Static Electric and magnetic moments of exotic nuclear structures

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Some references:
"Modern Theories of Nuclear Moments", B. Castel and I.S. Towner, Oxford Scientific Publishing, 1990
"The Nuclear Shell Model", K.L.G. Heyde, Springer Verlag, 1994 (2nd edition)
"Nuclear Shell Theories", de-Shalit and Talmi, Academic Press, New York, 1963
"Nuclear Magnetic and Quadrupole Moments for Nuclear Structure Research on Exotic Nuclei", G. Neyens, Reports on Progress in Physics 66 (2003) 633

Table of contents:

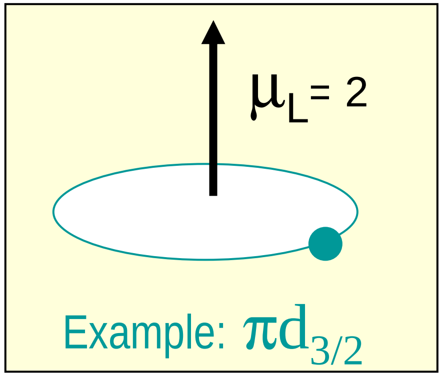
Introduction

The magnetic dipole moment

Orbiting charged particles induce a magnetic field

→ orbital magnetic moment with strength g_L
and proportional to **orbital angular momentum l**

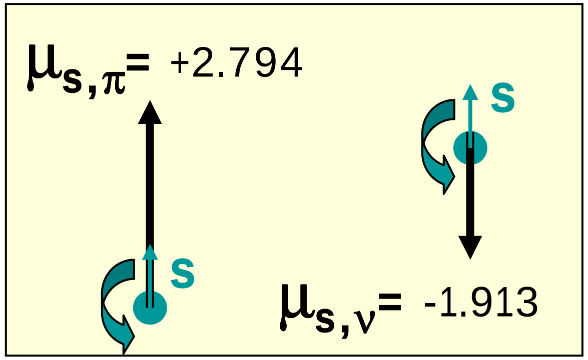
protons : $g_{L,\pi}$ = 1
neutrons have no charge → $g_{L,\nu}$ = 0



Intrinsic spin of the nucleons induces a magnetic field

→ spin magnetic moment with a strength g_S
and proportional to **instrinsic spin s=1/2**

protons : $g_{S,\pi}$ = + 5.587
neutrons : $g_{S,\nu}$ = - 3.826



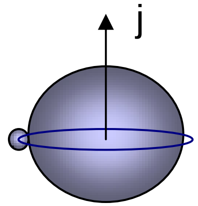
magnetic dipole operator : $\hat{\mu} = g_L \hat{L} + g_S \hat{S}$

Magnetic moment of a state with spin I
= expectation value of the z-component of the dipole operator

$$\mu(I) = \langle I, m=I \mid \hat{\mu}_z \mid I, m=I \rangle$$

Magnetic moments of ground states in odd nuclei

In the extreme single particle shell model



- nucleons moving in a central potential induced by the other nucleons
- all nucleons are paired to spin zero
- the ground state is determined by the non-paired nucleon

the magnetic moment of an odd-proton (or an odd-neutron) nucleus with spin I is determined by the magnetic moment of the unpaired proton (or neutron) in orbital j(n,l)

$$\mu(I) = \langle (ls)jm \mid \mu_z \mid (ls)jm \rangle_{m=j}$$

Magnetic dipole operator(=vector): a *composed* spherical tensor operator of rank 1

$\mu_z(L,S)$ acts on a state with spin **j** *composed out of **l** and **s***:
L acts on the orbital momentum **l**
S acts on the intrinsic spin **s**

→use angular momentum coupling algebra and properties of spherical tensor operators to calculate the magnetic moment of a single nucleon

Ref: de-Shalit and Talmi in "Nuclear Shell Theory"
Brink and Shatchler in "Angular Momentum Algebra"

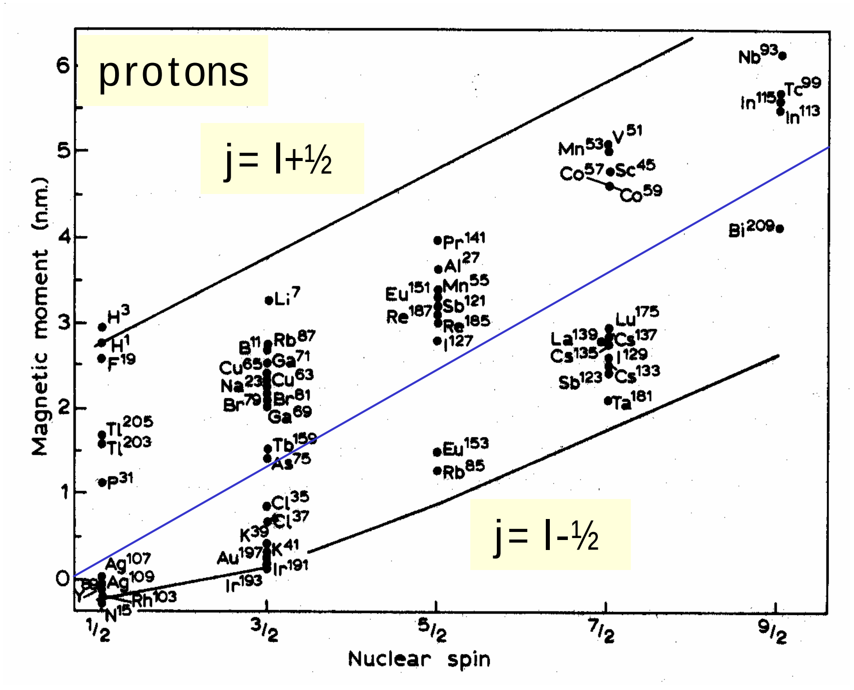
Free g-factors: the Schmidt moments

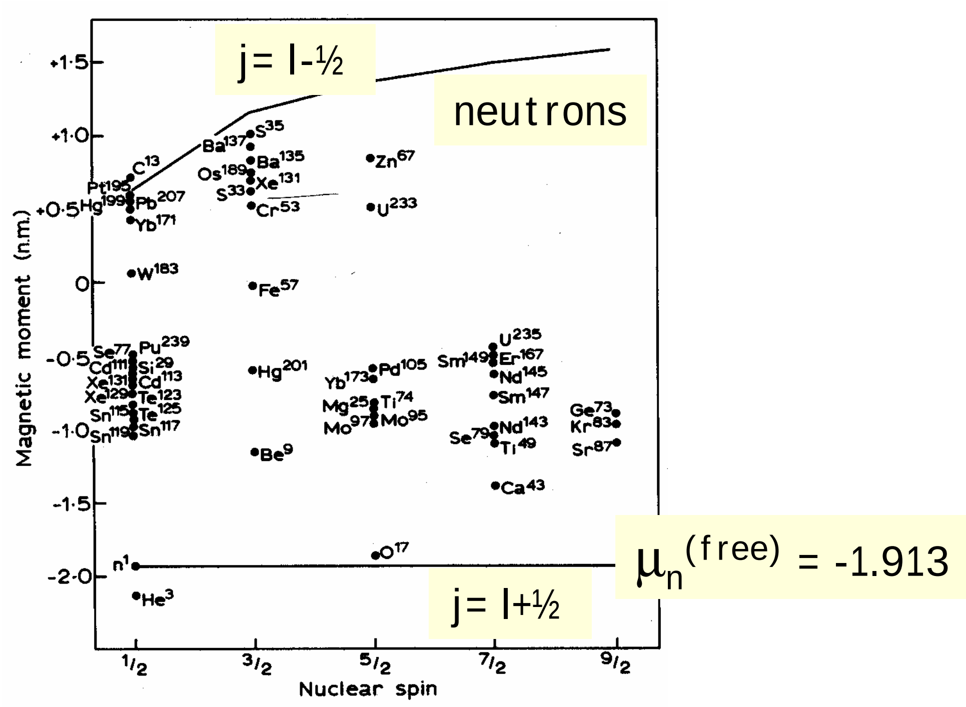
→ **Magnetic moment of a nucleon in a shell model orbit with spin j:**

$$\mu(j=l+\tfrac{1}{2}) = [\,(j-\tfrac{1}{2})\,g_L + \tfrac{1}{2}\,g_S\,]\,\mu_N$$

$$\mu(j=l-\tfrac{1}{2}) = \frac{j}{j+1}\,[\,(j+\frac{3}{2})\,g_L - \tfrac{1}{2}\,g_S\,]\,\mu_N$$

