Lattice QCD with mixed actions

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Outline

- Mixed action Lattice QCD: What? Why? Does it make sense?
- Brief overview: Status of mixed action QCD
  - Numerical simulations
  - (Mixed) Chiral Perturbation Theory
- Some issues with mixed action QCD
  - Size of the cut-off effects
  - Quark mass matching
  - Partial quenching effects
- What I would like to see next

Mixed action Lattice QCD: What is it ?

Fermions enter a numerical lattice calculation twice

1. generating the configurations (sea quarks) $$e^{-S_{eff}} = \det(D+m)e^{-S_g}$$

2. computing quark propagators (valence quarks) $$(D+m)^{-1}$$

these are two separate steps

→ Use different Dirac operators for sea and valence quarks $$e^{-S_{eff}} = \det(D_S+m_S)e^{-S_g}$$

"Mixed action Lattice QCD" $$(D_V+m_V)^{-1}$$

Mixed action Lattice QCD: What is it ?

Mixed action QCD is a generalization of partially quenched QCD !

- Partially quenched QCD $D_V = D_S$ $m_V \neq m_S$
- Mixed action QCD $D_V \neq D_S$ $m_V \neq m_S$

Theoretical formulation: sea and valence quarks + valence ghosts (similarly to PQQCD)

Difference: No full (unquenched) subsector in mixed action QCD

$$m_V = m_S \quad \not\longrightarrow \quad \text{unquenched QCD}$$

It is even not entirely obvious what $m_V = m_S$ means (later more)

Does it make sense ?

Any "good" lattice Dirac operator satisfies

$$D(p) = ip_\mu\gamma_\mu + \mathcal{O}(ap^2)$$

1. All "good" D give the same continuum limit $a \to 0$
2. Differences are $\mathcal{O}(a)$ and vanish in the continuum limit

$$D_S - D_V = \mathcal{O}(a)$$

Naive expectation: Using different Dirac operators for sea and valence quarks results in errors of $\mathcal{O}(a)$ which vanish in the continuum limit

Some lattice Dirac operators

- Wilson fermions (twisted or untwisted)
- Staggered fermions (no fourth root)
- Ginsparg-Wilson fermions (satisfying the Ginsparg-Wilson relation)
  - Overlap fermions
  - Domain wall fermions
- Approximate GW fermions
  - Chirally improved (CI) fermions
  - Fixed-point (FP) fermions

Note: Using different approximations of the same type of fermions is also a mixed action theory !

Example: Domain wall fermions with $L_{5,S} \neq L_{5,V}$

Why mixed action QCD ?

Theoretically "ideal"
lattice simulation: Dynamical Ginsparg-Wilson fermions

But: Too expensive (right now !)
also algorithmic issues (changing the top. sector)

→ talk by P. Hasenfratz

Cost efficient compromise: Mixed simulation with

- Cheap sea quarks (Wilson, twisted mass Wilson, staggered ...)
- Ginsparg-Wilson valence quarks

Why mixed action QCD ?

Advantages:

- Cost efficient alternative to expensive full GW simulations
- Exact chiral symmetry in the valence sector

  Beneficial for calculation of weak matrix elements ($K \rightarrow \pi\pi$, ...)
- Use other people's configs ⟶ ~ cost of quenched GW simulation

Drawbacks:

- Unitarity is lost at non-zero lattice spacing
- Afflicted with all "diseases" known in PQQCD
- No unquenched sector ($m_V = m_S$) at non-zero lattice spacing

Status - Numerical simulations

Staggered sea quarks (MILC configurations):

- LHPC collaboration, using domain-wall valence quarks
  Edwards et.al.
  - Pion and nucleon form factors
    Phys.Rev.D72:054506,2005
  - Nucleon axial charge
    hep-lat/0510062
  - Moments of parton distributions
    Nucl.Phys.Proc.Suppl.140:255-260,2005

