Symmetries of the nuclear shell model

P. Van Isacker, GANIL, France

Racah's SU(2) pairing model

Wigner's SU(4) symmetry

Elliott's SU(3) model of rotation

Generalized pairing models*

Generalized SU(3) models*

Nuclear shell model

- Many-body quantum mechanical problem:

$$H = \sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k<l}^{A} W(\xi_k, \xi_l) \qquad \left[\xi_k \equiv \{\vec{r}_k, \vec{s}_k, \vec{t}_k\}, \quad \vec{p}_k = -i\hbar\vec{\nabla}_k\right]$$

$$= \underbrace{\sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + V(\xi_k)\right]}_{\text{mean field}} + \underbrace{\left[\sum_{k<l}^{A} W(\xi_k, \xi_l) - \sum_{k=1}^{A} V(\xi_k)\right]}_{\text{residual interaction}}$$

- Independent-particle assumption. Choose *V* and neglect residual interaction:

$$H \approx H_{\mathrm{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + V(\xi_k)\right]$$

Independent-particle shell model

- Solution for one particle:

$$\left(\frac{p_k^2}{2m_k} + V(\xi_k)\right)\phi_i(k) = E_i \phi_i(k) \qquad \left[\phi_i(k) \equiv \phi_i\left(\vec{r}_k, \vec{s}_k, \vec{t}_k\right)\right]$$

- Solution for many particles:

$$\Phi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A) = \prod_{k=1}^{A} \phi_{i_k}(k)$$

$$H_{\mathrm{IP}} \Phi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A) = \left(\sum_{k=1}^{A} E_{i_k}\right) \Phi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A)$$

Independent-particle shell model

- Antisymmetric solution for many fermions (**Slater** determinant):

$$\Psi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A) = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_{i_1}(1) & \phi_{i_1}(2) & \ldots & \phi_{i_1}(A) \\ \phi_{i_2}(1) & \phi_{i_2}(2) & \ldots & \phi_{i_2}(A) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{i_A}(1) & \phi_{i_A}(2) & \ldots & \phi_{i_A}(A) \end{vmatrix}$$

- Example for $A=2$ fermions:

$$\Psi_{i_1 i_2}(1,2) = \frac{1}{\sqrt{2}}\left[\phi_{i_1}(1)\phi_{i_2}(2) - \phi_{i_1}(2)\phi_{i_2}(1)\right]$$

Hartree-Fock approximation

- Vary ϕ_i (*i.e. V*) to minize the expectation value of *H* in a Slater determinant:

$$\delta \frac{\int \Psi^*_{i_1 i_2 \ldots i_A}(1,2,\ldots,A)\, H\, \Psi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A)\, d\xi_1 d\xi_2 \ldots d\xi_A}{\int \Psi^*_{i_1 i_2 \ldots i_A}(1,2,\ldots,A)\, \Psi_{i_1 i_2 \ldots i_A}(1,2,\ldots,A)\, d\xi_1 d\xi_2 \ldots d\xi_A} = 0$$

- Application requires choice of *H*. Many global parametrizations (Skyrme, Gogny,…) have been developed.

Poor man's Hartree-Fock

- Choose a simple, analytically solvable *V* that approximates the microscopic HF potential:

$$H_{\mathrm{IP}} = \sum_{k=1}^{A} \left(\frac{p_k^2}{2m} + \frac{1}{2} m \omega^2 r_k^2 - \zeta_{\mathrm{ls}} \vec{l}_k \cdot \vec{s}_k - \zeta_{\mathrm{ll}} l_k^2 \right)$$

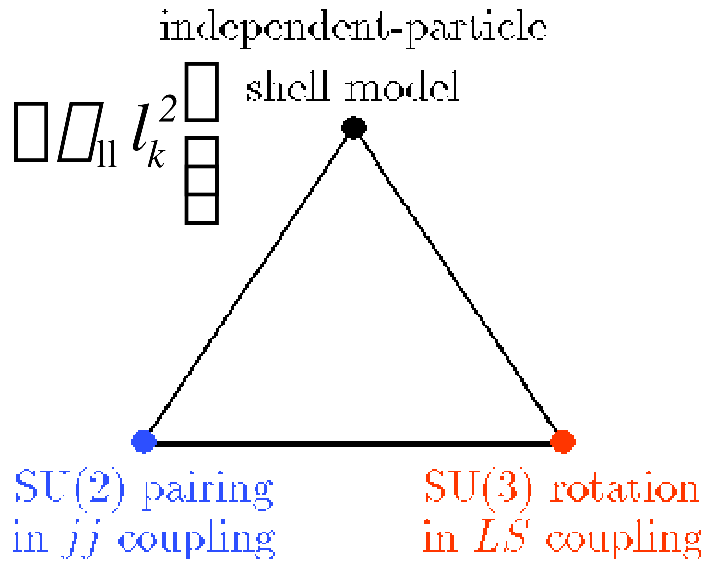
- Contains
  - Harmonic oscillator potential with constant ω.
  - Spin-orbit term with strength ζ_{ls}.
  - Orbit-orbit term with strength ζ_{ll}.
- Adjust ω, ζ_{ls} and ζ_{ll} to best reproduce HF.

Symmetries of the shell model

- Three bench-mark solutions:
  - No residual interaction ⇒ IP shell model.
  - Pairing (in *jj* coupling) ⇒ **Racah**'s SU(2).
  - Quadrupole (in *LS* coupling) ⇒ **Elliott**'s SU(3).
- Symmetry triangle:

$$H_{\mathrm{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{1}{2} m\omega^2 r_k^2 - \zeta_{\mathrm{ls}} \vec{l}_k \cdot \vec{s}_k - \zeta_{\mathrm{ll}} l_k^2 \right]$$

$$+ \sum_{k<l}^{A} V_{\mathrm{RI}}(\xi_k, \xi_l)$$

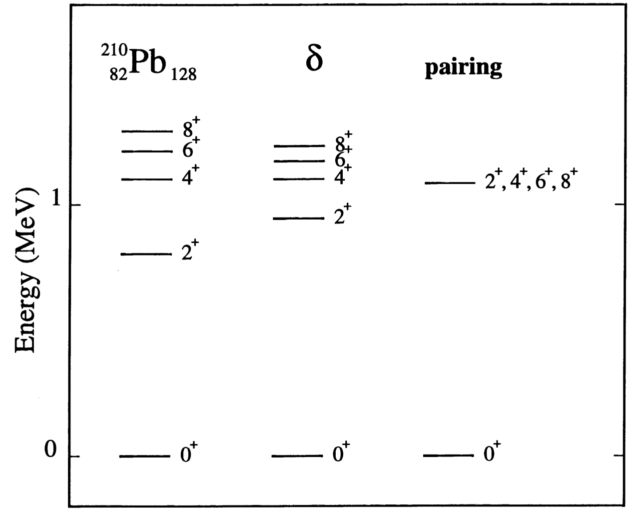


Racah's SU(2) pairing model

- Assume large spin-orbit splitting $\square_{\mathrm{ls}}$ which implies a *jj* coupling scheme.
- Assume pairing interaction in a single-*j* shell:

$$\langle j^2 JM_J | V_{\mathrm{pairing}} | j^2 JM_J \rangle = \begin{cases} -\frac{1}{2}(2j+1)g, & J = 0 \\ 0, & J \neq 0 \end{cases}$$

