Physics with Exotic Nuclei and Exotic Atoms at Relativistic Energies

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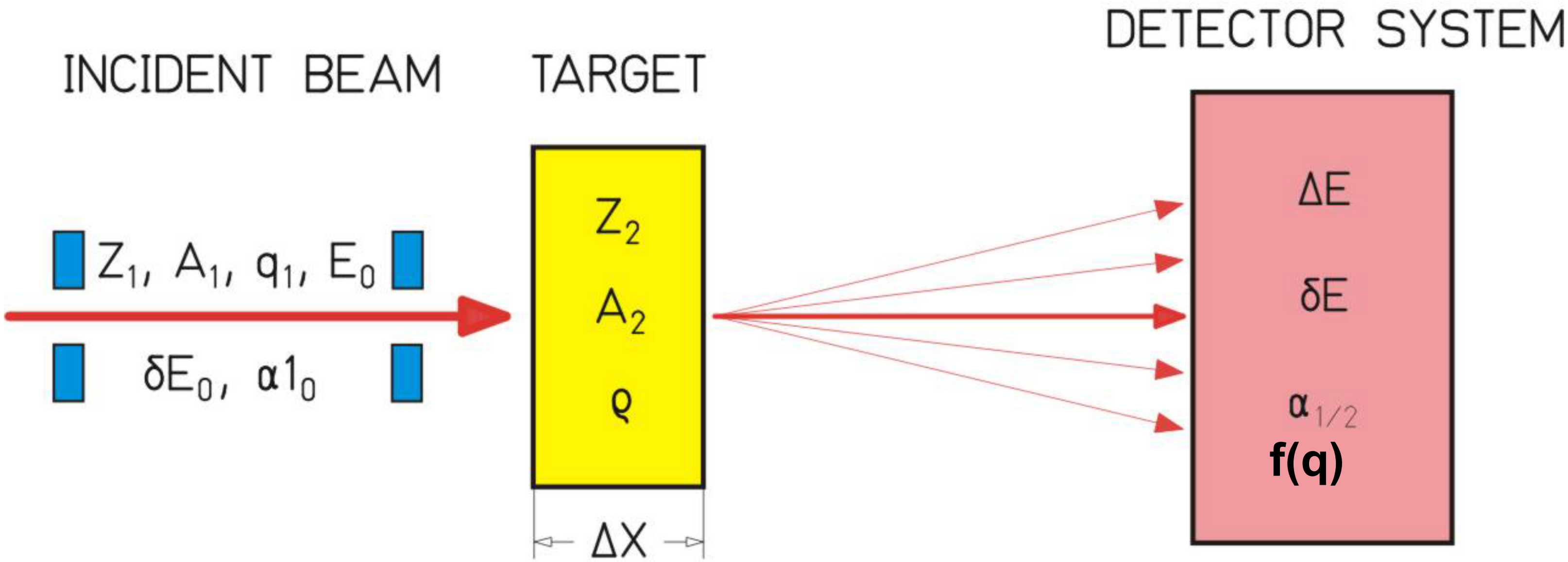
Euroschool Valencia, September 2003

- Introduction √
- Momentum Measurements, Ion Optics, √ Spectrometers
- **Atomic Interaction of Heavy Ions with Matter**

Literature: Nucl. Instr. Meth. B 195 (2002)
ICRU Report on Heavy Ion Stopping Powers submitted

Penetration of Heavy Ions through Matter

TRANSMISSION EXPERIMENT



STOPPING MEASUREMENT
- Target Investigations
- Energy-Angle Measurements

What will happen to the Projectiles and Target?

Macroscopic Scale :

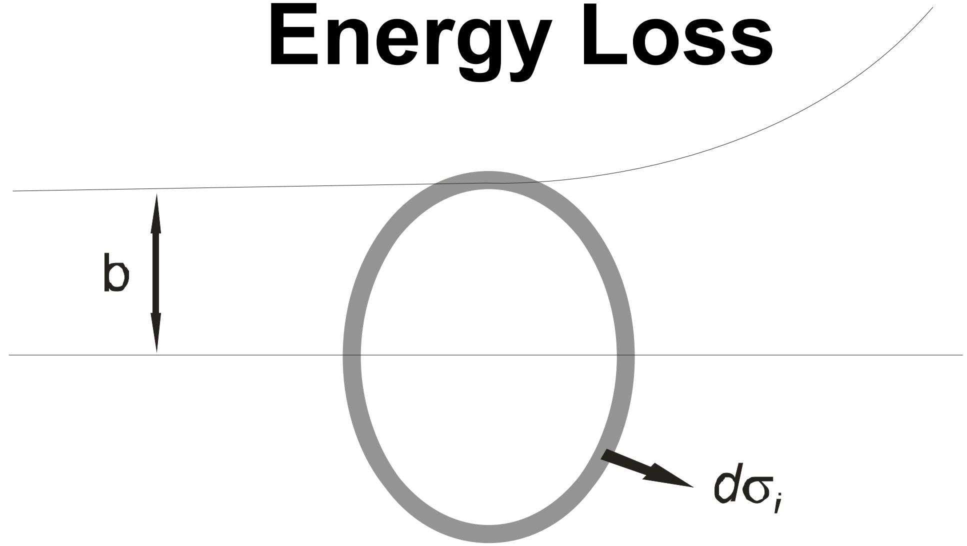
Projectile : energy loss, angular scattering, charge state

Target : increase in T, radiation damage (defect formation)

Microscopic Scale :

1) Inelastic atomic collisions (excitation and ionization)
2) Elastic collisions (scattering in a Coulomb field)
3) Generation of photons (Bremsstrahlung)
4) Nuclear reactions

Energy Loss



Cross section :

$$d\sigma_i = 2\pi b\,db$$

Probability hitting the target area :

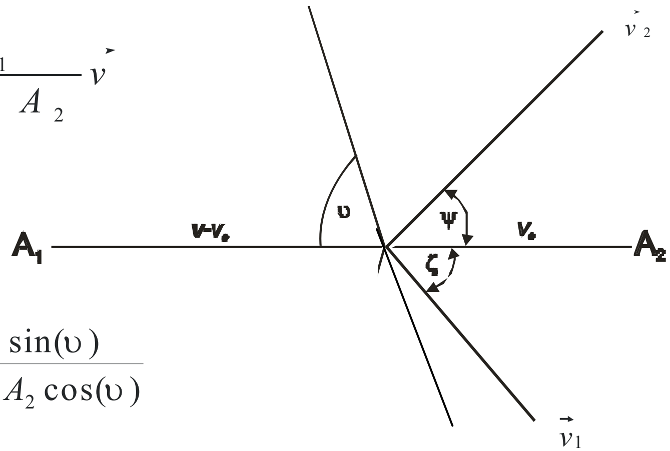
$$P_i = N\Delta x \sigma_i$$

Mean energy loss :

$$\langle \Delta E \rangle = N\Delta x \sum_i \sigma_i T_i$$

Energy Loss II

$$\vec{v}_c = \frac{A_1}{A_1 + A_2}\vec{v}$$



$$\tan(\zeta) = \frac{A_1 \sin(\upsilon)}{A_1 + A_2 \cos(\upsilon)}$$

Energy transfer to particle 2 :

$$T = 4\frac{A_1 A_2}{(A_1 + A_2)^2} E \sin^2\left(\frac{\upsilon}{2}\right), \qquad E = \frac{1}{2}A_1 v^2$$

Coulomb potential :

Collision diameter

$$V(r) = \frac{q_1 q_2}{r} \Rightarrow \tan\left(\frac{\upsilon}{2}\right) = \frac{a}{2b} \qquad a = \frac{q_1 q_2}{\frac{1}{2}\mu v^2}, \qquad \mu = \frac{A_1 A_2}{A_1 + A_2}$$

Rutherford scattering

Reduced mass

$$T(b) = \frac{2 q_1^2 q_2^2}{A_2 v^2}\frac{1}{b^2 + \left(\frac{a}{2}\right)^2}$$

