Low-energy constants of the $\Delta S = 1$ weak Hamiltonian from (quenched) lattice QCD

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Outline

- Introduction to K$\rightarrow \pi\pi$ decays
- SU(4) strategy for K$\rightarrow \pi\pi$ decays
- Renormalization of $H_{\rm eff}^{\Delta S=1}$ with GW fermions
- Numerical results for the SU(4) symmetric case
- Conclusions and outlook

K^0–$\bar{K}^0$ mixing

- In the basis $(K^0, \bar{K}^0)$, Hermiticity + CPT lead to

$$H = \left[M - \frac{i}{2}\Gamma\right] = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix}$$

where no conserved quantum numbers prevent $p, q \neq 0$

- If CP preserved by weak interactions ($p = q$)

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K}^0\rangle\right] \qquad \mathrm{CP}|K_1\rangle = |K_1\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K}^0\rangle\right] \qquad \mathrm{CP}|K_2\rangle = -|K_2\rangle$$

- CP violation removes the mass degeneracy

$$|K_\mathrm{S}\rangle = \frac{|K_1\rangle + \bar{\varepsilon}|K_2\rangle}{\sqrt{1+|\bar{\varepsilon}|^2}} \qquad |K_\mathrm{L}\rangle = \frac{|K_2\rangle + \bar{\varepsilon}|K_1\rangle}{\sqrt{1+|\bar{\varepsilon}|^2}} \qquad \bar{\varepsilon} = \frac{p-q}{p+q}$$

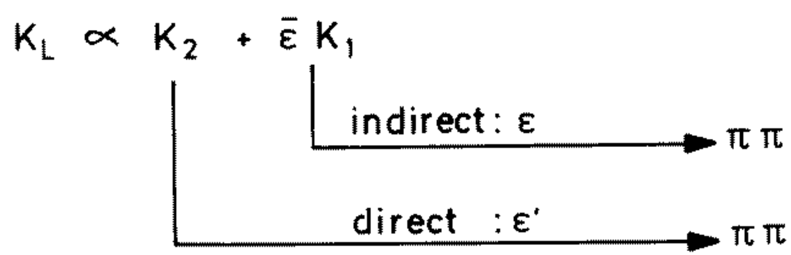
- $K \to \pi\pi$ amplitudes can be parameterized [CP violation implies $A_I \neq A_I^*$]

$$
\begin{aligned}
-i\,T[K^+ \to \pi^+\pi^0] &= \frac{1}{\sqrt{2}} A_2 e^{i\delta_2} \\
-i\,T[K^0 \to \pi^+\pi^-] &= \sqrt{\tfrac{1}{3}} A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{6}} A_2 e^{i\delta_2} \\
-i\,T[K^0 \to \pi^0\pi^0] &= -\sqrt{\tfrac{1}{3}} A_0 e^{i\delta_0} + \sqrt{\tfrac{2}{3}} A_2 e^{i\delta_2}
\end{aligned}
$$

$$
-iT[K^0 \to (\pi\pi)_I] = A_I e^{i\delta_I} \qquad T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2\, e^{i\delta_I} \sin\delta_I
$$

- CP violation can be parameterized as

$$
\varepsilon = \frac{T[K_L \to (\pi\pi)_0]}{T[K_S \to (\pi\pi)_0]} \simeq \bar{\varepsilon} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}
$$



$$
\begin{aligned}
\varepsilon' &= \frac{\varepsilon}{\sqrt{2}} \left(\frac{T[K_L \to (\pi\pi)_2]}{T[K_L \to (\pi\pi)_0]} - \frac{T[K_S \to (\pi\pi)_2]}{T[K_S \to (\pi\pi)_0]} \right) \\
&\simeq \frac{1}{\sqrt{2}} e^{i(\delta_2 - \delta_0 + \frac{\pi}{2})} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)
\end{aligned}
$$

- $\Delta \mathrm{I} = 1/2$ rule

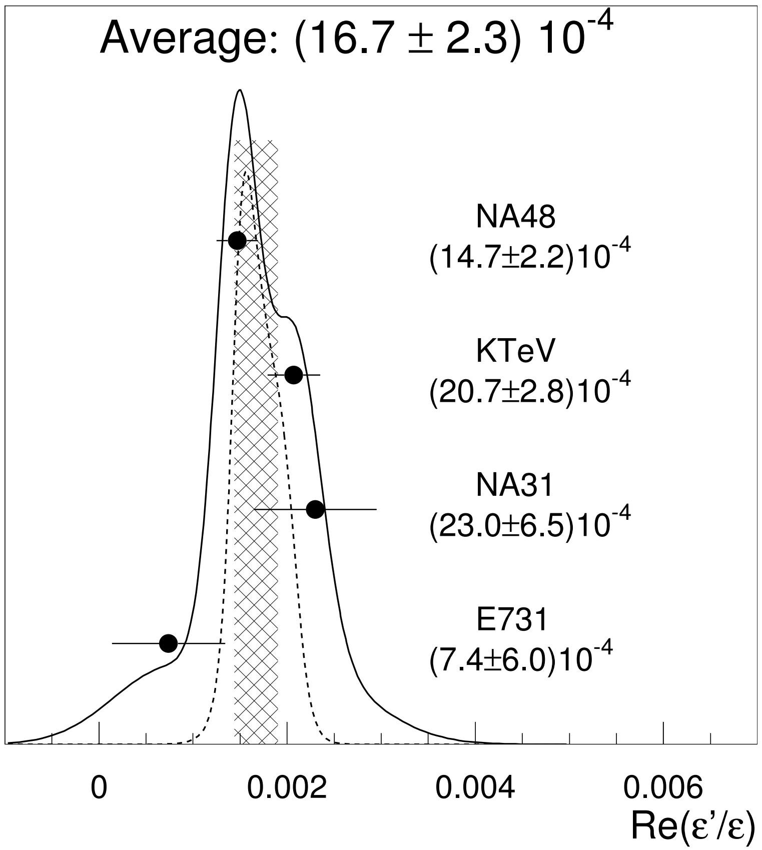
$$\left|\frac{A_0}{A_2}\right| \simeq 22.1$$

- Indirect CP violation

$$|\varepsilon| \quad = (2.282 \pm 0.017) \times 10^{-3}$$

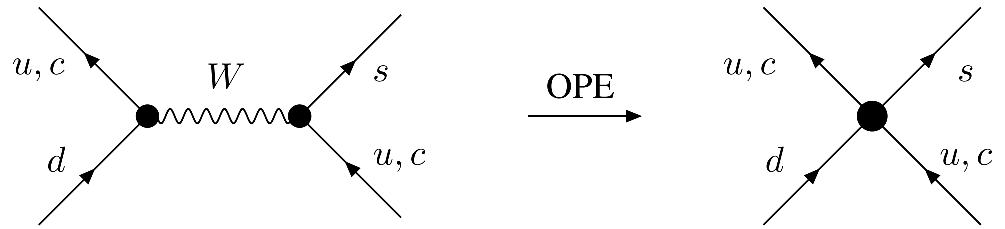
- Direct CP violation

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (16.7 \pm 2.3) \cdot 10^{-4}$$



The $H_{\rm eff}^{\Delta S=1}$ with an active charm

- By using the Operator Product Expansion



$$iA_I e^{i\delta_I} = \langle (\pi\pi)_I \mid H_{\rm eff}^{\Delta S=1} \mid K^0 \rangle$$

- The CP-conserving $\Delta S = 1$ eff. Hamiltonian is [Gaillard, Lee 74; Altarelli, Maiani 74]

$$H_{\rm eff}^{\Delta S=1} = \sqrt{2} G_F V_{ud} V_{us}^* \Big\{ \sum_{\sigma=\pm} k_1^\sigma Q_1^\sigma + k_2^\sigma Q_2^\sigma \Big\}$$

$$Q_1^\pm = \Big[(\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u) \Big] - \Big[u \to c \Big]$$

$$Q_2^\pm = (m_u^2 - m_c^2) \Big[m_d (\bar{s} P_+ d) + m_s (\bar{s} P_- d) \Big]$$