- Spectrum of ^{210}Pb:



SU(2) quasi-spin formalism

- The pairing hamiltonian,

$$H = E_0 - gS_+ \cdot S_-, \quad S_+ = \tfrac{1}{2}\sqrt{2j+1}\left(a_j^+ \times a_j^+\right)_0^{(0)}, \quad S_- = \left(S_+\right)^+$$

- …has a *quasi-spin* SU(2) algebraic structure:

$$\left[S_+, S_-\right] = \tfrac{1}{2}\left(2n_j - 2j - 1\right) \equiv 2S_z, \quad \left[S_z, S_\pm\right] = \pm S_\pm$$

- *H* has SU(2) ⊃ SO(2) dynamical symmetry:

$$-gS_+ \cdot S_- = -g\left(S^2 - S_z^2 + S_z\right)$$

- Eigensolutions of pairing hamiltonian:

$$-gS_+ \cdot S_- \left|SM_S\right\rangle = -g\left(S(S+1) - M_S(M_S - 1)\right)\left|SM_S\right\rangle$$

A. Kerman, Ann. Phys. (NY) **12** (1961) 300

Interpretation of pairing solution

- Quasi-spin labels S and M_S are related to nucleon number n_j and seniority υ:

$$S = \tfrac{1}{4}(2j - \upsilon + 1), \quad M_S = \tfrac{1}{4}(2n_j - 2j - 1)$$

- Energy eigenvalues in terms of n_j and υ:

$$\langle j^{n_j}\upsilon J M_J | -g S_+ \cdot S_- | j^{n_j}\upsilon J M_J \rangle = -\tfrac{1}{4} g (n_j - \upsilon)(2j - n_j + \upsilon + 3)$$

- Eigenstates have an S-pair character:

$$| j^{n_j}\upsilon J M_J \rangle \propto (S_+)^{(n_j - \upsilon)/2} | j^{\upsilon}\upsilon J M_J \rangle$$

- Seniority υ is the number of nucleons *not* in S pairs (pairs coupled to $J=0$).

G. Racah, Phys. Rev. **63** (1943) 367

Pairing and superfluidity

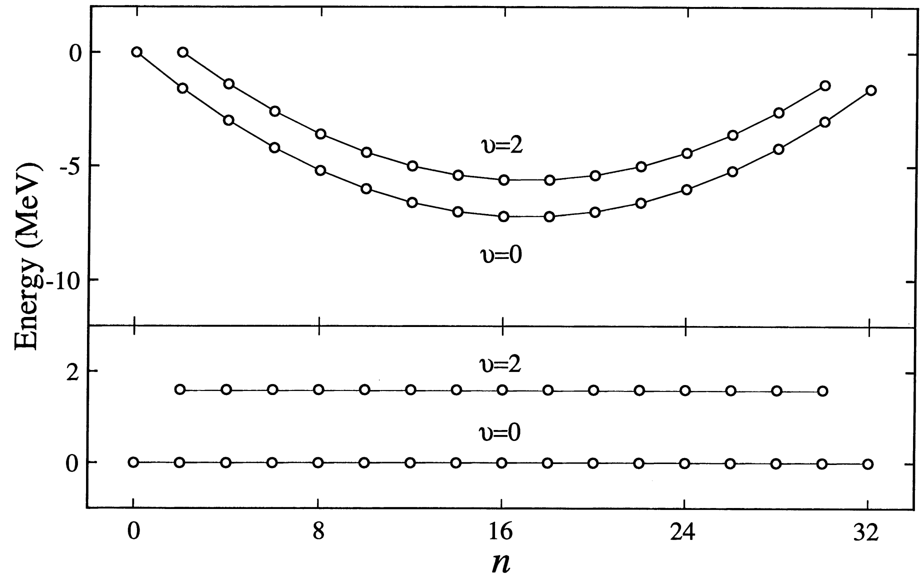
- Ground states of a pairing hamiltonian have *superfluid* character:
  - Even-even nucleus (□=*0*): $\left(S_+\right)^{n_j/2}|o\rangle$
  - Odd-mass nucleus (□=*1*): $a_j^+\left(S_+\right)^{n_j/2}|o\rangle$
- Nuclear superfluidity leads to
  - Constant energy of first 2^+ in even-even nuclei.
  - Odd-even staggering in masses.
  - Two-particle (2n or 2p) transfer enhancement.

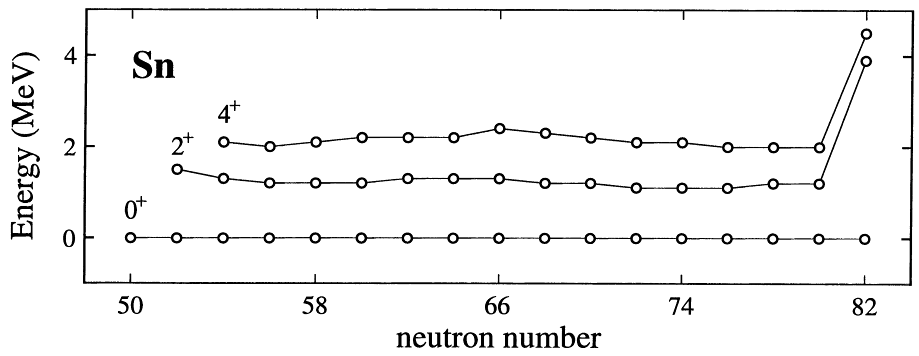
Superfluidity in semi-magic nuclei

- Even-even nuclei:
  - Ground state has $\upsilon=0$.
  - First-excited state has $\upsilon=2$.
  - Pairing produces constant energy gap:

$$E_{\mathrm{x}}\left(2_1^+\right) = \tfrac{1}{2}(2j+1)g$$

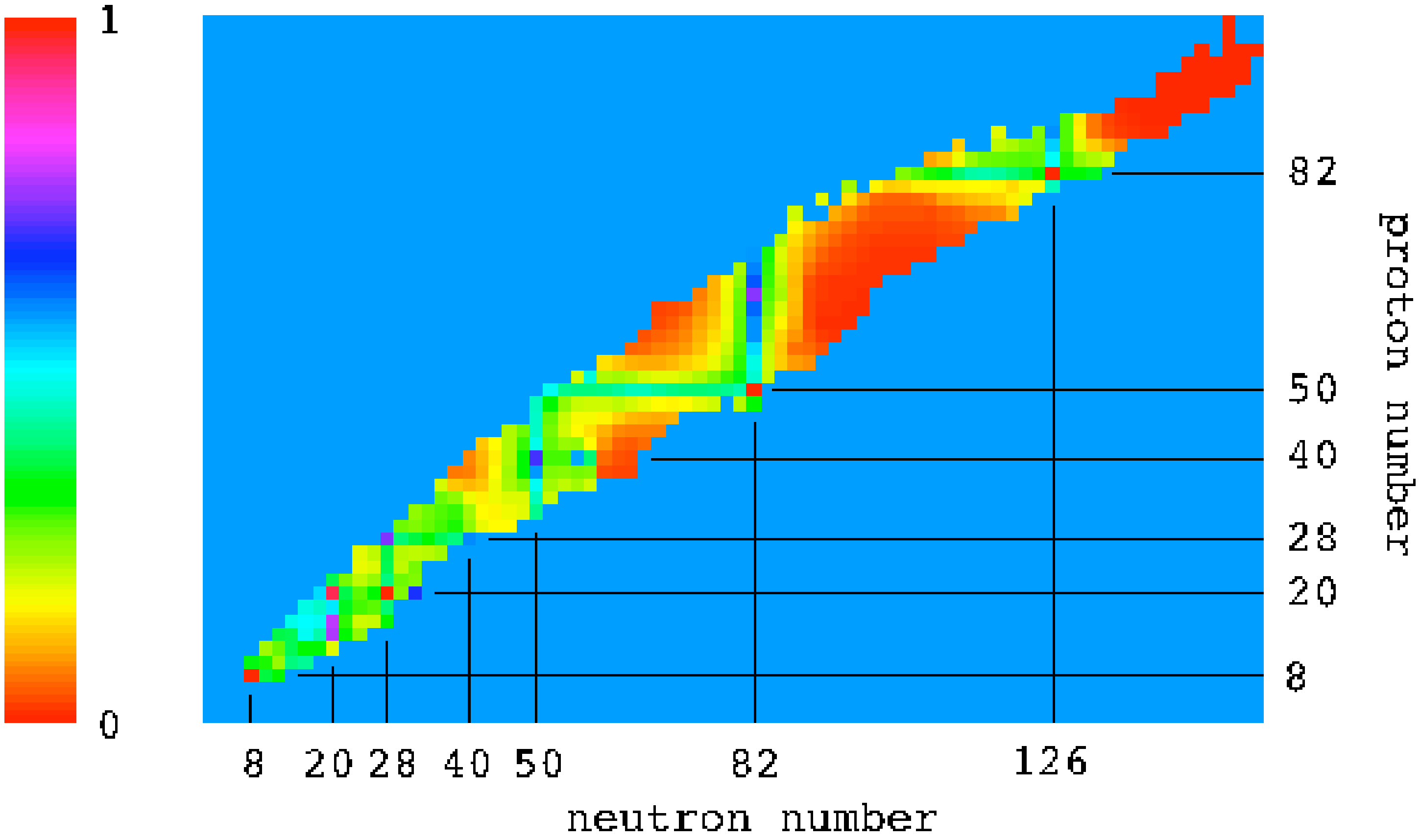
- Example of Sn nuclei:





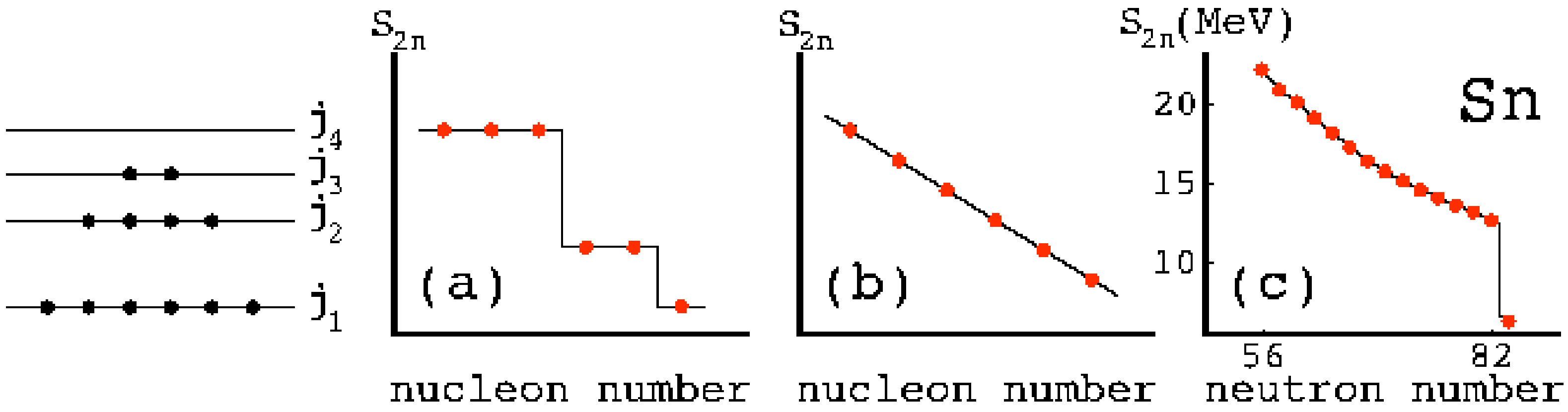
Shell structure of nuclei

- Direct evidence for nuclear shell structure is obtained from $E_x(2^+)$ (here scaled by $A^{1/3}$):



Superfluidity versus magicity

- Two-nucleon separation energies S_{2n}:
  - (a) Shell splitting dominates over interaction.
  - (b) Interaction dominates over shell splitting.
  - (c) S_{2n} in tin isotopes.



Generalized pairing models

- Trivial generalization from a single-*j* shell to several degenerate *j* shells:

$$S_+ = \frac{1}{2} \sum_j \sqrt{2j+1} \left(a_j^+ \times a_j^+ \right)^{(0)}_0$$

- Pairing with neutrons and protons:
  - *T*=*1* pairing: SO(5).
  - *T*=*0* & *T*=*1* pairing: SO(8).
- Non-degenerate shells:
  - **Talmi**'s generalized seniority.
  - **Richardson**'s integrable pairing model.

