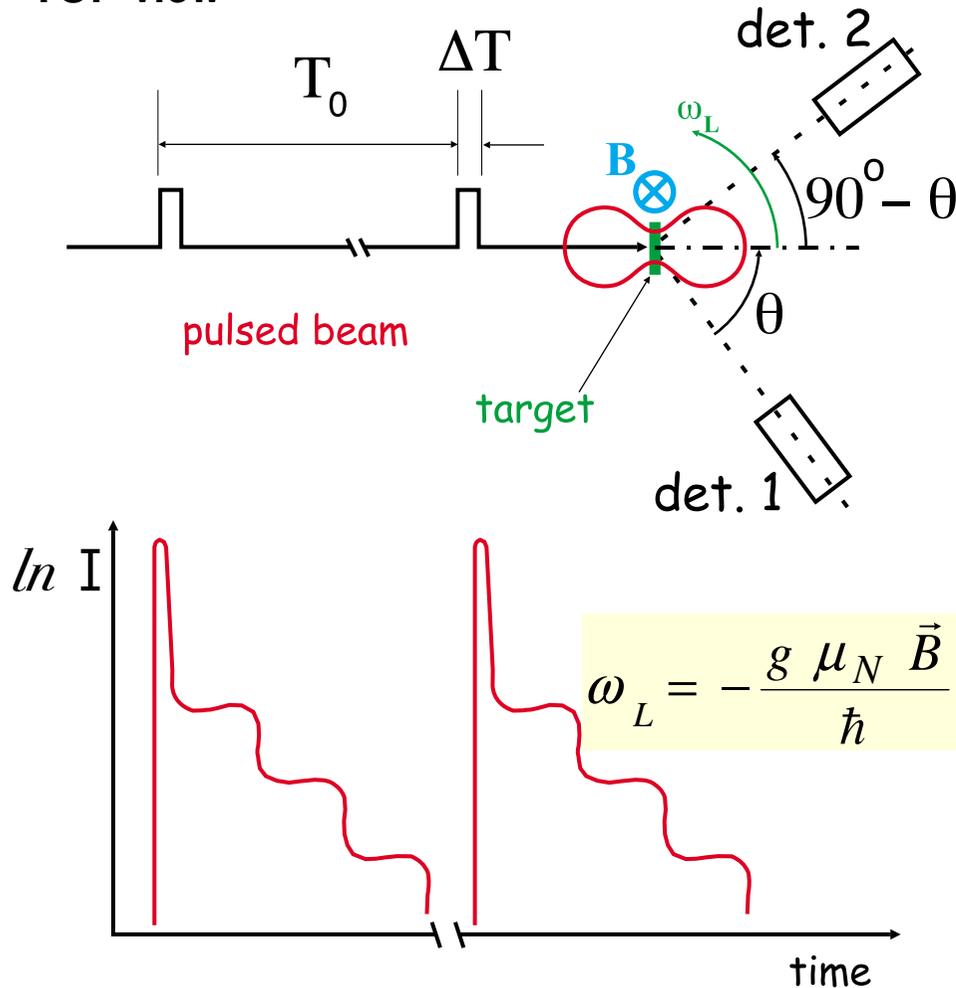


Measure the **quadrupole precession** of an oriented ensemble submitted to an EFG



TDPAD (Time Differential Perturbed Angular Distribution)

TOP view



Pulsed beam needed:

$$T_0 > 3\tau \quad \Delta T \ll \tau \quad (\text{duty cycle})$$

Detection : γ -detectors (HPGe)

At 90° wrt each other

In the plane of the orientation symmetry axis
Perpendicular to the static field

Registered signal:

