The interacting boson model

P. Van Isacker, GANIL, France

Dynamical symmetries of the IBM

Neutrons, protons and *F*-spin (IBM-2)

T=0 and *T=1* bosons: IBM-3 and IBM-4

The interacting boson model

- Nuclear collective excitations are described in terms of N s and d bosons.
- Spectrum generating algebra for the nucleus is U(6). All physical observables (hamiltonian, transition operators,…) are expressed in terms of the generators of U(6).
- Formally, nuclear structure is reduced to solving the problem of N interacting s and d bosons.

Justifications for the IBM

- Bosons are associated with *fermion pairs* which approximately satisfy Bose statistics:
$$S^+ = \sum_j \alpha_j \left(a_j^+ \times a_j^+\right)_0^{(0)} \to s^+, \quad D_m^+ = \sum_{jj'} \beta_{jj'} \left(a_j^+ \times a_{j'}^+\right)_m^{(2)} \to d_m^+$$
- Microscopic justification: The IBM is a truncation and subsequent bosonization of the *shell model* in terms of *S* and *D* pairs.
- Macroscopic justification: In the classical limit ($N \to \infty$) the expectation value of the IBM hamiltonian between coherent states reduces to a *liquid-drop* hamiltonian.

Algebraic structure of the IBM

- The U(6) algebra consists of the generators

$$\mathrm{U}(6) = \left\{ s^+ s, s^+ d_m, d_m^+ s, d_m^+ d_{m'} \right\}, \quad m, m' = -2, \ldots, +2$$

- The harmonic oscillator in 6 dimensions,

$$H = n_s + n_d = s^+ s + \sum_{m=-2}^{+2} d_m^+ d_m = C_1[\mathrm{U}(6)] \equiv N$$

- …has U(6) symmetry since

$$\forall g_i \in \mathrm{U}(6): \quad [H, g_i] = 0$$

- Can the U(6) symmetry be lifted while preserving the rotational SO(3) symmetry?

The IBM hamiltonian

- Rotational invariant hamiltonian with up to *N*-body interactions (usually up to 2):

$$H_{\mathrm{IBM}} = \varepsilon_s n_s + \varepsilon_d n_d + \sum_{ijklJ} \upsilon^{L}_{ijkl} \left(b_i^+ \times b_j^+\right)^{(L)} \cdot \left(\tilde{b}_k \times \tilde{b}_l\right)^{(L)} + \cdots$$

- For what choice of single-boson energies ε_s and ε_d and boson-boson interactions υ^{L}_{ijkl} is the IBM hamiltonian solvable?

- This problem is equivalent to the enumeration of all algebras *G* that satisfy

$$\mathrm{U}(6) \supset G \supset \mathrm{SO}(3) \equiv \left\{ L_\mu = \sqrt{10}\left(d^+ \times \tilde{d}\right)^{(1)}_\mu \right\}$$

Dynamical symmetries of the IBM

- U(6) has the following subalgebras:

$$\mathrm{U}(5)=\left\{\left(d^{+}\times\tilde{d}\right)^{(0)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(1)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(2)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(3)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(4)}_{\mu}\right\}$$

$$\mathrm{SU}(3)=\left\{\left(d^{+}\times\tilde{d}\right)^{(1)}_{\mu},\left(s^{+}\times\tilde{d}+d^{+}\times\tilde{s}\right)^{(2)}_{\mu}-\sqrt{\tfrac{7}{4}}\left(d^{+}\times\tilde{d}\right)^{(2)}_{\mu}\right\}$$

$$\mathrm{SO}(6)=\left\{\left(d^{+}\times\tilde{d}\right)^{(1)}_{\mu},\left(s^{+}\times\tilde{d}+d^{+}\times\tilde{s}\right)^{(2)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(3)}_{\mu}\right\}$$

$$\mathrm{SO}(5)=\left\{\left(d^{+}\times\tilde{d}\right)^{(1)}_{\mu},\left(d^{+}\times\tilde{d}\right)^{(3)}_{\mu}\right\}$$

- Three solvable limits are found:

$\mathrm{U}(6)\quad \mathrm{U}(5)\quad \mathrm{SO}(5)\quad \mathrm{SO}(3)$

$\mathrm{U}(6)\quad \mathrm{SU}(3)\quad \mathrm{SO}(3)$

$\mathrm{U}(6)\quad \mathrm{SO}(6)\quad \mathrm{SO}(5)\quad \mathrm{SO}(3)$

Dynamical symmetries of the IBM

- The general IBM hamiltonian is

$$H_{\mathrm{IBM}} = \varepsilon_s n_s + \varepsilon_d n_d + \sum_{ijklJ} \upsilon_{ijkl}^{L} \left(b_i^+ \times b_j^+\right)^{(L)} \cdot \left(\tilde{b}_k \times \tilde{b}_l\right)^{(L)}$$

- An *entirely equivalent* form of H_{IBM} is

$$\begin{aligned} H_{\mathrm{IBM}} = {} & \eta_1 C_1[\mathrm{U}(6)] + \eta_2 C_1[\mathrm{U}(5)] + \eta_0 C_1[\mathrm{U}(6)] C_1[\mathrm{U}(5)] \\ & + \kappa_1 C_2[\mathrm{U}(6)] + \kappa_2 C_2[\mathrm{U}(5)] + \kappa_3 C_2[\mathrm{SU}(3)] \\ & + \kappa_4 C_2[\mathrm{SO}(6)] + \kappa_5 C_2[\mathrm{SO}(5)] + \kappa_6 C_2[\mathrm{SO}(3)] \end{aligned}$$

- The coefficients η_i and κ_j are certain combinations of the coefficients ε_i and $\upsilon^L{}_{ijkl}$.

