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Inflation + quintessence: non-canonical models

(or "What happens if the inflaton or the quintessence field have a small speed of sound?")

The canonical models of inflation / quintessence have a $\mathcal{L}$:

$$\mathcal{L}_\phi^{CAN} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Here we want to concentrate on

$$\mathcal{L}_\phi^{NON-CAN} = P(X,\phi) \qquad X = -\partial_\mu\phi\,\partial^\mu\phi$$

To gain some intuition + motivation let us look at the case:

$$P(X)$$

What's that?

$$T_{\mu\nu} = 2P'(X)\,\partial_\mu\phi\,\partial_\nu\phi + P(X)\,g_{\mu\nu}$$

Compare with a perfect fluid: $T_{\mu\nu} = (\rho + p)\,U_\mu U_\nu + p\,g_{\mu\nu}$

If we identify: $\rho = 2P'X - P \qquad p = P \qquad U_\mu = \dfrac{\partial_\mu\phi}{\sqrt{X}}$

we are describing a perfect fluid! Unique relationship $p(\rho)$: barotropic

In particular, when $p = w\rho$ (w = const)

$$P = X^{\frac{1+w}{2w}} \qquad \left(\text{e.g. } P = X^2 \text{ for a radiation fluid!}\right)$$

let us do one check of it:

in Minkowski $\phi = ct$ is a solution $\forall c$
($\neq c$ represents $\neq$ values of ρ which is constant in Mink)

If I look at perturbations: $\phi = ct + \delta\phi(\vec{x}; t)$

$$X = (c + \delta\dot{\phi})^2 - (\nabla\delta\phi)^2 = c^2 + \delta\dot{\phi}^2 + 2c\,\delta\dot{\phi} - (\nabla\delta\phi)^2$$

at 2^{nd} order I get

$$\mathcal{L}_{2nd} = P'(c^2)\left[(\delta\dot{\phi})^2 - (\nabla\delta\phi)^2\right] + 2P''(c^2)\,c^2\,\delta\dot{\phi}^2$$

$$c_s^2 = \left.\frac{P'(x)}{P'(x) + 2xP''(x)}\right|_{x=c^2} = \frac{p'(x)}{\rho'(x)} = \frac{dp}{d\rho} = w$$

Indeed we are describing a $w =$ const fluid! Non-linear.

Useful to describe perturbations in RD or MD. Not so $\neq$ from perturbations during inflation

But here we want to concentrate on inflation + quintessence: K-inflation and K-essence

Usually we take $c_s^2 = 1$. What happens when $c_s^2 \ll 1$?

[Nothing strange to have $c_s^2 \neq 1$; the background spontaneously breaks Lorentz ...]

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The action: Though we are interested in $\neq$ aspects

Both in inflation and quintessence, we are interested to see the behaviour of perturbations around a background solution

Straightforward: $$S = \int d^4x \sqrt{-g}\, P(\phi; X)$$

$$\phi = \phi_0 (t + \pi(\vec{x}, t))$$

Plug back in the action and use background EOM

$$S = \int d^4x\, a^3 \left[\frac{1}{2}\left(\rho_\phi + p_\phi + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}\left(\rho_\phi + p_\phi\right)\frac{(\nabla\pi)^2}{a^2} \right.$$
$$\left. + 2M^4\left(\dot{\pi}^3 - \dot{\pi}\frac{(\nabla\pi)^2}{a^2}\right) - \frac{4}{3}\bar{M}^4\dot{\pi}^3 + \dots \right]$$

$$M^4 \equiv P_{XX} X_0^2 \qquad \bar{M}^4 \equiv P_{XXX} X_0^3$$

- $\rho_\phi + p_\phi$; M^4 ; $\bar{M}^4$ are arbitrary function of time (once I chose them I can, if I like, go back to P)
  - → What we are probing experimentally is the second action, not the first
  - → Field redefinition independence
  - → Sometimes only the second action makes sense (GC)

- $$c_s^2 = \frac{\rho_\phi + p_\phi}{\rho_\phi + p_\phi + 4M^4} = \left.\frac{\delta p}{\delta \rho}\right|_{\phi = \text{const}}$$ Velocity orthogonal

the same as for the fluid before

Inflation: $c_s^2 \to 0$ limit

$$M^4 \gg |\rho_\phi + p_\phi|$$

in this limit the cubic term becomes very large: non-Gaussianities

How large? $\quad \dot{\pi}^2 - c_s^2 (\nabla\pi)^2 \quad$ Freezing: $c_s k \simeq H$

$$NG = \frac{\mathcal{L}_3\ \dot{\pi}(\nabla\pi)^2}{\mathcal{L}_2} = \frac{\dot{\pi}(\nabla\pi)^2}{\dot{\pi}^2} \simeq \frac{1}{c_s^2} \underbrace{H\pi}_{\sim 10^{-5}}$$

Experimentally?

$N_{pixel}^{WMAP} \sim 2 \cdot 10^6 \qquad NG \lesssim 10^{-3} \qquad$ remarkable!