- NPLQCD collaboration, using domain-wall valence quarks
  Beane et.al.
  - I=2 $\pi\pi$ scattering length
    hep-lat/0506013

- UKQCD collaboration, using overlap valence quarks
  Bowler et.al.
  - Light hadron spectrum, decay constants, nucleon masses
    JHEP 08 (2005) 003

    Exploratory study !!! Only 10 configurations

Status - Numerical simulations

- In all these simulations:
  - One lattice spacing $a \approx 0.125 fm$

    No continuum extrapolation
  - Continuum ChPT is used for the chiral extrapolation

    Error due to neglected lattice spacing artifacts

→ All simulations have a preliminary and explorative character

Mixed Chiral Perturbation Theory

Lee, Sharpe, Singleton
OB, Shoresh, Rupak

Step 1: Construct Symanzik effective theory (continuum theory)

$$\longrightarrow \quad S_{\rm Sym} = S_4 + aS_5 + a^2 S_6 + \ldots$$

- $S_{\rm Sym}$: Most general expression compatible with locality and symmetries
- First term: Continuum PQQCD
- S_5 , S_6 : Higher dimensional operators of dim 5, and dim 6
- Simplest example: Wilson fermions (explicit chiral symmetry breaking)

$$\longrightarrow \quad S_5 = c\,\overline{\psi}_S \sigma_{\mu\nu} G_{\mu\nu} \psi_S$$

Pauli term

Mixed Chiral Perturbation Theory

Step 2: Symmetry group of PQQCD (massless limit)

$$G_{\rm PQQCD} = SU(N_S + N_V|N_V)_L \otimes SU(N_S + N_V|N_V)_R$$

- Assumption: spontaneously broken to vector subgroup

  → light pseudo scalar Goldstone bosons

  → described by a chiral Lagrangian

- Follow standard procedure to construct the chiral Lagrangian

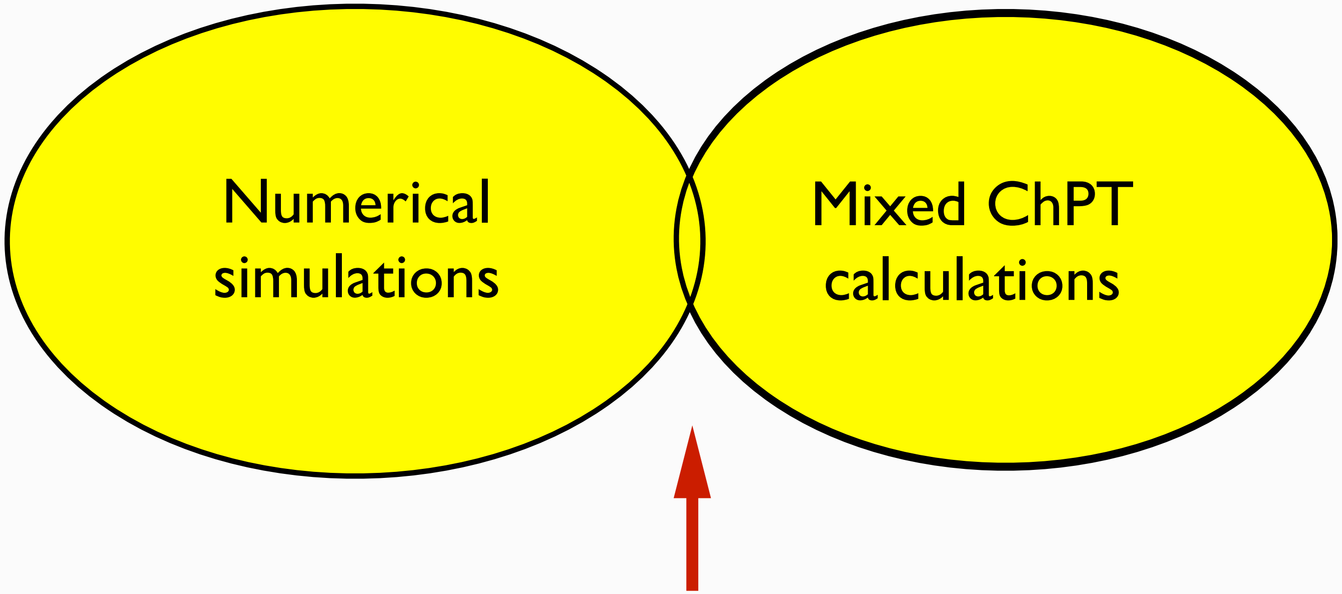
  Spurion analysis for mass and symmetry breaking terms in S_5 and S_6