The solvable IBM hamiltonians

- Without *N*-dependent terms in the hamiltonian (which are always diagonal)

$$H_{\mathrm{IBM}} = \square_1 C_1[\mathrm{U}(5)] + \square_1 C_2[\mathrm{U}(5)] + \square_2 C_2[\mathrm{SU}(3)] + \square_3 C_2[\mathrm{SO}(6)] + \square_4 C_2[\mathrm{SO}(5)] + \square_5 C_2[\mathrm{SO}(3)]$$

- If certain coefficients are zero, H_{IBM} can be written as a sum of commuting operators:

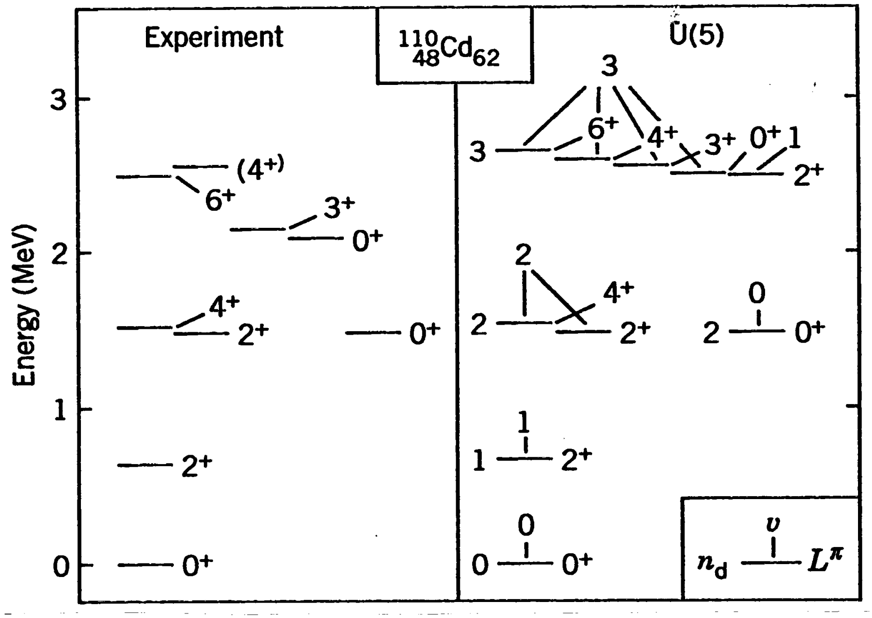
$$H_{\mathrm{U}(5)} = \square_1 C_1[\mathrm{U}(5)] + \square_1 C_2[\mathrm{U}(5)] + \square_4 C_2[\mathrm{SO}(5)] + \square_5 C_2[\mathrm{SO}(3)]$$

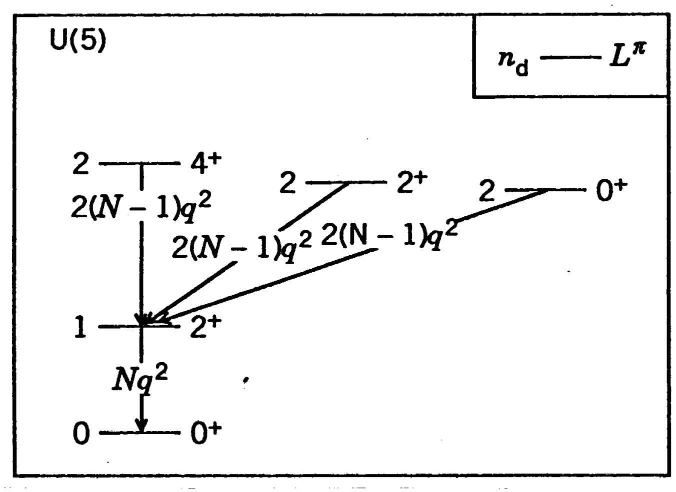
$$H_{\mathrm{SU}(3)} = \square_2 C_2[\mathrm{SU}(3)] + \square_5 C_2[\mathrm{SO}(3)]$$

$$H_{\mathrm{SO}(6)} = \square_3 C_2[\mathrm{SO}(6)] + \square_4 C_2[\mathrm{SO}(5)] + \square_5 C_2[\mathrm{SO}(3)]$$

The U(5) vibrational limit

- Spectrum of an anharmonic oscillator in 5 dimensions associated with the quadrupole oscillations of a droplet's surface.
- Conserved quantum numbers: n_d, □, L.

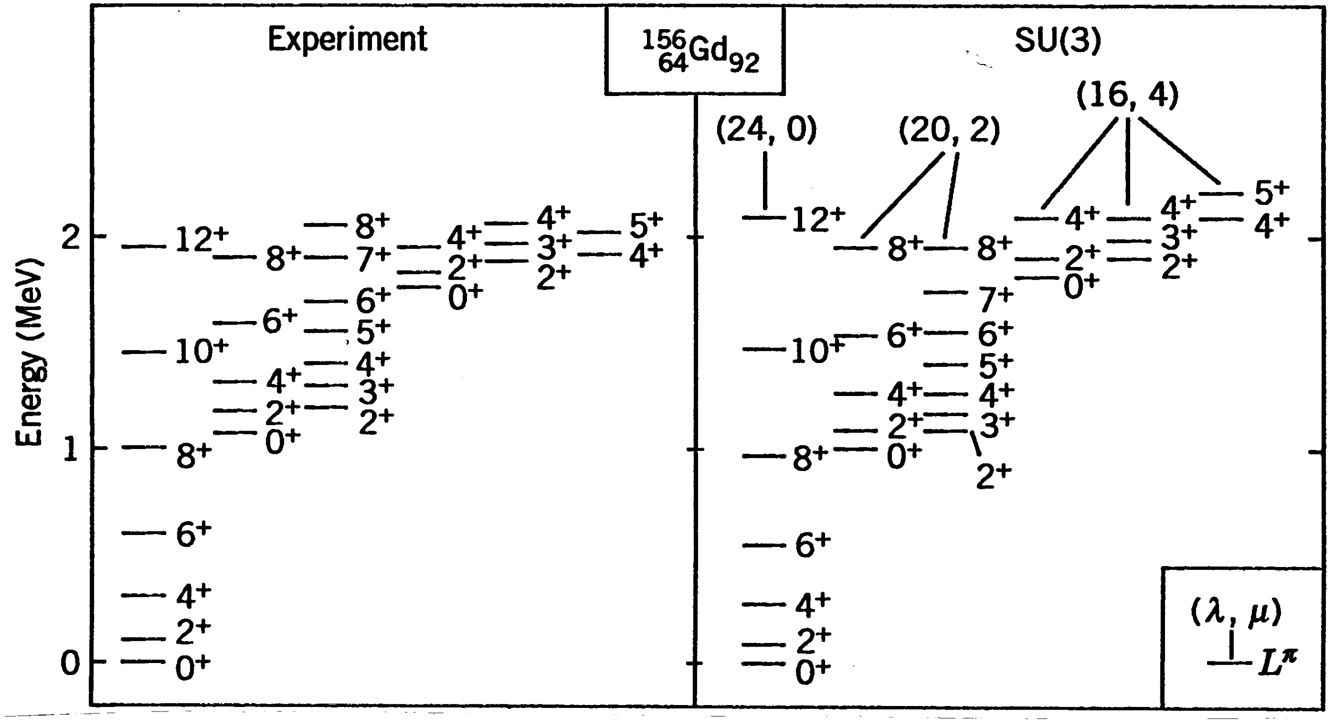




A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253
D. Brink *et al.*, Phys. Lett. **19** (1965) 413