γ -intensity as function of time

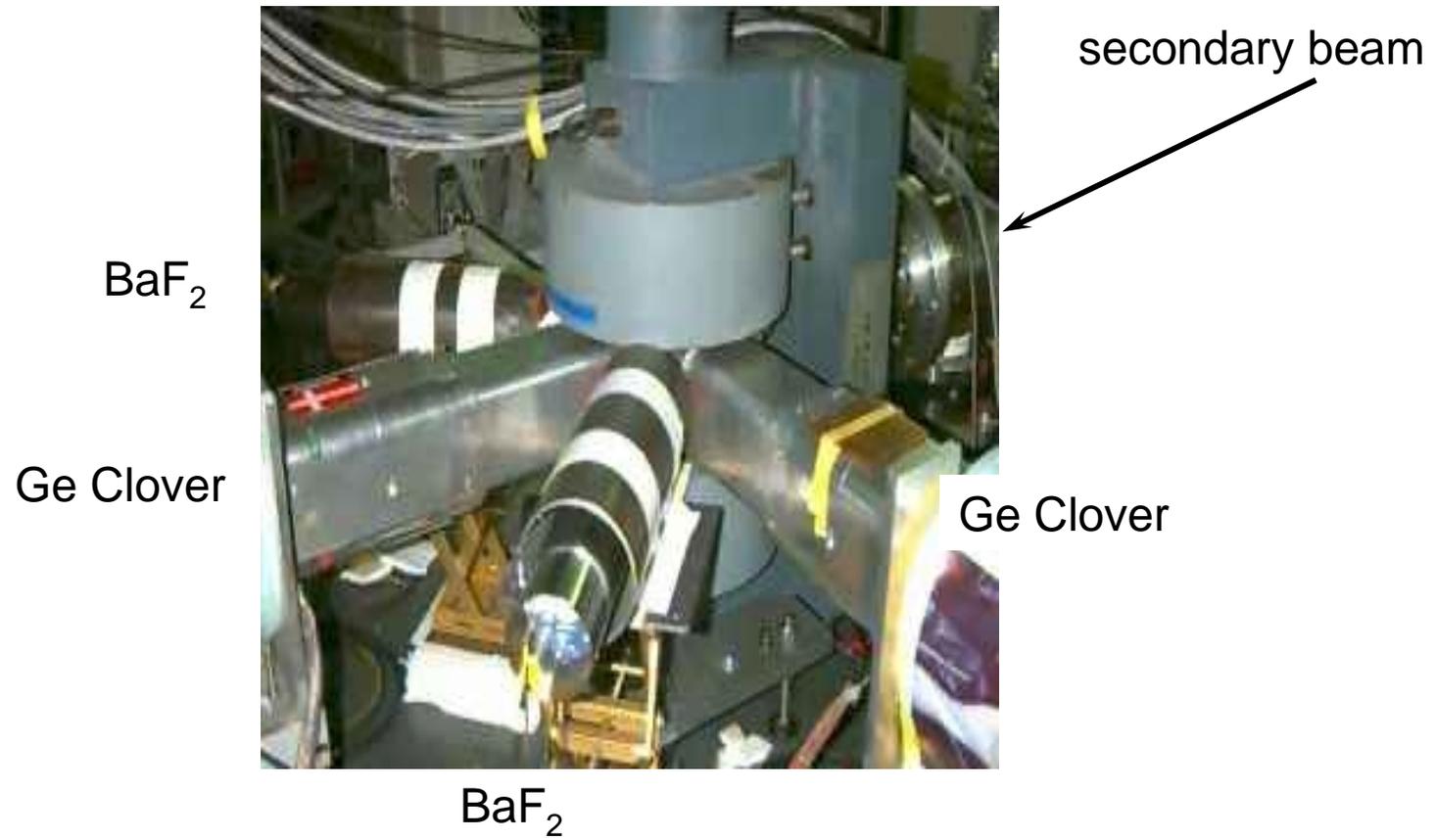
start = puls from beam

or ion arrival time

stop = detection of a γ

$$R(t) = \frac{N(0) - N(90)}{N(0) + N(90)}$$

TDPAD setup at GANIL



Experimental techniques to measure moments

time differential methods (TDPAD)

for short lifetimes only
(spin-spin relaxation time $\sim 10\text{-}100\ \mu\text{s}$)

Measure time integrated (= time integrated methods)

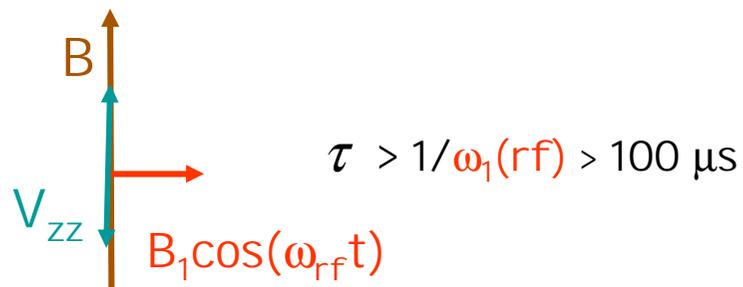
Measure the **change in anisotropy** as a function of an external variable
spin-lattice relaxation is longer (ms - sec)

→ Nuclear Magnetic Resonance (NMR) method

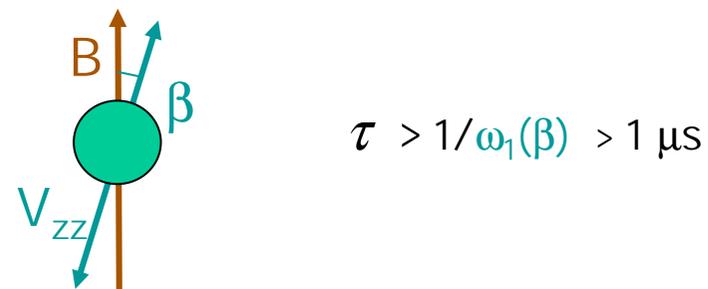
→ Nuclear Quadrupole Resonance (NQR) method

→ Level Mixing Resonance (LMR) method

A **radiofrequency (rf) magnetic field**
induces a perturbation of the population of
the quantum levels.
external variable = applied rf-frequency