If g_L and g_S are the free-nucleon g-factors for protons or neutrons → Schmidt moments





Experimental magnetic moments deviate sometimes strongly from these Schmidt values (from 0.5 – 1.5 μ_N) → mainly inwards

Effective g-factors

* the magnetic dipole operator as defined above is an approximation:

→ the unpaired nucleons in a nucleus are interacting with each other (e.g. through exchange of mesons: Meson Exchange Current correction to the dipole operator is needed, this is a small two-body operator correction)

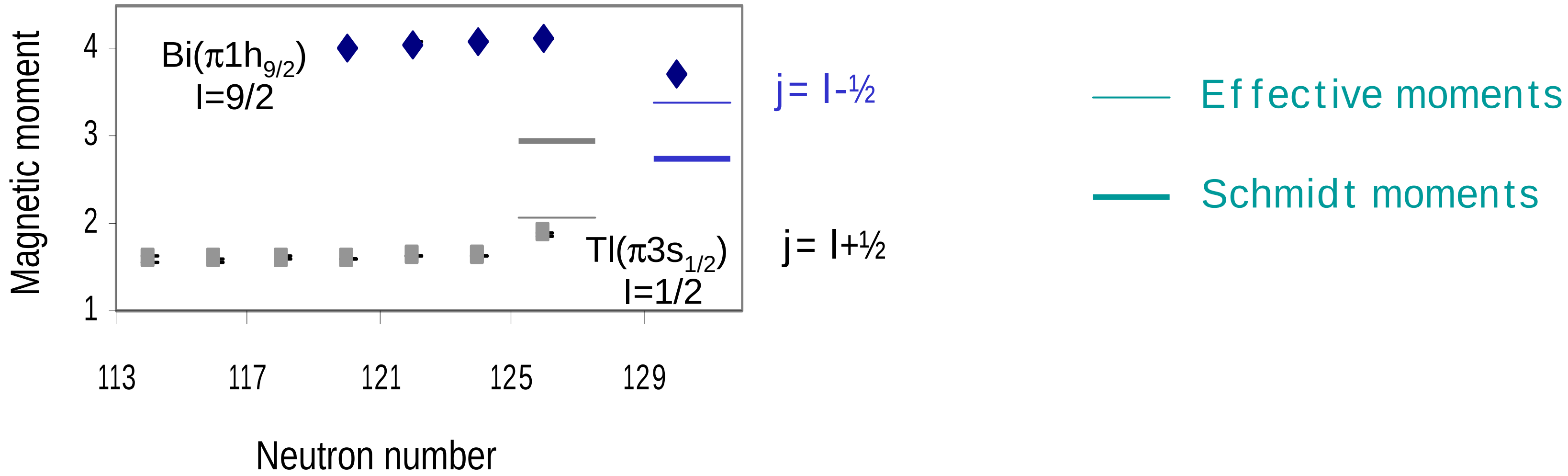
→ the valence nucleons interact with the nucleons in the core

effective g-factors should be used

typical values : $g_S^{eff} \sim 0.7\, g_S$ (Correction ~ 30%)

$g_L^{eff} \sim g_L$ (Correction less than 10%)

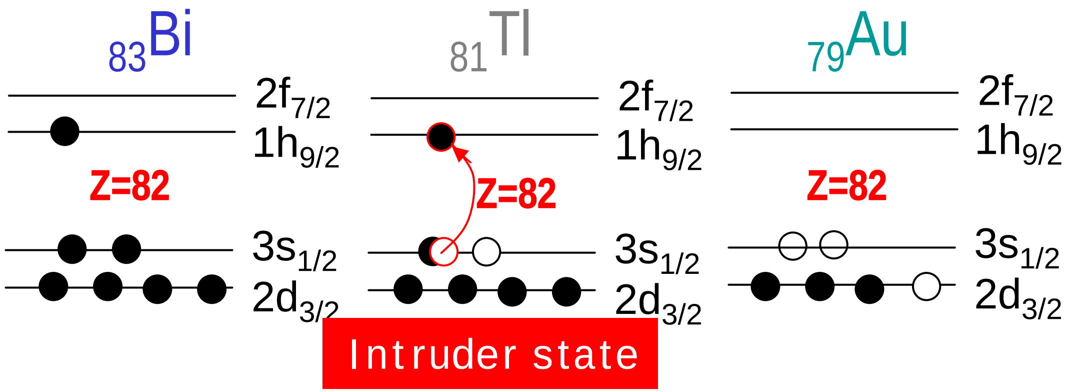
Example: magnetic moments in Pb-region

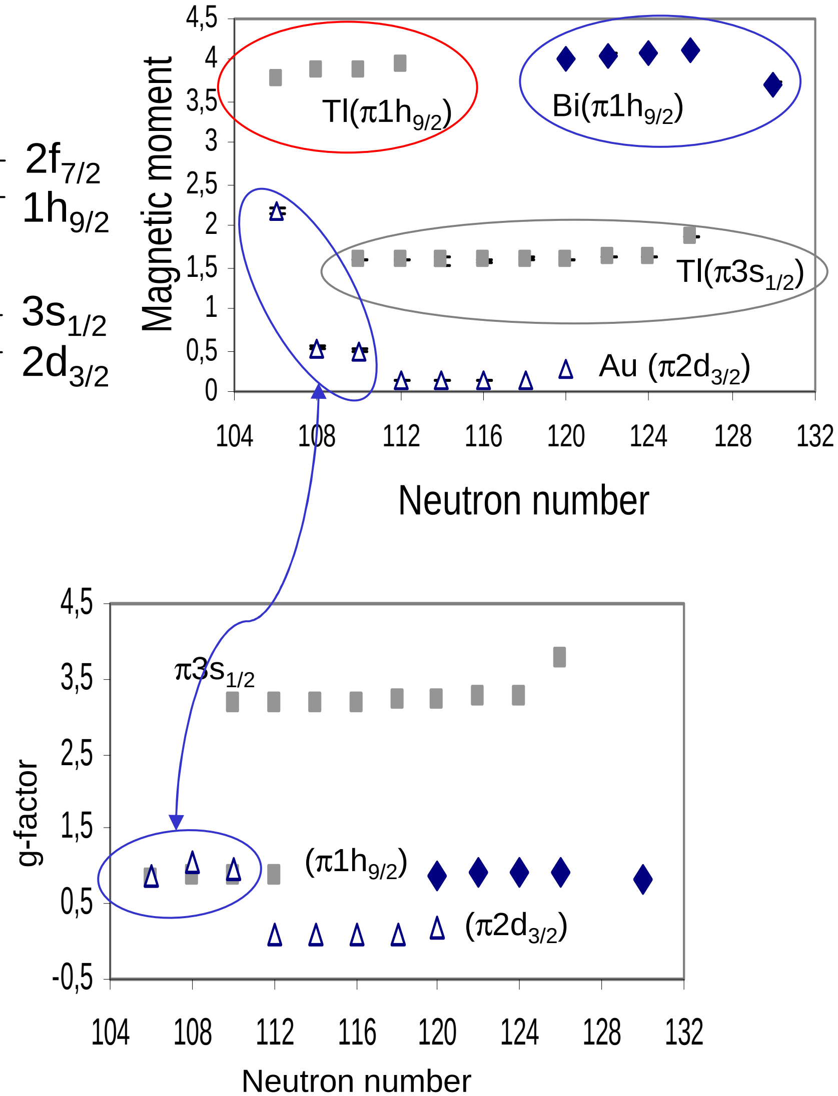


magnetic moments / g-factors (μ = g.I) allow testing an assigned shell model configuration

Example: Pb-region (Z=82)

Ref: G. Neyens, Rep. Prog. Phys. 66





For a nucleon in orbital with spin j,

and spin of the state is I → g(I) = g(j)

g-factor is a better indication to determine the occupied orbital

Consider two unpaired nucleons in a shell model orbital j, coupling to spin I.

