Universitat de València

Correccions Quàntiques en la Teoria Quiral de Ressonàncies

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A Rut. I a Pau, és clar.

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Introducció

Teories Efectives: la lupa dels físics teòrics

A qui se li acudiria buscar un carrer a València amb un mapa galàctic? O a l'inrevés, qui voldria comprovar la posició d'una galàxia mirant el plànol d'una ciutat? Aquest és, en definitiva, l'arrel de les teories efectives de camps: la dinàmica a escales de distàncies grans no depén dels detalls de la dinàmica a escales de distàncies petites; és a dir –tot considerant un exemple més físic– no té sentit tenir en compte el moviment de la Lluna al voltant de la Terra quan s'estudia el moviment de la nostra galàxia.

En realitat, la idea de les teories efectives de camps sempre ha estat implícita a l'hora d'estudiar els fenòmens de la natura. Hom considera els graus de llibertat adequats al sistema que s'està estudiant.

Com a exemple il·lustratiu, es pot estudiar la física de l'àtom. Una anàlisi que considere l'Electrodinàmica Quàntica (QED) no és la millor opció, és a dir, utilitzar els quarks i els leptons com a graus de llibertat no sembla ser massa útil. Un enfocament més convenient empraria electrons no relativistes que orbiten al voltant d'un nucli. Com a primera aproximació, hom podria considerar un nucli amb massa infinita: solament la massa de l'electró i la constant d'estructura fina caldrien per descriure el sistema. Si es desitjara una precisió major, s'hauria de considerar la massa finita del protó, si encara se'n vol més, l'spin i el moment magnètic... i així successivament. El punt principal és que s'haja triat la teoria efectiva adient.

Partint d'aquest plantejament es dedueix que una primera qüestió, i molt sovint no trivial, és saber triar els graus de llibertat adequats a l'escala que s'està considerant. Per tant, les teories efectives constitueixen les eines teòriques més convenients per descriure la dinàmica a baixes energies, on el terme 'baixes' es refereix a una determinada escala Λ. Com s'explica al llarg de la tesi, només es consideren els graus de llibertat rellevants, i.e. aquells estats amb $m \ll \Lambda$, mentre que les excitacions més pesades amb $M \gg \Lambda$ han estat integrades fora de l'acció. S'han d'emprar les interaccions necessàries entre els estats lleugers, que poden organitzar-se com una expansió en potències de l'energia sobre l'escala Λ.

Un tret important per classificar les teories efectives de camps és la intensitat de la teoria fonamental en la regió d'energies que s'estudia, és a dir, es distingeix el cas en què la teoria a altes energies està feblement o fortament acoblada. En el primer cas el valor dels acoblaments efectius pot obtenir-se pertorbativament en termes de la teoria subjacent. Tanmateix, en el segon cas hom no pot fer càlculs pertorbatius, així que aquesta possibilitat no està a l'abast en els models realistes. L'enfocament efectiu que farem en aquesta tesi és del segon tipus: solament certs lligams provinents de la teoria subjacent podran emprar-se, però el valor dels acoblaments no podrà obtenir-se teòricament de manera directa.

S'ha dit que la dinàmica a escales de distàncies grans no depén dels detalls de la dinàmica a escales de distàncies petites. Aquesta afirmació s'hauria d'aclarir: l'únic efecte de la teoria a altes energies consisteix a fixar el valor dels acoblaments a baixes energies, a banda de proveir les simetries que hauran de ser tingudes en compte en estudiar la física a grans distàncies.

La Teoria de Pertorbacions Quirals

El corriment d'α_s

Amb les evidències experimentals i teòriques actualment a l'abast hom accepta que la teoria gauge $SU(3)_C$, la Cromodinàmica Quàntica (QCD), descriu adequadament les interaccions hadròniques. Com que l'interés d'aquesta tesi se centra en QCD a energies entre la massa de la ρ i 2 GeV, potser siga convenient començar analitzant el comportament de la interacció forta a baixes energies.

QCD descriu la interacció forta entre quarks i gluons mitjançant una teoria gauge no abeliana $SU(3)_C$. Abans de fer càlculs pertorbatius a baixes o altes energies, s'ha d'analitzar el corriment de l'acoblament fort α_s, per poder confirmar si és possible considerar l'acoblament com una quantitat petita. És a dir, s'ha de renormalitzar la teoria i, en el cas d'utilitzar regularització dimensional, estudiar la dependència de α_s en l'escala μ. Assumint que l'acoblament fort és suficientment petit per fer càlculs pertorbatius, es troba que

$$\alpha_s(q^2) = \frac{12\pi}{(11N_C - 2n_f)\log(q^2/\Lambda_{QCD}^2)}\,, \tag{1}$$

amb N_C el nombre de colors i n_f el nombre de sabors. Es veu que l'evolució de l'acoblament amb l'escala sols depén d'un paràmetre Λ_{QCD}, que s'anomena *l'escala de QCD*. L'Eq. (1) permet comprovar la llibertat asimptòtica de QCD, és a dir, el corriment fa xicotet l'acoblament fort a altes energies, a diferència amb el que ocorre en QED. Si en el cas de QED aquest resultat s'interpreta com un apantallament de la càrrega degut a la presència de parells virtuals electró-positró, es pensa en un efecte d'anti-apantallament en QCD, que és conseqüència de la natura no abeliana de les interaccions gluòniques.