The SU(3) rotational limit

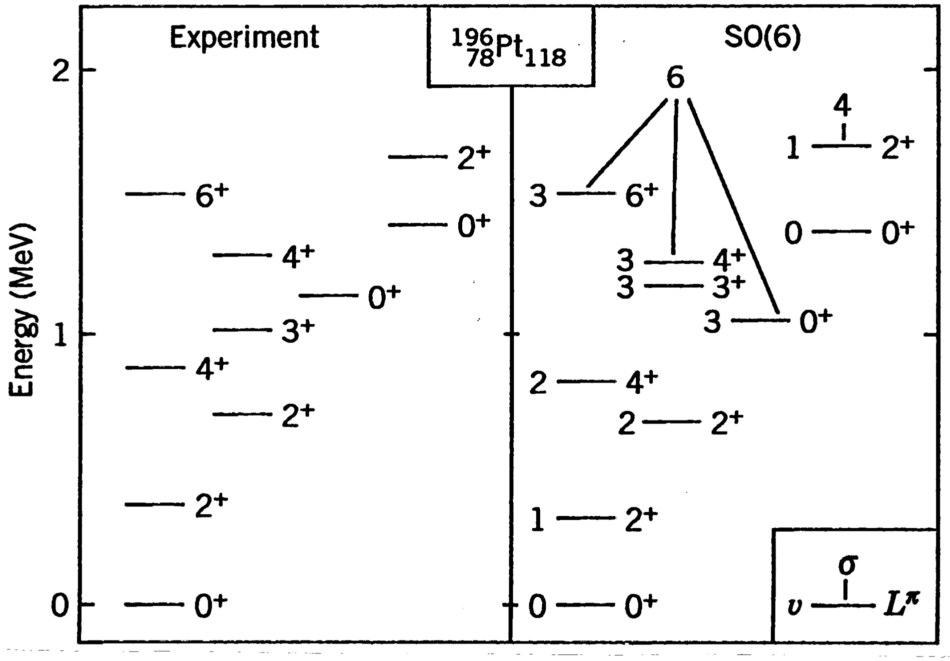
- Rotation-vibration spectrum with β- and γ-vibrational bands.
- Conserved quantum numbers: (λ,μ), *L*.

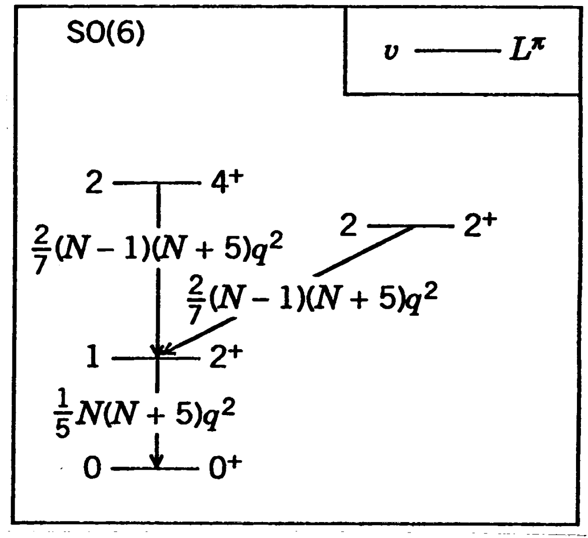


A. Arima & F. Iachello, Ann. Phys. (NY) **111** (1978) 201
A. Bohr & B.R. Mottelson, Dan. Vid. Selsk. Mat.-Fys. Medd. **27** (1953) No 16

The SO(6) □-unstable limit

- Rotation-vibration spectrum of a □-unstable body.
- Conserved quantum numbers: □, □, *L*.

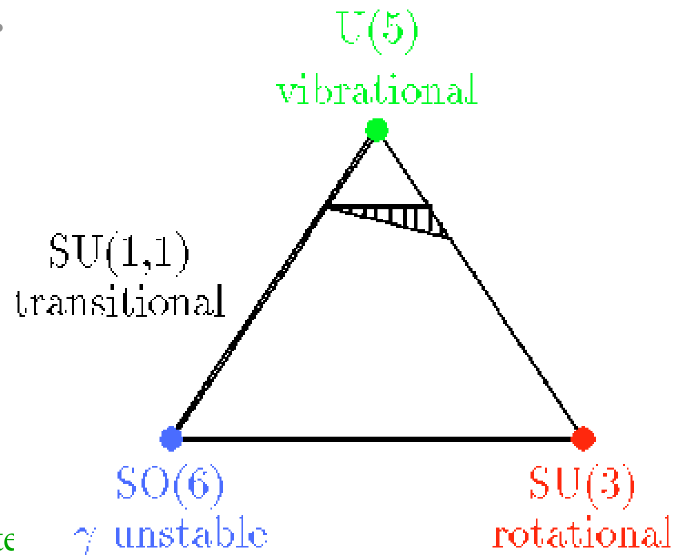




A. Arima & F. Iachello, Ann. Phys. (NY) **123** (1979) 468
L. Wilets & M. Jean, Phys. Rev. **102** (1956) 788

Synopsis of IBM symmetries

- Symmetry triangle of the IBM:
  - Three standard solutions: U(5), SU(3), SO(6).
  - SU(1,1) analytic solution for U(5) ▯ SO(6).
  - Hidden symmetries (parameter transformations).
  - Deformed-spherical coexistent phase.
  - Partial dynamical symmetries.
  - Critical-point symmetries?

