Shell-Model Applications in nuclear physics and astrophysics

Gabriel Martínez Pinedo

IEEC
Institut d'Estudis Espacials de Catalunya

*icrea
*institució catalana de recerca i estudis avançats

- Introduction
- Shell-Model Basics
- Effective Interaction
- Lanczos Strength Function
- Astrophysical Applications

Nucleus as a many body problem

- Nucleus is made of quarks interacting by exchange of gluons. We need to solve Quantum Chromodinamics.

Nucleus as a many body problem

- Nucleus is made of quarks interacting by exchange of gluons. We need to solve Quantum Chromodinamics.
- Nucleus is made of nucleons (neutrons and protons) that interact by meson exchange. Interaction could be described by a non-relativistic potential.

Nucleus as a many body problem

- Nucleus is made of quarks interacting by exchange of gluons. We need to solve Quantum Chromodinamics.
- Nucleus is made of nucleons (neutrons and protons) that interact by meson exchange. Interaction could be described by a non-relativistic potential.

We need to solve the Schrödinger equation:

$$\boldsymbol{H}\Psi = E\Psi$$

$$\boldsymbol{H} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i}{2m} + \sum_{i<j=1}^{A} V_{ij} + \sum_{i<j<k=1}^{A} V_{ijk}$$

Basic symmetries

- Invariance under translations. Potential depends only on relative coordinates.

Basic symmetries

- Invariance under translations. Potential depends only on relative coordinates.
- Invariance under rotations (space, spin, isospin).
  - ➢ States are eigenstates of total angular momentum ($\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$) and isospin.
  - ➢ Tensor operators (multipole operators).

$$[J_{\pm}, T^k_q] = \hbar\sqrt{k(k+1) - q(q \pm 1)}\, T^k_{q+1}$$

$$[J_0, T^k_q] = \hbar q$$

Basic symmetries

- Invariance under translations. Potential depends only on relative coordinates.
- Invariance under rotations (space, spin, isospin).
  - ➢ States are eigenstates of total angular momentum ($\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$) and isospin.
  - ➢ Tensor operators (multipole operators).

$$[J_\pm, T^k_q] = \hbar\sqrt{k(k+1) - q(q\pm 1)}\, T^k_{q+1}$$

$$[J_0, T^k_q] = \hbar q$$

- invariance under parity and time reversal

Example of Potential

Potential consistent with two-nucleon scattering data and deuteron structure

Potential AV8':

$$
\begin{aligned}
V(r) \;=\; & V_c(r) + V_\tau(r)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_\sigma(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\
& + V_{\sigma\tau}(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_t(r) S_{12} + V_{t\tau}(r) S_{12}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\
& + V_b(\boldsymbol{L} \cdot \boldsymbol{S}) + V_{b\tau}(r)(\boldsymbol{L} \cdot \boldsymbol{S})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)
\end{aligned}
$$

Solution only possible for light systems

Benchmark test calculations for ^{4}He

[H. Kamada, *et al.*, Phys. Rev. C 64, 044001 (2001)]

$$E_{\rm exp} = 28.296 \text{ MeV}$$

TABLE I. The expectation values $\langle T\rangle$ and $\langle V\rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

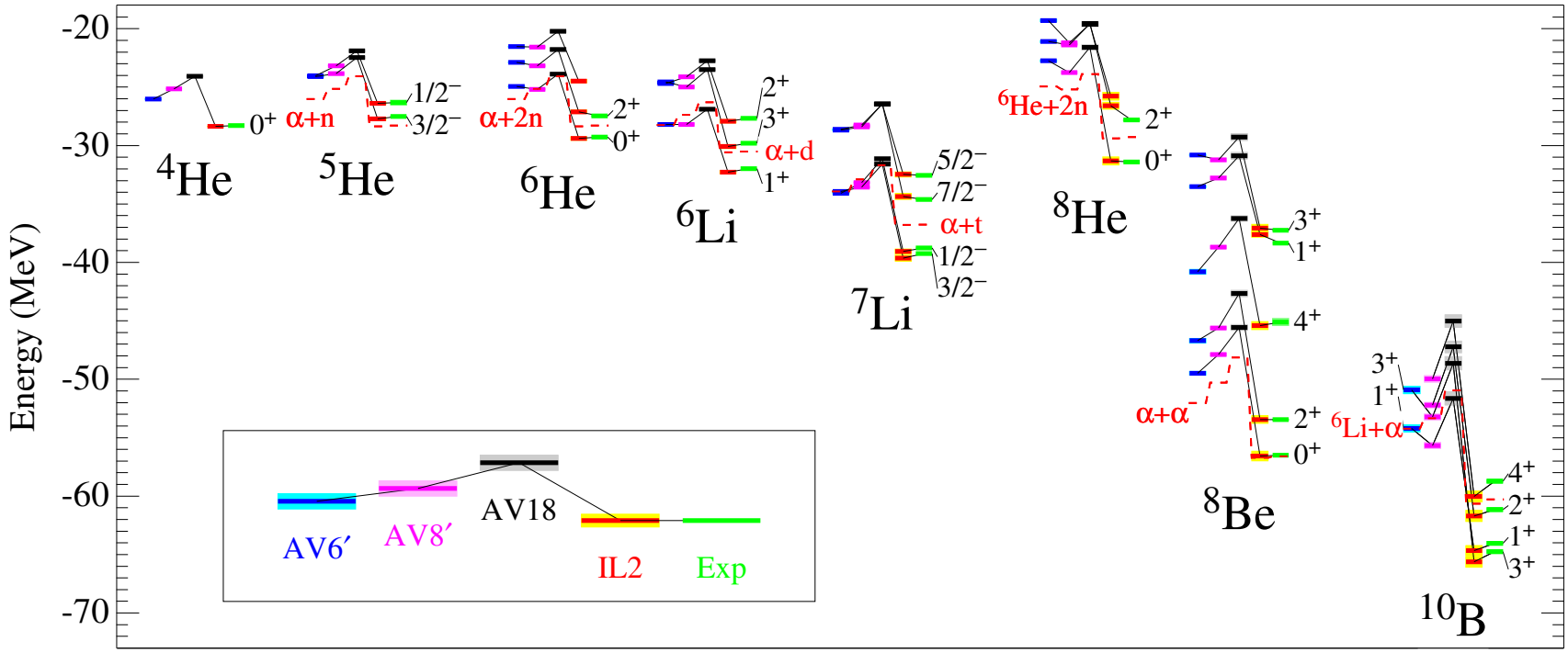
Method	$\langle T\rangle$	$\langle V\rangle$	E_b	$\sqrt{\langle r^2\rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

TABLE II. Expectation values of the eight potential operators in Eq. (24) in MeV.

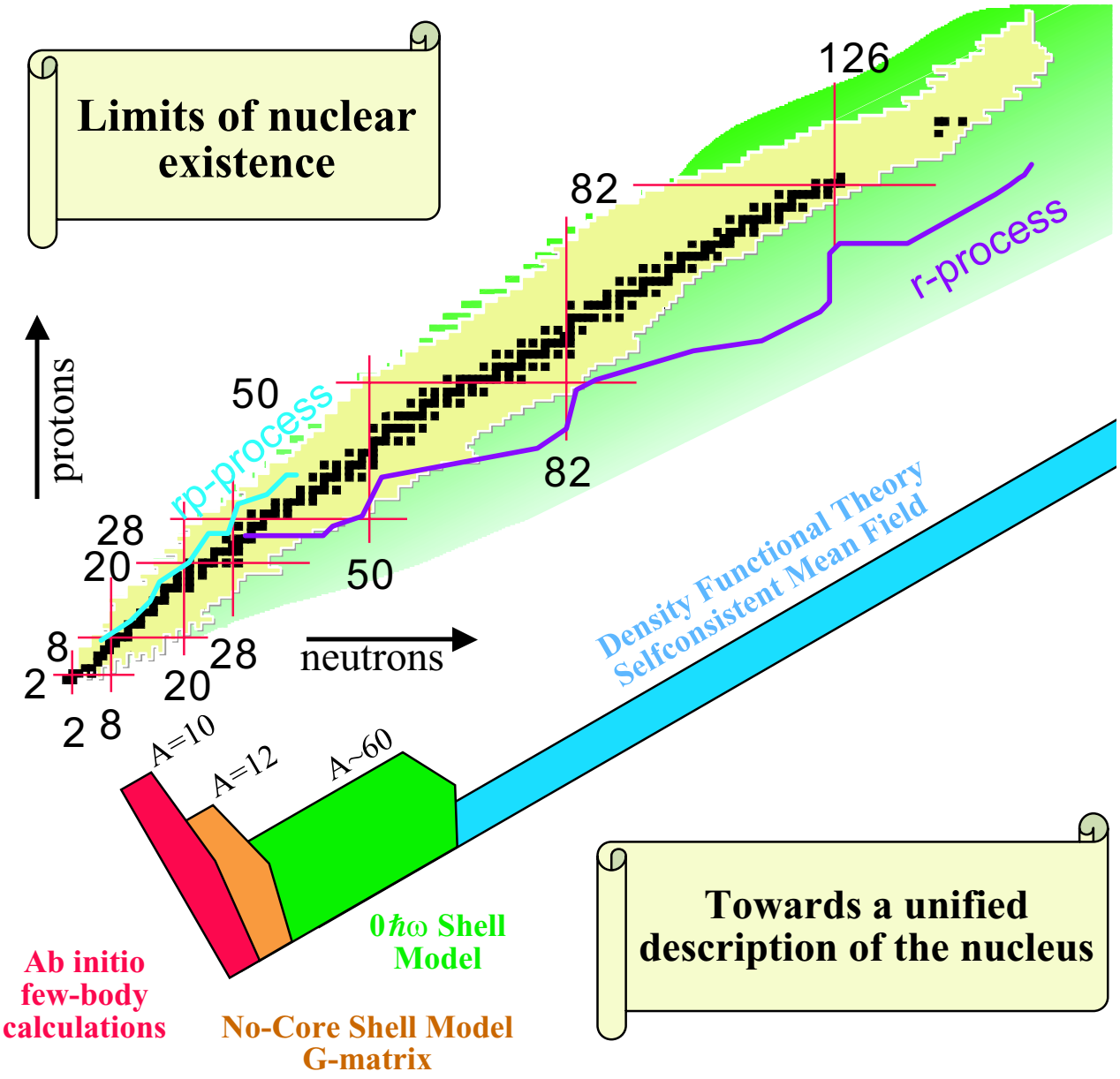
Method	$\langle V_c\rangle$	$\langle V_\tau\rangle$	$\langle V_\sigma\rangle$	$\langle V_{\sigma\tau}\rangle$
FY	16.54	−5.038	−9.217	−57.55
CRCGV	16.54	−5.035	−9.215	−57.51
SVM	16.54	−5.036	−9.213	−57.51
HH	16.57	−5.034	−9.255	−57.59
GFMC	16.5(5)	−5.03(6)	−9.21(7)	−57.3(5)
NCSM	16.16	−4.92	−9.77	−57.89

Method	$\langle V_t\rangle$	$\langle V_{t\tau}\rangle$	$\langle V_b\rangle$	$\langle V_{b\tau}\rangle$
FY	0.707	−69.06	10.79	−15.50
CRCGV	0.708	−68.99	10.60	−15.30
SVM	0.707	−69.03	10.78	−15.49
HH	0.702	−69.03	10.76	−15.46
GFMC	0.71(3)	−68.8(5)	10.62(15)	−15.40(15)
NCSM	0.68	−69.13	11.23	−15.80

GFMC calculations for light nuclei



Theoretical models



Shell-Model basics

- Shell-Model assumes the existence of shells. Magic numbers are obtained when a shell is completely fill.
- Shells results from the bunching (grouping) of levels coming from a independent particle average potential.

Independent-Particle Model

- Assume the existence of some single-particle wave functions that are the solution of a Schrödinger equation

$$\boldsymbol{h}\phi(r) = \{\boldsymbol{T} + \boldsymbol{U}\}\phi_a(r)\} = \varepsilon_a \phi_a(r)$$

The independent-particle motion hamiltonian is then:

$$\boldsymbol{H}_0 = \sum_{k=1}^{A} \boldsymbol{T}(k) + U(r_k)$$

Eigenfunctions are the product of single-particle wave functions:

$$\Phi_{a_1 a_2 \ldots a_A}(1, 2, \ldots, A) = \prod_{k=1}^{A} \phi_{a_k}(r_k)$$

System identical particles

Wave function should be antisymmetric. For two particles:

$$\Phi_{ab}(1,2) = \frac{1}{\sqrt{2}}[\phi_a(1)\phi_b(2) - \phi_a(2)\phi_b(1)] = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_a(1) & \phi_b(2) \\ \phi_a(2) & \phi_b(1) \end{vmatrix}$$

A-particle Wave function:

$$\Phi_{a_1 a_2 \ldots a_A}(1,2,...,A) = \sqrt{\frac{1}{A!}} \begin{vmatrix} \phi_{a_1}(r(1)) & \phi_{a_1}(r(2)) & \cdots & \phi_{a_1}(r(A)) \\ \phi_{a_2}(r(1)) & \phi_{a_2}(r(2)) & \cdots & \phi_{a_2}(r(A)) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{a_A}(r(1)) & \phi_{a_A}(r(2)) & \cdots & \phi_{a_A}(r(A)) \end{vmatrix}$$

Simple single-particle potential

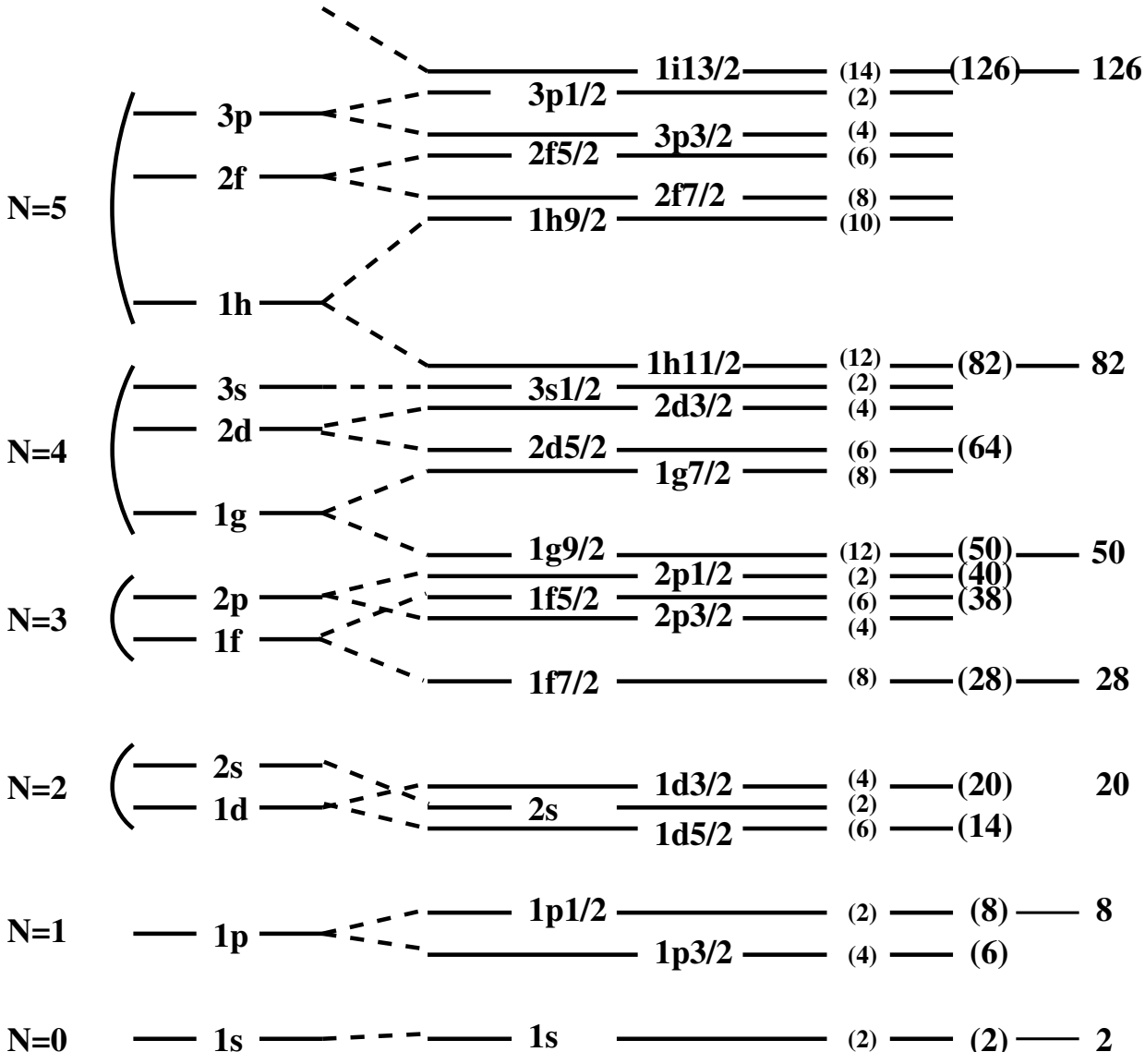
Empirical construction based in an harmonic oscillator potential plus a spin-orbit term to reproduce the magic numbers (M. Goeppert-Mayer and H. Jensen)

$$\boldsymbol{U}(r) = \frac{1}{2}m\omega^2 r^2 + D\boldsymbol{l}^2 + \boldsymbol{l}\cdot\boldsymbol{s}$$

$$\varepsilon_{nlj} = \hbar\omega[2(n-1)+l+3/2)]+Dl(l+1)+C\begin{cases} l+1 & j=l-1/2 \\ -l & j=l+1/2 \end{cases}$$

$$\hbar\omega = \frac{41}{A^{1/3}}\ \mathrm{MeV}$$

Spherical mean-field



Solution to the many-nucleon problem

$$\boldsymbol{H}\Psi(1,2,\ldots,A) = \left[\sum_{k=1}^{A} T(k) + \sum_{k<l=1}^{A} W(k,l)\right]\Psi(1,2,\ldots,A) = E\Psi(1,2,\ldots,A)$$

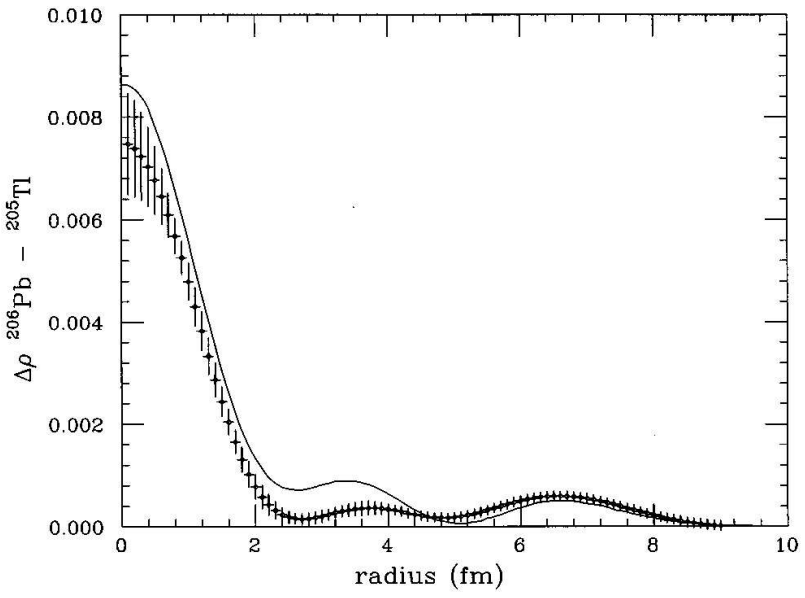
Hamiltonian rewritted:

$$\boldsymbol{H} = \sum_{k=1}^{A}[T(k)+U(k)] + \left[\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k)\right] = \boldsymbol{H}_0 + \boldsymbol{V}_{\rm res}$$

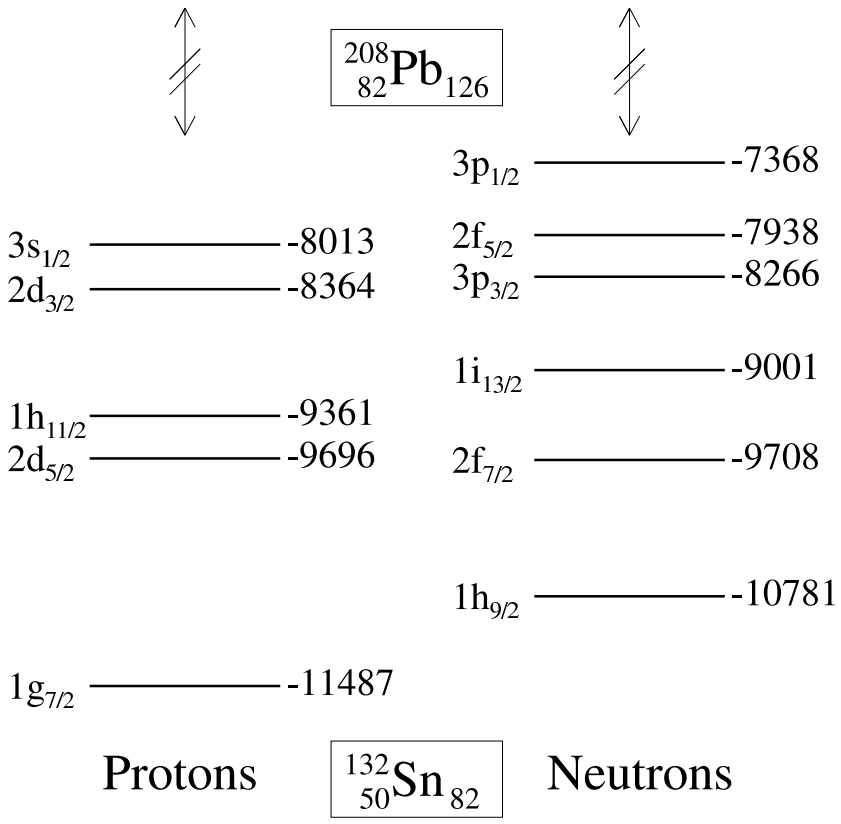
Hartree-Fock theory provides method to derive single-particle potential. The criterium is to search for the "best" A-particle slater determinant such us the value of $\boldsymbol{H}$ is minimum. Next, one assumes that the resulting residual interaction is small and that:

$$\Psi(1,2,\ldots,A) = \Phi_{a_1 a_2 \ldots a_A}(1,2,\ldots,A)$$

Validity shell structure

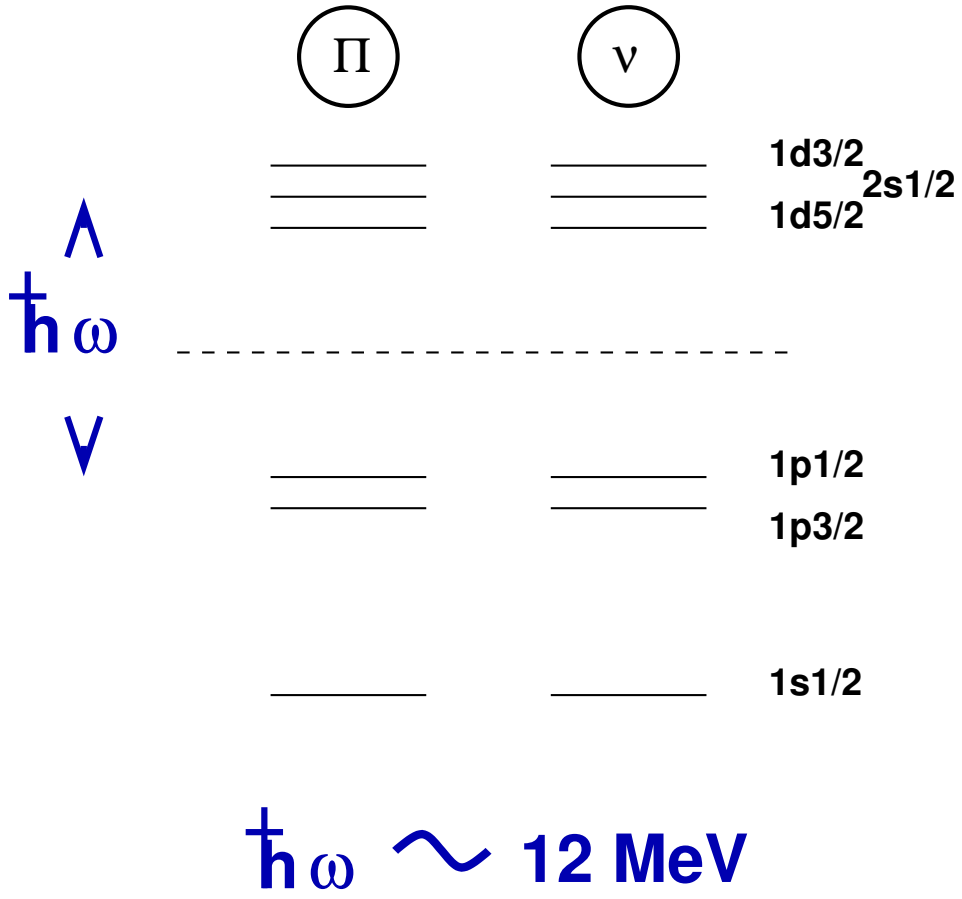


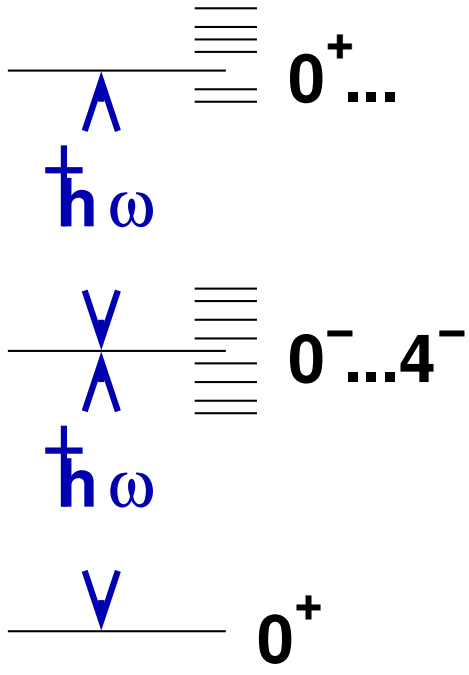
V. R. Pandharipande, I. Sick and P. K. A. deWitt Huberts, Rev. mod. Phys. 69, 981 (1997)