Encara que l'Eq. (1) solament és vàlida en la regió en què α_s és xicoteta, perquè ha estat obtinguda mitjançant un càlcul pertorbatiu, s'espera un creixement de l'acoblament a baixes energies, que condueix al confinament de QCD: els estats asimptòtics de QCD no poden ser els quarks. De fet, la fenomenologia suporta aquesta idea: pot suposar-se el confinament de quarks i gluons en hadrons.

Com a conseqüència d'això no podem treballar pertorbativament a baixes energies emprant el lagrangià de QCD. Un enfocament que utilitze teories efectives de camps sembla ser la manera adequada de treballar amb QCD a grans distàncies.

La simetria quiral

En absència de la massa dels quarks el lagrangià de QCD és invariant sota transformacions globals $G \equiv SU(n_f)_L \otimes SU(n_f)_R$, independentment per als components quirals dretà i esquerrà dels quarks.

Les simetries globals tenen influència en l'espectre, mentre que les locals determinen la interacció. Per això, la simetria global quiral, que hauria de ser una bona simetria en el sector dels quarks lleugers ($n_f = 3$), hauria de tenir implicacions en l'espectre hadrònic. Tanmateix, això no vol dir que s'haja d'observar necessàriament en l'espectre, ja que les simetries sempre tenen dues maneres d'implementar-se: o són manifestes –i donen lloc a una classificació en l'espectre– o es produeix un trencament espontani de la simetria, amb la conseqüent generació dels bosons de Goldstone [5].

El següent pas és anar, doncs, a la fenomenologia. Encara que els hadrons poden ser classificats amb representacions $SU(3)_V$, no es troben multiplets degenerats amb paritat oposada. De més a més, l'octet de pseudoescalars més lleuger ho és molt més que la resta dels estats hadrònics. Tot això condueix a considerar un trencament espontani $SU(3)_L \otimes SU(3)_R$ a $SU(3)_V$. Com que hi ha $n_f^2 - 1 = 8$ generadors axials trencats en el grup quiral, s'haurien d'observar vuit estats hadrònics amb $J^P = 0^-$ (π^+, π^-, π^0, η, K^+, K^-, K^0, $\overline{K}^0$). Les seues masses xicotetes són generades per la matriu de masses dels quarks, que explícitament trenca la simetria quiral. Tenint en compte aquest trencament explícit de la simetria, nosaltres anomenarem el multiplet dels pions *pseudo-bosons de Goldstone*.

El formalisme general per construir lagrangians efectius amb trencament espontani de simetria fou proposat per Callan, Coleman, Wess and Zumino [7]. En el cas de QCD a baixes energies és possible utilitzar aquestes idees per construir una teoria efectiva que descriga la interacció entre els pseudo-bosons de Goldstone. Com que hi ha un espai gran de separació entre la massa d'aquest multiplet i la resta d'hadrons, hom pot imaginar fàcilment una teoria efectiva de camps que continga sols aquests modes. Seguint aquestes idees, la Teoria de Pertorbacions Quirals (χPT) és la teoria efectiva de QCD per a molt baixes energies [8, 9, 10]. El seu éxit ha estat comprovat fenomenològicament [11].

Aquest marc es desenvolupa com una expansió pertorbativa en moments i masses dels pseudo-bosons de Goldstone. Llavors, el lagrangià pot ser organitzat en termes creixents en potències de masses o moments,

$$\mathcal{L}_{\chi PT} = \sum_{n=1} \mathcal{L}_{2n}^{\chi PT} , \qquad (2)$$

on els subíndexs indiquen el nombre de derivades o masses.

Encara que χPT no és renormalitzable *stricto sensu*, se segueix un procediment pertorbatiu de renormalització on les divergències generades amb els diagrames amb bucles són reabsorvides amb un nombre finit de nous operadors. Aquest mètode està ben definit ordre a ordre en l'expansió quiral. Així, per exemple, les divergències generades amb bucles construïts amb el lagrangià dominant $\mathcal{L}_2^{\chi PT}$ són reabsorvides mitjançant la renormalització dels acoblaments de $\mathcal{L}_4^{\chi PT}$ [9].

La Teoria Quiral de Ressonàncies

Millorant els lagrangians fenomenològics *à la* Weinberg

Un cop s'accepta que l'estudi de la dinàmica hadrònica està dificultat a baixes energies per la nostra incapacitat d'implementar QCD d'una manera no pertorbativa en aquests processos, es necessiten noves maneres de treballar amb QCD en aquest règim d'energies. Com ha estat argumentat anteriorment, les teories efectives de camps són una eina adequada per aquest fi [1, 2]. L'èxit de diferents teories efectives de QCD per a diferents règims, com ara la Teoria de Pertorbacions Quirals (χPT), la Teoria Efectiva del Quark Pesat (HQET) o la Cromodinàmica Quàntica No Relativista (NRQCD), és una bona prova d'això.

Tot i això, en la regió d'energies en què estem interessats, $M_\rho \lesssim E \lesssim 2$ GeV, la situació és més complicada. Encara que la simetria quiral encara dóna molts lligams dinàmics, l'habitual contatge de χPT es trenca amb la presència d'escales d'energia majors. A més a més, aquest règim està poblat de moltes ressonàncies i l'absència d'una separació entre les masses en l'espectre fa difícil un enfocament mitjançant teories efectives de camps, ja que no és clar quins graus de llibertat han estat integrats i, en qualsevol cas, des de quin llindar d'energies s'estan integrant els modes pesats. En tot cas, molts dels principals trets de les teories efectives de camps seran molt útils per portar a terme el nostre objectiu.