  → chiral Lagrangian with explicit a dependence

Status - Mixed ChPT

- Staggered sea quarks:
  - Pseudoscalar masses and decay constants
    OB, Bernard, Shoresh, Rupak 2005
  - I=2 $\pi\pi$ scattering length
    Chen et.al. 2005
  - Baryon masses
    Tiburzi 2005
  - Vector meson masses
    Grigoryan, Thomas 2005
  - Scalar correlator
    Prelovsek 2005
- Wilson sea quarks
  - Pseudoscalar masses
    OB, Shoresh, Rupak 2003
  - Baryon masses
    Tiburzi 2005
  - Nucleon properties (magnetic moments, ...)
    Beane, Savage 2003; Arndt, Tiburzi 2004
  - Vector meson masses
    Grigoryan, Thomas 2005
  - The role of the double pole
    Golterman, Izubuchi, Shamir 2005

Status summary



Unfortunately very little overlap

- No real 'confrontation' has been done so far !
- No results to assess the mixed action approach !

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Symanzik action for the mixed theory

$$S_{\rm Sym} = S_4 + aS_5 + a^2 S_6 + \dots$$

- First term: Continuum PQQCD
- S_5: Operators with sea fields only (for Wilson sea quarks)
- Three types of operators of $\mathcal{O}(a^2)$
  - Type 1: Involve sea fields only
  - Type 2: Involve valence fields only
  - Type 3: Involve both

Mixed 4 - fermion operators

General structure of mixed 4 - fermion operators

OB, Bernard, Rupak, Shoresh 2005

$$O^{(6)}_{\rm Mix} = \overline{\psi}_{\rm S}(\gamma_{Spin} \otimes t^a_{Color})\psi_{\rm S}\, \overline{\psi}_{\rm V}(\gamma_{Spin} \otimes t^a_{Color})\psi_{\rm V}$$

- Allowed operators are products of a sea-sea and a val-val bilinear
- In total there are four types of these operators

  γ_{Spin}: vector or axial vector

  t^a_{Color}: color group generator or identity

- For staggered sea quarks: The sea-sea bilinear is trivial in taste

  → same form as four Wilson sea quarks

Direct sea - valence coupling at $\mathcal{O}(a^2)$

LO chiral Lagrangian

$$\mathcal{L}_\chi = \frac{f^2}{8}\langle\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\rangle - \frac{f^2B}{4}\langle\Sigma M^\dagger + M\Sigma^\dagger\rangle + \frac{m_0^2}{6}\langle\Phi\rangle^2 + a^2\mathcal{V}.$$

- Singlet term explicitly left in the Lagrangian

  (for convenience, later we take $m_0 \to \infty$)

- $\mathcal{V}$: potential terms (no derivative) proportional to a^2

- $\mathcal{V}$ enters LO Lagrangian

  Underlying assumption: $m \approx a^2\Lambda_{\mathrm{QCD}}^3$

  This assumption can and should be checked (see later)