Limits of the IPM

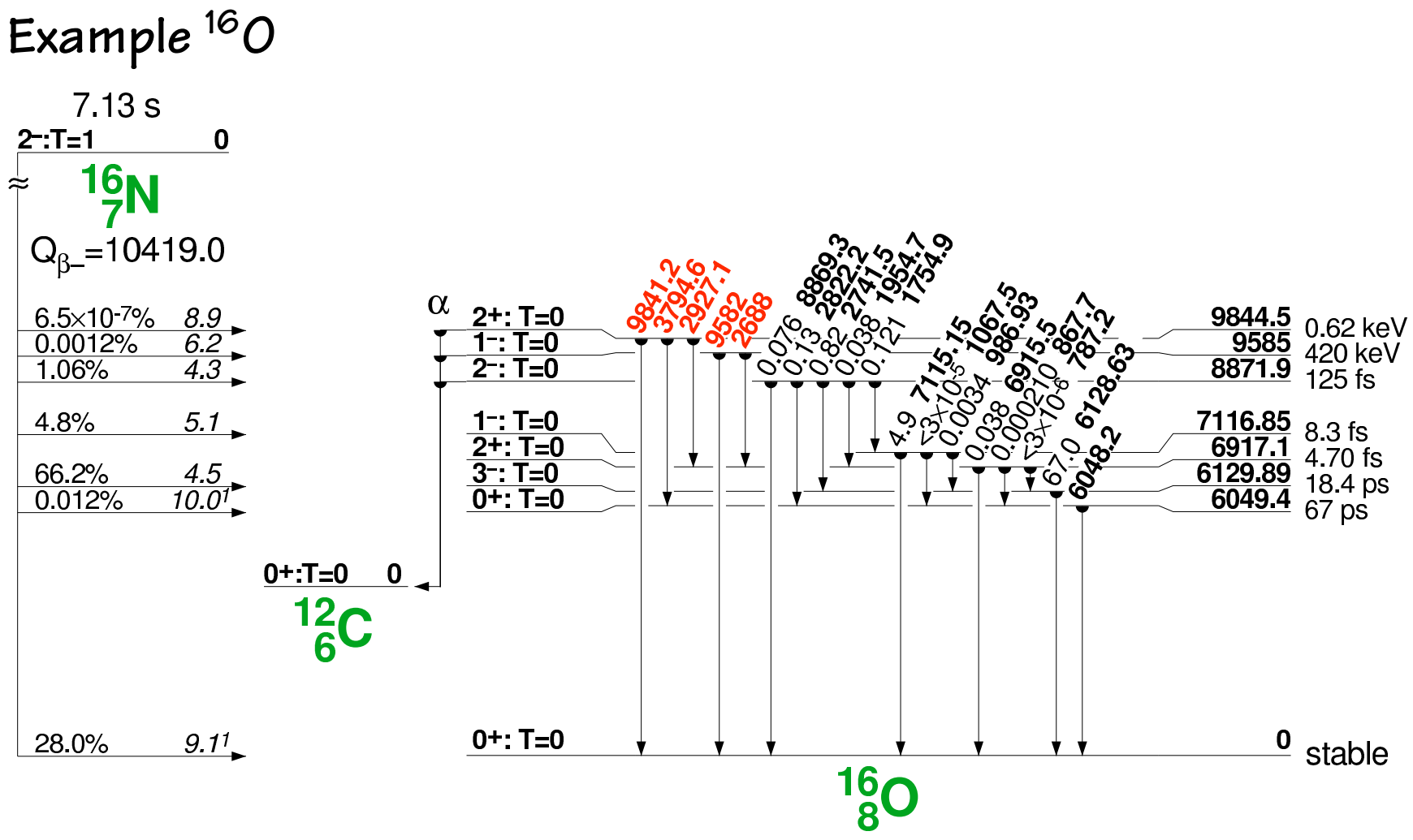
Example ^{16}O





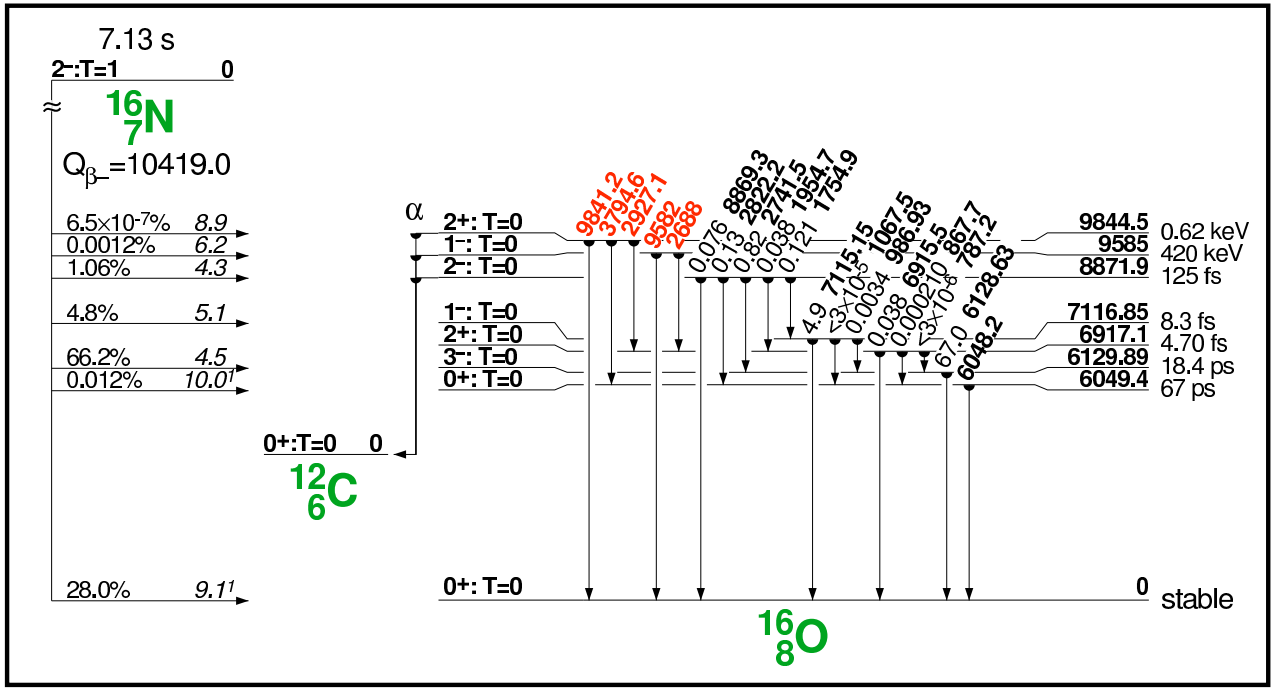
$\hbar\omega \sim 12$ MeV

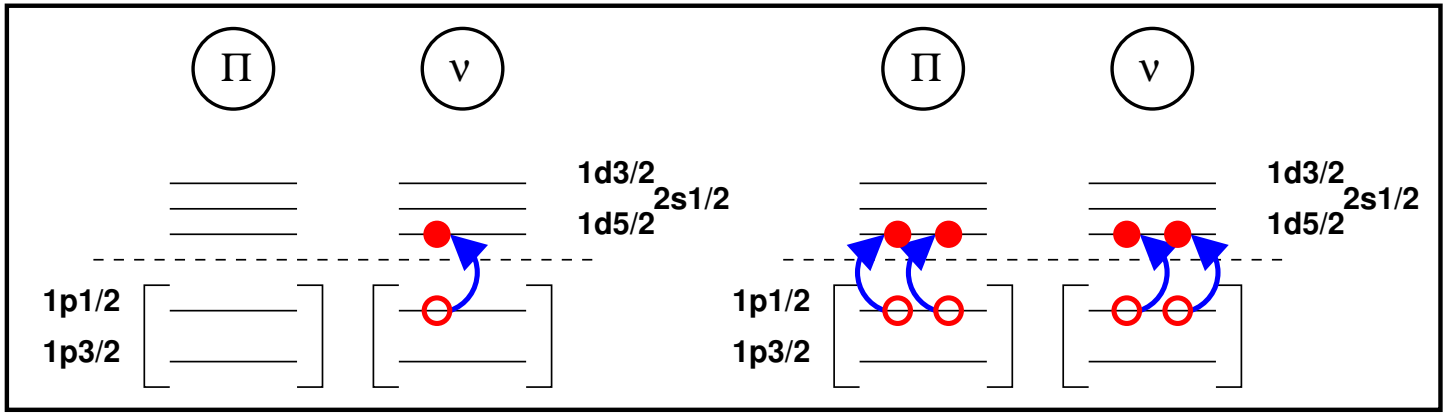
Limits of the IPM



Limits of the IPM

Example ^{16}O





Two-particle wave function

Let's consider the possible isospin values for two nucleons:

$$|T=1,T_z=1\rangle = |n\rangle|n\rangle, \quad |1,-1\rangle = |p\rangle|p\rangle$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}\left[|n\rangle|p\rangle + |p\rangle|n\rangle\right] \qquad q$$
$$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|n\rangle|p\rangle - |p\rangle|n\rangle\right]$$

N-particle system: $T =$ odd (symmetric), $T =$ even (antisymmetric).
We can also couple the angular momentum:

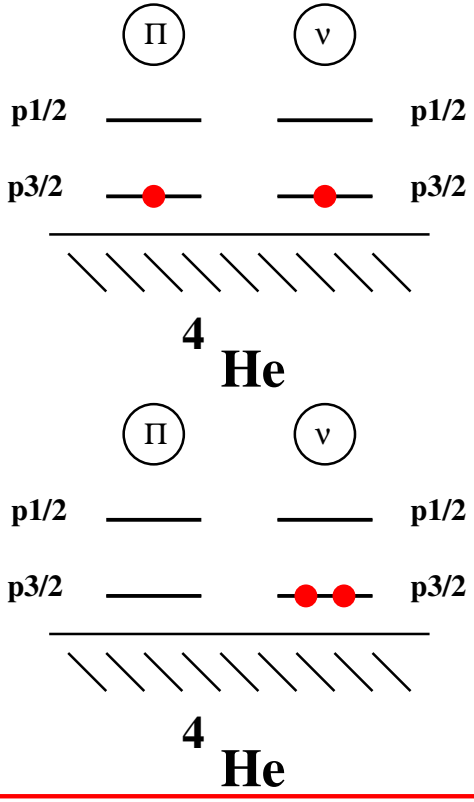
$$|JM\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | JM\rangle |j_1 m_1\rangle |j_2 m_2\rangle$$

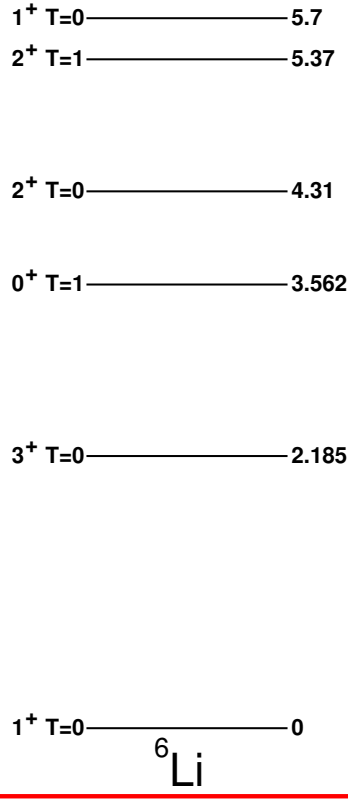
In $N = Z$ nuclei protons and neutrons occupy the same orbits. Low lying states fulfill ($J + T =$ odd number).

Example ^{6}Li

posible two-particle states $(j_1 j_2)^{JT}$

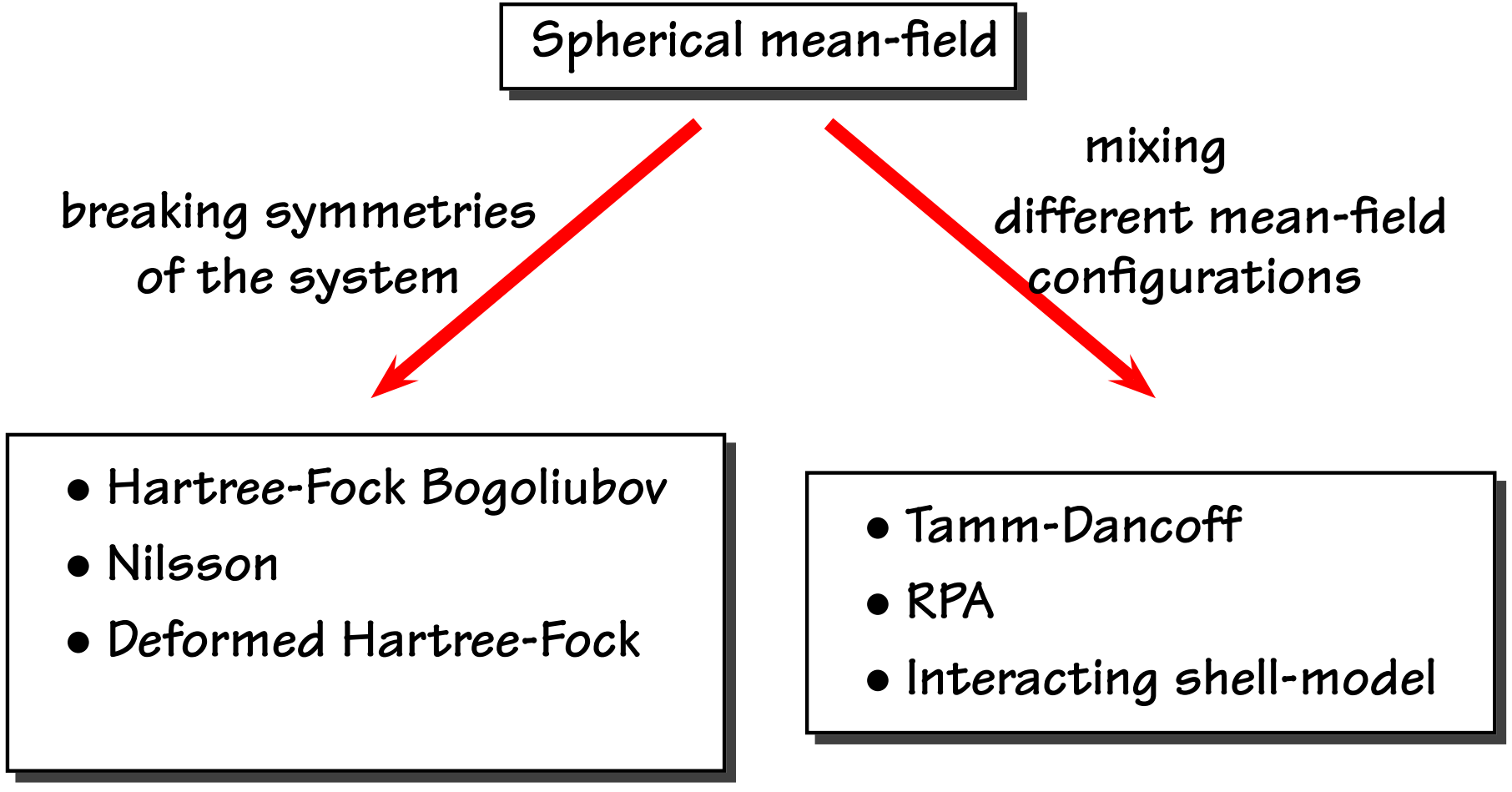
$$(p_{3/2}p_{3/2})^{J=1,3,5;T=0}, \quad (p_{3/2}p_{3/2})^{J=2,4,6;T=1}$$
$$(p_{3/2}p_{1/2})^{J=1,2;T=0} \quad (p_{3/2}p_{1/2})^{J=1,2;T=1}$$
$$(p_{1/2}p_{1/2})^{J=1;T=0} \quad (p_{1/2}p_{1/2})^{J=0;T=1}$$





Correlations (residual interaction)

In order to incorporate the correlations, one has to go beyond mean-field



Possible solution

- Take the basics formed by the A-particle Slater determinants:

$$\Phi_a = \Phi_{a_1 a_2 \ldots a_A}(1, 2, \ldots, A)$$

Possible solution

- Take the basics formed by the A-particle Slater determinants:

$$\Phi_a = \Phi_{a_1 a_2 \ldots a_A}(1, 2, \ldots, A)$$

- build the (infinite) matrix:

$$\langle \Phi_b | \boldsymbol{H} | \Phi_a \rangle$$

Possible solution

- Take the basics formed by the A-particle Slater determinants:

$$\Phi_a = \Phi_{a_1 a_2 \ldots a_A}(1, 2, \ldots, A)$$

- build the (infinite) matrix:

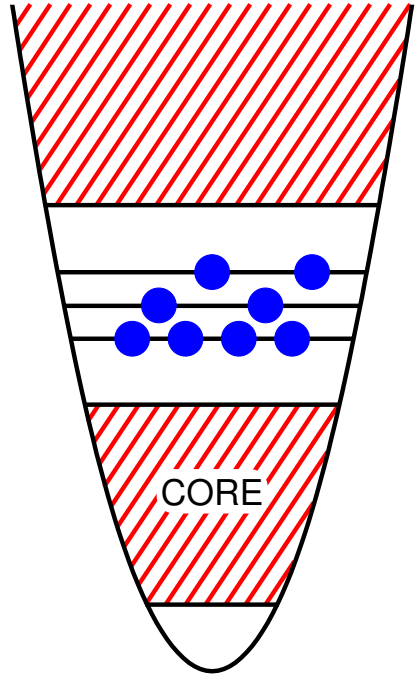
$$\langle \Phi_b | \boldsymbol{H} | \Phi_a \rangle$$

- diagonalize

Shell-Model approximation

The space of orbits generated by the mean-field potential are grouped in three blocks

- Inner core: orbits that are always full.
- Valence space: orbits that contain the physical degrees of freedom relevant to a given property. The distribution of the valence particles among these orbitals is governed by the interaction.
- External space: all the remaining orbits that are always empty.



Shell-Model approximation

The exact solution on the infinite Hilbert space spanned by the mean field orbits is approximated in the large scale shell-model calculation by the solution of the Schrödinger equation in the valence space using an effective interaction.

$$\boldsymbol{H}\Psi = E\Psi \rightarrow \boldsymbol{H}_{\rm eff}\Psi_{\rm eff} = E\Psi_{\rm eff}$$

In general, effective operators have to be introduced to account for the restrictions of the Hilbert space

$$\langle\Psi|\boldsymbol{O}|\Psi\rangle = \langle\Psi_{\rm eff}|\boldsymbol{O}_{\rm eff}|\Psi_{\rm eff}\rangle$$

Shell-Model calculation

A shell model calculation needs the following ingredients:

- A valence space

Shell-Model calculation

A shell model calculation needs the following ingredients:

- A valence space
- An effective interaction

Shell-Model calculation

A shell model calculation needs the following ingredients:

- A valence space
- An effective interaction
- A code to build and diagonalize the hamiltonian matrix.

Valence space

The choice of the valence space:

- In light nuclei the harmonic oscillator closures determine the natural valence spaces.

^{4}He $\longrightarrow$ ^{16}O $\longrightarrow$ ^{40}Ca $\longrightarrow$ ^{80}Zr

p shell — Cohen/ Kurath

sd shell — Brown/ Wildenthal

pf shell

$\uparrow$ Deformed

Valence space

- In heavier nuclei:
  $\longrightarrow jj$ closures due to the spin-orbit term show up

  N=28, 50, 82, 126

Valence space

- In heavier nuclei:
  $\longrightarrow jj$ closures due to the spin-orbit term show up

  N=28, 50, 82, 126

- the transition $HO \longrightarrow jj$: occurs between ^{40}Ca and ^{100}Sn where the protagonism shifts from the $1f_{7/2}$ to the $1g_{9/2}$

Valence space

- In heavier nuclei:
  $\longrightarrow jj$ closures due to the spin-orbit term show up

  N=28, 50, 82, 126

- the transition $HO \longrightarrow jj$: occurs between ^{40}Ca and ^{100}Sn where the protagonism shifts from the $1f_{7/2}$ to the $1g_{9/2}$
- A valence space can be adequate to describe some properties and completely wrong for others

^{48}Cr	$(f_{\frac{7}{2}})^8$	$(f_{\frac{7}{2}}p_{\frac{3}{2}})^8$	$(fp)^8$
Q(2^+) ($e.fm^2$)	0.0	-23.3	-23.8
E(2^+) (MeV)	0.63	0.44	0.80
E(4^+)/E(2^+)	1.94	2.52	2.26
BE2($2^+ \rightarrow 0^+$) ($e^2.fm^4$)	77	150	216
B(GT)	0.90	0.95	3.88

Second quantization

- Creation and destruction operators:

$$a_k^\dagger|0\rangle = |k\rangle, \qquad a_k|k\rangle = |0\rangle$$

vacuum $|0\rangle$, such $a_k|0\rangle\ \forall k$.
For fermions antisymmetry given by anti-commutation rules

$$\{a_i^\dagger, a_j^\dagger\} = \{a_i, a_j\} = 0, \qquad \{a_i^\dagger, a_j\} = \delta_{ij}$$

- Slater determinant:

$$\Phi_{\alpha_1\alpha_2\ldots\alpha_A}(1,2,\ldots,A) = a_{\alpha_A}^\dagger \cdots a_{\alpha_2}^\dagger a_{\alpha_1}^\dagger|0\rangle$$

Second quantization

- One body operators:

$$\mathcal{O} = \sum_{k=1}^{A} \boldsymbol{O}(k) \rightarrow \mathcal{O} = \sum_{\alpha\beta} \langle \alpha | \boldsymbol{O} | \beta \rangle a_\alpha^\dagger a_\beta$$

number of particles $N = \sum_\alpha n_\alpha = \sum_\alpha a_\alpha^\dagger a_\alpha$

- Two-body operators:

$$\mathcal{O} = \sum_{i<j=1}^{A} \boldsymbol{O}(i,j) \rightarrow \mathcal{O} = \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | \boldsymbol{O} | \gamma\delta \rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

The interaction in second quantization

$$\boldsymbol{H} = \sum_{\alpha} \varepsilon_{\alpha} a^{\dagger}_{\alpha} a_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\delta} a_{\gamma} = \boldsymbol{H}_0 + \boldsymbol{V}_{\rm res}$$

The interaction in second quantization

In general it is convenient to work in a couple basis:

Defining $\tilde{a}_{jm} = (-1)^{j+m} a_{j-m}$, and the coupling:

$$[a^\dagger_{j_1} a^\dagger_{j_2}]^J_M = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | JM \rangle a^\dagger_{j_1 m_1} a^\dagger_{j_2 m_2}$$

$$\begin{aligned} \boldsymbol{H} &= \sum_\alpha \varepsilon_\alpha n_\alpha - \frac{1}{4} \sum_{j_1 j_2 j_3 j_4 JT} \langle j_1 j_2 | V | j_3 j_4 \rangle^{JT} \\ & \sqrt{(2J+1)(2T+1)(1+\delta_{12})(1+\delta_{23})} \\ & \left[[a^\dagger_{j_1} a^\dagger_{j_2}]^{JT} \times [\tilde{a}_{j_3} \tilde{a} j_4]^{JT} \right]^{00} \end{aligned}$$

The shell-model interaction

All the information needed for a shell-model calculation are the independent particle energies (ε_α) and the two-body matrix elements $\langle j_1 j_2; JT|V|j_3 j_4; JT\rangle$

Example USD interaction ($d_{5/2}, s_{1/2}, d_{3/2}$)

```
USD
     3  205    1001     203
   -3.94780 -3.16354  1.64658
    0  205  205  205  205    0    5
    0.0000  -1.6321   0.0000  -1.5012   0.0000  -4.2256
   -2.8197   0.0000  -1.0020   0.0000  -0.1641   0.0000
    0  205 1001  205 1001    2    3
   -1.4474  -3.8598
   -0.8183   0.7626
    0  205  203  205  203    1    4
   -6.5058  -3.8253  -0.5377  -4.5062
    1.0334  -0.3248   0.5894  -1.4497
    0 1001 1001 1001 1001    0    1
    0.0000  -3.2628
   -2.1246   0.0000
    0 1001  203 1001  203    1    2
   -4.2930  -1.8194
    0.6066  -0.4064
    0  203  203  203  203    0    3
    0.0000  -1.4151   0.0000  -2.8842
   -2.1845   0.0000  -0.0665   0.0000
    ........................
```

Application

Spectrum of 0^+ states in ^{18}O

we build a 3×3 matrix:

$$\begin{array}{ccc} (d_{\frac{5}{2}})^2 & (d_{\frac{3}{2}})^2 & (s_{\frac{1}{2}})^2 \\ \begin{array}{l} -3.9478 \times 2 \\ +(-2.8197) \end{array} & -3.1856 & -.13247 \\ -3.1856 & \begin{array}{l} 1.64658 \times 2 \\ +(-2.1845) \end{array} & -1.0835 \\ -1.3247 & -1.0835 & \begin{array}{l} -3.1654 \times 2 \\ +(-2.1246) \end{array} \end{array}$$

whose eigenvalues produce the spectrum:

0_3^+ _____ 14.1

0_2^+ _____ 4.3

0_1^+ _____ 0.0

Large scale Shell-Model basis and codes

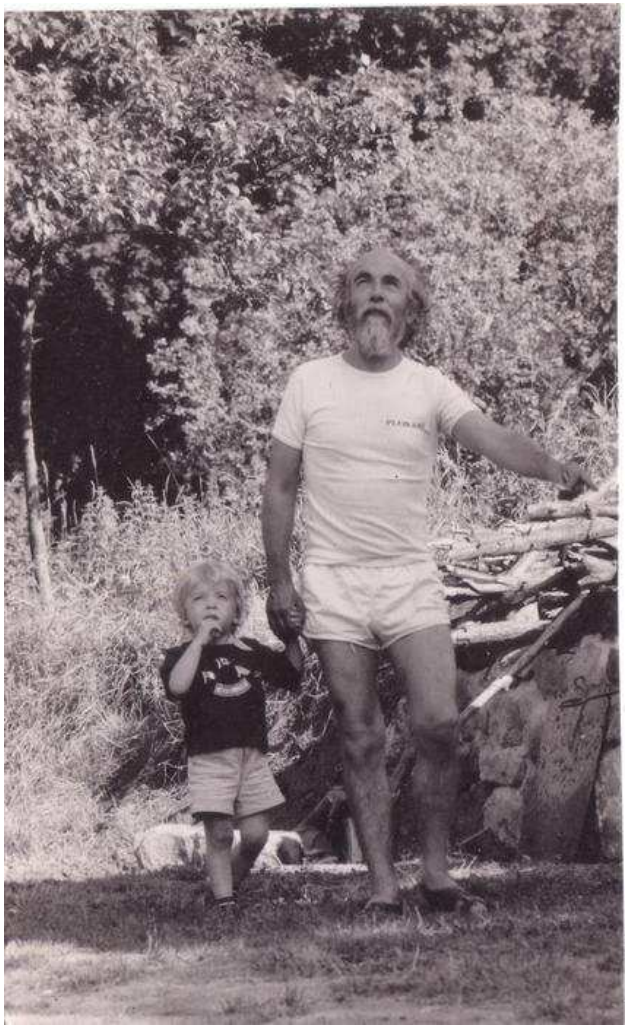
pf-shell valence space: $1f_{7/2}, 2p_{3/2}, 2p_{1/2}, 1f_{5/2}$

nucleus	m-scheme (ANTOINE)	jj-scheme (NATHAN)
^{48}Cr	1,963,461	41,355
^{54}Fe	345,400,174	5,220,621
^{56}Fe	501,113,392	7,413,488
^{56}Ni	1,087,455,228	15,443,684

Impossible to store Hamiltonian matrix!
Still possible to compute $H\Psi$.
Diagonalization using an iterative algorithm.

Shell-Model code ANTOINE

http://sbgat194.in2p3.fr/~theory/antoine/



Lanczos algorithm

Construction of a orthonormal basis:
Initial vector $|1\rangle$.

$$E_{12}|\mathbf{2}\rangle = (\boldsymbol{H} - E_{11})|\mathbf{1}\rangle$$
$$E_{23}|\mathbf{3}\rangle = (\boldsymbol{H} - E_{22})|\mathbf{2}\rangle - E_{12}|\mathbf{1}\rangle$$
$$\dots$$
$$E_{NN+1}|\mathbf{N+1}\rangle = (\boldsymbol{H} - E_{NN})|\mathbf{N}\rangle - E_{N-1N}|\mathbf{N-1}\rangle$$

where

$$E_{NN} = \langle \mathbf{N}|\boldsymbol{H}|\mathbf{N}\rangle, \qquad E_{NN+1} = E_{N+1N}$$

Lanczos algorithm

$$\begin{pmatrix} E_{11} & E_{12} & 0 & 0 & \dots & 0 \\ E_{12} & E_{22} & E_{23} & 0 & \dots & 0 \\ 0 & E_{23} & E_{33} & E_{34} & \dots & 0 \\ & & & & & \end{pmatrix}$$

- Diagonalize and obtain an approximation to the energy and wave function.
- Do a few more iterations
- Diagonalize again and check that $\Delta E = E_{N+k} - E_N < \epsilon$

Lanczos convergence

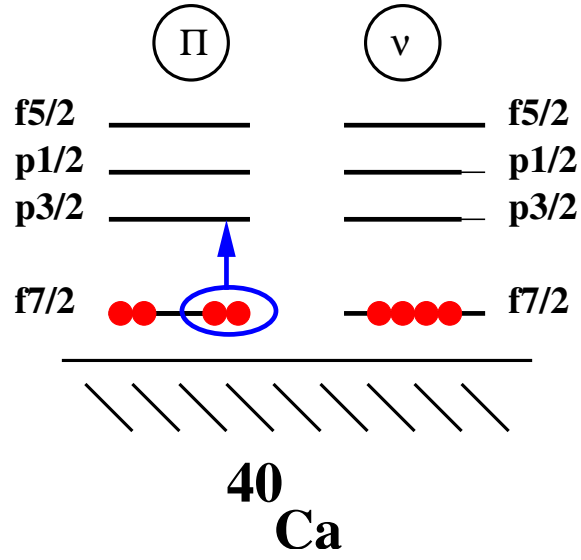
```
     RANDOM STARTING VECTOR
3
     -6.345165   11.335118   29.120687
6
    -21.344259   -7.802025    4.637278   16.927858   29.308309
9
    -30.092574  -19.653950   -9.343311    0.467972   10.265731
12
    -32.722076  -24.462806  -17.104890   -9.353111   -1.628857
15
    -32.930624  -26.709841  -22.335011  -15.957805   -9.401645
18
    -32.952147  -28.028244  -24.233122  -19.625844  -14.772679
21
    -32.953570  -28.413699  -25.350732  -22.676041  -18.180356
24
    -32.953655  -28.537584  -26.244093  -23.883982  -20.534298
27
    -32.953658  -28.559930  -26.542899  -24.362551  -22.197866
30
    -32.953658  -28.563001  -26.646165  -24.887184  -23.559799
33
    -32.953658  -28.564277  -26.912739  -26.199181  -24.299165
36
    -32.953658  -28.564535  -27.102898  -26.382496  -24.409357
39
    -32.953658  -28.564567  -27.148522  -26.416873  -24.529055
42
    -32.953658  -28.564570  -27.156735  -26.425250  -24.724078
45
    -32.953658  -28.564570  -27.158085  -26.427319  -24.910915
48
    -32.953658  -28.564570  -27.158371  -26.428021  -25.107898
```

Lanczos convergence

^{48}Cr

Dim (t=2) = 6×10^5

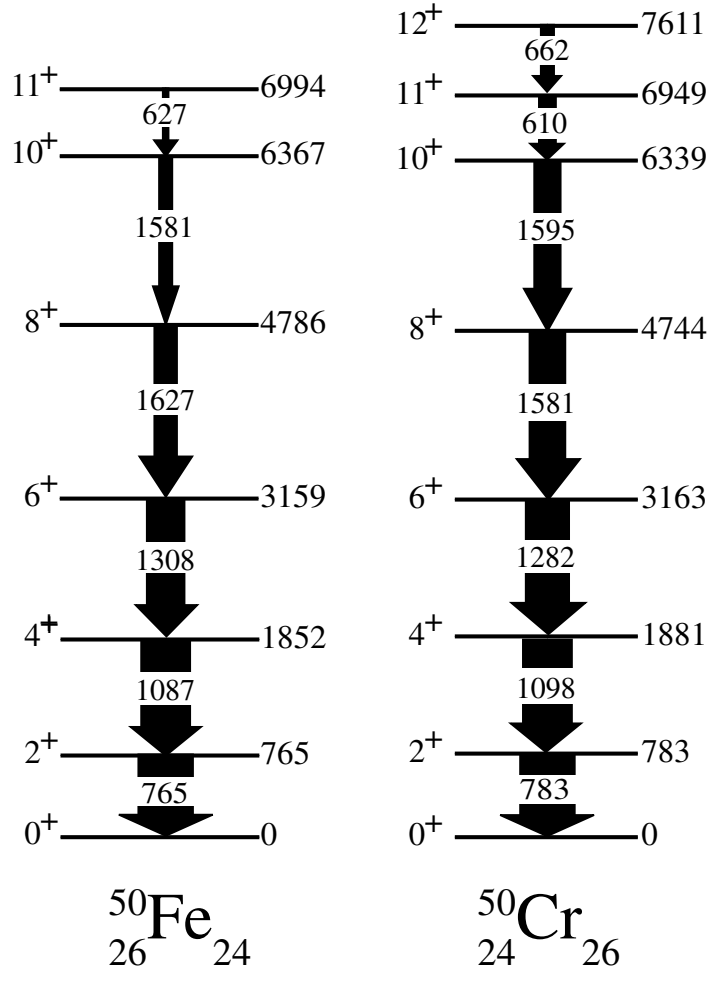
Dim (full space) = 2×10^6




```
          STARTING VECTOR :EIGENVECTOR OF A SMALLER SPACE
 ITER=  1     DIA=   -31.105920     NONDIA=     4.642871
3
   -32.578285  -21.260843    5.090417
6
   -32.929531  -27.208522  -16.116780   -1.200061   14.816894
9
   -32.952149  -28.024347  -22.702052  -13.782511   -3.514506
12
   -32.953553  -28.345536  -25.965169  -20.636169  -12.806719
15
   -32.953655  -28.528301  -26.951521  -22.532438  -18.004439
```

Isospin symmetry

Nuclear spectra are almost unchanged under the exchange of neutrons and protons.