Els principals ingredients del nostre esquema són els següents:

1. Hauríem de començar l'enfocament mitjançant els lagrangians fenomenològics proposats per Weinberg en la Ref. [8]. Ell suggerí construir el lagrangià més general possible, incloent tots els termes consistents amb els principis de simetria assumits, esperant que els càlculs dels elements de matriu donaren la matriu S més general que respectara analiticitat, unitarietat, descomposició en grups i les simetries. En el cas de QCD a baixes energies, un dels trets fonamentals seria la introducció dels estats lligats, els hadrons, com a graus de llibertat. Vegeu que la tria dels graus de llibertat és un punt fonamental per construir els lagrangians efectius.

   És important assenyalar que en aquests lagrangians generals, que anomenarem *lagrangians fenomenològics à la Weinberg*, no s'ha fet ús de cap informació dinàmica més enllà de principis generals. Aquest fet ens permet emprar informació addicional de la teoria subjacent de les interaccions fortes per millorar la nostra descripció.

2. QCD a gran N_C [14, 15] proporciona un bon escenari on treballar. El límit d'un nombre infinit de colors de quarks resulta ser un instrument molt útil per entendre moltes característiques de QCD i subministra un contatge per descriure les interaccions entre mesons. Assumint confinament, el límit $N_C \to \infty$ restringeix enormement la dinàmica dels mesons afirmant que les funcions de Green de la teoria vénen descrites per diagrames a ordre arbre d'un lagrangià efectiu amb vèrtexs locals i mesons com a graus de llibertat; les correccions en $1/N_C$ es calculen mitjançant bucles obtinguts amb el mateix lagrangià efectiu. L'expansió en $1/N_C$ dóna un bon esquema quantitatiu d'aproximació a la dinàmica hadrònica [16].

3. L'altre progrés en l'enfocament fenomenològic és dut a terme mitjançant les propietats de QCD a curtes distàncies. La majoria d'aquests lligams s'obtenen a partir de fer l'empalmament de les funcions de Green dels corrents de QCD avaluades amb la teoria efectiva amb aquells obtinguts utilitzant l'expansió pertorbativa OPE. L'altra font de lligams sorgeix de l'anàlisi dels factors de forma.

En resum, tenint en compte les dificultats per utilitzar un mètode formal amb teories efectives de camps en la regió de les ressonàncies, es treballarà mitjançant un enfoncament basat en les idees dels lagrangians fenomenològics explicats en la Ref. [8]. Aquest acostament pot desenvolupar-se emprant l'expansió en $1/N_C$ i els lligams obtinguts a partir del comportament a altes energies de QCD. Amb tot això s'han avançat les principals claus que destaquen en la Teoria Quiral de Ressonàncies (RχT) [17, 18, 19], el marc proposat en aquesta tesi per tractar amb la Cromodinàmica Quàntica a energies intermèdies, $M_\rho \lesssim E \lesssim 2$ GeV.

L'expansió en $1/N_C$

Treballar amb QCD a energies intermèdies seria més fàcil utilitzant un paràmetre d'expansió. Com s'ha explicat anteriorment, l'acoblament fort α_s no pot ser una solució, tenint en compte les equacions del grup de renormalització. En la regió de ressonàncies es trenca l'usual contatge quiral. Per tant, la teoria gauge $SU(3)_C$ amb quarks lleugers no té un paràmetre d'expansió obvi.

't Hooft suggerí que hom podria generalitzar QCD i emprar un grup gauge $SU(N_C)$ [14]. L'esperança era poder resoldre la teoria en el límit en què N_C és molt gran, i que el valor físic $N_C = 3$ fóra qualitativament i quantitativament pròxim al valor obtingut en aquest límit.

QCD se simplifica quan N_C és gran, i dóna lloc a una expansió sistemàtica en potències d'$1/N_C$. Triant que l'acoblament α_s és d'$\mathcal{O}(1/N_C)$, és a dir, considerant que $\alpha_s N_C$ és una constant en el límit del gran nombre de colors, i suposant confinament, els principals resultats són els següents:

1. En el límit $N_C \to \infty$ els mesons i els estats gluònics són lliures, estables i no interaccionen entre ells. Les masses dels mesons tenen límits suaus i el nombre d'estats mesònics és infinit.

2. Els decaïments dels mesons són d'$\mathcal{O}(1/\sqrt{N_C})$, mentre que les amplituds de col·lisió mesó-mesó són d'$\mathcal{O}(1/N_C)$.

3. A primer ordre en l'expansió $1/N_C$ la dinàmica dels mesons està descrita per la suma dels diagrames a ordre arbre d'un lagrangià efectiu local que inclou mesons físics en lloc de quarks i gluons. Açò convida a pensar ràpidament en el bon enfocament dels lagrangians fenomenològics proposats anteriorment.

4. En el límit $N_C \to \infty$ la regla de Zweig és exacta i, per tant, els mesons es classifiquen en nonets de sabors $U(3)$. Ha desaparegut en aquest límit l'anomalia axial i s'ha restablert així la simetria $U(3)_L \otimes U(3)_R$.

5. Sota hipòtesis raonables es pot provar que el grup de simetria de QCD en el límit $N_C \to \infty$, $U(n_f)_R \otimes U(n_f)_L$ es trenca espontàniament a $U(n_f)_V$ [20].

6. Els mesons són estats $q\overline{q}$ purs, és a dir, la física hadrònica del mar de parells $q\overline{q}$ està suprimida.

