Physics with Exotic Nuclei and Exotic Atoms at Relativistic Energies

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- Introduction √
- **Momentum Measurements, Ion Optics, Spectrometers**

Heavy Ion Momentum Measurements

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}},$$

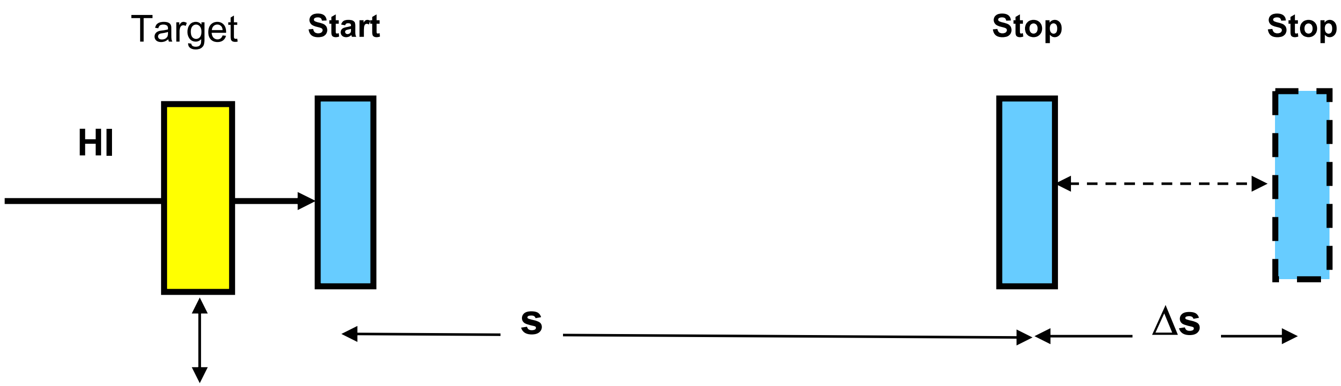
m_0 rest mass
v velocity of ions
c velocity of light

1. Time of flight measurement in a free drift space

2. Magnetic Analysis

3. Time of flight measurement in ion optical systems

I. Time-of-Flight Measurements



1. Time of flight measurement in a free drift space

$$\frac{\Delta p}{p} = \frac{\Delta v}{v}\left[\frac{1}{1-\frac{v^2}{c^2}}\right]$$

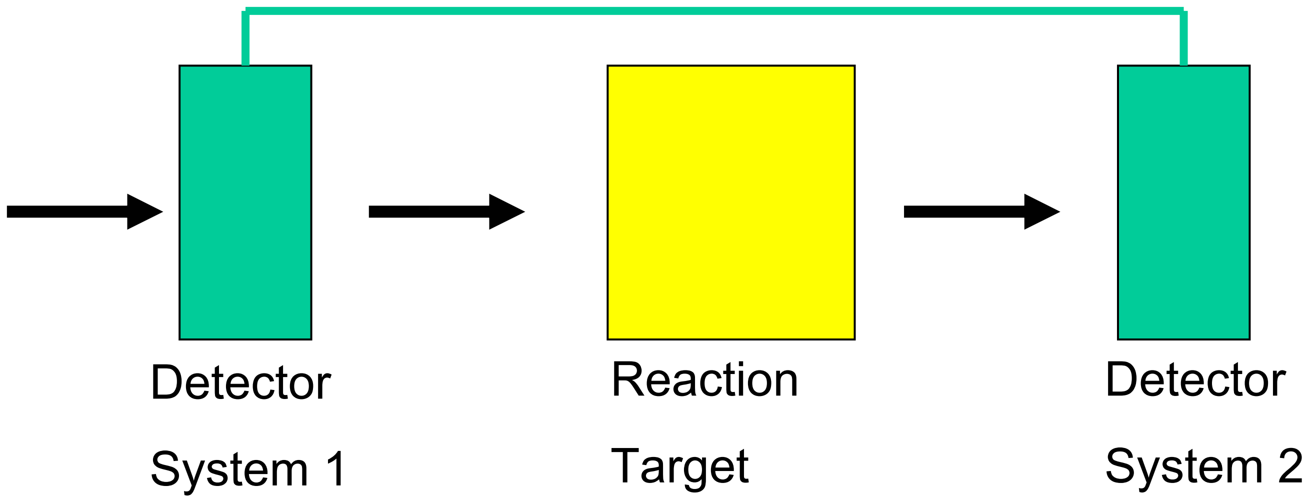
s = 30 m   Δ t = 100 ps

E (MeV/u)	Δp/p
100	5.26 E-4
500	1.79 E-3
1500	6.28 E-3
5000	4.00 E-2

Precision Momentum Measurements under the condition of an incident beam with large emittance

Solutions:

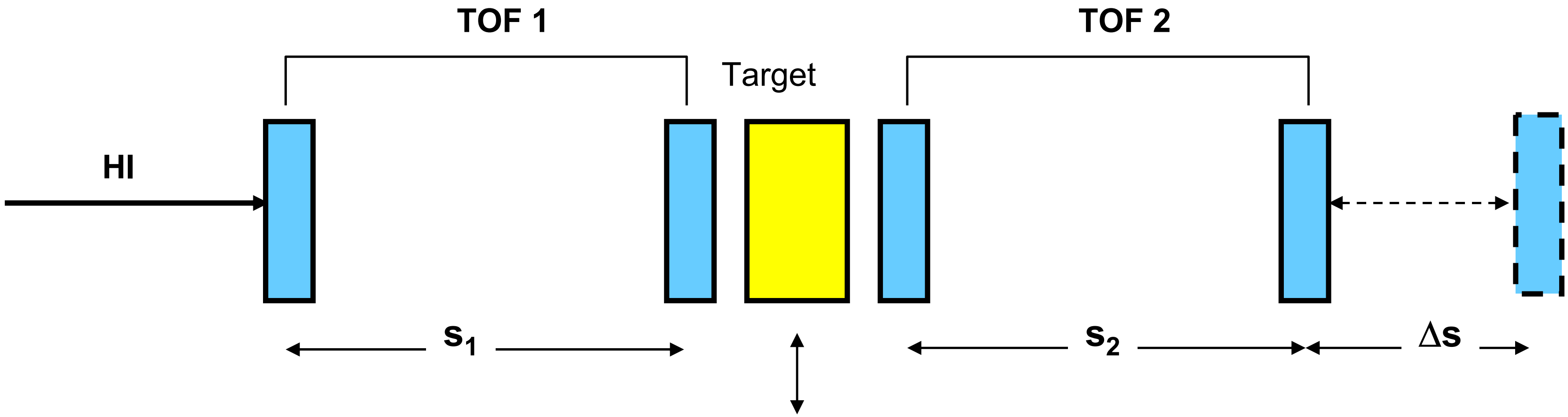
1. Coincidence Measurements in front and behind the reaction target (event-by-event)



2. Special ion optical systems

(Energy-loss spectrometer)

II. Double Time-of-Flight Measurements



- Coincidence measurements (event-by-event) allow precise velocity (e.g. energy-loss) determination independent of the incident beam spread

- Provides easy energy variation in front of D-TOF (inhomogeneous degrader, angle scattering)

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Motion of Charged Particles in Electromagnetic Fields

is governed by the Lorentz force

Lorentz Equation: $$\vec{F} = \frac{\mathrm{d}\,\vec{p}}{\mathrm{d}t} = Z_1 e(\vec{E} + \vec{\mathrm{v}} \times \vec{B})$$

Hamilton Function: $$H = Z_1 e\phi + c\left\{m_0^2 c^2 + (\vec{p} - Z_1 e\vec{A})^2\right\}^{1/2}$$

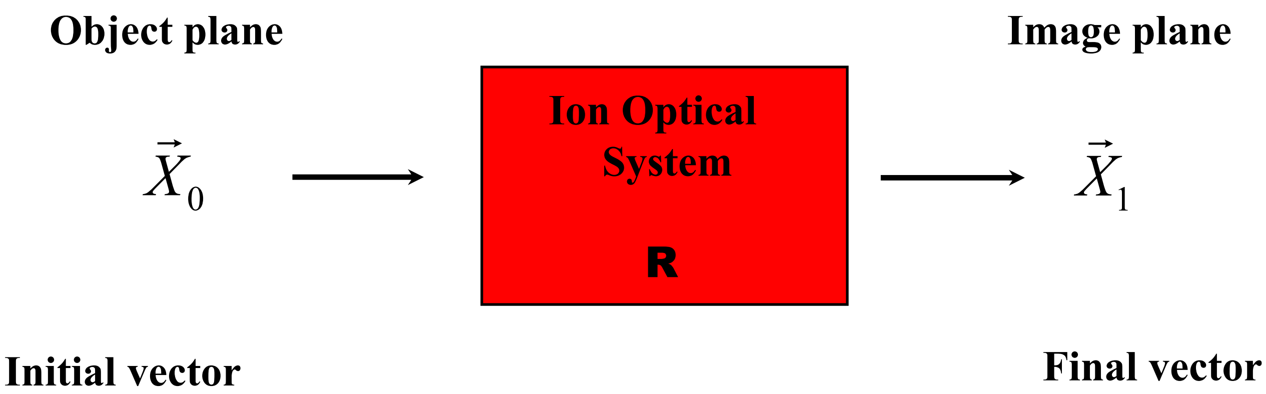
$$\vec{B} = \nabla \times \vec{A}, \qquad \vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla \phi$$

Canonical Equations: $$\frac{\mathrm{d}\,q_i}{\mathrm{d}t} = \frac{\partial H}{\partial p_i}, \qquad \frac{\mathrm{d}\,p_i}{\mathrm{d}t} = -\frac{\partial H}{\partial q_i}$$

Liouville's Theorem: $$\mathrm{d}\,\rho_p / \mathrm{d}\,t = 0$$

The particle density in phase space is invariant under the action of conservative forces.

Ion optical Imaging with Matrix Description



The transfer function that images the initial phase space to any desired position in the system can be represented by a Taylor series in matrix form.

$$\vec{X}_0 = \begin{pmatrix} x_0 \\ \theta_0 \\ y_0 \\ \varphi_0 \\ \ell_0 \\ \delta p_0 \end{pmatrix} \qquad \mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \qquad \vec{X}_1 = \begin{pmatrix} x_1 \\ \theta_1 \\ y_1 \\ \varphi_1 \\ \ell_1 \\ \delta p_1 \end{pmatrix}$$

$$\vec{X}_1 = \mathbf{R}\,\vec{X}_0$$

$\mathbf{R}$: Transfer matrix

Matrix Elements in First and Second Order Approximation

$$x_i = \sum_{j=1}^{6} R_{ij}\; x_j(0) + \sum_{j=1}^{6} \sum_{k}^{6} T_{ijk}\; x_j\; x_k$$

where

$$x_1 = x \qquad x_2 = x' = \theta \qquad x_3 = y \qquad x_4 = y' = \phi \qquad x_5 = l \qquad x_6 = \delta p$$

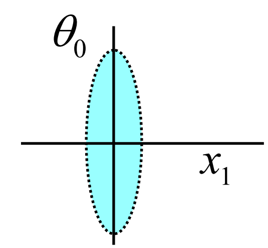
$$R_{11} =: \frac{\partial x_1}{\partial x_0} =: (x, x)$$ (Magnification)

$$R_{12} =: \frac{\partial x_1}{\partial \theta_0} =: (x, x')$$

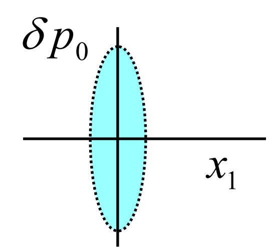
$$R_{16} =: \frac{\partial x_1}{\partial \delta p_0} =: (x, \delta)$$ (Dispersion)