Isospin representation

Neutron and proton are considered different states of the same particle: the nucleon

$$|n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |p\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Isospin operators:

$$\boldsymbol{t} = \frac{1}{2}\boldsymbol{\tau}, \boldsymbol{t}_{\pm} = \boldsymbol{t}_1 \pm \boldsymbol{t}_2$$

$$\boldsymbol{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\tau}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

States are eigenstates of total isospin ($\boldsymbol{T} = \sum_{i=1}^{A} \boldsymbol{t}^i$)

$$\boldsymbol{T}^2 = T(T+1)|T, T_z\rangle, \qquad T_z|T, T_z\rangle$$

$$T \geq T_z = \frac{|N-Z|}{2}$$

Realistic interactions

	Exp.	KB	KLS	Bonn A	Bonn B	Bonn C
		2_1^+ excitation energy				
^{44}Ca	1.16	1.45	1.43	1.31	1.25	1.26
^{46}Ca	1.35	1.45	1.42	1.26	1.22	1.23
^{48}Ca	3.83	1.80	1.60	1.23	1.30	1.41
^{50}Ca	1.03	1.41	1.35	1.27	1.10	1.17
$\langle (f_{7/2})^8 \vert \Psi_{GS} \rangle$		0.468	0.381	0.214	0.345	0.437

Realistic interactions

	Exp.	KB	KLS	Bonn A	Bonn B	Bonn C
		2_1^+ excitation energy				
^{44}Ca	1.16	1.45	1.43	1.31	1.25	1.26
^{46}Ca	1.35	1.45	1.42	1.26	1.22	1.23
^{48}Ca	3.83	1.80	1.60	1.23	1.30	1.41
^{50}Ca	1.03	1.41	1.35	1.27	1.10	1.17
$\langle (f_{7/2})^8 \vert \Psi_{GS} \rangle$		0.468	0.381	0.214	0.345	0.437
		^{56}Ni model space $(f_{\frac{7}{2}} p_{\frac{3}{2}})^{16}$				
^{56}Ni	2.70	0.39	0.31	0.43	0.42	0.42
$\langle (f_{\frac{7}{2}})^{16} \vert \Psi_{GS} \rangle$		0.04	0.015	0.018	0.011	0.019
$\langle n_{p_{3/2}} \rangle$		4.5	5.2	5.7	5.2	5.0

Realistic interactions

	Exp.	KB	KLS	Bonn A	Bonn B	Bonn C
		2_1^+ excitation energy				
^{44}Ca	1.16	1.45	1.43	1.31	1.25	1.26
^{46}Ca	1.35	1.45	1.42	1.26	1.22	1.23
^{48}Ca	3.83	1.80	1.60	1.23	1.30	1.41
^{50}Ca	1.03	1.41	1.35	1.27	1.10	1.17
$\langle (f_{7/2})^8 \vert \Psi_{GS} \rangle$		0.468	0.381	0.214	0.345	0.437
		^{56}Ni model space $(f_{\frac{7}{2}} p_{\frac{3}{2}})^{16}$				
^{56}Ni	2.70	0.39	0.31	0.43	0.42	0.42
$\langle (f_{\frac{7}{2}})^{16} \vert \Psi_{GS} \rangle$		0.04	0.015	0.018	0.011	0.019
$\langle n_{p_{3/2}} \rangle$		4.5	5.2	5.7	5.2	5.0

The realistic interactions do not reproduce the shell closure N or Z=28

Realistic interactions

	Exp.	KB	KLS	Bonn A	Bonn B	Bonn C
		2_1^+ excitation energy				
^{44}Ca	1.16	1.45	1.43	1.31	1.25	1.26
^{46}Ca	1.35	1.45	1.42	1.26	1.22	1.23
^{48}Ca	3.83	1.80	1.60	1.23	1.30	1.41
^{50}Ca	1.03	1.41	1.35	1.27	1.10	1.17
$\langle (f_{7/2})^8 \vert \Psi_{GS} \rangle$		0.468	0.381	0.214	0.345	0.437
		^{56}Ni model space $(f_{\frac{7}{2}} p_{\frac{3}{2}})^{16}$				
^{56}Ni	2.70	0.39	0.31	0.43	0.42	0.42
$\langle (f_{\frac{7}{2}})^{16} \vert \Psi_{GS} \rangle$		0.04	0.015	0.018	0.011	0.019
$\langle n_{p_{3/2}} \rangle$		4.5	5.2	5.7	5.2	5.0

The realistic interactions do not reproduce the shell closure N or Z=28

Why?

The structure of the Hamiltonian

From the work of M. Dufour and A. Zuker (PRC 54 1996 1641)
Separation theorem:
Any effective interaction can be split in two parts:

$$\boldsymbol{H} = \boldsymbol{H}_m(\text{monopole}) + \boldsymbol{H}_M(\text{multipole})$$

$\boldsymbol{H}_m$ contains all the terms that are affected by a spherical Hartree-Fock variatition, hence reponsible of the global saturation properties and of the evolution of the spherical single particle field.
Important property:

$$\langle CS \pm 1|\boldsymbol{H}|CS \pm 1\rangle = \langle CS \pm 1|\boldsymbol{H}_m|CS \pm 1\rangle$$

The structure of the Hamiltonian

For all the realistic G-matrices,

- $\boldsymbol{H}_m$ is not accurate enough.
- $\boldsymbol{H}_M$ is almost the same.

The monopole part has to be empirically corrected to reproduce the structure of the "simple" nuclei $|CS \pm 1\rangle$

Multipole Hamiltonian

Can be characterized as the sum of simple terms:

- $L = 0$ isovector and isoscalar pairing
- Elliot's quadrupole-quadrupole force.
- $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$
- Octupole and hexadecapole terms $r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$

All the interactions contains similar terms:

Interaction	$JT = 01$	$JT = 10$	$\lambda\tau = 20$	$\lambda\tau = 40$	$\lambda\tau = 11$
KB3	-4.75	-4.46	-2.79	-1.39	+2.46
FPD6	-5.06	-5.08	-3.11	-1.67	+3.17
GOGNY	-4.07	-5.74	-3.23	-1.77	+2.46

The Monopole Hamiltonian

H_m contains terms that depend in n and T. Assume that we have a constant potential and single particle energy:

$$E = n\varepsilon + \frac{n(n-1)}{2}V$$

The Monopole Hamiltonian

If we consider also isospin:

$$E = n\varepsilon + \frac{n(n-1)}{2}V + bT(T+1)$$

The Monopole Hamiltonian

In general we have different orbits each with different average interactions:

$$E = \sum_i n_i \varepsilon_i + \sum_i \frac{n_i(n_i-1)}{2} V_{ii} + \sum_{i \neq j} n_i n_j V_{ij}$$

The Monopole Hamiltonian

If we consider also the fact that we have isospin:

$$\boldsymbol{H}_m = \sum_i \varepsilon_i \boldsymbol{n}_i + \sum_{ij} \left[\frac{1}{1+\delta_{ij}} a_{ij} \boldsymbol{n}_i (\boldsymbol{n}_j - \delta_{ij}) + \frac{1}{1+\delta_{ij}} b_{ij} \left(\boldsymbol{T}_i \cdot \boldsymbol{T}_j - \frac{3\boldsymbol{n}_i}{4} \delta_{ij} \right) \right)$$

Where a and b are defined from the centroids (average interaction):

$$V_{ij}^T = \frac{\sum_J (2J+1) W_{ijij}^{JT}}{\sum_J (2J+1)}$$

$$a_{ij} = \frac{3V_{ij}^1 + V_{ij}^0}{4}, \qquad b_{ij} = V_{ij}^1 - V_{ij}^0$$

Influence of monopole hamiltonian

The evolution of effective spherical single particle energies with the number of particles in the valence space can be extracted from $\boldsymbol{H}_m$. In the case of identical particles the expresion is:

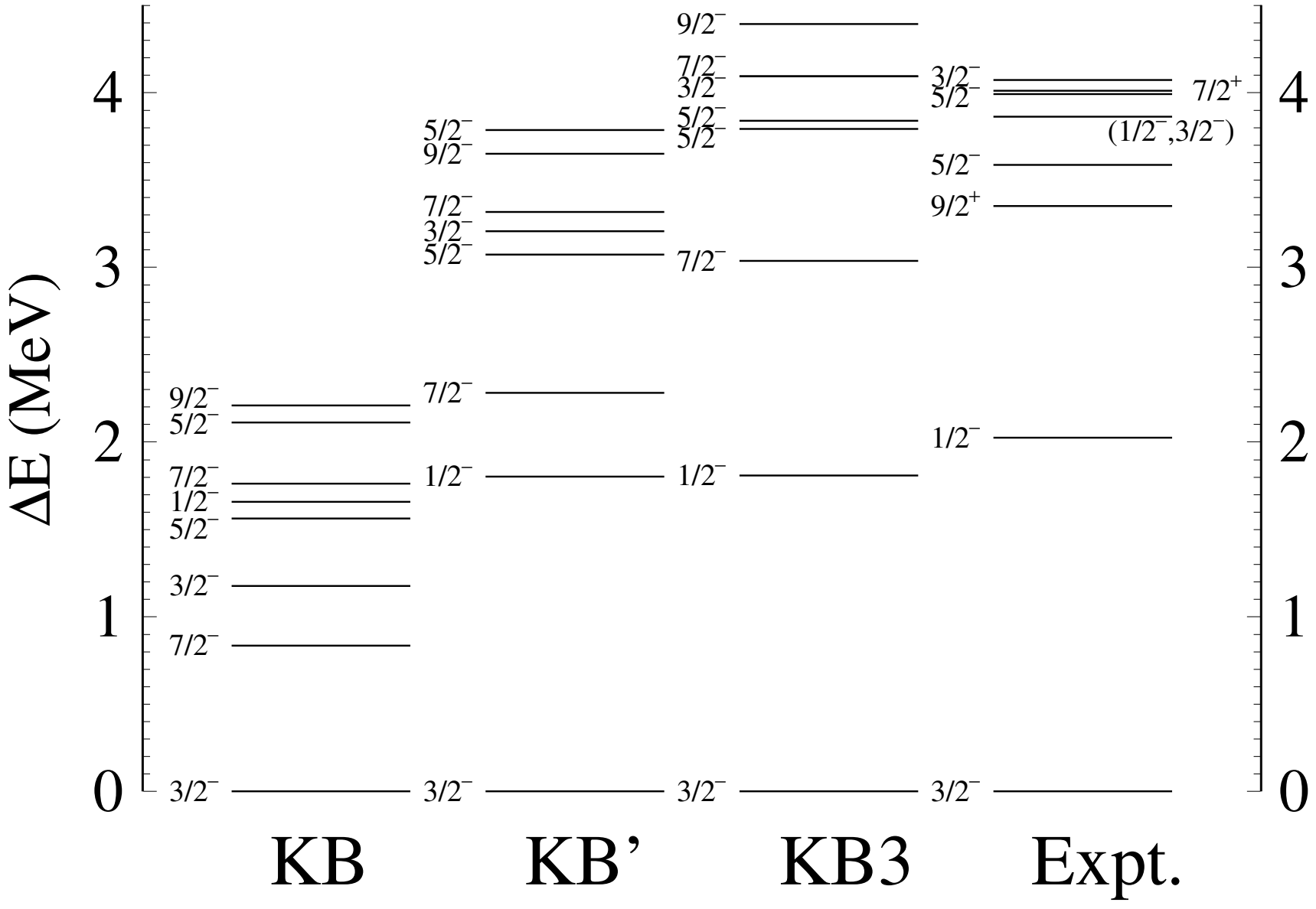
$$\varepsilon_j(n) = \varepsilon_j(n=1) \; + \sum_i V_{ij} n_i$$

The monopole hamiltonian $\boldsymbol{H}_m$ also governs the relative position of the various T-values in the same nucleus, via the terms:

$$b_{ij} \boldsymbol{T}_i \cdot \boldsymbol{T}_j$$

Even small defects in the centroids can produce large changes in the relative position of the different configurations due to the appearance of quadratic terms involving the number of particles in the different orbits

Effect of monopole corrections



Evolution SPE

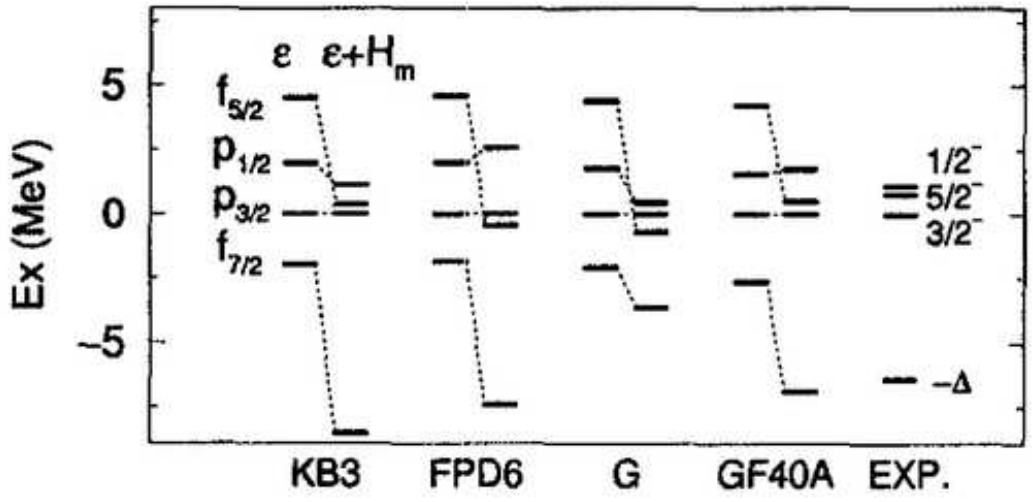
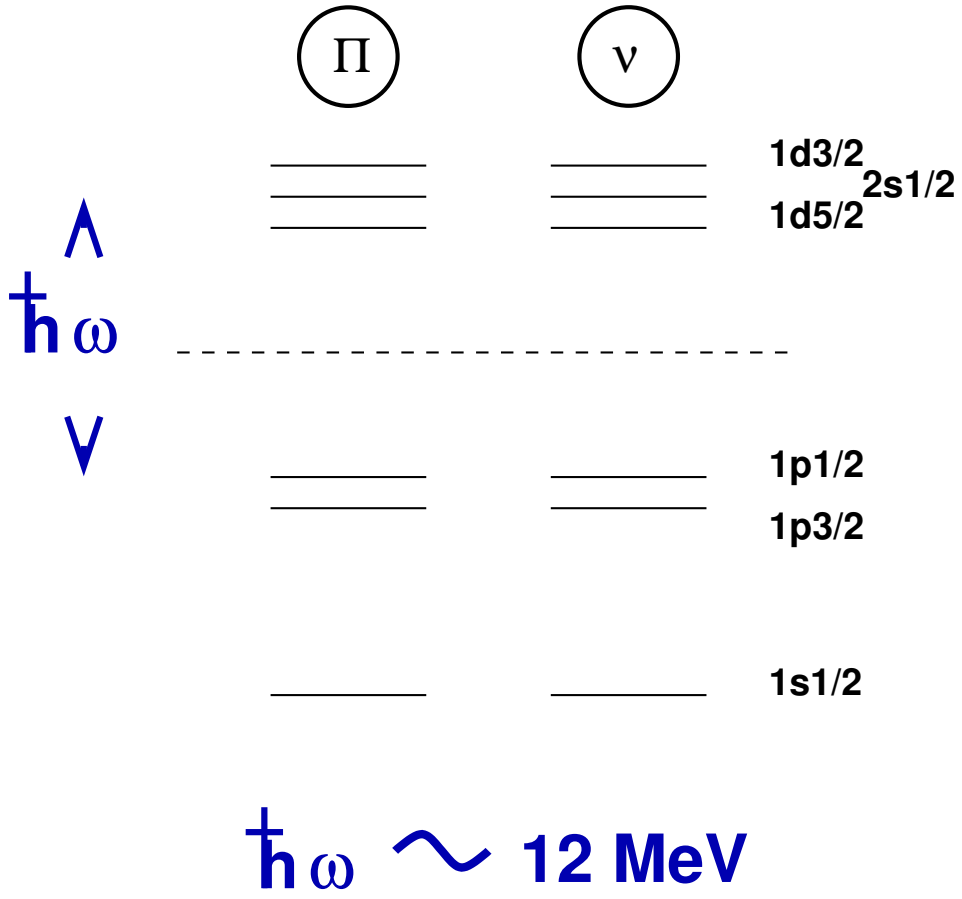
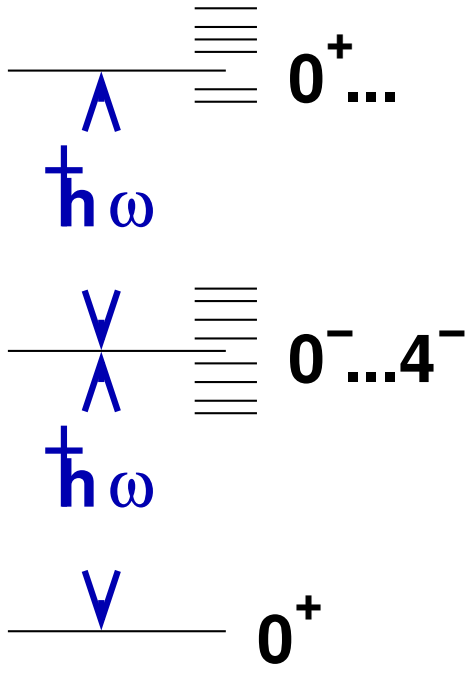


Figure 3. Comparison of bare single-particle energies (ε) and effective single-particle energies ($\varepsilon + H_m$) of ^{56}Ni. Far right column shows experimental energy levels of ^{57}Ni and the shell-gap evaluated by using the binding energies: $\Delta = 2BE(^{56}\mathrm{Ni}) - BE(^{57}\mathrm{Ni}) - BE(^{55}\mathrm{Ni})$. Data are taken from ref. [16]

Limits of IPM

Example ^{16}O

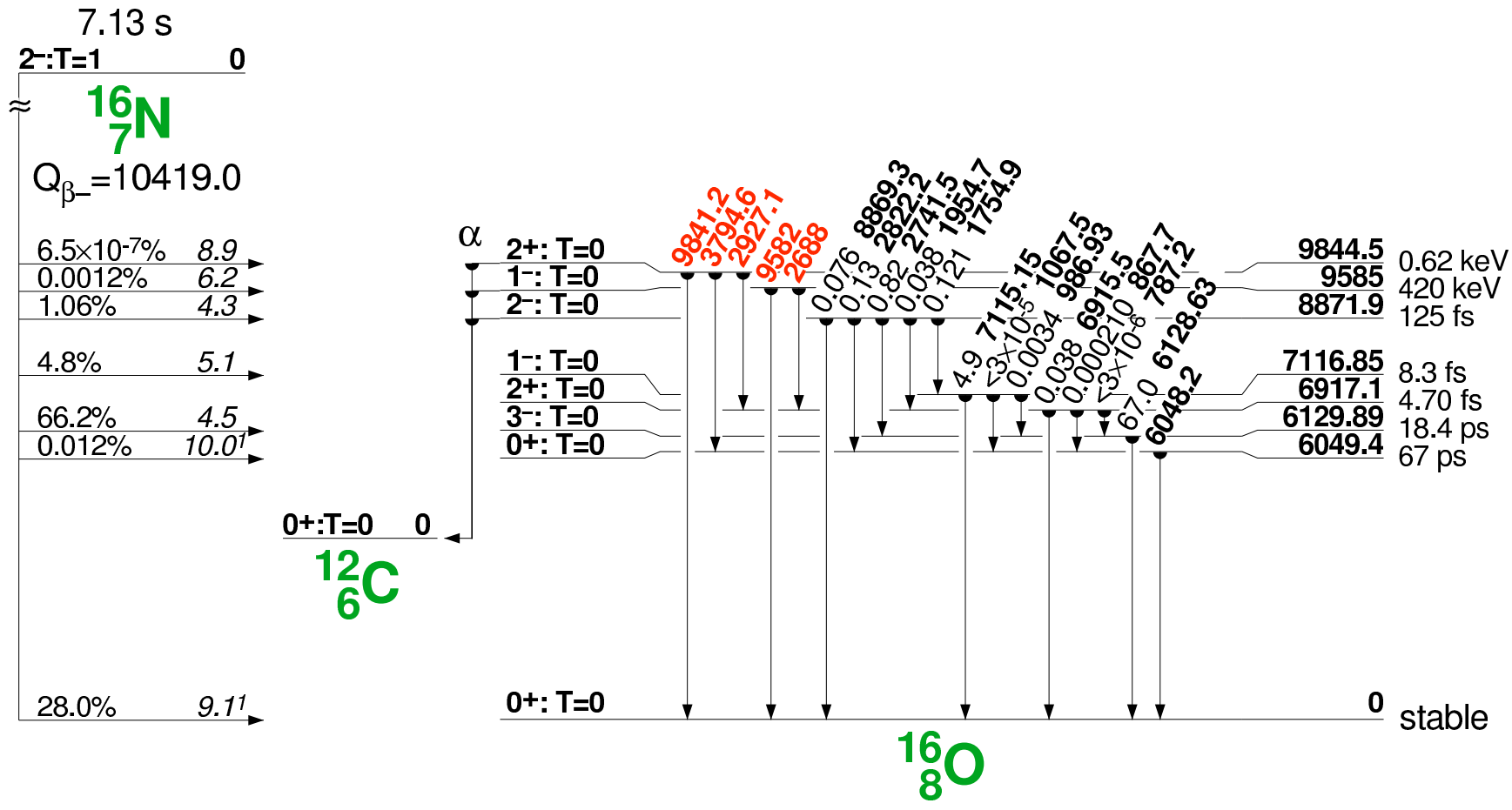




$\hbar\omega \sim 12$ MeV

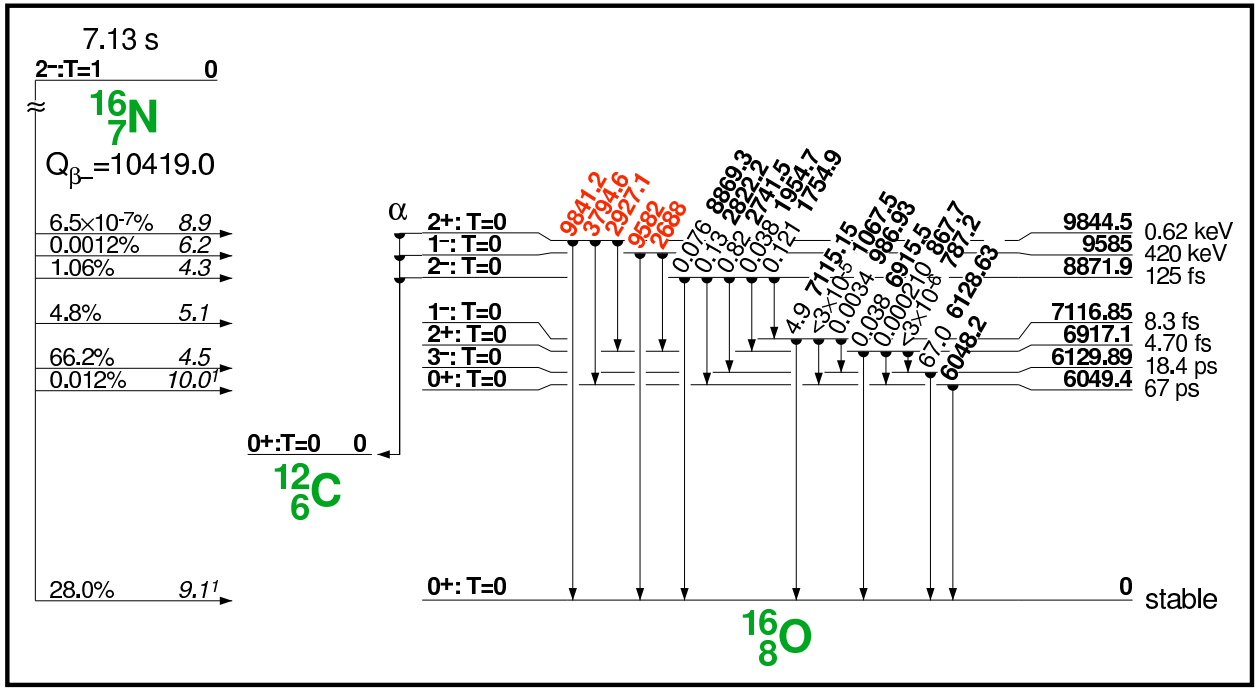
Limits of IPM

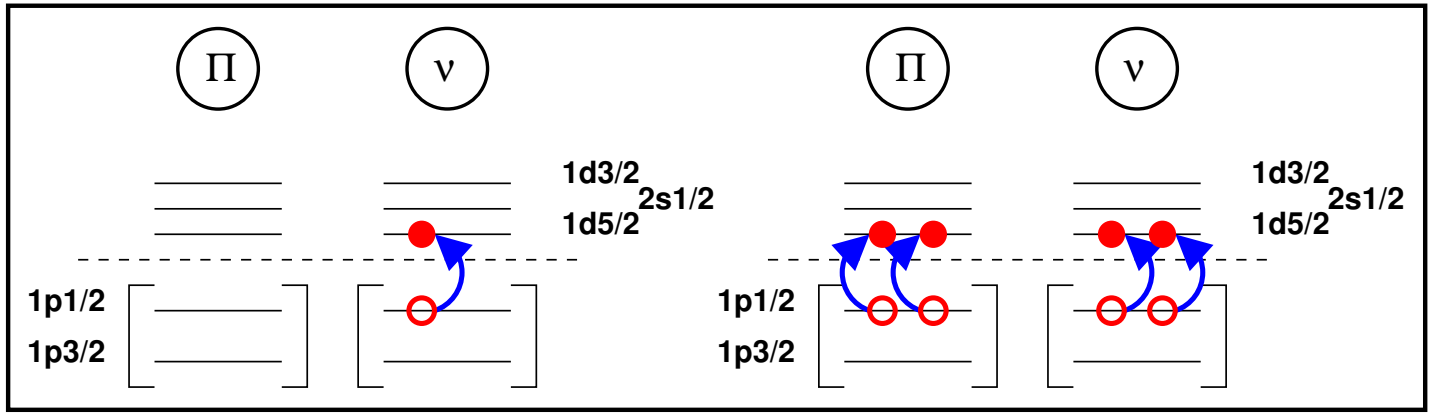
Example ^{16}O



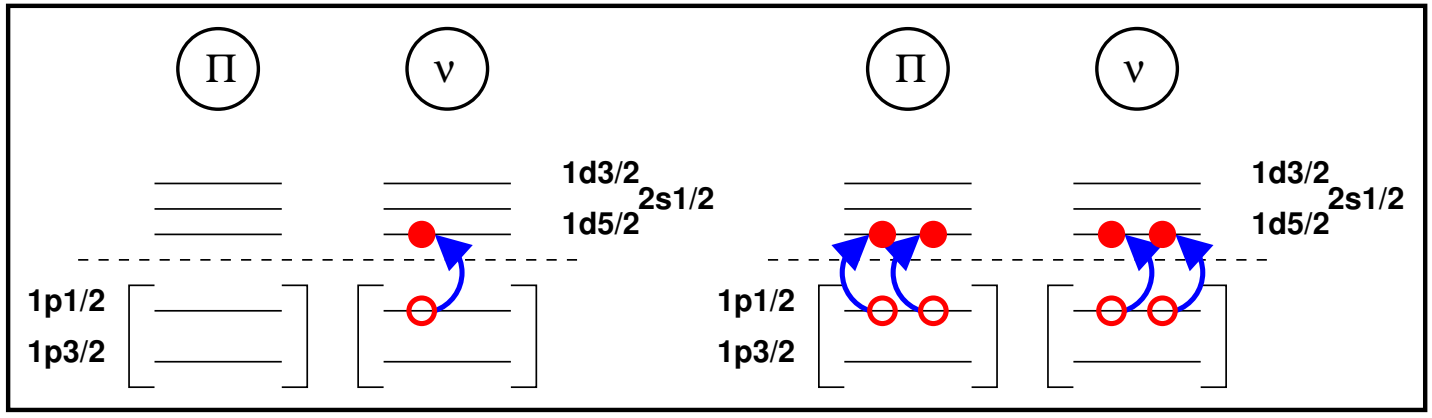
Limits of IPM

Example ^{16}O



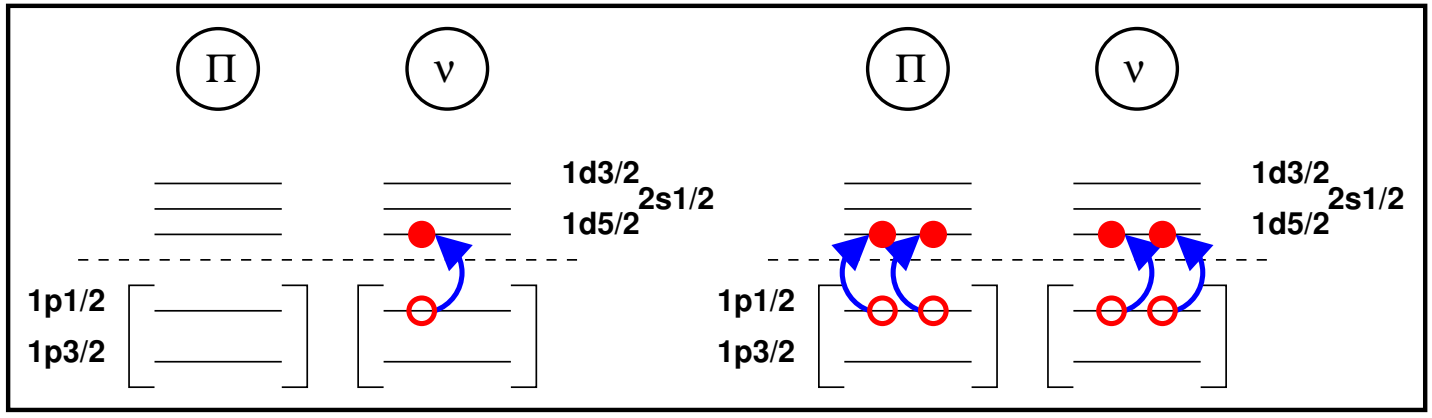


Monopole explanation



$$
\begin{aligned}
E_{0\mathrm{ph}} &= 12\varepsilon_p + \frac{12\cdot 11}{2} V_{pp} \\
E_{1\mathrm{ph}} &= 11\varepsilon_p + \varepsilon_r + 55 V_{pp} + 11 V_{pr} \\
E_{2\mathrm{ph}} &= 10\varepsilon_p + 2\varepsilon_r + 45 V_{pp} + 20 V_{pr} + V_{rr} \\
E_{4\mathrm{ph}} &= 8\varepsilon_p + 4\varepsilon_r + 28 V_{pp} + 32 V_{pr} + 6 V_{rr}
\end{aligned}
$$