Aquests resultats poden llegir-se de dues maneres. Hom podria dir que s'ha utilitzat l'expansió en $1/N_C$ per explicar alguns fets experimentals de la interacció forta. La possibilitat que nosaltres preferim és dir que aquestes característiques quantitatives són una prova que QCD a gran N_C és una bona aproximació a la natura [15]. Vegeu que afirmar que l'expansió en $1/N_C$ és una bona aproximació al món real és quelcom important des d'un punt de vista teòric, ja que aquesta expansió pot ser emprada com a paràmetre d'expansió en dur a terme el nostre enfocament efectiu.

Construint el lagrangià de la Teoria Quiral de Ressonàncies

L'objectiu és treballar amb QCD en la regió de les ressonàncies, $M_\rho \lesssim E \lesssim 2$ GeV, tot utilitzant els lagrangians fenomenològics *à la* Weinberg explicats adés, que seran implementats mitjantçant l'expansió en $1/N_C$. Hom ha de considerar el lagrangià més general possible, és a dir, el que inclou tots els termes consistents amb les simetries, utilitzant els hadrons com a graus de llibertat. Per a la construcció de la nostra teoria efectiva s'han de tenir en compte diferents assumptes:

1. Per tal de poder recuperar a molt baixes energies els resultats de χPT, considerar la simetria quiral és la millor opció. Tenint en compte els resultats de QCD a gran N_C, els mesons són ordenats en multiplets $U(3)$ i solament es consideren operadors amb una traça en l'espai de sabor [21, 23].

2. És un fet ben conegut que per donar-li sentit a qualsevol descripció efectiva cal empalmar els resultats amb els de la teoria subjacent (QCD en aquest cas). Cal ressaltar que el comportament asimptòtic de QCD es troba ja a energies $E \sim 2$ GeV. Aleshores RχT hauria de recuperar el comportament a altes energies de QCD. Aquesta exigència exclou interaccions amb moltes derivades, ja que aquestes tendeixen a violar el comportament asimptòtic guiat per QCD.

Això permet entendre l'éxit fenomenològic de les aproximacions habituals, on solament es consideren tensors quirals fins $\mathcal{O}(p^2)$. A més, aquest empalmament provoca alguns lligams entre els acoblaments efectius del lagrangià, reduint-se per tant el nombre de paràmetres desconeguts.

3. Encara que QCD a gran N_C és un instrument robust per dur a terme QCD a energies intermèdies, calen algunes aproximacions per poder construir el lagrangià efectiu. Com que el nombre de mesons és infinit en aquest límit, l'aproximació més comú és el tall en el nombre de ressonàncies, considerant solament els estats més lleugers. Se suposa que aquesta és una aproximació bona perquè les contribucions dels estats més pesats estan suprimides per llurs masses. La fenomenologia suporta aquesta aproximació.

   En la Ref. [17] solament es consideraren les contribucions de les ressonàncies més lleugeres amb cadascún dels nombres quàntics, la famosa aproximació d'una sola ressonància. Així mateix la nostra anàlisi serà feta sota aquesta aproximació.

   Com que el nombre de funcions de Green és infinit, és clar que no totes les condicions d'empalmament poden satisfer-se amb un nombre finit de ressonàncies: certes incerteses degudes al tall de l'espectre són introduïdes en la determinació dels paràmetres. De fet, es pot arribar a certes inconsistències en les relacions entre els acoblaments efectius. L'aproximació hadrònica mínima generalitza l'aproximació d'una sola ressonància, de tal manera que s'inclou el mínim nombre de ressonàncies que permet recuperar el comportament de QCD a altes energies en l'amplitud considerada [24]. Encara que aquesta opció és una aproximació de QCD, està ben suportada per la fenomenologia d'aquelles funcions de Green que són paràmetres d'ordre de la simetria quiral. En algunes situacions les desviacions respecte als resultats en el limit $N_C \to \infty$ estan sota control [24, 25].

4. S'ha demostrat [18] que els acoblaments del lagrangià subdominant de χPT, $\mathcal{L}_4^{\chi PT}$, estan saturats per l'intercanvi de ressonàncies generat pels termes lineals en els camps de ressonàncies. Per tant, la introducció explícita dels operadors de $\mathcal{L}_{pGB}^{(4)}$ conduiria a considerar dues vegades les contribucions de les ressonàncies. No s'ha realitzat una anàlisi similar a $\mathcal{O}(p^6)$ sistemàticament, encara que sembla una suposició raonable. La nostra teoria assumeix una saturació completa dels acoblaments del lagrangià de χPT.

5. A més a més de les peces cinètiques, solament els termes lineals en les ressonàncies foren considerats en la Ref. [17] perquè l'objectiu de l'article era obtenir les contribucions dominants de les ressonàncies als acoblaments quirals de $\mathcal{L}_4^{\chi PT}$. En el capítol 3 l'estudi d'un observable, a ordre subdominant en l'expansió en $1/N_C$, permet justificar la necessitat de considerar operadors amb més d'una ressonància, si es volen satisfer les condicions d'empalmament amb QCD [26].

Seguint el camí obert en la Ref. [17], s'han estudiat les contribucions dominants de les ressonàncies a alguns acoblaments quirals d'$\mathcal{O}(p^6)$, tot considerant diverses funcions de tres punts [27]. Un enfocament més sistemàtic i complet a aquest tema pot trobar-se en la Ref. [19].

L'empalmament amb QCD

Com hem senyalat adés, un ingredient bàsic per avançar en la construcció de la Teoria Quiral de Ressonàncies és considerar els lligams de QCD a altes energies, és a dir, l'empalmament entre RχT i la teoria completa. De fet, sense considerar els lligams asimptòtics de la dinàmica forta subjacent hi ha massa paràmetres desconeguts en el nostre enfocament efectiu, degut a la natura no pertorbativa de QCD a baixes energies. Cal emfatitzar la importància del nombre de paràmetres desconeguts per determinar el poder predictiu del lagrangià.