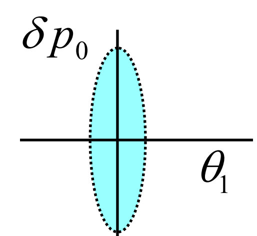
$$R_{26} =: \frac{\partial x'_1}{\partial \delta p_0} =: (x', \delta)$$

Image condition $R_{12} = 0$



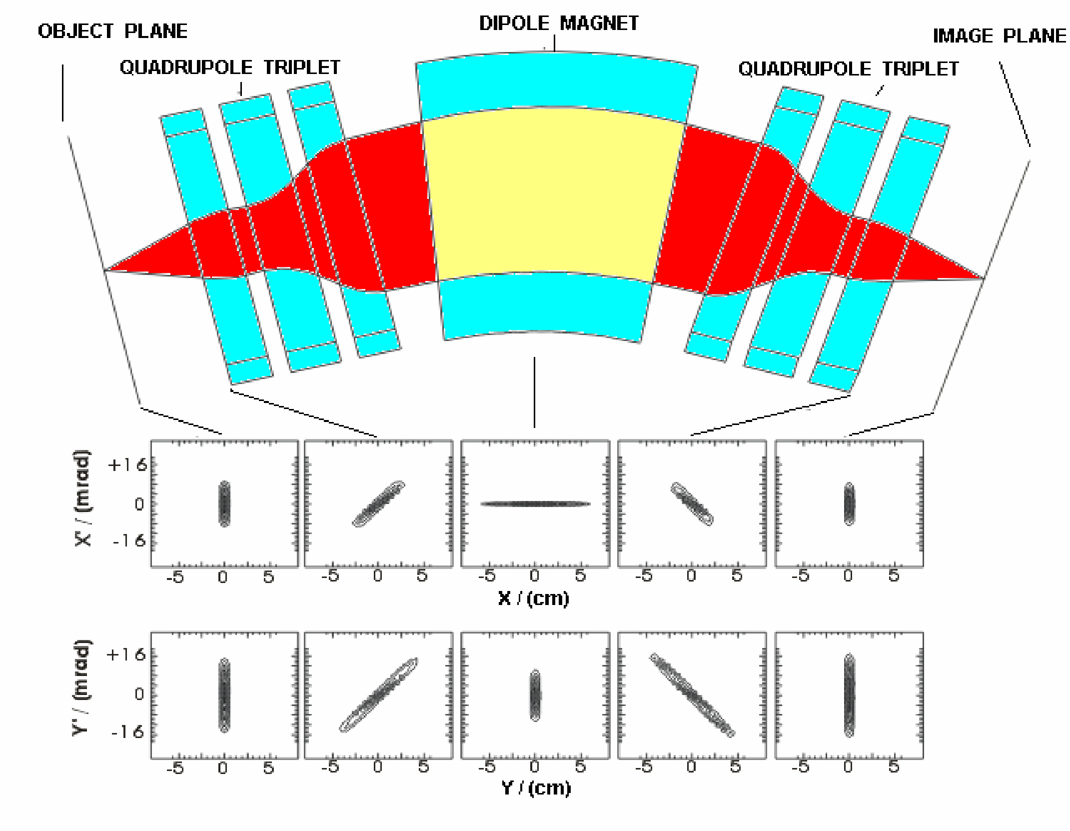
Achromatic condition $\begin{cases} R_{16} = 0 \\ R_{26} = 0 \end{cases}$





H. Wollnik, Optics of Charged Particles

Conventional Liouvillian Dispersive Ion Optical System



$$x''+k_x(s)x=\frac{1}{\rho}\frac{\Delta p}{p}, \quad y''+k_y(s)y=0$$

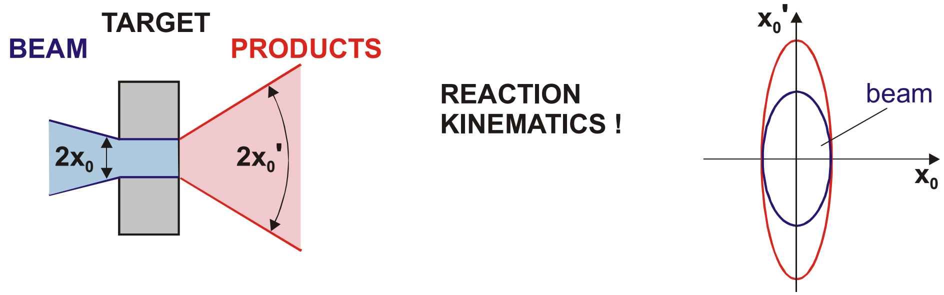
instead of a time variable the path length s is used along the central trajectory

