References:

(1) *Topology conserving gauge action and the Overlap-Dirac Operator* By H. Fukaya, S. Hashimoto, T. Hirohashi, K. Ogawa, T. Onogi (hep-lat/0510116) + proceedings: hep-lat/0510095, hep-lat/0509184

(2) *Exploring topology conserving gauge actions for lattice QCD* By W. Bietenholz, K. Jansen, K.-I. Nagai, S. Necco, L. Scorzato, S. Shcheredin (hep-lat/0511016) + proceedings: hep-lat/0509170, hep-lat/0412017, hep-lat/0409073

QUENCHED CASE:

It is possible to construct lattice SU(3) gauge actions which suppress small plaquette value $\longrightarrow$ suppress the topological transitions

-Lattice spacing: $a \simeq 0.07 - 0.15$ fm

-Lattice size: $L \simeq 1.1 - 1.5$ fm

-Lattice artefacts under control

Theoretical bound:

$$S_P(U_P) = 1 - \frac{1}{3}\mathrm{ReTr}\, U_P < \varepsilon = \frac{1}{30} \quad (\frac{1}{20.5})$$

P. Hernández, K. Jansen, M. Lüscher (1999)
H. Neuberger (2000)

Action:

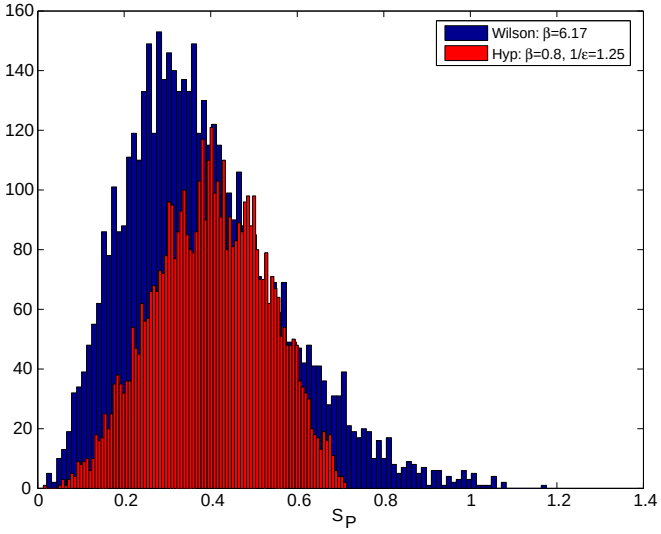
$$S = \beta \sum_P S^{\mathrm{hyp}}_{\varepsilon,n}(U_P)$$

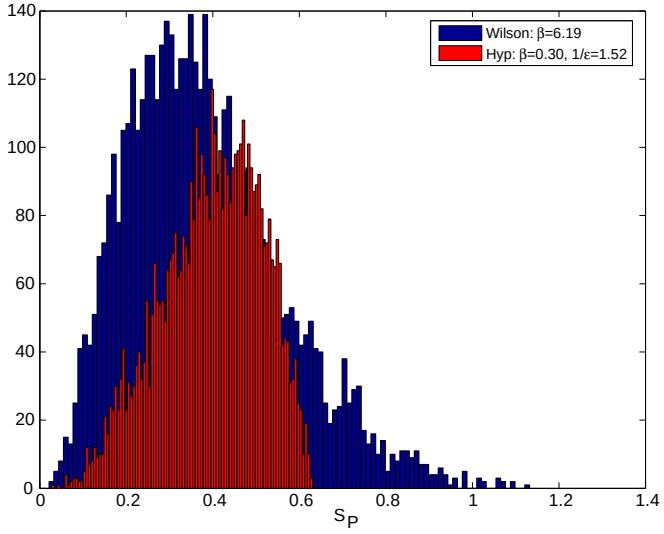
$$S^{\mathrm{hyp}}_{\varepsilon,n}(U_P) = \begin{cases} \frac{S_P(U_P)}{[1-S_P(U_P)/\varepsilon]^n} & \text{for } S_P(U_P) < \varepsilon \\ +\infty & \text{otherwise} \end{cases}$$