La majoria dels lligams a curtes distàncies emprats en la bibliografia resulten d'utilitzar els resultats dominants de l'expansió OPE de les funcions de Green dels corrents de QCD. L'altra possibilitat és considerar el comportament de Brodsky-Lepage dels factors de forma [28], i.e. exigir que els factors de forma a dos cossos dels corrents hadrònics s'anul·len a altes energies. Aquest comportament ha estat comprovat en el cas dels pseudo-bosons de Goldstone i dels fotons. El dubte apareix quan s'estudien factors de forma que involucren ressonàncies com a estats asimptòtics. Una de les principals motivacions d'aquesta tesi és intentar clarificar aquesta qüestió, relacionant els factors de forma a dos cossos amb les funcions de Green de dos punts a ordre subdominant en l'expansió en $1/N_C$ [29, 30].

Cal emfatitzar un altre punt respecte als lligams asimptòtics. Òbviament les relacions trobades depenen del lagrangià emprat: no tindrem els mateixos lligams si utilitzem el lagrangià de la Ref. [17], on únicament s'han considerat les interaccions lineals en els camps de ressonàncies [22], o si es considera un lagrangià més general que no limite el nombre de ressonàncies en els operadors [29, 30].

Correccions quàntiques en la Teoria Quiral de Ressonàncies

Des del començament la Teoria Quiral de Ressonàncies ha estat utilitzada tant en l'estudi de les contribucions de les ressonàncies en processos febles [53] com en l'estudi de factors de forma de mesons [46]. En ambdós casos el lagrangià de RχT s'ha utilitzat solament a ordre arbre i, en conseqüència, s'han obtingut les contribucions dominants en l'expansió en $1/N_C$ del nostre model.

Les correccions quàntiques sorgeixen quan es fan càlculs a un bucle amb la teoria i el seu control comença a caldre tant per la convergència de les prediccions com per redreçar el nostre coneixement no pertorbatiu de QCD:

1. Cal una resumació de Dyson-Schwinger d'ordres subdominants per descriure

les amplituds prop dels pics de les ressonàncies [47], la qual cosa condueix a càlculs fins a un bucle [26, 29, 30, 37, 54].

2. És convenient millorar les prediccions no pertorbatives de QCD per poder distingir els efectes de la nova física en alguns observables.

3. La determinació de les contribucions de les ressonàncies en els acoblaments quirals a ordre subdominant mitjançant aquestes correccions quàntiques en RχT és molt interessant, ja que permetria mantenir sota control l'escala de renormalització i les possibles incerteses degudes al corriment d'aquests acoblaments.

4. Des d'un punt de vista més teòric, les correccions quàntiques són fonamentals per trobar la descripció mitjançant una teoria de camps que permeta entendre les interaccions hadròniques més enllà de modelitzacions *ad hoc*.

RχT no és renormalitzable. A més a més la manca d'un paràmetre d'expansió en el lagrangià dificulta l'aplicació d'un programa de renormalització pertorbatiu basat en un contatge anàlog al cas de χPT. Tanmateix, com s'explica al llarg de la tesi, la situació no és molt diferent al cas de χPT [29, 30]. Com es mostra en el capítol 3, on el factor de forma del pió és calculat a ordre subdominant, dins de la teoria és possible construir un nombre finit d'operadors, els acoblaments dels quals puguen absorbir les divergències dels diagrames amb un bucle. L'únic requeriment, és clar, és que el procés de regularització de les divergències respecte les simetries del lagrangià.

Un primer pas en aquesta direcció fou l'estudi de les contribucions de les ressonàncies a nivell quàntic al corriment de l'acoblament de χPT $L_{10}(\mu)$ [37], tot i que no es va fer una anàlisi de les divergències ultraviolades i la seua renormalització corresponent.

Els diagrames de Feynman amb bucles que inclouen ressonàncies solament s'han analitzat en certs models amb simetries addicionals. Per exemple, la descripció que fa ús de la simetria local oculta [38] implica un comportament ultraviolat molt més simple [39]. Les correccions quàntiques a alguns paràmetres de ressonàncies s'han estudiat [40, 41] en el context de la χPT del mesó vectorial pesat [42], que adopta el límit $M_R \to \infty$ per garantir un bon contatge quiral; també amb aquest fi [43] s'ha utilitzat el model de Nambu-Jona-Lasinio [44].

Així mateix és molt important estudiar el comportament asimptòtic de la teoria una vegada s'han introduït els contratermes adients. Cal estudiar amb cura els lligams implicats per les propietats de QCD a altes energies en el cas de càlculs a ordre subdominant en $1/N_C$.

La renormalització formal de RχT a un bucle sembla ser un treball complicat, que demana prèviament l'anàlisi de certs ingredients. Aquesta tesi mira de ser un avançament en aquesta direcció:

1. Per tal de millorar l'enteniment del comportament de les correccions quàntiques, fóra convenient començar fent un càlcul d'una amplitud física ben determinada

que incloga diagrames amb un bucle. En el capítol 3 es mostra una investigació acurada del factor de forma del pió a ordre subdominant en l'expansió $1/N_C$.

És molt rellevant el límit a altes i baixes energies del resultat trobat. El límit a altes energies permet estudiar els lligams asimptòtics de QCD quan es treballa amb correccions quàntiques. Per la seua banda, el límit a baixes energies ens proporciona les contribucions subdominants de l'intercanvi de ressonàncies a l'acoblament quiral $\ell_6(\mu)$ ($L_9(\mu)$ en el cas de tres sabors).