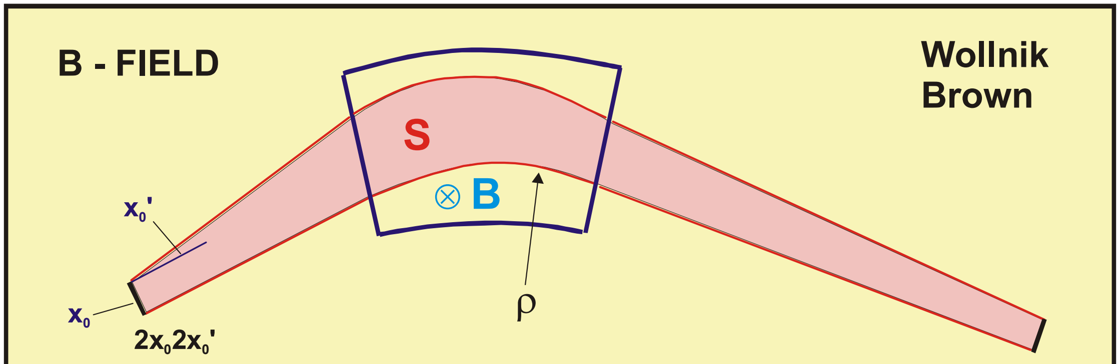
$$x''=\frac{\partial^2 x}{\partial s^2}, \quad y''=\frac{\partial^2 y}{\partial s^2}$$

$$\vec{R}=\begin{bmatrix} C_x & S_x & 0 & 0 & 0 & D_x \\ C'_x & S'_x & 0 & 0 & 0 & D'_x \\ 0 & 0 & C_y & S_y & 0 & 0 \\ 0 & 0 & C'_y & S'_y & 0 & 0 \\ (s,x) & (s,x') & 0 & 0 & 1 & (s,\delta p) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The phase-space volumes are constant in the x- and y subspaces (Liouville theorem).

Momentum Resolving Power





$$\frac{p}{\Delta p} = \frac{1}{2x_0 2x_0'} \cdot \frac{1}{B\rho} \int \vec{B}\, d\vec{f}$$

Magnetic

$$B\rho = \frac{p}{q}$$

Electrostatic

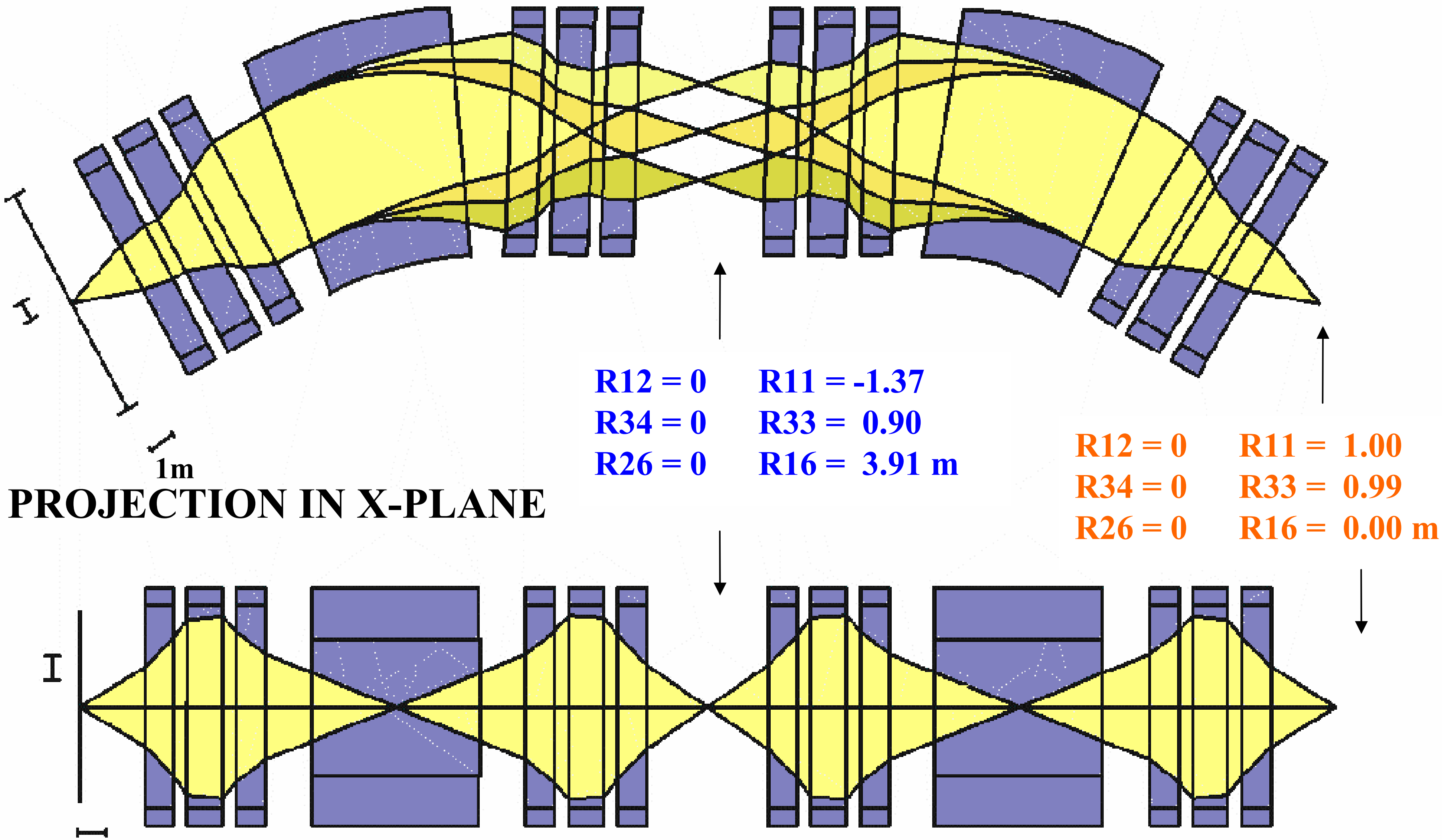
$$E\rho = \frac{p\,v}{q}$$

Rigidities

The optical resolving power is determined by the area **S** used in the dispersive Element.