- For $m_s \pm m_d \neq 0$

$$\bar{s}P_{\pm}d = \partial_\mu \left[\frac{1}{m_s - m_d} \bar{s}\gamma_\mu d \pm \frac{1}{m_s + m_d} \bar{s}\gamma_\mu\gamma_5 d \right]$$

  and it does not contribute in MEs which preserve four-momentum

- In physical matrix elements

$$H_{\rm eff}^{\Delta S=1} = \sqrt{2} G_F V_{ud} V_{us}^* \sum_{\sigma=\pm} k_1^\sigma Q_1^\sigma$$

- The Wilson coefficients are known at NLO in α_s [Buras et al. 92; Ciuchini et al. 94]

- A non-perturbative determination of the matrix elements $\langle (\pi\pi)_I | \widehat{Q}_1^{\pm} | K^0 \rangle$ of the properly renormalized operators is needed

A lot of activity in the past to compute A_0 and A_2

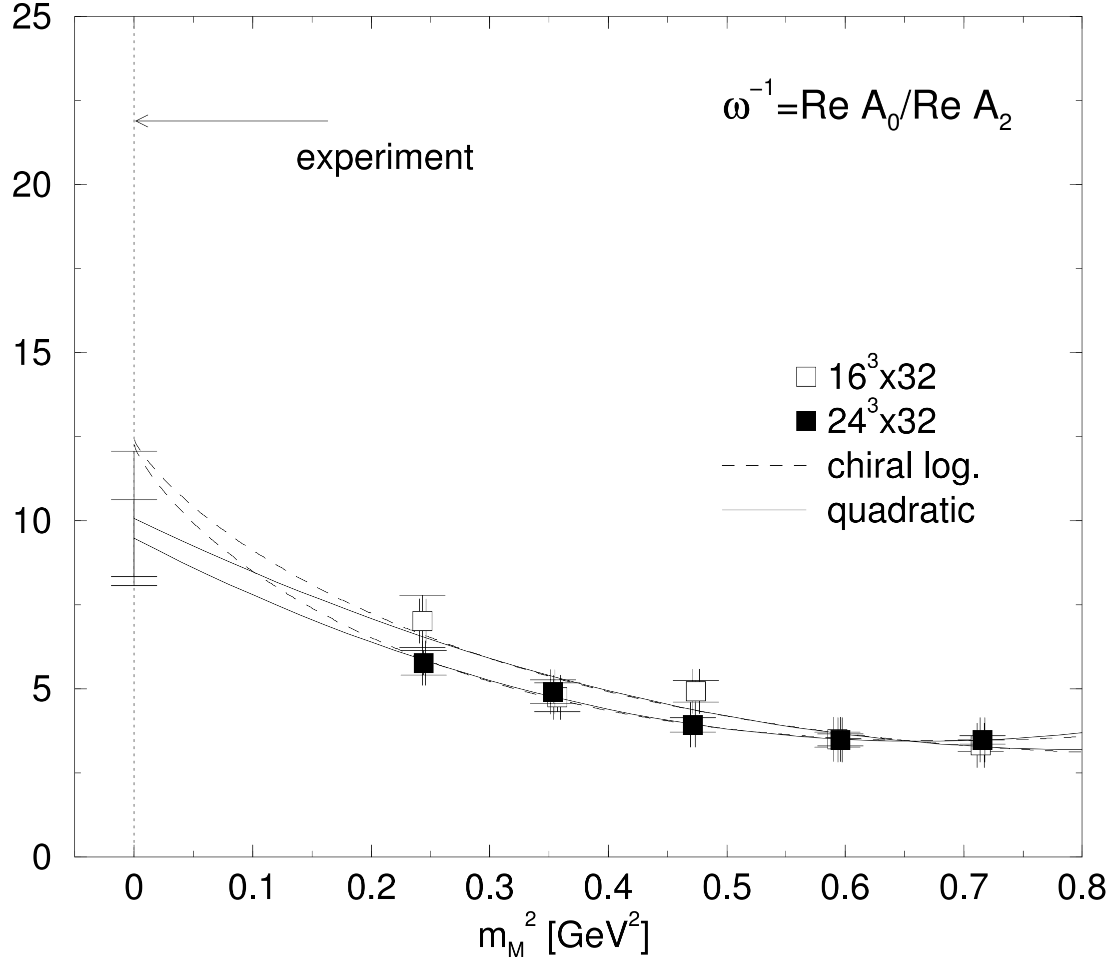
- Use of ChPT for weak decays already developed in the '80s
  [Georgi 84; Bernard et al. 85; Kambor et al. 91]

- Exploratory computations on the lattice: a statistical signal obtained for $K \to \pi$ matrix elements in the quenched approximation, systematics not under control
  [Kilcup, Pekurovsky 98; Blum et al. 01; Ali Khan et al. 01]

  - Approximate chiral symmetry
  - Charm integrated out: UV problem
  - Large quark masses

- Large amount of work done with models
  [see for recent review Bertolini et al. 00; Pallante et al. 01]

$|A_0/A_2|$ from $K \to \pi$ at LO in ChPT [CP-PACS collaboration (Ali Khan et al.) 01]



- Lightest pion mass $m_M \simeq$ 495 MeV

SU(4) strategy [L.G., Hernández, Laine, Lüscher, Weisz, Wittig 04]

- It is interesting to understand the origin of the $\Delta I = 1/2$ enhancement: is there a single mechanism or is an accumulation of several small factors?

- Important to disentangle possible sources of enhancement

  - Leading order contributions in ChPT
  - Charm mass dependence
  - Higher orders in ChPT (FSI, $m_s, \ldots$)

- Computing each of these contributions requires simulations with different numerical difficulties

- First goal: computation of LO LECs of $H_{\rm eff}^{\Delta S=1}$ as a function of m_c

- First step toward this goal: computation of LECs in the SU(4) symmetric limit

- The effective chiral Lagrangian of QCD at LO is

$$\mathcal{L} = \frac{\bar{F}^2}{4}\,\mathrm{Tr}\,\left\{\partial_\mu U^\dagger \partial_\mu U\right\} - \frac{\bar{\Sigma}}{2}\,\mathrm{Tr}\,\left\{U M^\dagger + M U^\dagger\right\}$$

- The CP-conserving $\Delta S = 1$ Hamiltonian reads

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \sqrt{2} G_F V_{ud} V_{us}^* \Big\{ \sum_{\sigma=\pm} \bar{g}_1^\sigma\, \mathcal{Q}_1^\sigma + \bar{g}_2^\sigma\, \mathcal{Q}_2^\sigma \Big\}$$

- At leading order in momentum expansion

$$\mathcal{Q}_1^\pm = \frac{\bar{F}^2}{4}\Big[(U\partial_\mu U^\dagger)_{us}(U\partial_\mu U^\dagger)_{du} \pm (U\partial_\mu U^\dagger)_{ds}(U\partial_\mu U^\dagger)_{uu}\Big] - [u \to c]$$

$$\mathcal{Q}_2^\pm = -\frac{\bar{\Sigma}}{2}(m_u^2 - m_c^2)\Big[m_s U_{ds} + m_d U_{ds}^\dagger\Big]$$

Matching to SU(3) and $K \to \pi\pi$ at LO in ChPT

- Matching SU(4) $\longrightarrow$ SU(3) in ChPT for values of m_c in the chiral regime
  [L.G. et al. 04; Hernández, Laine 04]

- SU(3) ChPT when m_c is heavy enough to decouple

$$\bar{g}_1^{\pm} \to g_1^{\pm}(m_c) \qquad\qquad \bar{g}_2^{\pm} \to g_2^{\pm}(m_c)$$

