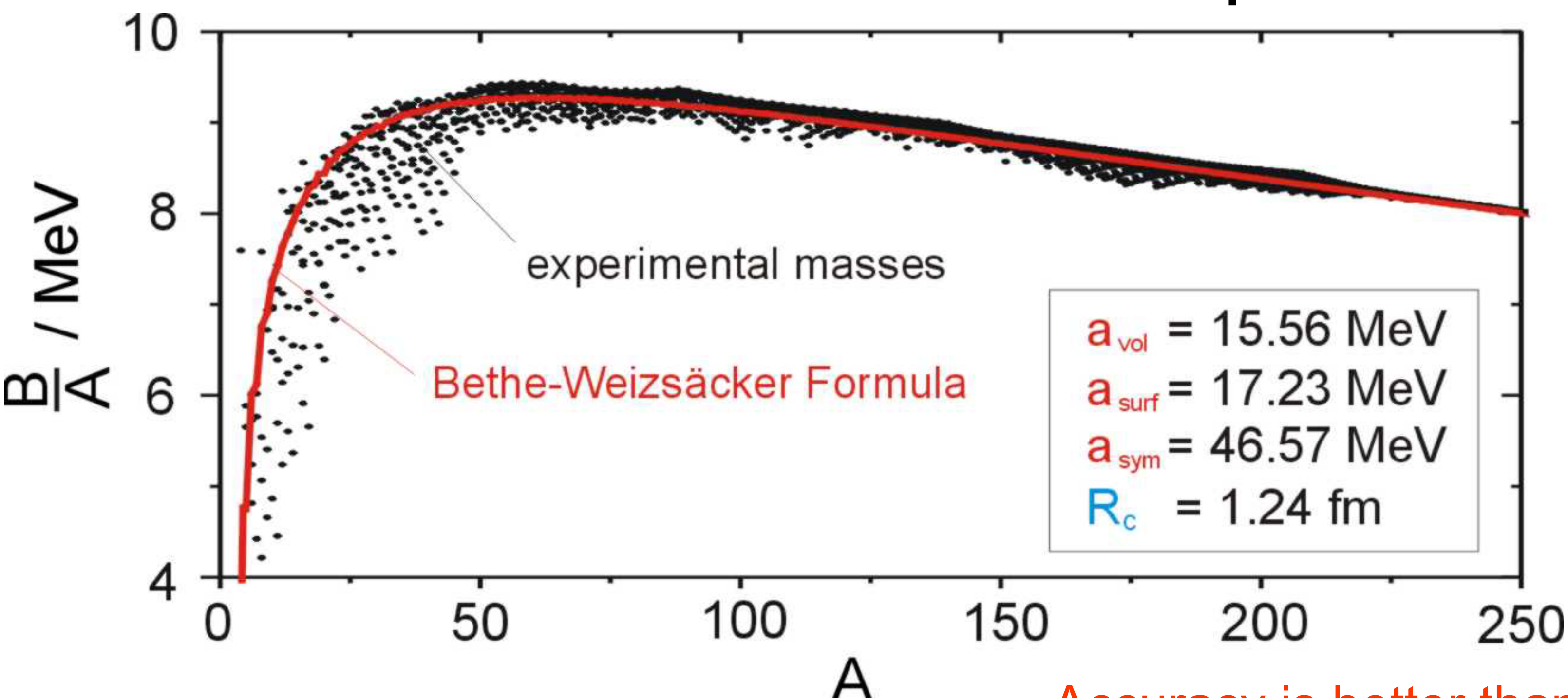
Macroscopic description

Nuclei are treated as a classical liquid drop

Bethe-Weizsäcker Formula: Z. Phys. 96, 431 (1935)

$$B(N, Z) = a_{vol} A - a_{surf} A^{2/3} - \frac{1}{2} a_{sym} \frac{(N-Z)^2}{A} - \frac{3}{5} \frac{Z^2 e^2}{R_c}$$