The potential

The potential can be written as a sum of two terms*

$$\mathcal{V} = \mathcal{U}_S + \mathcal{U}_V$$

- $\mathcal{U}_S$ contains sea fields only due to P_S : projector on sea fields
- $\mathcal{U}_S$ contains the same terms as
  - staggered ChPT (for staggered sea quarks)
    Lee, Sharpe, Bernard, Aubin

    example: $C_1 \langle \hat{\xi}_5 P_S \Sigma \hat{\xi}_5 P_S \Sigma^\dagger \rangle$ $\hat{\xi}_5$: taste matrix
  - Wilson ChPT (for Wilson sea quarks)
    Sharpe, Singleton, Rupak, Shoresh, OB, Aoki

    $$W_0^2 W_7' \langle P_S (\Sigma - \Sigma^\dagger) \rangle^2$$
- $\mathcal{U}_S$ does not contribute to val-val masses and decay constants at one loop

*for staggered fermions sometimes written as: $\mathcal{V} = \mathcal{U}_{\rm S} + \mathcal{U}'_{\rm S} + \mathcal{U}_{\rm V}$

The term $\mathcal{U}_{\rm V}$

$$\mathcal{U}_{\rm V} = -C_{\rm Mix}\langle\tau_3\Sigma\tau_3\Sigma^\dagger\rangle$$

$$\tau_3 = (1_S, -1_V)$$

- The potential $\mathcal{U}_{\rm V}$
  - stems from the mixed 4 fermion operators
  - contains only one term associated with one low-energy constant $C_{\rm Mix}$
  - preserves the SU(4) taste symmetry for staggered sea quarks
    (expected since the mixed 4-fermion operators are trivial in taste space)

- $\mathcal{U}_{\rm V}$ is present only in mixed theories

  Reason: smaller symmetry group of the mixed theory

$$G_{\rm Mixed} = G_{\rm Sea} \otimes G_{\rm Val} \neq G_{\rm PQQCD}$$

→ Mixed ChPT has more unknown low-energy constants ($C_{\rm Mix}$ at LO)

LO masses

Notation: Sea quark flavors: *S, S'*

Valence quark flavors: *V, V'*

Val-Val:

$$m_{VV'}^2 = B(m_V + m_{V'})$$

- Vanishes for zero quark mass because of exact chiral symmetry

LO masses

Sea-Sea:
(staggered)

$$m^2_{SS',b} = B(m_S + m_{S'}) + a^2\Delta(\xi_b)$$

ξ_b : Taste label

Taste splittings $\Delta(\xi_b)$ are the same as in Staggered ChPT

Sea-Sea: *
(Wilson)

$$m^2_{SS'} = B(m_S + m_{S'}) + 2a^2 c_2$$

c_2 same as in Wilson ChPT

*Quark masses contain shift linear in a ('shifted masses')

LO masses

Sea-Val: $$m_{SV}^2 = B(m_S + m_V) + a^2 \Delta_{\mathrm{Mix}}$$

- Mass does not vanish in the massless limit (no symmetry argument)
- Mixed meson mass depends on the new low-energy constant: $\Delta_{\mathrm{Mix}} \equiv \dfrac{16 C_{\mathrm{Mix}}}{f^2}$
- Mass shift is proportional to a^2 and C_{Mix}

  → direct measure for size of the cut-off artifacts

- Mass can be directly measured from the propagator of a mixed meson
- This mass will enter the 1-loop expression for the decay constant (see later)

Lessons I

- Mixed ChPT has more unknown low-energy constants than 'ordinary' ChPT
- Mixed lattice QCD has more observables (e.g. mixed meson mass)

→ Not necessarily less predictive

- Mixed meson mass is a direct measure for the size of the lattice spacing artifacts

→ Measure it to decide the appropriate power counting

$m \approx a^2\Lambda^3_{\rm QCD}$ means $B(m_S + m_V) \approx a^2\Delta_{\rm Mix}$

Quark mass matching

Quark mass matching: Required to reach unquenched QCD in the continuum limit

$$m_V = m_S \quad \xrightarrow{\text{e.g.}} \quad m^2_{VV'} = m^2_{SS'}$$

- Note: Matching is not unique
  - other choices possible
  - different matching conditions differ by O(a)