- At leading order in ChPT

$$\left|\frac{A_0}{A_2}\right| = \frac{1}{2\sqrt{2}}\left\{1 + 3\frac{g_1^-}{g_1^+}\right\}$$

and if compared with experimental results $g_1^-/g_1^+ \approx 20.5$

- Lecs determined by matching suitable quantities in massless QCD with ChPT
- A possible choice is

$$\langle \pi^+ | k_1^\pm \widehat{Q}_1^\pm + k_2^\pm \widehat{Q}_2^\pm | K^+ \rangle = \bar{g}_1^\pm \frac{\bar{F}^2 \bar{M}_K \bar{M}_\pi}{2} - \bar{g}_2^\pm (m_u^2 - m_c^2) \bar{M}_K^2$$

$$\langle 0 | k_1^\pm \widehat{Q}_1^\pm + k_2^\pm \widehat{Q}_2^\pm | K^0 \rangle = \frac{i}{\sqrt{2}} \bar{g}_2^\pm (m_u^2 - m_c^2) \bar{F} (\bar{M}_K^2 - \bar{M}_\pi^2)$$

- Computing $\bar{g}_1^\pm$ requires eight-diagrams only
- Penguin diagrams needed for $g_1^\pm(m_c)$

Ginsparg–Wilson fermions

- The "mildest way" of breaking standard chiral symmetry [Ginsparg Wilson 82]

$$\gamma_5 D + D\gamma_5 = \bar{a} D \gamma_5 D$$

- An exact symmetry at finite cut-off implied [Lüscher 98]

$$\delta q = \epsilon \hat{\gamma}_5 q \quad \delta \bar{q} = \epsilon \bar{q} \gamma_5 \qquad \hat{\gamma}_5 = \gamma_5 (1 - \bar{a} D)$$

- $U(1)_{\rm A}$ anomaly from the Jacobian à la Fujikawa [Lüscher 98]

$$J = \exp\{\epsilon \bar{a} \sum_x \mathrm{Tr}\, [\gamma_5 D(x,x)]\}$$

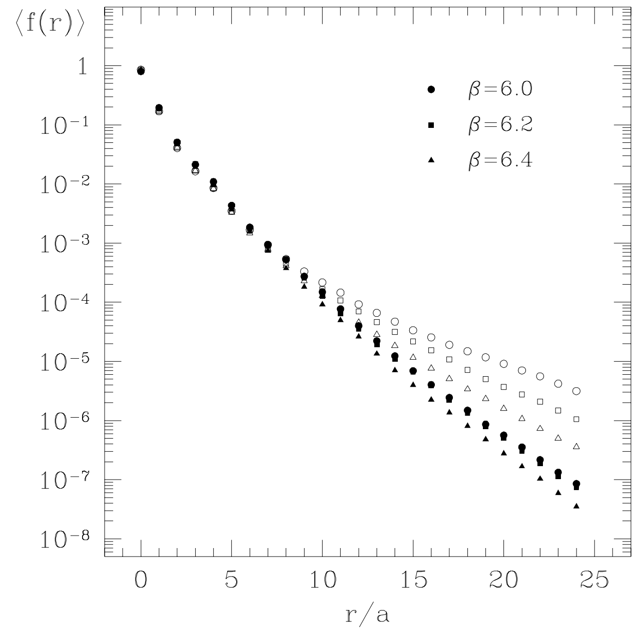
- The topological charge density defined as [Neuberger 97, Hasenfratz et al. 98, Lüscher 98]

$$a^4 Q(x) = -\frac{\bar{a}}{2} \mathrm{Tr}[\gamma_5 D(x,x)] \qquad n_+ - n_- = \mathrm{index}(D) = \sum_x Q(x)$$

and for smooth gauge configurations $Q(x) \to -\frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\, [F_{\mu\nu}(x) F_{\rho\sigma}(x)]$

- After 15 years from the GW relation, a Dirac operator that satisfies the GW relation, is local and leads to the correct continuum limit was found [Neuberger 97]

Hernández, Jansen, Lüscher 99



$$D = \frac{1}{\bar{a}}\left(1 + \frac{X}{\sqrt{X^\dagger X}}\right)$$

with

$$X = D_W - 1/\bar{a} \qquad \bar{a} = a/(1+s)$$

$$f(r) = \max\left\{ \left|\left| \frac{X}{\sqrt{X^\dagger X}}(x,y) \right|\right| \;\middle|\; ||x-y|| = r \right\}$$

- A family of GW regularizations for $0 < s < 2$

$$\langle f(r) \rangle \propto e^{-\mu r/a} \qquad r/a \gg 1$$

- Numerical treatment challenging and expensive

Exact chiral symmetry: some advantages and remarks

- No mixing among operators of different chirality:
  - No additive quark mass renormalization
  - Simplified mixing for composite operators
  - $O(a^2)$ discretization effects
- Very light quark masses can be reached
- Bilinears with correct chiral properties

$$O^{\Gamma}_{\alpha\beta}(x) = \bar{\psi}_\alpha(x)\Gamma\tilde{\psi}_\beta(x) \qquad \tilde{\psi}_\beta(x) = \Big[(1 - \tfrac{\bar{a}}{2} D)\psi_\beta\Big](x)$$

- Apparently no simple transformation under CP. But at non-zero physical distance

$$O^{\Gamma}_{\alpha\beta}(x) = \frac{1}{(1 - \frac{\bar{a}}{2} m_\beta)} \bar{\psi}_\alpha(x)\Gamma\psi_\beta(x) \qquad \Longrightarrow \qquad O^{\Gamma}_{\alpha\beta}(x) \xrightarrow{\text{CP}} \frac{1 - \frac{\bar{a}}{2} m_\alpha}{1 - \frac{\bar{a}}{2} m_\beta} O^{\Gamma}_{\beta\alpha}(\tilde{x})$$

Renormalization pattern for $Q_1^\pm$ [Capitani, L. G. 00]

- To select operators with $d \leq 6$:

  - Flavour symmetry
  - P, C symmetries
  - Chiral symmetry

- At a non-zero physical distance (on-shell) one operator is left

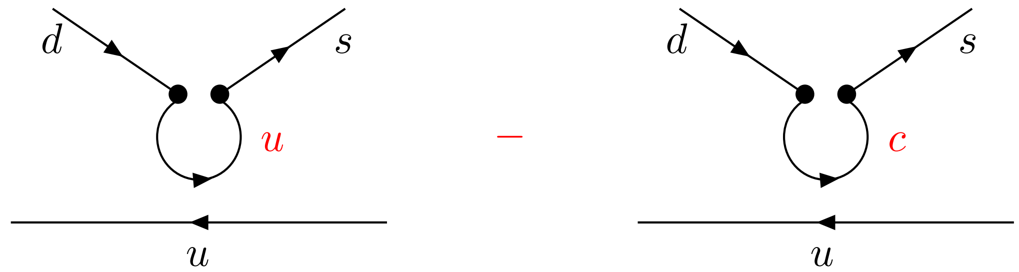
$$Q_2 = (m_u^2 - m_c^2)\Big[m_d(\bar{s}P_+\tilde{d}) + m_s(\bar{s}P_-\tilde{d})\Big]$$

- No power divergent subtractions are needed with GW fermions: UV ren. solved

$$\widehat{Q}_1^\pm = Z_1^\pm\Big\{Q_1^\pm + z^\pm Q_2\Big\}$$

- Note the quadratic GIM mechanism

Active charm with Wilson fermions [Maiani et al. 87; Dawson et al. 97]