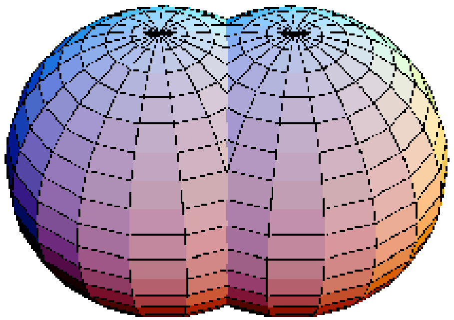


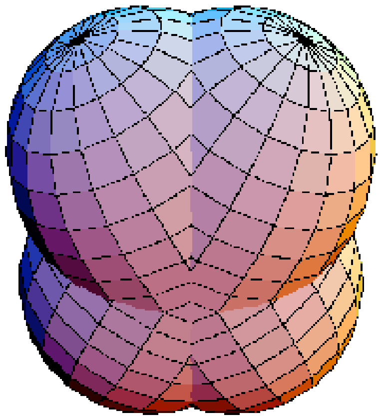
Extensions of the IBM

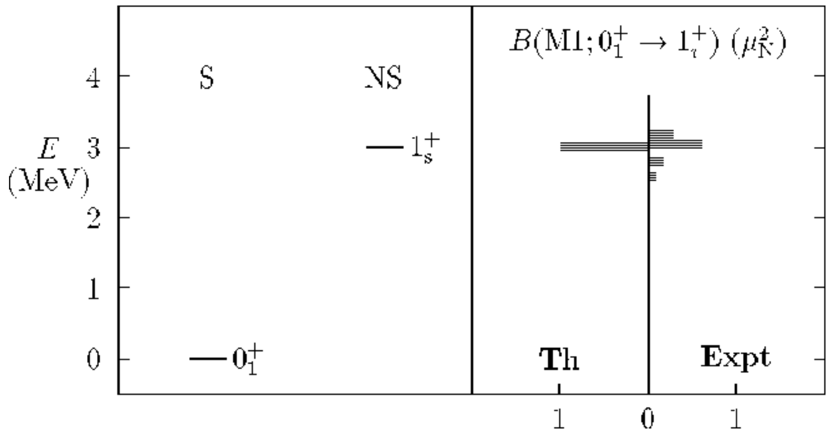
- Neutron and proton degrees freedom (IBM-2):
  - *F*-spin multiplets ($N_{\square}+N_{\square}$=constant).
  - Scissors excitations.
- Fermion degrees of freedom (IBFM):
  - Odd-mass nuclei.
  - Supersymmetry (doublets & quartets).
- Other boson degrees of freedom:
  - Isospin $T=0$ & $T=1$ pairs (IBM-3 & IBM-4).
  - Higher multipole (*g*,…) pairs.

Scissors excitations

- Collective displacement modes between neutrons and protons:
  - *Linear* displacement (giant dipole resonance): $\boldsymbol{R}_{\pi}$-$\boldsymbol{R}_{\nu}$ $\Rightarrow$ *E1* excitation.
  - *Angular* displacement (scissors resonance): $\boldsymbol{L}_{\pi}$-$\boldsymbol{L}_{\nu}$ $\Rightarrow$ *M1* excitation.



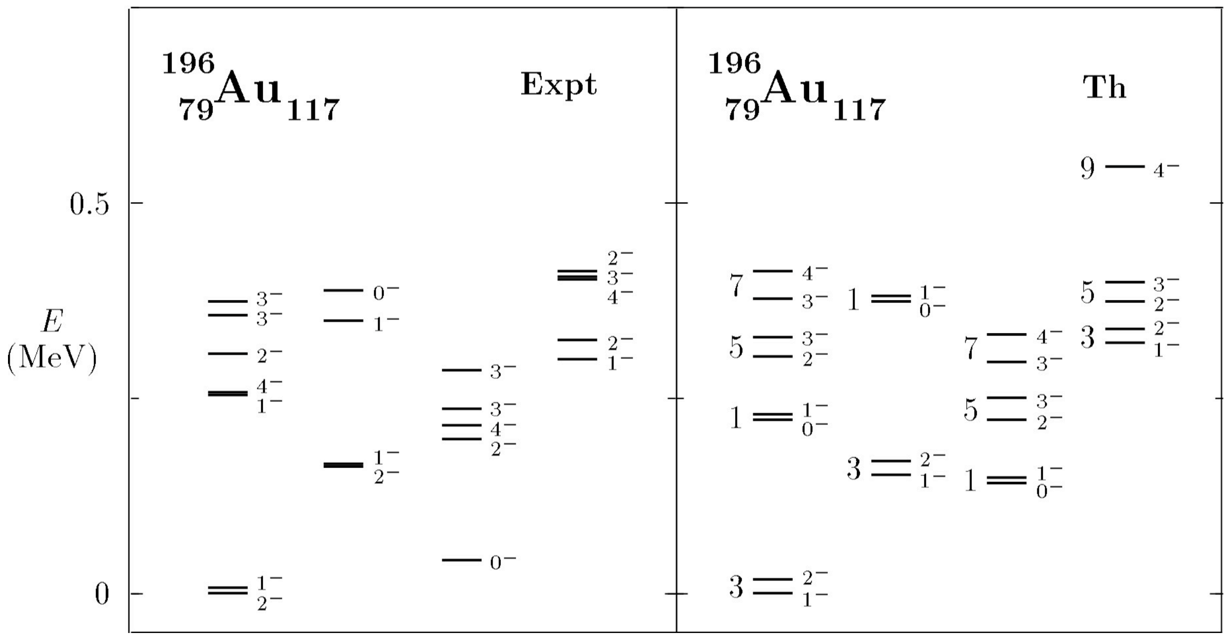




N. Lo Iudice & F. Palumbo, Phys. Rev. Lett. **41** (1978) 1532
F. Iachello, Phys. Rev. Lett. **53** (1984) 1427
D. Bohle *et al.*, Phys. Lett. B **137** (1984) 27

Supersymmetry

- A simultaneous description of even- and odd-mass nuclei (*doublets*) or of even-even, even-odd, odd-even and odd-odd nuclei (*quartets*).
- Example of ^{194}Pt, ^{195}Pt, ^{195}Au & ^{196}Au:



F. Iachello, Phys. Rev. Lett. **44** (1980) 772
P. Van Isacker *et al.*, Phys. Rev. Lett. **54** (1985) 653
A. Metz *et al.*, Phys. Rev. Lett. **83** (1999) 1542

Isospin invariant boson models

- Several versions of IBM depending on the fermion pairs that correspond to the bosons:
  - IBM-1: single type of pair.
  - IBM-2: $T=1$ nn ($M_T=-1$) and pp ($M_T=+1$) pairs.
  - IBM-3: full isospin $T=1$ triplet of nn ($M_T=-1$), np ($M_T=0$) and pp ($M_T=+1$) pairs.
  - IBM-4: full isospin $T=1$ triplet and $T=0$ np pair (with $S=1$).
- Schematic IBM-*k* has only *S* ($L=0$) pairs, full IBM-*k* has *S* ($L=0$) and *D* ($L=2$) pairs.

IBM-4

- Shell-model justification in *LS* coupling:

particle number	spatial symmetry	L	spin–isospin symmetry	$(\lambda\mu\nu)$	(S,T)
2	□□ (S)	$0^2, 2^2, 4$	□ □ (A)	(010)	(0,1) (1,0)
	□ □ (A)	$1, 2, 3$	□□ (S)	(200)	(0,0) (1,1)

- Advantages of IBM-4:
  - Boson states carry *L*, *S*, *T*, *J* and $(\lambda\mu\nu)$.
  - Mapping from the shell model to IBM-4 ⇒ shell-model test of the boson approximation.
  - Includes np pairs ⇒ important for *N*~*Z* nuclei.