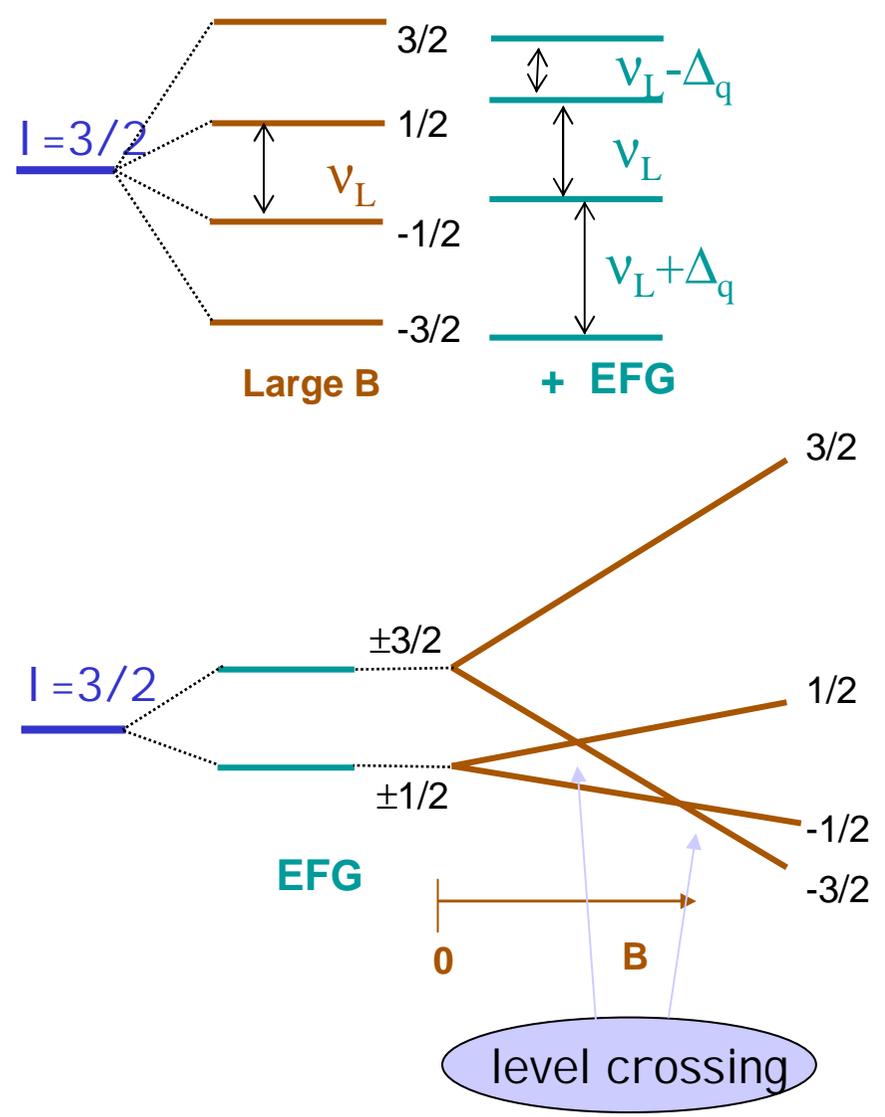


The **misalignment of two static interactions** induces a perturbation of the population of the quantum levels.
external variable = static field strength



Combined interactions: magnetic dipole + electric quadrupole

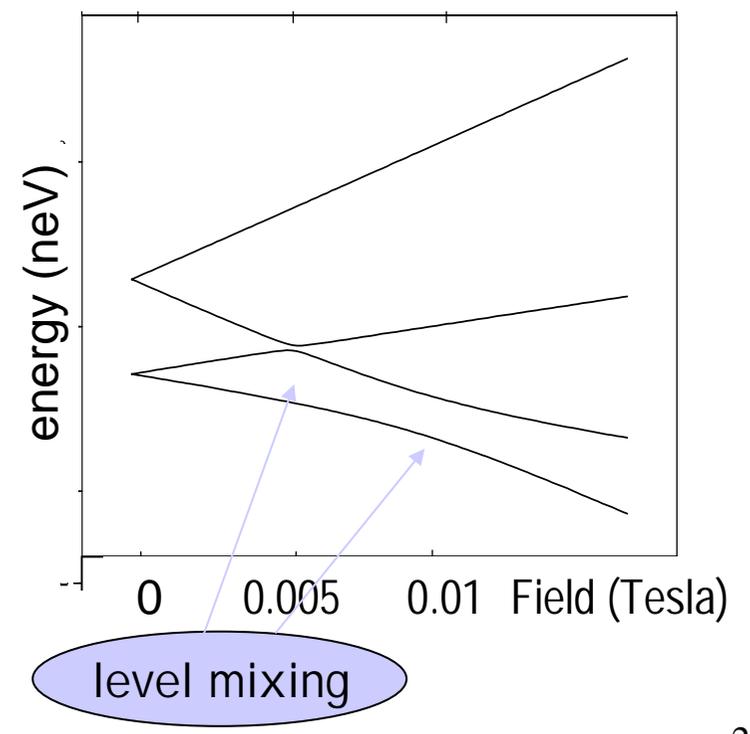
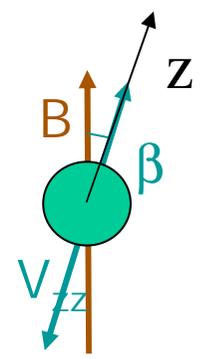
Crystal c-axis parallel to magnetic field