- Parity-odd and parity-even components renormalize differently
- For parity conserving sector, using flavour and CPS

$$
\begin{aligned}
[\widehat{Q}_1^{\pm}]^{\mathrm{PC}} &= \mathcal{Z}_1^{\pm}[\widetilde{\mathcal{Q}}_1^{\pm}]^{\mathrm{PC}} \\
[\widetilde{Q}_1^{\pm}]^{\mathrm{PC}} &= [Q_1^{\pm}]^{\mathrm{PC}} + \sum_j b_j^{\pm} O_j^{\pm} + z_\tau^{\pm} Q_\tau + \frac{z_s^{\pm}}{a^2} Q_s
\end{aligned}
$$

where

$$Q_\tau = (m_u - m_c)\bar{s}\sigma_{\mu\nu}F_{\mu\nu}d \qquad\qquad Q_s = (m_u - m_c)\bar{s}d$$

and $O_j^{\pm}$ are 4-fermion operators with wrong chirality

- With a broken chirality the GIM mechanism is only linear

The $H_{\rm eff}^{\Delta S=1}$ with charm integrated out

- The effective Hamiltonian reads

$$H_{\rm eff}^{\Delta S=1} = \sqrt{2} G_F V_{ud} V_{us}^* \sum_{i=1}^{10} h_i \widehat{Q}_i$$

where a basis for QCD-penguin operators is

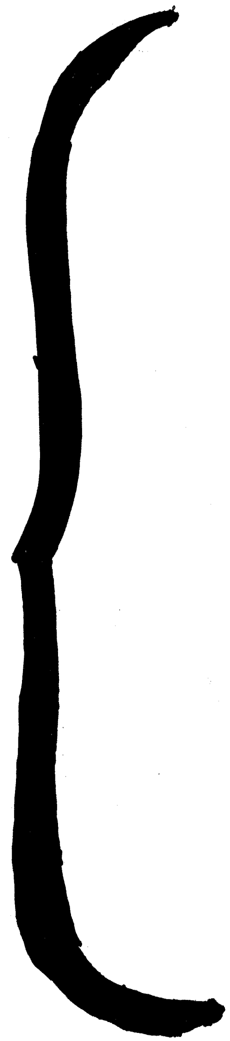
$$Q_{3,5} = (\bar{s}\gamma_\mu P_- \tilde{d}) \textstyle\sum_q (\bar{q}\gamma_\mu P_\mp \tilde{q}) \qquad\qquad Q_{4,6} = (\bar{s}^\alpha \gamma_\mu P_- \tilde{d}^\beta) \textstyle\sum_q (\bar{q}^\beta \gamma_\mu P_\mp \tilde{q}^\alpha)$$

- At a non-zero physical distance two more operators can mix

$$\begin{aligned} Q_\sigma &= m_d (\bar{s} F_{\mu\nu} \sigma_{\mu\nu} P_+ \tilde{d}) + m_s (\bar{s} F_{\mu\nu} \sigma_{\mu\nu} P_- \tilde{d}) \\ Q_m &= m_d (\bar{s} P_+ \tilde{d}) + m_s (\bar{s} P_- \tilde{d}) \end{aligned}$$

- Power-divergences even with chiral symmetry

$$\widehat{Q}_i = Z_{ij} \Big[Q_j + z_j^\sigma Q_\sigma + \frac{z_j^m}{a^2} Q_m \Big]$$



ChPT and quenched approximation

- No asymptotic expansion at fixed N_c in quenched QCD

- An extra $1/N_c$ expansion needed to define quenched ChPT
  [Bernard, Golterman 92; Sharpe 92]

$$\begin{aligned}\mathcal{L}^{\mathrm{Quen}} &= \frac{F^2}{4}\mathrm{Str}\left\{\partial_\mu U^\dagger \partial_\mu U\right\} - \frac{m\Sigma}{2}\mathrm{Str}\left\{U + U^\dagger\right\} \\ &+ \frac{m_0^2}{2N_c}\Phi_0^2 + \frac{\alpha}{2N_c}(\partial_\mu \Phi_0)^2\end{aligned}$$

- Infrared divergences appear associated with Φ_0

- When QChPT is applied, it is assumed that the form of the correlators in the specified kinematical region is close to the one in QCD and that it can be parameterized as predicted by the quenched effective theory

Light quarks in a box: ε regime of QCD [Gasser, Leutwyler 87]

- When $F^2 M^2 L^4 \simeq 1$, $L \gg 1/(4\pi F)$ and for $p^2 \simeq 1/L^2$

$$\frac{M}{\Lambda_\chi} \sim \frac{p^2}{\Lambda_\chi^2} \sim \frac{1}{(4\pi L F)^2} = \epsilon^2$$

and QCD Green's functions can be expanded in powers of ϵ

- At LO the reordered chiral expansion gives

$$\mathcal{S} = \frac{F^2}{4} \int d^4 x \text{Tr} \left\{ \partial_\mu U^\dagger \partial_\mu U \right\} - \frac{m \Sigma V}{2} \text{Tr} \left\{ U_0 + U_0^\dagger \right\}$$

where the the $O(1)$ zero-mode fluctuations have to be treated exactly

$$\int dU_0 \exp\left[\frac{m\Sigma V}{2} \text{Tr} \left\{U_0 + U_0^\dagger\right\}\right] \qquad U = U_0 \exp\left(i\sqrt{2}\xi(x)/F\right) \qquad \int \xi(x) = 0$$

- The expansion can also be performed in fixed-topology sectors

- $L_4 \rightarrow L_8$ do not enter at NLO!

Numerical challenges to simulate the ε-regime

- Large condition number of the Dirac operator ($\propto 1/V$)

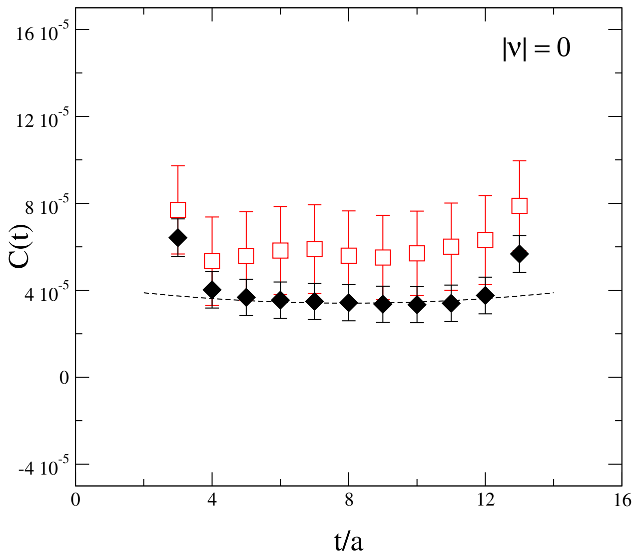
$$m \sim |\lambda_1| \sim \frac{1}{V}$$

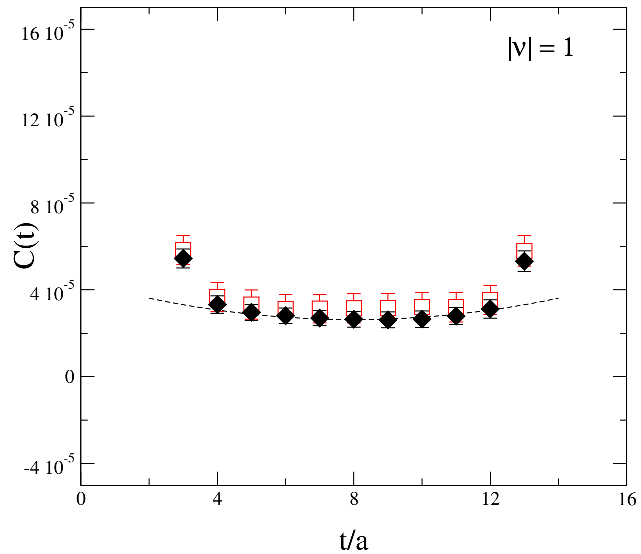
  requires low-mode preconditioning in the conjugate gradient solver
  [L. G., Hoelbling, Lüscher and Wittig 02]

- Large fluctuations in the observables due to large contributions from only few modes require low-mode averaging [L.G., Hernández, Laine, Weisz, Wittig 04]

- Low-mode averaging reduces the variance by averaging the low-mode contributions from any point to any point and computing the rest locally