The g-factor of this state g(I), in terms of the single-nucleon g-factor g(j), is derived easily using again angular momentum coupling rules:

$$g(I) = g(j)$$

This can be generalized:

the g-factor of a n-nucleon configuration in an orbital j is equal to the 1-nucleon g-factor, independent on n and on I.

→ in a series of isotopes or isotones, it allows to attribute a certain spin to a state, based on a g-factor measurement (exotic nuclei !).

→ within one nucleus, the g-factor is a very sensitive tool to check whether the configuration within a sequence of spin-states (0,2,4,6,8,...) produced by the gradual alignment of two identical nucleons, is pure, down to the lowest excitation energy.

Magnetic moments in odd-odd nuclei: additivity rule

magnetic dipole operator is assumed to be additive:

the dipole operator acting on a nucleus of A nucleons
= sum of individual nucleon dipole operators

$$\hat{\mu} = \sum_{i=1}^{A} \left(g_{L,i}\, \hat{l}_i + g_{S,i}\, \hat{s}_i \right)$$

The magnetic moment of an odd-odd nucleus with spin I,

assuming it consists of a *weak coupling between an odd proton (j_1) and an odd neutron (j_2)*,

IN TERMS OF THE SINGLE NUCLEON MAGNETIC MOMENS $\mu(j_1)$ and $\mu(j_2)$

is calculated using the decoupling rules for angular momenta

$$\mu(I) = I\left[\tfrac{1}{2}\left(\frac{\mu_1}{j_1} + \frac{\mu_2}{j_2}\right) + \tfrac{1}{2}\left(\frac{\mu_1}{j_1} - \frac{\mu_2}{j_2}\right) \frac{j_1(j_1+1) - j_2(j_2+1)}{I(I+1)} \right]$$

Ref: K. Heyde, The Nuclear Shell Model

Knowing the experimental magnetic moments of nearby odd nuclei, one can calculate the magnetic moment of a nuclear state with unpaired nucleons, assuming a particular configuration.

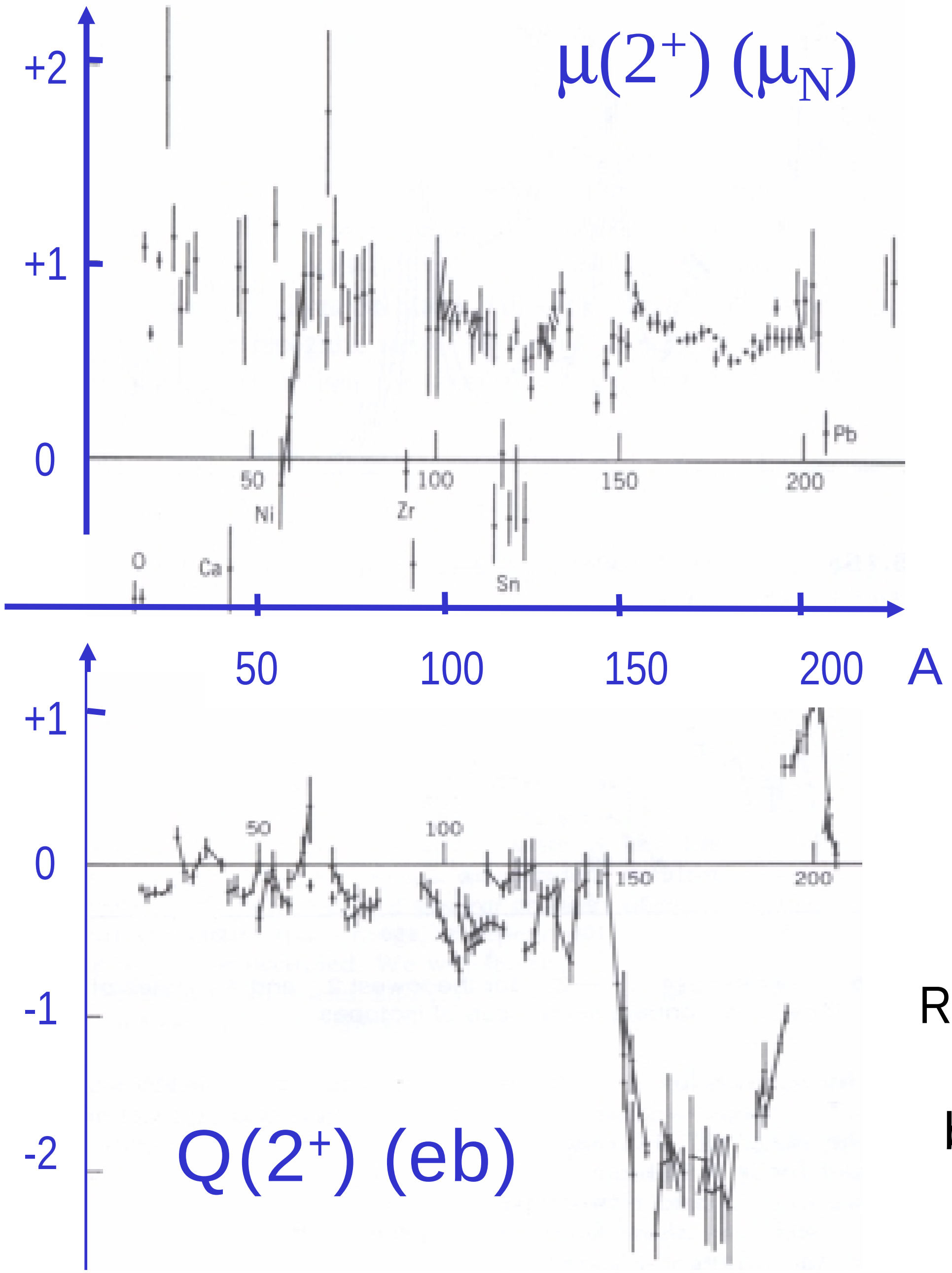
Deviations of the experimental value from this calculated value, are then an indication for the fact that the assumed configuration is probably not pure.

Alternatively: using effective g-factors for the single nucleon values, and assuming a particular shell model configuration, allows to **test the validity of an assumed configuration.**

A more sophisticated calculation can be performed using a shell-model code, which calculates the wave function more accurately.

In regions where such calculations are reproducing well experimental values → deviations from the calculated values can be used as an indication for changes in the shell structure.

However, magnetic moments are not very sensitive to deformation or collectivity



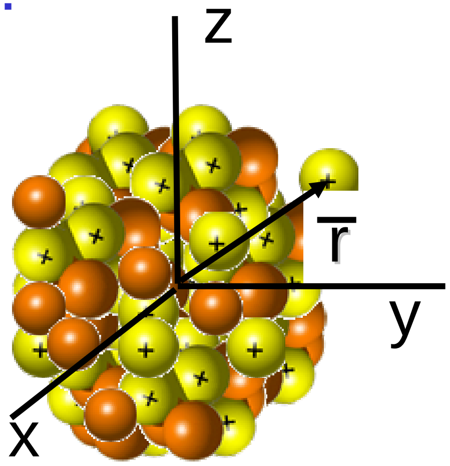
No signature in the magnetic moments !

Region of well-deformed nuclei
A = 150 - 200
between Z=50-82, N=82-126
|Q| > 1 eb

The electric quadrupole moment

The non-spherical distribution of the charges in a nucleus give rise to a quadrupole moment. The quadrupole moment operator is defined as:

$$\hat{Q} = e\sum_{i=1}^{A}(3\,z_i^2 - r_i^2)$$



It is a spherical tensor of rank 2:

$$Q_2 = e\sum_{i=1}^{A} r_k^2\, Y_2(\theta_k,\phi_k)\sqrt{\frac{16\pi}{5}}$$

The Spectroscopic Quadrupole Moment of a state with spin I
= expectation value of the z-component of the quadrupole operator

$$Q(I) = \langle\, I m \,|\, \hat{\mathbf{Q}}_z \,|\, I m \,\rangle_{m=I}$$

or with spherical tensor notation:

$$Q(I) = \langle I m \,|\, Q_2^{\,0} \,|\, I m\rangle_{m=I}$$

Quadrupole moments in odd nuclei

In the extreme single particle shell model

the quadrupole moment of an odd-proton (or an odd-neutron) nucleus with spin I is determined by the single particle moment of the unpaired proton (or neutron) in the orbital j.