Energy Loss III

Thomson formula :

$$\frac{d\sigma}{dT} = -\frac{2\pi q_1^2 q_2^2}{A_2 v^2}\frac{1}{T^2}$$

The same result is obtained if the scattering is treated quantum mechanically

$$\langle \Delta E \rangle = N \Delta x \int T d\sigma = -N \Delta x \frac{2\pi q_1^2 q_2^2}{A_2 v^2} \int_{T_{min}}^{T_{max}} \frac{dT}{T}$$

Elastic : $q_1 = Z_1 e$ $q_2 = Z_2 e$ N number of A_2 target atoms

Inelastic : $q_1 = Z_1 e$ $q_2 = -e$ NZ_2 number of electrons with mass m_e

Bohr theory (1913) :

$$-\frac{dE}{dx} = \frac{4\pi Z_1^2 e^4}{m_e v^2} N Z_2 \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

Integration would lead to infinite energy loss

Collision time $t = \frac{b}{v\gamma}$

Oscillation period of an electron $\tau = \frac{2\pi}{\omega}$

$$b_{min} = \frac{2|Z_1|e^2}{m_e v^2 \gamma^2}, b_{max} = \frac{\gamma v}{\omega}$$

$$T_{max} = 2 m_e v^2 \gamma^2 ; \frac{2 Z_1^2 e^4}{m_e v^2} \frac{4}{b_{min}^2} = 2 m_e v^2 \gamma^2$$

Stopping Power (Force)

$$-\frac{dE}{dx} = -\lim_{\Delta x \to 0} \frac{\langle \Delta E \rangle}{\Delta x}$$

$$-\frac{dE}{dx} = \left(-\frac{dE}{dx}\right)_{elastic} + \left(-\frac{dE}{dx}\right)_{inelastic}$$

Elastic : $q_1 = Z_1 e$ $q_2 = Z_2 e$ N number of A_2 target atoms

Inelastic : $q_1 = Z_1 e$ $q_2 = -e$ NZ_2 number of electrons with mass m_e

$$\langle \Delta E \rangle = N \Delta x \int T d\sigma = -N \Delta x \frac{2\pi q_1^2 q_2^2}{A_2 v^2} \int_{T_{min}}^{T_{max}} \frac{dT}{T}$$

T energy transfer in a single collision

$$\left(\frac{dE}{dx}\right)_{elastic} = -\frac{2\pi Z_1^2 Z_2^2 e^4}{A_2 v^2} N \ln\left(\frac{T_{max}}{T_{min}}\right)$$

$$\left(\frac{dE}{dx}\right)_{inelastic} = -\frac{2\pi Z_1^2 e^4}{m_e v^2} Z_2 N \ln\left(\frac{T_{max}}{T_{min}}\right)$$

Z_1, A_1 : atomic number and mass of projectile

Z_2, A_2 : atomic number and mass of target

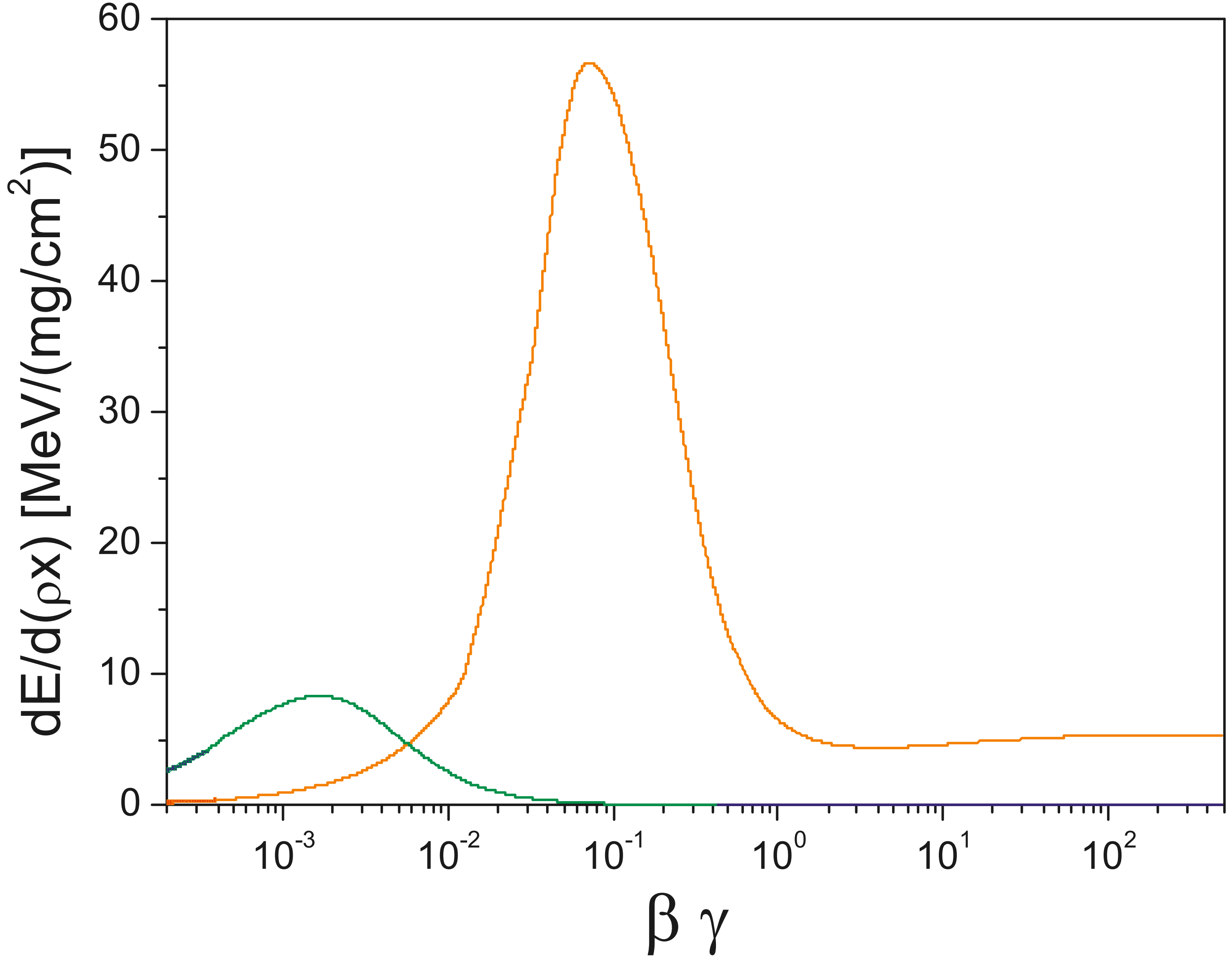
v : projectile velocity

N : targets atoms / cm^3

$$\frac{\left(\frac{dE}{dx}\right)_{elastic}}{\left(\frac{dE}{dx}\right)_{inelastic}} \approx \frac{Z_2 m_e}{A_2} \approx \frac{1}{4000}$$

larger in the high energy region

Stopping Power

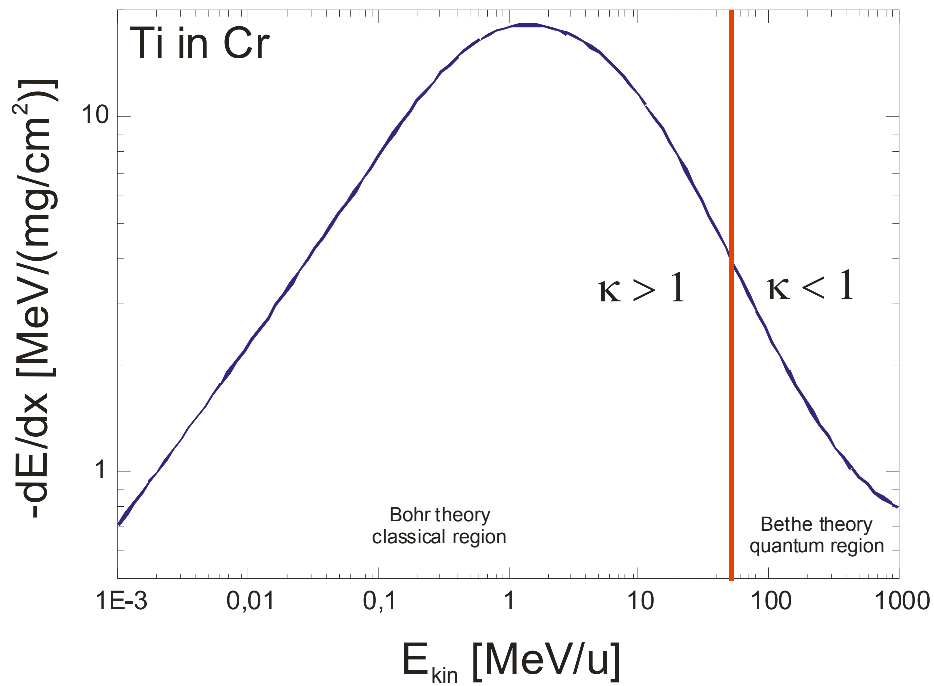


Stopping Power

Bohr :