deformation dependent



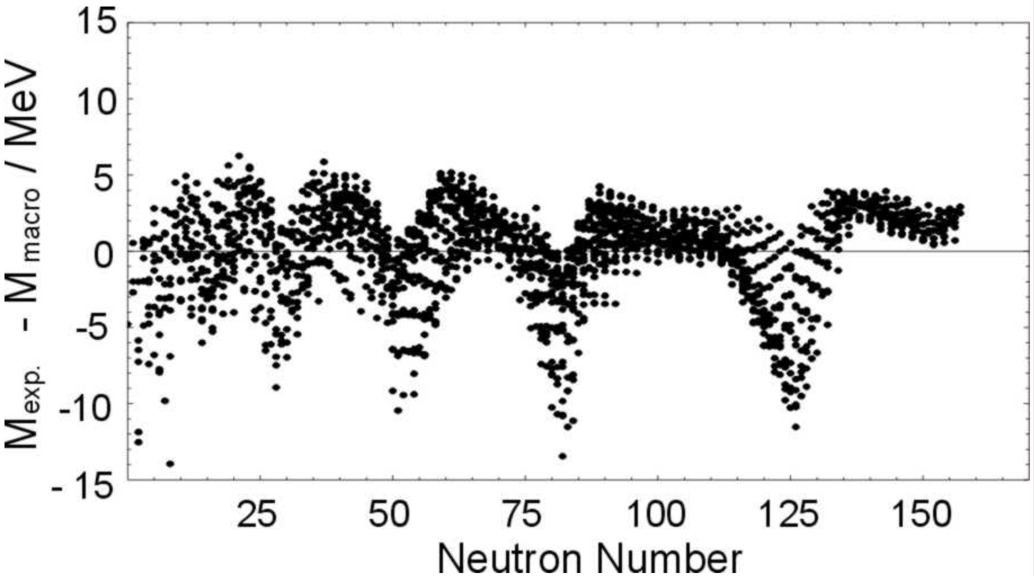
Accuracy is better than 1 % !

Microscopic corrections to the binding energy 1

$$\boxed{B(N,Z)=B(N,Z)_{macro}+E_{corr}^{shell}+E_{corr}^{pair}}$$

1. Shell correction

magic numbers: 2,8,20,28,50,82,126



V.M. Strutinski, Nucl. Phys. A95, 420 (1967)

energy in a single-particle potential

$$E_{shell}=\sum_{\nu=1}^{n/2}\varepsilon_\nu=\int_{-\infty}^{\varepsilon_F}\rho(\varepsilon)d\varepsilon$$

level density

"smooth" microscopic energy

$$\widetilde{E}_{shell}=\int_{-\infty}^{\widetilde{\varepsilon}_F}\widetilde{\rho}(\varepsilon)d\varepsilon$$

"smooth" level density

shell correction energy

$$\boxed{E_{corr}^{shell}=E_{shell}-\widetilde{E}_{shell}}$$

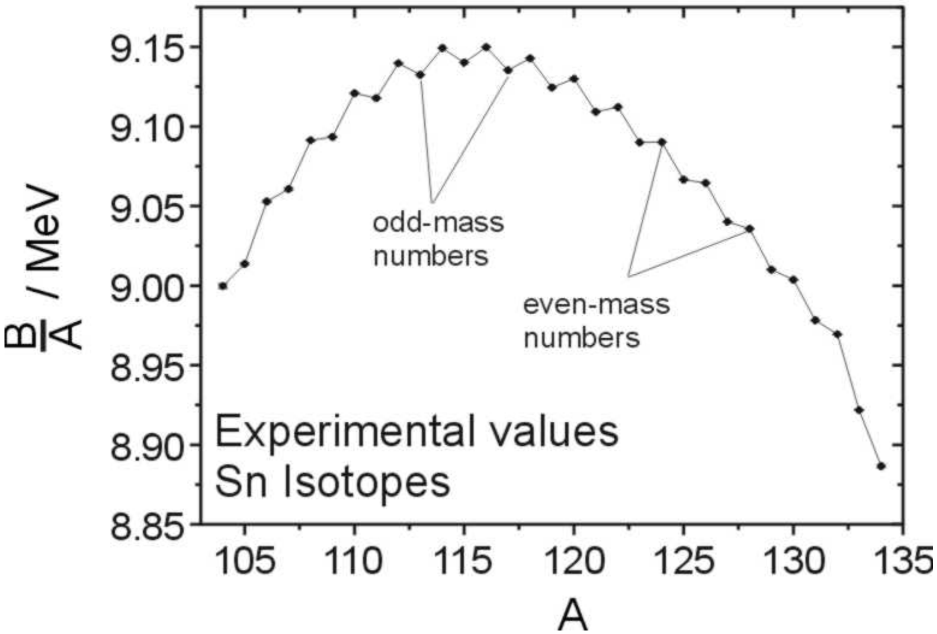
Nobel Prize 1963 M. Goeppert-Mayer, J.H.D. Jensen

