Current phenomenological aspects of Kaon decays

Gino Isidori [*INFN-Frascati*]

Plan of the talk :

- The extraction of V_{us} from K_{l3} & K_{l2} decays → Measurement of fundamental SM couplings
- $\pi\pi$ phase shifts from $K\to3\pi$ decays → Deep insight about the structure of the QCD vacuum [$\pi\pi$ phase shifts $\Leftrightarrow \langle 0|\, q_L q_R |0\rangle$]
- Rare K decays → Unique window about the flavour structure of physics beyond the SM

The extraction of V_{us} from K decays

- V_{us} is a *fundamental coupling* of the SM Lagrangian which could possibly play a deeper role in a *theory of flavour*...

$$\mathscr{L}_{c.c.} = (g/\sqrt{2})\, W_\mu^+ \, \bar{u}_L^{\,i} \, (V_{CKM})_{ij} \, \gamma^\mu d_L^{\,j} + \text{h.c.}$$

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

- ...plays a major role in the most precise test of CKM unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 - 0.001 \pm 0.001 \qquad \left[\begin{array}{l} V_{ud} = 0.9738 \pm 0.0003 \\ V_{us} = 0.2250 \pm 0.0010 \\ V_{ub} = 0.0037 \pm 0.0005 \end{array}\right.$$

CKM 2005

- ... recent precise data [KTeV, NA48, KLOE, E865] have pushed the error completely on the theory side [$K \to \pi$ / $K \to 0$ form factors] $\Rightarrow$ interesting challenge for CHPT & Lattice QCD !

The extraction of V_{us} from K decays

This subject has indeed stimulated a substantial phenomenological activity in the last few years:

- Marciano 2004
- MILC coll. 2004
- Knecht-Neufeld-Rupertsberger-Talavera 2000

$K_{\mu 2}$

- VC-Knecht-Neufeld-Rupertsberger-Talavera 2002
- VC-Neufeld-Pichl 2004
- Andre 2004
- Moussallam-Descotes 2005

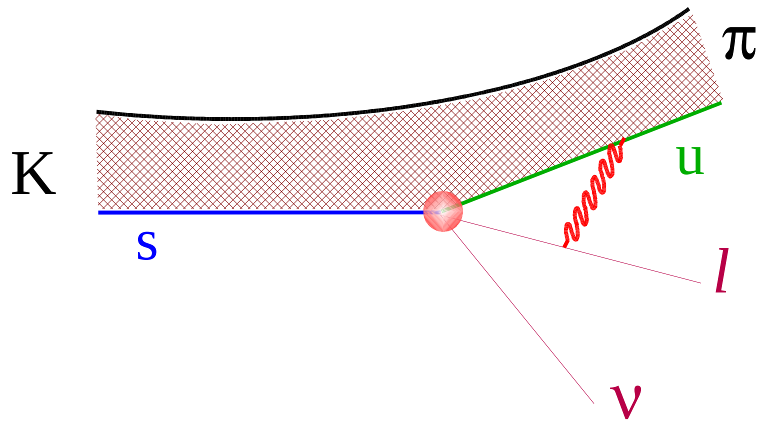
K_{l3} EM

- Bijnens-Talavera 2003
- Post-Schilcher 2002
- Leutwyler-Roos 1984
- Jamin-Oller-Pich 2004
- Becirevic et al 2004
- VC-Ecker-Eidemuller-Kaiser-Pich-Portoles 2005

K_{l3} ~~SU(3)~~

Cirigliano [KAON 2005]

I. V_{us} from K_{l3} decays:



Master formula :

$$\Gamma(K^{i}_{l3+n\gamma}) = C_i \times |V_{us}|^2 \times |f_+(0)|^2 \times I(\lambda'_+, \lambda''_+, \lambda'_0, \ldots) \times [\, 1 + \delta_{e.m.} + \delta_{SU(2)} \,]$$

green = exp. inputs
red = th. inputs

$$\Gamma(K^i_{l3+n\gamma}) = C_i \times |V_{us}|^2 \times |f_+(0)|^2 \times I(\lambda'_+, \lambda''_+, \lambda'_0, \ldots) \times [\, 1 + \delta_{e.m.} + \delta_{SU(2)} \,]$$

vector form factor at $t = (p_K - p_\pi)^2 = 0$

$$\langle \pi(p_\pi)|\bar{s}\gamma_\mu u|K(p_K)\rangle = f_+^{K\pi}(t)\,(p_K + p_\pi)_\mu + f_-^{K\pi}(t)\,(p_K - p_\pi)_\mu$$

$$f_0(t) \equiv f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

$$f_{+,0}(t) = f_+(0)\left(1 + \lambda'_i \frac{t}{M_\pi^2} + \lambda''_i \frac{t^2}{M_\pi^4} + \ldots\right)$$

The slopes (λ_i) are a key ingredient to compute the phase-space integrals $I(\lambda_i)$ and to cross-check the th. calculations of $f_+(0)$

CVC $\Rightarrow$ $f_+(0) = 1$ in the SU(3) limit $m_s = m_u = m_d$

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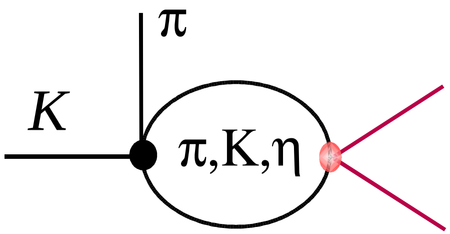
vector form factor at
$t = (p_K - p_\pi)^2 = 0$

$m_u \neq m_d$ corrections computed with good precision in CHPT:



Leutwyler, Roos '84
Cirigliano *et al*. '01

$$\langle \pi(p_\pi) | \bar{s}\gamma_\mu u | K(p_K) \rangle = f_+^{K\pi}(t)\,(p_K + p_\pi)_\mu + f_-^{K\pi}(t)\,(p_K - p_\pi)_\mu$$

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$f_+(0; K^0 \to \pi^-)$ conventionally chosen as common normalization

CVC $\Rightarrow$ $f_+(0) = 1$ in the SU(3) limit $m_s = m_u = m_d$

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in addition to $m_u \neq m_d$, also electromagnetic corrections distinguish the four possible decay channels:

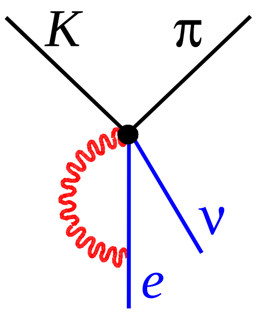
$K^0 \to \pi^- e^+ \nu$

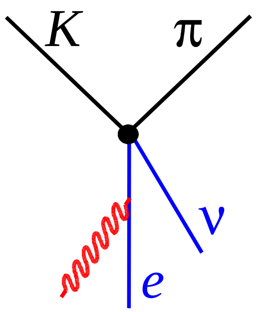
$K^0 \to \pi^- \mu^+ \nu$

$K^+ \to \pi^0 e^+ \nu$

$K^+ \to \pi^0 \mu^+ \nu$

Good control within CHPT:





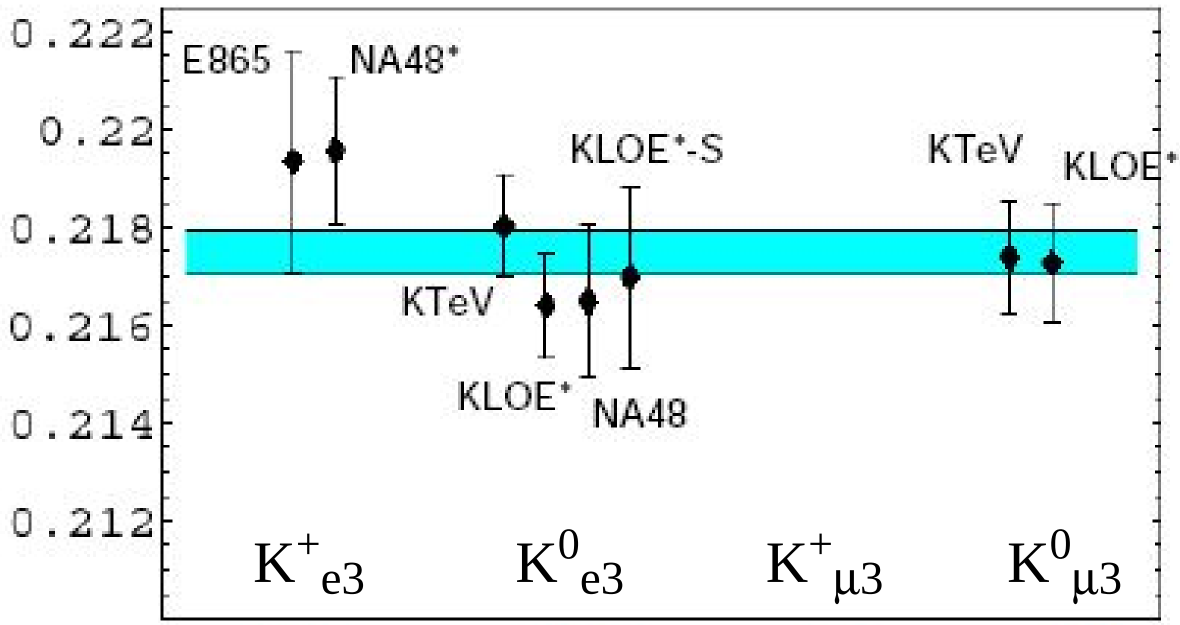
- Sizable model-independent logs $\Rightarrow O[\alpha \log(m_K/m_e)] = O(1\%)$
- Small uncertainty due to unknown local operators: $O(\alpha/\pi) \sim O(0.1\%)$

Cirigliano *et al.* '01-'04

Andre '04, Gatti '05

$$\Gamma(K^i_{l3+n\gamma}) = C_i \times |V_{us}|^2 \times |f_+(0)|^2 \times I(\lambda'_+, \lambda''_+, \lambda'_0, \ldots) \times [\,1 + \delta_{e.m.} + \delta_{SU(2)}\,]$$

$f_+(0)\, V_{us}$



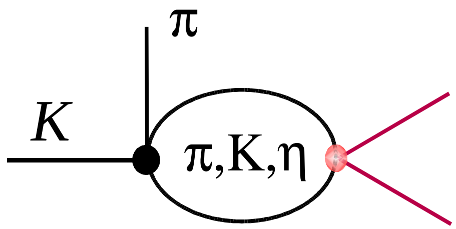
consistent treatment of IB corrections
+
new generation of experiments

➡ $f_+(0)\, V_{us} = 0.2173 \pm 0.0008$ [0.4 %]

a fashinating challenge for a precise estimate of $f_+(0)$!

What do we know about $[\, f_+(0) - 1 \,]$?

- no linear corrections in $(m_s - m_u)$ [Ademollo-Gatto, '64]
- $| f_+(0) | < 1$ [Furlan *et al.* '65] $$| f_+(0) |^2 = 1 - \sum_{n \neq \pi^+} \left| \langle K^0 | Q^{us} | n \rangle \right|^2$$
- Within CHPT: $$f_+(0) = 1 + f_4 + f_6 + \ldots$$



$O(p^4)$: finite non-polynomial term induced by meson loops
$[\sim m_P \log m_P \Rightarrow \sim (m_s - m_u)^2 / m_s \,]$
small compared to naïve expectations: $f_4 = -2.3\,\%$

Leutwyler & Roos '84

$O(p^6)$: appearance of $(m_s - m_u)^2 / \Lambda^4_\chi$ (a priori unknown) local terms
main source of uncertainty
[original (gu)estimate by LR: $f_6 = -1.6 \pm 0.8\,\%$]

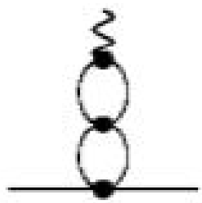
$\longrightarrow f_+(0)_{LR} = 0.961 \pm 0.008$

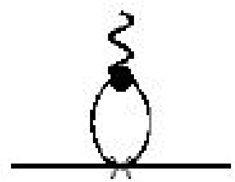
General decomposition of f_6 within CHPT:

$$f_6 = f_{p^6}^{2-\mathrm{loops}}(\mu) + f_{p^6}^{L_i \times \mathrm{loop}}(\mu) + f_{p^6}^{\mathrm{tree}}(\mu)$$

Post-Schilcher '02
Bijnens-Talavera '03

$$8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2}\left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho)\right]$$





$$f_{p^6}^{L_i \times \mathrm{loop}}(M_\rho) = -0.0020$$

$$f_{p^6}^{2-\mathrm{loops}}(M_\rho) = 0.0113$$

N.B.: large & positive 2-loop contribution with sizable scale dependence

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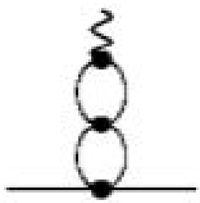