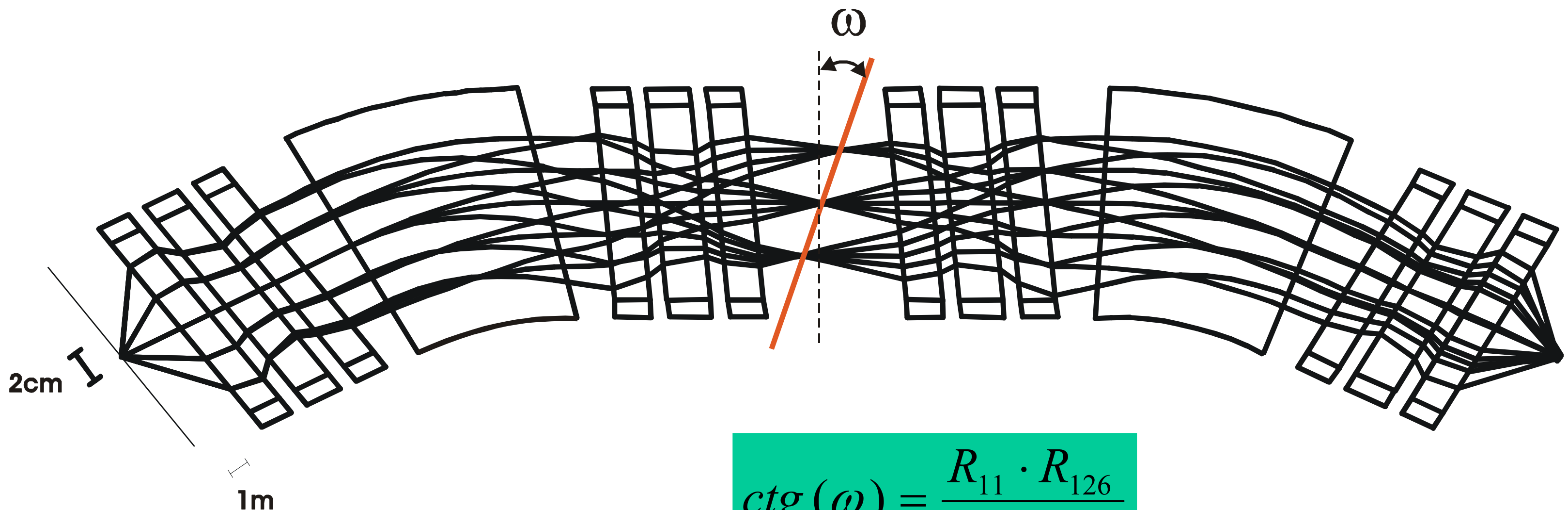
$$R_\Sigma = \frac{1}{2x_0 2x_0'} \left(\sum \frac{B_i S_i}{B\rho} + \sum \frac{E_i S_i}{E\rho} \right)$$

Two-Stage Achromatic System

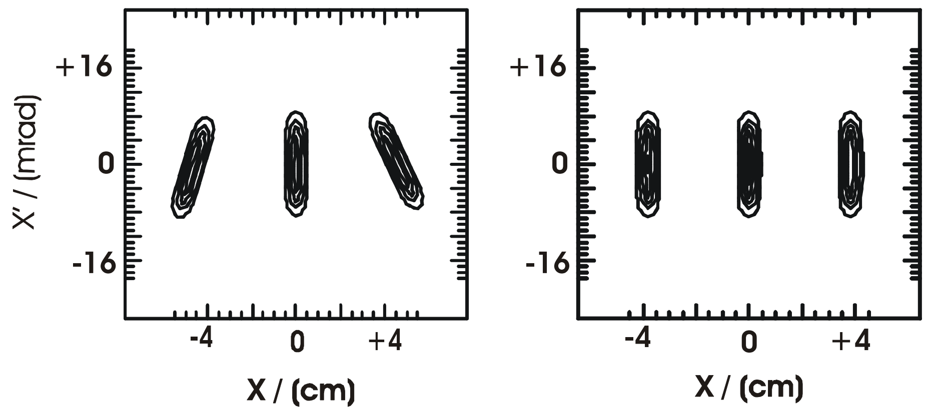


PROJECTION IN Y-PLANE

Image Aberrations



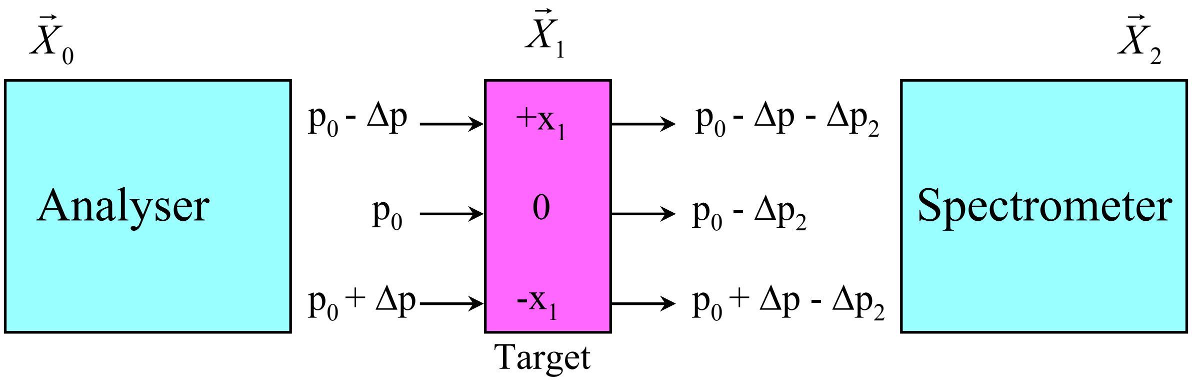
$$ctg\,(\omega) = \frac{R_{11} \cdot R_{126}}{R_{16}}$$