2. Tot utilitzant els correladors amb correccions quàntiques, en el capítol 4 es justifica la necessitat d'estudiar el comportament a altes energies dels factors de forma que inclouen ressonàncies com a estats asimptòtics. Tenint en compte la importància de l'empalmament en qualsevol teoria efectiva, és fonamental tenir clar quins són els lligams que s'han d'utilitzar en la nostra modelització de la interacció forta a energies intermèdies.

   S'analitzen tots els factors de forma a dos cossos que poden definir-se en el sector de paritat intrínseca parella de la Teoria Quiral de Ressonàncies sota l'aproximació d'una única ressonància. Una vegada s'han incorporat aquests lligams s'espera evitar el mal comportament asimptòtic a grans moments d'aquelles contribucions que vénen de diagrames amb ressonàncies com a estats intermedis.

   A més a més, una vegada es disposa d'amplituds ben comportades a curtes distàncies, càlculs a un bucle poden predir les contribucions de les ressonàncies als acoblaments quirals amb la dependència d'escala sota control. Seguint aquest camí, es presenta una predicció subdominant de $L_8(\mu)$.

3. En el capítol 5 s'estudia la funció generatriu a nivell subdominant obtinguda amb RχT quan només s'incorporen ressonàncies escalars i pseudoescalars i sols acoblaments bilineals en els camps de les ressonàncies. S'obtenen així tots els operadors necessaris per renormalitzar la teoria.

Chapter 1

Effective Field Theories

1.1 The Magnifying Glass of the Theoretical Physicists

Who would look for a street in València with a galactic map? Or the other way around, who would want to check the position of a galaxy by using the street plan of a city? All in all, this is the key of an Effective Field Theory (EFT): the long-distance dynamics do not depend crucially on the details of the short-distance dynamics. In other words, considering the Moon movement around the Earth in order to study our galaxy movement has no sense.

In fact, the idea of effective field theories has been always implicit when describing Nature. One takes into account the suitable degrees of freedom for the problem at hand.

As an illustrative example the physics of the atom can be examined. An analysis considering Quantum Electrodynamics (QED) seems to be a bit useless, i.e. using quarks as degrees of freedom is not the best choice. A better approach would make use of non-relativistic electrons orbiting around the nucleus. As a first approximation, one could consider an infinite mass for the nucleus: only the electron mass and the fine structure constant would be required to describe the system. If more precision is needed, the finite mass of the proton can be taken into account, if even more precision is demanded the spin and the magnetic moment... and so on. The main idea is that the right effective theory of the system has been chosen.

From this setting one can deduce that a first question, and often not naive, is to choose the appropriate degrees of freedom at the scale under consideration. That is, effective theories are the suitable theoretical tools to describe low-energy dynamics, where the term 'low' refers to a determined scale Λ. As it will be explained in the next section, only the relevant degrees of freedom, i.e. those states with $m \ll \Lambda$, are considered, while heavier excitations with $M \gg \Lambda$ have been integrated out from the action. One has to use suitable interactions among the light states, which can be organized as an expansion in powers of energy over the scale Λ.

A remarkable feature to classify effective field theories is the strength of the underlying theory in the region at hand, i.e. one can distinguish the case in which the high-energy theory is weakly or strongly coupled. In the first case the value of the effective couplings can be obtained perturbatively in terms of the underlying couplings. However, in the second case one cannot perform perturbative calculations, so this possibility is not at hand for realistic models. The effective approach we are going to use within this work is of the second kind, that is, only different restrictions coming from the underlying theory can be used, but the value of the couplings cannot be obtained directly.

It has been claimed above that the low-energy dynamics do not depend on the details of the high-energy region. This sentence should be clarified: the only effect of the high-energy theory is to fix the value of the couplings and to provide the symmetries that must be considered in order to describe the long-distance scenario.

To prepare this first chapter, we have made extensive use of several reviews [1, 2].

1.2 Integration of the Heavy Modes

We want to present from a more formal point of view what has been explained in the previous section, by following path integrals methods. Assuming that the theory at high energies is known, the effective action Γ_{eff}, which encodes all the information at low energies, reads

$$e^{i\,\Gamma_{\text{eff}}[\Phi_l]} \;=\; \int [\mathrm{d}\Phi_h]\; e^{iS[\Phi_l,\Phi_h]}\,, \tag{1.1}$$

where Φ_l and Φ_h refer to the light and heavy fields respectively and $S[\Phi_l,\Phi_h]$ is the action of the underlying theory. Thus the effective lagrangian is defined through the expression

$$\Gamma_{\text{eff}}[\Phi_l] \;=\; \int \mathrm{d}^4x\; \mathcal{L}_{\text{eff}}[\Phi_l]\,. \tag{1.2}$$

It is possible to compute the effective action $\Gamma_{\text{eff}}[\Phi_l]$, at least formally, using the saddle point technique. The heavy field Φ_h can be expanded around some field configuration $\overline{\Phi}_h$ as follows

$$\begin{aligned} S[\Phi_l,\Phi_h] \;&=\; S[\Phi_l,\overline{\Phi}_h] + \int \mathrm{d}^4x\, \frac{\delta S}{\delta\Phi_h(x)}\bigg|_{\Phi_h=\overline{\Phi}_h} \Delta\Phi_h(x) \\ &\quad + \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\; \frac{\delta^2 S}{\delta\Phi_h(x)\delta\Phi_h(y)}\bigg|_{\Phi_h=\overline{\Phi}_h} \Delta\Phi_h(x)\Delta\Phi_h(y) \;+\; \dots\,, \end{aligned} \tag{1.3}$$

where the definition $\Delta\Phi_h(x) \equiv \Phi_h(x) - \overline{\Phi}_h$ has been used. It can be chosen $\overline{\Phi}_h$ so that

$$\frac{\delta S[\Phi_l,\Phi_h]}{\delta\Phi_h(x)}\bigg|_{\Phi_h=\overline{\Phi}_h} \;=\; 0\,. \tag{1.4}$$