J.P. Elliott & J.A. Evans, Phys. Lett. B **195** (1987) 1

IBM-4 with *L*=*0* bosons

- Schematic IBM-4 with bosons
  - $L=0$, $S=1$, $T=0$ □ $J=1$ (p boson, □=+1).
  - $L=0$, $S=0$, $T=1$ □ $J=0$ (s boson, □=+1).
- Two applications:
  - Microscopic (but schematic) study of the influence of the spin-orbit coupling on the structure of the superfluid condensate in $N=Z$ nuclei.
  - Phenomenological mass formula for N~Z nuclei.

Boson mapping of SO(8)

- Pairing hamiltonian in non-degenerate shells,

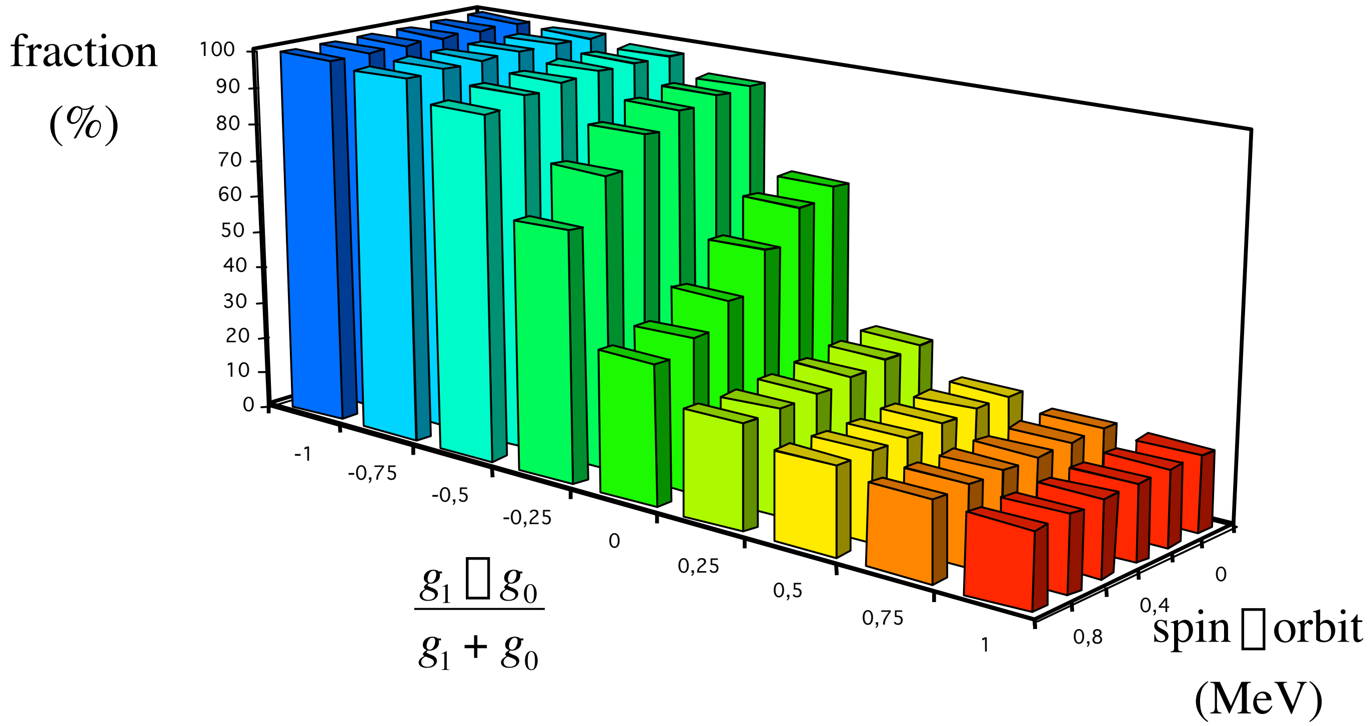
$$H = \sum_j \varepsilon_j n_j - g_0 S_+^{10} \cdot S_-^{10} - g_1 S_+^{01} \cdot S_-^{01}$$

- …is non-solvable in general but can be treated (numerically) via a boson mapping.
- Correspondence $S_+{}^{10} \rightarrow p^+$ and $S_+{}^{01} \rightarrow s^+$ leads to a schematic IBM-4 with $L=0$ bosons.
- Mapping of shell-model pairing hamiltonian completely determines boson energies and boson-boson interactions (*no* free parameters).

P. Van Isacker *et al.*, J. Phys. G **24** (1998) 1261

Pair structure and spin-orbit force

- Fraction of *p* bosons in the lowest *J=1*, *T=0* state for *N=Z=5* in the *pf* shell:



O. Juillet & S. Josse, Eur. Phys. A **2** (2000) 291

Mass formula for *N~Z* nuclei

- Schematic IBM-4 with *L=0* bosons has U(6) algebraic structure.
- The symmetry lattice of the model:

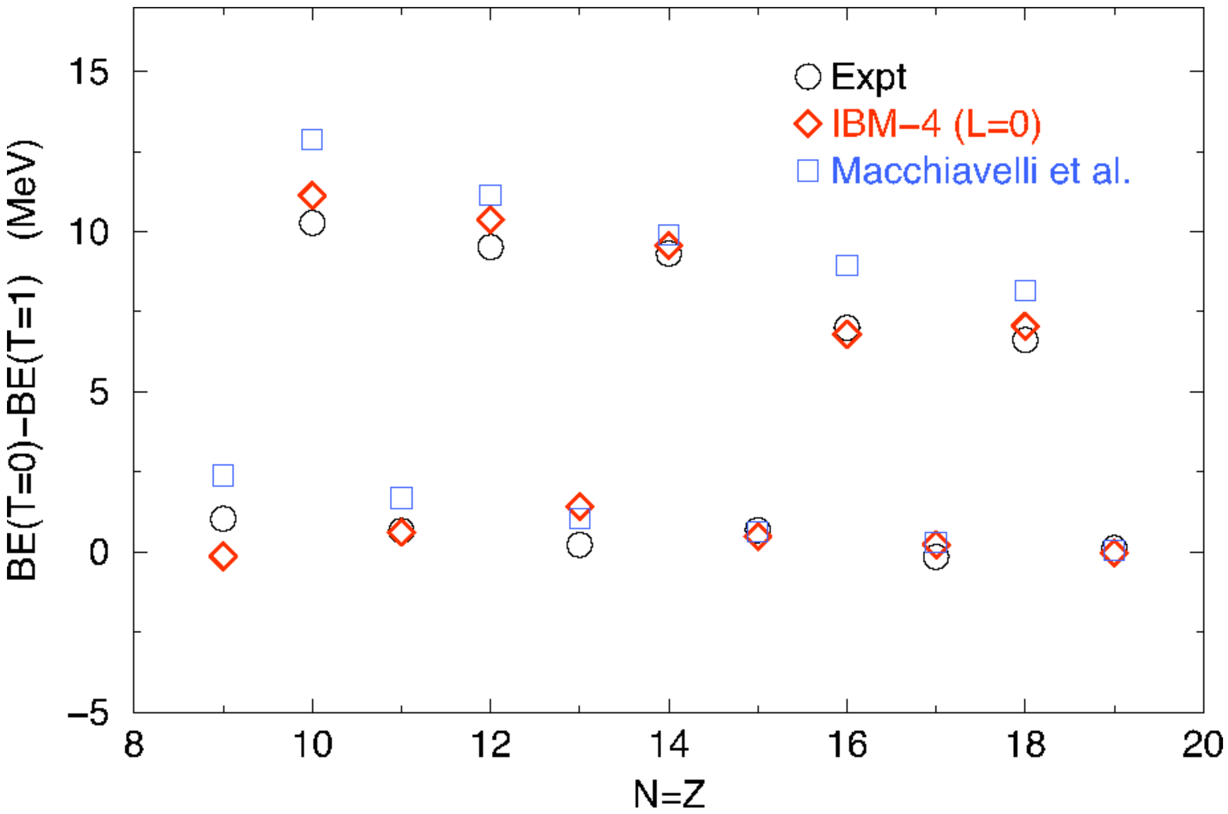
$$\mathrm{U}(6) \quad \begin{Bmatrix} \mathrm{U}_S(3) \quad \mathrm{U}_T(3) \\ \mathrm{SU}(4) \end{Bmatrix} \quad \mathrm{SO}_S(3) \quad \mathrm{SO}_T(3)$$

- Simple IBM-4 hamiltonian suggested by microscopy with *adjustable* parameters:

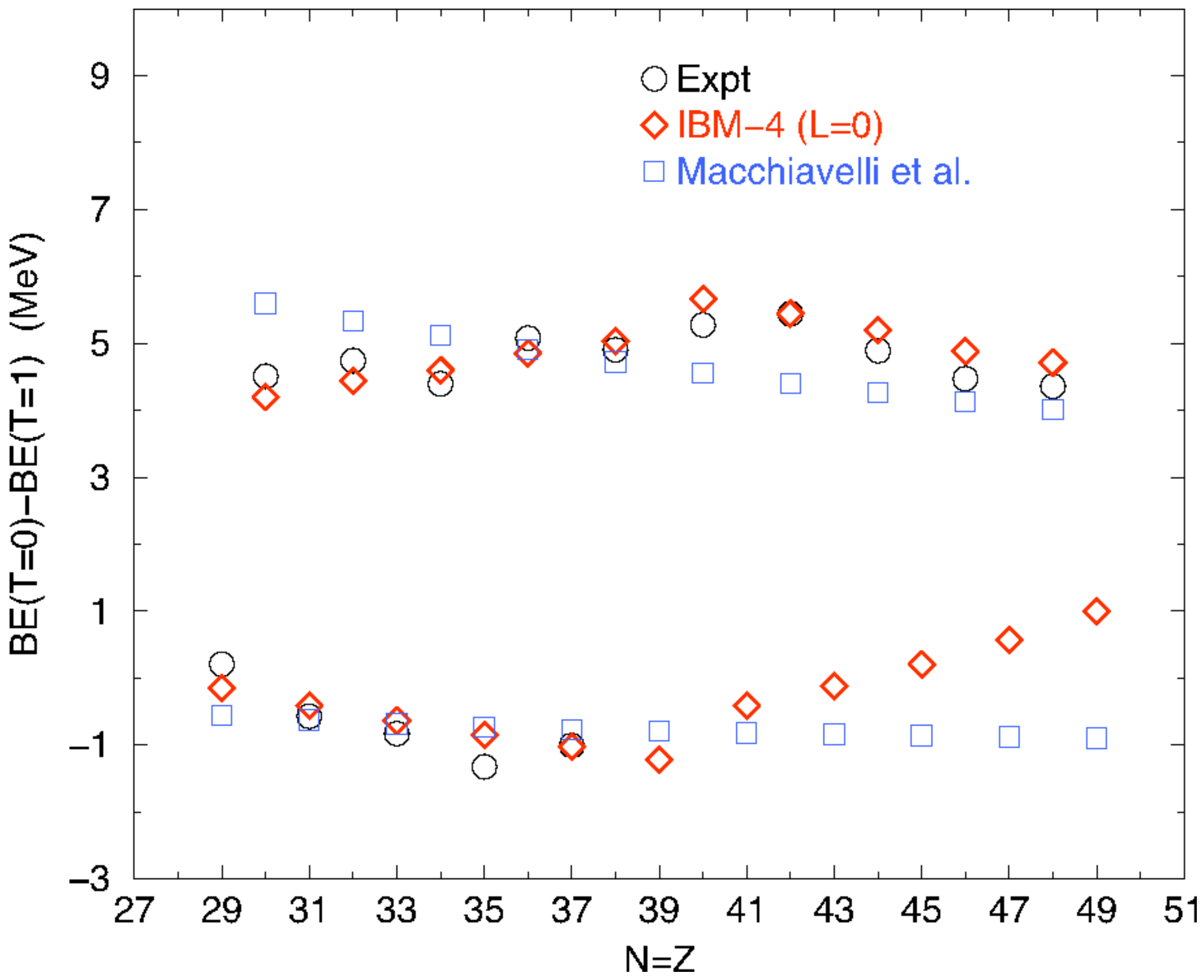
$$H = aC_1[\mathrm{U}(6)] + bC_2[\mathrm{U}(6)] + cC_2[\mathrm{SO}_T(3)] + dC_2[\mathrm{SU}(4)] + eC_2[\mathrm{U}_S(3)]$$