Classical valid if $\kappa = \frac{2Z_1 v_0}{v} > 1$

$$\left(-\frac{dE}{dx}\right)_{inelastic} = \frac{4\pi Z_1^2 e^4}{m_e v^2} Z_2 N \ln\left[\frac{m_e v^3 \gamma^2}{|Z_1| e^2 \omega}\right]$$



Bethe :

$$\left(-\frac{dE}{dx}\right)_{inelastic} = \frac{4\pi Z_1^2 e^4}{m_e v^2} Z_2 N \left(\ln\left[\frac{2 m_e v^2 \gamma^2}{I}\right] - \beta^2\right), \quad \kappa < 1$$

I : mean ionization potential

Energy Loss Straggling

Energy straggling :

$$\Omega^2 = \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2$$

Average square fluctuation

$$\Omega^2 = N \Delta x \int T^2 d\sigma$$

Energy straggling :

$$\Omega^2_{Bohr} = 4\pi Z_1^2 Z_2 e^4 \gamma^2 N \Delta x$$

Bohr Formula

Energy Loss in Thick Targets

Range :

$$R = \int_0^{E_i} \frac{1}{\left(\frac{dE}{dx}(E)\right)} dE$$

Energy loss in thick targets :

$$d = \int_{E_f}^{E_i} \frac{1}{\left(\frac{dE}{dx}(E)\right)} dE = R(E_i) - R(E_f)$$

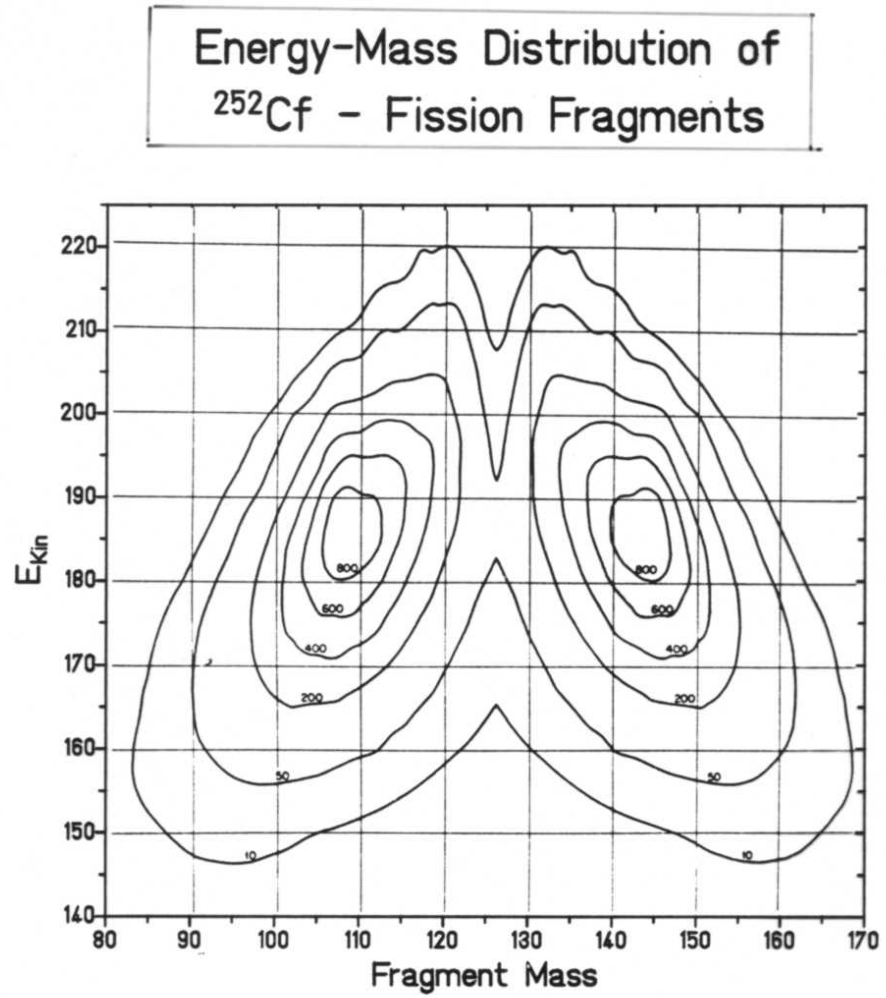
$$\Delta E = E_i - E_f$$

Range Straggling

Range straggling :

$$\sigma_R^2 = \int_0^{E_i} \frac{\frac{d\sigma^2}{dx}(E)}{\left[\frac{dE}{dx}\right]^3} dE \quad \textbf{variance}$$

The First Heavy Ion Experiments were done with Fission Fragments



Problem:

Large Phase Space

Population

First Charge-State Studies with Swift Heavy Ions

N. Bohr

Phys. Rev. 59, 270 1941
"Velocity-Range Relation for Fission Fragments"

"The Problem of primary importance is the estimate of the number of electrons carried with the fragment nucleus on its way through the gas"

VELOCITY CRITERION:
Electron capture and loss balance each other when the velocity of the most weakly bound remaining electron on the ion equals the collision velocity