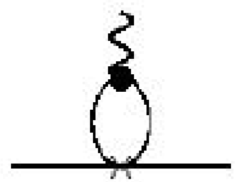

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model-independent relation which links this $O(p^6)$ CT combination to λ'_0, λ''_0 & F_K/F_π

Bijnens & Talavera, '03

Not useful -at the moment- to derive a competitive estimate of f_6 :

$$\delta f_6 \sim 1\% \quad \Leftrightarrow \quad \delta(F_K/F_\pi) \sim 1\% \oplus \delta\lambda'_0 \sim 0.1\% \oplus \delta\lambda''_0 \sim 0.01\%$$

but useful cross-check for any method used to compute f_6

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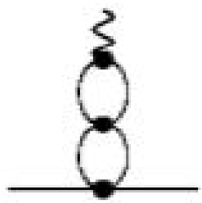

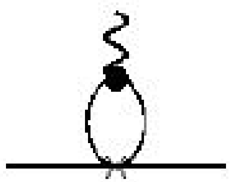

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Two basic strategies for model-dependent estimates:

I. Identification of f_6^{tree} with the old LR estimate of f_6^{full}

$\Rightarrow$ $f_+(0)_{\mathrm{BT}} = 0.976 \pm 0.010$ [Bijnens & Talavera, '04]

II. Expansion of ⟨ T(*SPP*) ⟩ based on large N_C [finite resonance stauration]

$\Rightarrow$ $f_6^{\mathrm{tree}} = -\dfrac{(M_K^2 - M_\pi^2)^2}{2\, M_S^4}\left(1 - \dfrac{M_S^2}{M_P^2}\right)^2$ $\Rightarrow$ $f_+(0)_{\mathrm{large\ N}} = 0.984 \pm 0.012$

[Cirigliano, Ecker, Eidemuller, Kaiser, Pich, Portoles '05]

On the lattice we don't need to decompose local and non-local contributions [no scale-dependence problem] but it can still be convenient trying to extract directly f_6, or better $\Delta f = 1 + f_4 - f_+(0)$

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Strategy of our recent quencehd analysis:

- Evaluation of $f_0(q^2_{max})$ with high precision [double ratio method - FNAL '99]
- Extrapolation $f_0(q^2_{max}) \to f_0(0)$ [extraction of the $f_0(q^2)$ slopes]
- Construction of Δf [subtraction of the $O(p^4)$ qCHPT logs] & chiral extrapolation

[Becirevic, G.I, Lubicz, Martinelli, Mescia, Simula, Tarantino, Villadoro '04]

↓

$$f_+(0) = 1 + f_4 - (\Delta f)^{(q)}$$

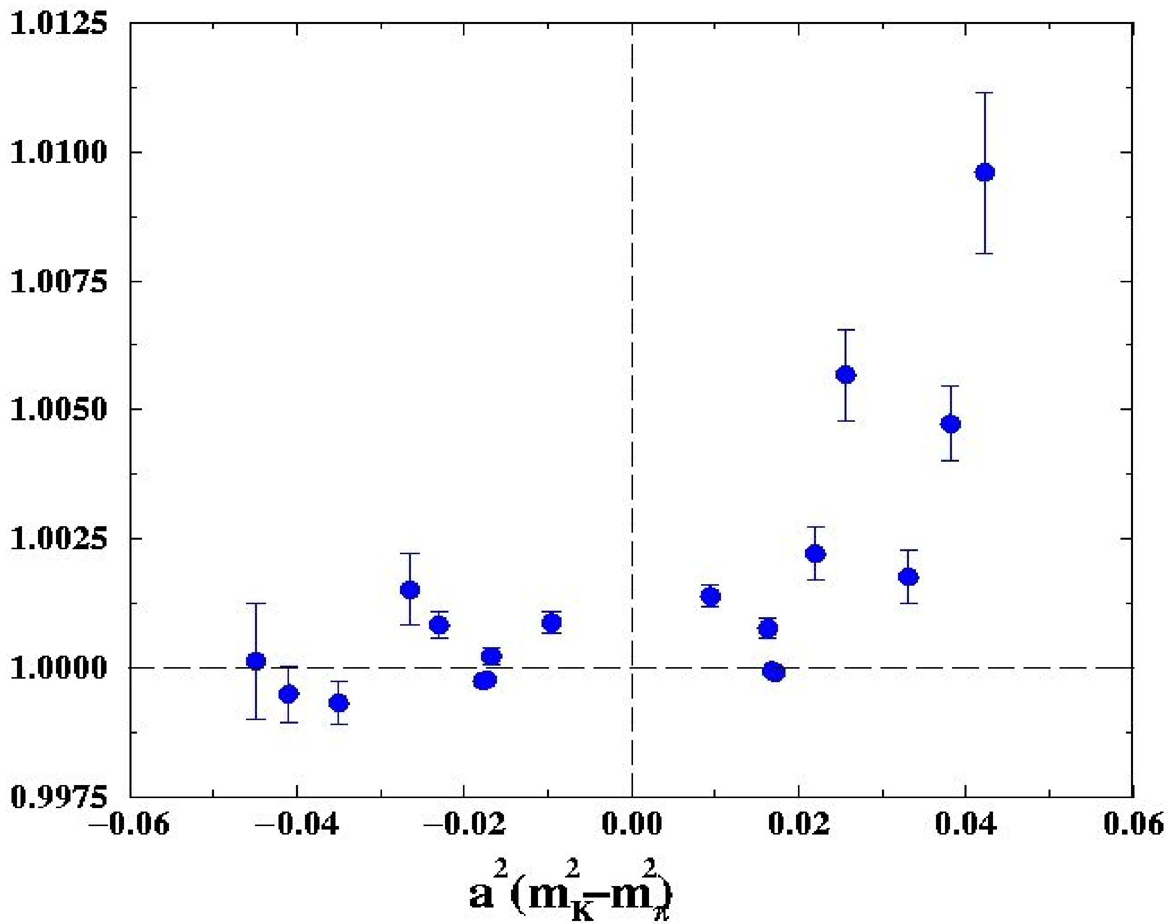
↓

$$f_+(0)^{\text{lattice}}_{\text{semi-quench}} = 0.960 \pm 0.005_{stat} \pm 0.007_{syst}$$

- Evaluation of $f_0(q^2_{max})$ with high precision [double ratio method - FNAL '99]

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle\langle K|\bar{u}\gamma_0 s|\pi\rangle}{\langle\pi|\bar{u}\gamma_0 u|\pi\rangle\langle K|\bar{s}\gamma_0 s|K\rangle} = \frac{(M_K+M_\pi)^2}{4M_K M_\pi}\left[f_0(q^2_{max}; M_K, M_\pi)\right]^2$$

$f_0(q^2_{max}; M_K, M_\pi)$



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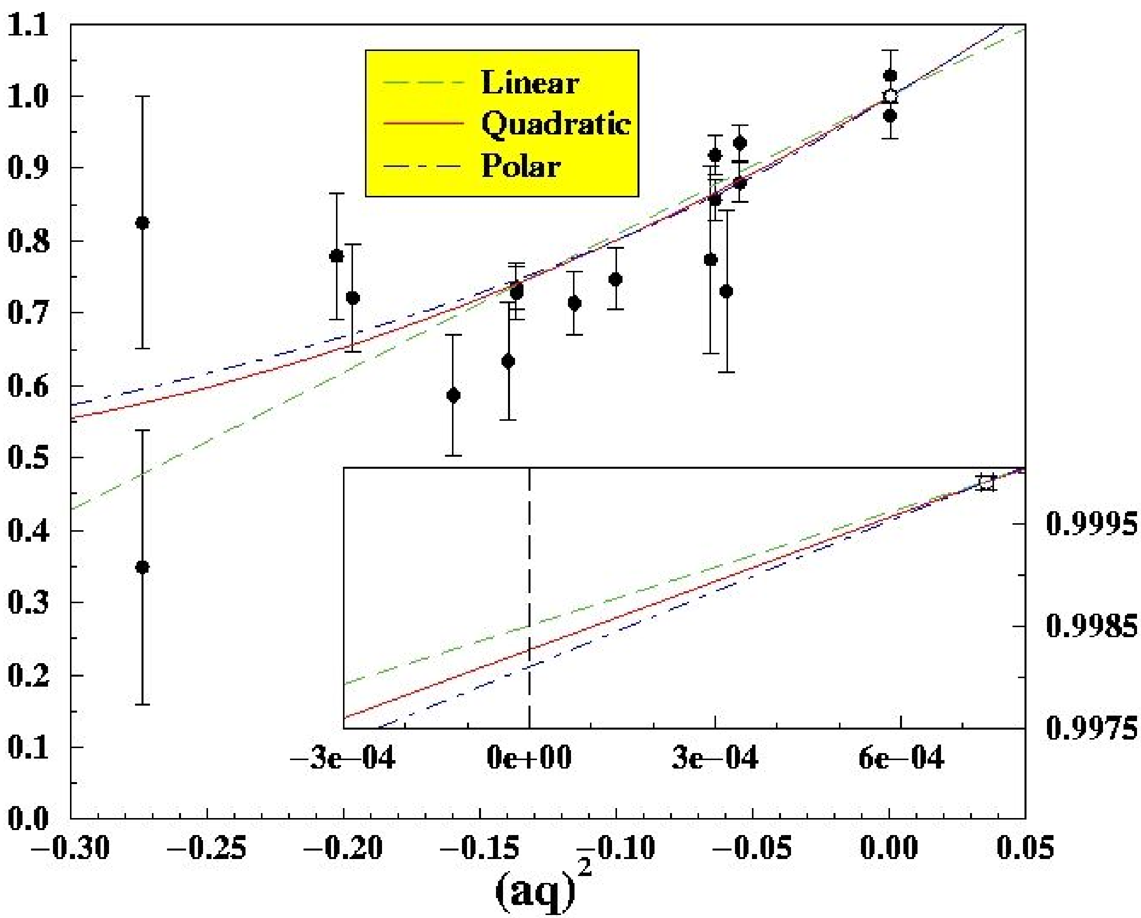

prediction for the physical slope
$[\lambda'_0 = 0.012 \pm 0.002]$

in agreement with recent exp. data
$[\lambda'_0 = 0.0136 \pm 0.0007$ KTeV '05$]$

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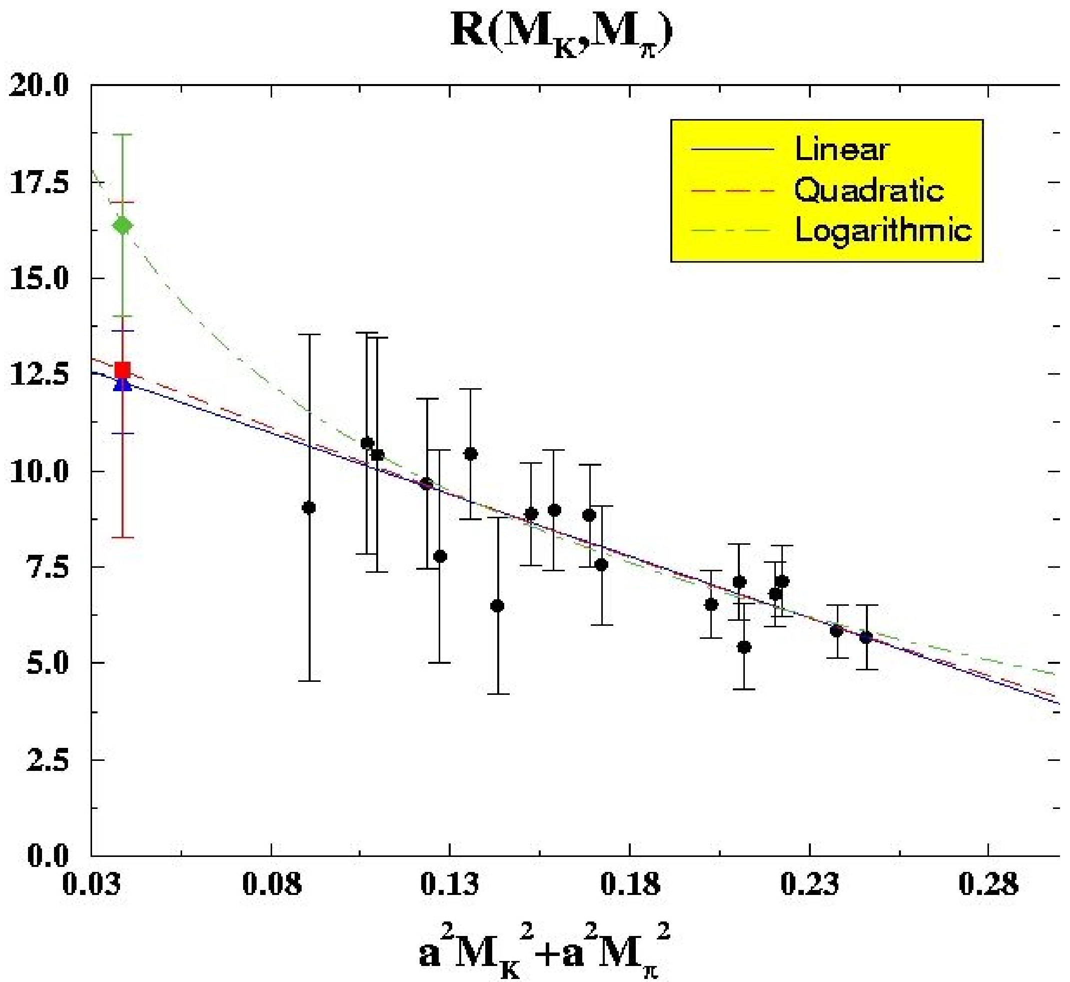
- Construction of Δf [subtraction of the $O(p^4)$ qCHPT logs] & chiral extrapolation

$$R(M_K, M_\pi) \equiv \frac{\Delta f}{(\Delta M^2)^2}$$

$$= \frac{1 + f^q_{p^4}(M_K, M_\pi) - f_0(0; M_K, M_\pi)}{(\Delta M^2)^2}$$

$O(p^4)$ non-local contribution computed in qCHPT (no CT as in the full teory)