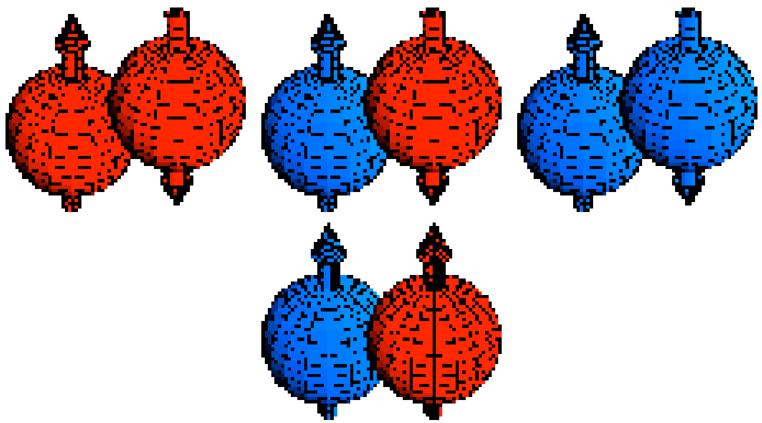
Pairing with neutrons and protons

- For neutrons and protons *two* pairs and hence *two* pairing interactions are possible:
  - Isoscalar (*S=1,T=0*):

$$\square S_+^{10}\cdot S_-^{10},\quad S_+^{10}=\sqrt{l+\tfrac{1}{2}}\left(a^+_{l\frac{1}{2}\frac{1}{2}}\times a^+_{l\frac{1}{2}\frac{1}{2}}\right)^{(010)},\quad S_-^{10}=\left(S_+^{10}\right)^+$$

  - Isovector (*S=0,T=1*):

$$\square S_+^{01}\cdot S_-^{01},\quad S_+^{01}=\sqrt{l+\tfrac{1}{2}}\left(a^+_{l\frac{1}{2}\frac{1}{2}}\times a^+_{l\frac{1}{2}\frac{1}{2}}\right)^{(001)},\quad S_-^{01}=\left(S_+^{01}\right)^+$$



Neutron-proton pairing hamiltonian

- A hamiltonian with two pairing terms,

$$H = -g_0 S_+^{10} \cdot S_-^{10} - g_1 S_+^{01} \cdot S_-^{01}$$

- …has an SO(8) algebraic structure.
- *H* is solvable (or has dynamical symmetries) for $g_0=0$, $g_1=0$ and $g_0=g_1$.

SO(8) ‘quasi-spin’ formalism

- A closed algebra is obtained with the pair operators $\boldsymbol{S}_{\pm}$ with in addition

$$n = 2\sqrt{2l+1}\left[a^{+}_{l\frac{1}{2}\frac{1}{2}} \square\, a^{+}_{l\frac{1}{2}\frac{1}{2}}\right]^{(000)}_{000}, \quad Y_{\square\square} = \sqrt{2l+1}\left[a^{+}_{l\frac{1}{2}\frac{1}{2}} \square\, a^{+}_{l\frac{1}{2}\frac{1}{2}}\right]^{(011)}_{0\square\square}$$

$$S_{\square} = \sqrt{2l+1}\left[a^{+}_{l\frac{1}{2}\frac{1}{2}} \square\, a^{+}_{l\frac{1}{2}\frac{1}{2}}\right]^{(010)}_{0\square 0}, \quad T_{\square} = \sqrt{2l+1}\left[a^{+}_{l\frac{1}{2}\frac{1}{2}} \square\, a^{+}_{l\frac{1}{2}\frac{1}{2}}\right]^{(001)}_{00\square}$$

- This set of 28 operators forms the Lie algebra SO(8) with subalgebras

$$\mathrm{SO}(6) \square \mathrm{SU}(4) = \{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{Y}\}, \quad \mathrm{SO}_S(5) = \{n, \boldsymbol{S}, \boldsymbol{S}^{10}_{\pm}\},$$

$$\mathrm{SO}_T(5) = \{n, \boldsymbol{T}, \boldsymbol{S}^{01}_{\pm}\}, \quad \mathrm{SO}_S(3) = \{\boldsymbol{S}\}, \quad \mathrm{SO}_T(3) = \{\boldsymbol{T}\}$$

B.H. Flowers & S. Szpikowski, Proc. Phys. Soc. **84** (1964) 673

Solvable limits of the SO(8) model

- Pairing interactions can expressed as follows:

$$S_+^{01}\cdot S_-^{01} = \tfrac{1}{2}C_2[\mathrm{SO}_T(5)] - \tfrac{1}{2}C_2[\mathrm{SO}_T(3)] - \tfrac{1}{8}(2l-n+1)(2l-n+7)$$

$$S_+^{01}\cdot S_-^{01} + S_+^{10}\cdot S_-^{10} = \tfrac{1}{2}C_2[\mathrm{SO}(8)] - \tfrac{1}{2}C_2[\mathrm{SO}(6)] - \tfrac{1}{8}(2l-n+1)(2l-n+13)$$

$$S_+^{10}\cdot S_-^{10} = \tfrac{1}{2}C_2[\mathrm{SO}_S(5)] - \tfrac{1}{2}C_2[\mathrm{SO}_S(3)] - \tfrac{1}{8}(2l-n+1)(2l-n+7)$$

- Symmetry *lattice* of the SO(8) model:

$$\mathrm{SO}(8) \supset \left\{\begin{array}{c} \mathrm{SO}_S(5) \quad \mathrm{SO}_T(3) \\ \mathrm{SO}(6) \simeq \mathrm{SU}(4) \\ \mathrm{SO}_T(5) \quad \mathrm{SO}_S(3) \end{array}\right\} \supset \mathrm{SO}_S(3) \quad \mathrm{SO}_T(3)$$

- ⇒ Analytic solutions for $g_0=0$, $g_1=0$ and $g_0=g_1$.

Superfluidity of *N*=*Z* nuclei

- Ground state of a *T=1* pairing hamiltonian for identical nucleons is superfluid, $(S_+)^{n/2}|\mathrm{o}\square$
- Ground state of a *T=0* & *T=1* pairing hamiltonian with equal number of neutrons and protons has *different* superfluid character:

$$\left(\cos\square S_+^{10}\cdot S_+^{10}\ \square\sin\square S_+^{01}\cdot S_+^{01}\right)^{n/4}|\mathrm{o}\rangle$$

- □ Condensate of □'s (□ depends on g_0/g_1).
- Observations:
  - Isoscalar component in condensate survives only in *N*~*Z* nuclei, if anywhere at all.
  - Spin-orbit term *reduces* isoscalar component.