Monopole explanation



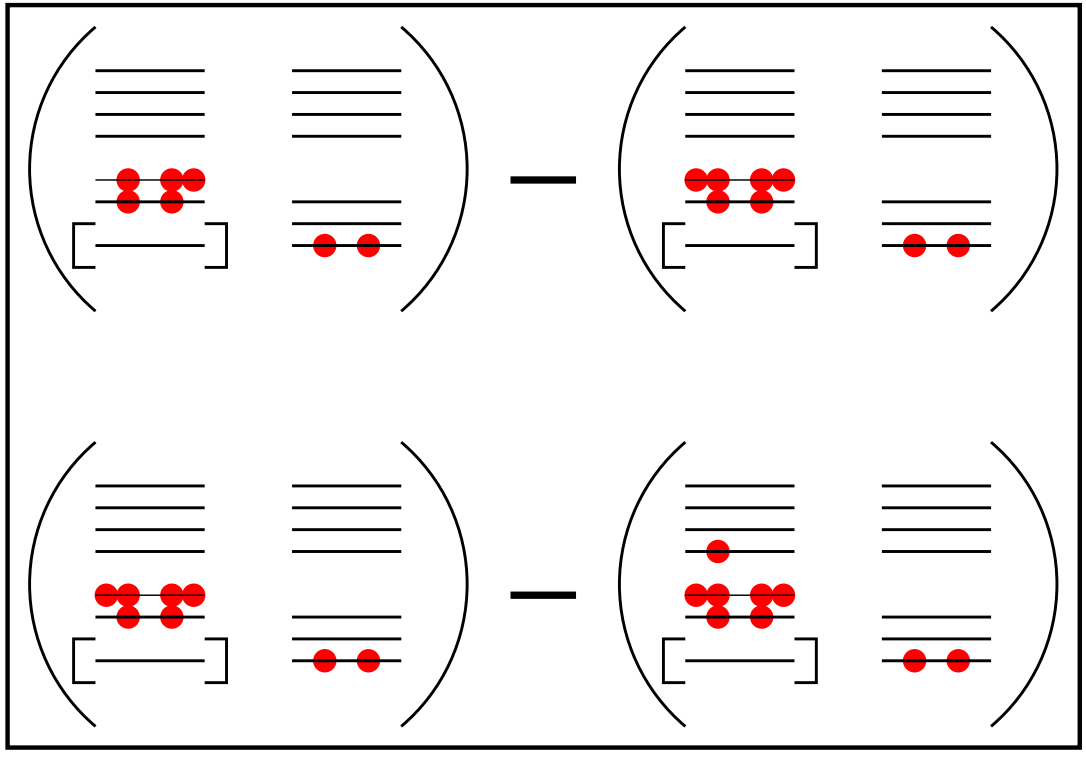
$$\Delta_{1\mathrm{ph}} = \varepsilon_r - \varepsilon_p + 11(V_{pr} - V_{pp})$$

$$\Delta_{2\mathrm{ph}} = 2(\varepsilon_r - \varepsilon_p) + V_{pp} + 20(V_{pr} - V_{pp}) + (V_{rr} - V_{pp})$$

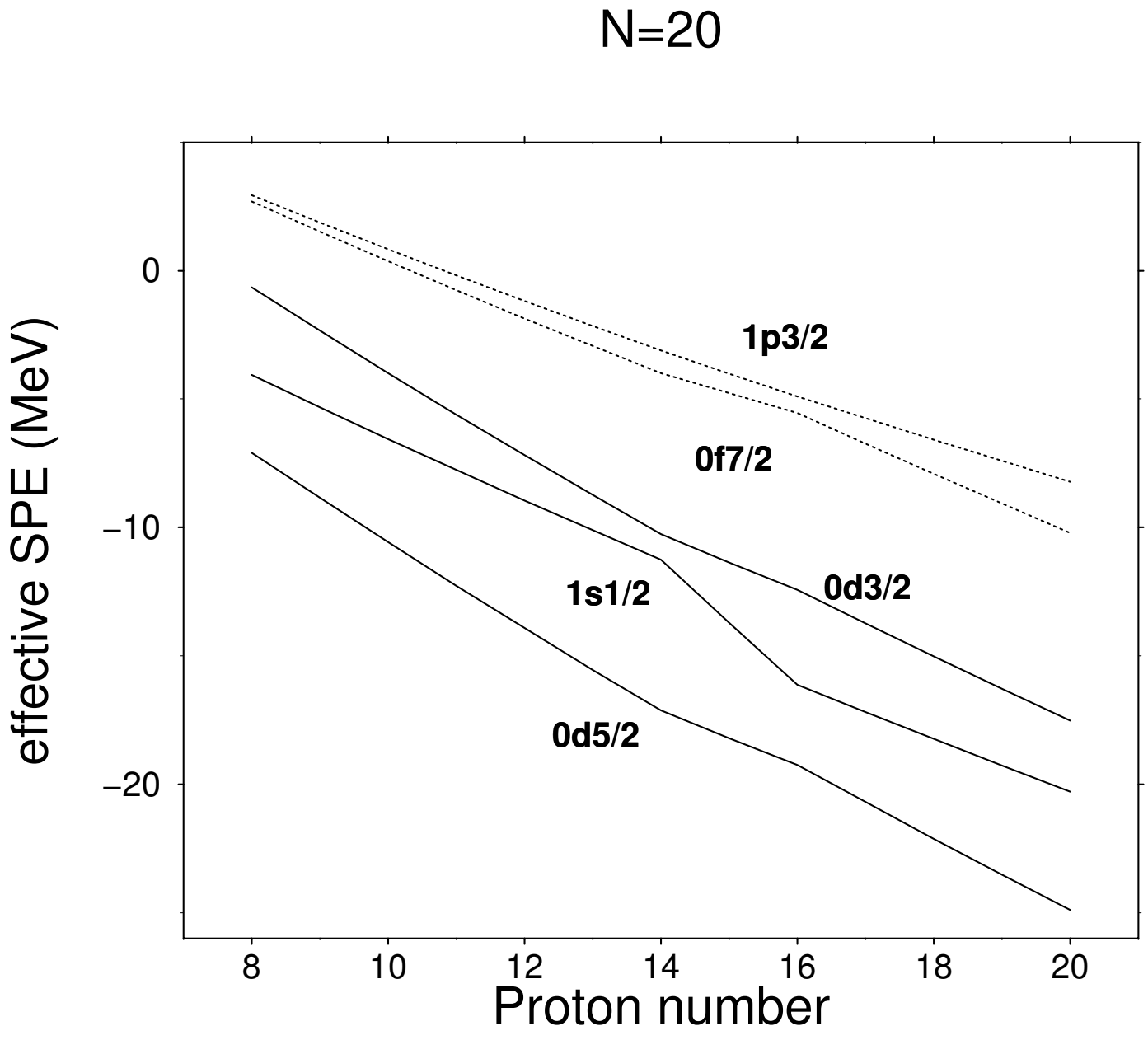
$$\Delta_{4\mathrm{ph}} = 4(\varepsilon_r - \varepsilon_p) + 32(V_{pr} - V_{pp}) + 6(V_{rr} - V_{pp})$$

Evolution SPE far off stability

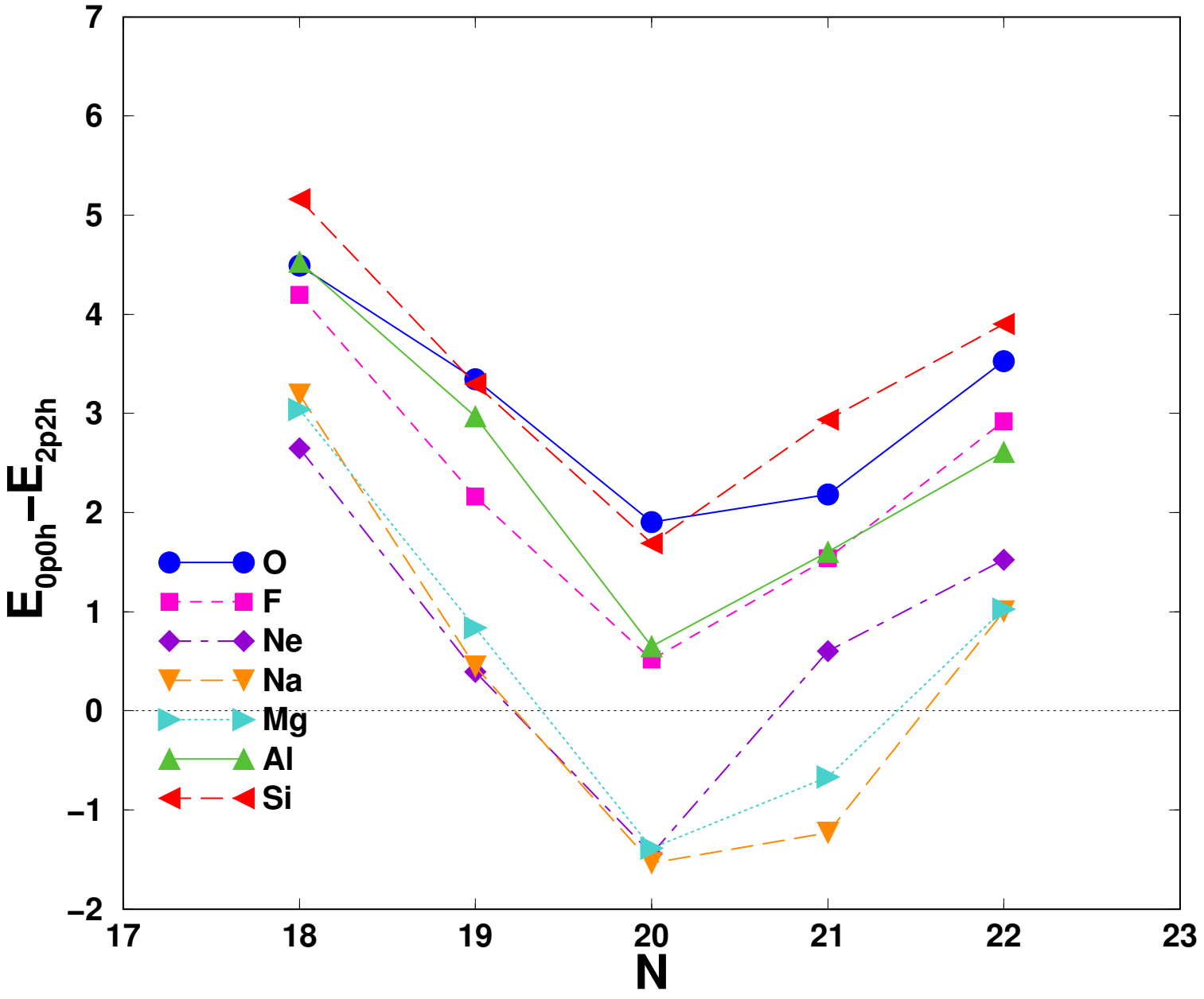
Given H_m, the effective single particle energies [from Otsuka *et al*, Phys. Rev. Lett. 87, 82502 (2001)],



Evolution SPE far off stability



Vanishing of shell closure at N=20



Computation of transition operators

Given a one-body transition operator $\mathcal{O}$, how do we compute

$$\langle \Psi_f | \mathcal{O} | \Psi_i \rangle$$

here Ψ_i and Ψ_f are many-body wave functions obtained from shell-model diagonalization

- One body operators:

$$\mathcal{O} = \sum_{i=1}^{A} o(r(i)) \longrightarrow \mathcal{O} = \sum_{i,j} \langle i | \boldsymbol{O} | j \rangle a_i^\dagger a_j$$

- We need to know
  - ➢ the value of our one body operator between single particle wave functions $\langle i | \boldsymbol{O} | j \rangle$
  - ➢ the one body density matrix elements $\langle \Psi_f | a_i^\dagger a_j | \Psi_i \rangle$

Computation of transition operators

- for $\langle i|\boldsymbol{O}|j\rangle$, one needs (eventually) to know the radial part of the wave function: usually harmonic oscillator, sometimes wood-saxon.

$$\langle i|\boldsymbol{O}|j\rangle = \int d^3r\ \phi_i^*(\boldsymbol{r})\boldsymbol{O}(\boldsymbol{r})\phi_j(\boldsymbol{r})$$

- for the one body density matrix elements (same procedure as for the hamiltonian):

$$a_5^\dagger a_2|001011\rangle = |011001\rangle$$

now we know the procedure to compute:

- EL transitions: $r^L Y_{L0}$
- β decay :
  - ➢ Fermi decay : $\tau_\pm$
  - ➢ Gamow-Teller decay: $\sigma\tau_\pm$

Example calculation

β decay half-life calculation

- Determine initial state $|\Psi_i\rangle$.
- Determine all posible final states $|\Psi_f\rangle$.
- Compute matrix elements $\langle\Psi_f|\boldsymbol{O}|\Psi_i\rangle$

$$\lambda_f = \frac{\ln 2}{K} f(Z, W_0^f)[B_f(F) + B_f(GT)]$$

- Determine total decay rate:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \sum_f \lambda_f$$

Lanczos Strength Functions

- $|\Psi\rangle$ ground state given nucleus.
- Sum rule state (or doorway): $|\Omega\rangle = \mathbf{\Omega}|\Psi\rangle$.
- Total sum rule (sum over all final states of the matrix element squared) is the norm of state $|\Omega\rangle$:

$$\langle\Omega|\Omega\rangle = \langle\Psi|\mathbf{\Omega}^\dagger\mathbf{\Omega}|\Psi\rangle = \sum_i |\langle i|\mathbf{\Omega}|\Psi\rangle|^2$$

- We can think of the state $|\Omega\rangle$ as a (probability) distribution over the eigenvalues $|i\rangle$ of $\boldsymbol{H}$ with values $|\langle i|\Omega\rangle|^2 = |\langle i|\mathbf{\Omega}|\Psi\rangle|^2$

Lanczos Strength Functions

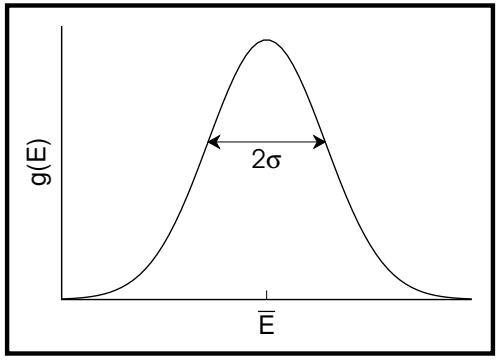
Any distribution can be characterized by the moments of the distribution.

$$\bar{E} = \langle\Omega|\boldsymbol{H}|\Omega\rangle = \sum_i E_i |\langle i|\boldsymbol{\Omega}|\Psi\rangle|^2$$

$$m_n = \langle\Omega|(\boldsymbol{H}-\bar{E})^n|\Omega\rangle = \sum_i (E_i-\bar{E})^n |\langle i|\boldsymbol{\Omega}|\Psi\rangle|^2$$

Gaussian distribution characterized by two moments ($\bar{E}, \sigma^2 = m_2$)

$g(E) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(E-\bar{E})^2}{2\sigma^2})$



Lanczos Strength Functions

In general we only need a finite number of momenta. We can define a basis of $|\alpha\rangle$ states.

$$m_n = \langle\Omega|(\boldsymbol{H}-\bar{E})^n|\Omega\rangle = \sum_{\alpha}^{N}(E_\alpha-\bar{E})^n|\langle\alpha|\boldsymbol{\Omega}|\Psi\rangle|^2 \quad (\forall n \leq M)$$

$$E_\alpha \approx \langle\alpha|\boldsymbol{H}|\alpha\rangle$$

With N states we can reproduce $2N$ moments of the distribution.

Lanczos Strength Functions

Example: Beta half-life

$$\lambda \sim \sum_i f(Z, W_0^i) |\langle i|\mathbf{\Omega}|\Psi\rangle|^2$$

$$f(Z, W_0^i) \approx \int_1^{W_0^i} W^2 (W_0^i - 1)^2 dW \approx \frac{(W_0^i)^5}{30}$$

Lanczos Strength Functions

Lanczos method provides a natural way of determining the basis $|\alpha\rangle$.

Initial vector $|\mathbf{1}\rangle = \frac{|\Omega\rangle}{\sqrt{\langle\Omega|\Omega\rangle}}$.

$$
\begin{aligned}
E_{12}|\mathbf{2}\rangle &= (\boldsymbol{H} - E_{11})|\mathbf{1}\rangle \\
E_{23}|\mathbf{3}\rangle &= (\boldsymbol{H} - E_{22})|\mathbf{2}\rangle - E_{12}|\mathbf{1}\rangle \\
&\cdots \\
E_{N\,N+1}|\mathbf{N+1}\rangle &= (\boldsymbol{H} - E_{N\,N})|\mathbf{N}\rangle \\
&\quad - E_{N-1\,N}|\mathbf{N-1}\rangle
\end{aligned}
$$

where

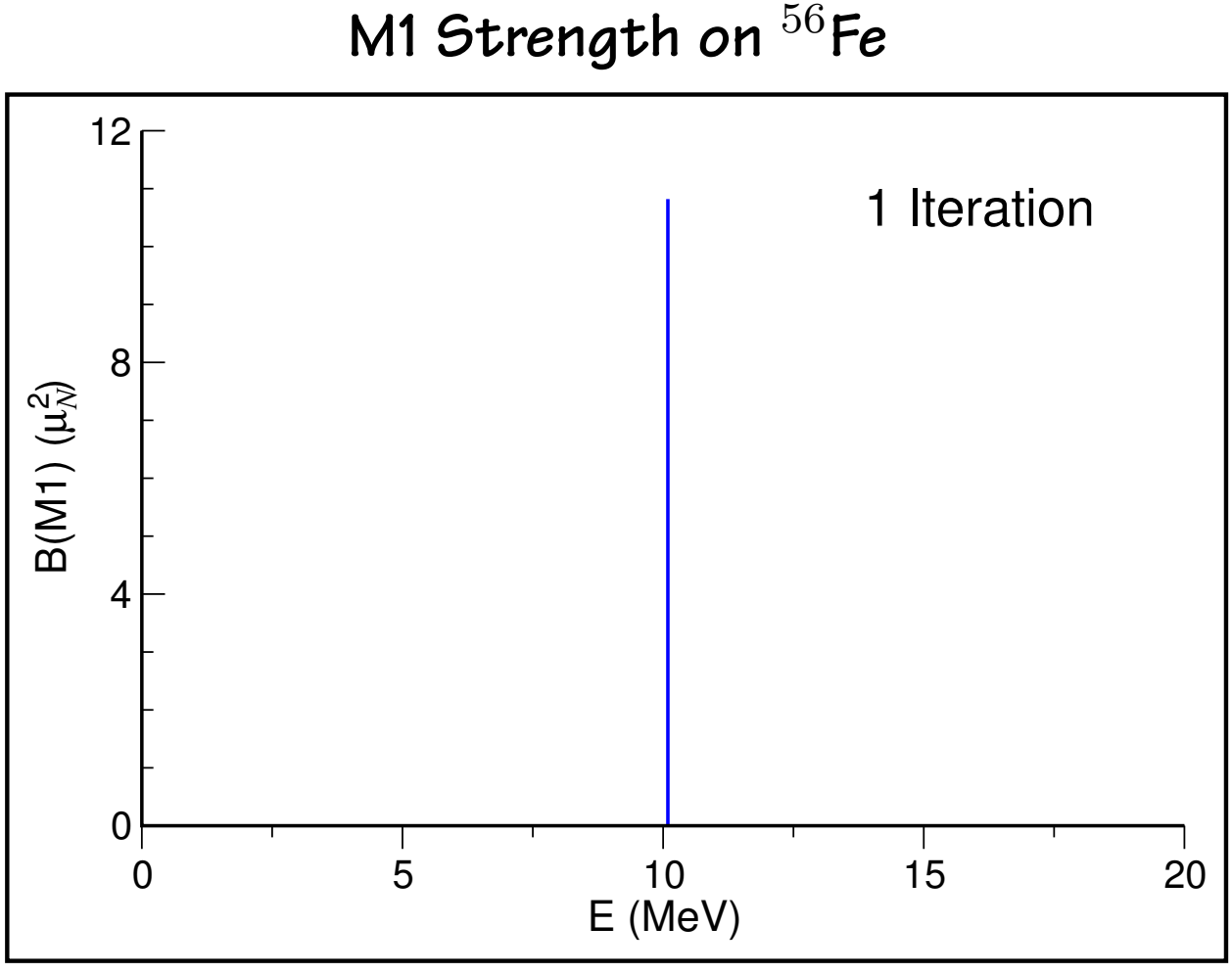
$$E_{N\,N} = \langle\mathbf{N}|\boldsymbol{H}|\mathbf{N}\rangle, \quad E_{N\,N+1} = E_{N+1\,N}$$

Each Lanczos iteration gives information about two new moments of the distribution.

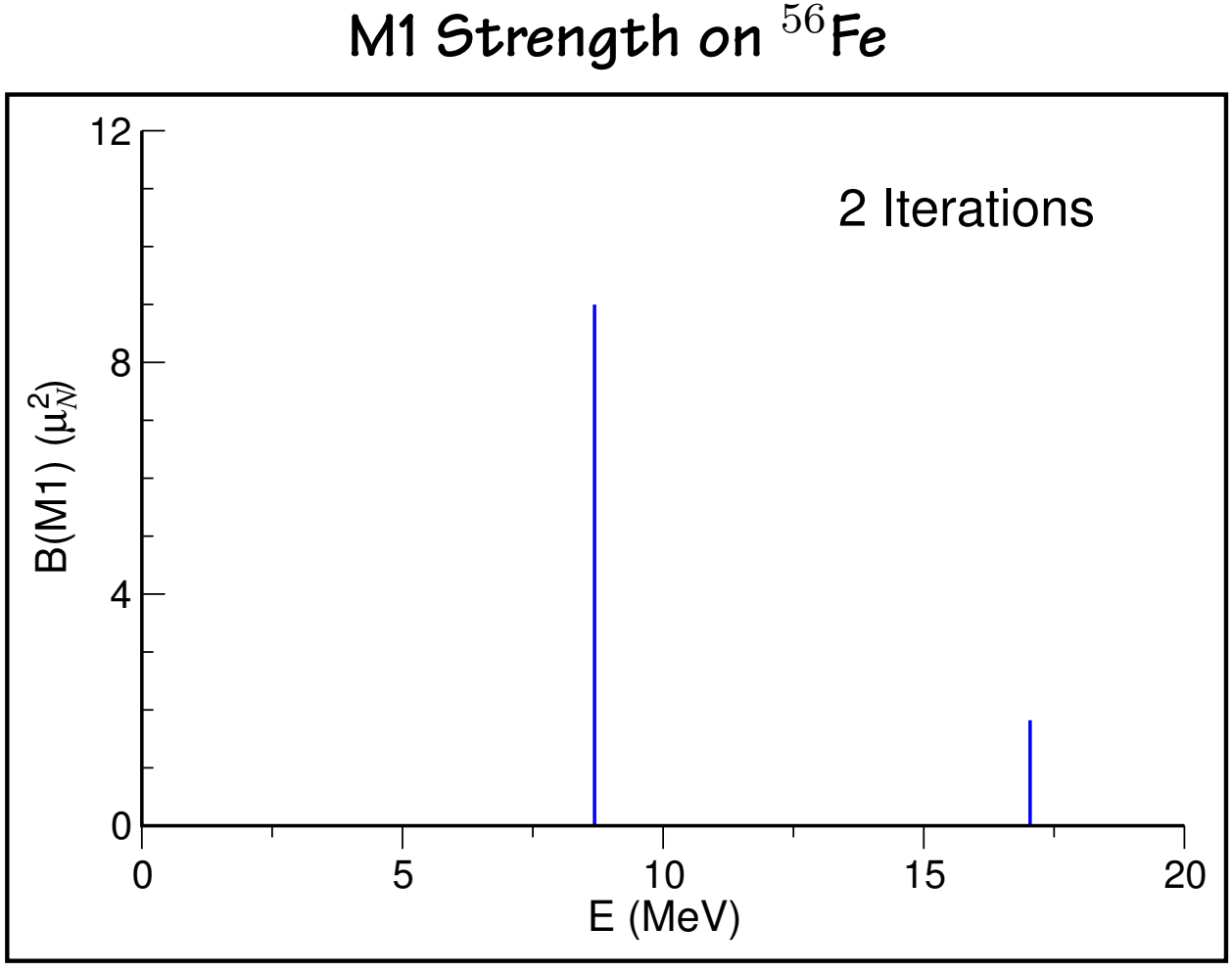
$$
\begin{aligned}
E_{11} &= \langle\mathbf{1}|\boldsymbol{H}|\mathbf{1}\rangle = \bar{E} \\
E_{12}^2 &= \langle\Omega|(\boldsymbol{H} - E_{11})^2|\Omega\rangle = m_2 \\
E_{22} &= \frac{m_3}{m_2} + \bar{E} \\
E_{23}^2 &= \frac{m_4}{m_2} - \frac{m_3^2}{m_2^2} - m_2
\end{aligned}
$$

Diagonalizing Lanczos matrix after N iterations gives an approximation to the distribution with the same lowest $2N$ moments.