Quark mass matching

Suppose we want: $m^2_{VV'} = m^2_{SS'}$ for $\begin{matrix} S \to u \\ S' \to d \end{matrix}$ (degenerate)

- Wilson sea quarks

$$m^2_{SS'} = m^2_{\pi^\pm} = m^2_{\pi^0}$$

- Twisted mass Wilson sea quarks

$$m^2_{SS'} = m^2_{\pi^\pm} \neq m^2_{\pi^0}$$

- Staggered sea quarks

$$m^2_{SS',b} = m^2_{\pi_5^\pm} \neq m^2_{\pi_{\mu 5}^\pm} \neq m^2_{\pi_{\mu\nu}^\pm} \neq m^2_{\pi_\mu^\pm} \neq m^2_{\pi_I^\pm}$$

Question: Is there a preferred way to match ?*
Does mixed ChPT give us a hint?

*Staggered sea quarks: This question is independent of the 4th root trick!

Staggered sea quarks: 1-loop pion mass

OB, Bernard, Rupak, Shoresh

$$\frac{(m_{VV}^{\rm NLO})^2}{2Bm_V} = 1 + \frac{1}{16\pi^2 f^2}\,\frac{2}{3}\left(R_1^{[2,2]}(\{\mathcal{M}_{X,I}^{[2]}\};\{\mu_I^{[2]}\})\,\tilde{l}(m_X^2) + \sum_{j=1}^{2} D_{j,1}^{[2,2]}(\{\mathcal{M}_{X,I}^{[2]}\};\{\mu_I^{[2]}\})\,\ell(m_j^2)\right) + \text{analytic}$$

$l(m^2), \tilde{l}(m^2)$: chiral logs

R, D : residue functions

ratios of products
involving LO masses

Example:

$$R_1^{[2,2]} = \frac{(m_{U_I}^2 - m_{VV}^2)(m_{S_I}^2 - m_{VV}^2)}{m_{VV}^2 - m_{\eta_I}^2}$$

Question

Can we choose the quark masses such that the result resembles the full (unquenched) theory ?

Can we bring the coefficients of the chiral logs to continuum form ?

Answer: Yes !

This is achieved by choosing (at this order)

$$m^2_{VV} = m^2_{\pi^\pm_I}$$

Matching to the taste singlet pion

Explicit result after matching

$$\frac{(m^{\rm NLO}_{\pi^+_{VV}})^2}{2Bm_V} = 1 + \frac{1}{16\pi^2 f^2}\left(l(m^2_{\pi^0_I}) - \frac{1}{3}\, l(m^2_{\eta_I})\right)$$

$$+\frac{16B}{f^2}\,(2L_8 - L_5)\,(2m_V) + \frac{32B}{f^2}\,(2L_6 - L_4)\,(2m_S + m_{S'}) + a^2\mathcal{C}$$

- Simplified residues, but still not unquenched

  The mixed result has always some remnant of partial quenching

  But: correct result in the naive continuum limit

- Masses in the chiral logs are taste singlet masses (sea -sea)

Decay constant

OB, Bernard, Rupak, Shoresh 2005

$$\frac{f^{\mathrm{NLO}}_{\pi^+_{VV}}}{f} = 1 + \frac{1}{16\pi^2 f^2}\Big[-2l\left(m^2_{SV}\right) - l\left(m^2_{S'V}\right)\Big]$$

$$+\frac{8B}{f^2}L_5\left(2m_V\right) + \frac{16B}{f^2}L_4\left(2m_S + m_{S'}\right) + a^2\mathcal{F}$$

- No residue functions (cancellations for degenerate valence quark masses)

  → no obviously preferred way to define a 'full' pion

- Mixed meson masses in the chiral logs

  → log behaviour is different compared to the pion mass result

  (also true for Wilson sea quarks)