- Already several variants and applications: [DeGrand, Schaefer 04; Fukaya et al. 05; Ogawa, Hashimoto 05]





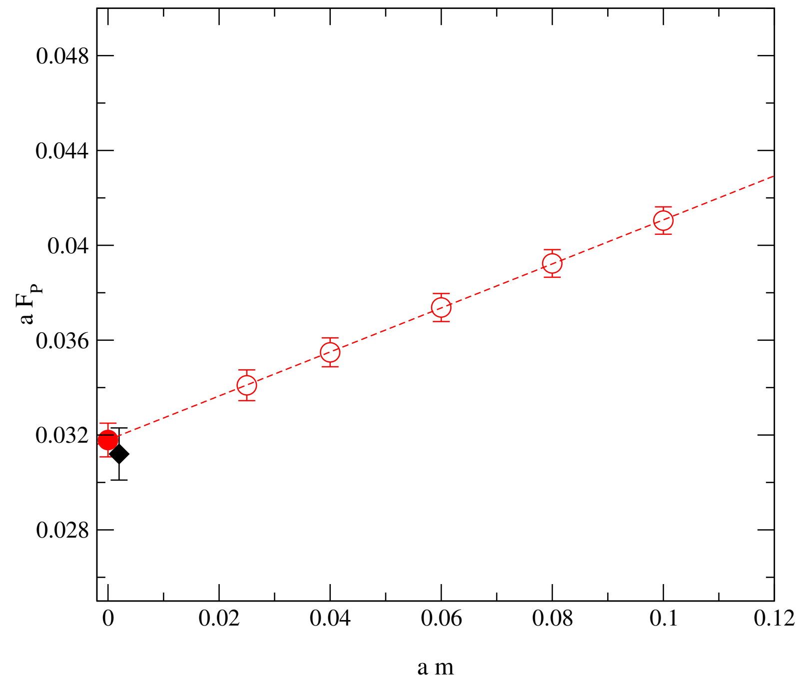
- The correlator of two left-currents (flavor index omitted)

$$C(t) = \sum_{\vec{x}} \langle J_0(x) J_0(0) \rangle \qquad J_0(x) = \bar{\psi}(x)\gamma_0 P_- \tilde{\psi}(x)$$

fits the QChPT prediction with $F = 102(4)$ $(\mu = m\Sigma V)$

$$\mathcal{C}(t) = \frac{F^2}{2T}\left\{1 + \frac{T}{F^2 L^3} b(\mu) h_1(\frac{t}{T})\right\} \qquad h_1\left(\frac{t}{T}\right) = \frac{1}{2}\left[\left(\left|\frac{t}{T}\right| - \frac{1}{2}\right)^2 - \frac{1}{12}\right]$$

Comparing results from p and ε regime: L_5 [L.G., Hernández, Laine, Weisz, Wittig 04]

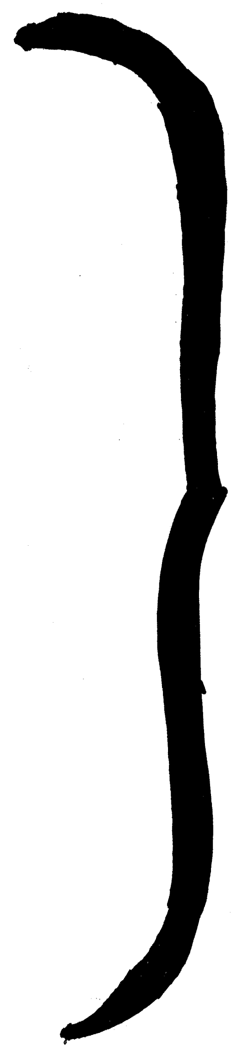


- In the p-regime

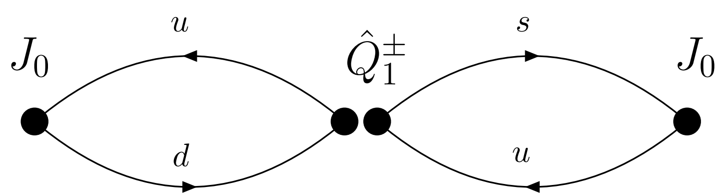
$$F_P = F\left\{1 + \frac{M_\pi^2}{2(4\pi F)^2}\alpha_5\right\} \qquad \alpha_5 = 8(4\pi)^2 L_5$$

and by fitting linearly $F = 104(2)$ MeV and $\alpha_5 = 1.66(8)$

- Excellent agreement between the two values of F!



Numerical computation of $\bar{g}_1^{\pm}$ in quenched QCD [L.G., P. Hernández, M. Laine, C. Pena, J. Wennekers, H. Wittig in prepar



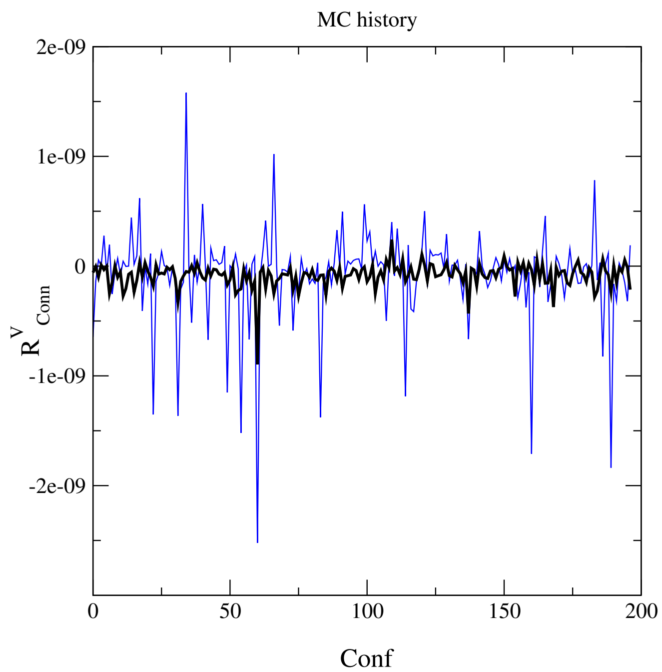
- For $y_0 \ll 0$ and $x_0 \gg 0$

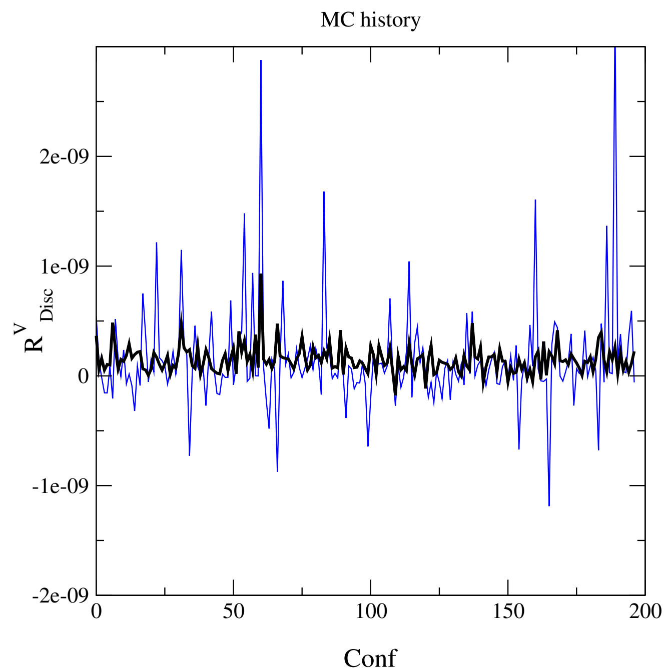
$$R_{\pm}(x_0, y_0) = \frac{\sum_{\vec{x},\vec{y}} \langle J_0(x) \hat{Q}_1^{\pm}(0) J_0(y) \rangle}{\sum_{\vec{x}} \langle J_0(x) J_0(0) \rangle \cdot \sum_{\vec{y}} \langle J_0(0) J_0(y) \rangle}$$