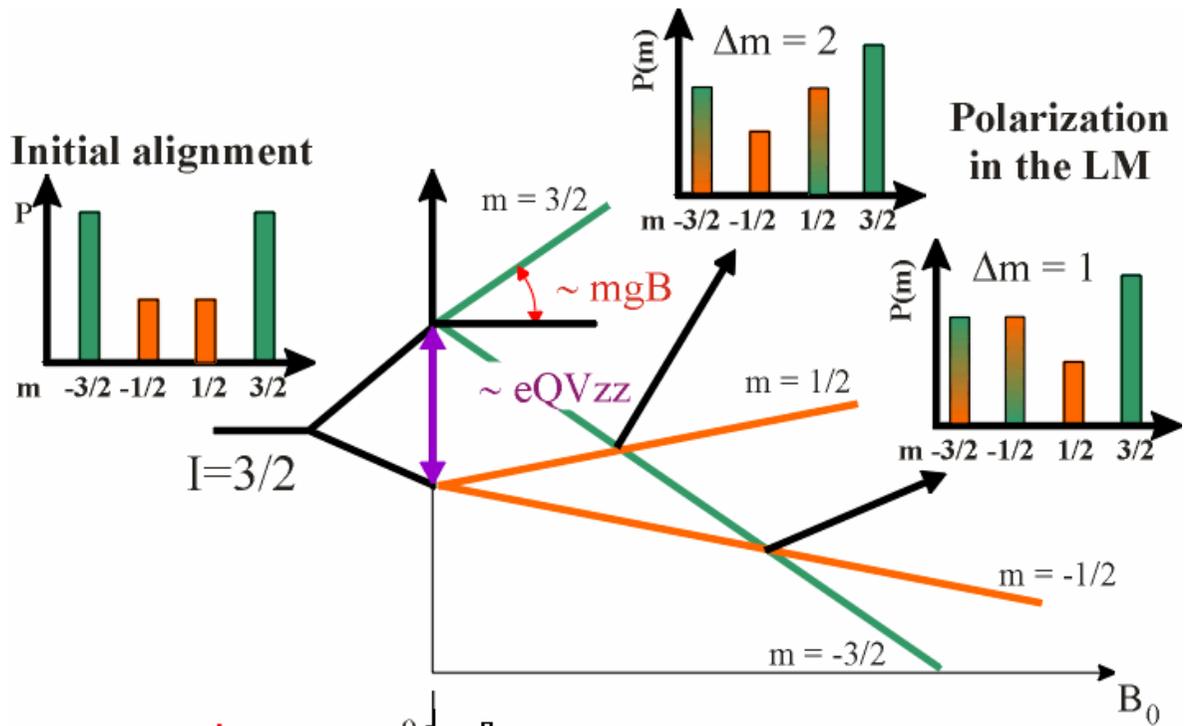
Crystal c-axis makes angle β with B

$$H = -\bar{\mu}_z \bar{B}_z + \bar{\mu}_x \bar{B}_x + \frac{\hbar}{\omega_Q} (3\bar{I}_z^2 - \bar{I}^2)$$

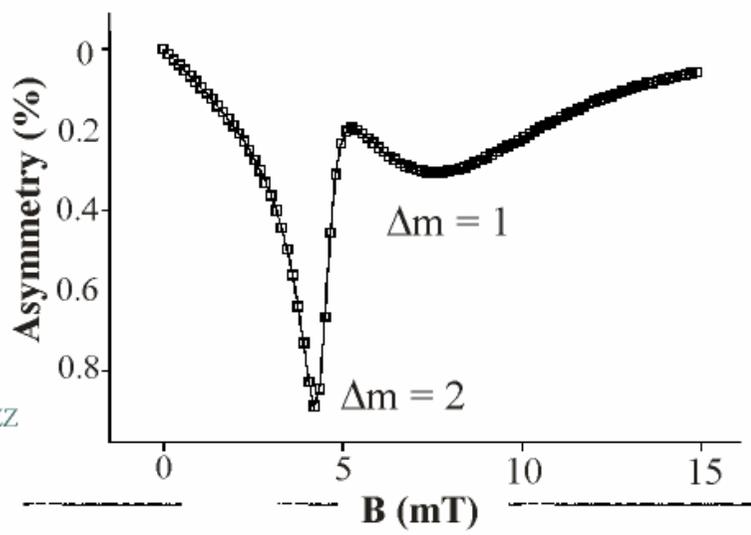
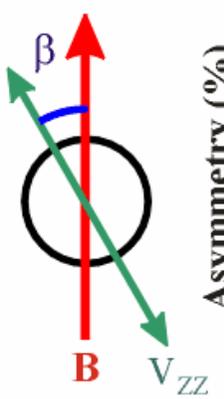
$$E_m = \hbar\omega_Q[(3m^2 - I(I+1))] - \hbar\omega_L m \cos\beta + \hbar\omega_L m \sin\beta$$



β-LMR (Level Mixing Resonance)



Transfer of alignment into polarization



- From the position ν_Q/μ
- From the amplitude orientation angle β
- From the width spin I
- Number of resonances

at crossing field:

$$E_{m1} = E_{m2}$$

If angle β occurs between V_{zz} and $B \rightarrow$ breaking of axial symmetry (small)
Make a two-level approximation, apply perturbation theory

$$|N_1\rangle = \frac{1}{\sqrt{2}} \{ |m_1\rangle - |m_2\rangle \}$$
$$|N_2\rangle = \frac{1}{\sqrt{2}} \{ |m_1\rangle + |m_2\rangle \}$$