E. Baldini-Neto *et al.*, Phys. Rev. C **65** (2002) 064303

Binding energies of *sd* N=Z nuclei



Binding energies of *pf* $N=Z$ nuclei

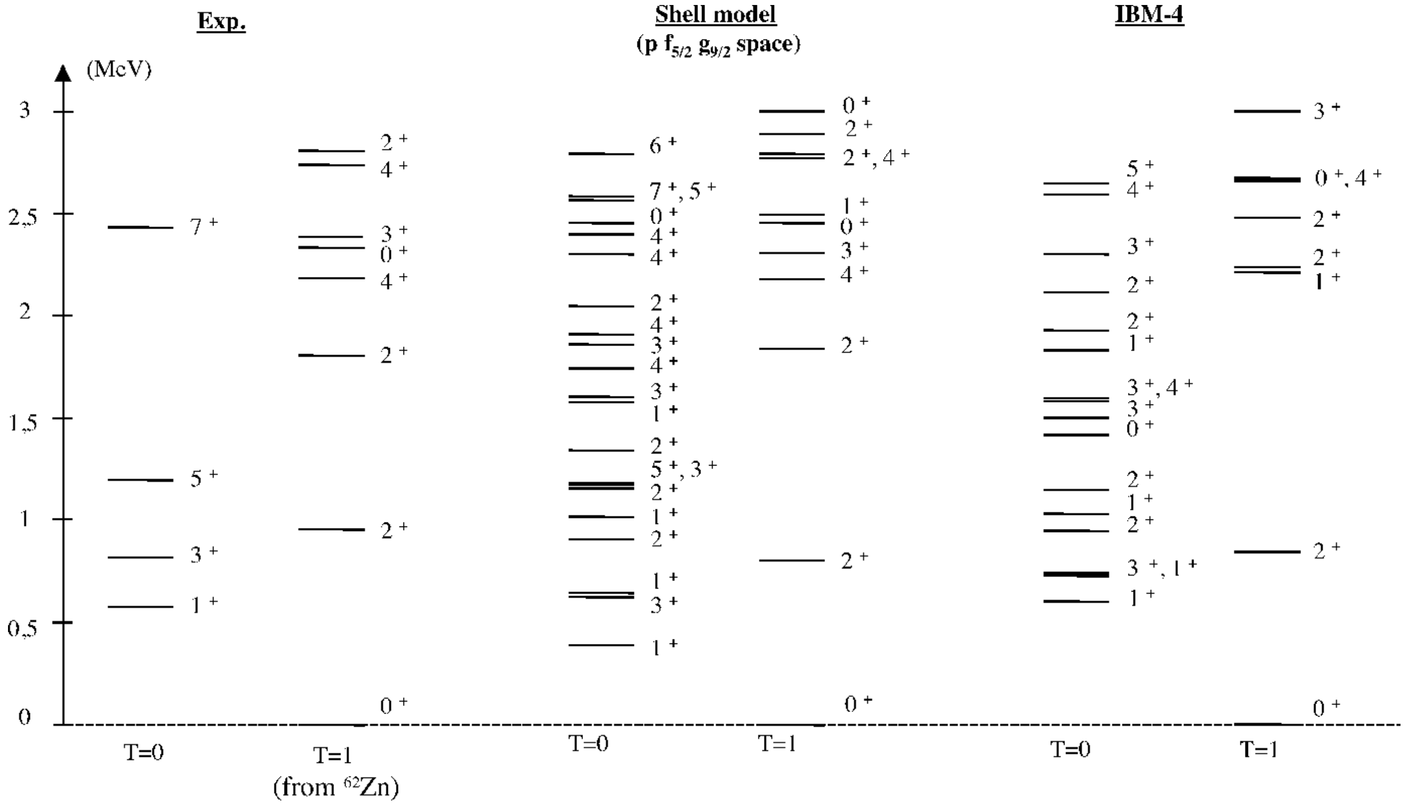


IBM-4 with $L=0$ & 2 bosons

- Full IBM-4 with bosons
  - $L=0$ & 2, $S=1$, $T=0$ □ $J=1^2,2,3$ (p^2,d,f bosons).
  - $L=0$ & 2, $S=0$, $T=1$ □ $J=0,2$ (s,d bosons).
- Aim: detailed spectroscopy for N~Z nuclei.
- IBM-4 hamiltonian derived from *realistic* shell-model interaction (*no* free parameters).
- Mapping relies on existence of approximate shell-model pseudo-SU(4) symmetry.
- Applications to nuclei between ^{56}Ni and ^{70}Br.

Shell-model and IBM-4 results

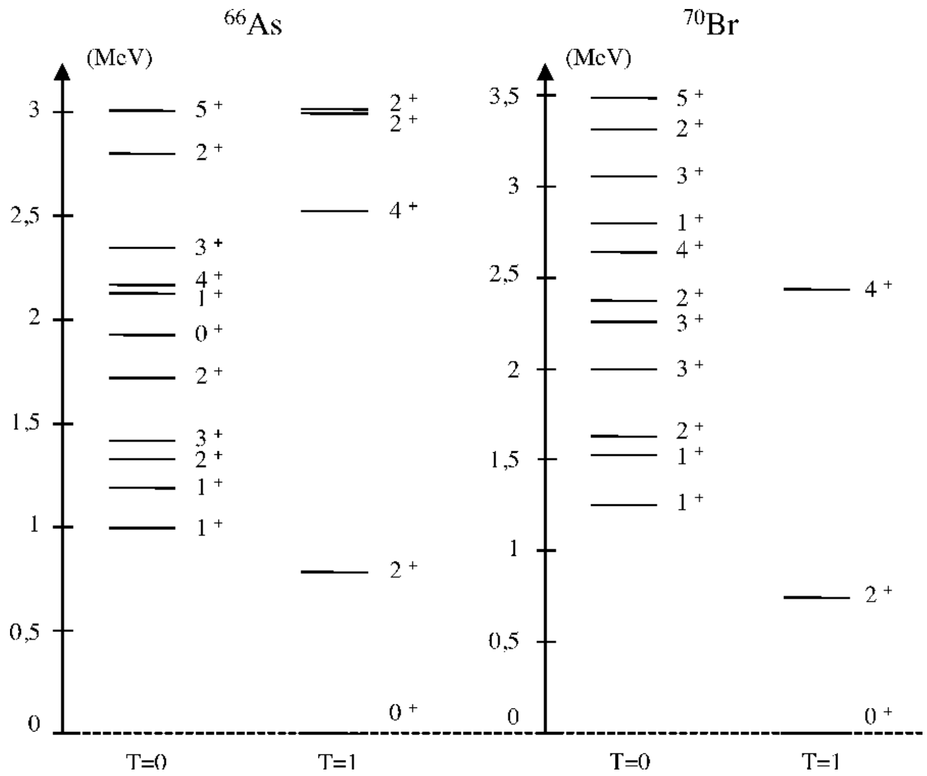
- Realistic interaction in $pf_{5/2}g_{9/2}$ space.
- Example: $A=62$.