Double pole effects with Wilson sea quarks

Golterman, Izubuchi, Shamir 2005

$$\mathcal{L}_\chi[a^2] = -a^2 W_0^2 \Big(W_7' \langle P_S(\Sigma - \Sigma^\dagger)\rangle^2 + \ldots \Big)$$

$W_0,\, W_7'$: low-energy constants

P_S : projector on the sea fields

This term contributes to the flavor neutral two-point function:

$$G_{ij}(p) = \left(\delta_{ij} - \frac{1}{N_S}\right)\frac{1}{p^2 + m_{VV}^2} - \frac{R}{(p^2 + m_{VV}^2)^2}$$

Residue of the double pole:

$$R = \frac{m_{SS}^2 - m_{VV}^2}{N_S} + \frac{32}{f^2} a^2 W_0^2 W_7'$$

Double pole effects with Wilson sea quarks

Golterman, Izubuchi, Shamir 2005

We have two choices for the quark mass matching

- Tune m_V such that $R = 0$ ⟶ $m_{VV}^2 \neq m_{SS}^2$

  Sign of W_7' determines whether $m_{VV} > m_{SS}$ or $m_{VV} < m_{SS}$

- Tune m_V such that $m_{VV}^2 = m_{SS}^2$ ⟶ $R \neq 0$

  ⟶ double pole (= partial quenching) effects in various quantities for example

  - I=0 $\pi\pi$ scattering
    Bernard, Golterman 1996
  - a_0 propagator
    Bardeen et.al. 2002
  - nucleon - nucleon potential
    Beane, Savage 2002

Lessons 2

- Mixed lattice QCD suffers from partial quenching effects
- The size of these effects depends
  - on the quark mass matching
  - on the size of the lattice spacing ($\approx a^2$)
  - on the observable

→ Study the size of these effects in actual simulations !

Example: The scalar propagator

Connected scalar correlator

$$C(t) = \sum_{\vec{x}} \langle 0 | \overline{d}u(\vec{x},t)\, \overline{u}d(\vec{0},0) | 0 \rangle$$
$$= A e^{-m_{a_0} t} + B(t) + \ldots$$

- $B(t)$: contribution from two-pseudoscalar states ($\pi\eta,\ KK, \pi\eta'$)