If I=j, then $$Q(I) = -e_j \frac{(2j-1)}{2(j+1)} \langle r_j^2 \rangle$$ with e_i the charge of the valence particle

With free-nucleon charges:

e_π=+1 : proton has a negative quadrupole moment

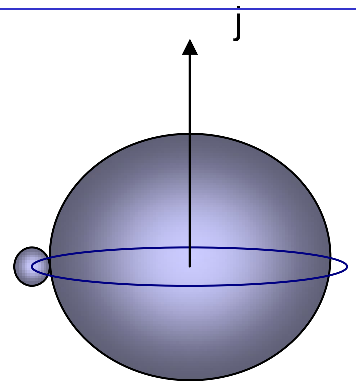
e_ν= 0 : neutron has no quadrupole moment (only charged particles have a quadrupole moment)

In a nucleus, due to the interaction of the valence nucleons with the core nucleons, the neutrons can induce a quadrupole moment.

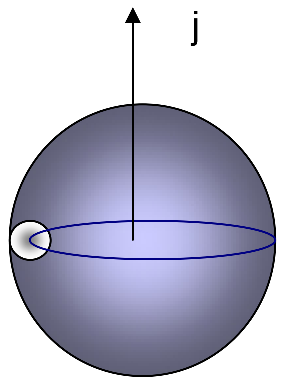
And also the proton quadrupole moment will be influenced by the interaction with the core nucleons.

One says that the valence neutrons and protons 'polarize' the core.

→ **effective charges !**



Polarization towards oblate deformation



Polarization towards oblate deformation

Effective charges

$100\ efm^2 = 1\ eb$

Single particle Q-moments are < 0.5 eb

Nuclei near shell closures are considered to be spherical: the wave function is described by individual nucleons moving in a spherical potential → the core does not contribute to the nuclear quadrupole moment.

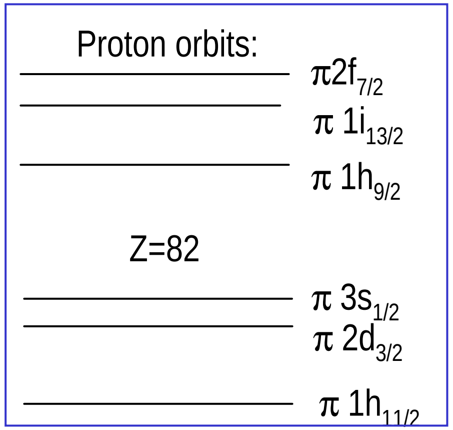
Due to particle-core interactions, the valence nucleon can polarize the core to small oblate or prolate shapes. This effect is taken into account by introducing **an 'effective charge'** for protons and neutrons.

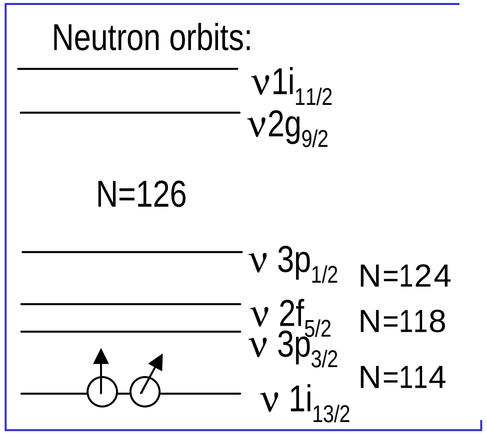
$$Qs(j) = e_{eff}/e \ \ Qs.p.(j)_{free}$$

Effective charges have been determined in several regions of the nuclear chart, and are found to be of the order

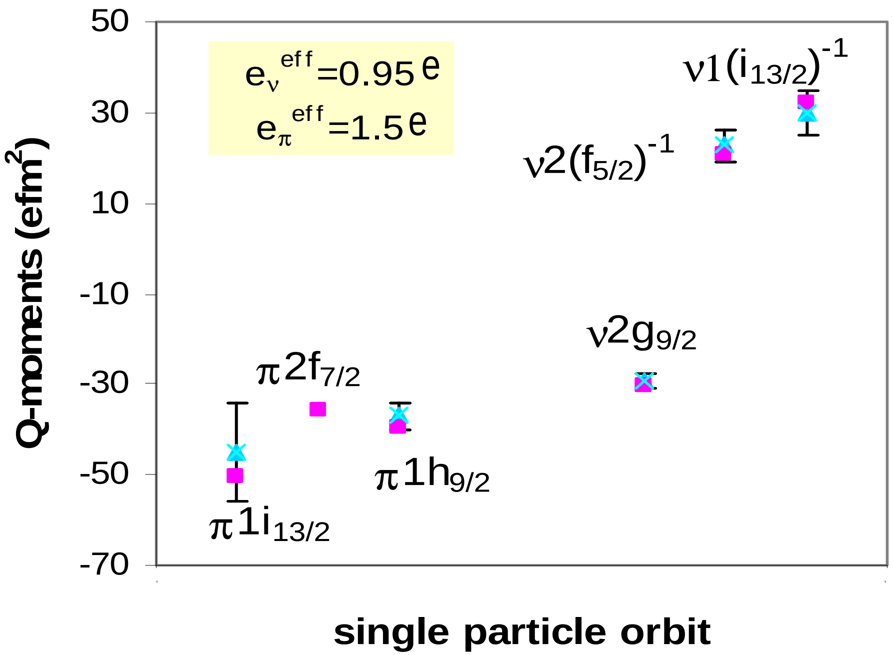
$$\mathbf{e_{\pi}^{eff} = 1.3 - 1.6\ e}$$

$$\mathbf{e_{\nu}^{eff} = 0.1\text{-}0.95\ e}$$





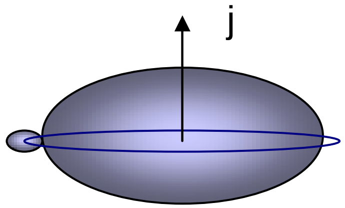
Orbitals in the Pb-region



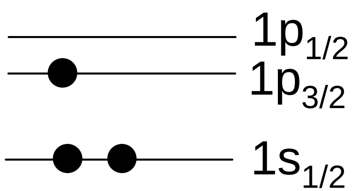
Core Polarization (sperical nuclei) / Prolate and oblate shapes (deformed nuclei)

The valence neutrons and protons 'polarize' the core. To minimize the energy of the nucleus, the valence nucleons try to overlap as much as possible with the core (the 'strong interaction' between nucleons is 'attractive').