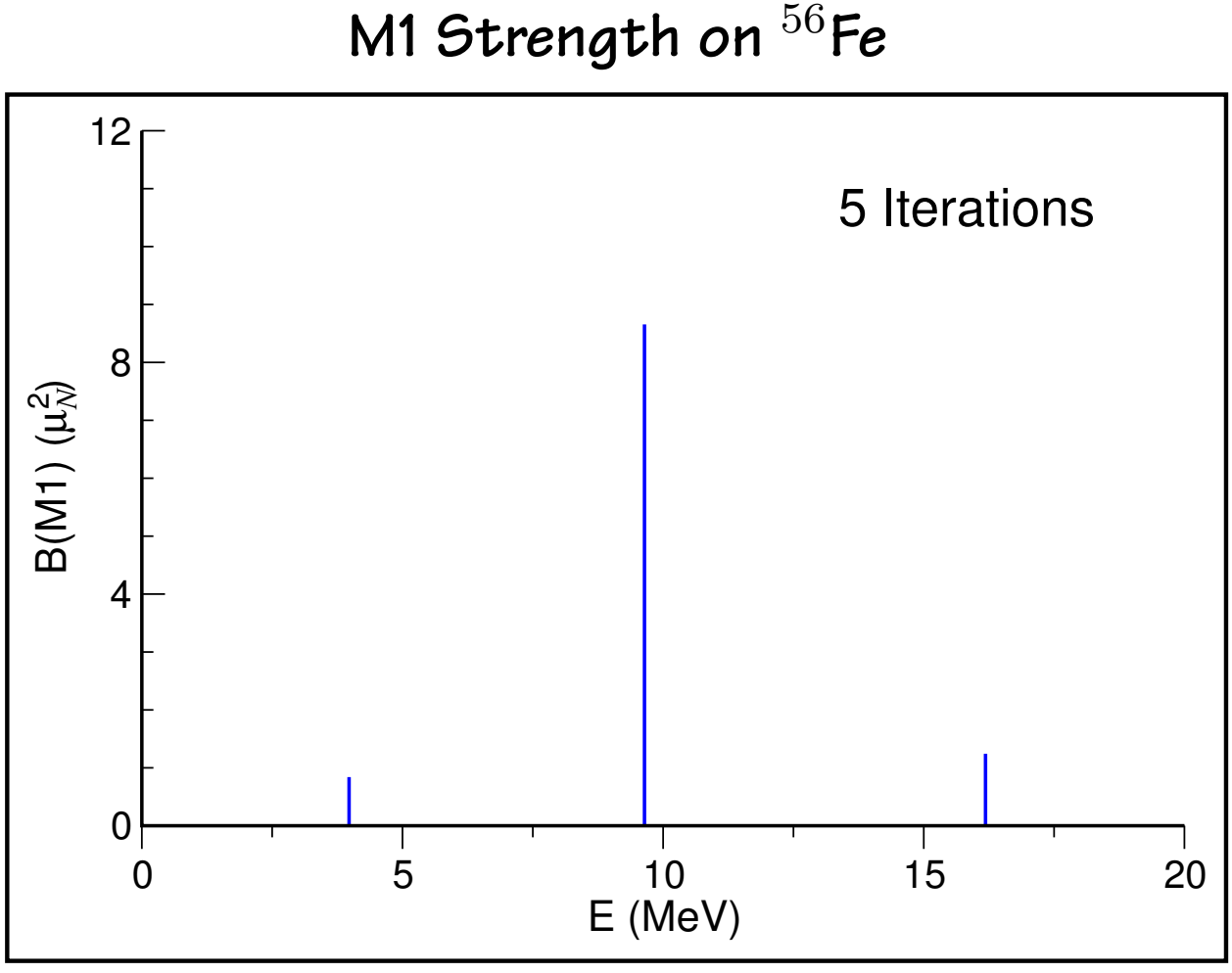
Evolution Strength Distribution



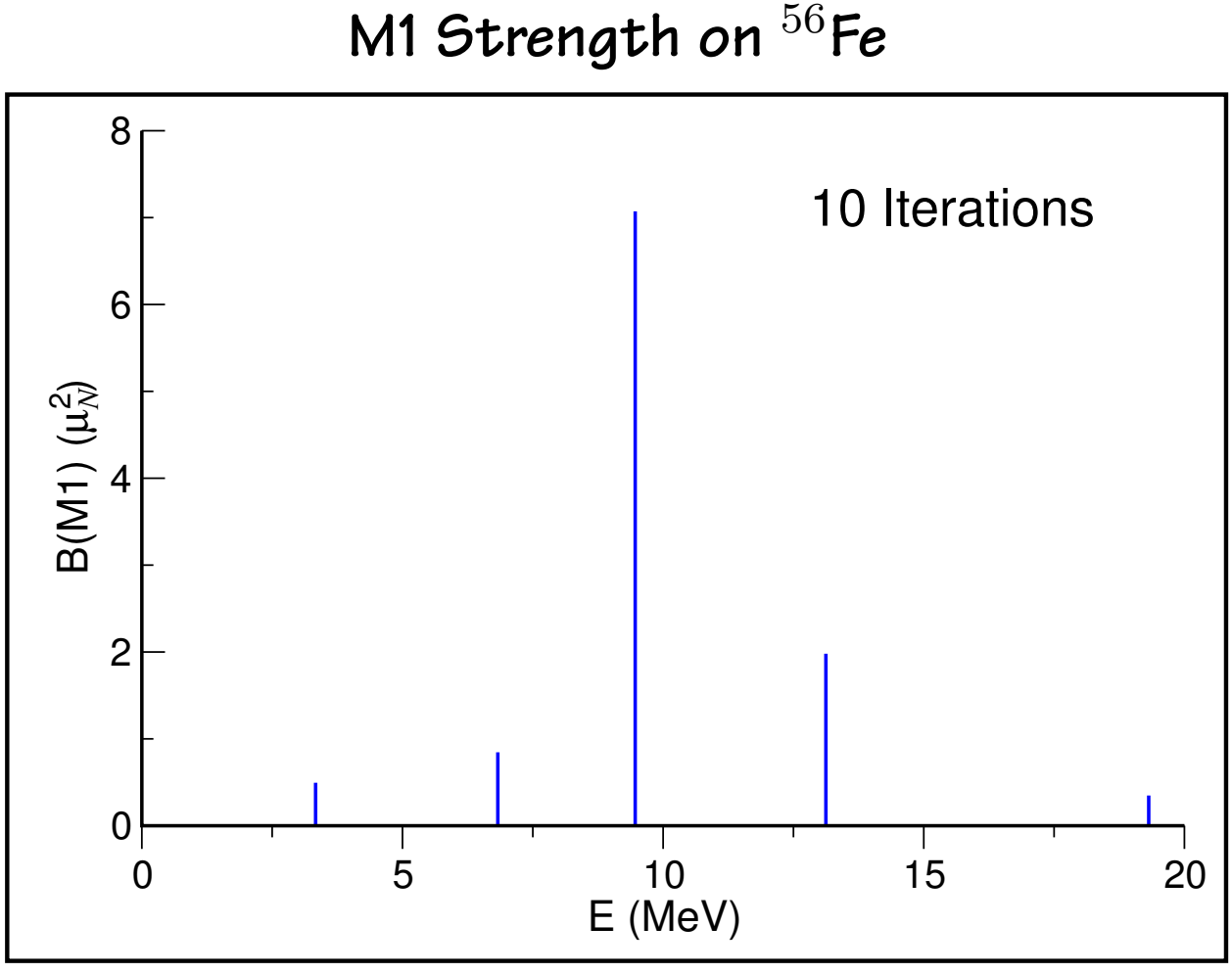
Evolution Strength Distribution



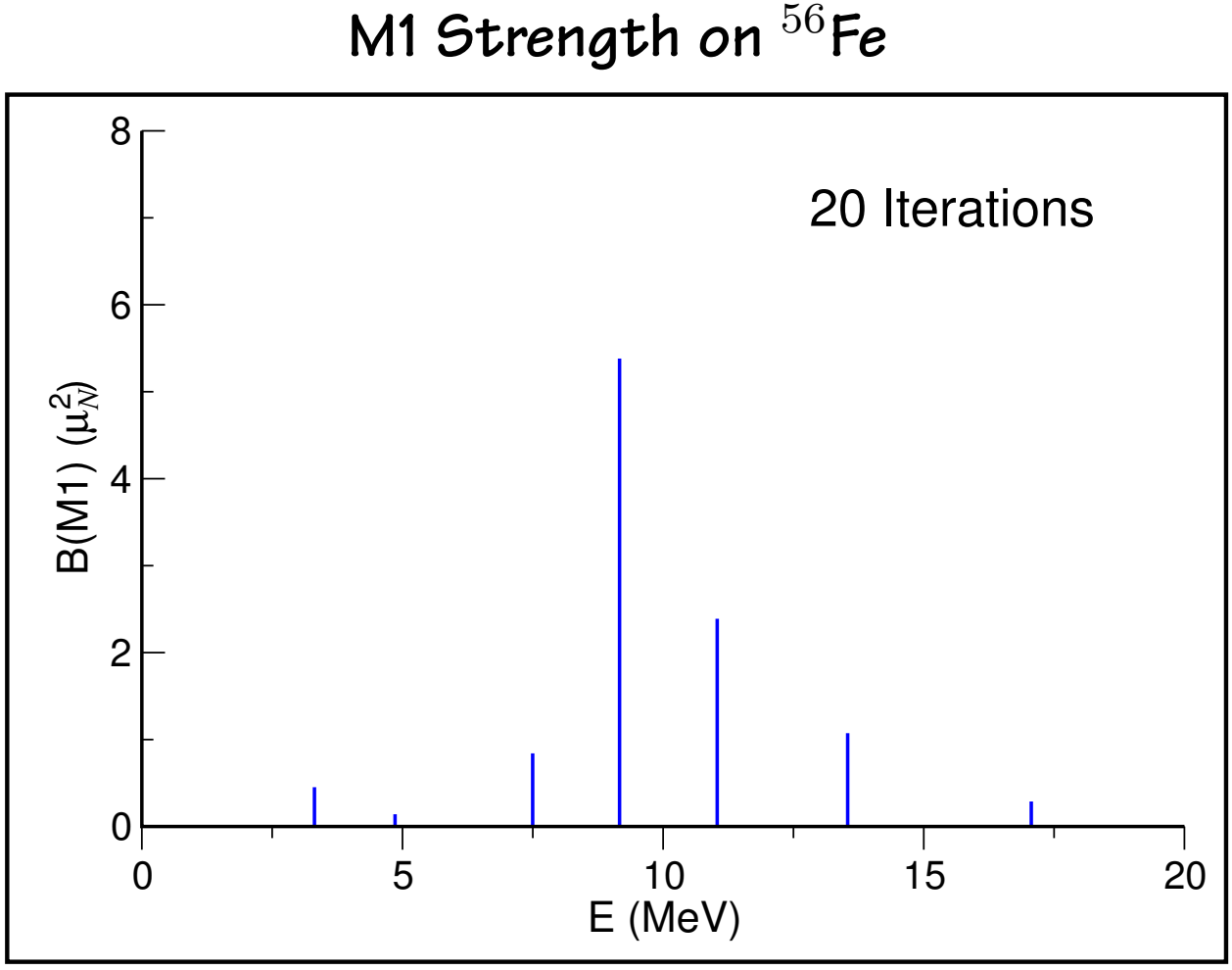
Evolution Strength Distribution



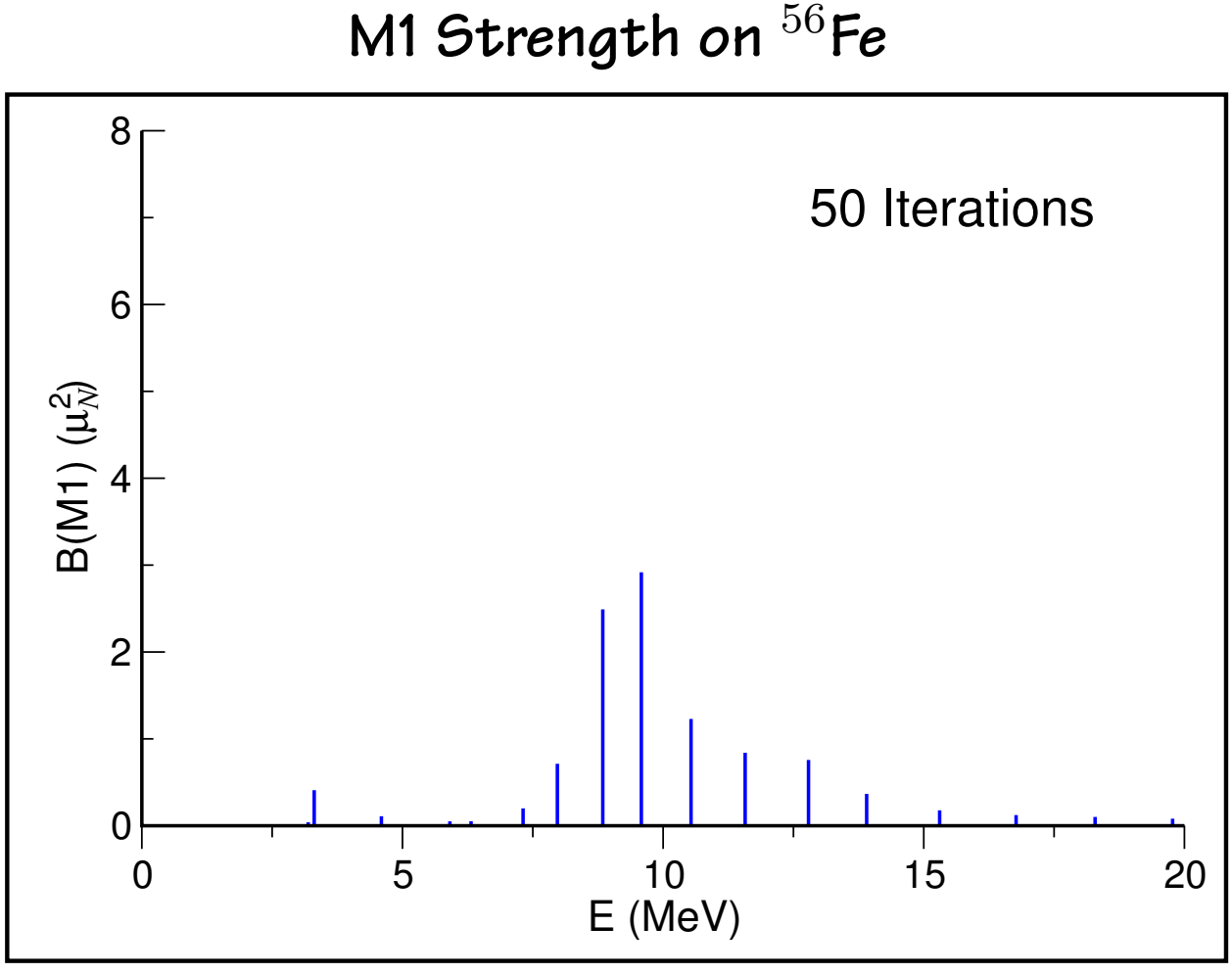
Evolution Strength Distribution



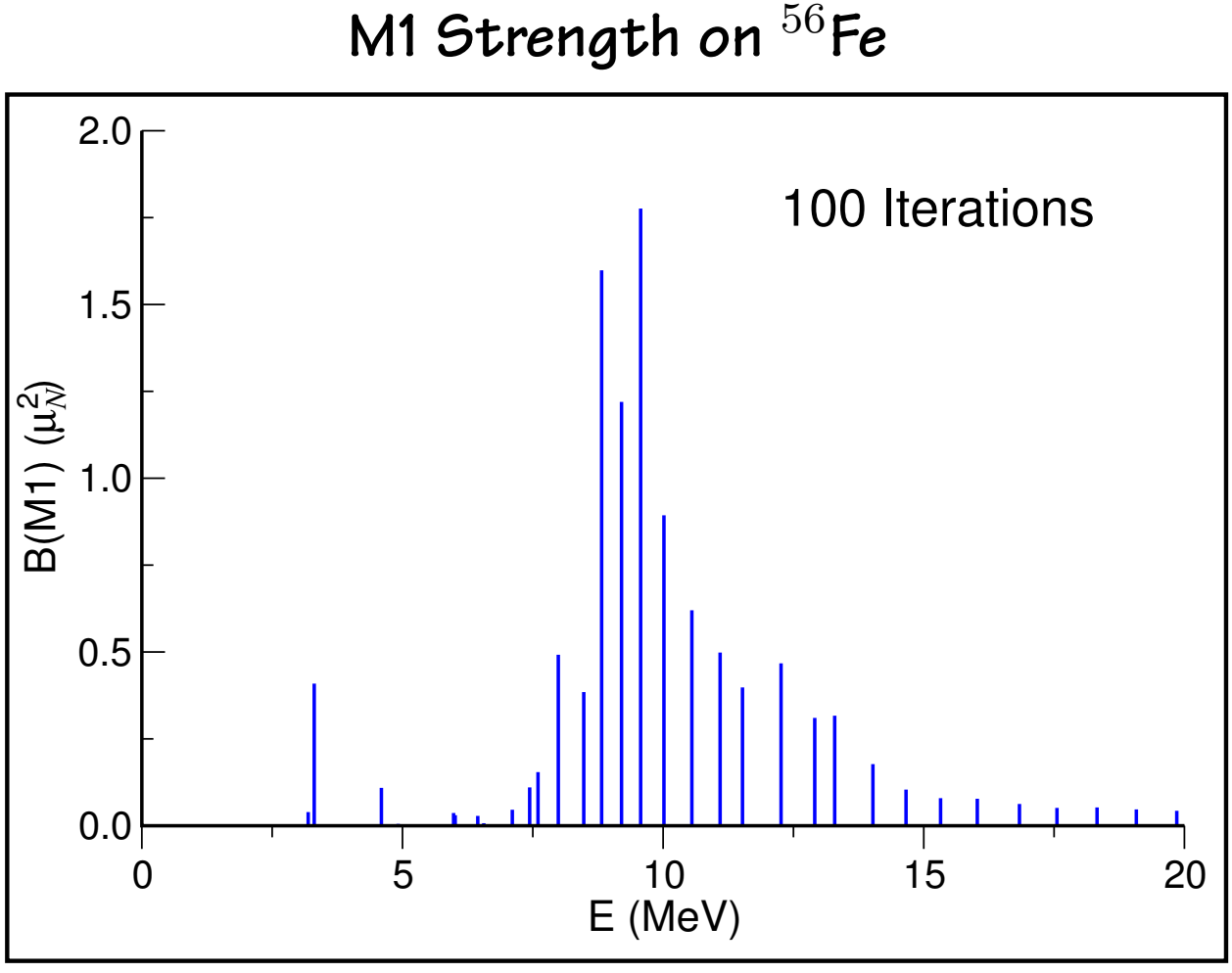
Evolution Strength Distribution



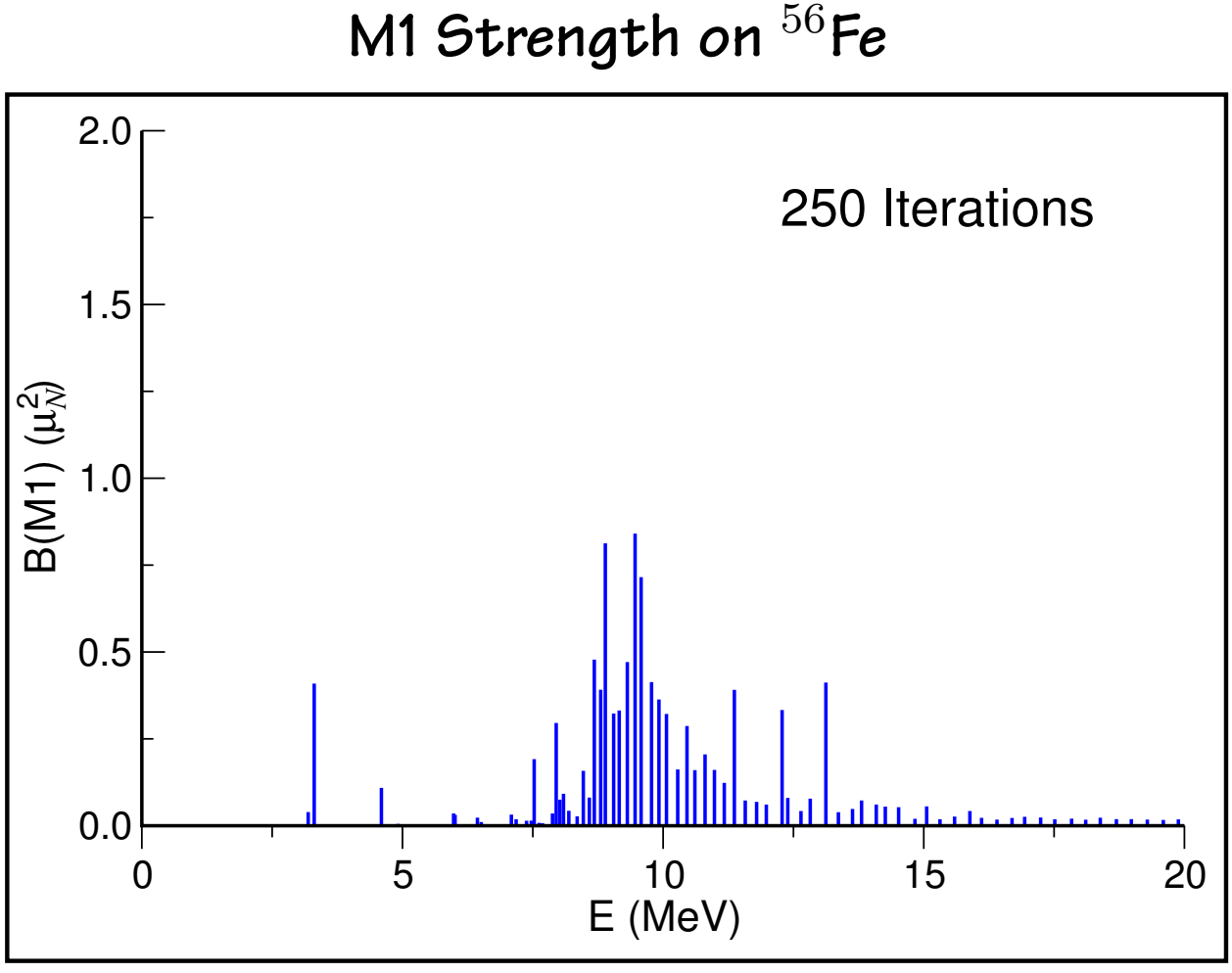
Evolution Strength Distribution



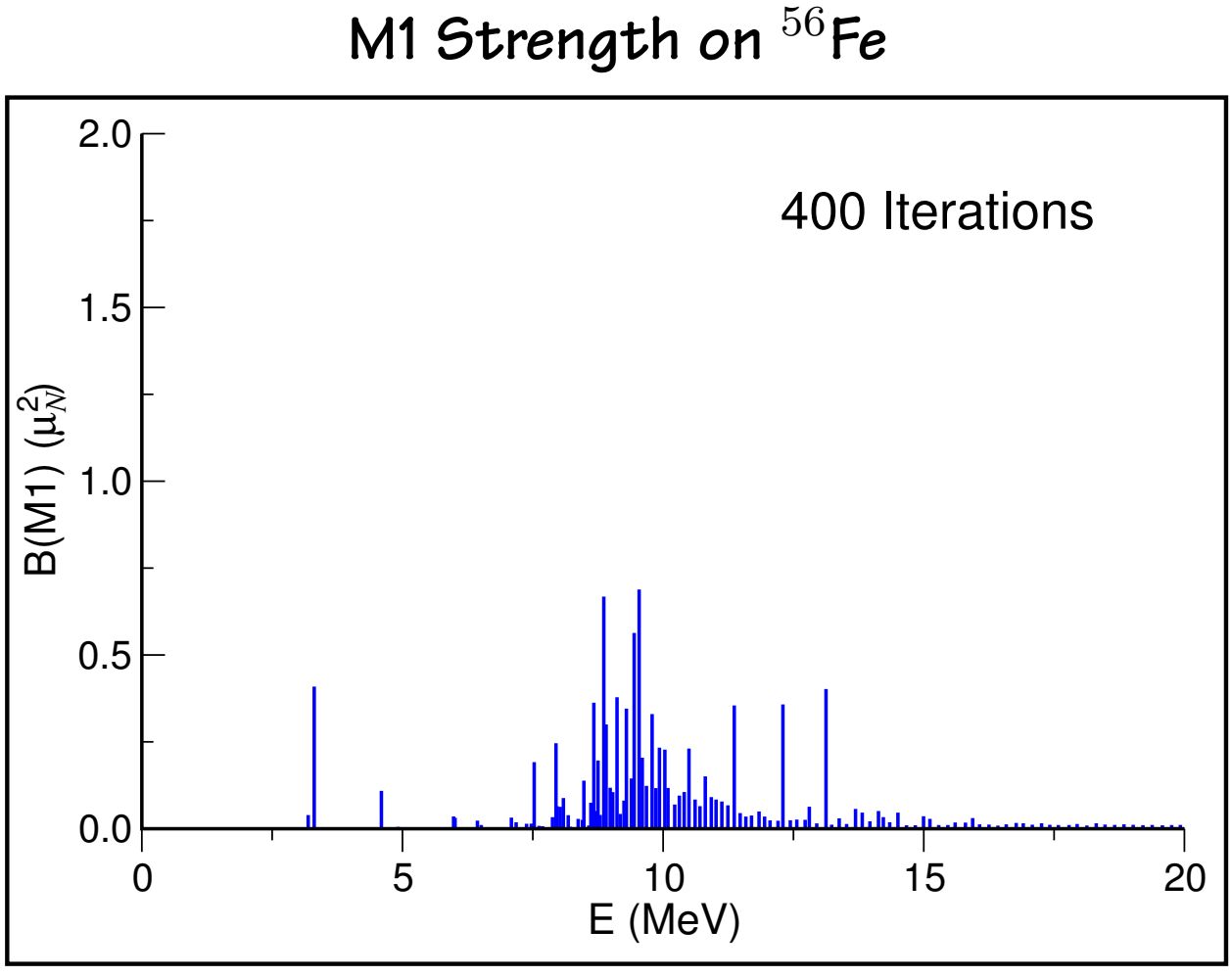
Evolution Strength Distribution



Evolution Strength Distribution

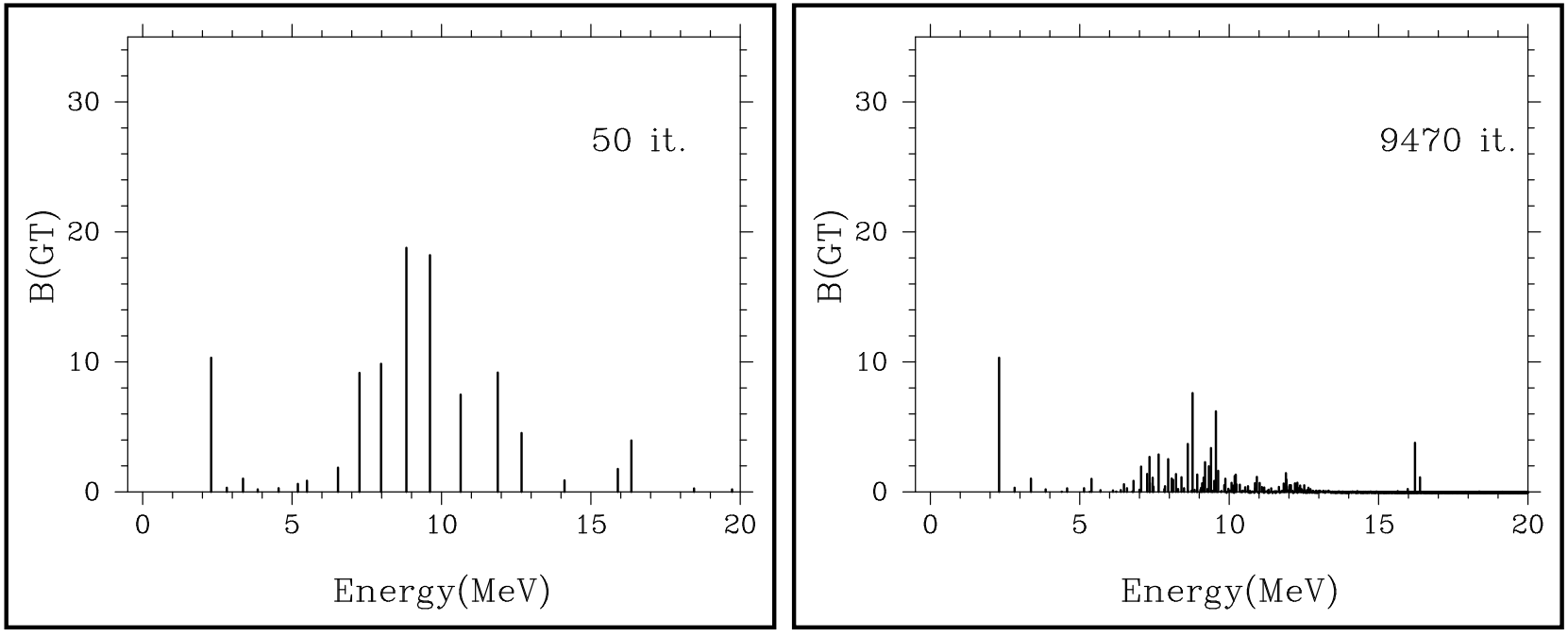


Evolution Strength Distribution



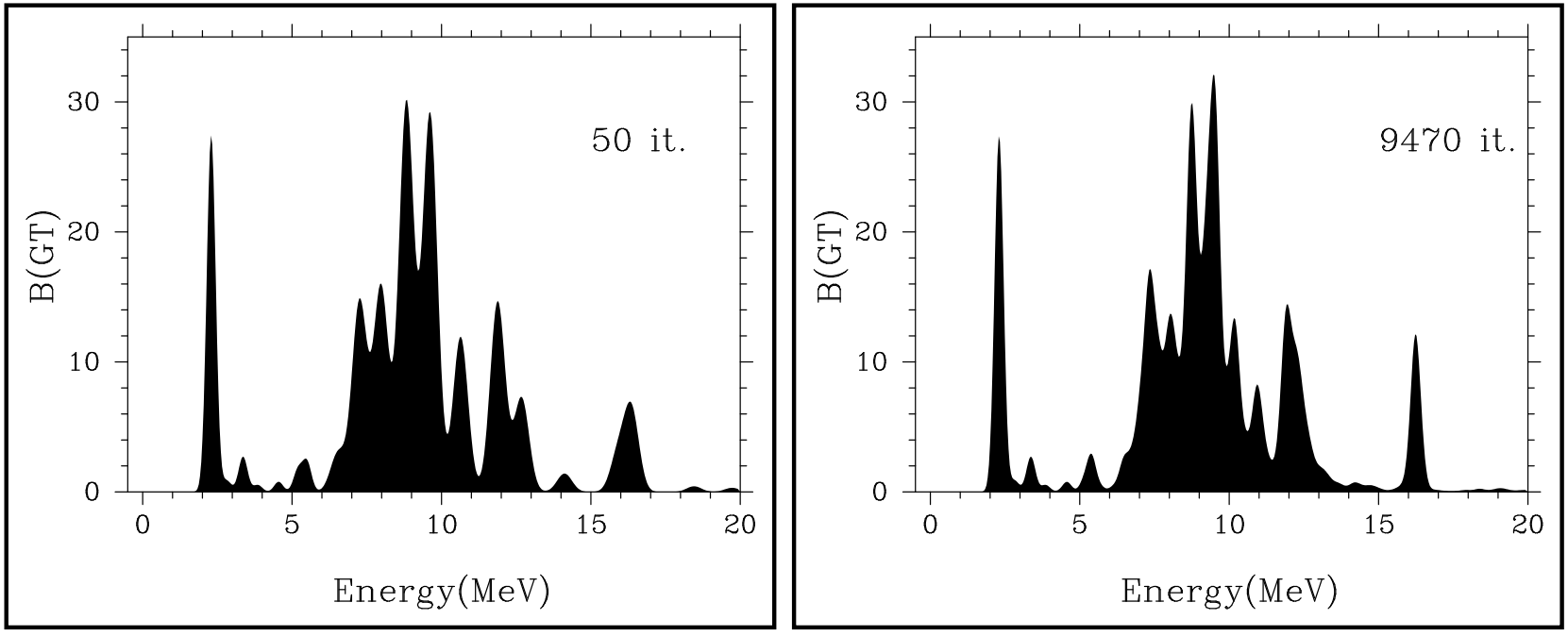
Evolution of Strength Distribution

GT Strength on ^{48}Sc



Evolution of Strength Distribution

GT Strength on ^{48}Sc



Fermi matrix elements

$$B(F) = \frac{1}{2J_i+1} \sum_{M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \boldsymbol{t}_{\pm}^{k} |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(F) = [T_i(T_i+1) - T_{z_i}(T_{z_i} \pm 1)]\delta_{J_i,J_f}\delta_{T_i,T_f}\delta_{T_{z_f},T_{z_i}\pm 1}$$