Higher order Calculation

Uncorrected **Corrected**

Principle of an Energy-Loss Spectrometer



Point - to - point image condition: ($R_{12} = (x, x') = 0$)

$$x_1 = {}^1R_{11} x_0 + {}^1R_{16} \overbrace{\left(\frac{p - p_0}{p_0} \right)}^{\delta}$$

$$x_2 = {}^1R_{11} x_1 + {}^2R_{16} \left(\delta - \frac{\Delta p_2}{p_0} \right)$$

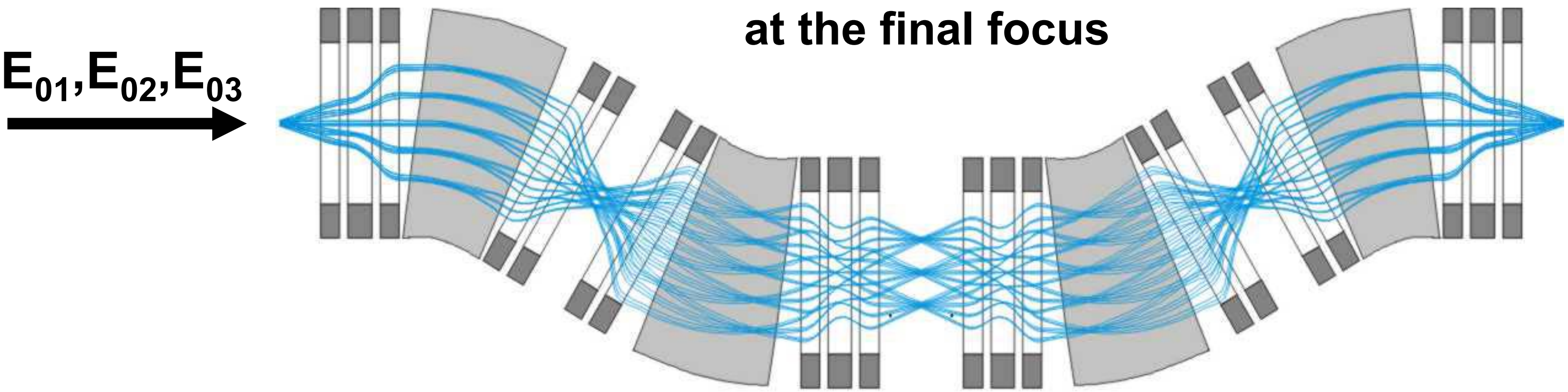
$$x_2 = {}^2R_{11}\,{}^1R_{11} x_0 + \underbrace{\left({}^2R_{11}\,{}^1R_{16} + {}^2R_{16} \right)}_{=0} \delta - {}^2R_{16} \frac{\Delta p_2}{p_0}$$

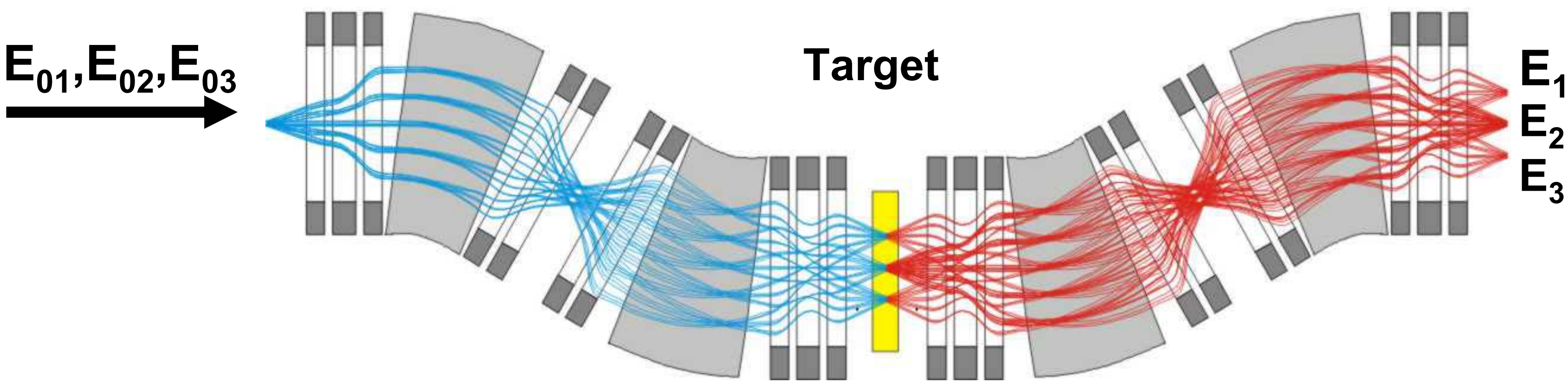
Image size of the final focus is independent of the incident momentum spread if

$${}^2R_{16} = -{}^2R_{11}\,{}^1R_{16}$$

The FRS as an Energy-Loss Spectrometer

Incident energy shifts do not show up at the final focus





Ion Interaction with Matter inside Ion-Optical Systems

Non-Liouvillian Phase-Space Modelling

If e.g. matter is included in the ion optical system we have to deal with non-Liouvillean systems