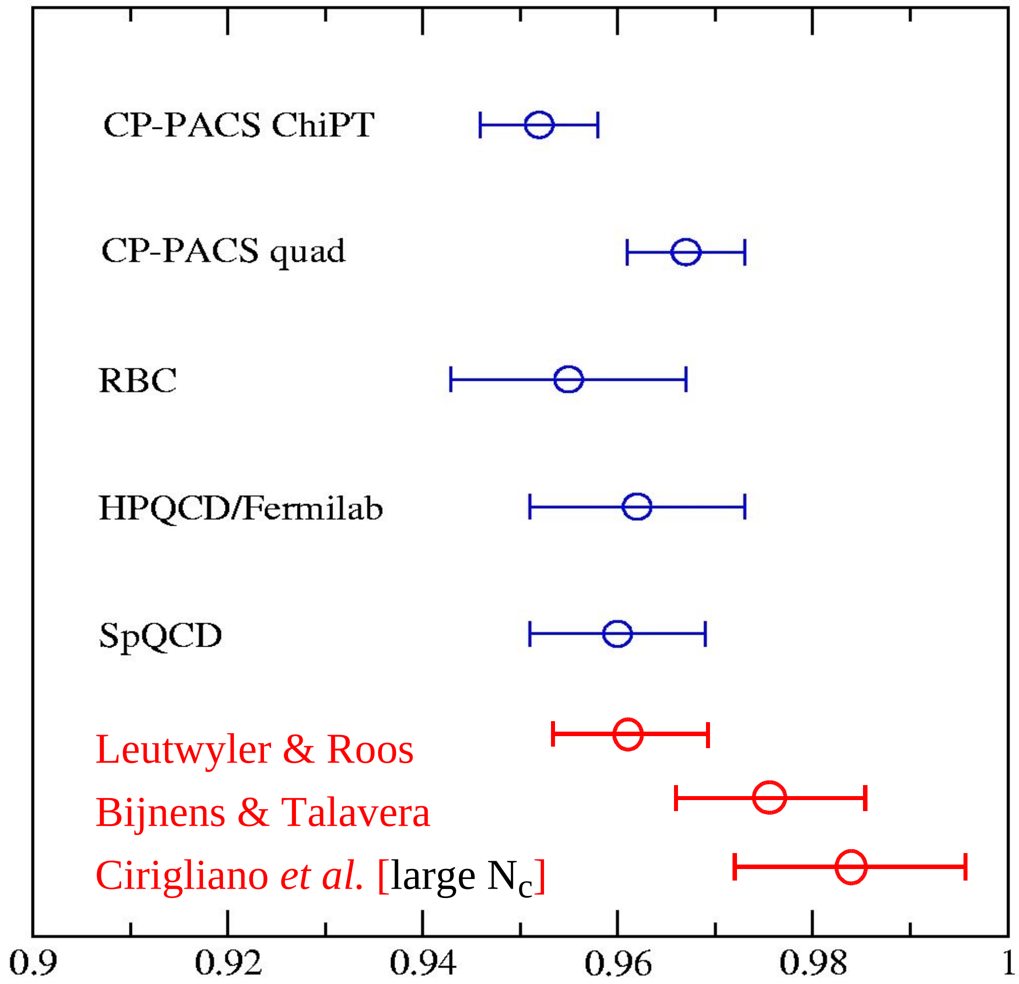


$$\Delta f_{phys} = 0.017 \pm 0.005_{stat} \pm 0.007_{syst}$$

Several new preliminary lattice results [based on different techniques] announced last summer:

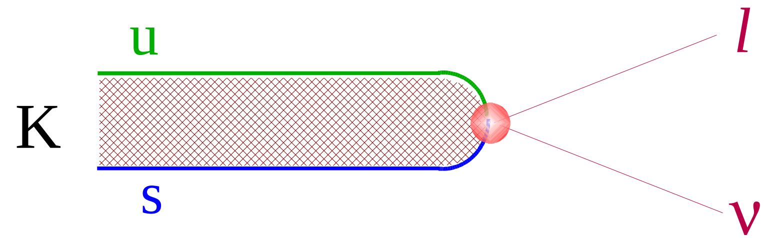
C.Dawson [Lattice 2005]

- All these new lattice numbers should be considered as preliminary for various reasons.
- Quenched, $N_f = 2$, and $N_f = 2 + 1$ all agree.
  - also with the Leutwyler-Roos
- Chiral extrapolation for all measurements is over a large range.
- Lattice spacing, Volume effects small ?



Reliable predictions with σ<1% seems to be within the reach in the near future

II. V_{us} from K_{l2} decays:



Master formula :

$$\frac{\Gamma(K_{\mu 2+n\gamma})}{\Gamma(\pi_{\mu 2+n\gamma})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{|F_K|^2}{|F_\pi|^2} \times \frac{M_K^2\,(1-m_\mu^2/M_K^2)}{M_\pi^2\,(1-m_\mu^2/M_\pi^2)} \times [\,1+\delta_{e.m.}\,]$$

green = exp. inputs
red = th. inputs

0.007 ± 0.002

Knecht *et al.* '00

essentially parameter-free prediction of CPHT at $O(e^2p^4)$

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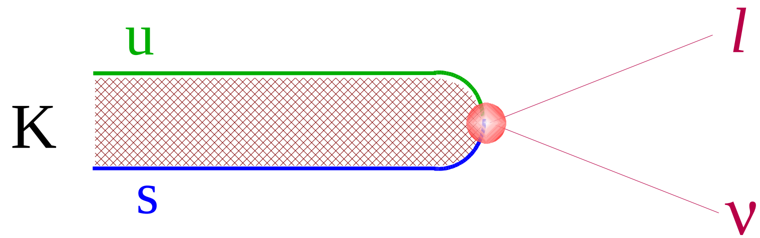

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- Non-trivial to reach 1% accuracy [no Ademollo-Gatto protection] → $\frac{\sigma[SU(3)\text{-brk}]}{[SU(3)\text{-brk}]} \sim 5\,\%$
- No competition from non-Lattice approaches
- Recent MILC results ⇒ *competitive* value of V_{us} :

$$F_K/F_\pi = 1.198 \pm 0.003^{+0.016}_{-0.005} \;\Rightarrow\; |V_{us}| = 0.2245^{+0.0011}_{-0.0031}$$

Summary about V_{us} from K_{l3} & K_{l2} decays:

$$|V_{us}|_{K_{l3}} = 0.2261 \pm 0.0008 \pm 0.017_{f_+(0)} \times \left[\frac{f_+(0)}{0.961}\right]$$

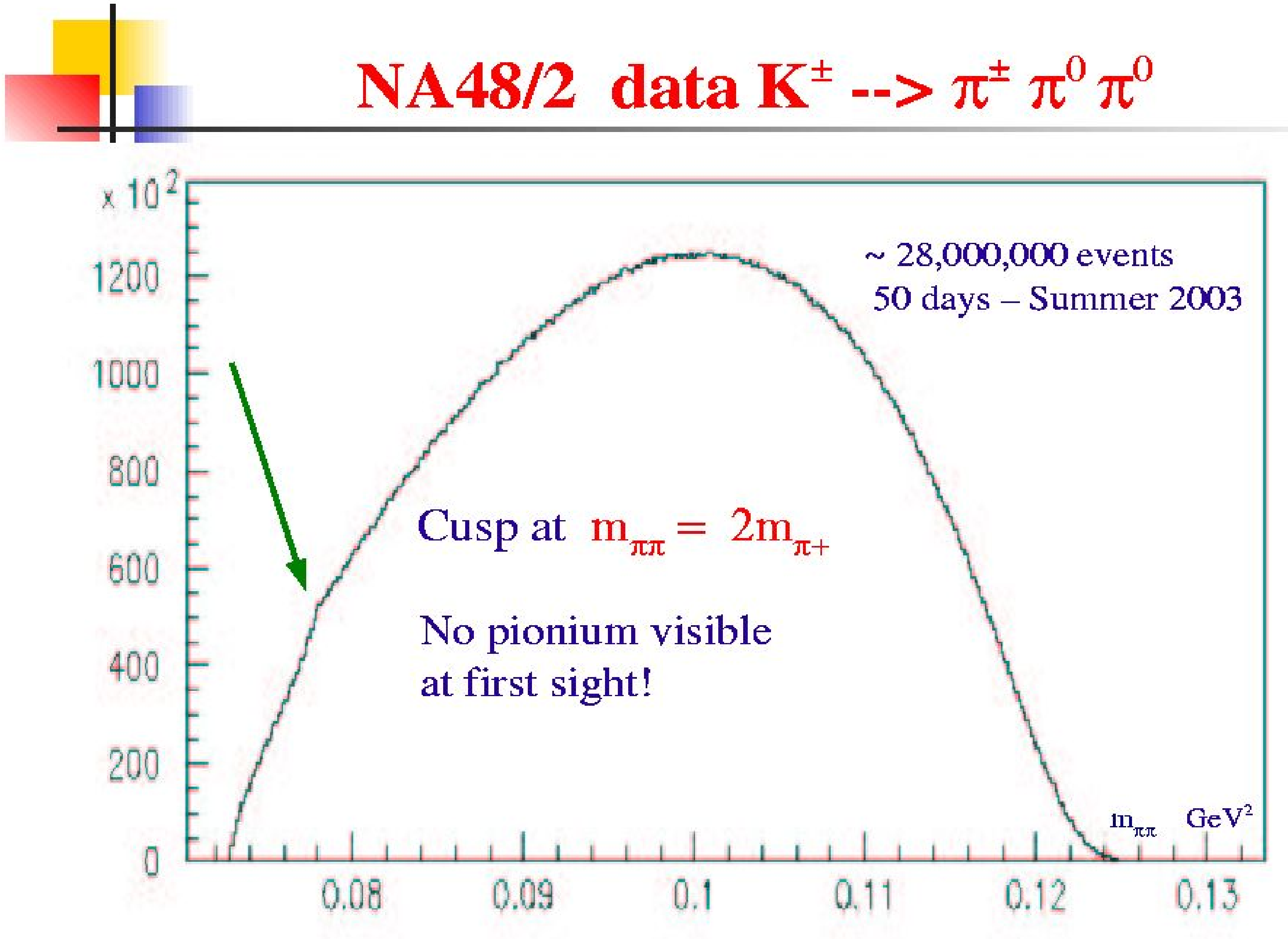
$$|V_{us}|_{K_{l2}} = 0.2245 \pm 0.0004 \,{}^{+0.0011}_{-0.0033}{}_{F_K/F_\pi} \times \left[\frac{F_K/F_\pi}{1.198}\right]$$

$$|V_{us}|_{unit} = [\, 1 - V_{ud}^2 - V_{ub}^2 \,]^{1/2} = 0.2274 \pm 0.0013$$

...still a lot of room for improvements !