- In the ε-regime the NLO QChPT prediction is

$$\mathcal{R}_{\pm}(x_0, y_0) = \frac{\bar{g}_1^{\pm}}{2} \left\{ 1 \pm \frac{2}{F^2 L^2} \frac{T}{L} \tilde{\beta} \right\}$$

- In the p-regime corresponding QChPT formula in finite volume



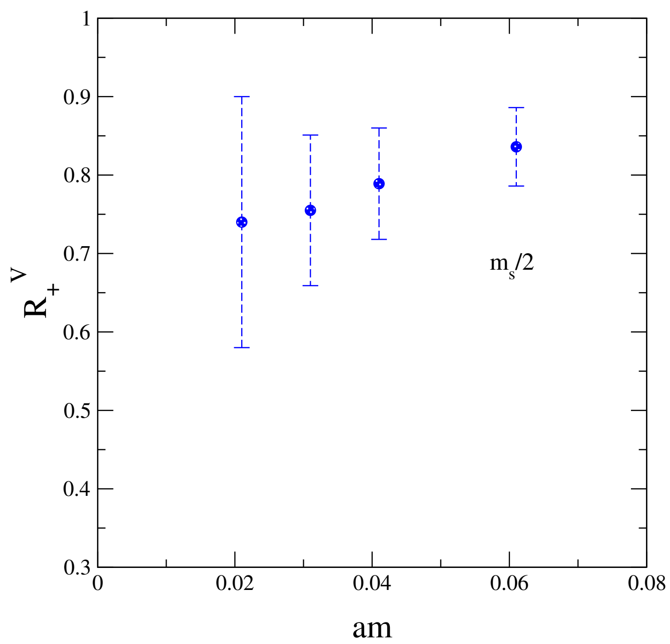


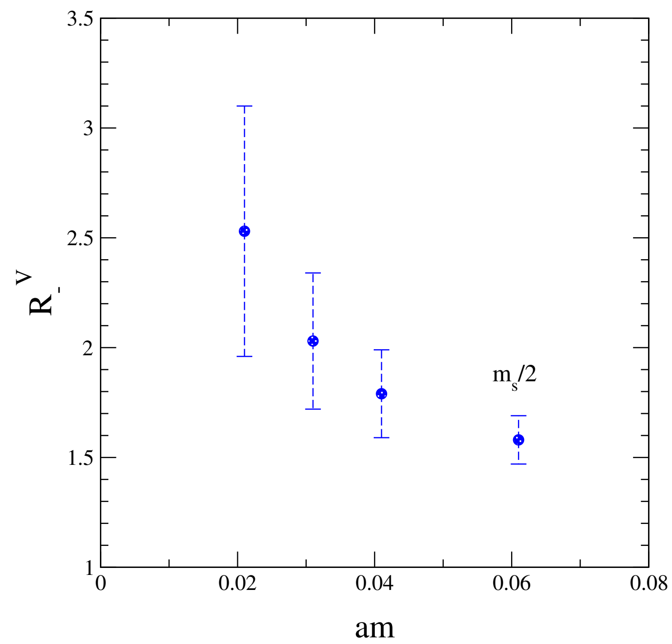
- Simulation parameters

$$\beta = 5.8485 \qquad \frac{V}{a^4} = 16^3 \cdot 32 \qquad a \approx 0.125 \text{ fm} \qquad V \approx 2^3 \cdot 4 \text{ fm}^4$$

- Quark masses $[m \sim m_s/6 - m_s/2]$

$$am = 0.020, 0.030, 0.040, 0.060 \qquad \mathrm{N_{confs}} = 197$$





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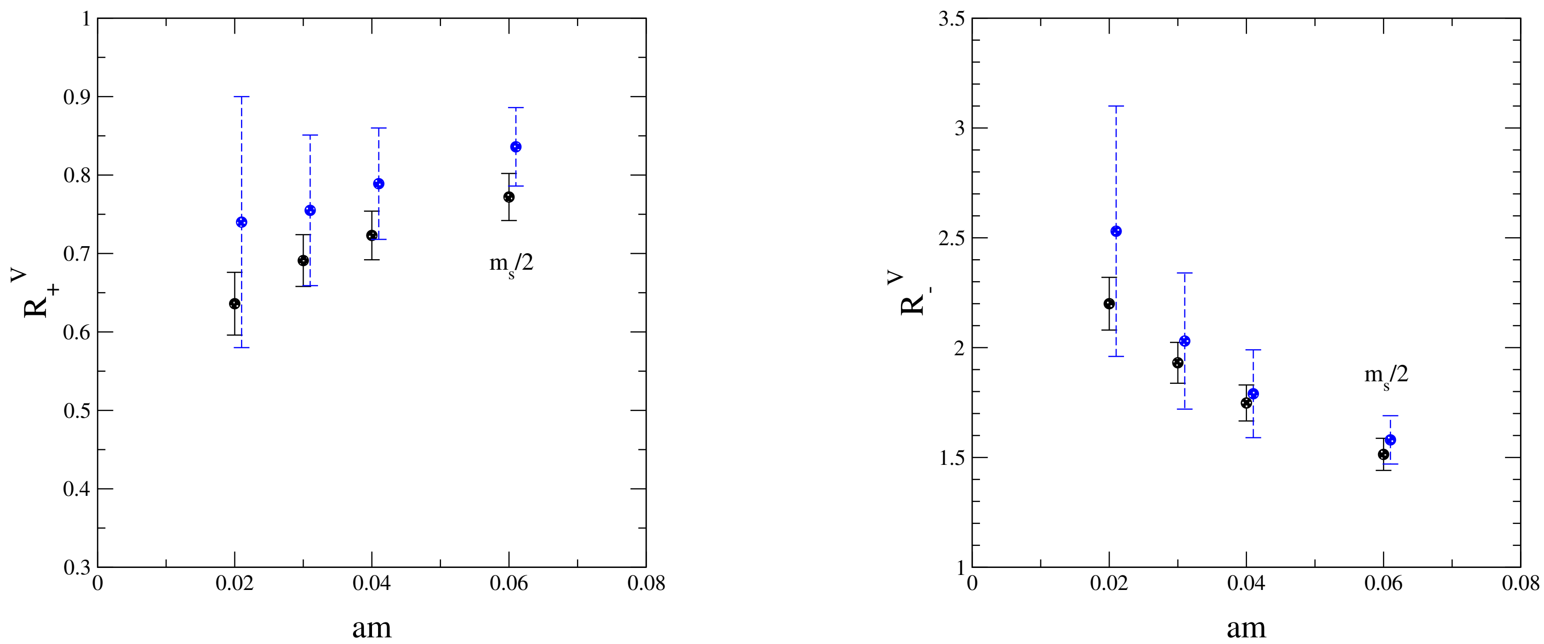