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i} + Q \qquad \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$$

the phase-space density ρ_p is not conserved any more $\frac{d\rho_p}{dt} = -\rho p \sum_{i=1}^{3} \frac{\partial Q_i}{\partial p_i}$

where

$\frac{\partial |Q_i|}{\partial p_i} > 0$ causes an increase

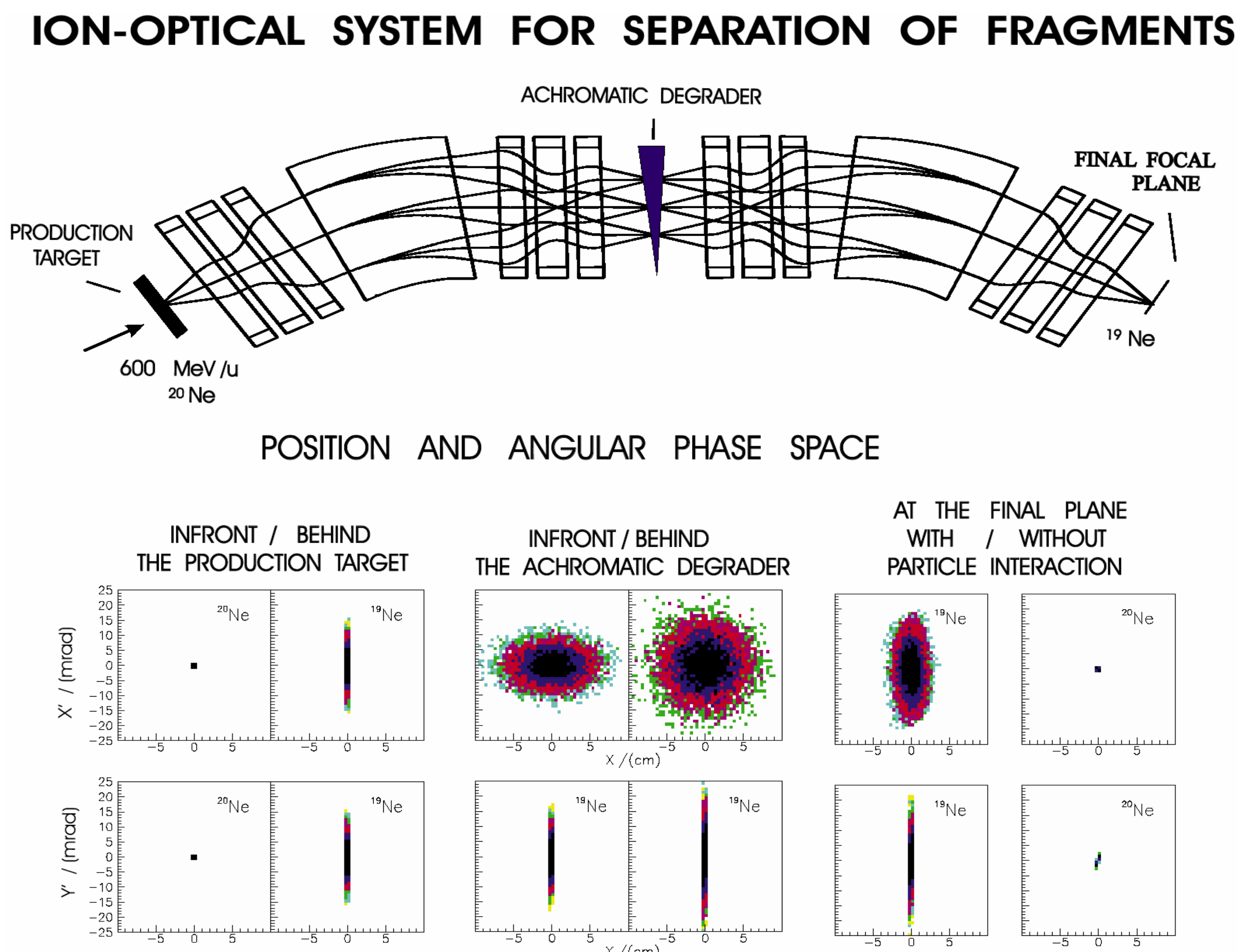
of phase-space density

$\frac{\partial |Q_i|}{\partial p_i} < 0$ causes a decrease

H. Geissel et al. Nucl. Instr. Meth. A282 (1989) 247

Matter Paced Inside an Achromatic Ion-Optical Systems

Separation of Projectile Fragments



Shaped Matter as Ion-Optical Elements

Matrix description

The transformation of a beam matrix by an ion-optical system and a degrader:

$$\boldsymbol{\sigma}(2) = \mathbf{R}\,\boldsymbol{\sigma}(1)\,\mathbf{R}^T$$

The degrader separates the ions according to the slowing down characteristics:

$$\frac{\partial v}{\partial x} = \frac{1}{m_0 c^2 \beta}\frac{d\,E}{d\,x}; \qquad v = \beta\,\gamma$$

The degrader has a thickness variation *d*(α) along the dispersive coordinate x

$$W_x = -\frac{1}{\mathrm{v}_2}\frac{\partial v_2}{\partial x}\alpha, \quad W_v = \frac{v_1}{v_2}\frac{\partial v_2/\partial x}{\partial v_1/\partial x}$$

$$\sqrt{\sigma_x(2)}_{achrom.} = \left[(C_{x1}C_{x2} + W_v)^2\sigma_x(1) + D_{x2}^2\sigma_{\delta v}\right]^{1/2}$$

$$\sqrt{\sigma_x(2)}_{monoenerg.} = \left[C_{x1}^2 C_{x2}^2 (1+W_v)^2\sigma_x(1) + C_{x2}^2 D_{x1}^2\sigma_x(0) + D_{x2}^2\sigma_{\delta v}\right]^{1/2}$$

Mass and Element Resolving Powers

$$R_m = \frac{\frac{1}{v_1}\frac{\partial v_1}{\partial x}\left[\frac{v_2}{\partial v_2/\partial x} - \frac{v_1}{\partial v_1/\partial x} + d\right]}{\left[4\frac{C_{x1}^2\sigma_x(1)}{D_{x1}^2} + 4W_v^{-2}\sigma_{\delta v}\right]^{1/2}}$$

$$R_x = \frac{-\frac{1}{v_1}\frac{\partial v_1}{\partial x}\left[\frac{v_2}{\partial v_2/\partial x} - \frac{v_1}{\partial v_1/\partial x} + 2d\right]}{\left[4\frac{C_{x1}^2\sigma_x(1)}{D_{x1}^2} + 4W_v^{-2}\sigma_{\delta v}\right]^{1/2}}$$