$$q_{eff} = Z_1^{1/3} \frac{v}{v_0}$$

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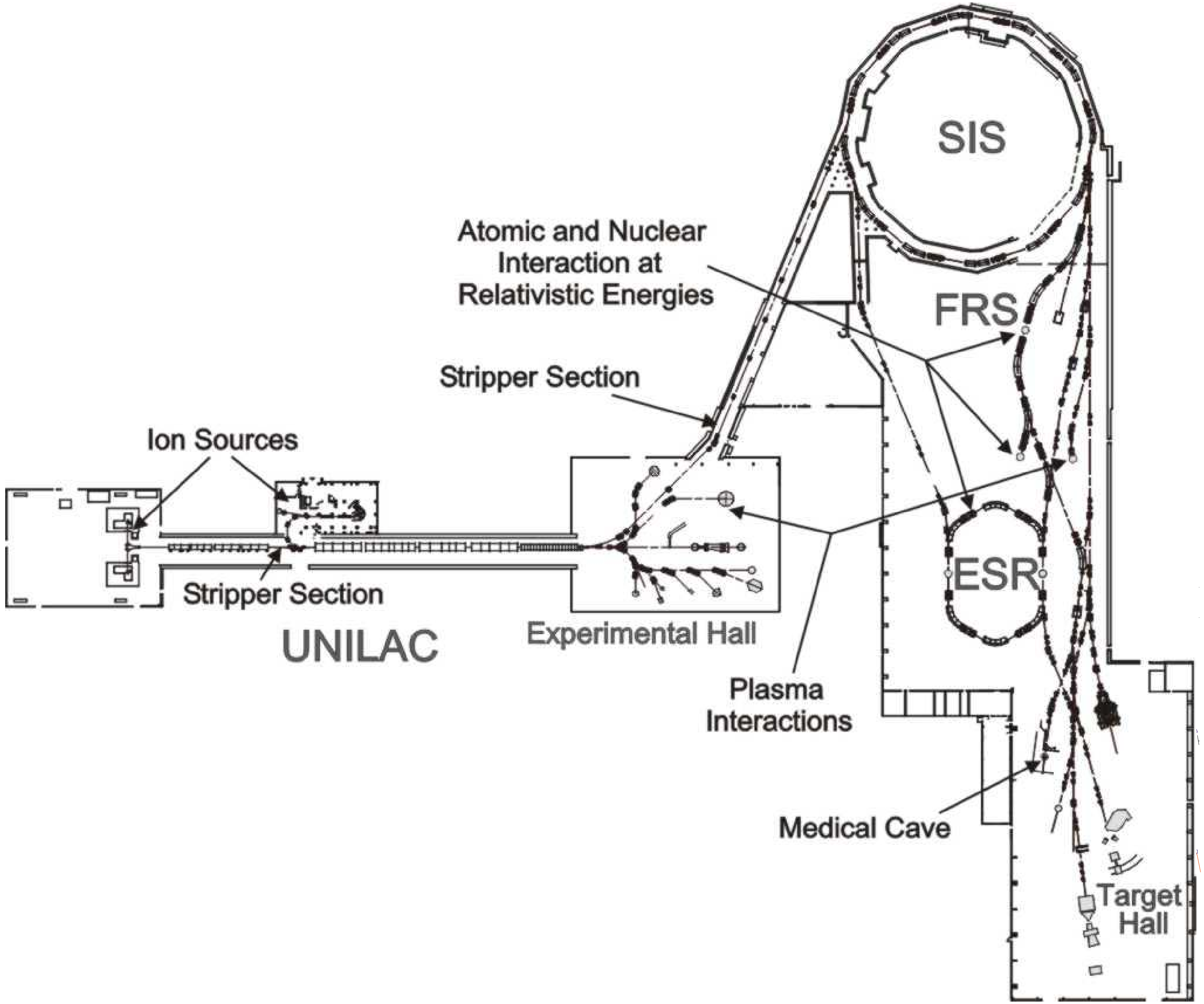
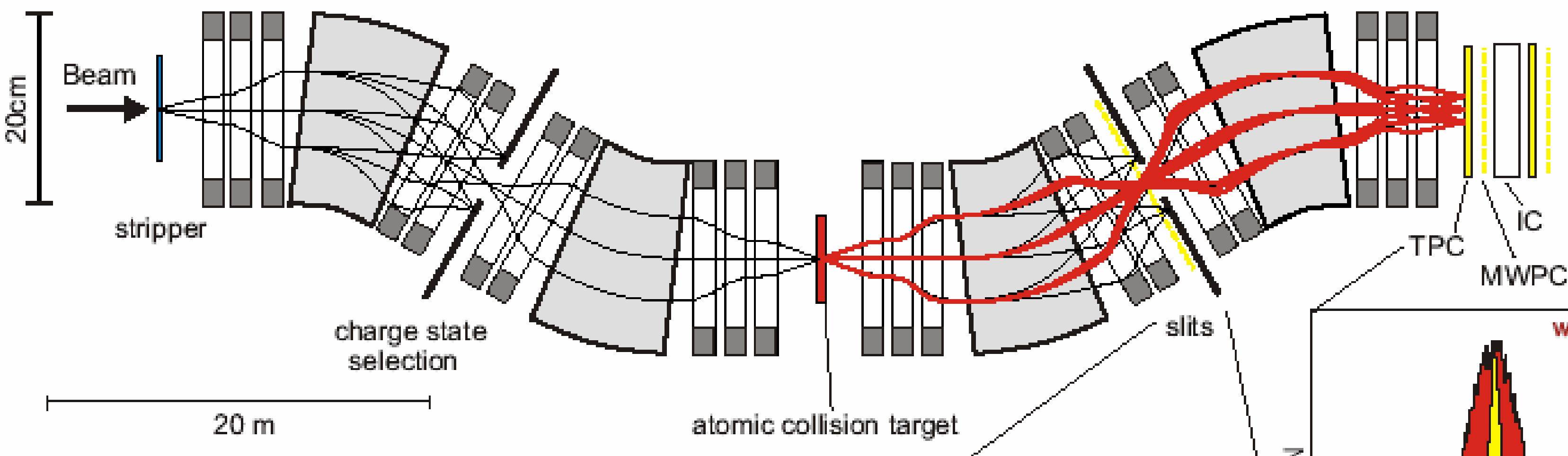
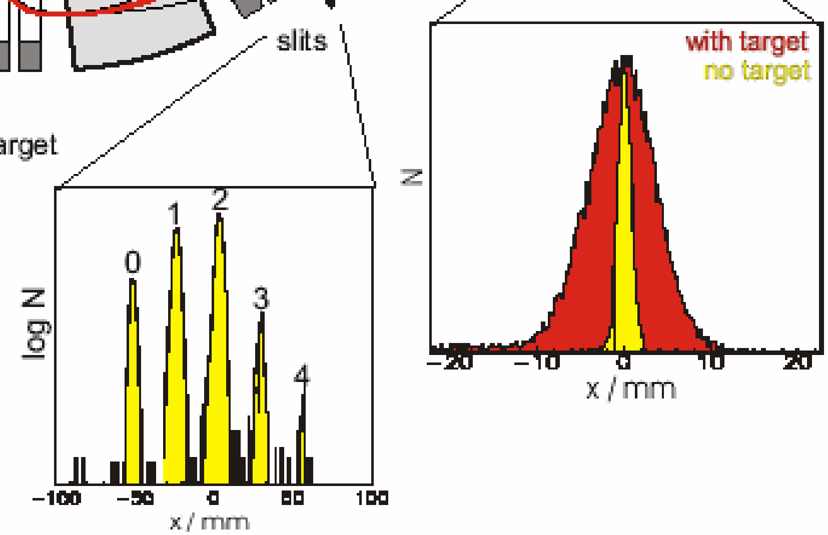
Atomic Collision Studies with the FRS