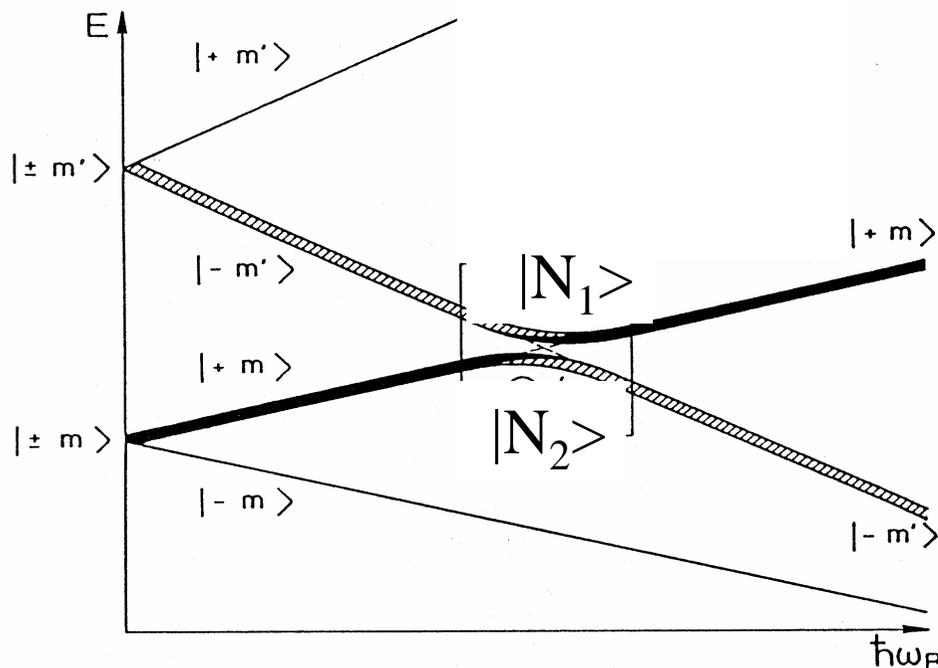


$$p(N_1) = p(N_2)$$



Conclusion: change in orientation

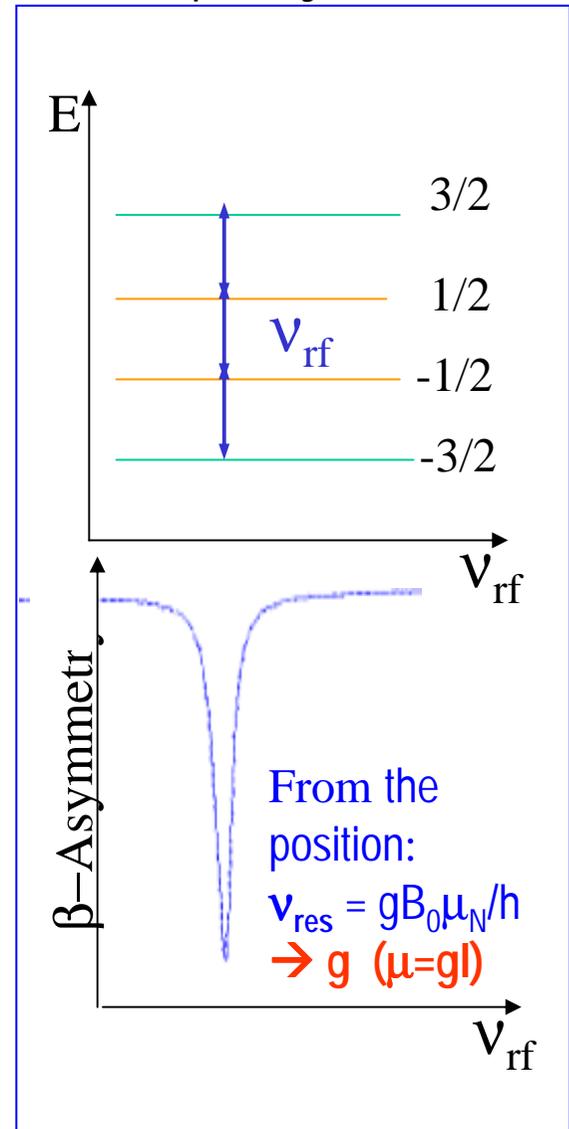
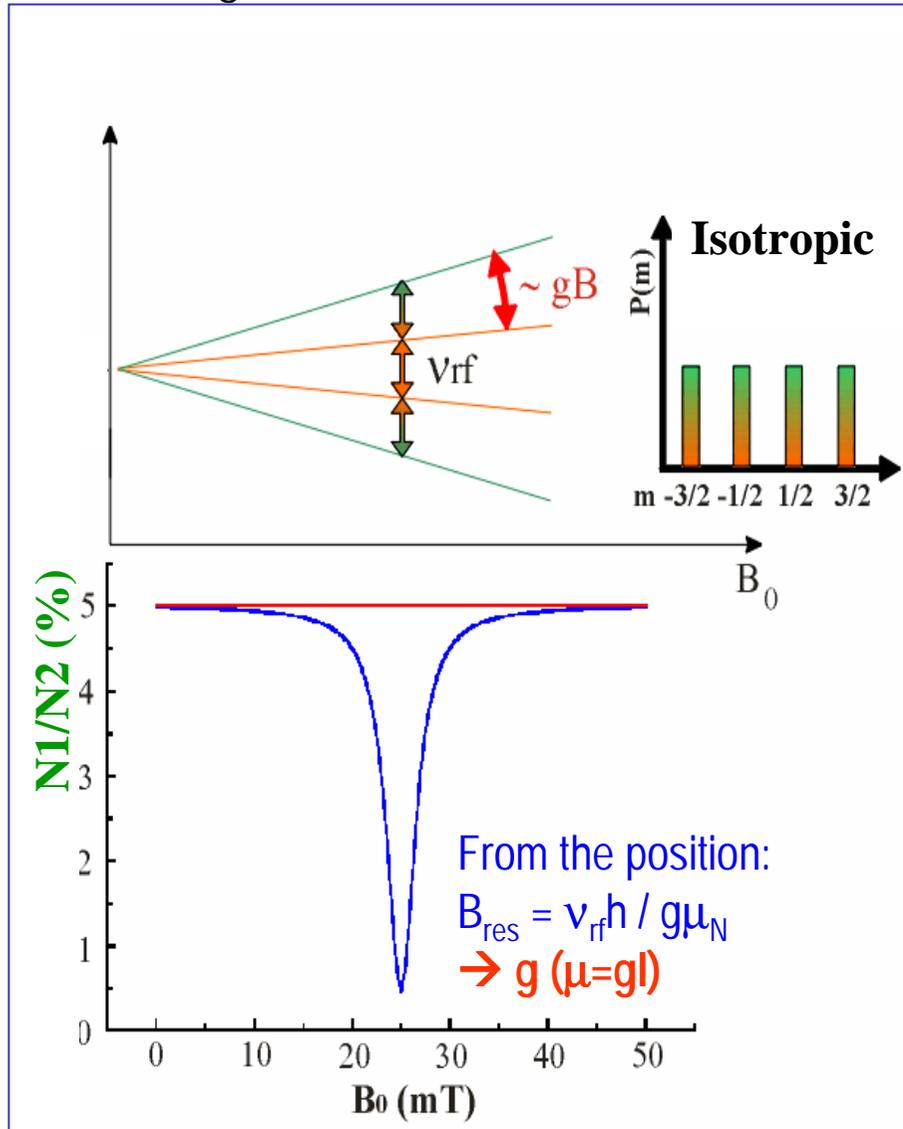
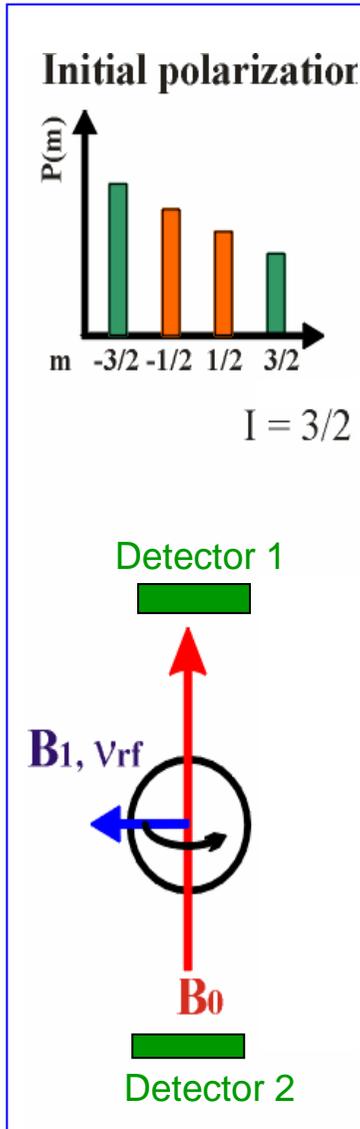
Visible as a resonance in the asymmetry of β -decay



β-NMR (Nuclear Magnetic Resonance)

Magnetic field as variable

Rf-Frequency as variable



β-NQR (Nuclear Quadrupole Resonance)