particle in an orbit

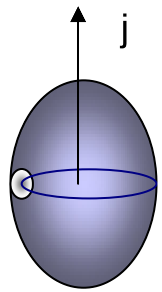


$e_i > 0$ → $Q < 0$
a particle polarizes the core to oblate shape

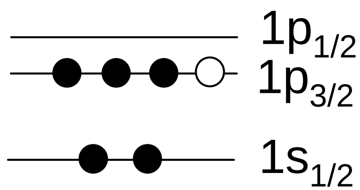


$Q(1p_{3/2}) < 0$

hole in an orbit



$Q > 0$
a hole polarizes the core to prolate shape
→ $e_h = -e_p$



$Q(1p^3_{3/2}) = Q(1p^{-1}_{3/2})$
$= - Q(1p_{3/2})$

Note: core polarization is mainly important in near-spherical nuclei
In deformed nuclei the nuclear core itself is deformed

$$Q = Q_{sp\ (< 0.5\ eb)} + Q_{core\ (> 0.5\ eb)}$$

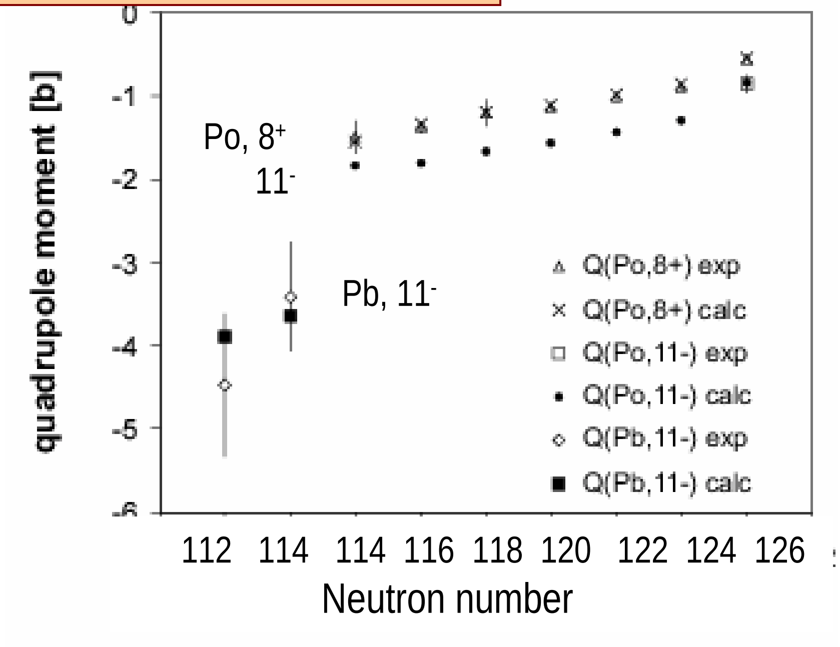
Core polarization in near-sperical nuclei: examples from the Pb-region

(1) Quadrupole moment increases (modulus) when moving away from shell closure

(2) Quadrupole moments of intruder states are **factor 2** larger than for normal states

The core-polarization and intruder deformations
→ can be fully attributed to the interaction of the single particle (spherical) orbits with a collective quadrupole excitation of the core (particle-core coupling).

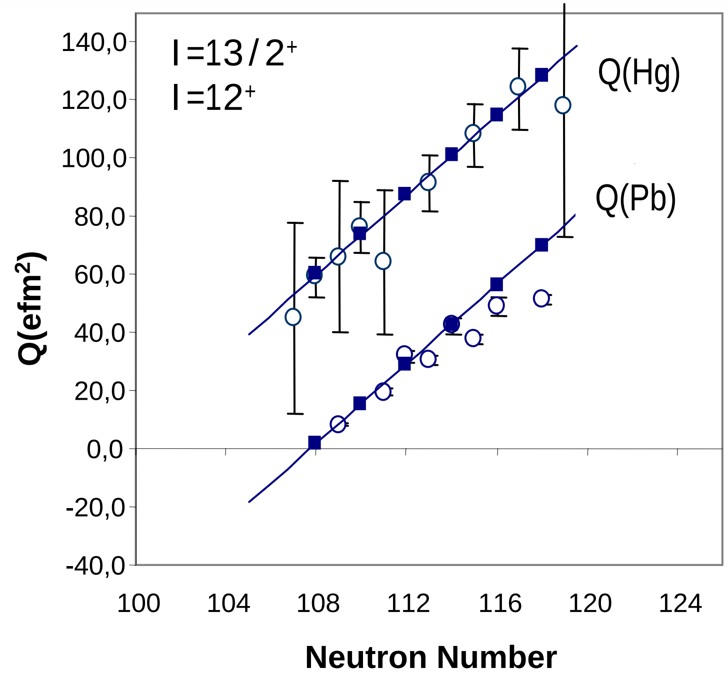
Ref. K. Vyvey et al., Phys. Lett. B 538(2002)33



(3) Quadrupole moment increases when there are holes in the Z=82 shell

Q(Hg) >> Q(Pb)

Ref: G. Neyens, Rep. Prog. Phys. 66 (2003)



Quadrupole moment and deformation: collective models

Nuclei in-between shell closures are often well deformed: there properties can be described, assuming that the nucleus is a deformed liquid drop. Assuming that the liquid drop has an axially symmetric deformation (elipsoidal shape), it's radius can be expanded in spherical harmonics:

$$R(\theta) = R_0 \, [\, 1 + \beta \, Y_2^0(\theta) \,]$$

β is called the deformation parameter.
R_0 is chosen such that the nuclear volume is independent of the nuclear deformation. In a first approximation, the radius can be calculated as

$$R_0 = 1.2 \ A^{1/3}$$

In this same context, the **intrinsic quadrupole moment** of a deformed elipsoidal charge distribution, can be calculated as:

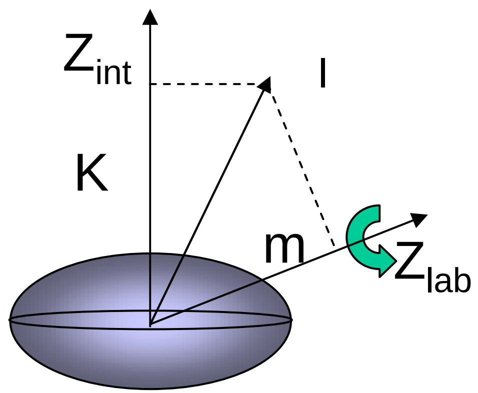
$$Q_0 = \frac{3}{\sqrt{5\pi}} \ eZR^2 \, \beta \, (1 + 0.36 \, \beta + \ldots)$$

Intrinsic and spectroscopic quadrupole moments

The **spectroscopic quadrupole moment** is the experimental observable, which in case of axially deformed nuclei, can be related to **the intrinsic quadrupole moment**, and thus to the nuclear charge deformation parameter β

In the ROTATIONAL MODEL, assuming axially symmetric deformation:

$$Q = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0$$



Z_{int} = intrinsic axis of axial symmetry
K =projection of spin onto this symmetry axis
M = projection of the spin onto the laboratory axis

If the nuclear spin is along the axial symmetry axis, then K=I and:

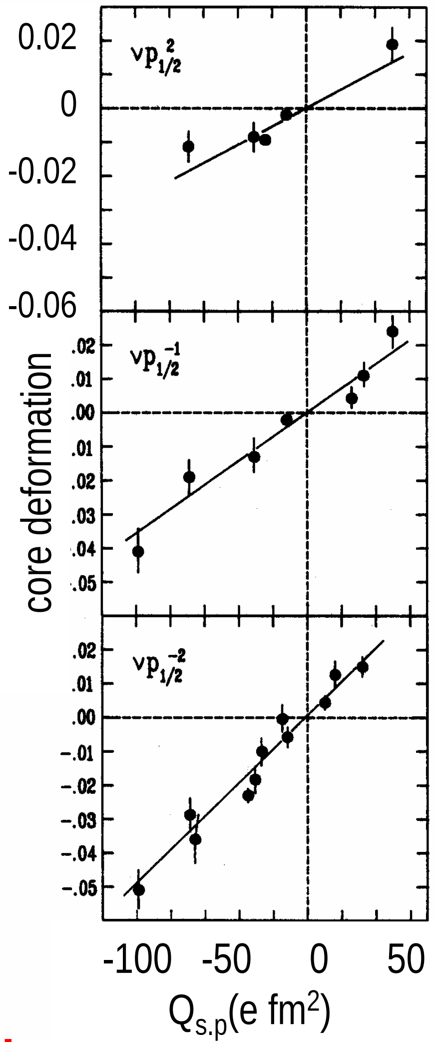
$$Q = \frac{I(2I-1)}{(I+1)(2I+3)} Q_0$$

This relation can be used to deduce and intrinsic quadrupole moment from a measured quadrupole moment, and from that a nuclear charge deformation.