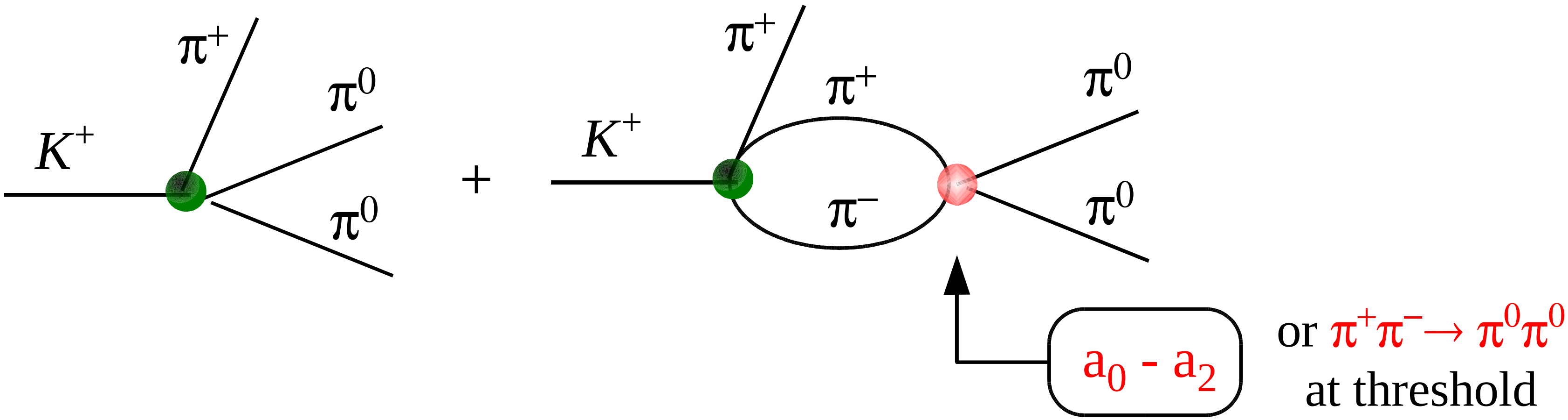
▶ $\pi\pi$ phase shifts from $K\to3\pi$ decays

The high resolution of the NA48/2 experiment has allowed to observe
- for the first time - a subtle & interesting phenomenon in $K \to 3\pi$ decays:



I. Mannelli,
CERN March '05

As pointed out by Cabibbo [P.R.L. '04] the origin of this discontinuity is due to the following interference/re-scattering effect:



- The decay amplitude is an analytic function of the di-pion invariant mass $s = (M_{\pi^0\pi^0})^2$

- The existence of a real intermediate state implies a discontinuity across the real axis for $s > s_0 = (2m_{\pi^+})^2 > (2m_{\pi^0})^2$

$$T(s+i\varepsilon) - T(s-i\varepsilon) = i\,\rho_{\pi\pi}(s)\, V_{K\to 3\pi}(s)\, V_{\pi\pi\to\pi\pi}(s)\, \Theta(s-s_0)$$

$$\rho_{\pi\pi}(s) \sim v_{\pi+\pi-}(s) \sim (s-s_0)^{1/2}$$

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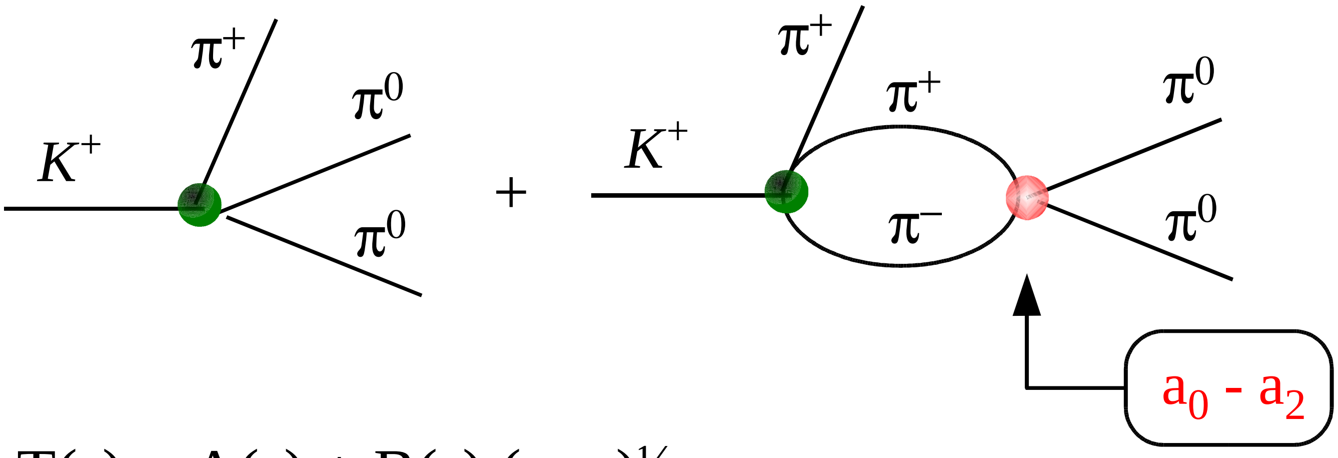

or $\pi^+\pi^- \to \pi^0\pi^0$ at threshold

$s > s_0 \qquad T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$

Analytic continuation below threshold

Sign determined by the iε prescription in pion propagators

$s < s_0 \qquad T(s) = A(s) + i\,B(s)\,(s_0-s)^{1/2}$

A(s) & B(s) regular around s_0

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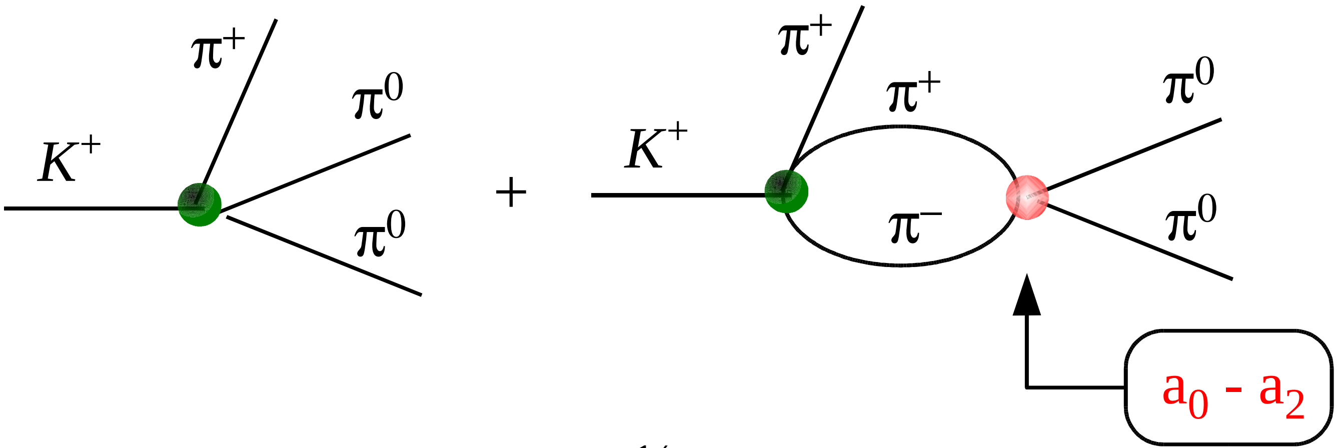

or $\pi^+\pi^- \to \pi^0\pi^0$ at threshold

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$$|T(s)|^2 = [\,\mathrm{Re}A(s) + \mathrm{Re}\,B(s)\,(s-s_0)^{1/2}\,]^2 + [\,\mathrm{Im}A(s) + \mathrm{Im}\,B(s)\,(s-s_0)^{1/2}\,]^2$$

$s < s_0$ $\quad T(s) = A(s) + i\,B(s)\,(s_0-s)^{1/2}$

$$|T(s)|^2 = [\,\mathrm{Re}A(s) - \mathrm{Im}\,B(s)\,(s_0-s)^{1/2}\,]^2 + [\,\mathrm{Im}A(s) + \mathrm{Re}\,B(s)\,(s_0-s)^{1/2}\,]^2$$

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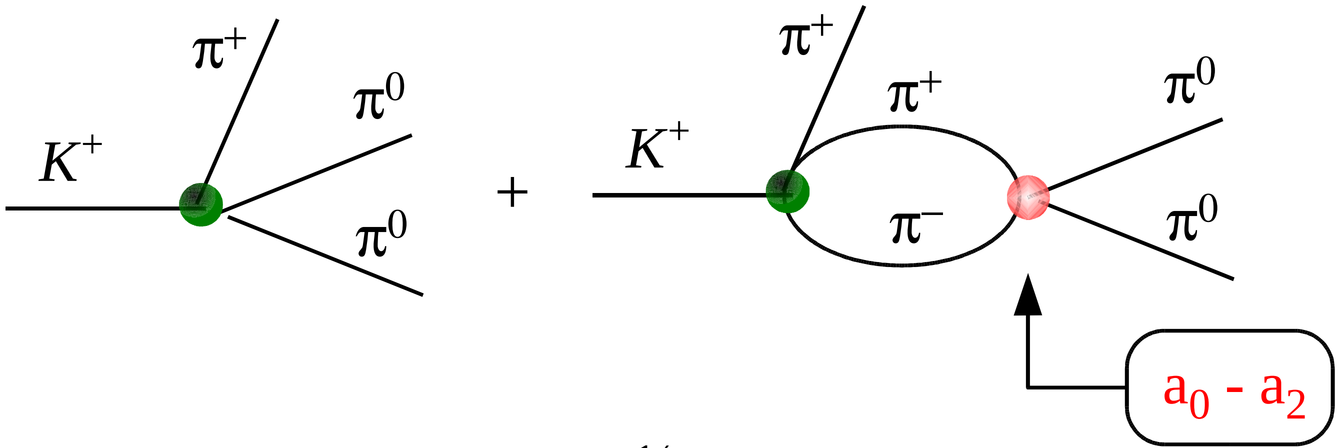

or $\pi^+\pi^- \to \pi^0\pi^0$ at threshold

$s > s_0$ $\quad T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$

$$|T(s)|^2 = [\, \mathrm{Re}A(s) + \cancel{\mathrm{Re}\,B(s)\,(s-s_0)^{1/2}}\,]^2 + [\, \cancel{\mathrm{Im}A(s)} + \mathrm{Im}\,B(s)\,(s-s_0)^{1/2}\,]^2$$

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in the limit where A(s) = real & B(s) = imaginary (real one-loop couplings) we have a square-root behaviour below the threshold

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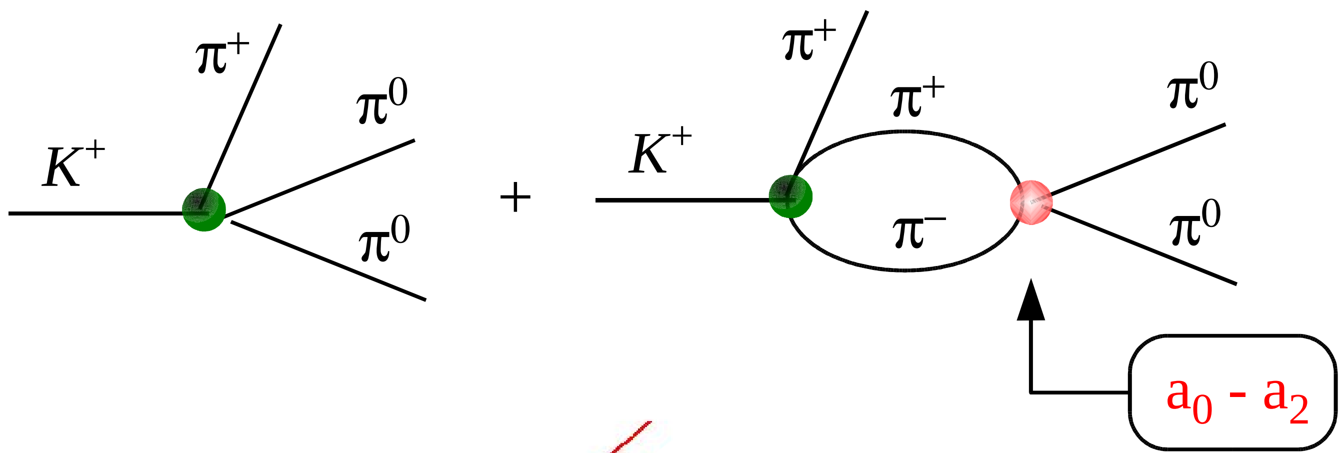

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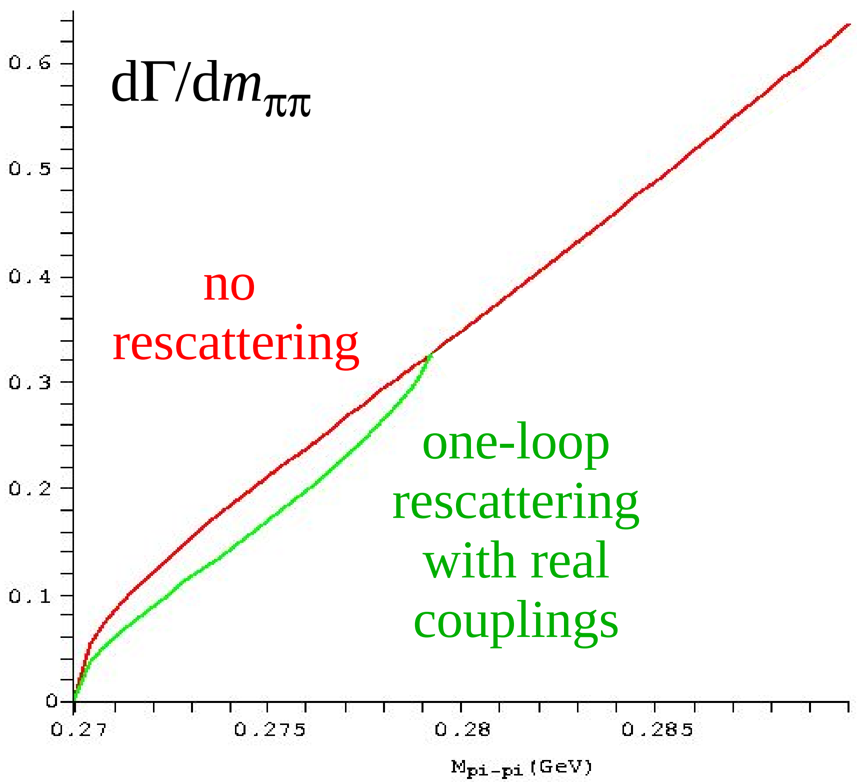

Key questions:

- What do we learn from this effect?
- To which approximation the one-loop result is correct, or at which level of accuracy can we extract a_0 - a_2 from the measured spectrum ?