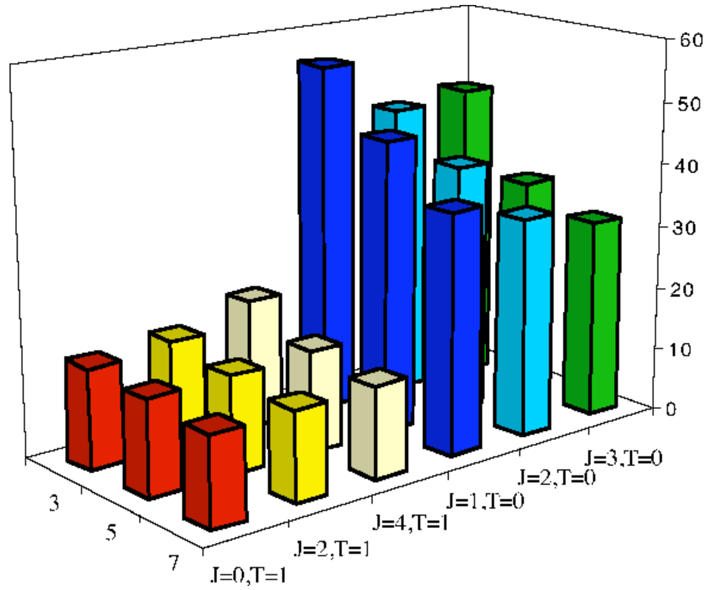


O. Juillet *et al.*, Phys. Rev. C **63** (2001) 054312

IBM-4 results

- Realistic interaction in $pf_{5/2}g_{9/2}$ space.
- Example: *A=66 & 70.*

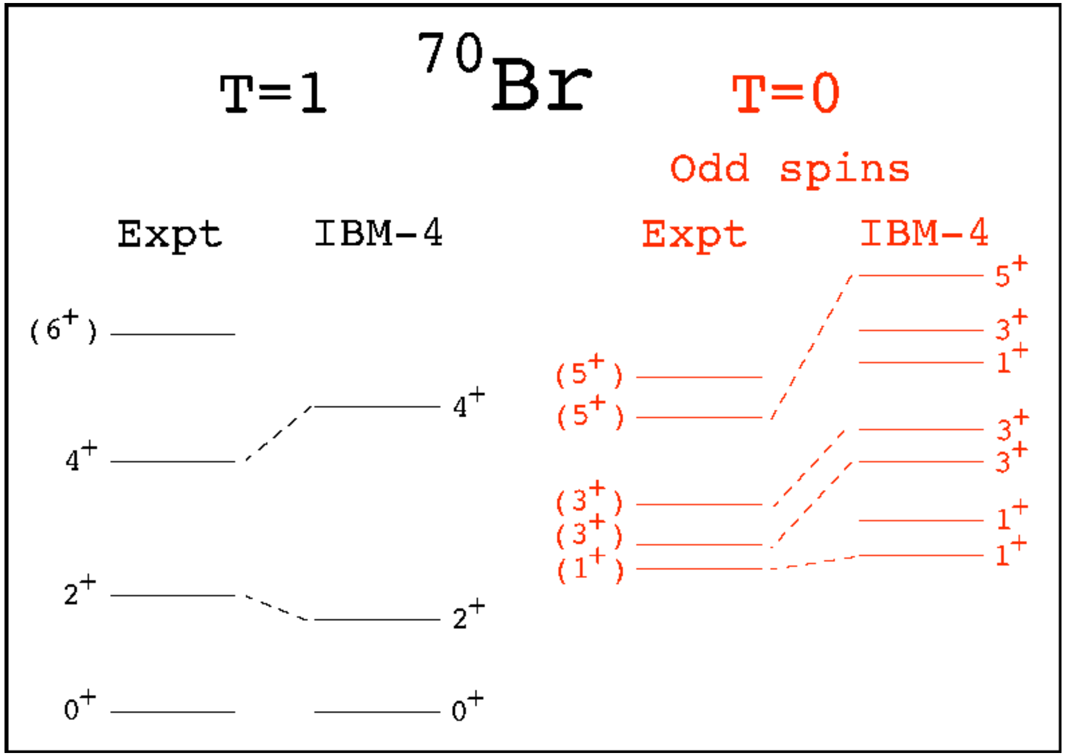




O. Juillet *et al.*, Phys. Rev. C **63** (2001) 054312

Spectroscopy of *N~Z* nuclei

- Many experiments on odd-odd nuclei: ^{46}V, ^{50}Mn, ^{54}Co, ^{58}Cu, ^{62}Ga, ^{66}As, ^{70}Br, ^{74}Rb.
- Evolution of $T=0$ versus $T=1$ states.
- Example: ^{70}Br.



D. Jenkins *et al.*, Phys. Rev. C **65** (2002) 064307

Algebraic many-body models

- The integrability of any quantum many-body (bosons and/or fermions) system can be analyzed with algebraic methods.
- Two nuclear examples:
  - Pairing *vs*. quadrupole interaction in the nuclear shell model.
  - Spherical, deformed and γ-unstable nuclei with *s*,*d*-boson IBM.

$$\mathrm{U}(6) \supset \left\{ \begin{array}{c} \mathrm{U}(5) \supset \mathrm{SO}(5) \\ \mathrm{SU}(3) \\ \mathrm{SO}(6) \supset \mathrm{SO}(5) \end{array} \right\} \supset \mathrm{SO}(3)$$

Other fields of physics

- Molecular physics:
  - U(4) vibron model with *s*,*p*-bosons.

$$\mathrm{U}(4) \supset \left\{ \begin{array}{c} \mathrm{U}(3) \\ \mathrm{SO}(4) \end{array} \right\} \supset \mathrm{SO}(3)$$

  - Coupling of many SU(2) algebras for polyatomic molecules.
- Similar applications in hadronic, atomic, solid-state, polymer physics, quantum dots…
- Use of *non-compact* groups and algebras for scattering problems.

F. Iachello, 1975 to now

Quantum dots

- Aggregate of electrons confined by a harmonic potential to a (usually) circular 2-dimensional region.
- Electrons interact via a repulsive Coulomb force (maybe screened $\sim e^{-\Box r}/r$).
- Coulomb force is *spin scalar* □ *LS* coupling.
- Physically meaningful classification could be

$$\mathrm{U}(2\Box)\quad \mathrm{U}(\Box)\quad \mathrm{SU}_S(2)\quad \cdots\quad \mathrm{SO}_L(2)\quad \mathrm{SO}_S(2)\quad \mathrm{SO}_J(2)$$