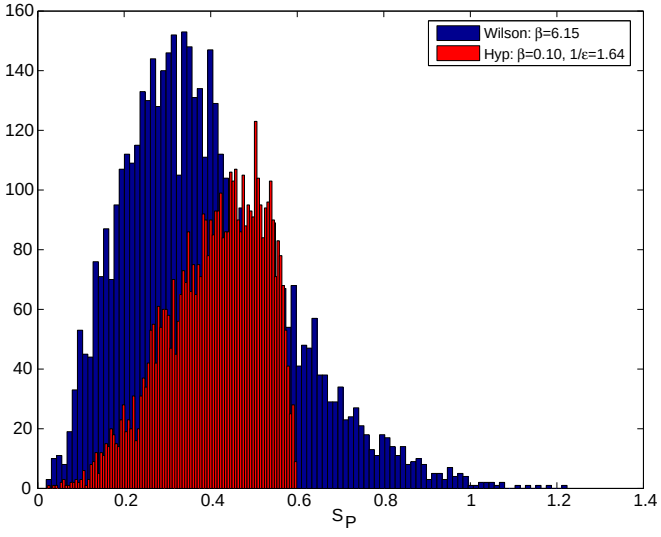
In practice:
(1) $1/\varepsilon = 0,\ 2/3,\ 1$
(2) $1/\varepsilon = 0,\ 1,\ 1.18,\ 1.25,\ 1.52,\ 1.64$;

In (2) other actions are proposed, without a "wall" for $S_P > \varepsilon$

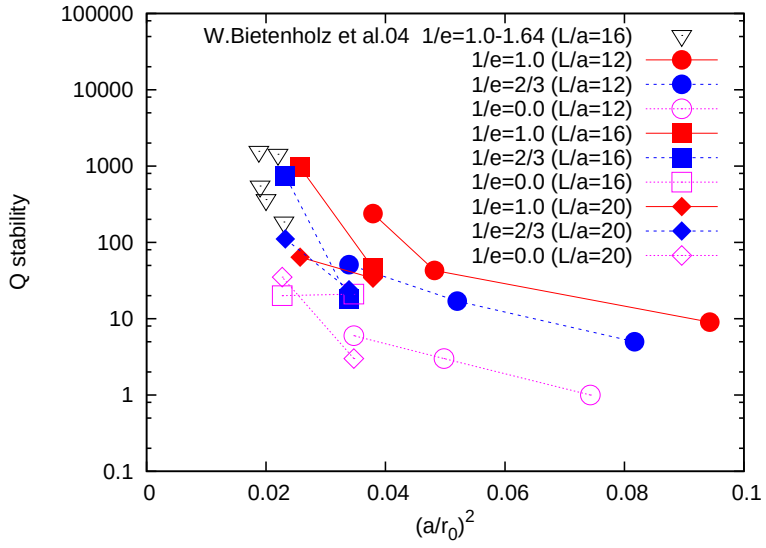
Plaquette histograms (2):