- What do we learn from this effect? ⇒ New way to measure the a_I

The S-wave ππ scattering lengths (a_I) have a very special role in CHPT

defined, in the I-spin limit, by $T(\pi\pi_{I,I_3} \to \pi\pi_{J,J_3}) = 4 a_I v_{\pi\pi}(s)\, \delta_{I_3 J_3} \delta_{IJ} + O(v^3)$

$O(p^2)$: $a_0\, m_\pi = \dfrac{7 m_\pi^2}{32\pi F_\pi^2} = 0.16$ $a_2\, m_\pi = \dfrac{-m_\pi^2}{16\pi F_\pi^2} = -0.05$

Weinberg '79

$O(p^4)$: $a_0\, m_\pi = 0.20 \pm 0.01$ $a_2\, m_\pi = -0.044 \pm 0.002$

Gasser &Leutwyler '83

$O(p^6)$: $a_0\, m_\pi = 0.217 \pm 0.005$ $a_2\, m_\pi = -0.0445 \pm 0.0010$

Bijens, Colangelo, Ecker, Gasser & Leutwyler, '99

Roy eqs. [beyond $O(p^6)$]:

Colangelo *et al.* '01

$a_0\, m_\pi = 0.220 \pm 0.005$

$(a_0 - a_2)\, m_\pi = 0.265 \pm 0.004$

1.5 % relative error !

An almost unique example of a very precise prediction (obtained by means of analytic methods), for a truly non-perturbative quantity (from the point of QCD)

- What do we learn from this effect? ⇒ New way to measure the a_I

$$a_0\, m_\pi = 0.220 \pm 0.005 \qquad a_2\, m_\pi = -\,0.0445 \pm 0.0010$$

$$(a_0 - a_2)\, m_\pi = 0.265 \pm 0.004$$

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- The smallness of $a_{0,2}$ is a direct consequence of the pseudo-Goldstone boson nature of the pions [⇒ key issue for the cusp effect]

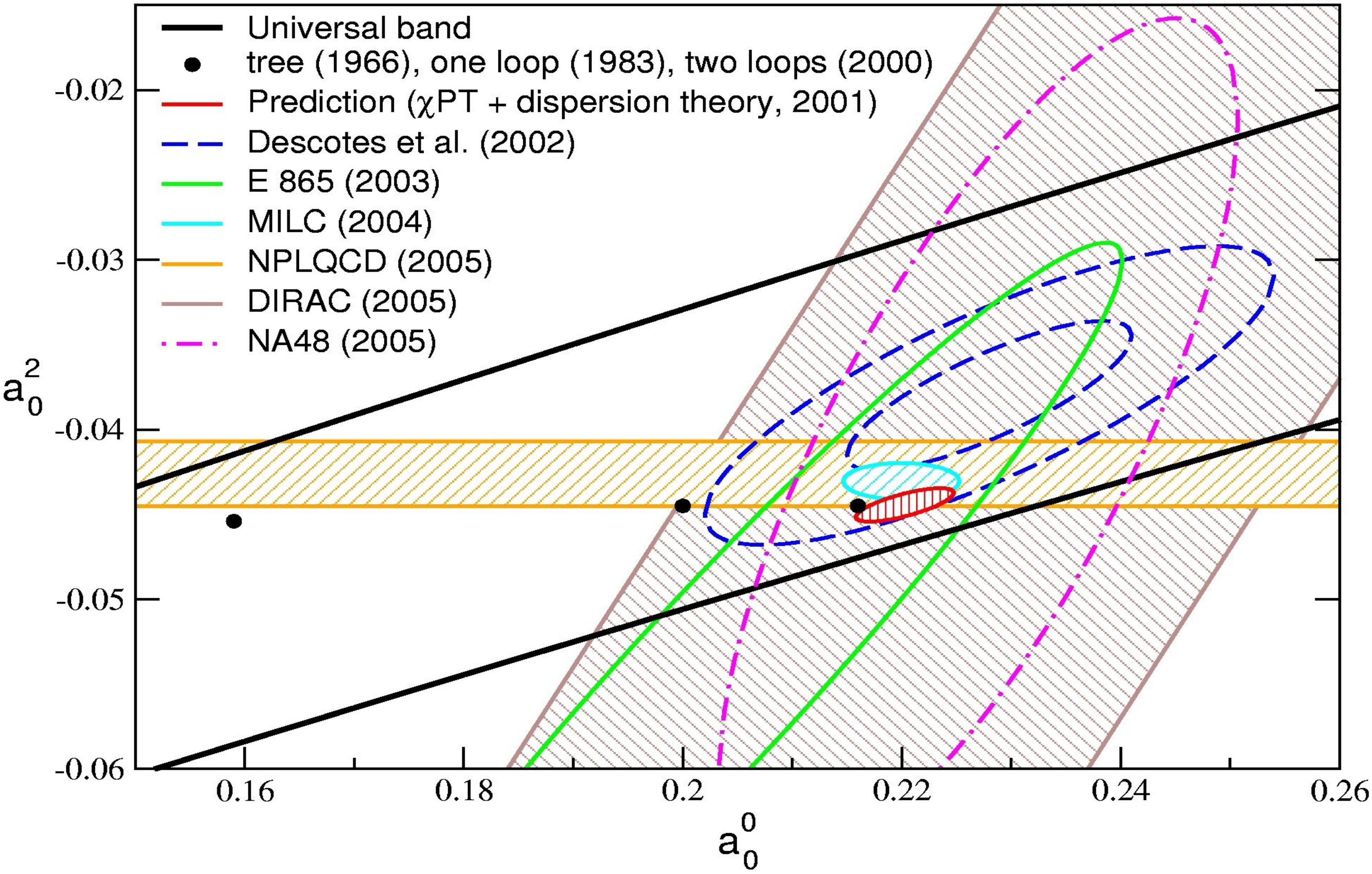
- The only way to modify these predictions is to modify the basic assumptions of CHPT. Different values are obtained assuming a more complicated structure for the QCD vacuum, e.g. assuming that m_π^2 is not ~ linear in m_q [Stern *et al.* '00]

$$\langle 0 |\, \bar{q}_L q_R | 0 \rangle = -B\, F^2 \quad \Rightarrow \quad m_\pi^2 = B\,(m_u + m_d)[1 + O(m_q)]$$

The measure of the a_I is a direct probe of the QCD vacuum !

Present status:

Colangelo *et al.* '05



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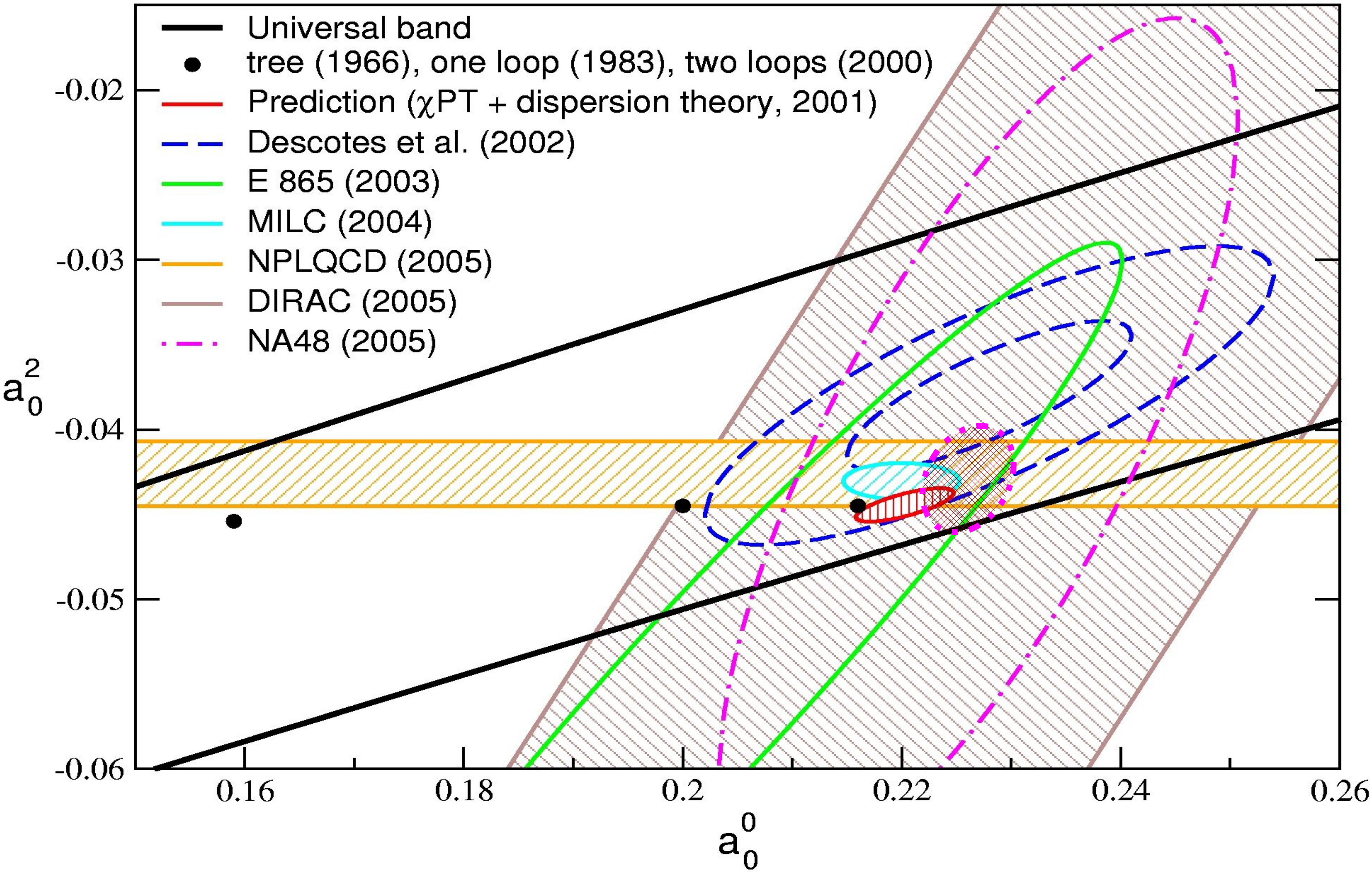

Possible future impact of a 2% meas. of $(a_0\text{-}a_2)$ & 5% meas. of a_2 from $K \to 3\pi$

- Toward a precise theoretical description of the cusp effect

In the last few years there has been a substantial progress in CHPT calculation for non-leptonic K decays [complete $e^0 p^4$, $(m_u - m_d)p^2$, $e^2 p^2$] and, particularly, in $K \to 3\pi$ [Bijnens, Dhonte, Persson, '02; Gamiz, Scimemi, Prades, '03-'04; Bijnens, Borg, '04-'05]

However, full CHPT calculations are problematic for this type of study:

- slow convergence of the chiral expansion
- too many free parameters in the sector of weak interactions

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However, full CHPT calculations are problematic for this type of study:

- slow convergence of the chiral expansion
- too many free parameters in the sector of weak interactions

...but a full calculation is not necessary in this case !