Superfluidity versus magicity

- Superfluid ground state for degenerate shells:

$$\left(\sum_j S_+(j)\right)^{n/2}|o\rangle$$

- 'Superfluid' ground state for non-degenerate shells (Richardson-Gaudin-Dukelsky):

$$\prod_{\alpha=1}^{n/2}\left(\sum_j \frac{1}{2\varepsilon_j - e_\alpha} S_+(j)\right)|o\rangle$$

$$1 - g\sum_j \frac{2j+1}{2\varepsilon_j - e_\alpha} + 4g\sum_{\beta(\neq\alpha)} \frac{1}{e_\beta - e_\alpha} = 0, \quad \alpha = 1,2,\ldots,n/2$$

R.W. Richardson, Phys. Lett. **5** (1963) 82

Wigner's SU(4) symmetry

- Assume the nuclear hamiltonian is invariant under spin *and* isospin rotations:

$$\left[H_{\text{nucl}}, S_{\square}\right] = \left[H_{\text{nucl}}, T_{\square}\right] = \left[H_{\text{nucl}}, Y_{\square\square}\right] = 0$$

$$S_{\square} = \underset{k=1}{\overset{A}{\square}} s_{\square}(k), \quad T_{\square} = \underset{k=1}{\overset{A}{\square}} t_{\square}(k), \quad Y_{\square\square} = \underset{k=1}{\overset{A}{\square}} s_{\square}(k) t_{\square}(k)$$

- Since $\{S_{\square}, T_{\square}, Y_{\square\square}\}$ form an SU(4) algebra:
  - H_{nucl} has SU(4) symmetry.
  - Total spin $\boldsymbol{S}$, total orbital angular momentum $\boldsymbol{L}$, total isospin $\boldsymbol{T}$ and SU(4) labels ($\square\square\square$) are conserved quantum numbers.

E.P. Wigner, Phys. Rev. **51** (1937) 106
F. Hund, Z. Phys. **105** (1937) 202

Physical origin of SU(4) symmetry

- SU(4) labels specify the separate spatial and spin-isospin symmetry of the wavefunction:

particle number	spatial symmetry	L	spin–isospin symmetry	$(\lambda\mu\nu)$	(S,T)
1	$\square$	$0,2$	$\square$	(100)	$(\frac{1}{2},\frac{1}{2})$
2	$\square\square$ (S)	$0^2,2^2,4$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ (A)	(010)	(0,1) (1,0)
	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ (A)	$1,2,3$	$\square\square$ (S)	(200)	(0,0) (1,1)

Note: S stands for symmetric, A for antisymmetric.

- Nuclear interaction is short-range attractive and hence *favours maximal spatial symmetry*.

Breaking of SU(4) symmetry

- *Non-dynamical* breaking of SU(4) symmetry as a consequence of
  - Spin-orbit term in nuclear mean field.
  - Coulomb interaction.
  - Spin-dependence of residual interaction.
- Evidence for SU(4) symmetry breaking from
  - Masses: rough estimate of nuclear BE from

    $$B(N,Z) \propto a + b g(\square\square\square) = a + b\langle \square\square\square | C_2[\mathrm{SU}(4)] | \square\square\square \rangle$$

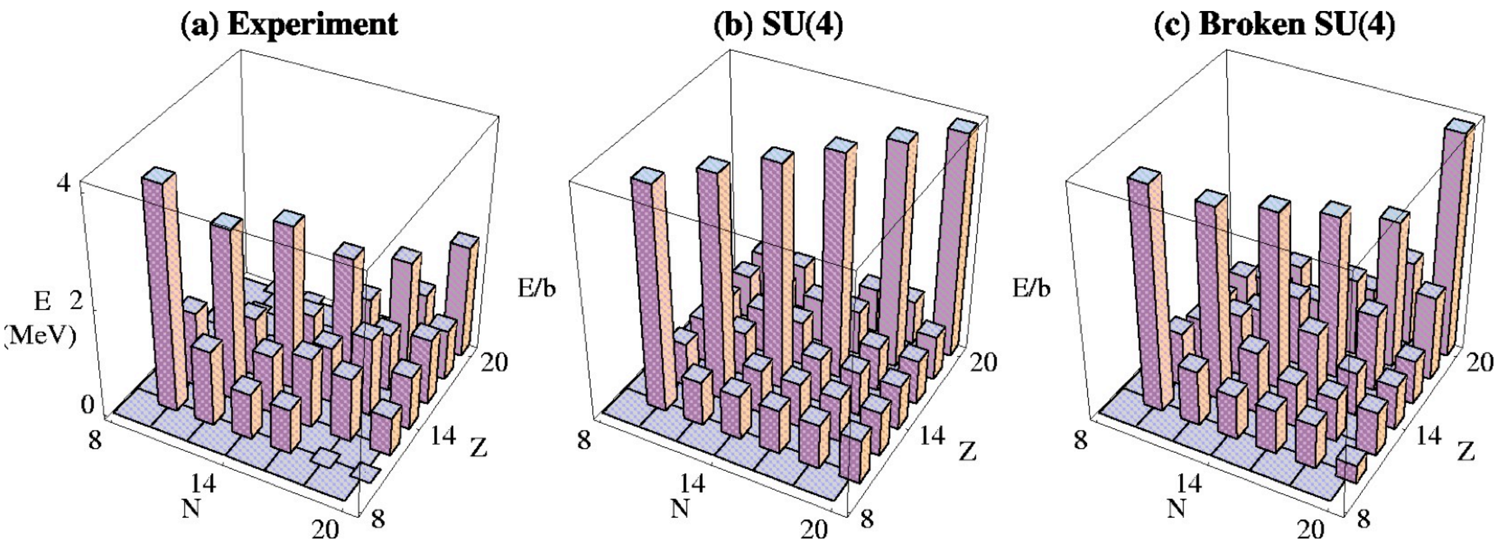
  - □ decay: **Gamow-Teller** operator $Y_{\square,\pm 1}$ is a generator of SU(4) □ selection rule in (□□□).