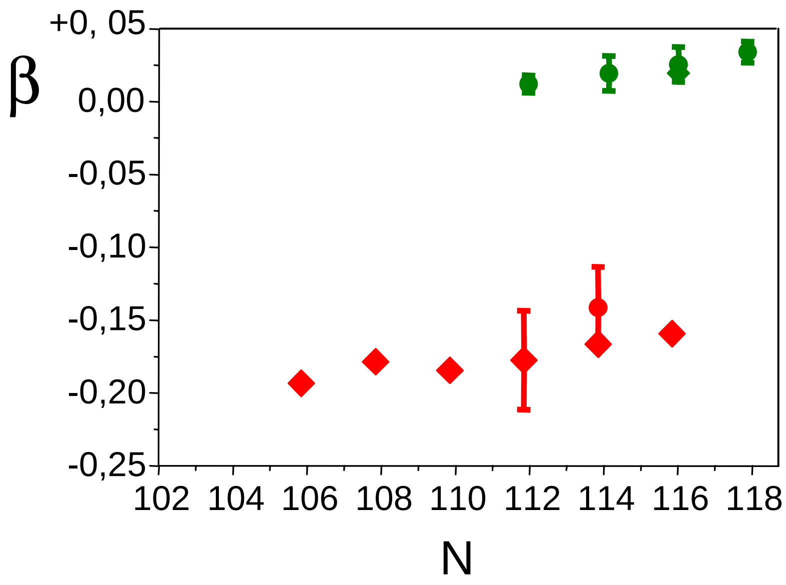
Deformations deduced form experimental data: examples for near-spherical nuclei

In the Pb-region, an increase of the core deformation (due to core polarization) is found with increasing single particle Q-moments. This is visualized by plotting β_{core} versus Q_{sp}, where β_{core} is deduced from Q_{exp} by taking out the single particle contribution.

Ref. G. Scheveneels, Ph.D. Thesis, K.U. Leuven



In the neutron deficient Pb-region, intruder states (proton 2p-2h excitations) occur. Intruder states are deformed (β~ 0.15-0.2) contrary to the quasi-spherical neutron states.



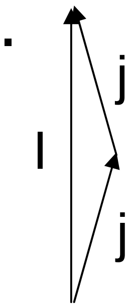
Pb, $I^{\pi}=12^{+}$, $\nu(1i^{-2}_{13/2})$

Pb, $I^{\pi}=11^{-}$, $\pi(3s^{-2}_{1/2}\,1h_{9/2}\,1i_{13/2})$

The holes in Z=82 induce large deformation, due to additional degrees of freedom (increased Q-Q interaction between protons and neutrons). (Calculation: HFB, N.A. Smirnova, PLB 2003, accepted)

Quadrupole moment for 2 identical particles

Consider **two unpaired nucleons in a shell model orbital j, coupling to spin I.**



The quadrupole moment $Q_s(I)$, **in terms of the single particle Q-moment $Q_{s.p.}(j)$,**

is derived easily using angular momentum coupling rules.
For 2 particles in an orbit: use the Clebsch-Gordon coefficients (or 3J-symbols)

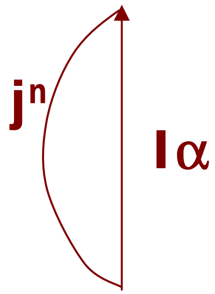
$$Q((j)^2;I) = \frac{\begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}}{\begin{pmatrix} j & 2 & j \\ -j & 0 & j \end{pmatrix}} (-1)^{I+1} \begin{Bmatrix} j & I & j \\ I & j & 2 \end{Bmatrix} Q_{s.p.}(j)$$

Ref: de-Shalit and Talmi in “Nuclear Shell Theory”

Ref: K. Heyde, “The Nuclear Shell Model”

Quadrupole moments of multi-particle configurations

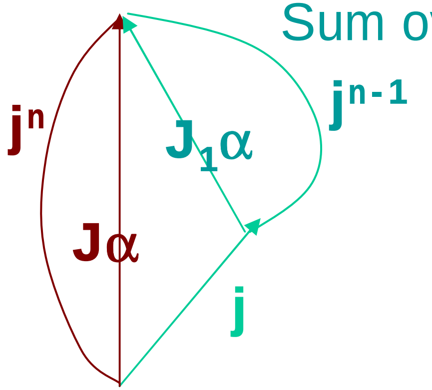
Consider **n nucleons in a shell model orbital j, of which α are unpaired (α=seniority) and coupling to a spin I.**



Ref: de-Shalit and Talmi in "Nuclear Shell Theory"

The quadrupole moment $Q_s(I)$, **in terms of the single particle Q-moment $Q_{s.p.}(j)$,**

is derived again using angular momentum coupling rules. For n particles in an orbit, the Coefficients of Fractional Parentage (CFP-coefficients) need to be used



Sum over all possible J_1, α

$$Q(j^n,\alpha;I) = \frac{2j+1-2n}{2j+1-2\alpha}\; Q(j^2;I)$$

Q is a linear function of n, with $Q(j^2,I)$ determining the slope

Quadrupole moments of states with n nucleons in an orbital j: example

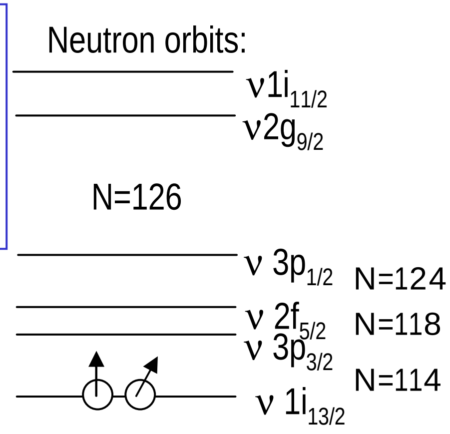
In the neutron deficient Pb-nuclei (Z=82), isomeric states appear at low energy :

I=13/2+ (an unpaired neutron hole in $\nu 1i_{13/2}$ orbit)

I=12+ (2 unpaired neutron holes in $\nu 1i_{13/2}$ orbit)

The first such isomers appears at N=118

Similar isomers appear in Hg (Z=80)



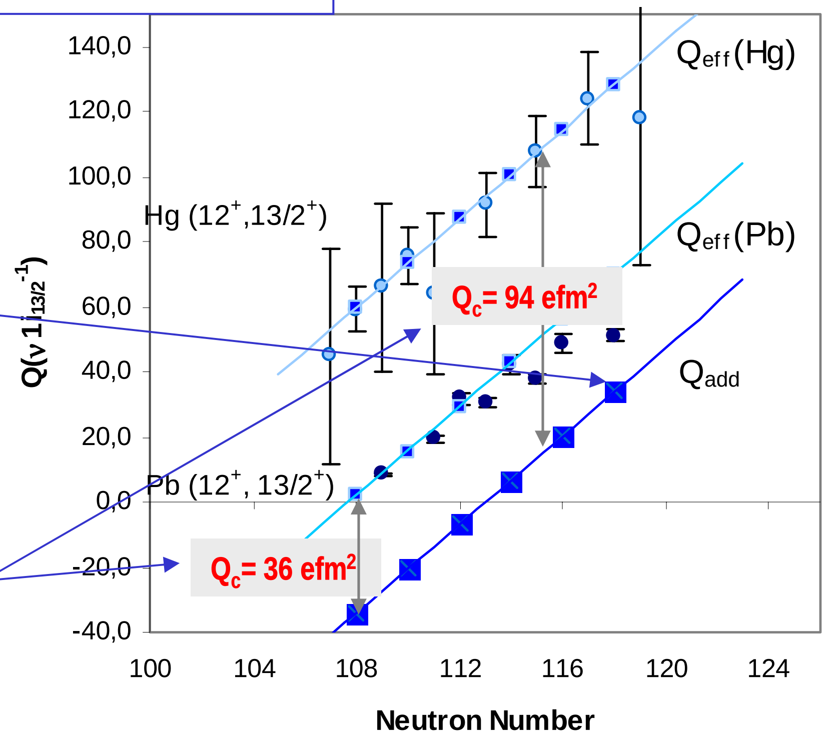
Additivity rule:

$$Q_{add}(i_{13/2}^{-n},\alpha=2;12^{+}, N=118-n) = \frac{2j+1-2n}{2j+1-2\alpha}\, Q(i_{13/2}^{-2};12^{+};N=118)$$