"for their discoveries concerning nuclear shell structure"

Microscopic corrections to the binding energy 2

$$B(N,Z) = B(N,Z)_{macro} + E^{shell}_{corr} + E^{pair}_{corr}$$

2. Pairing correction



Odd-Even staggering of nuclear masses
Heisenberg, Z. Phys. 78, 156 (1932)

Nobel Prize 1972: J.Bardeen, L.Cooper,J.Schrieffer
"for their jointly developed theory of superconductivity"

total energy of a nucleus:

$$E_{pair} = \langle \Psi_{BCS} | H_{pair} | \Psi_{BCS} \rangle$$

where pairing hamiltonian:

$$H_{pair} = \sum_{\nu=1}^{n} \varepsilon_\nu a^+_\nu a_\nu - G \sum_{\nu,\nu'>0} a^+_\nu a^+_{\bar{\nu}} a_{\bar{\nu}'} a_{\nu'}$$

and BCS wavefunctions:

$$\Psi_{BCS} = \prod_{\nu>0} (u_\nu + v_\nu a^+_\nu a^+_{\bar{\nu}}) |0\rangle$$

pairing correction energy:

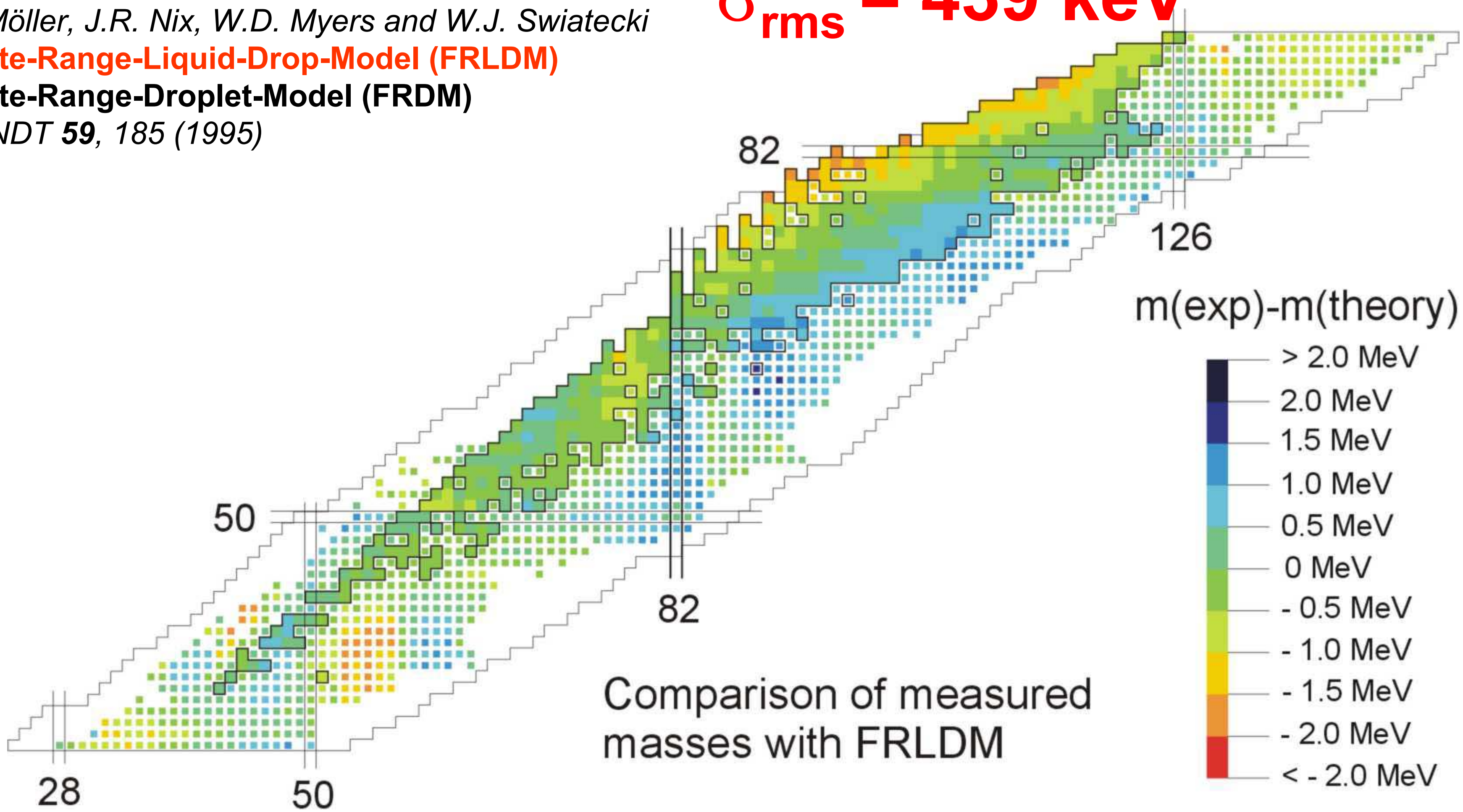
$$\boxed{E^{pair}_{corr} = E_{shell} - E_{pair}}$$