SU(4) breaking from masses

- Double binding energy difference δV_{np}

$$\delta V_{\mathrm{np}}(N,Z) = \tfrac{1}{4}\left[B(N,Z) - B(N-2,Z) - B(N,Z-2) + B(N-2,Z-2)\right]$$

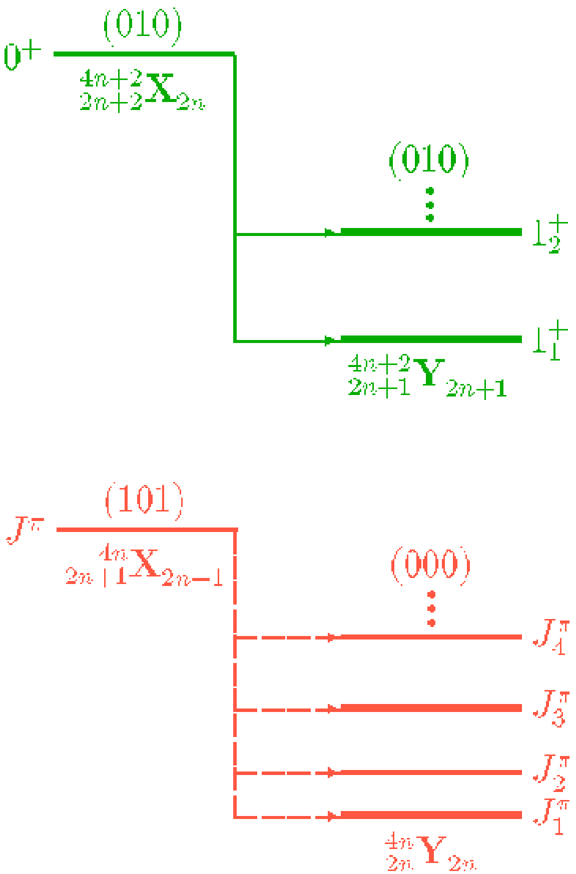
- δV_{np} in *sd*-shell nuclei:

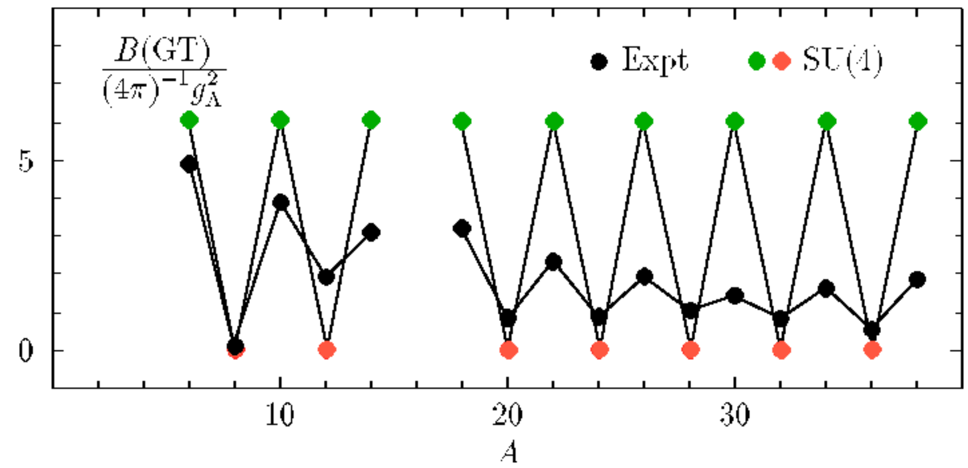


P. Van Isacker *et al.*, Phys. Rev. Lett. **74** (1995) 4607

SU(4) breaking from β decay

- Gamow-Teller decay into odd-odd or even-even N=Z nuclei:





P. Halse & B.R. Barrett, Ann. Phys. (NY) **192** (1989) 204

Elliott's SU(3) model of rotation

- Harmonic oscillator mean field (*no* spin-orbit) with residual interaction of quadrupole type:

$$H = \sum_{k=1}^{A}\left(\frac{p_k^2}{2m} + \frac{1}{2}m\omega^2 r_k^2\right) - g\,Q\cdot Q,$$

$$Q_\mu = \sqrt{\frac{4\pi}{5}}\left(\sum_{k=1}^{A} r_k^2 Y_{2\mu}(\hat{r}_k) + \sum_{k=1}^{A} p_k^2 Y_{2\mu}(\hat{p}_k)\right)$$

- State labelling in *LS* coupling:

$$\begin{array}{ccccccc} \mathrm{U}(4\Omega) & \supset\Big(\mathrm{U}_L(\Omega) & \supset \mathrm{SU}_L(3) & \supset \mathrm{SO}_L(3)\Big) & \otimes\Big(\mathrm{SU}_{ST}(4) & \supset \mathrm{SU}_S(2) & \otimes \mathrm{SU}_T(2)\Big) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ [1^M] & (\tilde{f}_1\tilde{f}_2\tilde{f}_3) & (\bar{\lambda}\bar{\mu}) & L & (\lambda'\mu'\nu') & S & T \end{array}$$

J.P. Elliott, Proc. Roy. Soc. A **245** (1958) 128; 562

Importance and limitations of SU(3)

- Historical importance:
  - Bridge between the spherical shell model and the liquid droplet model through mixing of orbits.
  - Spectrum generating algebra of Wigner's SU(4) supermultiplet.
- Limitations:
  - *LS* (Russell-Saunders) coupling, *not jj* coupling (zero spin-orbit splitting) □ beginning of *sd* shell.
  - *Q* is the *algebraic* quadrupole operator □ no major-shell mixing.

Generalized SU(3) models

- How to obtain rotational features in a *jj*-coupling limit of the nuclear shell model?
- Several efforts since Elliott:
  - Pseudo-spin symmetry.
  - Quasi-SU(3) symmetry (Zuker).
  - Effective symmetries (Rowe).
  - FDSM: fermion dynamical symmetry model.
  - ...