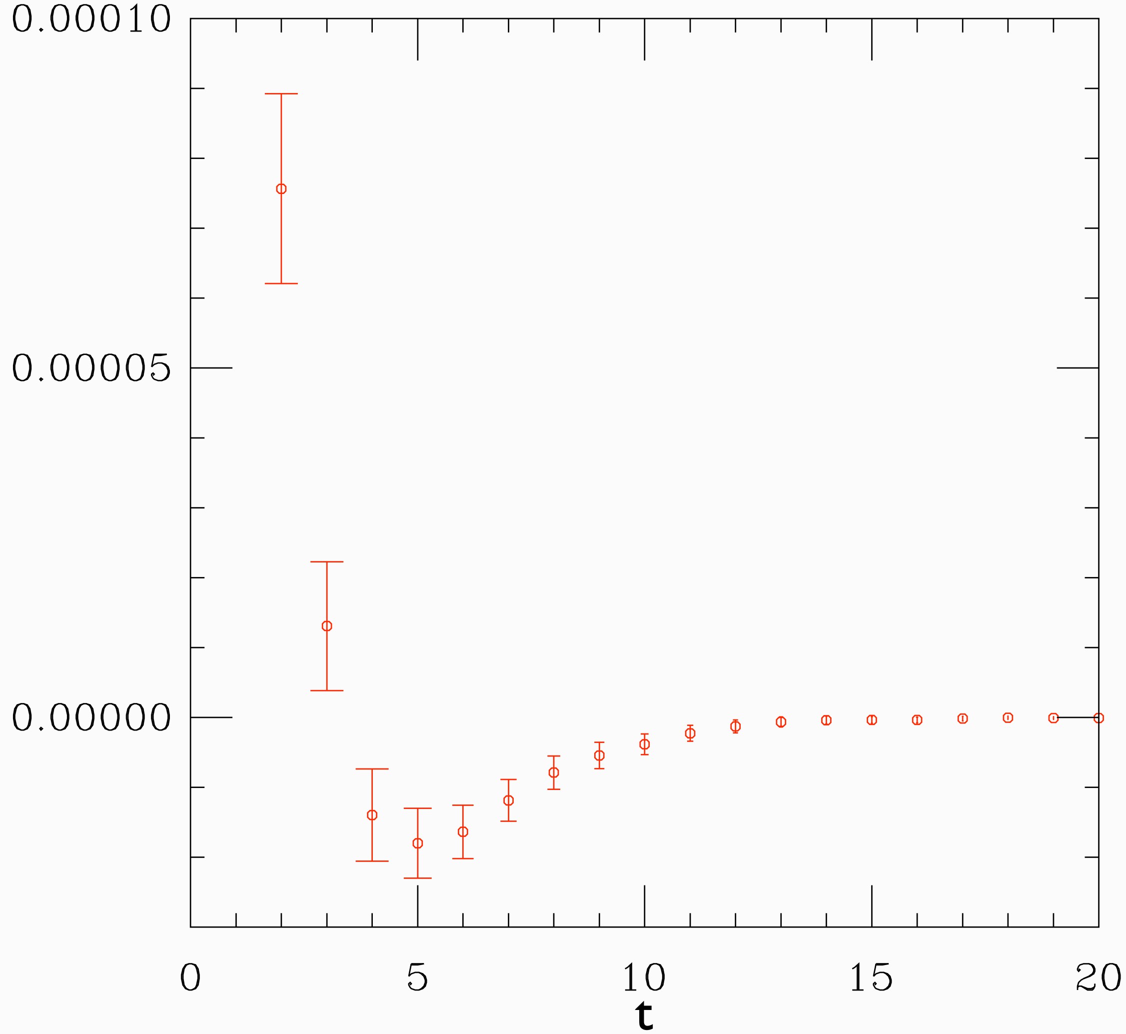
  $\ldots$: excited scalar states, multi-hadron states (less important)

- A LO continuum PQ ChPT analysis shows
  Prelovsek et.al. 2004
  - $B(t)$ = sum of exponentials for $m_V = m_S$
  - $B(t)$ gives a negative contribution for large t if $m_V < m_S$
  - The scalar correlator can become negative for $m_V < m_S$

A negative scalar correlator is a signal for partial quenching !

The scalar propagator with staggered sea quarks

LHP collaboration



plot by K. Orginos

Quark mass matching using the Goldstone pion

$$m_{VV}^2 = m_{\pi_5^\pm}^2$$

→ negative propagator

clear sign of partial quenching !