With this choice Eq. (1.1) turns out to be

$$e^{i\,\Gamma_{\text{eff}}[\Phi_l]} \;=\; e^{i\,S[\Phi_l,\overline{\Phi}_h]} \int [\mathrm{d}\Phi_h]\, e^{i\int \mathrm{d}^4x\,\mathrm{d}^4y \left\{\frac{1}{2}\Delta\Phi_h(x)\,A(x,y)\,\Delta\Phi_h(y)+\dots\right\}}\,, \tag{1.5}$$

where

$$A(x,y) \equiv \left. \frac{\delta^2 S}{\delta\Phi_h(x)\delta\Phi_h(y)} \right|_{\Phi_h=\overline{\Phi}_h} . \quad (1.6)$$

By a formal Gaussian integration and assuming that the heavy field is a boson,

$$\Gamma_{\text{eff}}[\Phi_l] \equiv \sum_{k=0}^{\infty} \Gamma^{(k)} = S[\Phi_l, \overline{\Phi}_h[\Phi_l]] + \frac{i}{2}\text{Tr}\,(\log A[\Phi_l]) + \dots . \quad (1.7)$$

The expansion in Eq. (1.3) turns out to be an expansion in the number of loops, that is the first term corresponds to a tree level integration of the heavy field Φ_h, as clearly seen from Eq. (1.5).

Although the above expansion is quite general and, in principle, it can always be performed, the calculations are very often complicated or cannot be obtained perturbatively. For instance, not always the degrees of freedom of the effective theory are present in the fundamental one. However, as it has been claimed in the former section, some information for the effective low-energy action can be obtained from symmetry constraints coming from the underlying theory.

1.3 Renormalizability and Effective Theories

Usually it is claimed that a quantum field theory should be renormalizable in order to be able to perform radiative corrections to the tree level result, i.e. that the lagrangian should contain only terms with dimension $\leq D$, with D the dimension of the space-time. Otherwise one needs an infinite number of counterterms, hence an infinite number of unknown parameters, so that the theory has no predictive power.

However, an effective field theory lagrangian contains already an infinite number of terms. The lagrangian can be organized by taking into account their dimension,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\leq D} + \mathcal{L}_{D+1} + \mathcal{L}_{D+2} + \dots , \quad (1.8)$$

where $\mathcal{L}_{\leq D}$ contain all terms with dimension $\leq D$, $\mathcal{L}_{D+1}$ contains terms with dimension $D+1$, and so on. The usual renormalizable lagrangian is just the first term, $\mathcal{L}_{\leq D}$. Although there are an infinite number of terms in $\mathcal{L}_{\text{eff}}$, the predictive power has not disappeared while one works at a given precision. As operators with higher dimensions are incorporated, a higher precision ϵ is reached,

$$\epsilon \lesssim \left(\frac{E}{\Lambda}\right)^{D_i^{\max}-4} , \quad (1.9)$$

where $D_i^{\max}$ is the considered highest dimension. Accordingly, once a given precision is decided, the number of operators and thus couplings needed is finite. In other words, a non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy.

With only the first term of Eq. (1.8) the effective lagrangian turns out to be a classical ‘renormalizable’ theory. In fact, the Standard Model is an effective theory in which only the first piece of the expansion is considered. It is supposed to exist a more general theory where, either with the degrees of freedom already present in the usual framework or with completely new ones, there are heavier modes. In consequence it is not surprising to find corrections to the Standard Model, consequence of these new modes, i.e. New Physics or Physics Beyond the Standard Model come simply from higher scales. There are two ways to know these new scales, either experiments at very high energies or improving the precision at the present energies.

1.4 The Decoupling Theorem

Intuitively, decoupling means that low-energy physics is "blind" to high-energy physics. Assuming a theory with light particles and a heavy particle of mass M, one can demonstrate that, under given conditions, the effects of the heavy particle in the low-energy dynamics only appears through corrections proportional to a negative power of M or through renormalization. The Appelquist-Carazonne theorem is the rigorous formulation of this phenomenon [3].

Let us consider a theory with a light field ϕ and a heavy field Φ with masses m and M respectively. $\Gamma^n(g, m, M, \mu; k_1, \ldots, k_n)$ is the vertex of n light particles with momenta k_i, which is derived from the classical action $S[\phi, \Phi]$, where g denotes the different couplings and μ is the renormalization scale. If now we consider the action $\widetilde{S}[\phi]$, which is obtained from $S[\phi, \Phi]$ by omitting the terms with heavy fields and replacing the original light particle mass and couplings by new parameters $\widetilde{m}$ and $\widetilde{g}$, the vertex of n light particles can be considered again, $\widetilde{\Gamma}^n(\widetilde{g}, \widetilde{m}, \mu; k_1, \ldots, k_n)$. Supposing some mass independent renormalization scheme, the theorem proves that

$$\Gamma^n(g, m, M, \mu; k_1, \ldots, k_n) \;=\; Z^{n/2}\, \widetilde{\Gamma}^{(n)}(\widetilde{g}, \widetilde{m}, \mu; k_1, \ldots, k_n) + \mathcal{O}(\frac{1}{M}) \,, \quad (1.10)$$

where the new couplings, mass and scale of fields, $\widetilde{g}(g, M, \mu)$, $\widetilde{m}(g, m, M, \mu)$ and $Z(g, M, \mu)$, depend now on the heavy scale; obviously the form of these functions depend on the renormalization scheme.