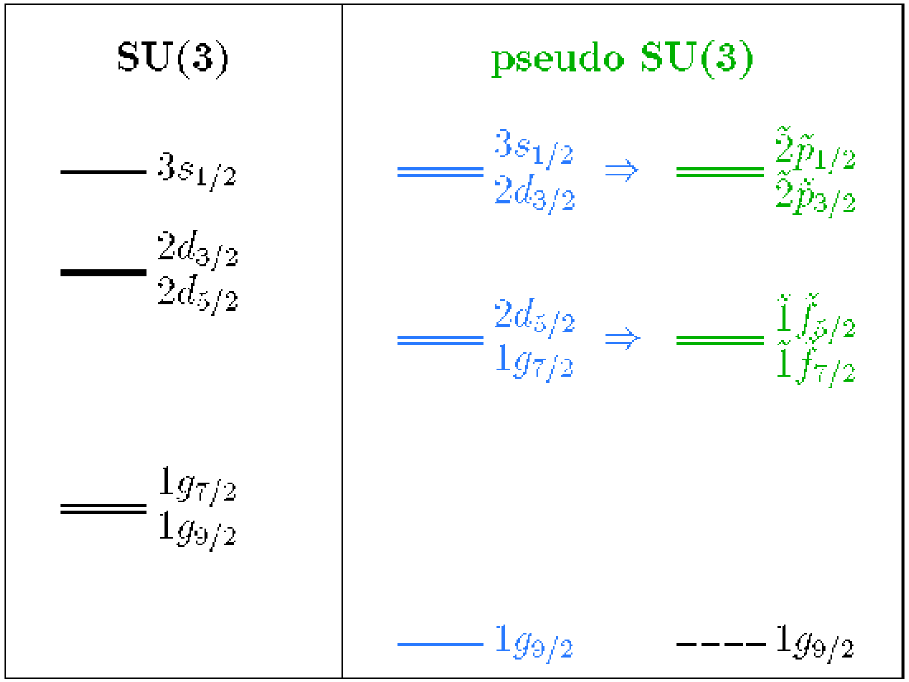
Pseudo-spin symmetry

- Apply a *helicity* transformation to the spin-orbit + orbit-orbit nuclear mean field:

$$u_k^{\square 1}\left(\square_{\rm ls}\,\vec{l}_k\cdot\vec{s}_k+\square_{\rm ll}\,l_k^2\right)u_k=\left(4\square_{\rm ll}\,\square\,\square_{\rm ls}\right)\vec{l}_k\cdot\vec{s}_k+\square_{\rm ll}\,l_k^2+{\rm c}^{\rm te}$$

$$u_k=2i\frac{\vec{s}_k\cdot\vec{r}_k}{r_k}$$

- Degeneracies occur for $4\square_{\rm ll}=\square_{\rm ls}$.



K.T. Hecht & A. Adler, Nucl. Phys. A **137** (1969) 129
A. Arima *et al.*, Phys. Lett. B **30** (1969) 517
R.D. Ratna *et al.*, Nucl. Phys. A **202** (1973) 433
J.N. Ginocchio, Phys. Rev. Lett. **78** (1998) 436

Pseudo-SU(4) symmetry

- Assume the nuclear hamiltonian is invariant under *pseudo*-spin and isospin rotations:

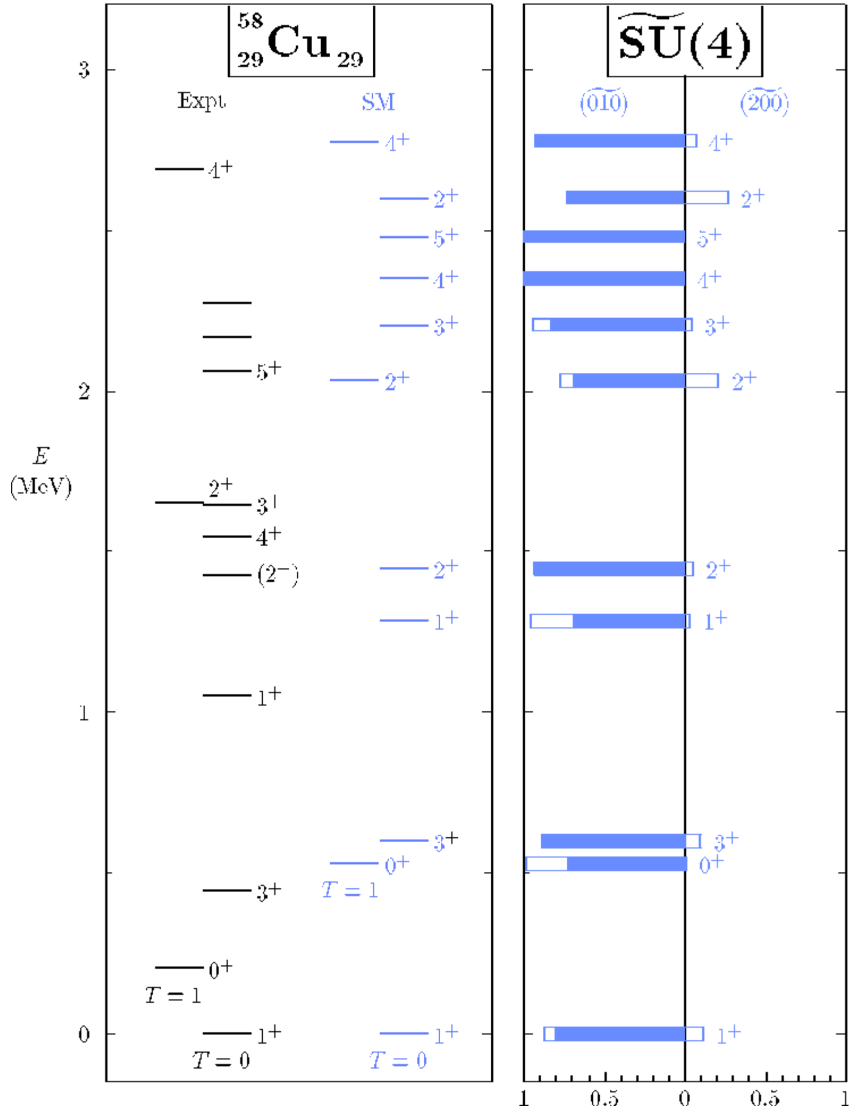
$$\left[H_{\mathrm{nucl}}, \tilde{S}_{\mu}\right] = \left[H_{\mathrm{nucl}}, T_{\mu}\right] = \left[H_{\mathrm{nucl}}, \tilde{Y}_{\mu\nu}\right] = 0$$

$$\tilde{S}_{\mu} = \sum_{k=1}^{A} \tilde{s}_{\mu}(k), \quad T_{\mu} = \sum_{k=1}^{A} t_{\mu}(k), \quad \tilde{Y}_{\mu\nu} = \sum_{k=1}^{A} \tilde{s}_{\mu}(k) t_{\nu}(k)$$

- Consequences:
  - Hamiltonian has pseudo-SU(4) symmetry.
  - Total pseudo-spin, total pseudo-orbital angular momentum, total isospin and pseudo-SU(4) labels are conserved quantum numbers.

D. Strottman., Nucl. Phys. A **188** (1971) 488

Test of pseudo-SU(4) symmetry



- Shell-model test of pseudo-SU(4).
- Realistic interaction in $pf_{5/2}g_{9/2}$ space.
- Example: ^{58}Cu.