What happens for

$$m_{VV}^2 = m_{\pi_I^\pm}^2$$

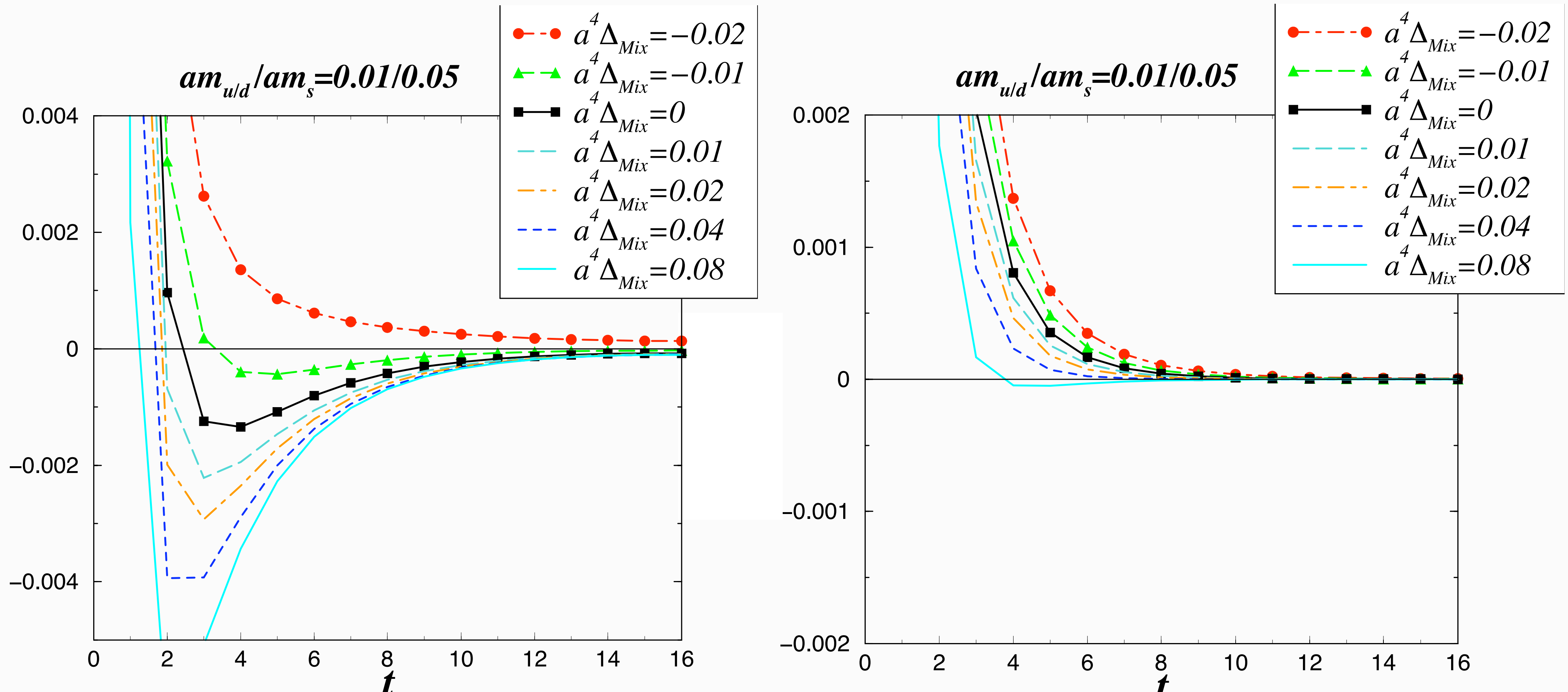
?

$B(t)$ in mixed ChPT

Prelovsek 2005

plots from hep-lat/0510080

$B(t)$ depends on C_{Mix} → here free parameter



Matching: $m_{VV}^2 = m_{\pi_5^\pm}^2$ $\qquad$ $m_{VV}^2 = m_{\pi_I^\pm}^2$

→ Effective theory predicts negative contribution $B(t)$ for the matching with the Goldstone pion

Lessons 3

- Mixed lattice QCD has certain 'diseases' (partial quenching effects)
- BUT: Mixed ChPT - provided it is the correct low-energy theory ! - must be able to reproduce these sicknesses

→ Exploit the characteristic signatures of the diseases to test the validity of the effective theory

Similar idea: check for curvature due to chiral logs

- Provided we have established that the effective description is correct :

→ Use the effective theory to account for the disease and extract the physical result

Example: Scalar correlator and extraction of m_{a_0}
Prelovsek et.al. 2004

What I would like to see next

- Measure the mass of the mixed meson and check the size of the a^2 effects

  Useful measure (?) $$R = \frac{m_{SV}^2}{m_{VV}^2}$$

- Measure the pion decay constant and
  - Compare to mixed ChPT

    Can we see the chiral log with the mixed meson mass ?
  - Take the continuum limit and compare to the unquenched result
- Measure other ‘simple’ quantities (like the scalar correlator)

After these steps I would move on to more complicated observables ...