

$$S_{\text{sym}} = S_4 + a S_5 + a^2 S_6 + \dots$$