More precisely:

M^4 induces equilateral NG (derivatives term favor interaction among modes $\sim$ wavelength)

$$f_{NL}^{equil} = \frac{85}{324} \frac{1}{c_s^2} \qquad -125 < f_{NL}^{equil} < 435$$

$$\Longrightarrow c_s^2 > 0.011$$

Second independent operator, 4-point function ...

Violation of NEC

$$T_{\mu\nu} n^\mu n^\nu \geq 0 \quad \forall n^\mu, \text{ null} \quad \xRightarrow{\text{FRW}} \quad \rho + p \geq 0$$

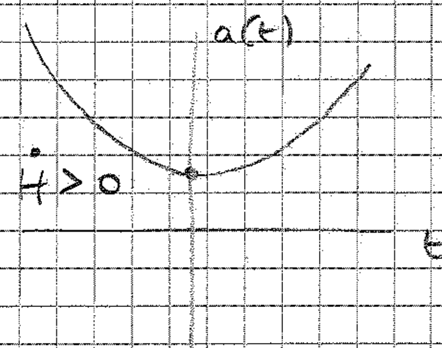
$$w \equiv \frac{p}{\rho} \qquad w \geq -1$$

$$\dot\rho = -3H(\rho + p)$$

NEC $\Rightarrow$ ρ decreases as the Universe expands

$\dot H \leq 0$

If ~~NEC~~ :
- No need for a Big Bang
- One can even have $H \to 0$ in our far past: start the Universe
- Bouncing cosmologies (as alternative to inflation)



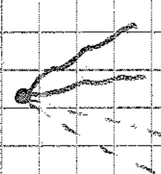
- Quintessence with $w < -1$

We must understand what is wrong with ~~NEC~~ : instabilities

$\longrightarrow$ For a minimal model $(M^4 = \bar M^4 = 0)$

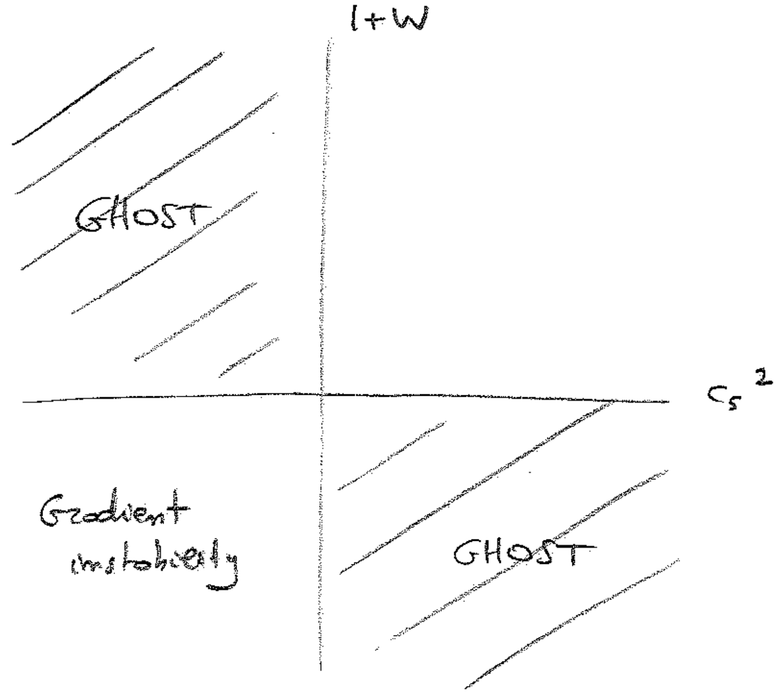
π field has wrong sign of $\dot\pi^2$ and wrong sign of $(\nabla\pi)^2$

GHOST



Vacuum quickly decays into positive and negative energy states

More generally:



To make a long story short, I can have $W<-1$ only with $c_s^2 \simeq 0$. In this regime higher order terms becomes relevant

Ghost condensate limit

Quintessence with $c_s^2 = 0$.

Does dark energy cluster?

For $1+w \neq 0$ quintessence energy density is time-dependent + spatially dependent: perturbations

Clustering of quintessence is relevant on scales below the sound horizon:

$$L_s = a \int \frac{c_s \, dt}{a}$$

When $c_s = 1$ $L_s \simeq H_0^{-1}$. Quintessence perturbations only at horizon scales.

ISW - galaxy correlation for example

For $c_s = 0$, $L_s = 0$ and quintessence clusters at all scales!

Indeed:

$$\nabla_\mu T^{\mu\nu} = \nabla_\mu \left[(\rho + p) u^\mu u^\nu + p g^{\mu\nu} \right]$$

If I project this equation $\perp$ 4-velocity I have the Euler equation:

$$u^\mu \nabla_\mu u^\nu = - \frac{1}{\rho_Q + p_Q} \left(g^{\nu\sigma} + u^\nu u^\sigma \right) \nabla_\sigma p_Q$$

$$P(X; \phi) \qquad \text{it only depends} \quad P(X; \phi = \text{const})$$

remember $c_s^2 = \left. \dfrac{\delta p}{\delta \rho} \right|_{\phi = \text{const}}$

$$\Longrightarrow u^\mu \nabla_\mu u^\nu = 0 \qquad \text{as DM!}$$