N.N. Bogoliubov (Nuovo Cimento 7, 794 (1958))- J.G. Valatin (Phys. Rev. 122, 1012 (1961)) description leads the same results

Examples of Macro-Micro Mass Models

P. Möller, J.R. Nix, W.D. Myers and W.J. Swiatecki
Finite-Range-Liquid-Drop-Model (FRLDM)
Finite-Range-Droplet-Model (FRDM)
ADNDT ***59****, 185 (1995)*

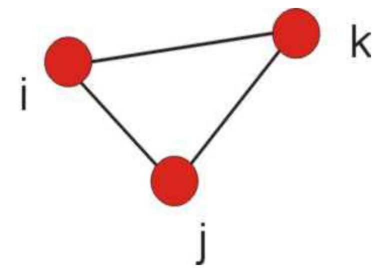
σ_{rms} **= 439 keV**



Comparison of measured masses with FRLDM

Microscopic Description of Atomic Nuclei

Assumption: nucleons move independently in a mean field created by all other nucleons in the nucleus



Hartree-Fock method:

Schroedinger equation for the nuclear many-body system:

$$\boxed{H\Psi = E\Psi}$$

nuclear hamiltonian

total binding energy

fully antisymmetric wave function

$$\Psi(1,2) = -\Psi(2,1)$$

$$\boxed{H = \sum_{i=1}^{A} T_i + \sum_{i<j} V^{eff}(i,j)}$$

kinetic energy operator

effective potential

Two types of effective potential V(i,j):

I) Skyrme, zero-range force
advantage: easy to calculate
disadvantage: interacting two nucleons have to be in one point

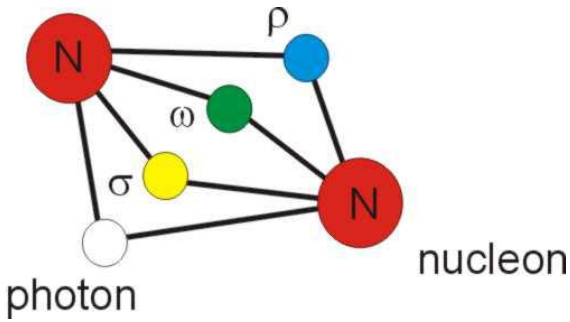
II) Gogny, finite-range force
advantage: realistic interaction length
disadvantage: no large-scale calculations could be performed yet

Self-consistent solution:

Wave functions depend on the effective potential and vice versa
Unknown wave functions and unknown potential are found iteratively

Relativistic Mean Field

The interaction is mediated by three types of mesons and a photon



Dirac equation for nucleons + Klein-Gordon equations for mesons and photon

Mesons generate a nuclear and the photon generates the Coulomb contributions to the mean field in which the nucleons move

The wave function of the nucleus is constructed from a product of single-particle spinors which are the solutions of the Dirac equation