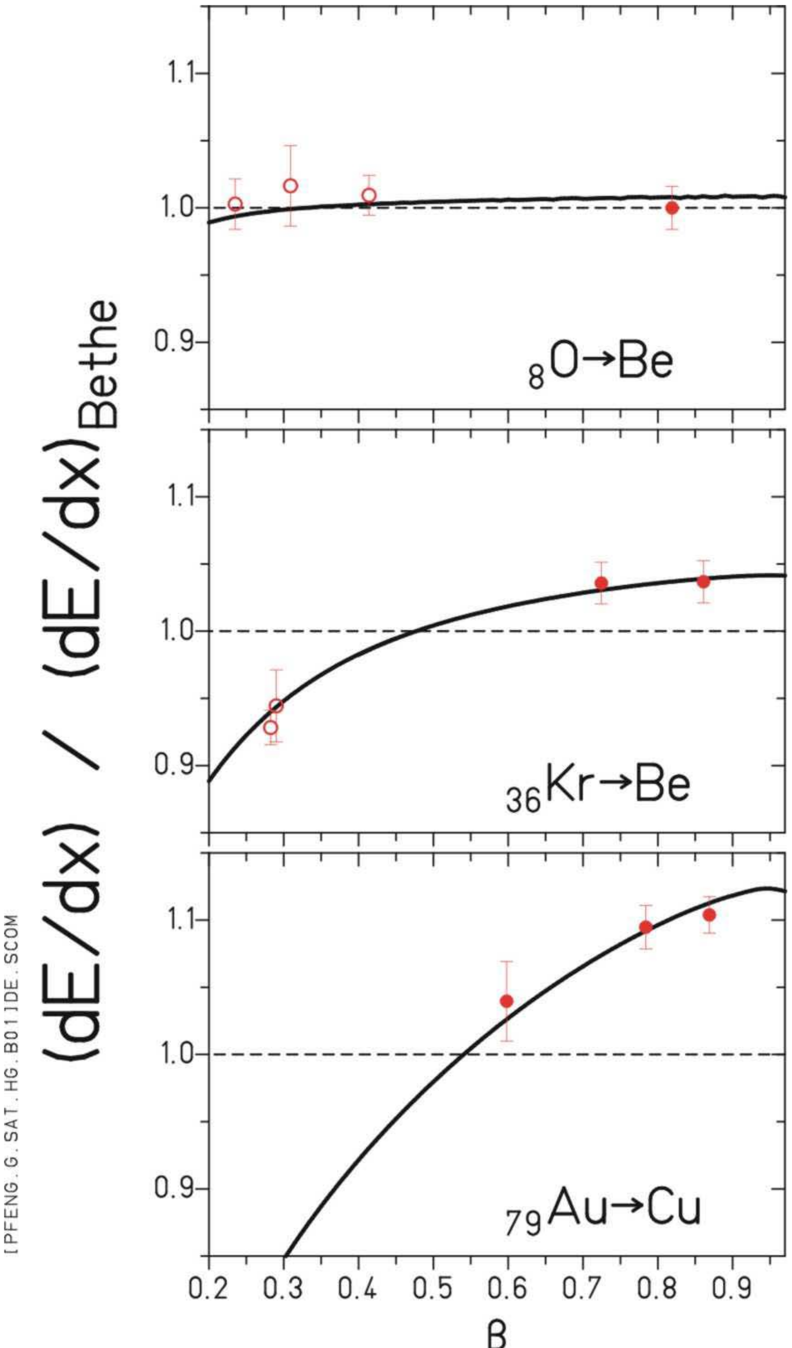
Atomic Collision Studies with the FRS



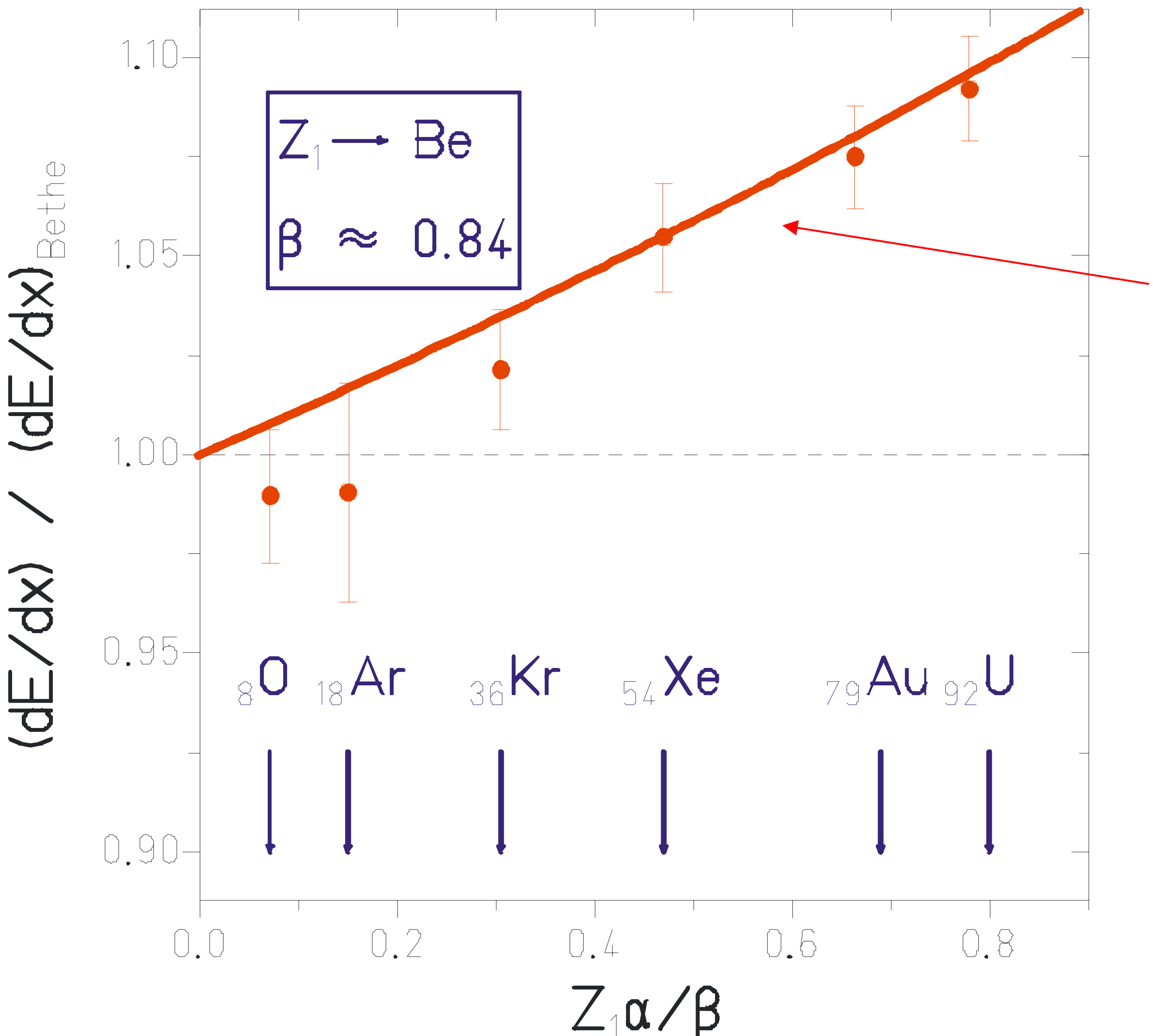


- Charge state distributions
- Energy loss from magnetic rigidity analysis
- Energy-loss straggling via position distribution in dispersive focal plane

How well can the Bethe theory describe the stopping power of bare heavy ions?



Failure of the Bethe Theory

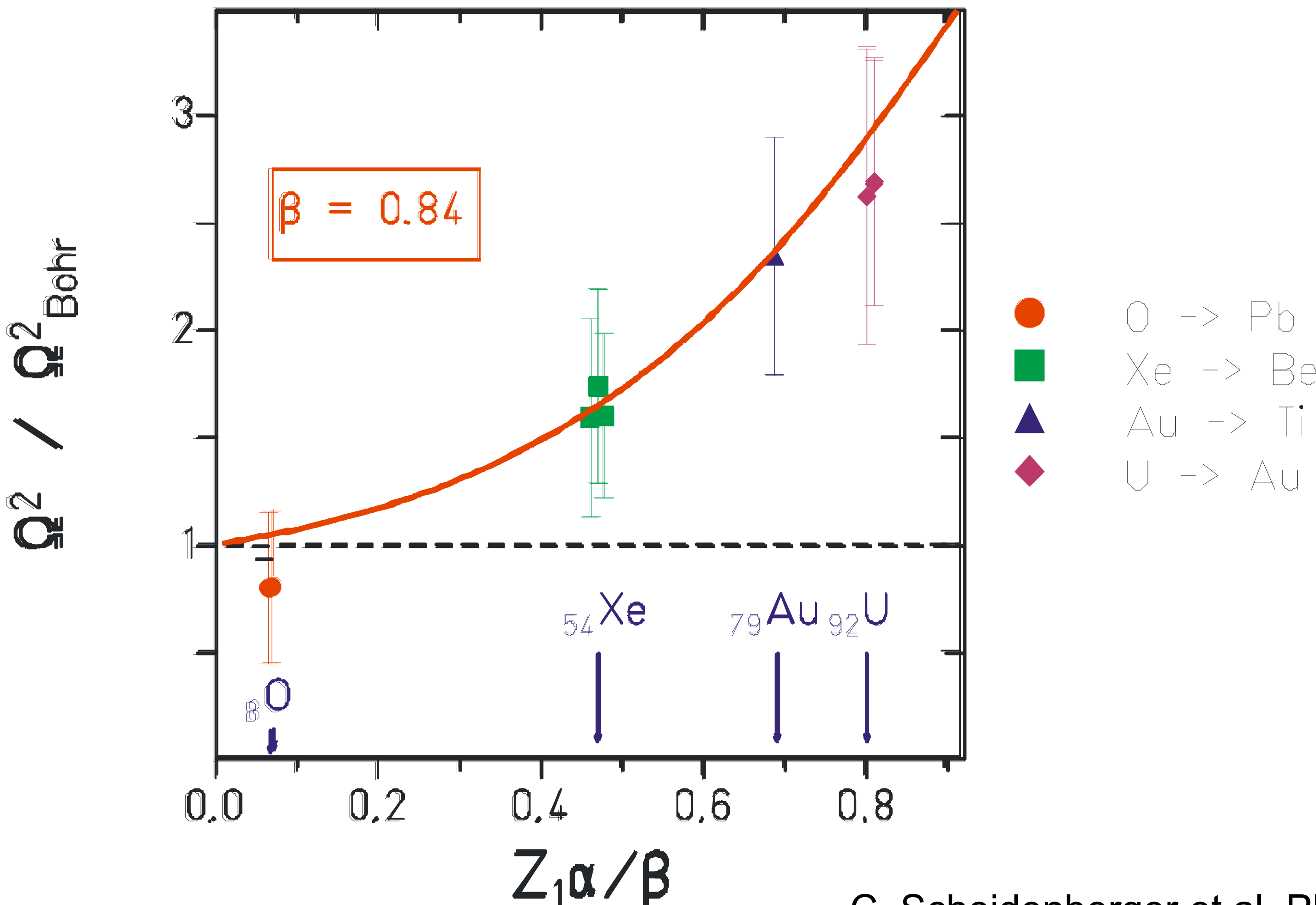


New Theory developed by J. Lindhard and A. Soerensen after the experimental were published!