- possible to perform a systematic expansion in powers of the a_I of the amplitudes which determine the coefficient of the singularity

We can use an *ad hoc* construction which maximize the available experimental info on $K \to 3\pi$ and use only:

Cabibbo & G.I., JHEP '05

- Unitarity & analyticity
- Smallness of the a_I
- Smallness of $v_{\pi\pi} = (s\text{-}s_0)^{1/2}$

$$T(s) = A(s) + B(s)\,(s\text{-}s_0)^{1/2}$$

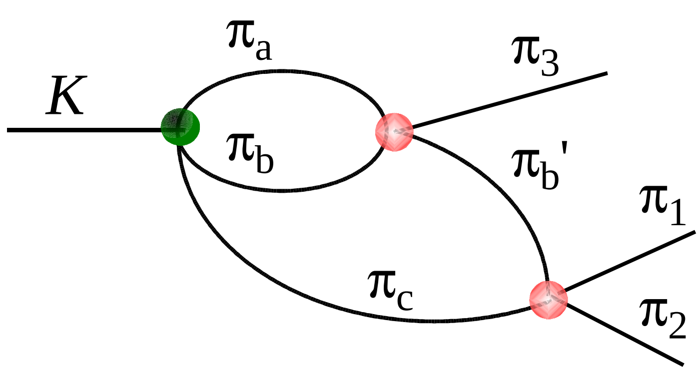
A(s) & B(s) regular around s_0

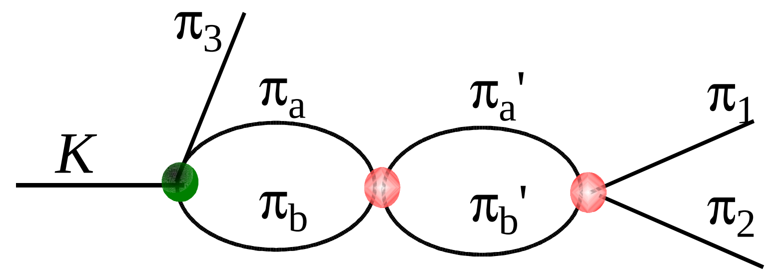
$$T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$$

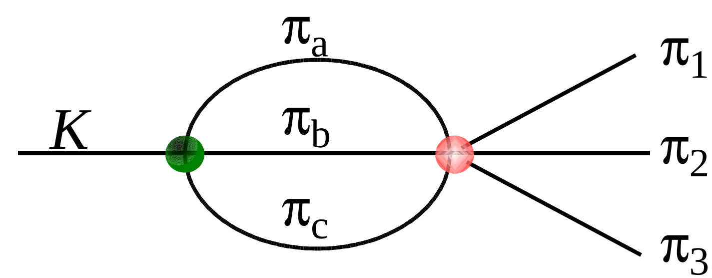
$\mathrm{Re}A(s) = O(1)$ exp. data
$\mathrm{Im}A(s) = O(a_I)$ one-loop

$\mathrm{Im}B(s) = O(a_I)$ one-loop
$\mathrm{Re}B(s) = O(a_I^2)$ two-loop

relevant 2-loop topologies:







Analysing the discontinuities of these diagrams, we determine - in powers of the a_I up to $O(a_I^2)$ - the coefficients of the $(s-s_0)^{1/2}$ terms in

$$|T(s)|^2 = [\,\mathrm{Re}A(s) - \mathrm{Im}\,B(s)\,(s_0-s)^{1/2}\,]^2 + [\,\mathrm{Im}A(s) + \mathrm{Re}\,B(s)\,(s_0-s)^{1/2}\,]^2 \qquad s < s_0$$

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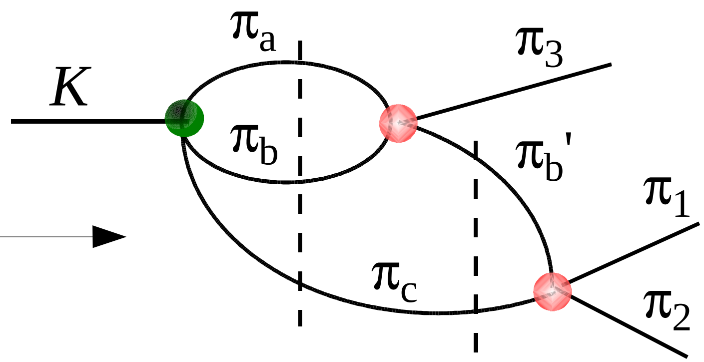

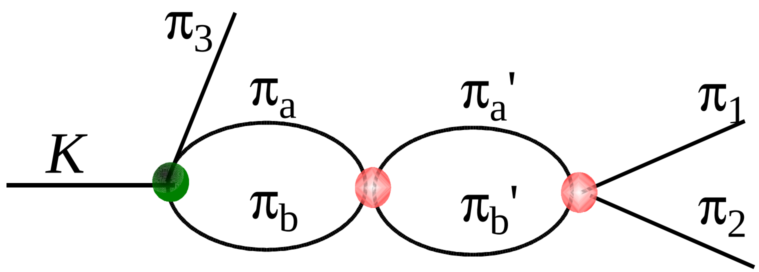

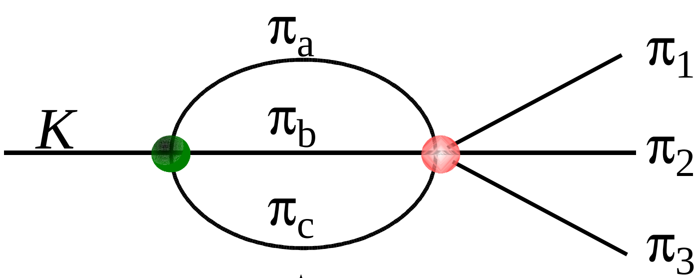

The largest effect is associated to this double cut (new phase due to pions with “large” relative velocity)

This diagram is the only one we cannot express in terms of the $a_I \Rightarrow$ evaluation by means of CHPT $\Rightarrow$ negligible contribution

$$T(s) = A(s) + B(s)\,(s-s_0)^{1/2}$$

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$\mathrm{Im}A(s) = O(a_I)$ one-loop

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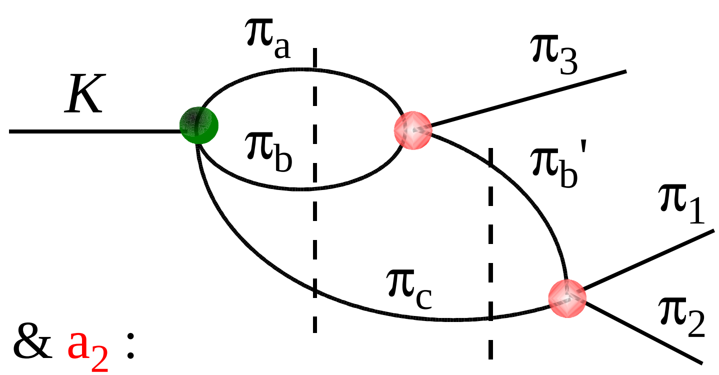


N.B.: at $O(a_I^2)$ the parameterization depends in a different way from (a_0-a_2) & a_2 :

$$\pi^0\pi^0 \to \pi^0\pi^0 \qquad \mathrm{Re}\,\mathcal{M}_{00} = \frac{8a_{00}m_{\pi^+}}{\pi} \qquad (\pi^+\pi^- \ \text{threshold}) \qquad a_{00} \xrightarrow{\text{I-spin}} \frac{a_0+2a_2}{3}$$

$$\pi^+\pi^0 \to \pi^+\pi^0 \qquad \mathrm{Re}\,\mathcal{M}_{+0} = \frac{8a_{+0}m_{\pi^+}}{\pi} \qquad (\pi^+\pi^0 \ \text{threshold}) \qquad a_{+0} \xrightarrow{\text{I-spin}} \frac{a_2}{2}$$

$$\pi^+\pi^- \to \pi^0\pi^0 \qquad \mathrm{Re}\,\mathcal{M}_{x} = \frac{8a_{x}m_{\pi^+}}{\pi} \qquad (\pi^+\pi^- \ \text{threshold} \qquad a_{x} \xrightarrow{\text{I-spin}} \frac{a_0-a_2}{3}$$

$$\pi^+\pi^- \to \pi^+\pi^- \qquad \mathrm{Re}\,\mathcal{M}_{+-} = \frac{8a_{+-}m_{\pi^+}}{\pi} \qquad (\pi^+\pi^- \ \text{threshold}) \qquad a_{+-} \xrightarrow{\text{I-spin}} \frac{2a_0+a_2}{6}$$

$$\pi^+\pi^+ \to \pi^+\pi^+ \qquad \mathrm{Re}\,\mathcal{M}_{++} = \frac{8a_{++}m_{\pi^+}}{\pi} \qquad (\pi^+\pi^+ \ \text{threshold}) \qquad a_{++} \xrightarrow{\text{I-spin}} a_2$$

possible to determine both combinations

What is still missing ?

- The present theoretical analysis has been done in an effective theory where m_{π^+}-m_{π^0} is the only source of I-spin breaking ⇒ we need to include in a systematic way other sources of I-spin breaking, most notably, e.m. corrections

What is still missing ?

- The present theoretical analysis has been done in an effective theory where m_{π^+}-m_{π^0} is the only source of I-spin breaking $\Rightarrow$ we need to include in a systematic way other sources of I-spin breaking, most notably, e.m. corrections
- On general grounds, e.m. effects are expected to produce small (few %) corrections, except very close to the $\pi^+\pi^-$ threshold, where they are responsible for the formation of a bound state: pionium [this is one of the reasons why a consistent inclusion of e.m. effect is a non-trivial task]

Probability of pionium formation:

$$\frac{\Gamma(K^+ \to \pi^+ + A_{\pi\pi})}{\Gamma(K^+ \to \pi^+\pi^+\pi^-)} \approx 7.4 \times 10^{-6}$$

Silagadze, '94

Life-time of the pionium:

$$\Gamma(A_{\pi\pi}) \approx 2 \times 10^{-7}\ \mathrm{MeV}$$

$$[\tau(A_{\pi\pi}) \approx 3 \times 10^{-15}\ \mathrm{s}]$$

Deser *et al.*, '54
:
Gasser *et al.*, '01

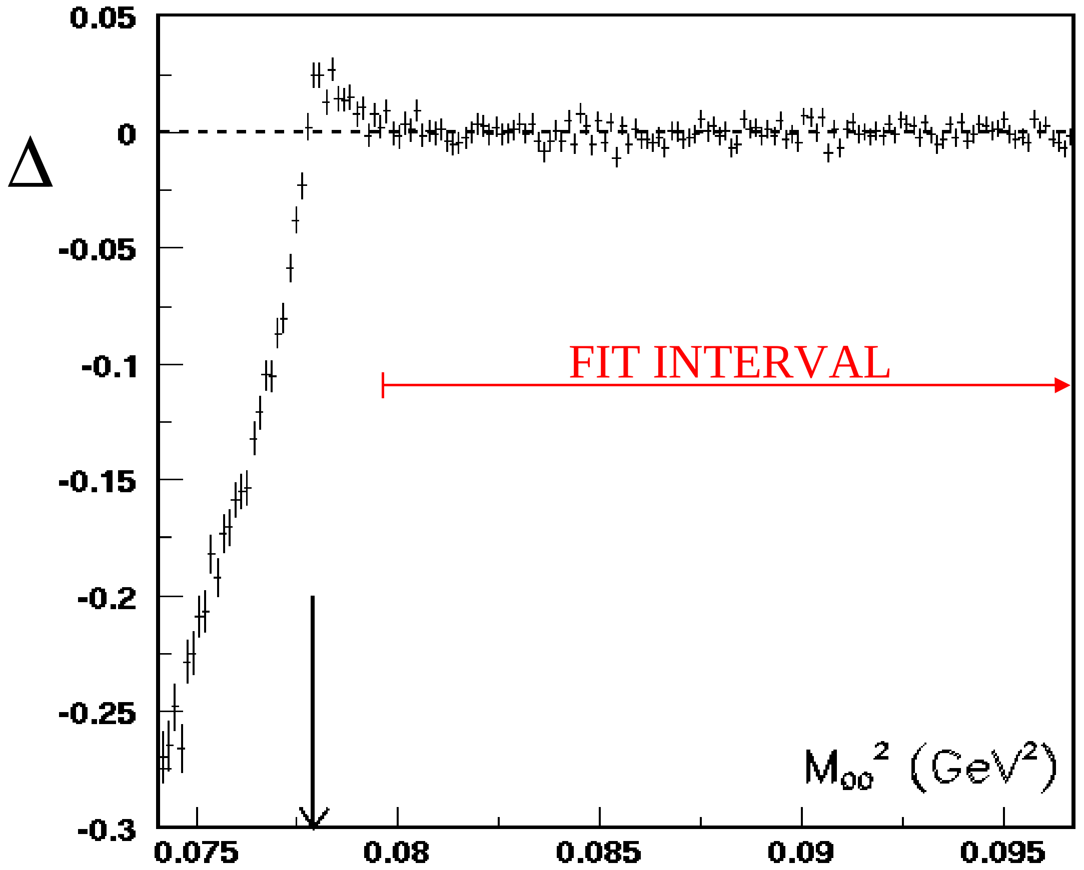
Conservative estimate of a 5% theoretical error in the extraction of the a_I using the present $O(e^0 a_I^2)$ parameterization

on-going activity by the Bern group to reduce this errror

Preliminary NA48 results on a_0 - a_2 [*courtesy of Luigi di Lella*]

$$\Delta \equiv \textbf{(data – fit) /data versus } M_{oo}^2$$

Without the re-scattering effect....