Initial polarizator

$I = 3/2$

Rf-Frequency as variable

$I = 3/2$

B + EFG

Partial destruction of P

β -Asymmetry

ν_{rf}

ν_L

Δ_q

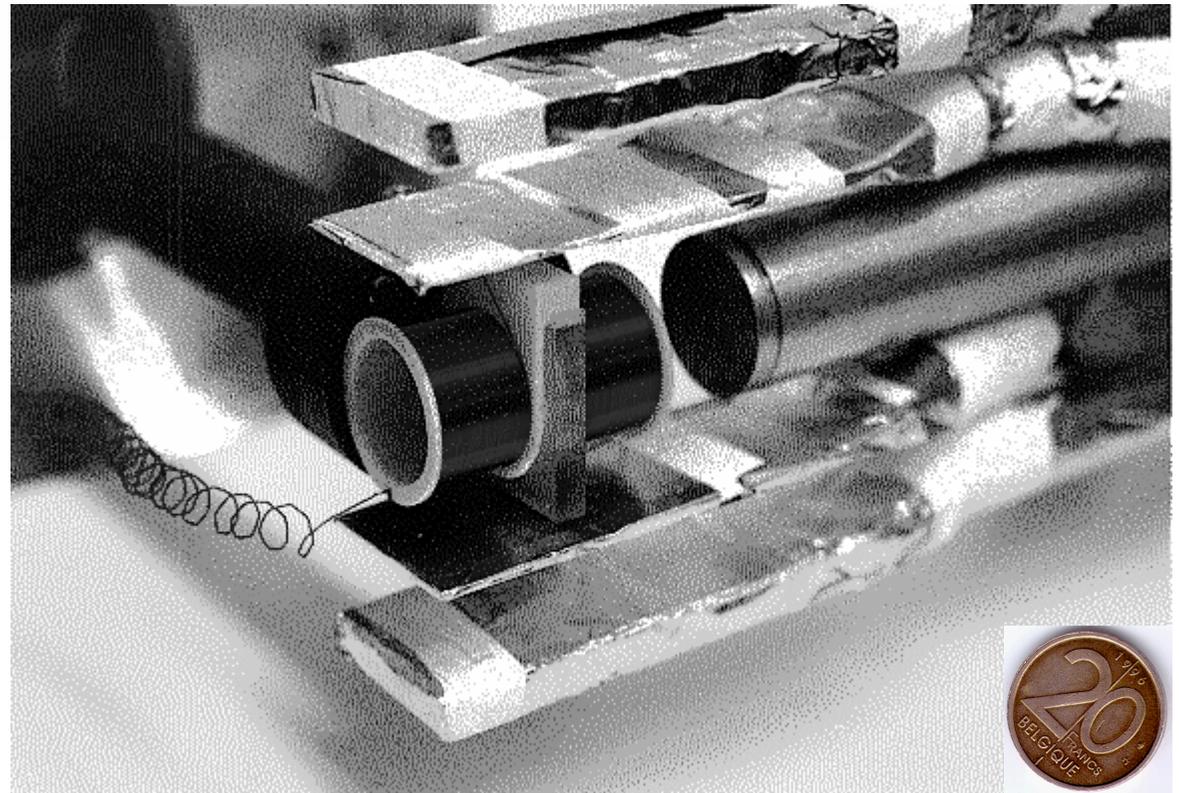
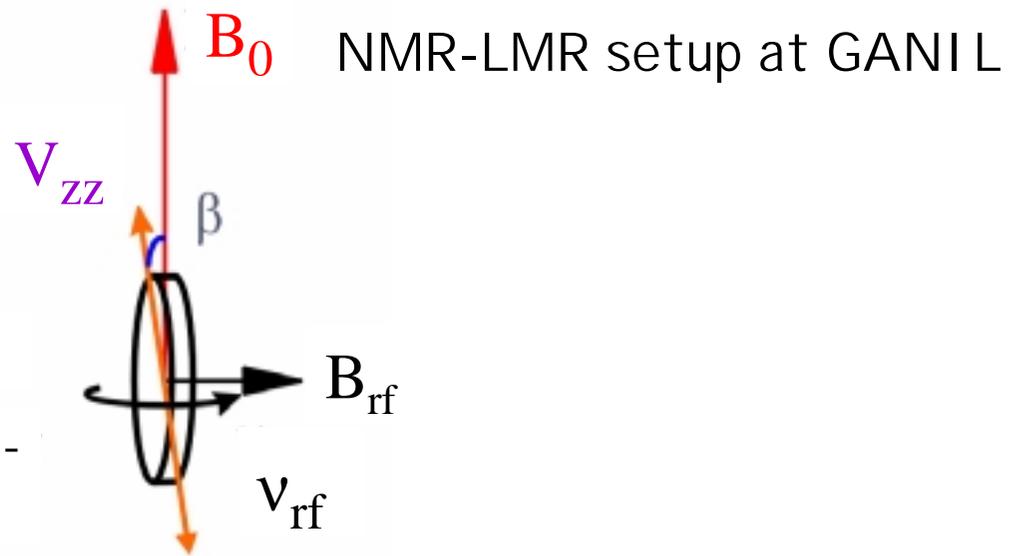
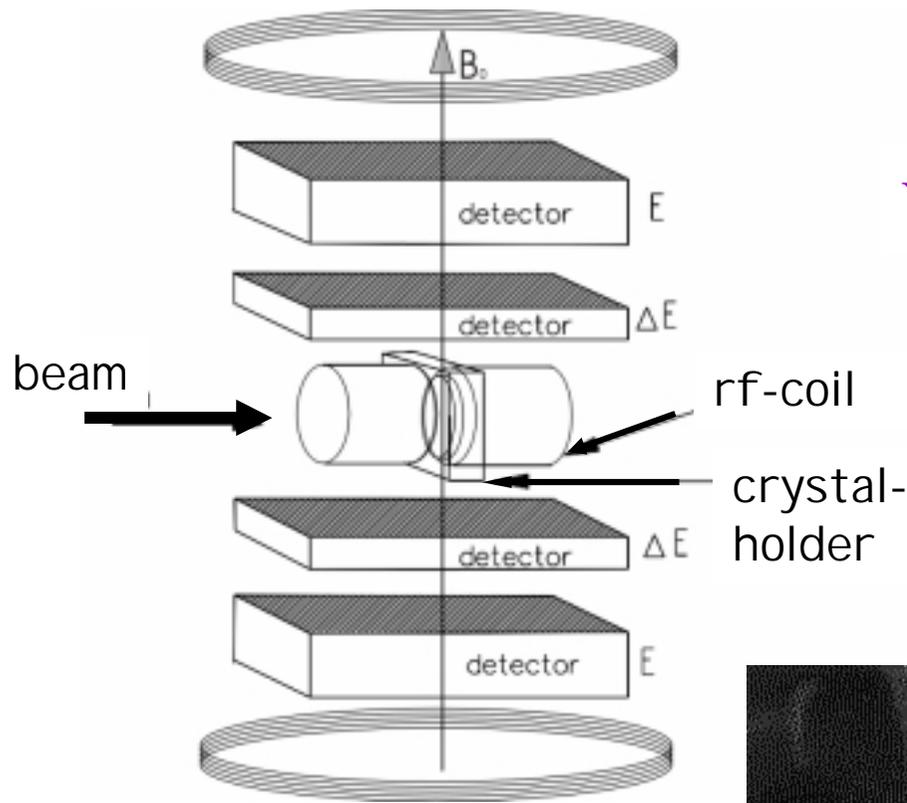
From the positions:
 $\nu_L \rightarrow g (\mu = gI)$ $\Delta_q = 1/2 \nu_Q \rightarrow Q, V_{zz}$