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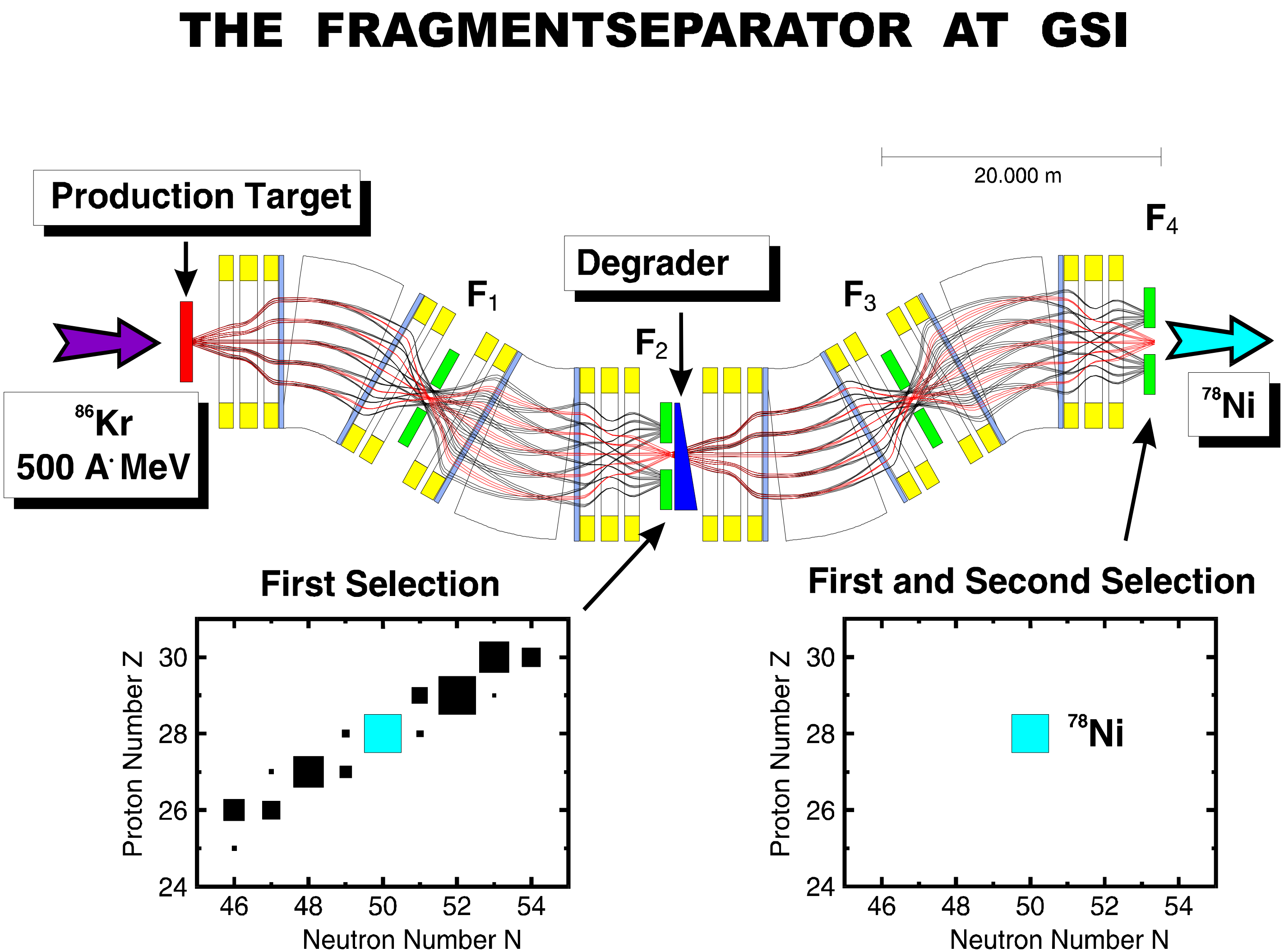
Separation Performance of the FRS



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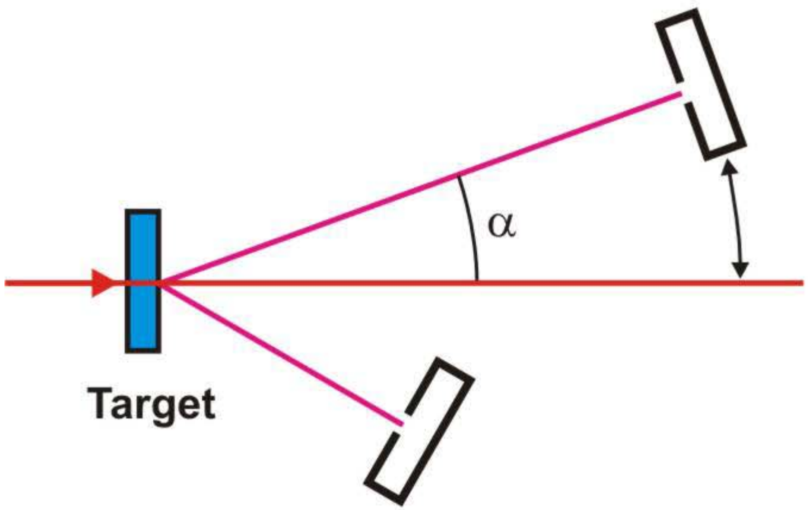
Measurement of Angular Scattering

a) 2 Detector Method



b) Spectrometer Method

$$\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ \delta_0 \end{pmatrix}$$

$$x = R_{11}x_0 + R_{12}x_0' + R_{13}\delta_0$$

$$R_{11} = R_{13} = 0$$

R. Anne et al. Nucl. Instr. Meth.34 (1988) 295