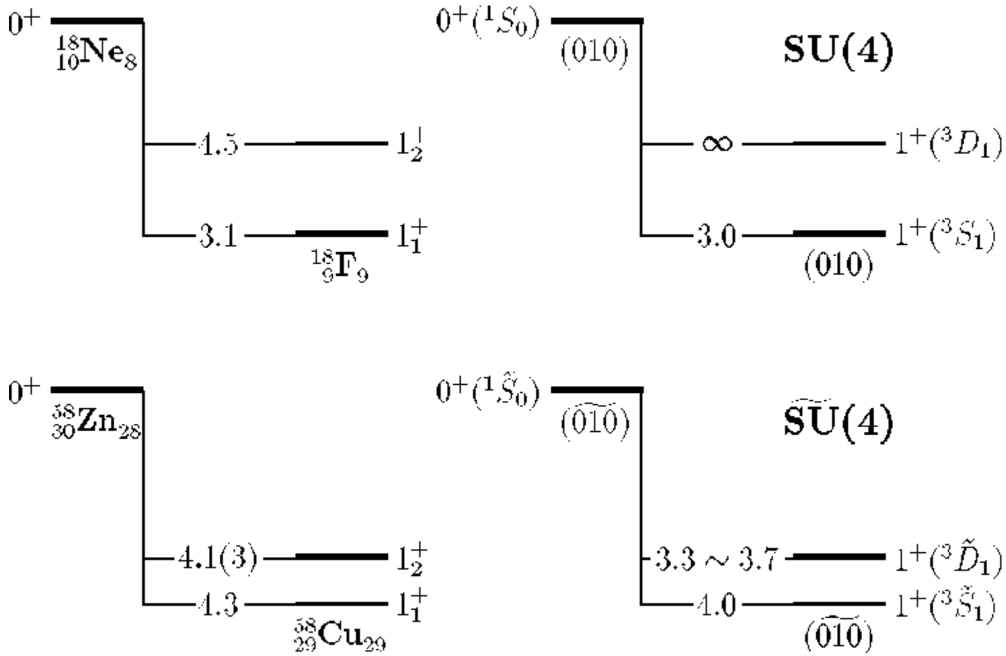
P. Van Isacker *et al.*, Phys. Rev. Lett. **82** (1999) 2060

Pseudo-SU(4) and β decay

- Pseudo-spin transformed Gamow-Teller operator is *deformation dependent*:

$$\tilde{s}_{\mu} t_{\nu} \equiv u^{-1} s_{\mu} t_{\nu} u = -\frac{1}{3} s_{\mu} t_{\nu} + \sqrt{\frac{20}{3}} \frac{1}{r^{2}} \left[\left(\vec{r} \times \vec{r} \right)^{(2)} \times \vec{s} \right]_{\mu}^{(1)} t_{\nu}$$

- Test: β decay of ^{18}Ne *vs.* ^{58}Zn.

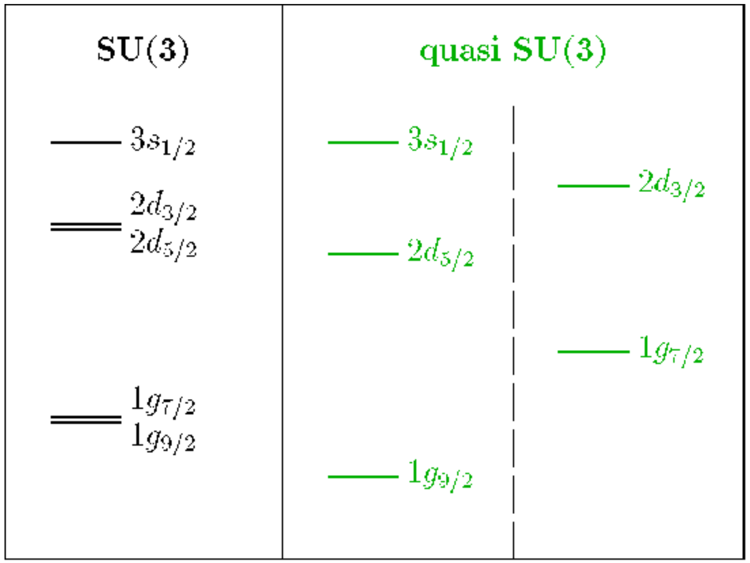


A. Jokinen *et al.*, Eur. Phys. A. **3** (1998) 271

Quasi-SU(3) symmetry

- Similar matrix elements of *Q* in *LS* and *jj* coupling lead to approximate decoupling of *j=l-1/2* and *j=l+1/2* spaces.

		LS coupling		$l \gg \lvert m \rvert$
$\langle lm \lvert \hat{Q}_{20} \rvert lm \rangle$	$=$	$\frac{l(l+1)-3m^2}{(2l-1)(2l+3)}$	$\rightarrow$	$\frac{1}{4}$
$\langle lm \lvert \hat{Q}_{20} \rvert l+2\ m \rangle$	$=$	$\frac{3[(l+1)^2-m^2]^{1/2}[(l+2)^2-m^2]^{1/2}}{2(2l+1)^{1/2}(2l+3)(2l+5)^{1/2}}$	$\rightarrow$	$\frac{3}{8}$
		jj coupling		$j \gg \lvert m \rvert$
$\langle jm \lvert \hat{Q}_{20} \rvert jm \rangle$	$=$	$\frac{j(j+1)-3m^2}{4j(j+1)}$	$\rightarrow$	$\frac{1}{4}$
$\langle jm \lvert \hat{Q}_{20} \rvert j+1\ m \rangle$	$=$	$\frac{3m[(j+1)^2-m^2]^{1/2}}{4j(j-1)(j+2)}$	$\rightarrow$	$\frac{3m}{4j} \sim 0$
$\langle jm \lvert \hat{Q}_{20} \rvert j+2\ m \rangle$	$=$	$\frac{3[(j+1)^2-m^2]^{1/2}[(j+2)^2-m^2]^{1/2}}{8(j+1)(j+2)}$	$\rightarrow$	$\frac{3}{8}$



A. Zuker *et al.*, Phys. Rev. C **52** (1995) R1741

Fermion dynamical symmetry model

- Construct shell-model hamiltonian in terms of *S* and *D* pairs such that the *S*,*D*-space decouples from the full shell-model space.
- Generalization of pseudo-spin (***j***=***k***+***i***):

$$a^{+}_{km_k im_i} = \sum_{jm_j} \langle km_k\, im_i | jm_j \rangle a^{+}_{l\frac{1}{2},jm_j}$$

- Many possible combinations of single-particle orbits via an appropriate choice of *k* and *i*.
- Limitation: structure of pairs is algebraically imposed.

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C.L. Wu *et al.*, Phys. Rev. C **36** (1987) 1157