χ^2 = 13574 / 148 **d.o.f.** ⇒ 120 / 110 **d.o.f.**

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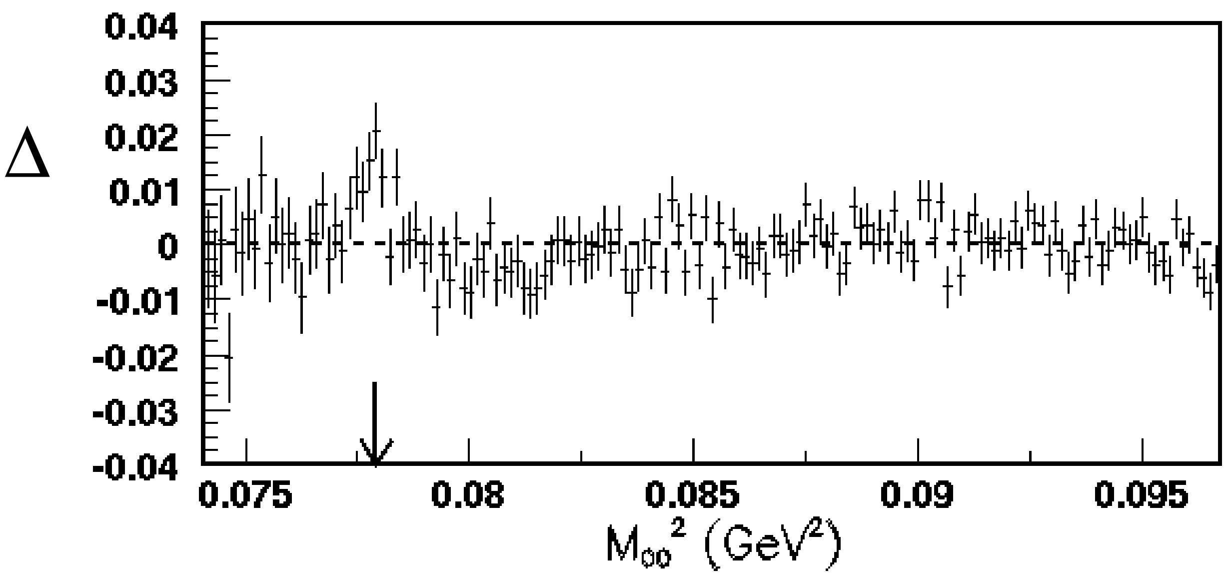
One-loop parameterization [Cabibbo, P.R.L. '04]

One additional parameter with respect to the PDG fit: $(a_0\text{-}a_2)m_\pi$



$\chi^2 = 217$ / 147 **d.o.f.**

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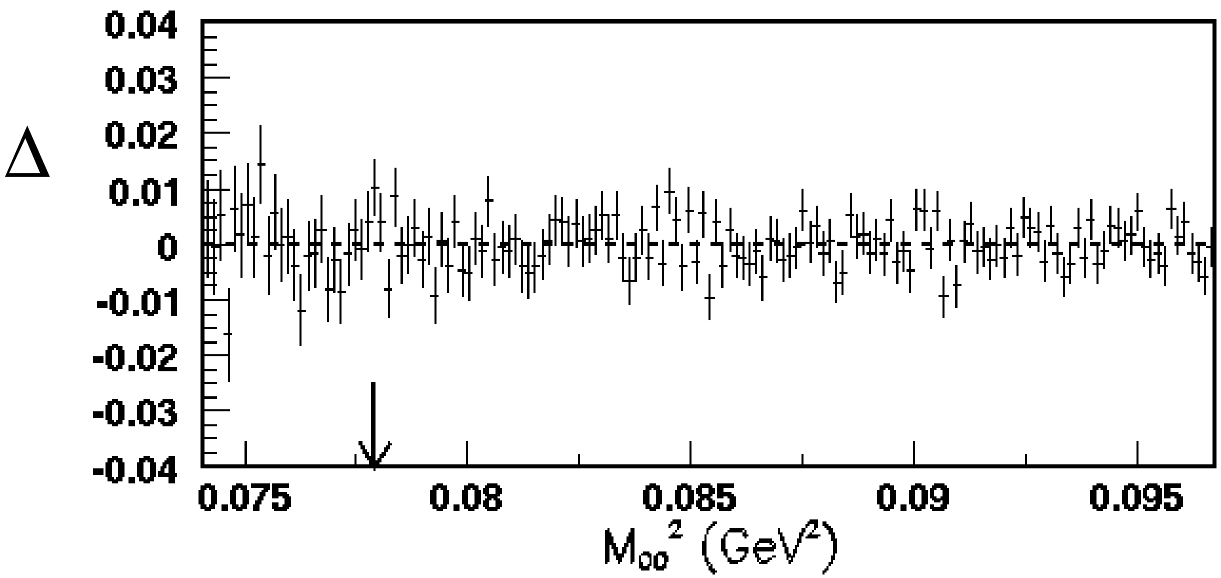

χ^2 = 156 / 146 **d.o.f.**

- *Preliminary NA48 results* on a_0 - a_2 [*courtesy of Luigi di Lella*]

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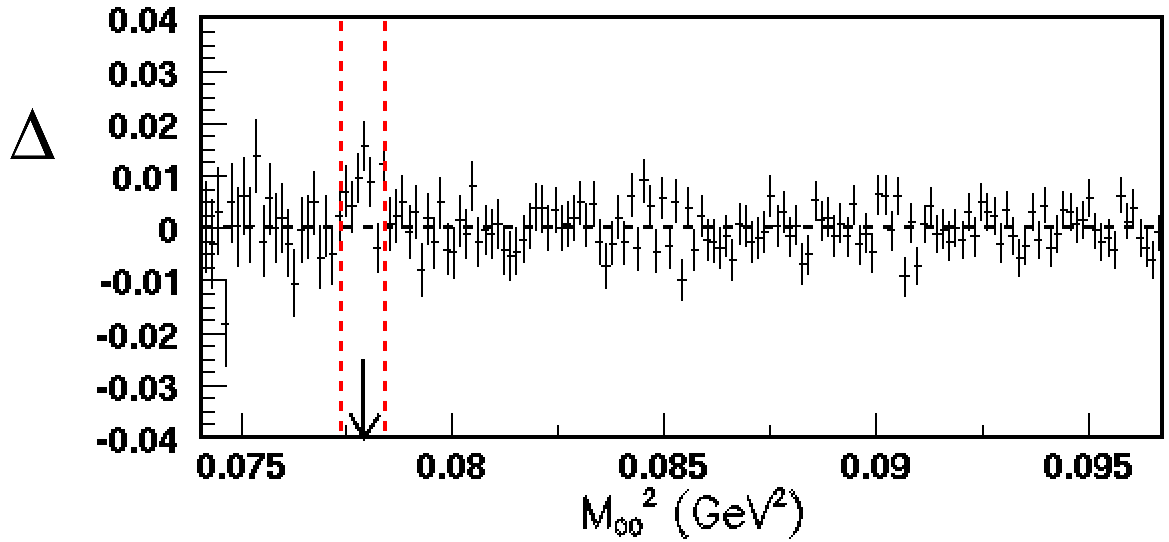
Fit excluding 7 bins centered around $M_{00} = 2m_{\pi^+}$



Excess of events in excluded bins $\Rightarrow$ evidence for pionium

Statistical significance $\sim 2.5\,\sigma$

[**agreement with th. expecttions**]

$\chi^2 = 141$ / 139 **d.o.f.** $\Rightarrow$ $(a_0\text{-}a_2)m_\pi = 0.270 \pm 0.009_{stat}$

...to be compared with the th. expectation [standard vacuum]: $(a_0\text{-}a_2)\, m_\pi = 0.265 \pm 0.004$

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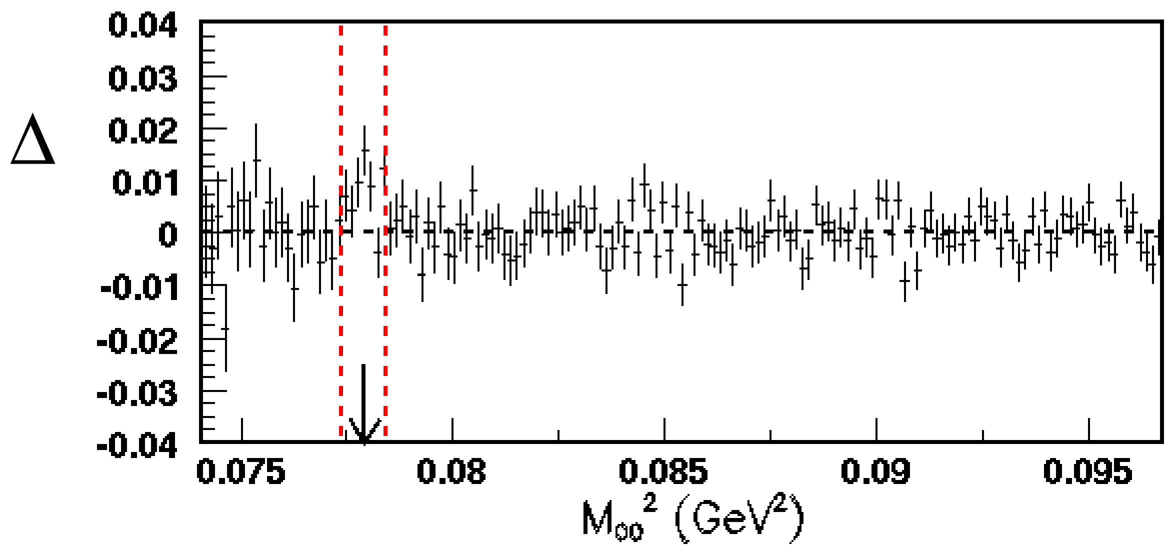

Substantial amount of work still needed to decrease the th. error...

...but these preliminary results are extremely encouraging !

In a few years this method could allow a very precise determination of S-wave $\pi\pi$ scattering lengths

$\chi^2 = 141 / 139$ **d.o.f.** $\Rightarrow$ $(a_0\text{-}a_2)m_\pi = 0.270 \pm 0.009_{stat} \pm 0.014_{th}$

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Rare K decays [*hopefully the future of kaon physics ...*]

The information coming form rare FCNC K decays
and in particular from the following four *golden* modes:

$K^+ \to \pi^+ \nu\nu$ $K_L \to \pi^0 \nu\nu$ $K_L \to \pi^0 e^+e^-$ $K_L \to \pi^0 \mu^+\mu^-$

is a key element to investigate the
flavour structure of *physics beyond the SM*

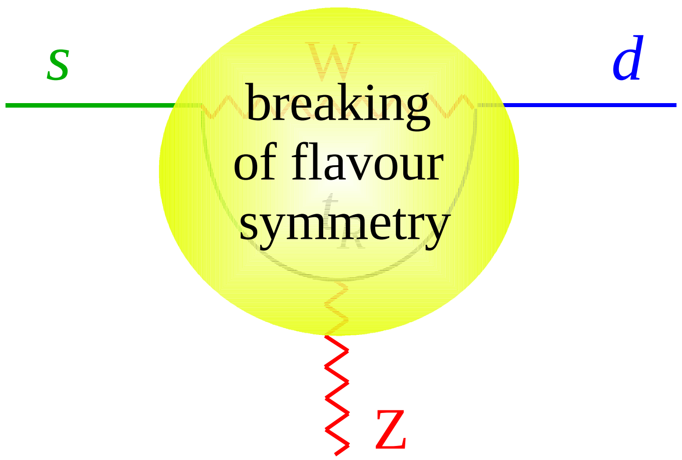
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm gauge}(A_i, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi_i, A_i, \psi_i ; Y, v) + \sum_{d\geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^{\ d}(\phi_i, A_i, \psi_i)$$

$\mathcal{L}_{SM}$ = renormalizable part of a more general $\mathcal{L}_{eff}$

A brief reminder about the interest of rare K decays mediated by Flavor Changing Neutral Currents:

- No SM tree-level contribution
- Strong suppression within the SM because of CKM hierarchy

$$s \rightarrow d + l^+l^-, \nu\nu$$



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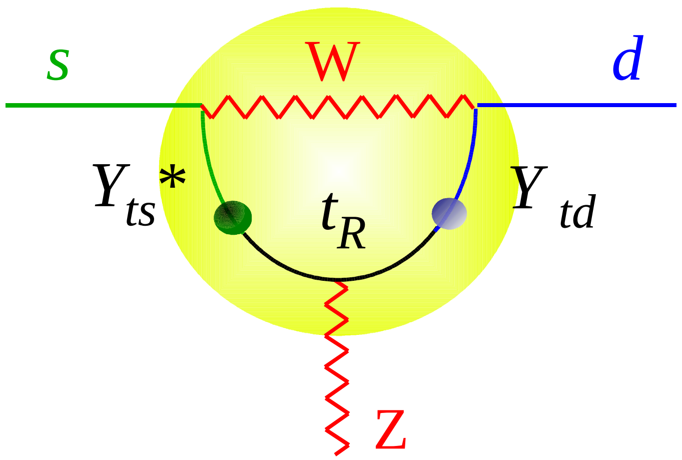