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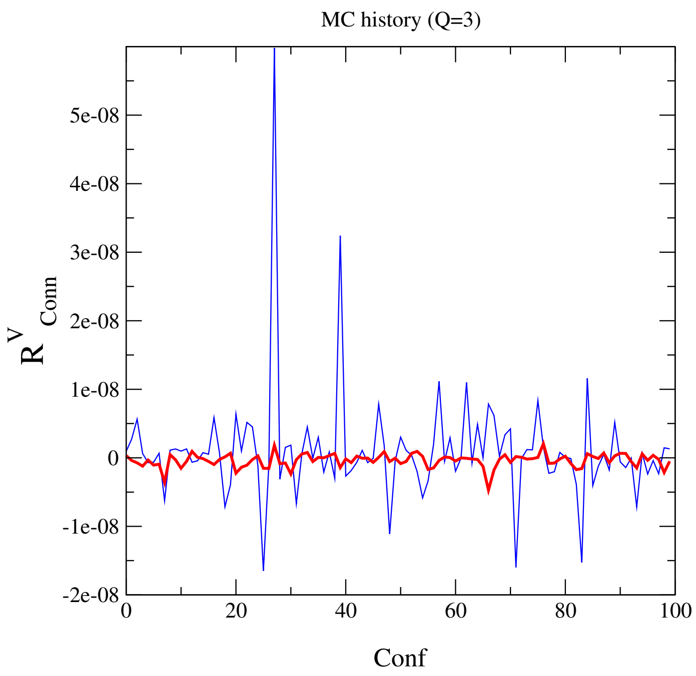
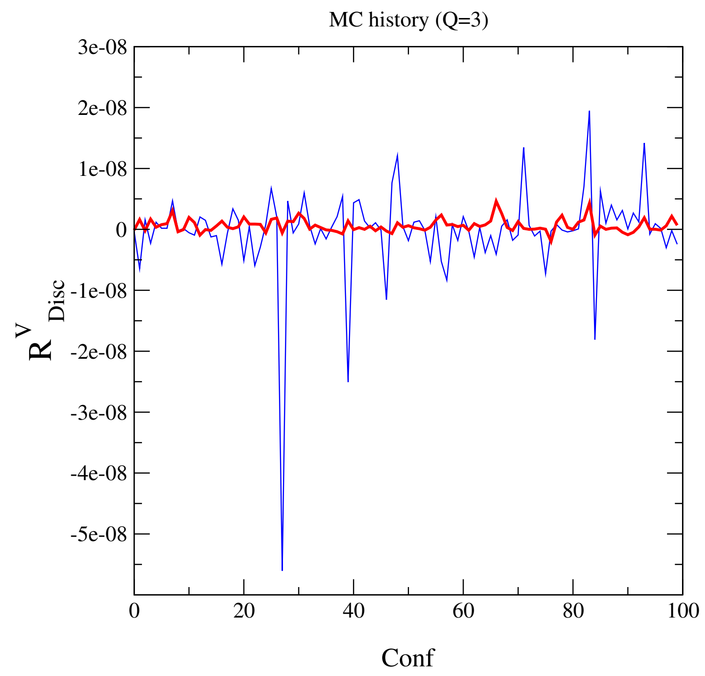
$$am = 0.020, 0.030, 0.040, 0.060 \qquad \mathrm{N_{confs}} = 197$$

ε-regime computation





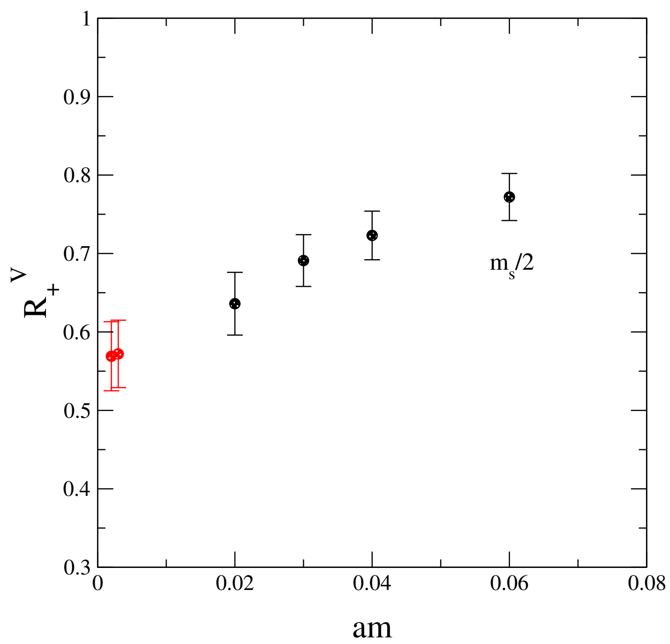
- Simulation parameters

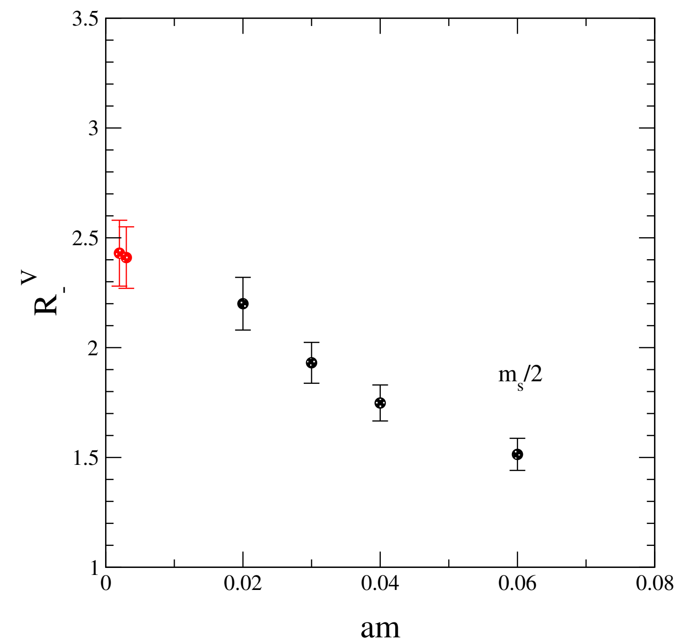
$$\beta = 5.8485 \qquad \frac{V}{a^4} = 16^3 \cdot 32 \qquad a \approx 0.125 \text{ fm} \qquad V \approx 2^3 \cdot 4 \text{ fm}^4$$

- Quark masses $[m \sim m_s/60 - m_s/40]$

$$am = 0.002, 0.003 \qquad \text{N}_{\text{confs}} \sim 630$$

ε-regime computation





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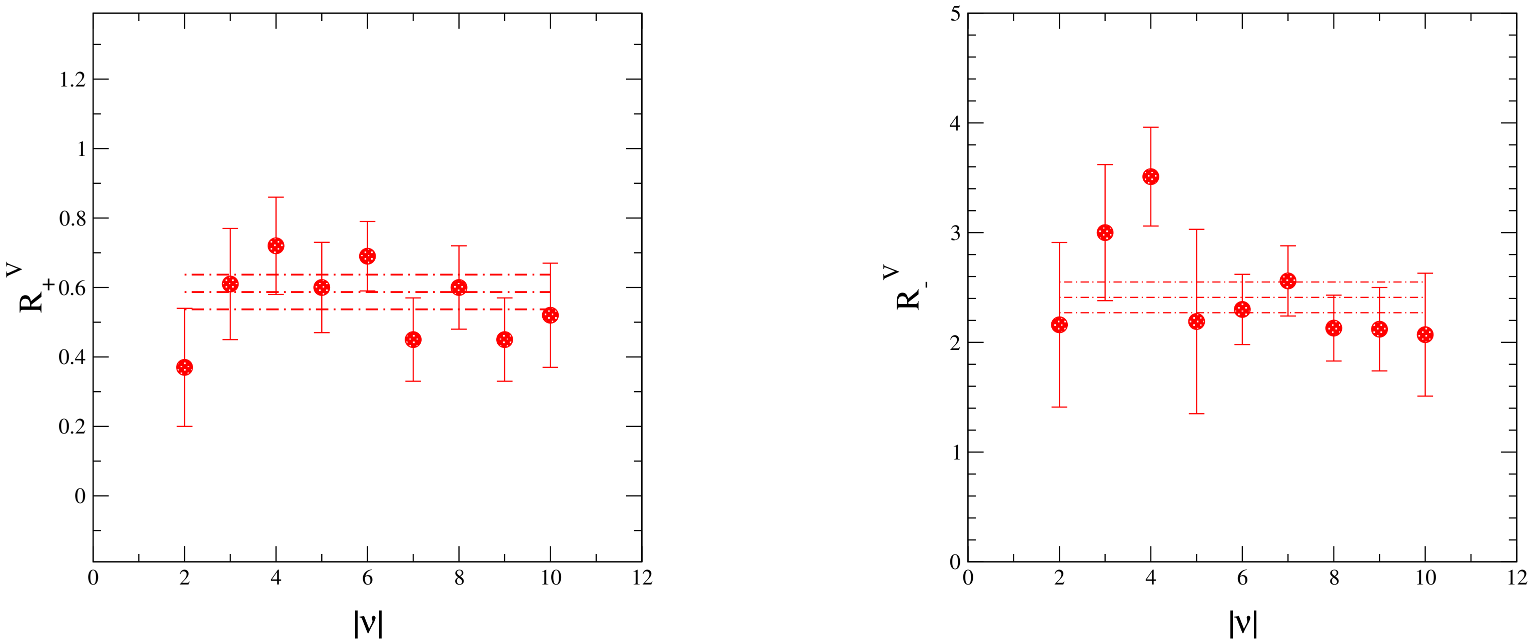