Q(12⁺,n=2) chosen to reproduce the slope of the experimental data points → quantitative agreement

An additional **core-quadrupole moment** is needed to reproduce the experimental values also qualitatively

$$Q_{exp} = Q_{add} + Q_{core}$$



Ref: G. Neyens, Rep. Prog. Phys. 66 (2003)

Quadrupole moments of multi-particle configurations

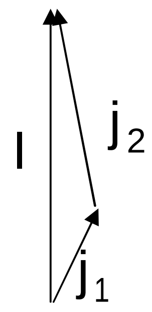
The quadrupole moment of an odd-odd nucleus with spin I, assuming it consists of a *weak coupling between the odd proton (in orbital j_1) and the odd neutron (in orbital j_2)*, is calculated

IN TERMS OF THE SINGLE-NUCLEON QUADRUPOLE MOMENTS $Q(j_1)$ AND $Q(j_2)$,

using the decoupling rules for angular momenta and tensor algebra:

Ref: K. Heyde, "The Nuclear Shell Model"

$$Q(I) = \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} (-1)^{j1+j2+I} \; (2I+1) \times \left[\begin{Bmatrix} j_1 & I & j_2 \\ I & j_1 & 2 \end{Bmatrix} \frac{Q(j_1)}{\begin{pmatrix} j_1 & 2 & j_1 \\ -j_1 & 0 & j_1 \end{pmatrix}} + \begin{Bmatrix} j_1 & I & j_2 \\ I & j_2 & 2 \end{Bmatrix} \frac{Q(j_2)}{\begin{pmatrix} j_2 & 2 & j_2 \\ -j_2 & 0 & j_2 \end{pmatrix}} \right]$$



3J-symbol
~ Clebs-Gordon coeffient

6J-symbol
~ Racah coeffient

Similarly, this can be used to calculate the Q-moment of a configuration containing valence neutrons with total spin I_n that are weakly interacting with valence protons coupled to spin I_p.

Quadrupole moment of a composed state

Suppose a nuclear state with spin I, described by a configuration consisting of neutrons coupled to spin I_p and protons coupled to spin I_n:

$$|\Psi\rangle = |(I_p, I_n)\ IM\rangle$$

The quadrupole moment of this state is defined as: $Q(I) = \langle IM \mid Q_2^0 \mid IM\rangle_{M=I}$ with Q_2^0 the additive one-body quadrupole operator. The quadrupole operator is thus supposed to act either on the protons or on the neutrons:

$$Q_2^0 = Q_2^0(\pi) + Q_2^0(\nu)$$

The state I can be decomposed into it's proton and neutron part:

$$|(I_p, I_n) IM\rangle = \sum_{\substack{M_p, M_n \\ M_p + M_n = M}} \langle I_p M_p, I_n M_n | IM\rangle \ |I_p M_p\rangle\, |I_n M_n\rangle$$

The Q-moment in terms of it's proton and neutron Q-moments $Q(\pi)$ and $Q(\nu)$ is then:

$$Q(I) = \langle (I_p I_n) IM | Q_2^0(\pi) | (I_p I_n) IM\rangle + \langle (I_p I_n) IM | Q_2^0(\nu) | (I_p I_n) IM\rangle$$

$$= \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \left\{ \langle (I_p I_n) I \,||\, Q_2(\pi) \,||\, (I_p I_n) I\rangle + \langle (I_p I_n) I \,||\, Q_2(\nu) \,||\, (I_p I_n) I\rangle \right\}$$

W.-E.

$$= \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \left\{ f_\pi(I_p, I_n, I)\ Q(I_p) + f_\nu(I_p, I_n, I)\ Q(I_n) \right\}$$

Tensor reduction rules
(weak interaction)

Quadrupole moments of composed states: an example

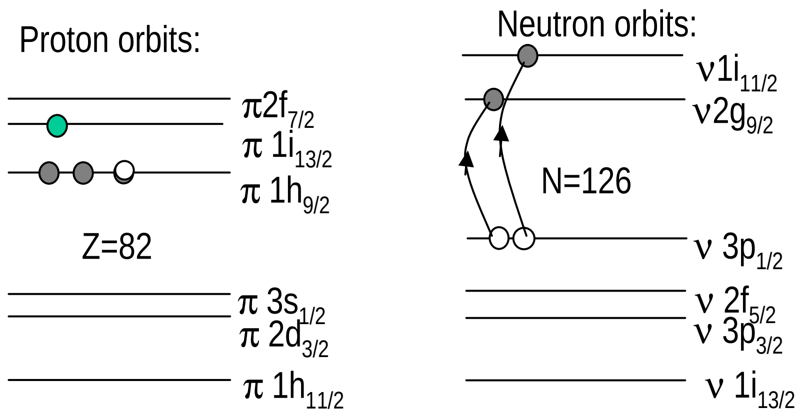
High-spin isomeric states in the Pb-region