Phys. Rev. A53 (1996) 2443

C. Scheidenberger et al. PRL 73 (1994) 50

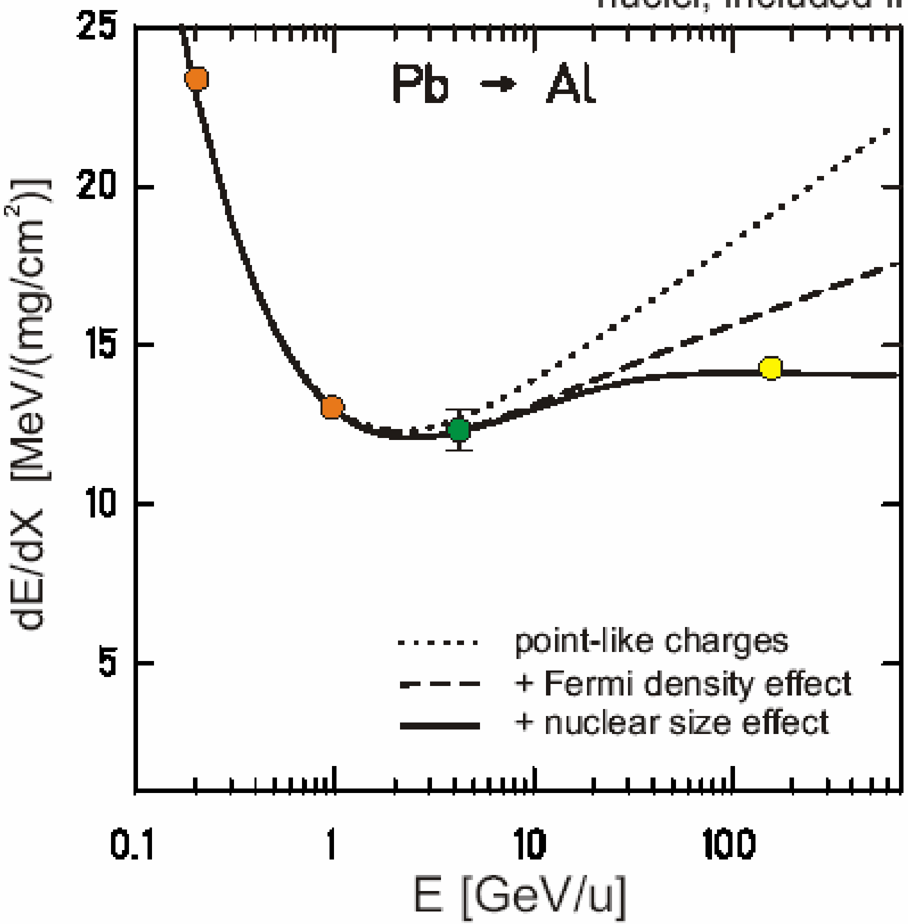
Energy-Loss Straggling Experiments with Relativistic Heavy Ions



C. Scheidenberger et al. PRL 73 (1994) 50

Nuclear Size and Density Effect

- Fermi-density effect: polarisation of medium leads to screening of the projectile
- Nuclear size effect: De Broglie wave length becomes comparable to size of the nucleus, deviation from point-like nuclei, included in LS-theory

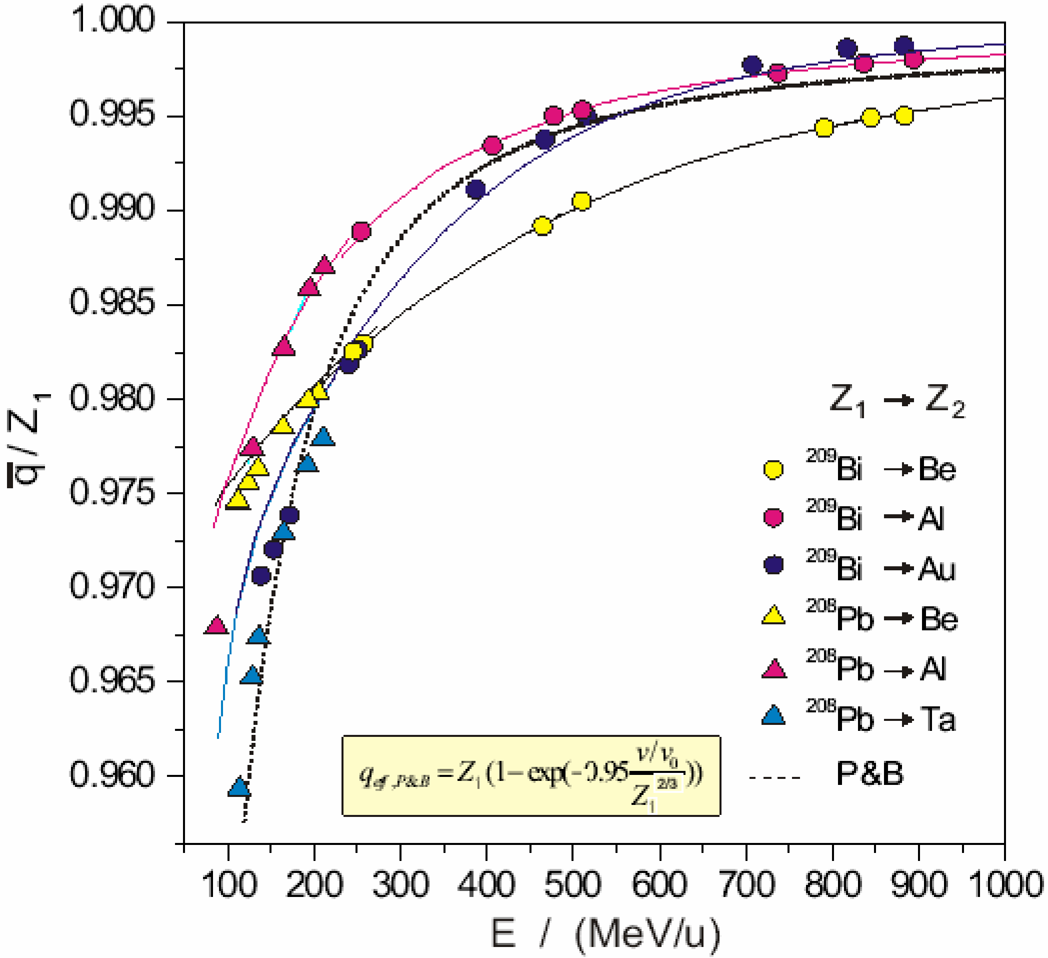


SIS/FRS, Scheidenberger, Weick et al.

AGS, G. Arduini et al.

CERN-SPS, S. Datz et al.