Fermi matrix elements

$$B(F) = \frac{1}{2J_i+1} \sum_{M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \boldsymbol{t}_{\pm}^{k} |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(F) = [T_i(T_i+1) - T_{z_i}(T_{z_i} \pm 1)]\delta_{J_i,J_f}\delta_{T_i,T_f}\delta_{T_{z_f},T_{z_i}\pm 1}$$

Energetics:

$$E_{\rm IAS} = Q_\beta + {\rm sign}(T_{z_i})[E_C(Z+1) - E_C(Z) - (m_n - m_H)]$$

Fermi matrix elements

$$B(F) = \frac{1}{2J_i+1} \sum_{M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \boldsymbol{t}_{\pm}^{k} |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(F) = [T_i(T_i+1) - T_{z_i}(T_{z_i} \pm 1)]\delta_{J_i,J_f}\delta_{T_i,T_f}\delta_{T_{z_f},T_{z_i}\pm 1}$$

Energetics:

$$E_{\rm IAS} = Q_\beta + {\rm sign}(T_{z_i})[E_C(Z+1) - E_C(Z) - (m_n - m_H)]$$

Selection rule:

$$\Delta J = 0 \qquad \Delta T = 0 \qquad \pi_i = \pi_f$$

Fermi matrix elements

$$B(F) = \frac{1}{2J_i+1} \sum_{M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \boldsymbol{t}_{\pm}^{k} |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(F) = [T_i(T_i+1) - T_{z_i}(T_{z_i} \pm 1)]\delta_{J_i,J_f}\delta_{T_i,T_f}\delta_{T_{z_f},T_{z_i}\pm 1}$$

Energetics:

$$E_{\rm IAS} = Q_\beta + {\rm sign}(T_{z_i})[E_C(Z+1) - E_C(Z) - (m_n - m_H)]$$

Selection rule:

$$\Delta J = 0 \qquad \Delta T = 0 \qquad \pi_i = \pi_f$$

Sum rule (sum *over* all the final states):

$$S(F) = S_-(F) - S_+(F) = 2T_{z_i} = (N-Z)$$

Gamow-Teller matrix elements

back

$$B(GT) = \frac{g_A^2}{2J_i+1} |\langle J_f; T_f T_{z_f} || \sum_{k=1}^{A} \sigma^k t_{\pm}^k || J_i; T_i T_{z_i} \rangle|^2$$

$$g_A = -1.2720 \pm 0.0018$$

Selection rule:

$$\Delta J = 0, 1 \ (\text{no } J_i = 0 \rightarrow J_f = 0) \qquad \Delta T = 0, 1 \qquad \pi_i = \pi_f$$

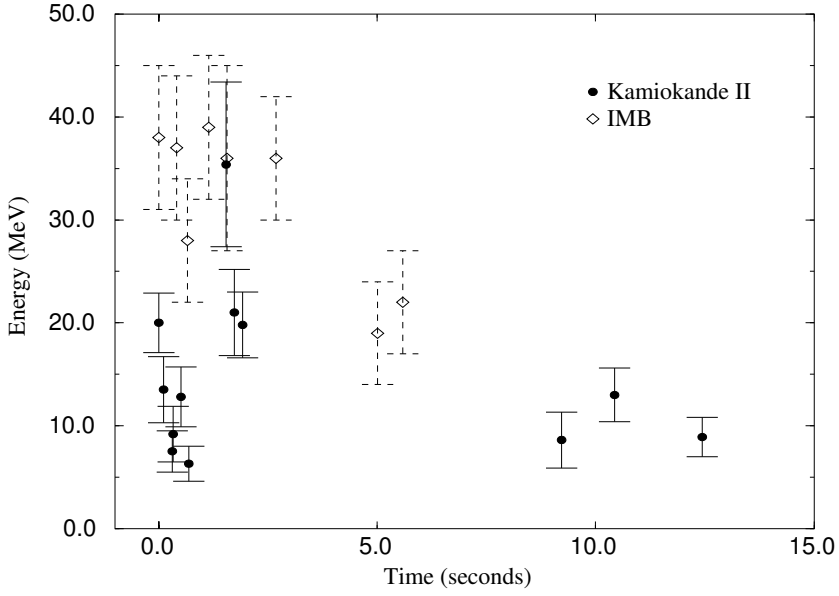
Ikeda sum rule:

$$S(GT) = S_-(GT) - S_+(GT) = 3(N - Z)$$

SN1987A

Type II supernova in LMC ($\sim$ 55 kpc)

neutrinos from SN1987A

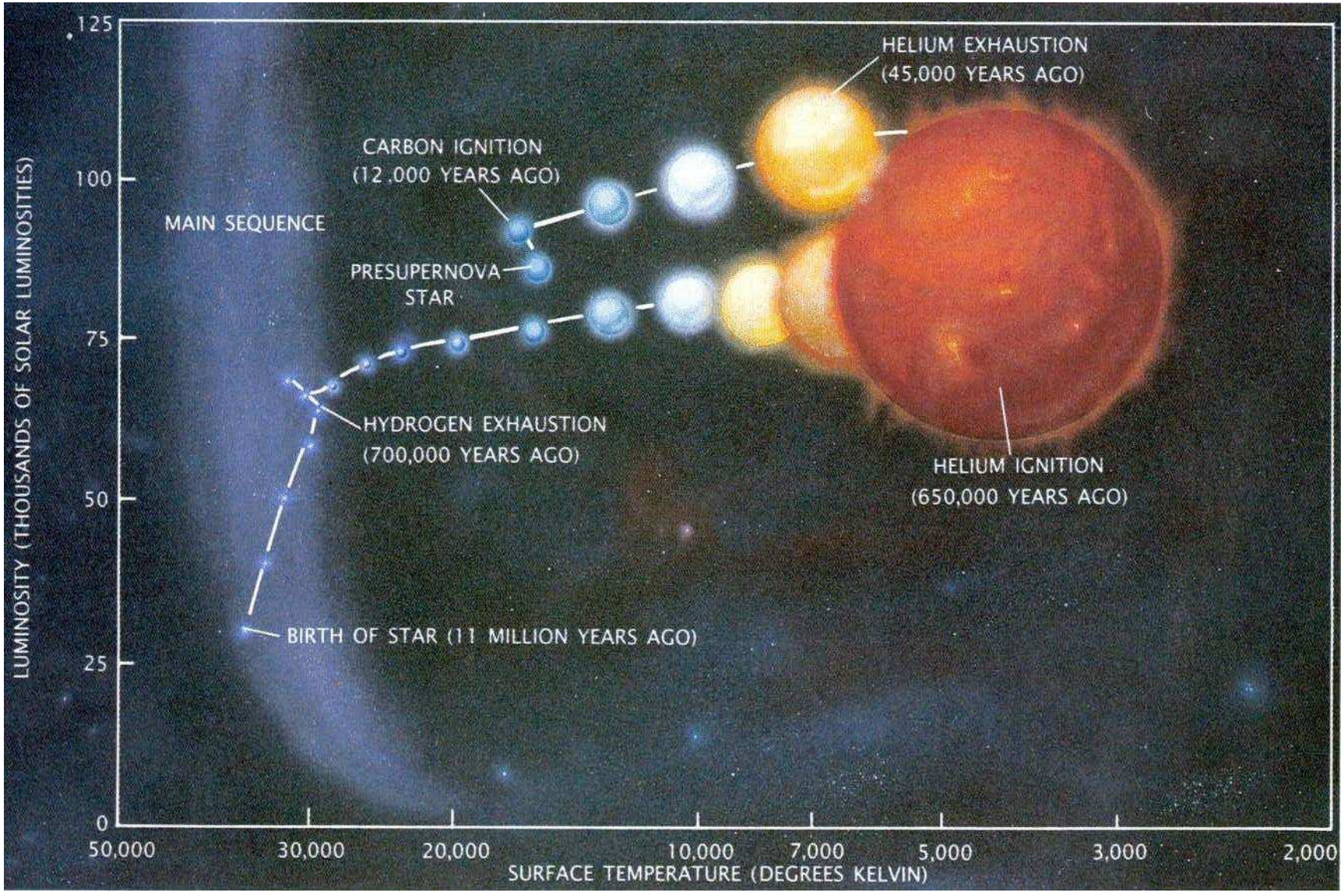


- $E_{\rm grav} \approx 10^{53}$ erg
- $E_{\rm rad} \approx 8 \times 10^{49}$ erg
- $E_{\rm kin} \approx 10^{51}$ erg $= 1$ foe

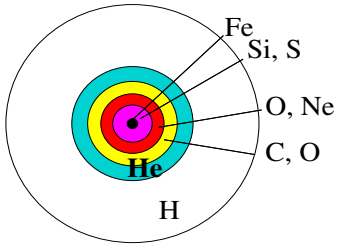
$$E_\nu \approx 2.7 \times 10^{53} \text{ erg}$$

Evolution Massive Stars

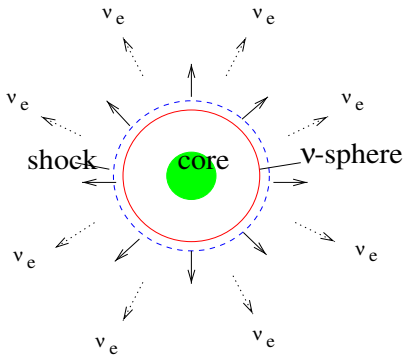
Evolution 20 solar mass star



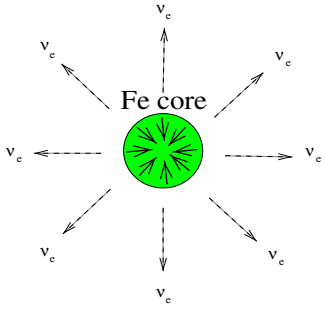
Late stages core evolution



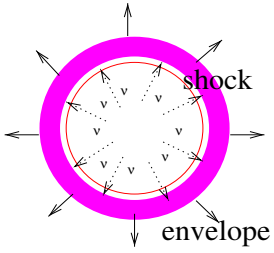
PRE-SUPERNOVA



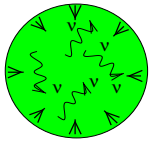
NEUTRINO BREAKOUT



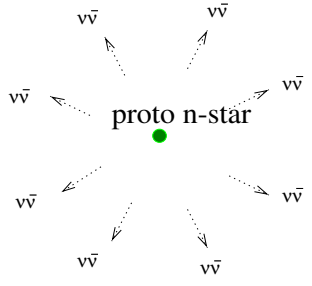
COLLAPSE



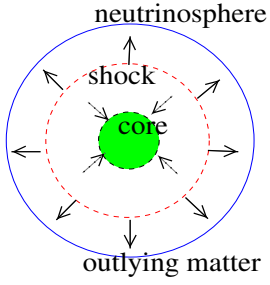
EXPLOSION



NEUTRINO TRAPPING

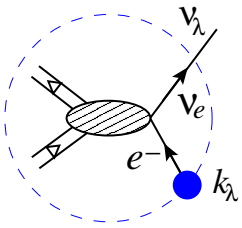


COOLING



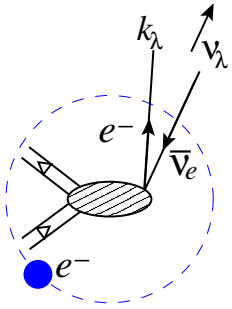
CORE BOUNCE

Semileptonic Weak Processes in Stars



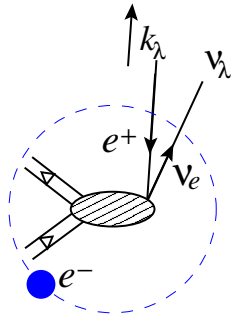
orbital e^- capture

$q_\lambda = \nu_\lambda - k_\lambda$



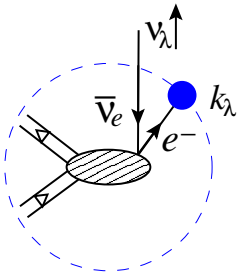
β^- decay

$q_\lambda = \nu_\lambda + k_\lambda$



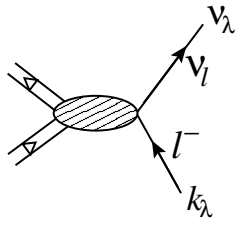
β^+ decay

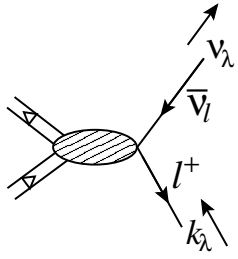
$q_\lambda = \nu_\lambda + k_\lambda$



bound-state β^- decay

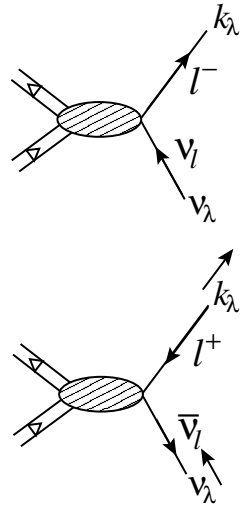
$q_\lambda = \nu_\lambda + k_\lambda$





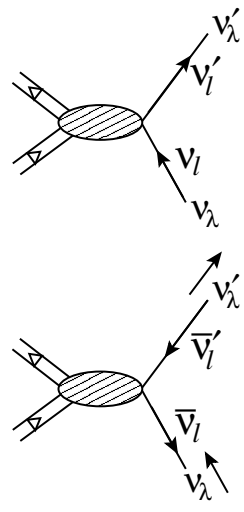
continuum charged (anti)lepton capture

$q_\lambda = \nu_\lambda - k_\lambda$



(anti)neutrino capture

$q_\lambda = k_\lambda - \nu_\lambda$

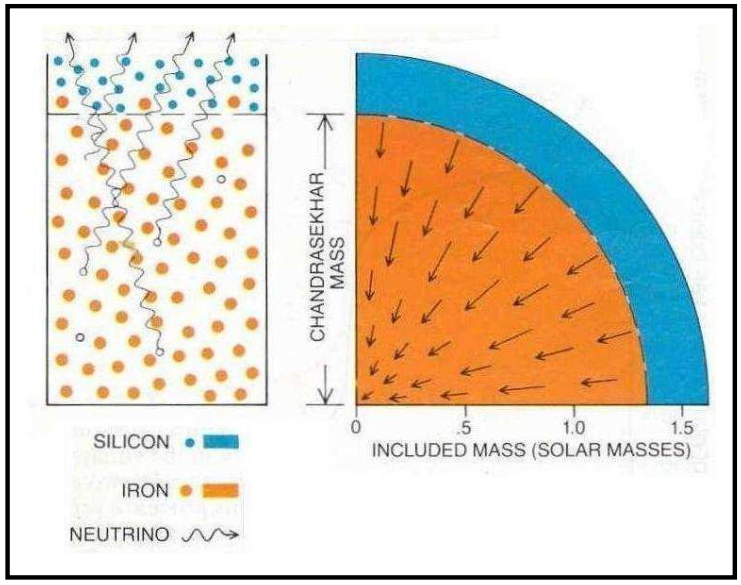


(anti)neutrino scattering

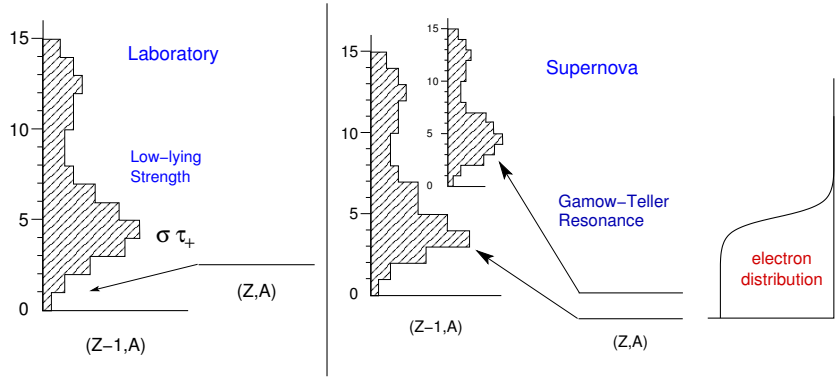
$q_\lambda = \nu'_\lambda - \nu_\lambda$

$$O_{\mathrm{F}} \sim e^{i\boldsymbol{qr}}\boldsymbol{\tau} \qquad O_{\mathrm{GT}} \sim e^{i\boldsymbol{qr}}\boldsymbol{\sigma\tau}$$

Presupernova evolution



- $T = 0.1$–0.8 MeV, $\rho = 10^7$–10^{10} g cm^{-3}.
  Composition of iron group nuclei (A = 45–65)
- Important processes:
  - ➢ electron capture:
    $e^- + (N, Z) \rightarrow (N + 1, Z - 1) + \nu_e$
  - ➢ β^- decay:
    $(N, Z) \rightarrow (N - 1, Z + 1) + e^- + \bar{\nu}_e$
- Dominated by allowed transitions (Fermi and Gamow-Teller)
- Evolution decreases number of electrons (Y_e) and Chandrasekar mass ($M_{\rm Ch} \approx 1.4(2Y_e)^2$ M$_\odot$)



- Fenomenological model (Fuller, Fowler, Newman, 1985)
- Charge exchange reactions (n, p), $(d, {}^2\mathrm{He})$
- Microscopic model (Shell-Model) (Langanke & Martínez-Pinedo, 2001)

GT in charge exchange reactions

GT strength could be measured in CE reactions:

- GT_- proved in (p, n), $(^3\text{He}, t)$.
- GT_+ proved in (n, p), $(t, ^3\text{He})$, $(d, ^2\text{He})$.

Mathematical relationship ($E_p \geq 100$ MeV/nucleon):

$$\frac{d\sigma}{d\Omega dE}(0^\circ) \approx S(E_x)B(GT)$$

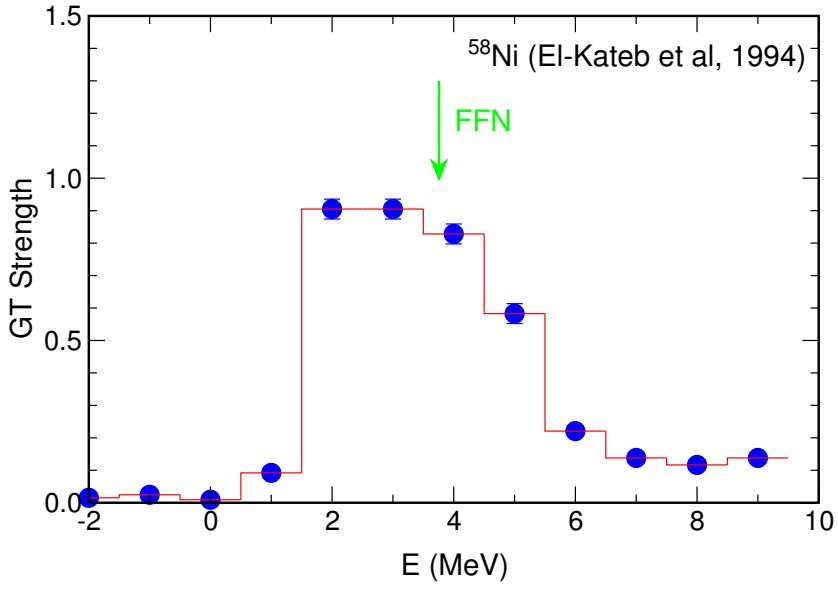
$$B(GT) = \left(\frac{g_A}{g_V}\right)^2 \frac{\langle f||\sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_\pm^k||i\rangle^2}{2J_i+1}$$

Ikeda sum rule:

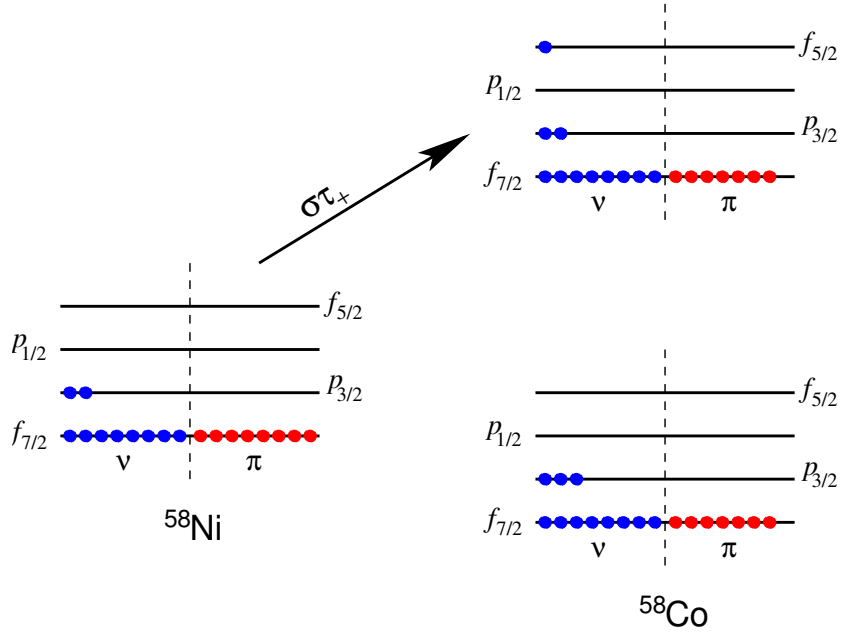
$$S_- - S_+ = 3(N-Z)$$

Independent Particle Model

GT_+ strength in ^{58}Ni measured in (n, p).

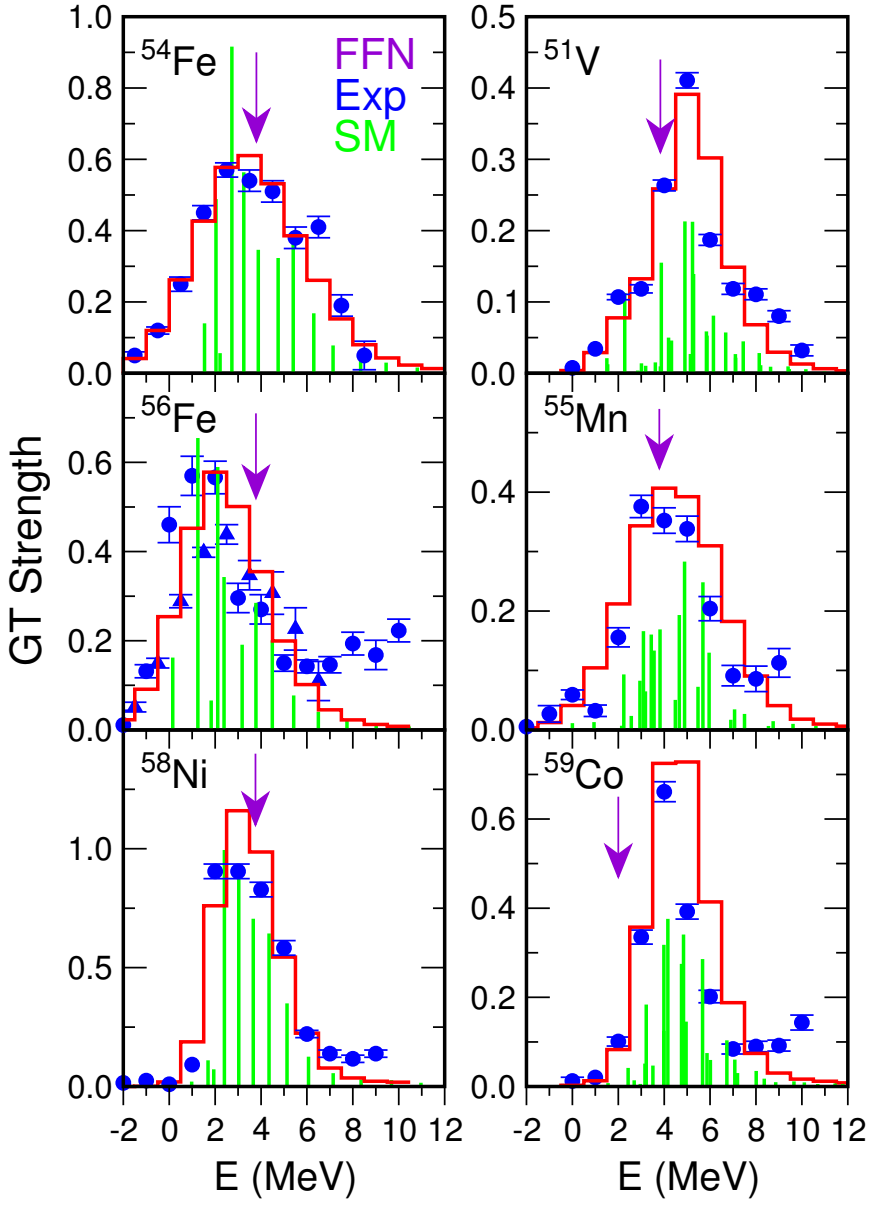


Independent particle model (FFN).

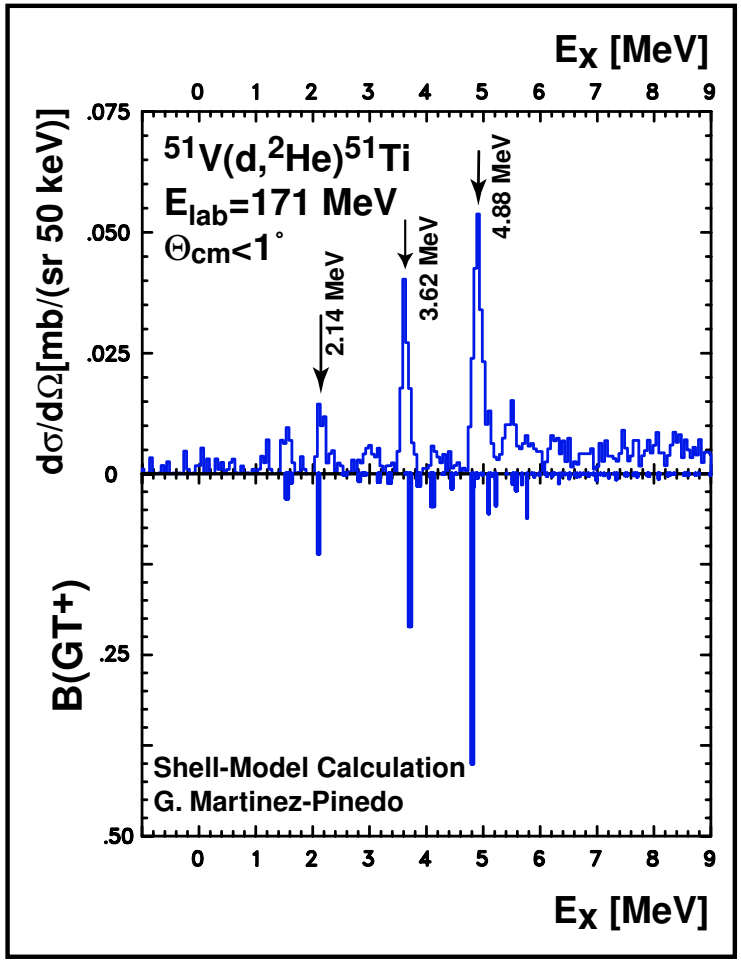


Gamow-Teller strength

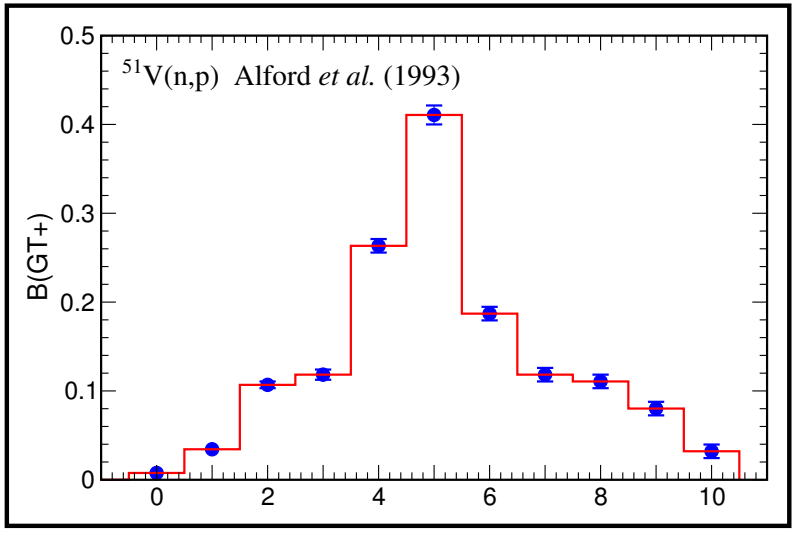
GT_+ strength measured in charge-exchange (n, p) experiments (TRIUMF).