The system is solved selfconsistently

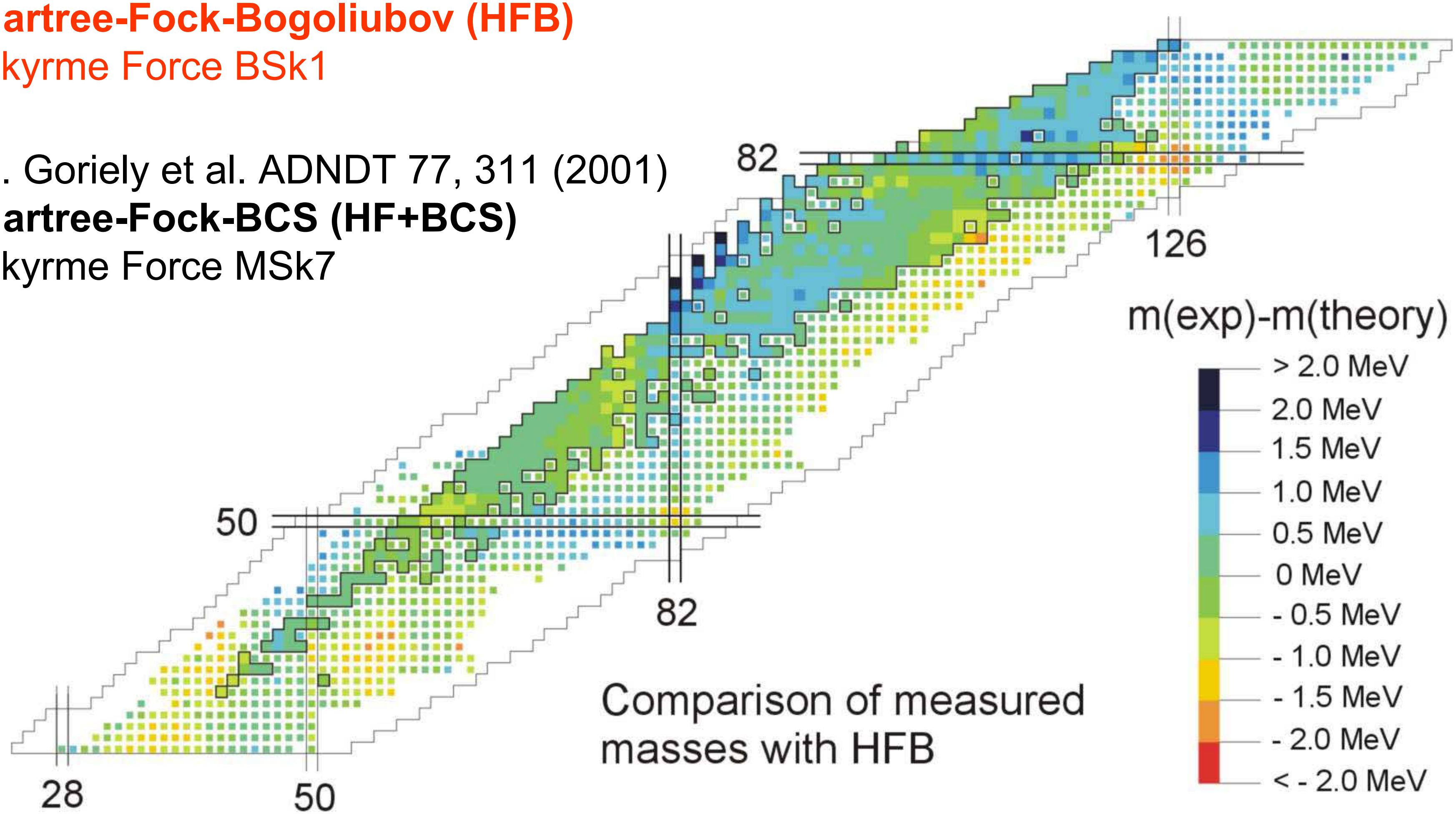
Advantages:

- **Spin-orbit interaction comes out automatically with the right order of magnitude**
- **Finite-range interaction is used**