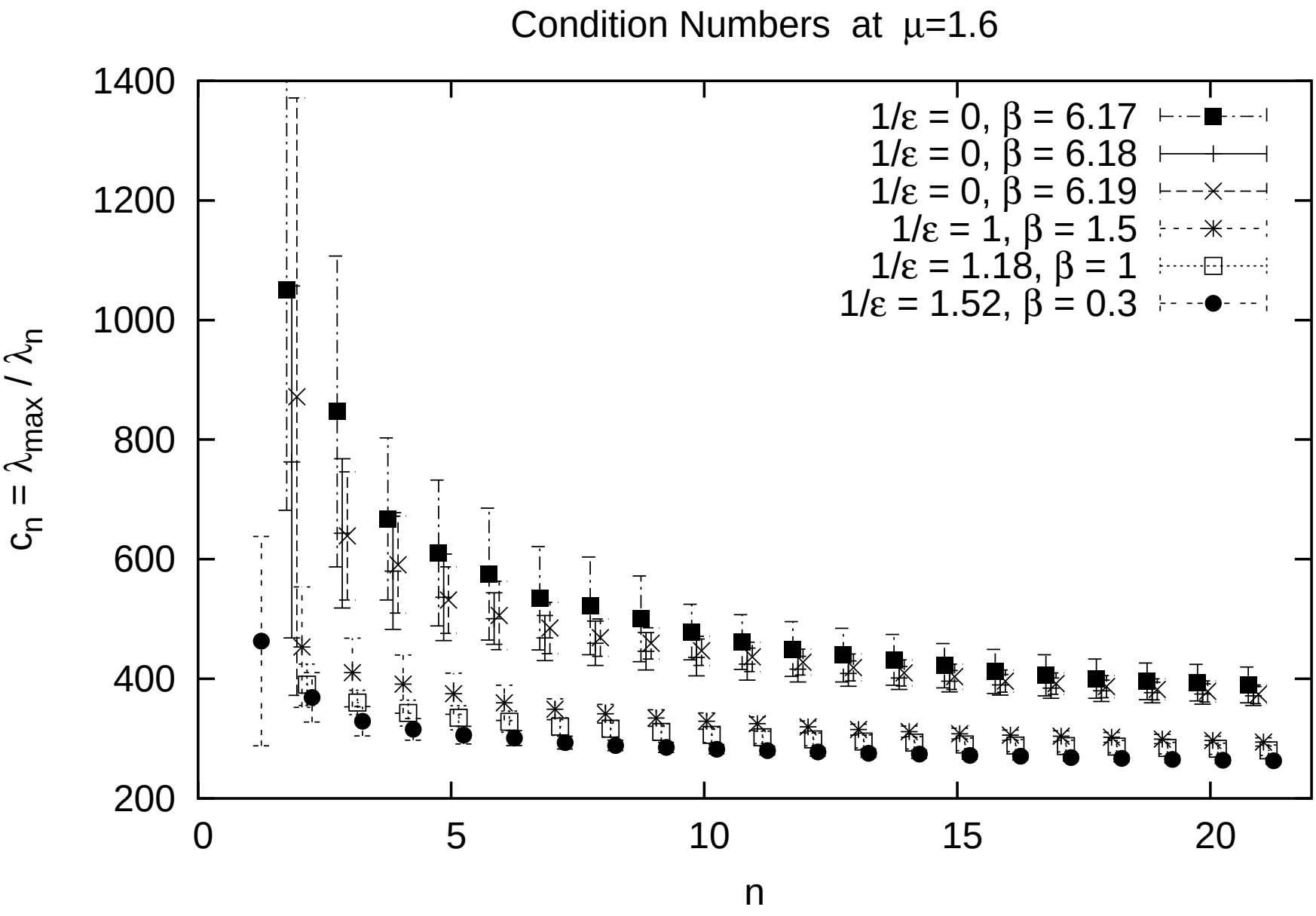
Topological stability (1)



$$Q_{stab} = \frac{N_{trj}}{\tau_{plaq} \times \#Q}$$

-Additional property: reduction of the condition number for the Neuberger Dirac operator:
if the first $n = O(20)$ eigenmodes of Q^2 are subtracted, the gain is only moderate (but n can be reduced considerably by keeping $c_n \sim$ constant)

$\longrightarrow$ suitable for simulations in the ϵ-regime


Condition Numbers at μ=1.6
1400
1200
1000
800
600
400
200
0
5
10
15
20
n
$c_n = \lambda_{max} / \lambda_n$
1/ε = 0, β = 6.17
1/ε = 0, β = 6.18
1/ε = 0, β = 6.19
1/ε = 1, β = 1.5
1/ε = 1.18, β = 1
1/ε = 1.52, β = 0.3

UNQUENCHED CASE:

Appearence of near-zero modes is suppressed $\longrightarrow$ suppression of reflection/refraction

(1) Experiments on 4^4 lattice:

* The distribution of low-lying eigenmodes exhibit smaller density
* But: no clear suppression of reflection/refraction is observed

$\longrightarrow$ need for further investigations