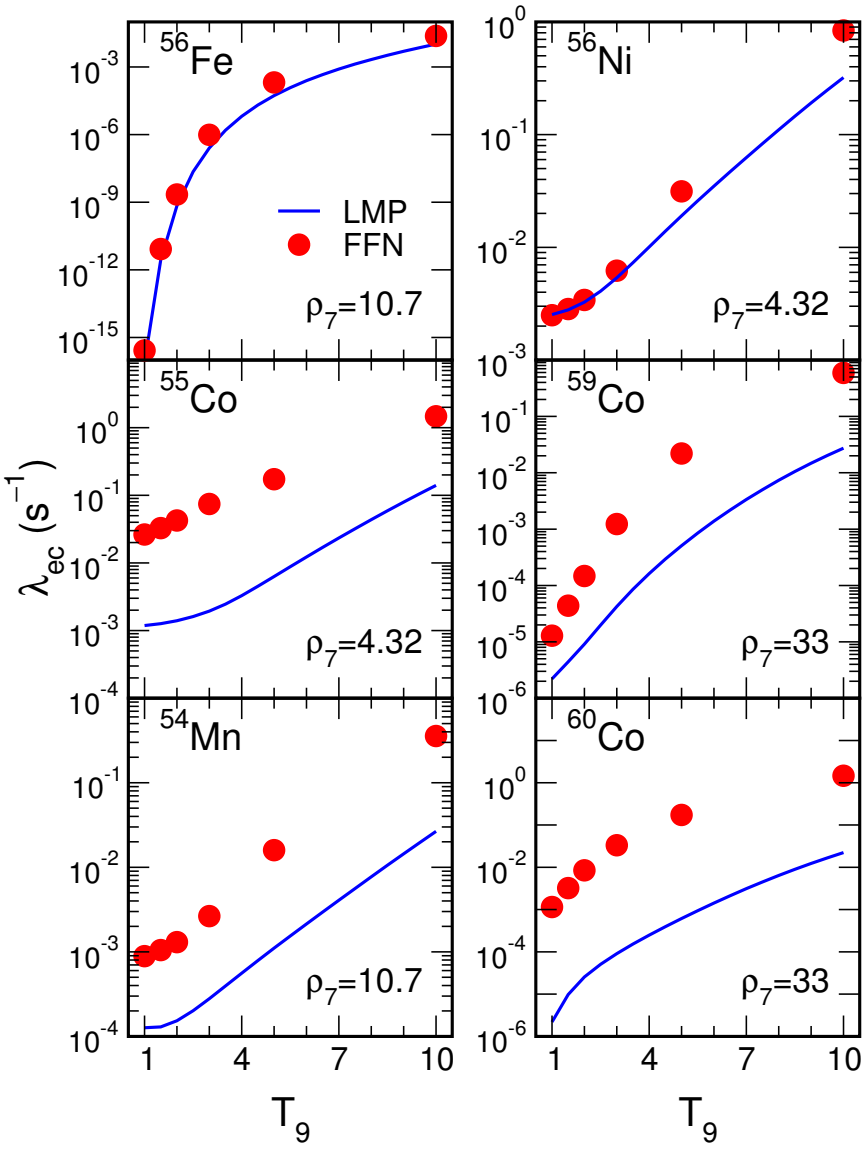
GT_+ strength measured in $(d,^2He)$



High resolution Gamow-Teller distributions on ^{51}V,^{58}Ni (^{64}Ni,...) measured at KVI (Groningen) by EuroSupernova Collaboration.

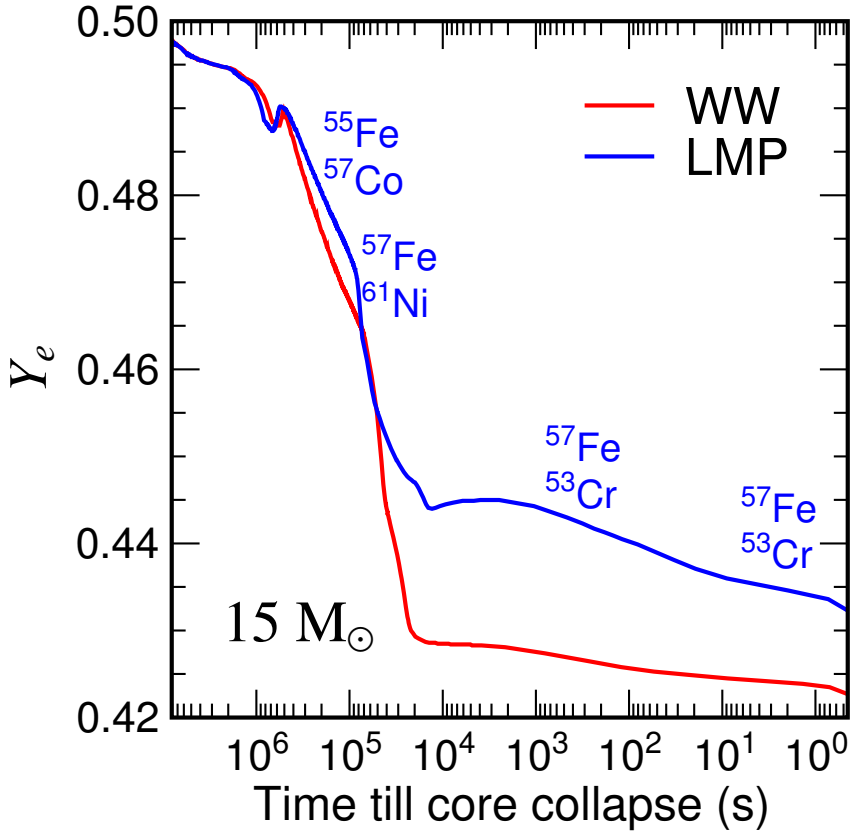


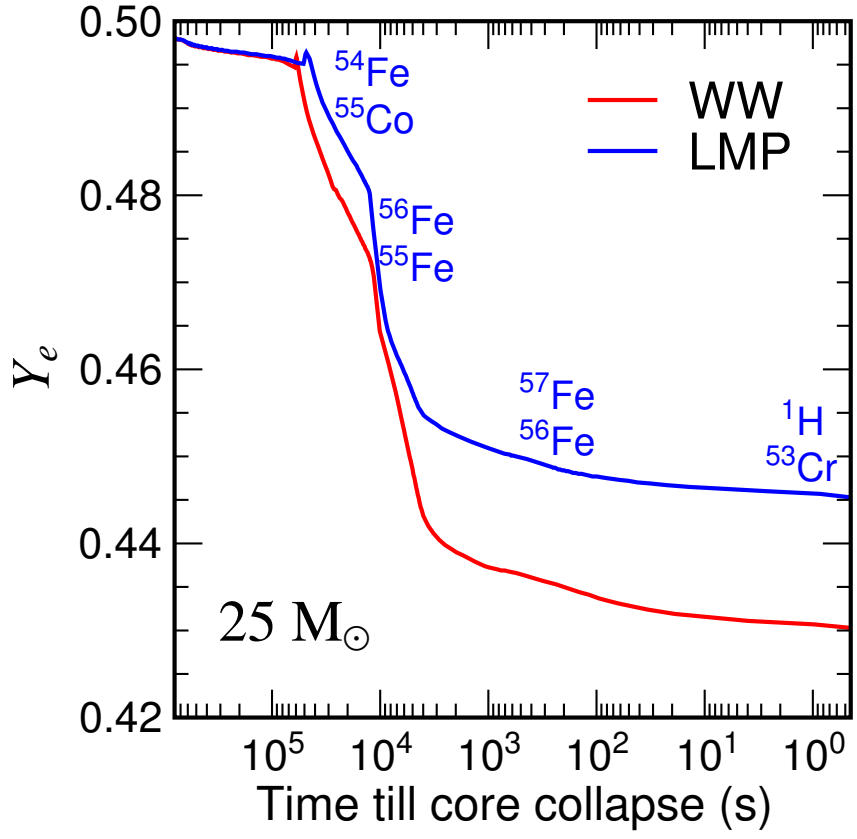
Shell-model (LMP) vs FFN rates



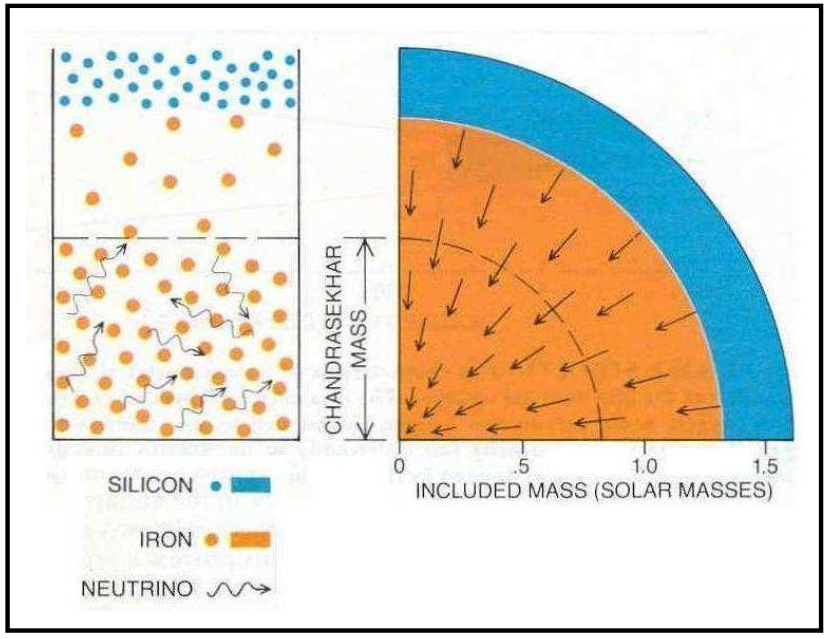
Most important nuclei

Most important nuclei to determine the electron capture rate





Collapse phase



Important processes:

- Neutrino transport (Boltzman equation):
  $\nu + A \rightleftarrows \nu + A$ (trapping)
  $\nu + e^- \rightleftarrows \nu + e^-$ (thermalization)

  cross sections $\sim E_\nu^2$
- electron capture on protons:
  $e^- + p \rightleftarrows n + \nu_e$

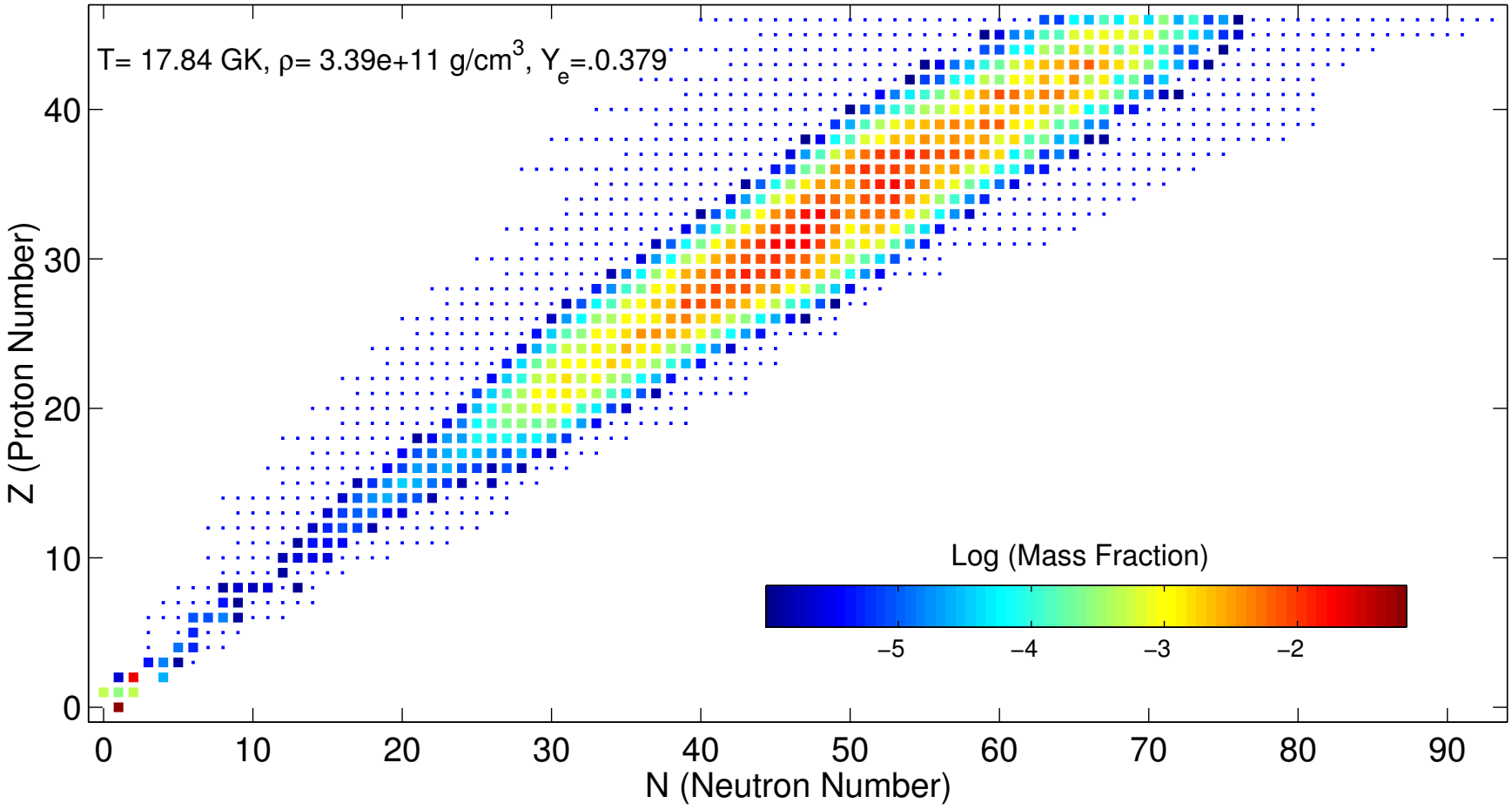
What is the role of electron capture on nuclei?

$$e^- + (N, Z) \rightleftarrows (N + 1, Z - 1) + \nu_e$$

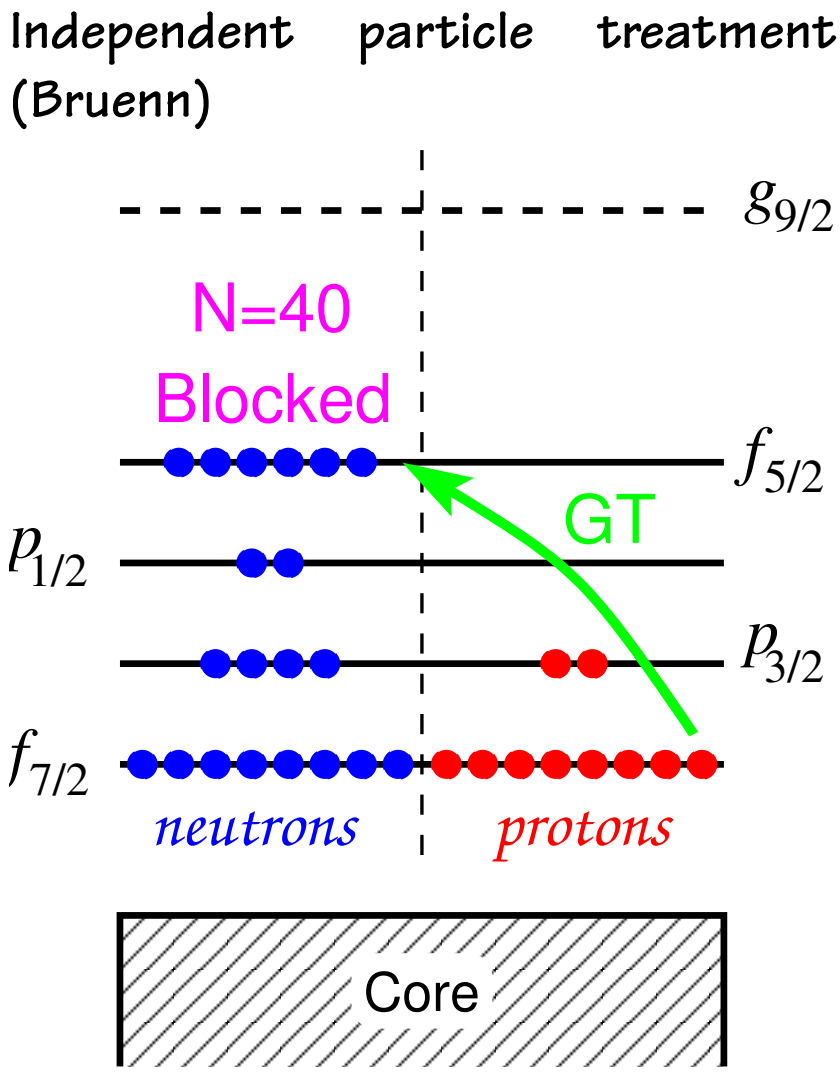
What is the role of inelastic neutrino-nucleus scattering?

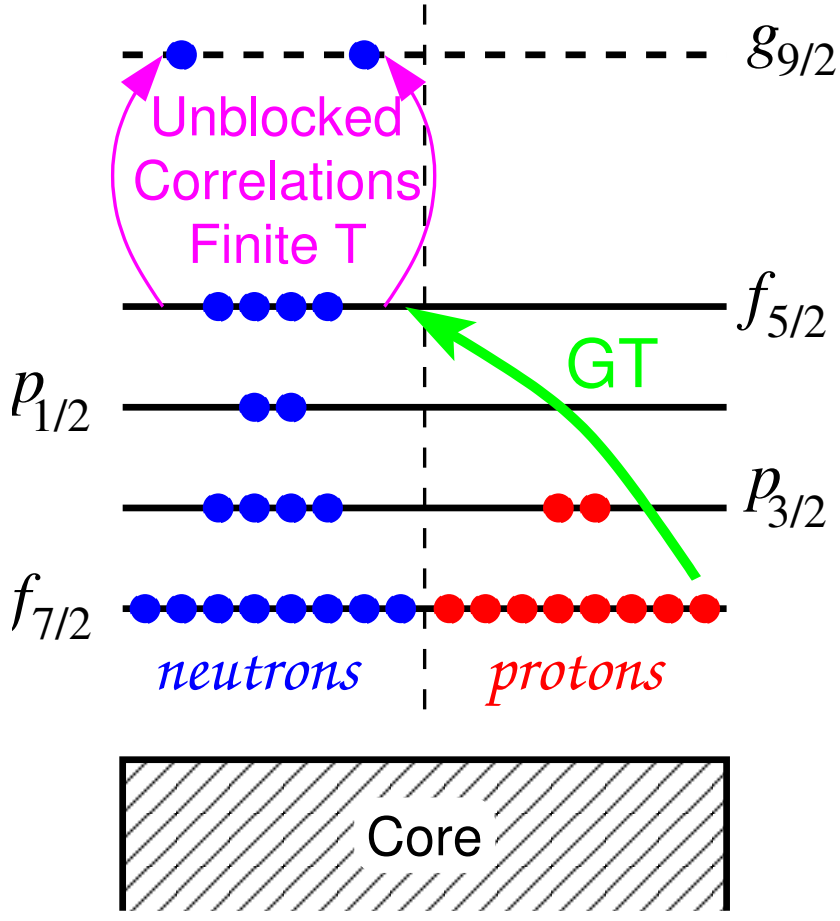
$$\nu + A \rightleftarrows \nu + A^*$$

Collapse abundances



(Un)blocking electron capture at N=40

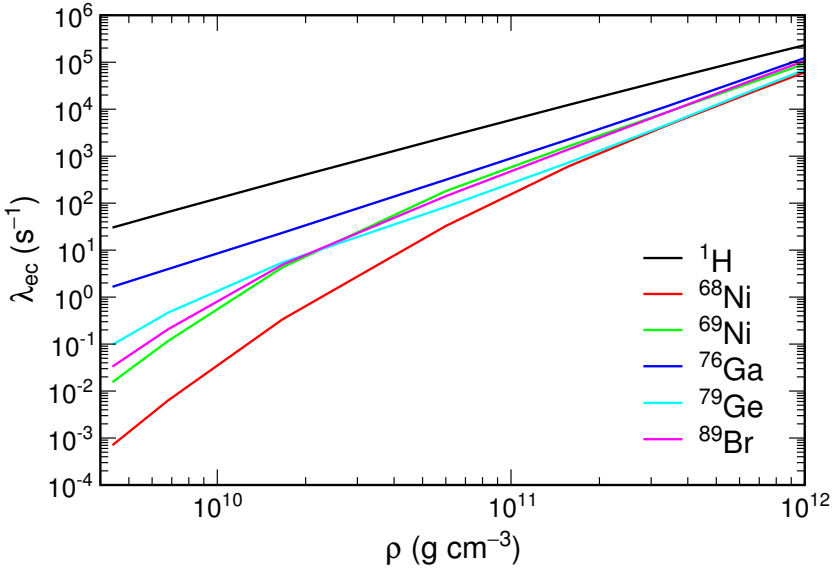




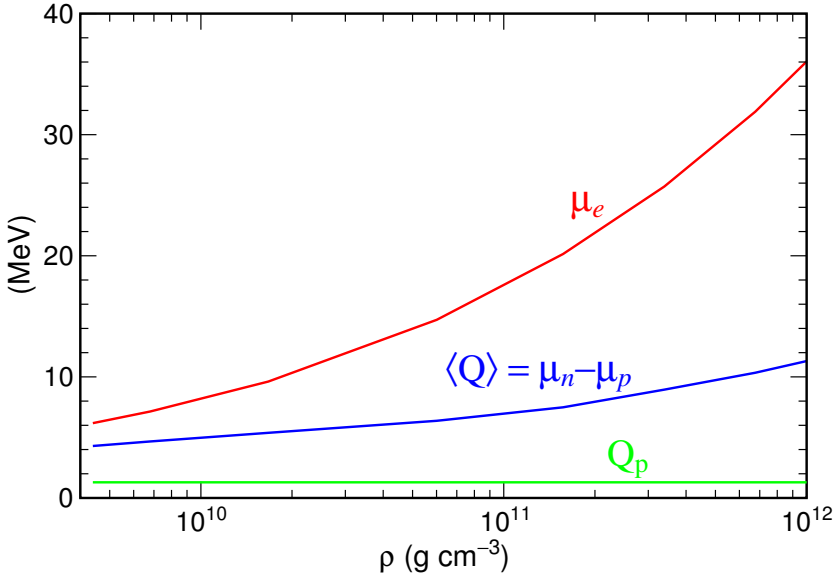
Weak Rates for nuclei with A=65–112 computed using the Shell Model Monte Carlo plus RPA approach

Electron capture: nuclei vs protons

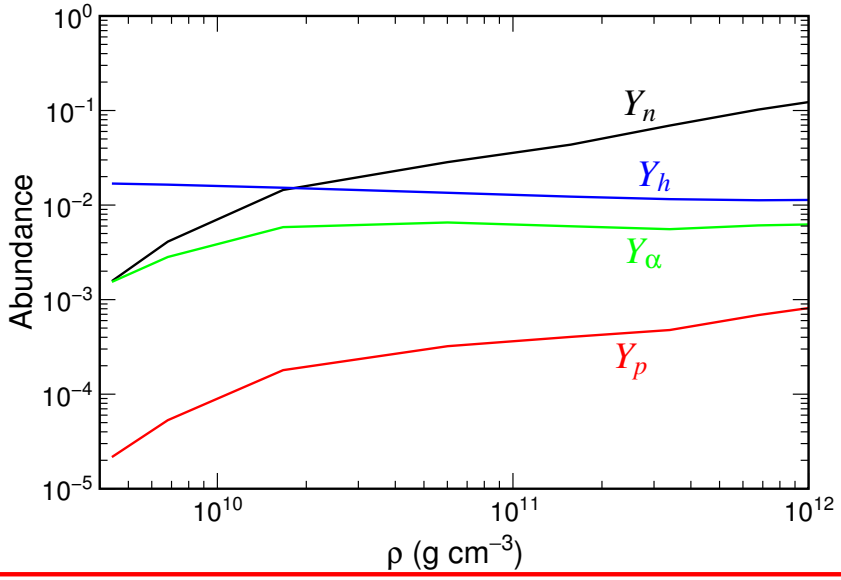
Electron capture rates



Energetics

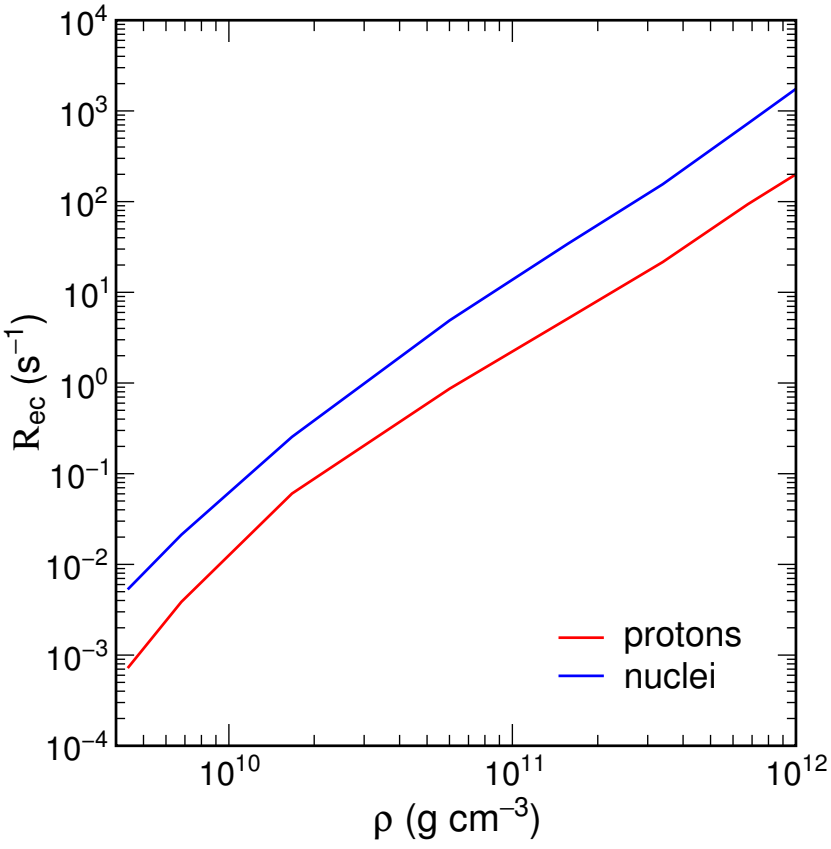


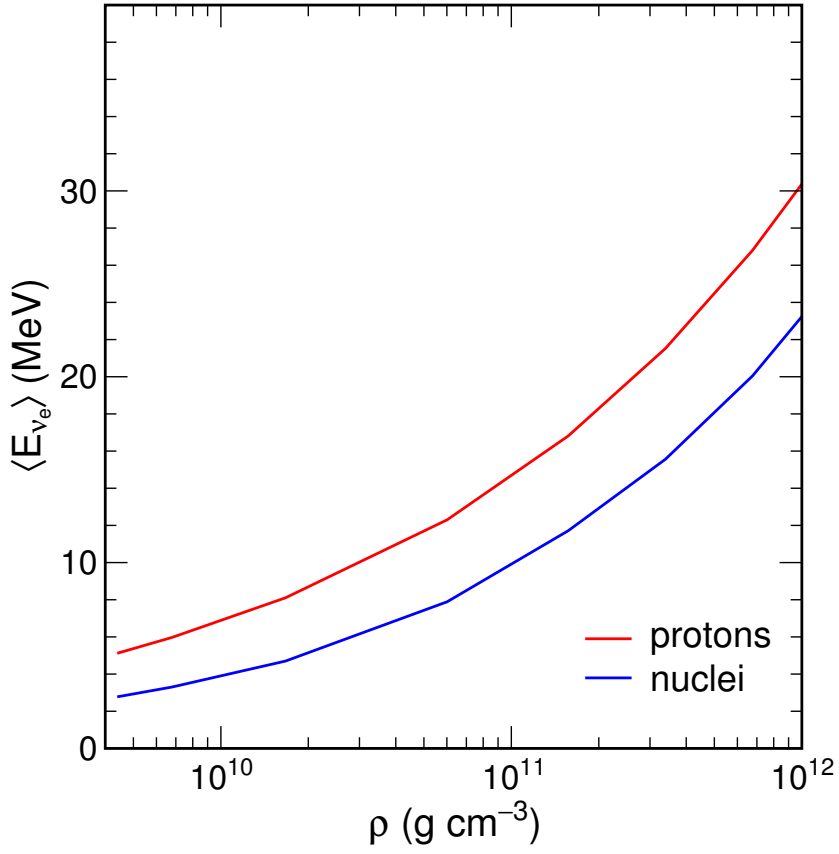
Abundances



$$R_h = \sum_i Y_i \lambda_i = Y_h \langle \lambda_h \rangle$$
$$R_p = Y_p \lambda_p$$