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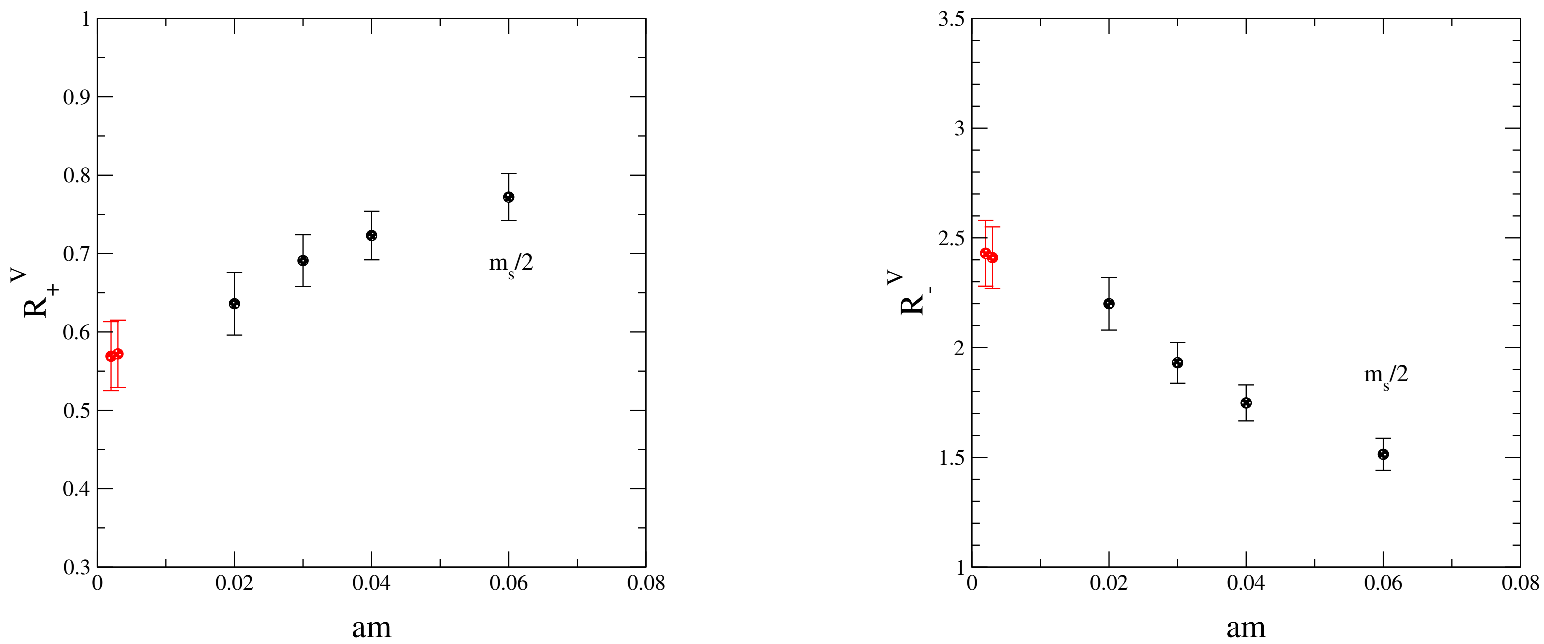
Some remarks (PRELIMINARY) [L.G., P. Hernández, M. Laine, C. Pena, J. Wennekers, H. Wittig in preparation]



- Low-mode averaging allows to get a good signal up to the ε regime
- Statistical errors for $R_\pm$ in the ε regime under control with $500 - 1000$ confs
- Chiral corrections tend to be large
- Data suggests a significant enhancement in $|A_0/A_2|$ already in SU(4)

Conclusions and outlook

- A SU(4) strategy to disentangle contributions from different sources in $K \to \pi\pi$ decays has been proposed
- First goal: computation of the $\Delta S = 1$ low-energy constants vs m_c
- By using GW fermions:

  the UV problem is solved, no power divergences

  the chiral regime of QQCD is reachable. Low-mode preconditioning and averaging techniques solve the numerical problem and allow to get a good statistical signal up to the ε-regime
- The NLO QChPT corrections have been computed in the ε regime
- The numerical computation of the low-energy constants in the SU(4) symmetric theory is almost completed
- Physics results soon!

Low-lying spectrum of the Dirac operator

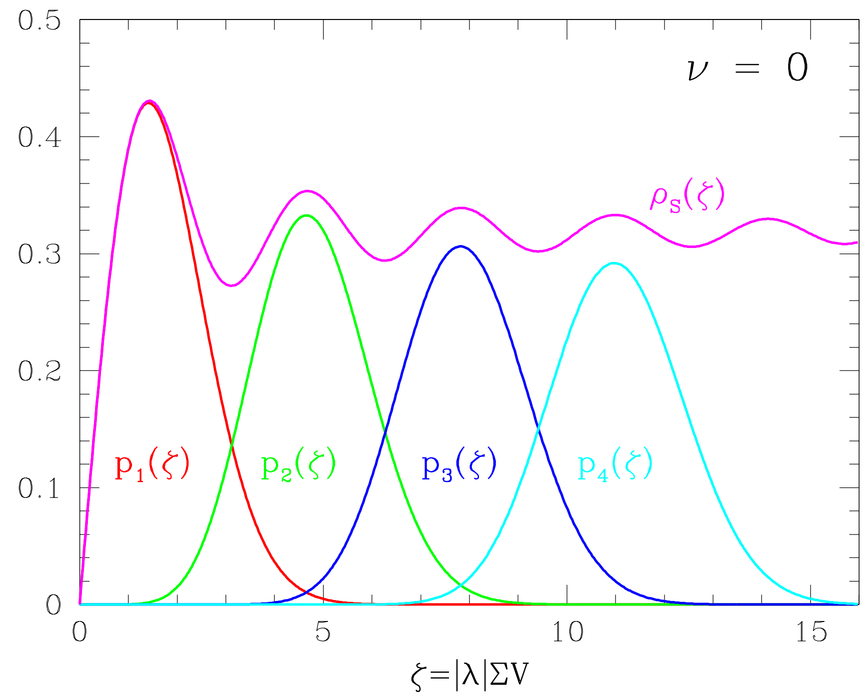
- In a random matrix model

$$\langle \lambda_i \rangle = \int D[W] \lambda_i \det(D[W] + m)^{N_{\rm sea}} e^{\frac{N}{2}{\rm Tr}\,\{W^\dagger W\}}$$

- The spectral density at fix topology

$$\rho_{k,\nu}(\zeta) = \lim_{N\to\infty} \langle \delta(\zeta - \lambda_k N)\rangle_\nu$$

and the joint distributions are well defined and known [Nishigaki et. al. 98; Damgaard, Nishigaki 00]



- The microscopic spectral density $\rho_s(\zeta)$ of the QCD Dirac operator can be computed in ChPT and it agrees with the one in RMT [Leutwyler, Smilga 92; Shuryak, Verbaarschot 93; Osborn et al 99]

- This is a strong hint that RMT is not needed at all also for $\rho_{k,\nu}(\zeta)$ [Akemann, Damgaard 04]