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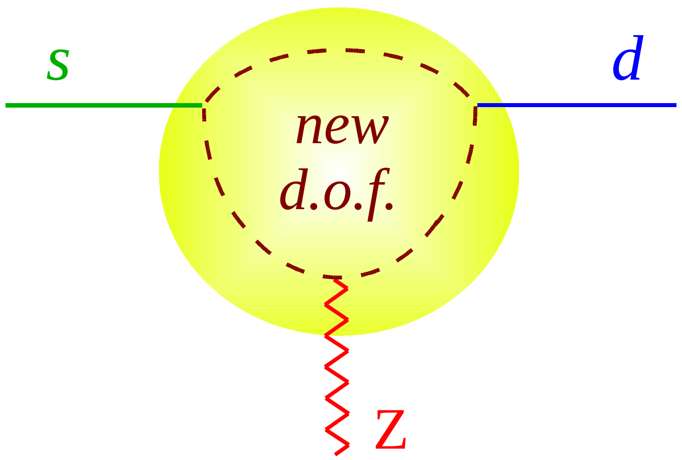

Strong sensitivity to the *flavour structure* of *physics beyond the SM*

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⬇

Strong sensitivity to the *flavour structure* of *physics beyond the SM*

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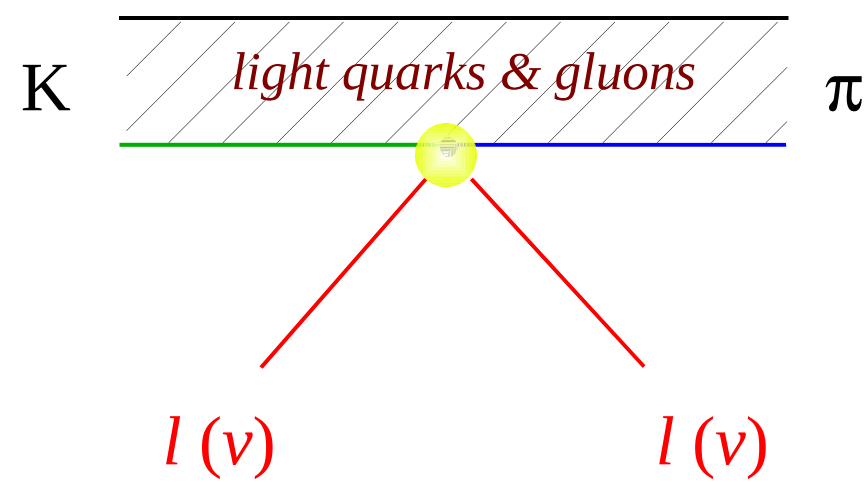
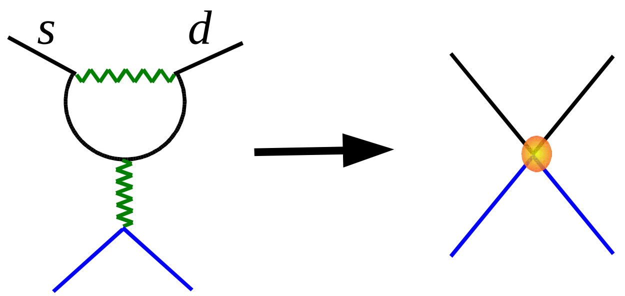
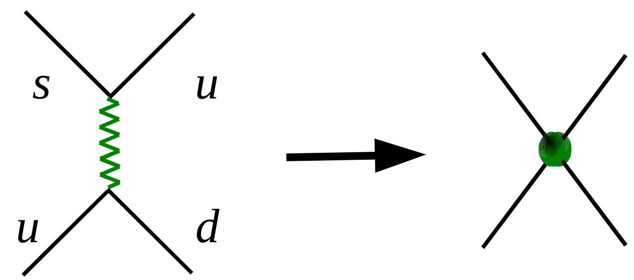


In this perspective, very important to understand at which level of precision the low-energy hadronic amplitudes are determined by dynamics around or above the electroweak scale [*short distance* dynamics] $\Rightarrow$ help from Lattice & CHPT...!

The most efficient tool to separate the various scales of the problem is the construction of an effective Hamiltonian integrating out the heavy d.o.f

$$\mathcal{H}_{eff} = \Sigma_i\, C_i(\mu)\, Q_i$$





FCNC operators:

$$Q_\nu = (s\,d)_{V-A}(\nu\,\nu)_{V-A}$$

$$Q_{9V} = (s\,d)_{V-A}(l\,l)_V$$

$$Q_{10A} = (s\,d)_{V-A}(l\,l)_A$$

Four-quark operators:

$$Q_1 = (s\,d)_{V-A}(u\,u)_{V-A}$$

$$Q_2 = (s\,u)_{V-A}(u\,d)_{V-A}$$

$$\vdots$$

The interesting short-distance info is encoded in the $C_i(M_W)$ (*initial conditions*) of the FCNC operators

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FCNC operators:

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$$\mathcal{H}_{eff} = \Sigma_i\, C_i(M_W)\, Q_i$$
$$\downarrow$$
$$\mathcal{H}_{eff} = \Sigma_i\, C_i(\mu\sim 1\ \mathrm{GeV})\, Q_i$$

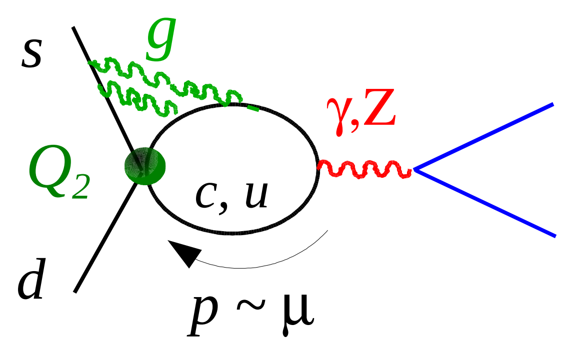
Four-quark operators:

$$Q_1 = (s\,d)_{V-A}(u\,u)_{V-A}$$
$$Q_2 = (s\,u)_{V-A}(u\,d)_{V-A}$$
$$\vdots$$

Two main sources of long-distance effects
[*dilution of the interesting short-distance info*]:

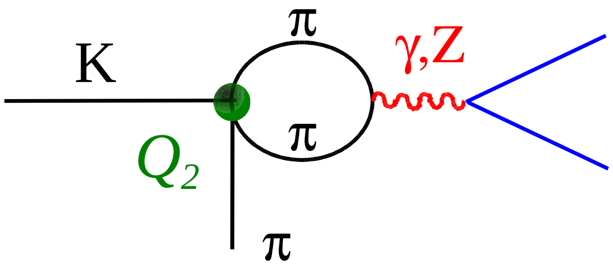
- Mixing of the four-quark Q_i into the FCNC Q_i [perturbative RGE running down to $\mu \sim m_c$]

e.g.:

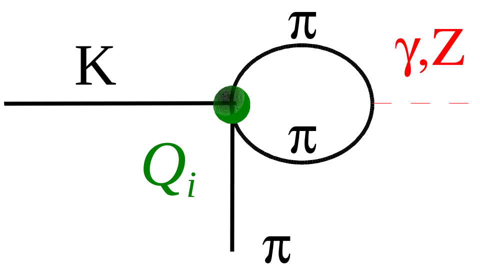


- Non-perturbative matrix elements of the four-quark operators [hadronic level]

e.g.:



Most serious difficulties associated to the non-perturbative part:



$$i \int d^4x\, e^{iqx} \langle \pi^i(p) | T\{ J^{\mu}_{V,A}(x) H_{\Delta S=1}(0) \} | K^i(k) \rangle$$

Wide literature: Ecker, Pich & de Rafael '87

⋮

recent renewed interest:

Buchalla, D'Ambrosio, G.I. '03

Friot, Grenat, de Rafael '04

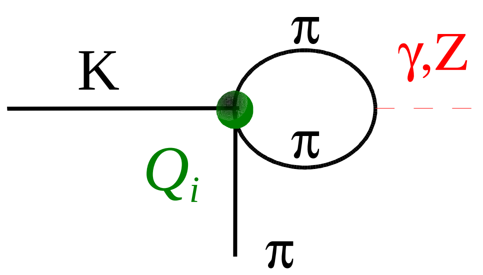
G.I., Mescia, Smith '05

G.I., Martinelli, Turchetti '05

Well-known CHPT expansion
for the vecotr current
non-trivial LECs starting at $O(G_F\, e\, p^4)$

Less-trivial CHPT expansion
for the axal current
non-trivial LECs starting at $O(G_F^2 p^2)$

Most serious difficulties associated to the non-perturbative part:



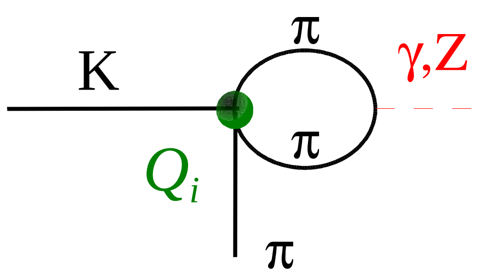
$$\longleftrightarrow \quad i\int d^4x\, e^{iqx} \langle \pi^i(p)| T\{ J^\mu_{V,A}(x) H_{\Delta S=1}(0)\} |K^i(k)\rangle$$

Key phenomenological observation:

because of different CP and electroweak structures, these long-distance effects play a very different (numerical) role in different processes:

- CPC one-γ processes [$K^+ \to \pi^+ l^+ l^-$ $\quad K_S \to \pi^0 l^+ l^-$] $\Rightarrow$ 100% long-distance
- CPV one-γ processes [$K_L \to \pi^0 l^+ l^-$] $\Rightarrow$ ~50% long-distance
- CPC one-Z processes [$K^+ \to \pi^+ \nu\nu$ $\quad K_S \to \pi^0 \nu\nu$] $\Rightarrow$ ~5% long-distance

Most serious difficulties associated to the non-perturbative part:



$\longleftrightarrow$

$$i \int d^4x\, e^{iqx} \langle \pi^i(p) | T\{ J^\mu_{V,A}(x) H_{\Delta S=1}(0) \} | K^i(k) \rangle$$

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both experimentally observed
[neutral mode: NA48/1 '03]

↓

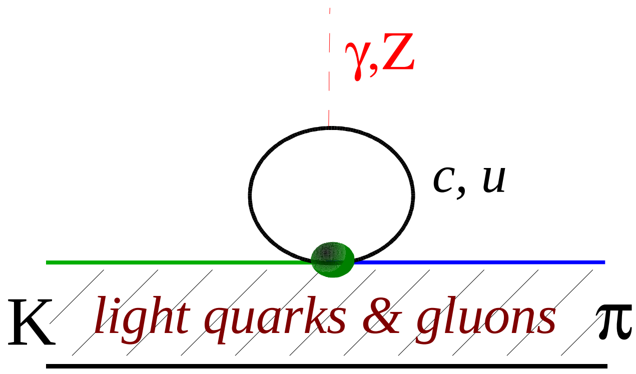
complete picture
-within CHPT-
of the vector couplings

Still missing:

Model-independent infos about the (short-distance) sign of the vector CT

Model-independent infos about sign & magnitude for the axial CT

LATTICE !



$$i\int d^4x\, e^{iqx}\langle \pi^i(p)|T\{J^\mu_{V,A}(x)H_{\Delta S=1}(0)\}|K^i(k)\rangle$$

This Green function could be studied [at least in principle...] on the Lattice with present techniques - even with Wislon-type actions :

- physical cuts avoided for $q^2 < m_\pi^2$ [sufficient to fix the LECs]
- no $x \to 0$ singularities in the vector case because of gauge invariance
- $x \to 0$ singularities in the axial case canceled by GIM + twisted mass
- power-like divergences due to mixing of $Q_i(\Delta S=1)$ with lowest-dim operators eliminated by appropriate spectral analysis

G.I., Martinelli, Turchetti '05

Summary of the *irreducible theoretical uncertainties* on rare K decays:

	short-distance (e.w.) contrib. to the total rate $(\Gamma - \Gamma_{\text{no s.d.}}) / \Gamma$	present irreducible th. error on the s.d. amplitude extracted from BR only	total BR withinSM (central value)
$K_L \to \pi^0 \nu\nu$	$> 99\%$	1%	3×10^{-11}
$K^+ \to \pi^+ \nu\nu$	88%	4%	8×10^{-11}
$K_L \to \pi^0 e^+e^-$	38%	15%	3.5×10^{-11}
$K_L \to \pi^0 \mu^+\mu^-$	28%	30%	1.5×10^{-11}

↑

Lattice studies of the corresponding non-perturbative effects would provide an important help to validate these error and -possibly- to further reduce them

Conclusions

Kaon physics is fantastic laboratory for particle physics, not only to confronts Lattice and CHPT but, more in general, to confronts with experiments our understanding of fundamental interactions...