3 rf simultaneously

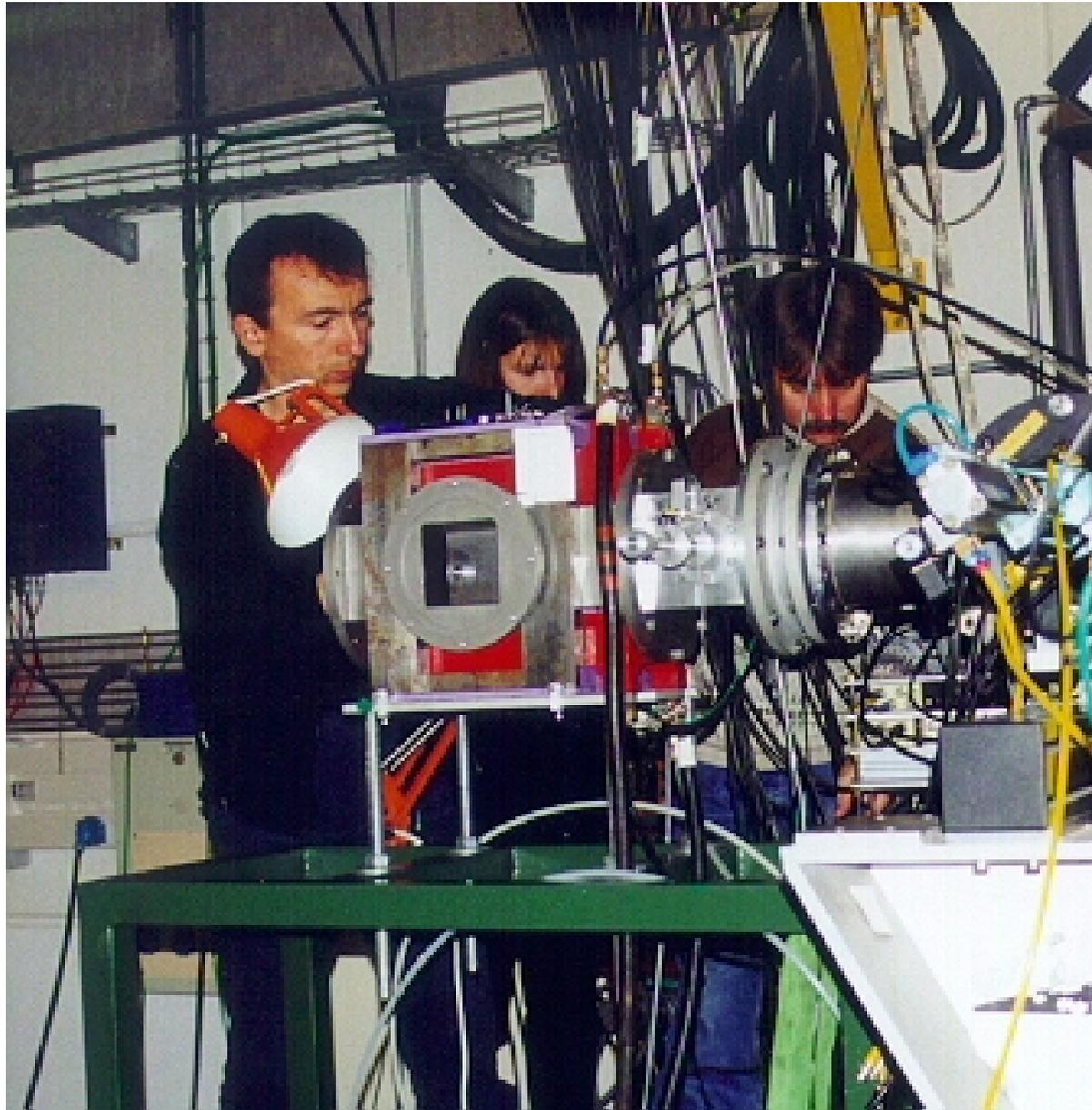
B + EFG

From the position:
 $\rightarrow \nu_Q (QV_{zz})$

Δ_q



Sideview set-up



NMR-LMR setup at GANIL