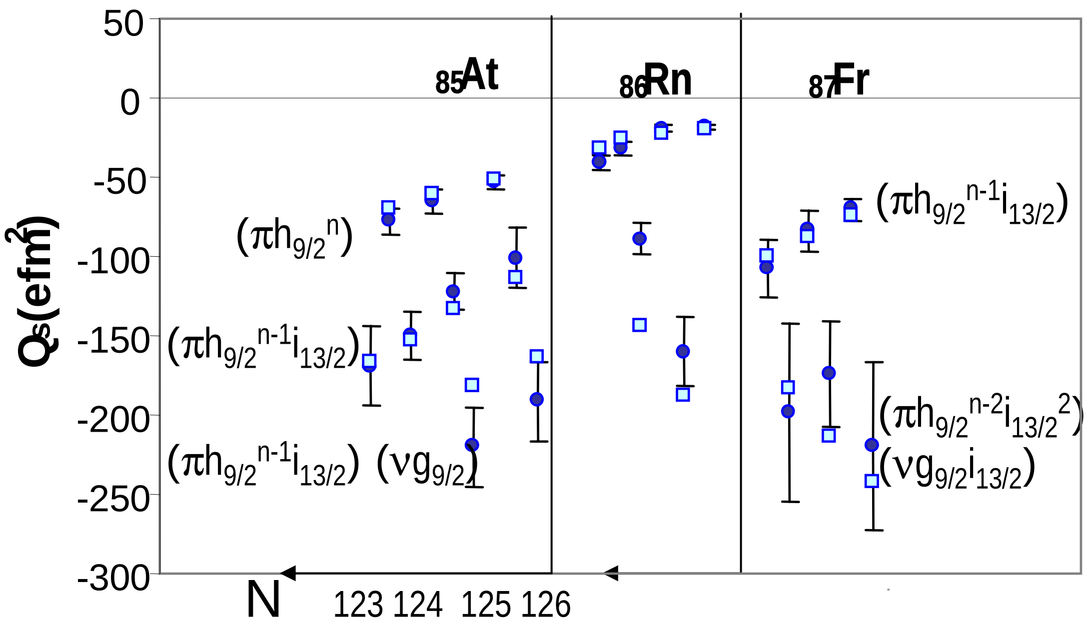
At (Z=85) : n=3

Rn (Z=86) : n=4

Fr (Z=86) : n=5 → mid shell

Neutrons: core excited states (intruders)
1p1h in N=125
2p2h in N=124,126





Dots: experimental data
Squares: additivity rule

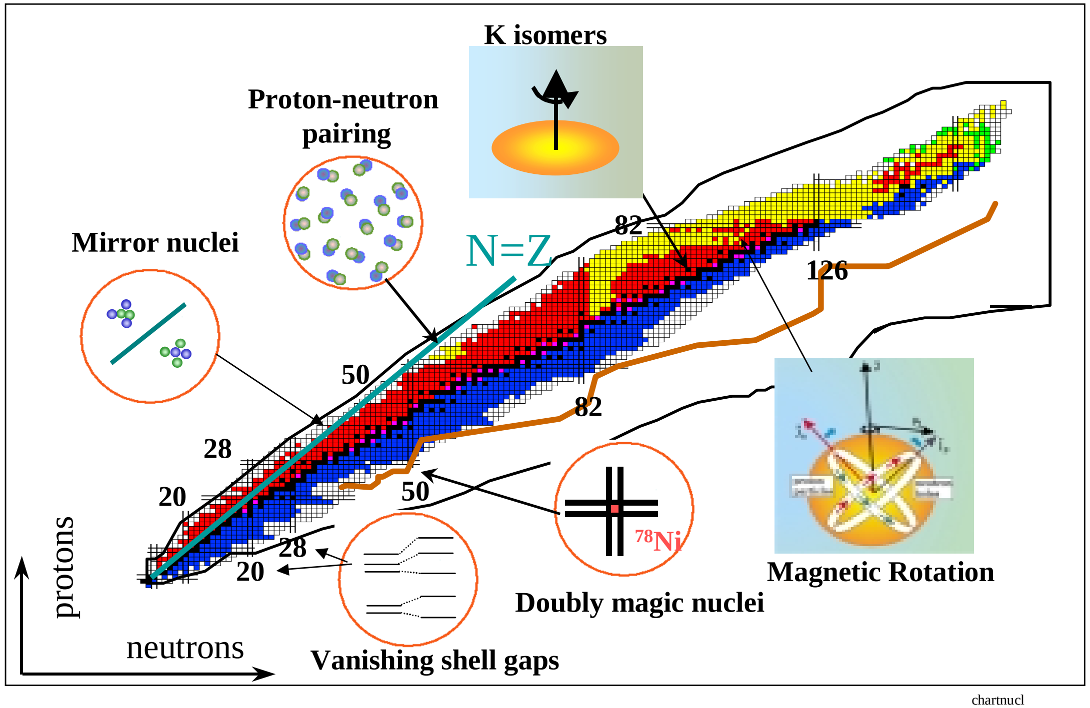
The fact that also the intruder state Q-moments are well-reproduced by the additivity rule,
is an indication that neutron core-excitations induce not much additional core polarization.

Ref: G. Neyens, Rep. Prog. Phys. 66 (2003)

Conclusion: why measure nuclear moments ?

Investigate properties of exotic nuclei ...

→ study influence of the N/Z degree of freedom on the strong nuclear force



Shell ordering ?
New / disappearing shell gaps ?
New quantal rotation ?

... nuclear moments → magnetic moment μ ⇨ single particle configurations (mixing)

→ the quadrupole moment Q ⇨ collective properties (deformation, core polarization)

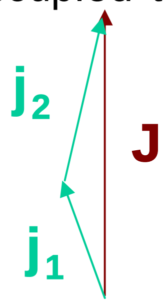
complementary information is needed to understand changes in shell structure !

Needed Impulsmoment Algebra

Ref: de-Shalit and Talmi in "Nuclear Shell Theory"

Ref: K. Heyde, "The Nuclear Shell Model"

decoupling of two angular momenta $|j_1m_1\rangle$, $|j_2m_2\rangle$ coupled to a state $|JM\rangle$



Clebsch-Gordon coefficients – 3J-symbols:

$$|(j_1,j_2)JM\rangle = \sum_{\substack{m_1,m_2 \\ m_1+m_2=M}} \langle j_1m_1, j_2m_2|JM\rangle \; |j_1m_1\rangle\,|j_2m_2\rangle$$

$$\langle j_1m_1, j_2m_2|JM\rangle = (-1)^{M-j1-j2}\sqrt{2J+1}\begin{pmatrix} j_1 & j_2 & J \\ m_1 & m_2 & -M \end{pmatrix}$$

Some Impulsmoment Algebra

Ref: de-Shalit and Talmi in “Nuclear Shell Theory”

Ref: K. Heyde, “The Nuclear Shell Model”

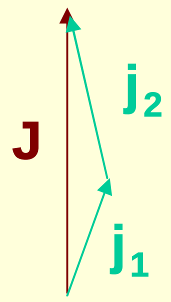
decoupling of two angular momenta $|j_1m_1\rangle$, $|j_2m_2\rangle$ coupled to a state $|JM\rangle$

Clebsch-Gordon coefficients – 3J-symbols:

$$|(j_1,j_2)JM\rangle = \sum_{\substack{m_1,m_2\\ m_1+m_2=M}} \langle j_1m_1,\, j_2m_2|JM\rangle \quad |j_1m_1\rangle\,|j_2m_2\rangle$$

$$\downarrow$$

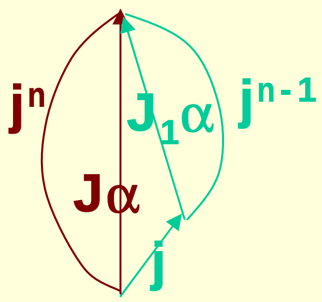
$$\boxed{(-1)^{M-j1-j2}\sqrt{2J+1}\begin{pmatrix} j_1 & j_2 & J\\ m_1 & m_2 & -M\end{pmatrix}}$$



decoupling of a state $|JM\rangle$, composed of n particles in orbit j with seniority α into two components $|jm\rangle$, $|J_1m_1\rangle$

Coefficients of Fractional Parentage

$$|(j^n)\alpha JM\rangle = \sum_{J_1,\alpha_1} [j^{n-1}(\alpha_1J_1),j;J|\}(j^n)\alpha J\ ] \quad |j^{n-1}(J_1),j;JM\rangle$$



Tensor reduction rules

Reduction of an expectation value:

The Wigner-Eckard Theorem:

$$\langle jm |\, T_k^n \,| j'm' \rangle = (-1)^{j-m} \begin{pmatrix} j & k & j' \\ -m & n & m' \end{pmatrix} \langle j \,||\, T_k \,||\, j \rangle$$

reduced matrix element

Expectation value of a 1-body operator acting on a composed state $|(j_1\, j_2)\, JM\rangle$:

$$\langle (j_1 j_2)JM |\ T_k^n(1)\ | (j_1' j_2')J'M' \rangle =$$

$$\sqrt{2J+1}\sqrt{2J'+1} \begin{Bmatrix} J & j_1 & j_2 \\ j_1' & J' & k \end{Bmatrix} (-1)^{j1+j2+J'+k} \langle j_1 \,||\, T_k(1) \,||\, j_1' \rangle\ \delta_{j2,j2'}$$

$$\langle (j_1 j_2)JM |\ T_k^n(2)\ | (j_1' j_2')J'M' \rangle =$$

$$\sqrt{2J+1}\sqrt{2J'+1} \begin{Bmatrix} j_1 & j_2 & J \\ k & J' & j_2' \end{Bmatrix} (-1)^{j1+j2'+J+k} \langle j_2 \,||\, T_k(2) \,||\, j_2' \rangle\ \delta_{j1,j1'}$$

Go to the reduced matrix element (W-E-theorem)

→ then the **spectroscopic quadrupole moment Qs** becomes:

$$Q(I) = \langle I m \mid Q_2^0 \mid I m \rangle_{m=I} = \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \sqrt{\frac{16\pi}{5}} \; \langle I \,\|\, \sum_{k=1}^{A} e_k\, r_k^2\, Y_2(\theta_k,\phi_k) \,\|\, I \rangle$$

$$\mathbf{Q_s} = Q(I) = \sqrt{\frac{I(2I-1)}{(I+1)(2I+3)}} \; \underbrace{\sqrt{\frac{16\pi}{5}} \; \langle I \,\|\, \sum_{k=1}^{A} e_k\, r_k^2\, Y_2(\theta_k,\phi_k) \,\|\, I \rangle}_{\text{Intrinsic quadrupole moment}}$$

Remark: different conventions related to the Wigner-Eckhart theorem and reduced matrix elements, lead to a factor $\sqrt{2I+1}$ difference in above expression.

For a single nucleon in an orbital with angular momentum j:

evaluate $\langle j \,\|\, Y_2 \,\|\, j \rangle$ and $\langle r^2 \rangle$