Reaction rates

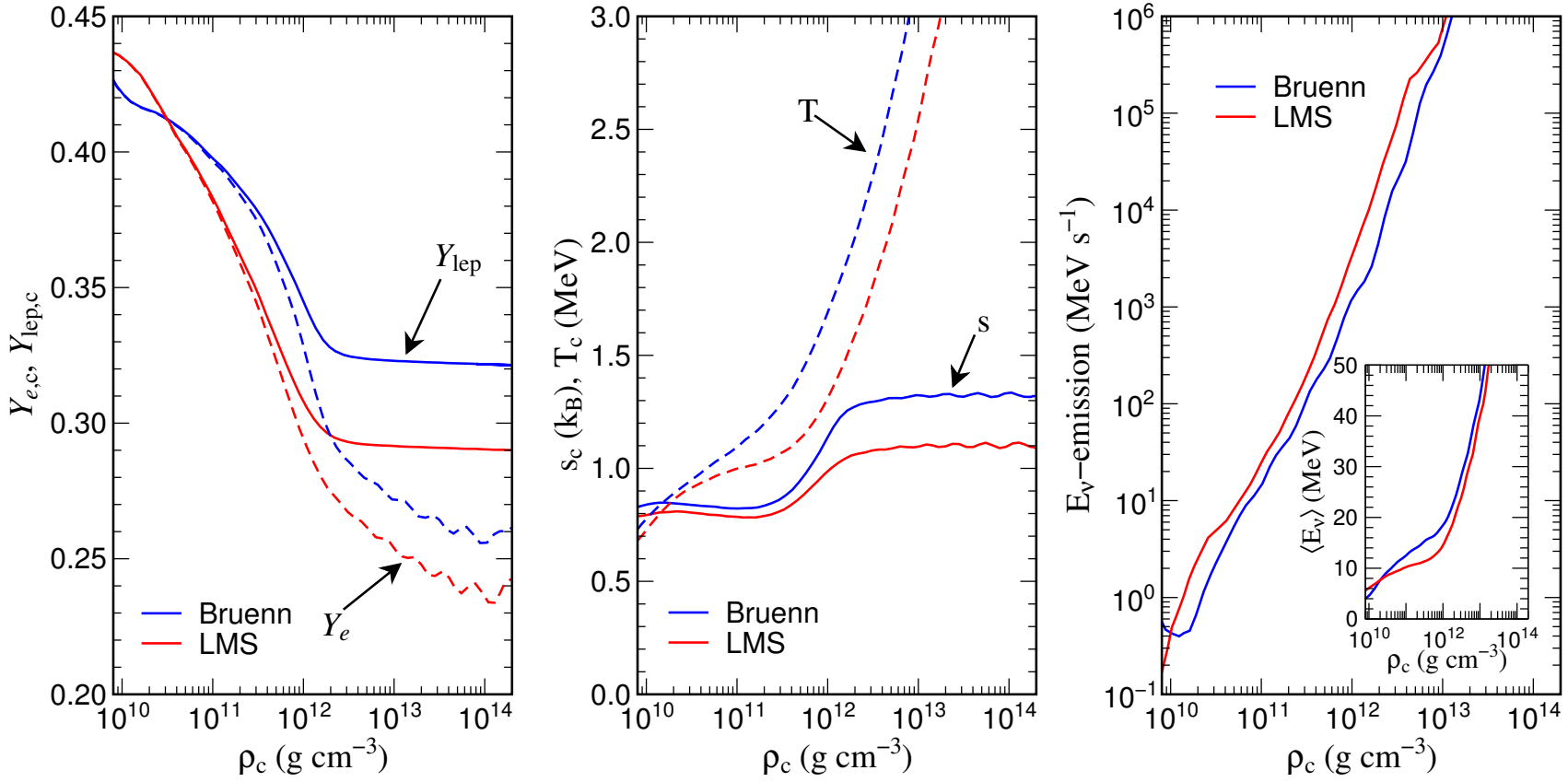




Electron capture on nuclei dominates over capture on protons

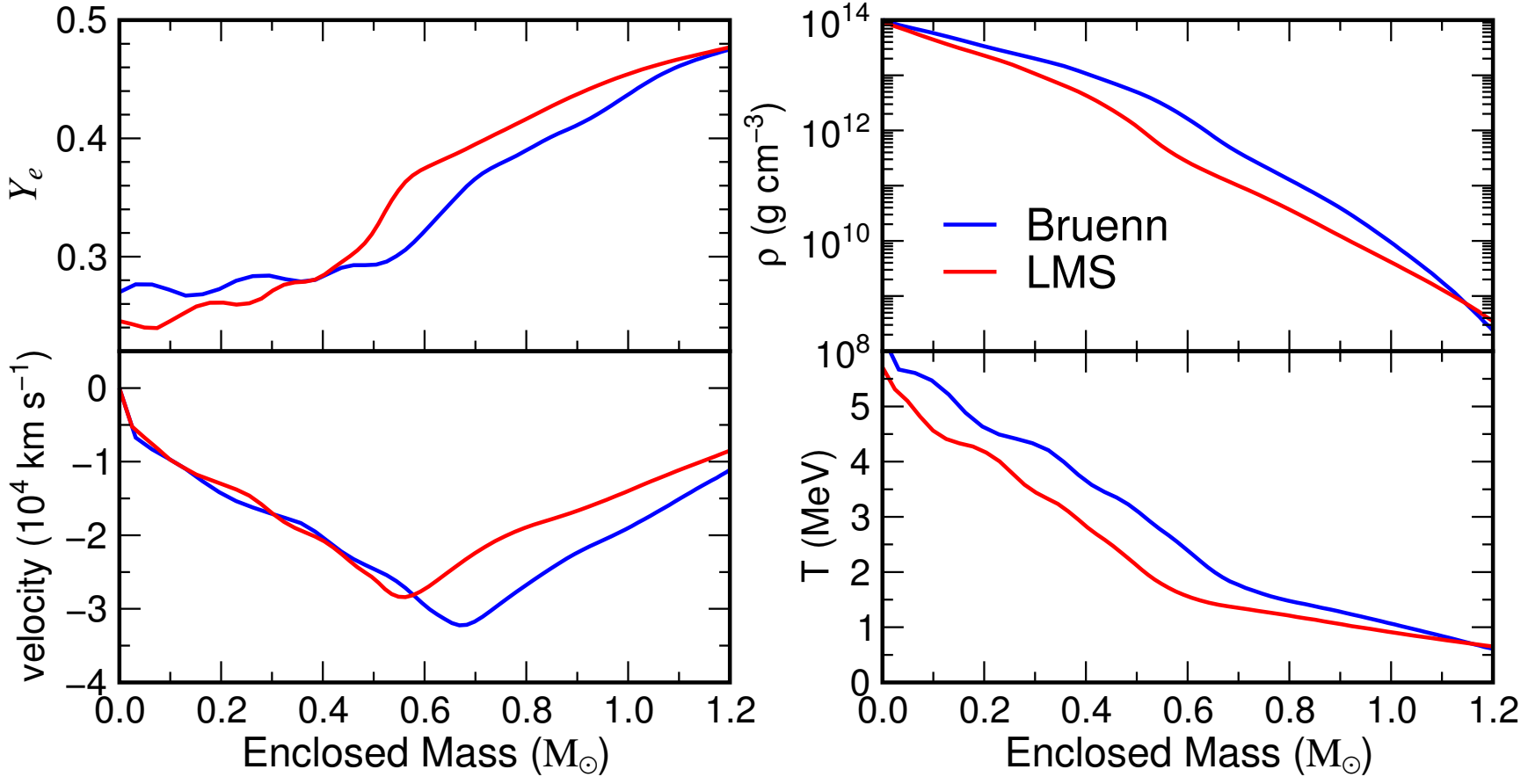
Consequences

With Rampp & Janka (General Relativic model)
15 $M_\odot$ presupernova model from A. Heger & S. Woosley



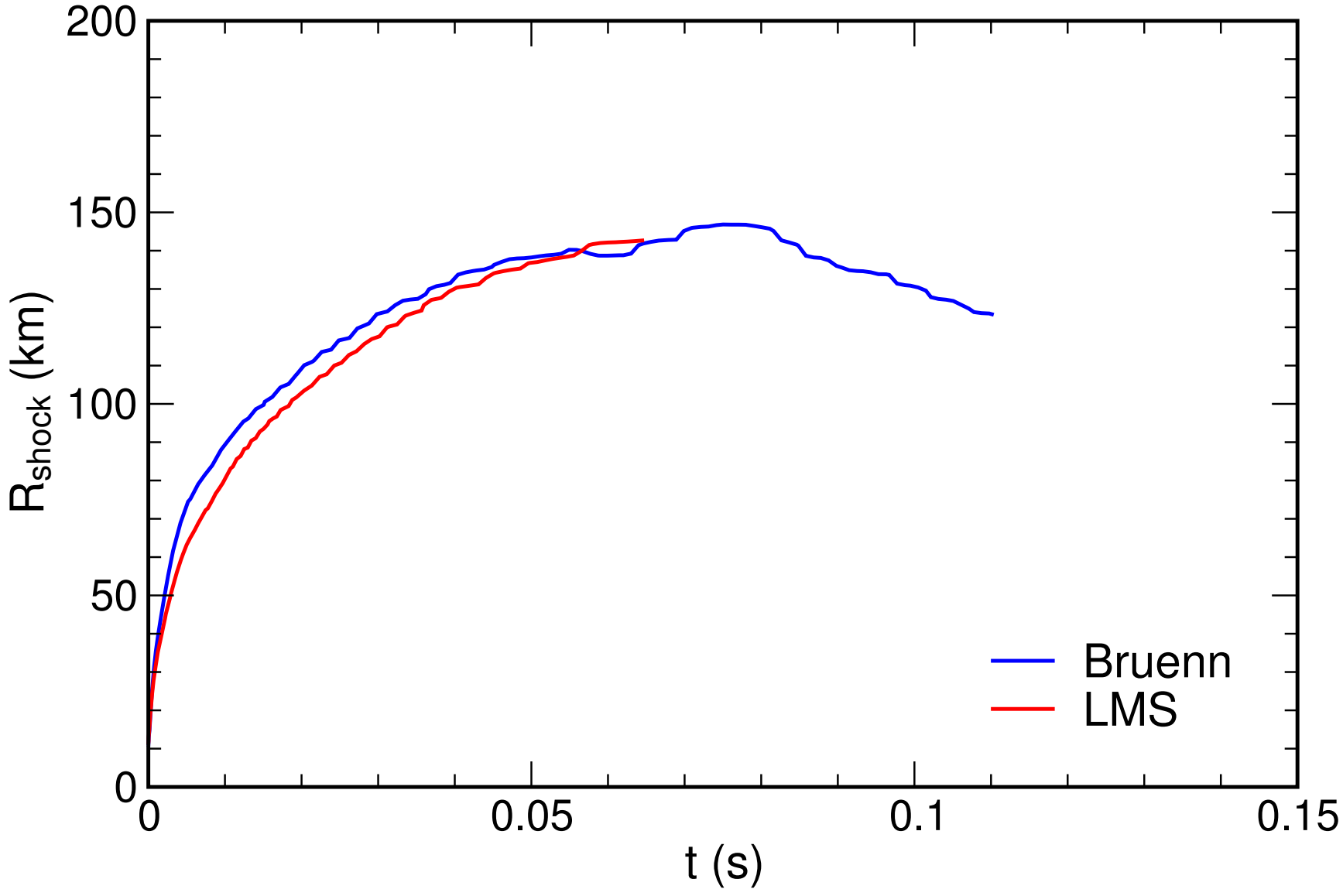
Consequences

With Hix, Liebendörfer, Mezzacappa, Messer
(Newtonian Gravity)

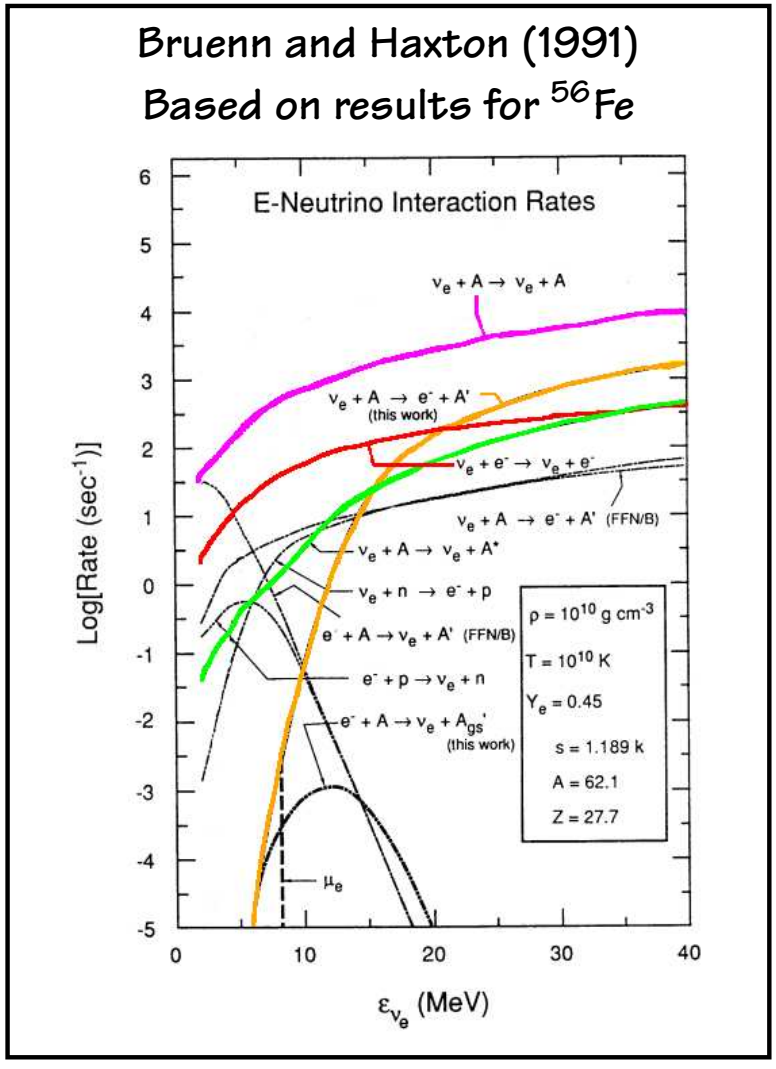


Shock evolution

With Rampp & Janka (General Relativic model)



Neutrino interactions in the collapse



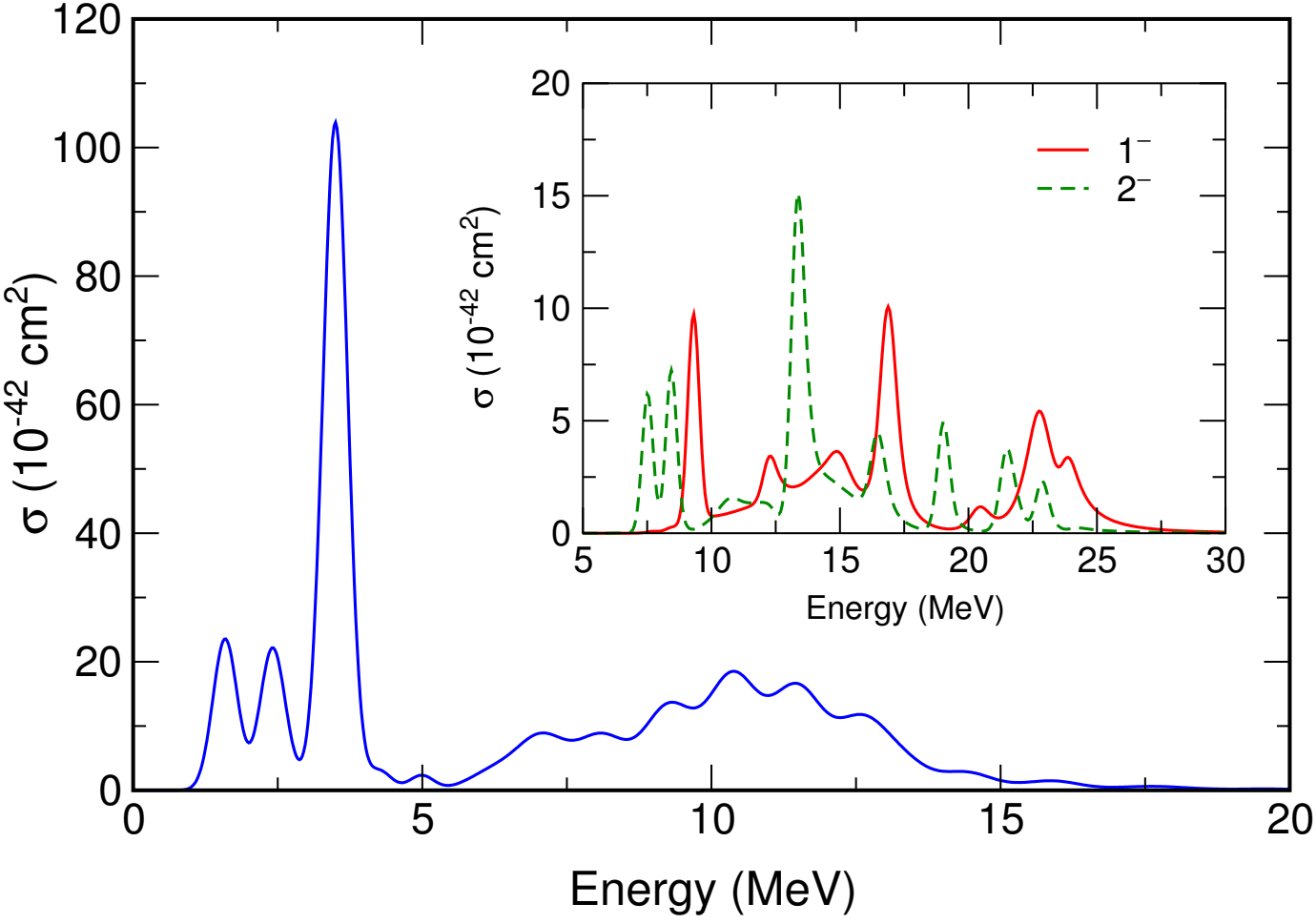
- **Elastic scattering:**
  $\nu + A \rightleftarrows \nu + A$ (trapping)
- **Absorption:**
  $\nu_e + (N, Z) \rightleftarrows e^- + (N-1, Z+1)$
- **ν-e scattering:**
  $\nu + e^- \rightleftarrows \nu + e^-$
- **Inelastic ν-nuclei scattering:**
  $\nu + A \rightleftarrows \nu + A^*$

ν_e absorption cross section on ^{56}Fe

$^{56}\mathrm{Fe}(\nu_e, e^-)^{56}\mathrm{Co}$ **measured by KARMEN collaboration:**

$\sigma_{\mathrm{exp}} = 2.56 \pm 1.08(\mathrm{stat}) \pm 0.43(\mathrm{syst}) \times 10^{-40}\ \mathrm{cm}^2$

$\sigma_{\mathrm{th}} = 2.38 \times 10^{-40}\ \mathrm{cm}^2$

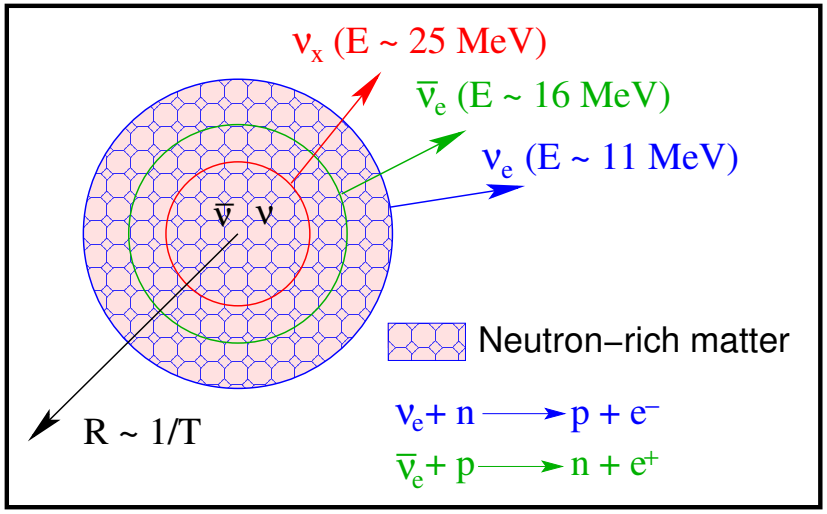


Neutrino nucleosynthesis

Neutrinos interact with abundant nuclear species

- Neutral current (ν, ν'): Nucleus excited to particle unbound states that decay by particle emission.
- Charged current (ν_e, e^-) and $(\bar{\nu}_e, e^+)$.

Product	Parent	Reaction
^{11}B	^{12}C	$(\nu, \nu' n), (\nu, \nu' p)$
^{15}N	^{16}O	$(\nu, \nu' n), (\nu, \nu' p)$
^{19}F	^{20}Ne	$(\nu, \nu' n), (\nu, \nu' p)$
^{138}La	^{138}Ba	(ν, e^-)
	^{139}La	$(\nu, \nu' n)$
^{180}Ta	^{180}Hf	(ν, e^-)
	^{181}Ta	$(\nu, \nu' n)$



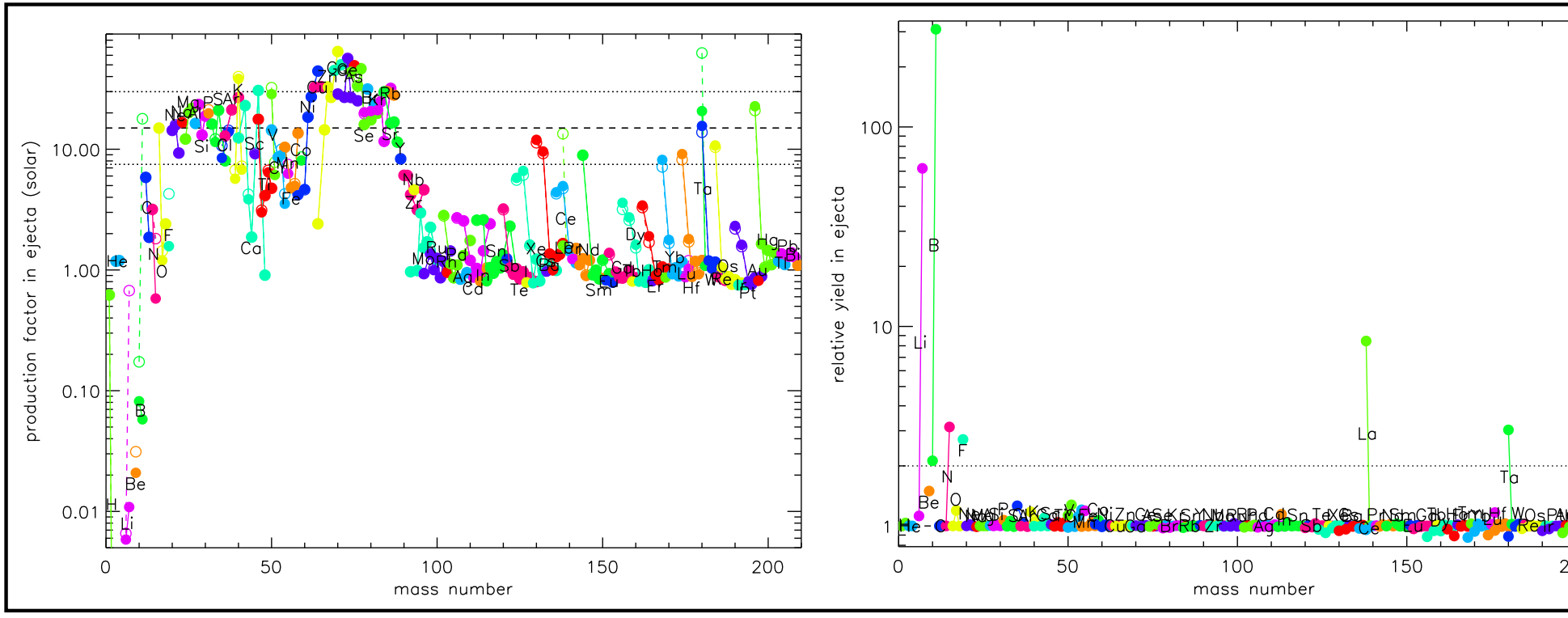
^{11}B/^{10}B traces galactic evolution:

- Big ratio (100) predicted by ν-process.
- Cosmic rays spallation reactions on C give a ratio of 2.
- Solar system ratio is 4.

^{138}La, ^{180}Ta production is sensitive to neutrino oscillations.

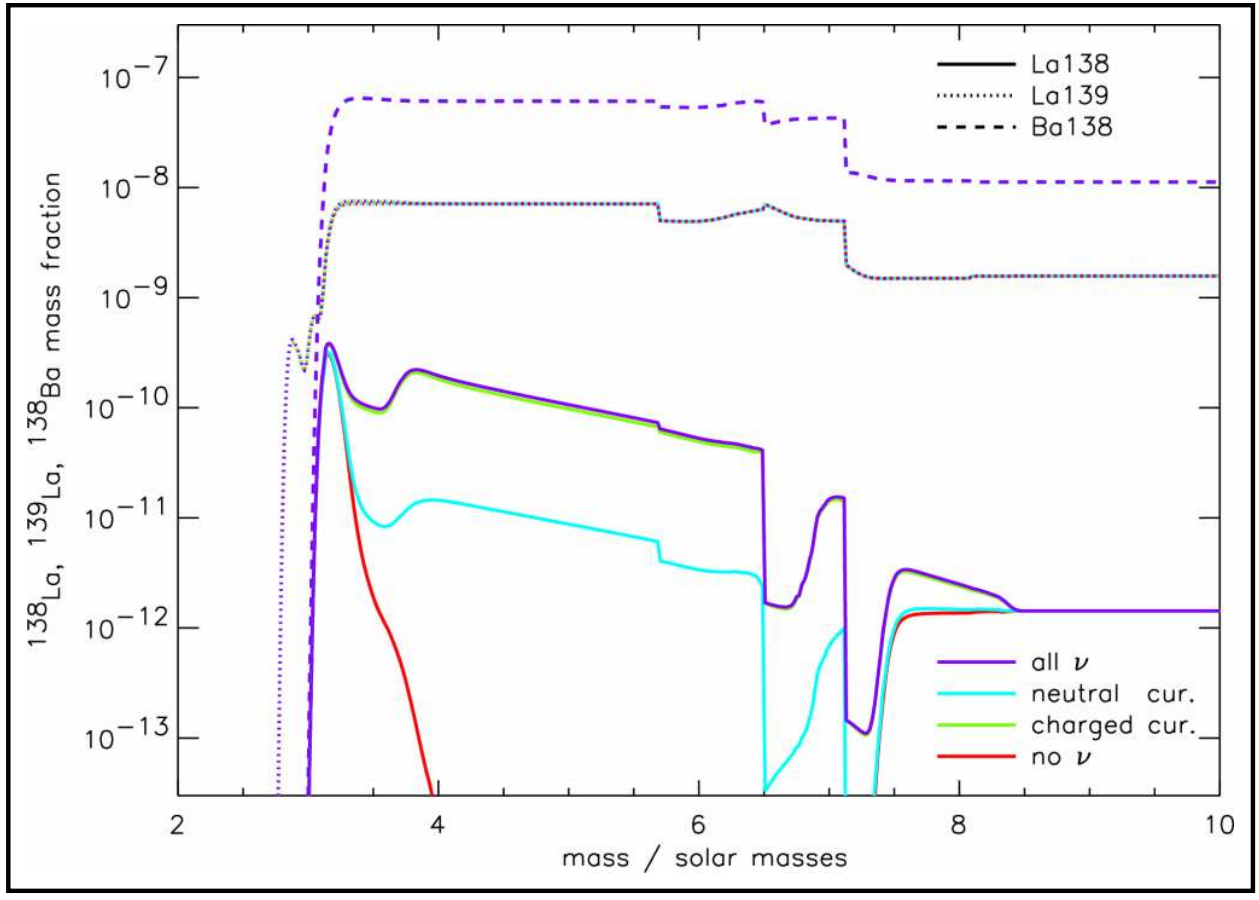
Nucleosynthesis with and without ν

With Heger (Los Alamos)



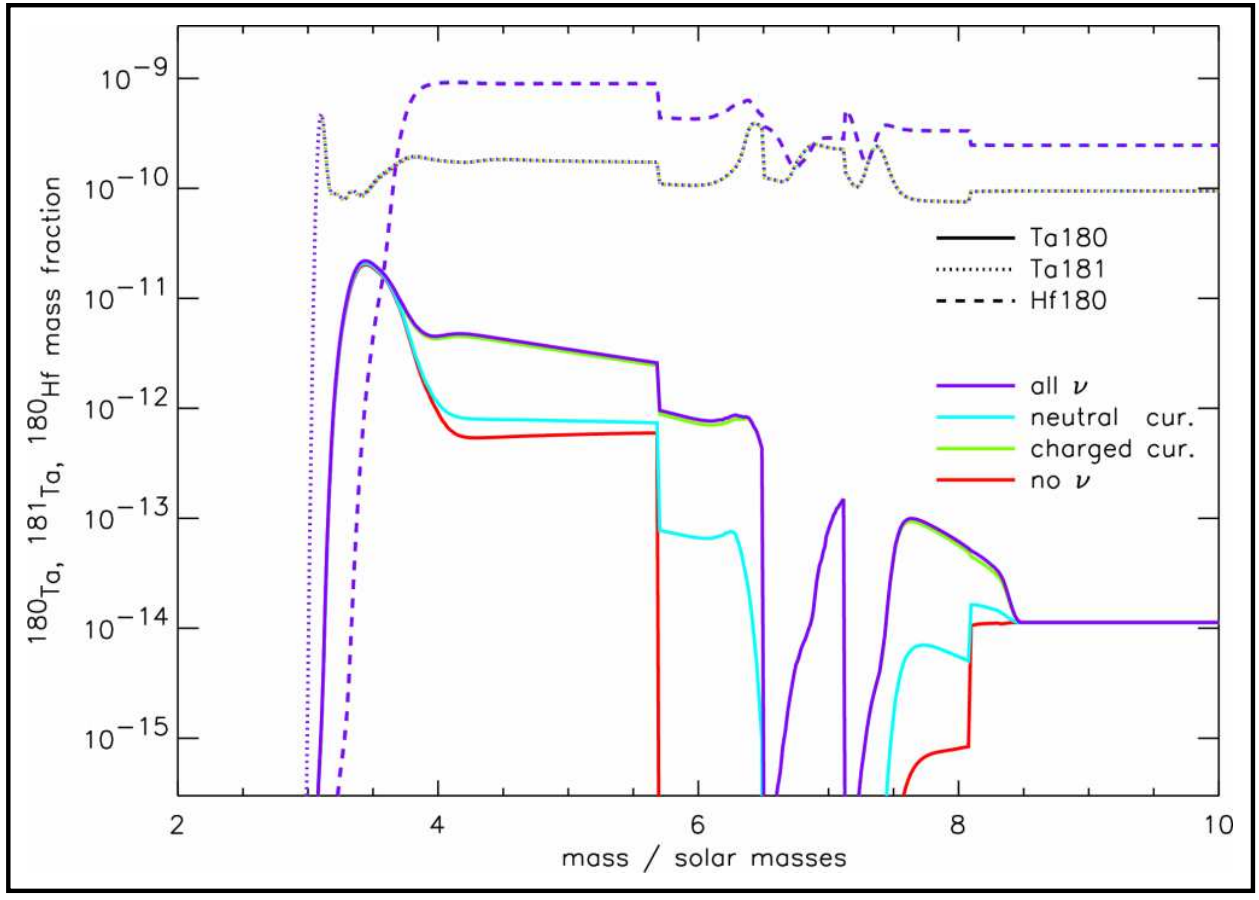
The production of ^{138}La

Produced by $^{139}\mathrm{La}(\gamma, n)$, $^{138}\mathrm{Ba}(\nu_e, e^-)$



The production of ^{180}Ta

Produced by $^{181}\mathrm{Ta}(\gamma, n)$, $^{180}\mathrm{Hf}(\nu_e, e^-)$



Presupernova abundances