Mean Charge States



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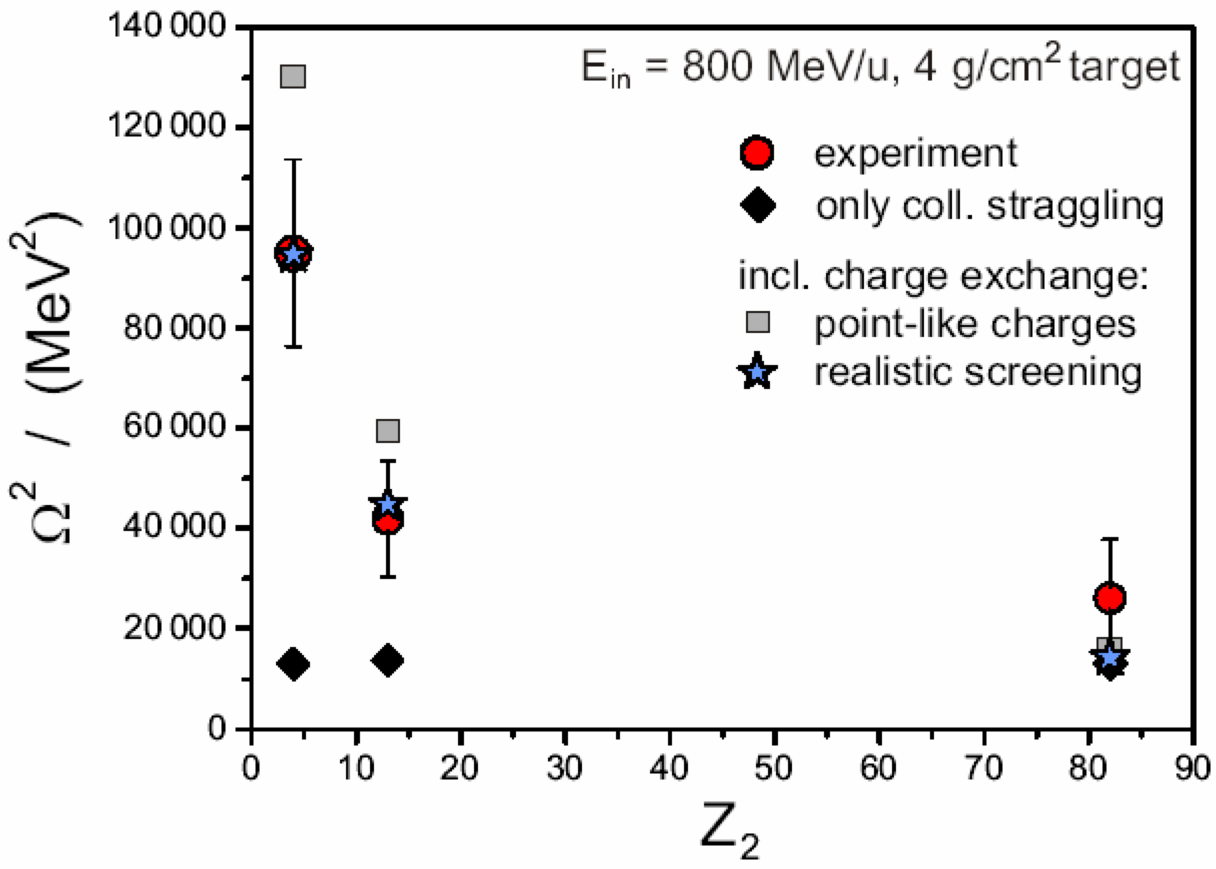
Energy Straggling -- Screening



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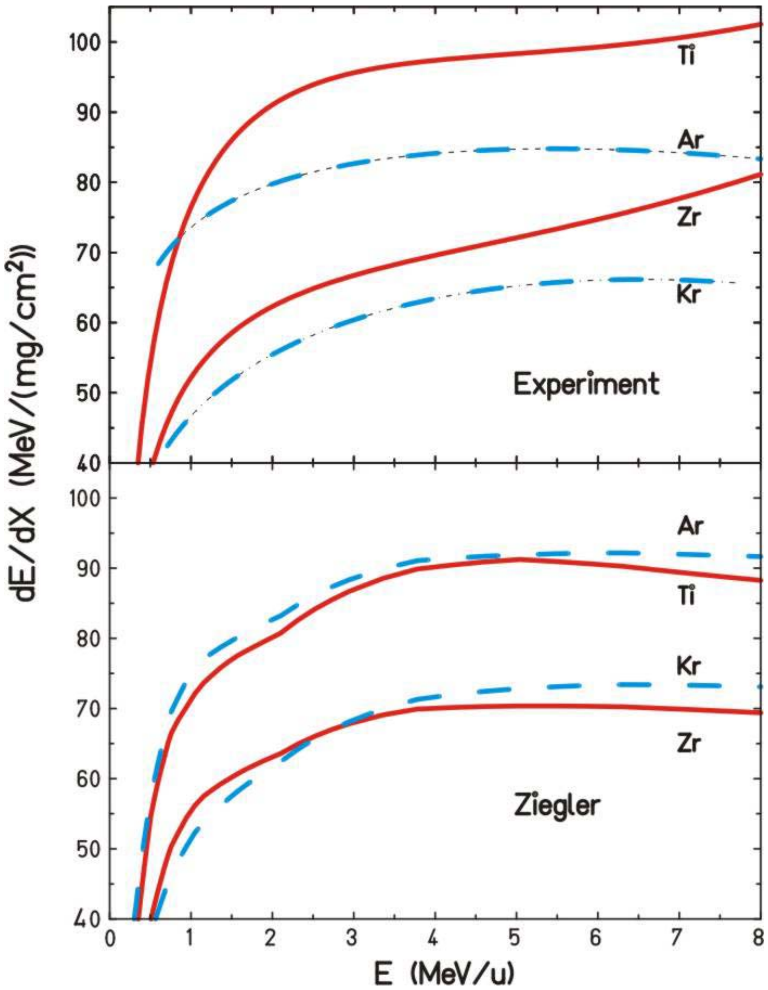
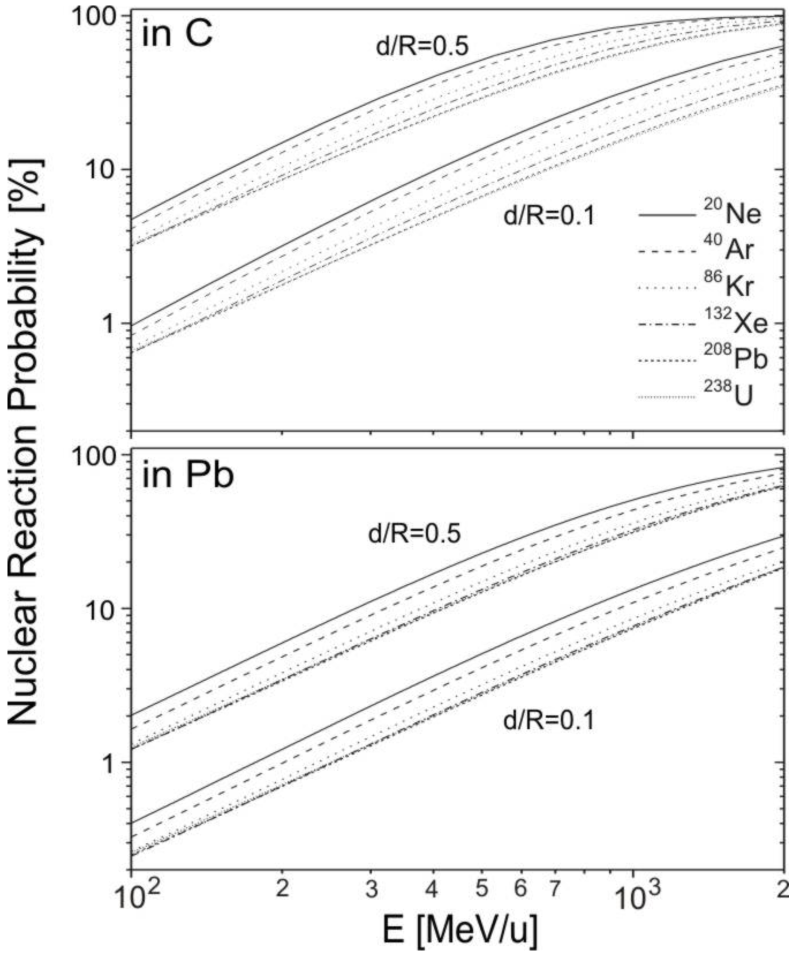
Gas-Solid Difference in Stopping Powers



The experiment reveals a gas-solid difference in the stopping of partially stripped heavy ions.

Most widely used code does not include the experimental observation.