Examples of Microscopic Mass Models

M. Samyn et al. Nucl. Phys. A700, 142 (2002)
Hartree-Fock-Bogoliubov (HFB)
Skyrme Force BSk1

S. Goriely et al. ADNDT 77, 311 (2001)
Hartree-Fock-BCS (HF+BCS)
Skyrme Force MSk7

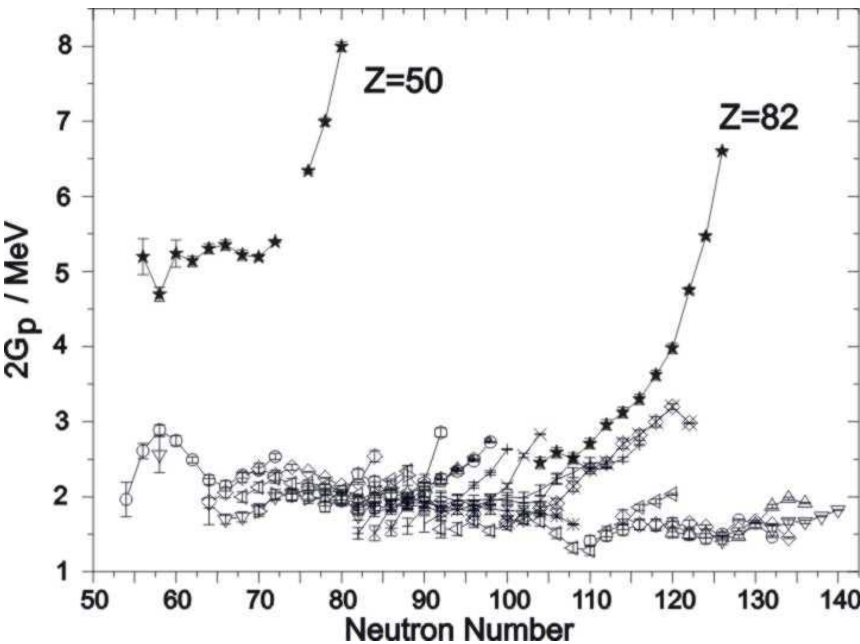


Nuclear Properties from Experiment and Theory

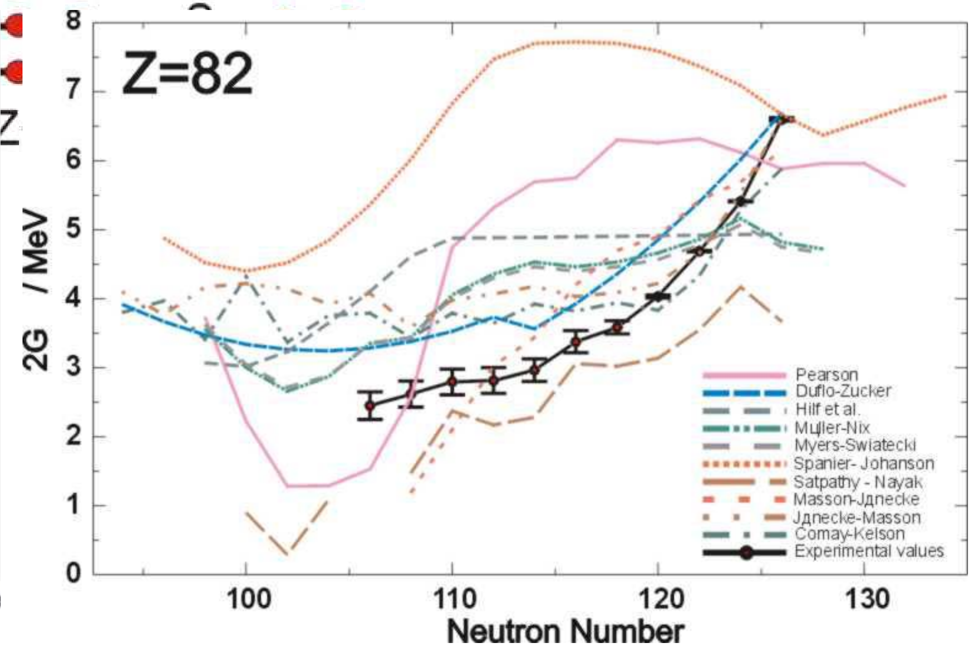
1. Shell structure far away of the valley of beta-stability

Proton Shell Gap $$2G_p=(m(Z-2,N)-2m(Z,N)+m(Z+2,N))\cdot c^2$$

Experiment:



Theory:



Explanation:

Masses of three nuclides enter in the definition of Proton Shell Gap
i.e.a nuclear structure change within these nuclides can lead to wrong conclusions

Predictive Power of Mass Models

The rms-error is often used to estimate how reliable are the predictions of a mass model

$$\sigma_{rms}^2 = \frac{1}{N}\sum_{i=1}^{N}\left(m_i^{exp} - m_i^{theory}\right)^2$$

Unique way to test mass models is to perform new mass measurements. Predictive powers of a few mass models are tested on the 310 new masses measured at GSI

Mass model	σ_{rms} /keV
FRDM	372
FRLDM	696
Spanier-Johanson	1210
Myers-Swiatecki	395
Duflo-Zucker	668
HF+BCS	960
HFB	650

No mass model exist presently which has the predictive power better then 300 keV