

$$e^{-S_{eff}} = \det(D_S + m_S) e^{-S_g}$$