Nuclear Reactions in Matter



macroscopic :

$$\sigma_{reac} = \pi R^2 (1 - \frac{B}{E_{cm}});$$ semiempirical,

$$R = r_0 [A_t^{1/3} + A_p^{1/3} + 1.85 \frac{A_t^{1/3} A_p^{1/3}}{A_t^{1/3} + A_p^{1/3}} - C(E)] + ...$$

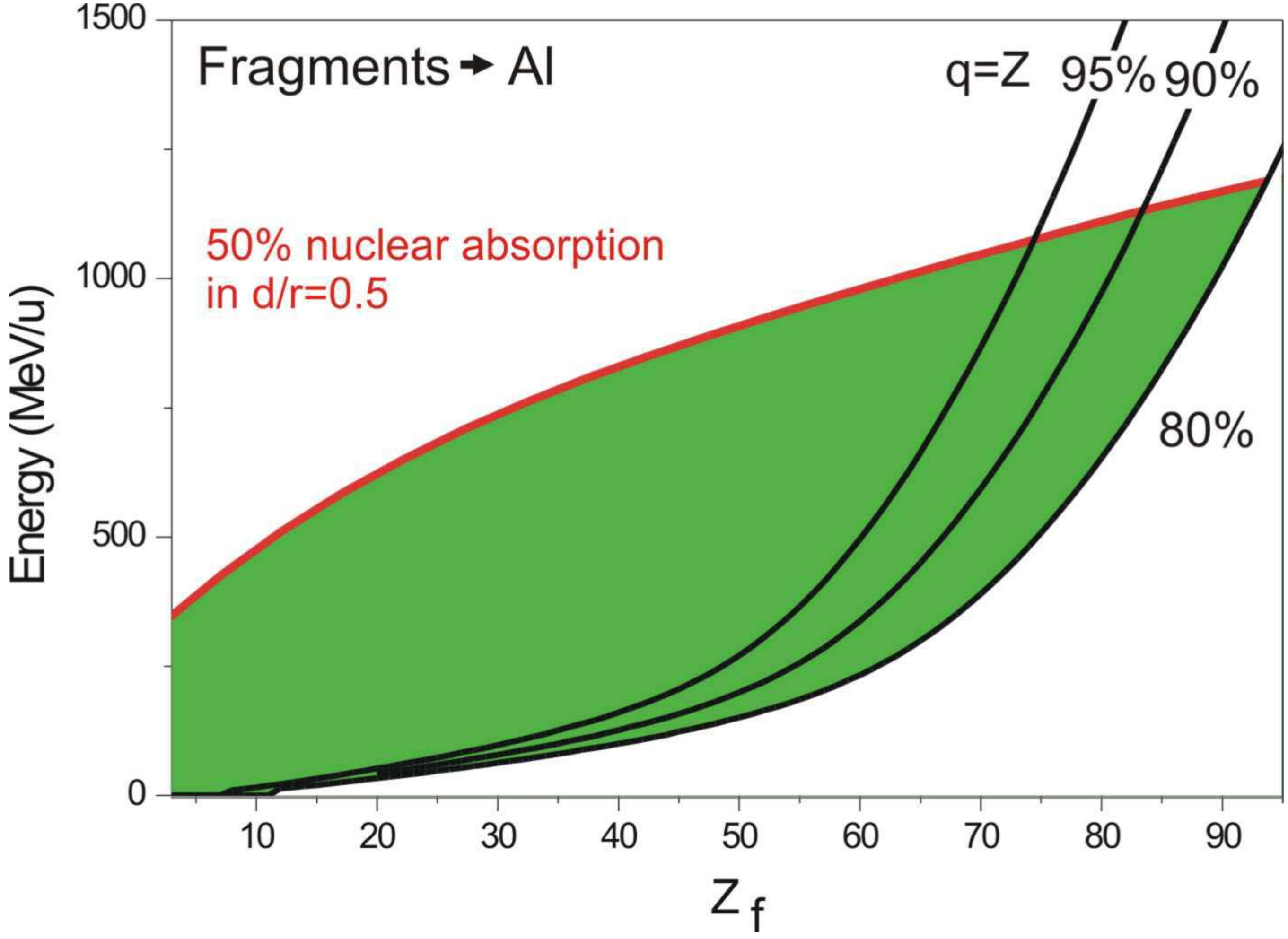
microscopic :

$$\sigma_{reac} = \int_0^\infty 2\pi\, (1 - T(p))\, b\, db$$

$$T(b) = \exp\{-\sigma_{NN}(E) \int d^2 r\ \rho_t(r)\ \rho_p(r - b)$$

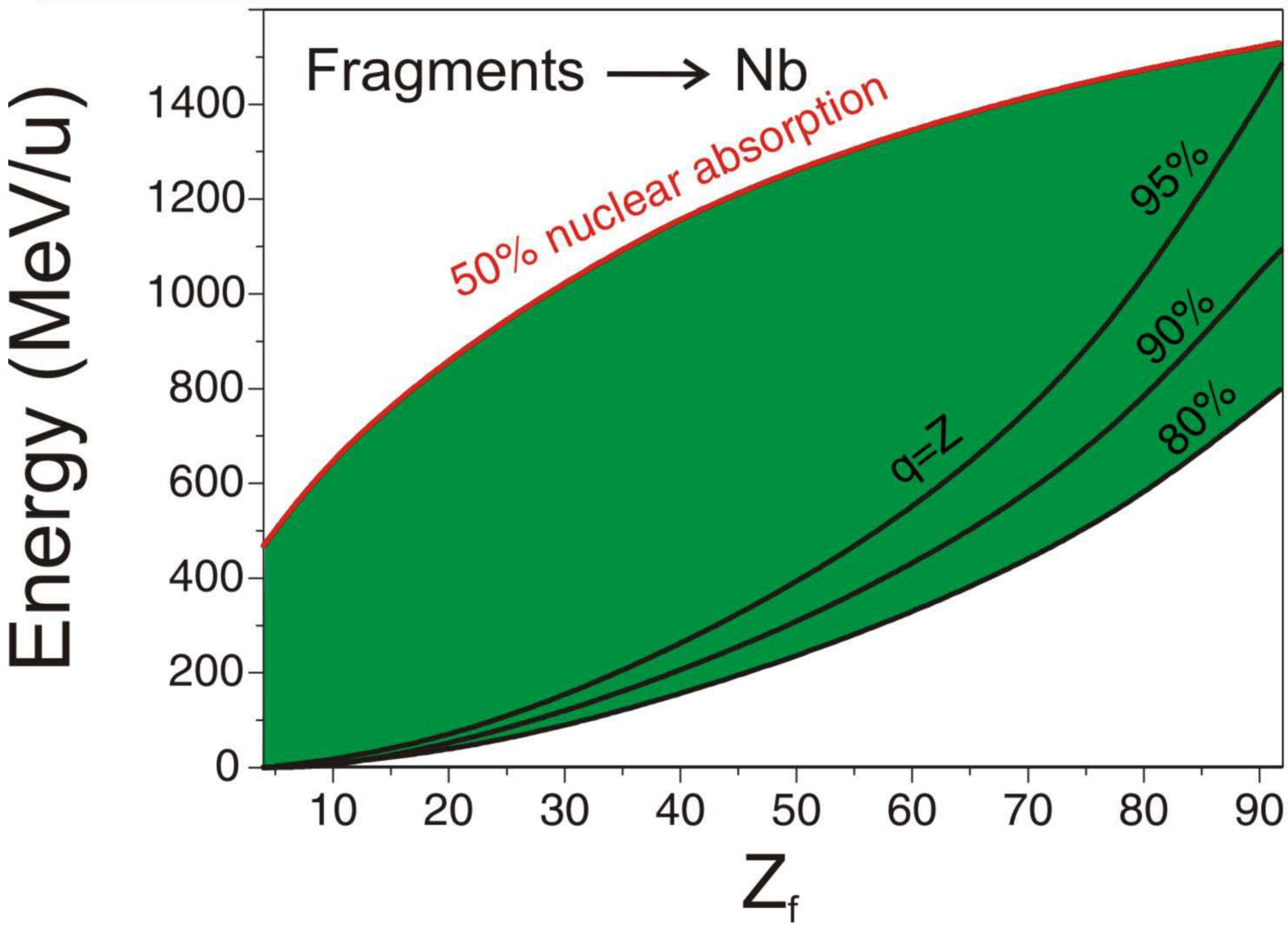
Densities of Target and Projectile

Ionisation Degree
Nuclear Reactions in Matter



Ionisation Degree

Operating Domain of the Bρ-ΔE-Bρ Separation



H. Geissel et al. NIM B 204 (2003) 71