As it has been indicated before, there are some conditions in order to be able to grant the validity of the theorem: the underlying theory has to be renormalizable, it should not have spontaneous symmetry breaking nor chiral fermions.

1.5 Matching

It is known that the effects of a heavy particle in the low-energy theory are present through higher-dimension operators, i.e. non-renormalizable ones which are suppressed by inverse powers of the heavy particle mass. The same physical predictions in the full and effective theories should be expected around the heavy-threshold region. Thus, both descriptions are related through a matching condition: the two theories (with and without the heavy field) should give rise to the same S matrix elements for processes involving light particles.

It is important to stress that while the matching conditions have not been taken into account, one is not dealing really with the effective field theory, that is, the matching procedure is a fundamental step to develop effective approaches.

Quantum Chromodynamics (QCD) is an appropriate way to understand this process. Considering the QCD lagrangian with $n_f - 1$ light quark flavors plus one heavy quark of mass M, one assumes that at $\mu < M$ one can integrate out the heavy quark. Accepting the decoupling, the resulting effective field theory consists of the original pieces without the heavy quark plus a tower of higher-dimensional operators suppressed by powers of $1/M$. The matching conditions will relate this effective field theory to the original QCD lagrangian with n_f flavors:

$$\mathcal{L}^{n_f}_{\rm QCD} \iff \mathcal{L}^{n_f-1}_{\rm QCD} + \sum_{i=1} \frac{c_i}{M^i}\mathcal{O}_i \,. \tag{1.11}$$

At low energies these extra operators are usually neglected, being reduced the effective lagrangian to the normal QCD lagrangian with $n_f - 1$ quark flavors. As it has been explained before, the two QCD theories have different renormalization properties: the running of the corresponding couplings $\alpha_s^{n_f}$ and $\alpha_s^{n_f-1}$ is different. The two effective couplings are related trough a matching condition:

$$\alpha_s^{n_f}\left(\mu^2\right) = \alpha_s^{n_f-1}\left(\mu^2\right)\left\{1 + \sum_{k=} C_k\left(\log\frac{\mu}{M}\right)\left(\frac{\alpha_s^{n_f-1}(\mu^2)}{\pi}\right)^k\right\}. \tag{1.12}$$

Since the QCD running coupling is not a physical observable, there can be different parameters and there is no reason why they should be the same at the matching point. The physical observables are those which should be equal at the matching point: they would be the same independently from the effective field theory at hand. In fact these matching conditions require a discontinuous coupling like Eq. (1.12)

1.6 Chiral Perturbation Theory

1.6.1 The QCD Lagrangian and the Running of α_s

With the present overwhelming experimental and theoretical evidence it is known that the $SU(3)_C$ gauge theory correctly describes the hadronic world [4]. Later we are going to be interested in QCD at energies between the ρ mass and 2 GeV, therefore we will start by studying the behaviour of the strong interaction at low energies. QCD describes the strong interaction between quarks and gluons through a non-Abelian $SU(N_C)$ gauge theory, with $N_C = 3$. The lagrangian reads

$$\begin{aligned}
\mathcal{L}_{QCD} &= \overline{q}\left(i\not{D} - \mathcal{M}\right)q - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \mathcal{L}_{FP} + \mathcal{L}_{GF}, \\
D_\mu &= \partial_\mu - ig_s G^a_\mu \frac{\lambda_a}{2} \\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu,
\end{aligned} \tag{1.13}$$

where $a = 1, \ldots, N_C^2 - 1 = 8$, G_μ^a are the gluon fields and g_s is the strong interaction coupling constant. The quark field q represents a column vector in both color and flavor spaces, $\mathcal{M}$ is the quark mass matrix in flavor space. The λ_a are the Gell-Mann matrices, so that $\lambda_a/2$ are the $SU(3)_C$ generators in the fundamental representation and f^{abc} are the structure constants. The Faddeev-Popov term $\mathcal{L}_{FP}$ includes the lagrangian for the ghost fields and $\mathcal{L}_{GF}$ refers to the gauge-fixing term.

Before working perturbatively at low or high energies, one has to explore the running of the g_s strong coupling, in order to confirm if it is possible to consider the coupling as a small quantity. That is, one has to renormalize the theory and, in the case of using dimensional regularization, study the dependence in g_s on the scale μ. Assuming that the strong coupling is small one can calculate the beta function at one-loop level,

$$\beta_{QCD} \,=\, \mu \frac{\partial g_s}{\partial \mu} \,=\, -\left(11 N_C - 2 n_f\right) \frac{g_s^3}{48\pi^2}\,, \tag{1.14}$$

so that, at least at this order, β_{QCD} is negative for $n_f \leq 16$, with n_f the number of flavors. Eq. (1.14) implies that the renormalized coupling constant varies with the scale, which is usually called the "running" of the coupling constant. Integrating this equation, it is obtained that

$$\alpha_s(q^2) \,=\, \frac{12\pi}{(11 N_C - 2 n_f) \log(q^2/\Lambda_{QCD}^2)}\,, \tag{1.15}$$

where $\alpha_s \equiv g_s^2/4\pi$. Written in this form, the evolution of the coupling with the scale only depends on a single parameter Λ_{QCD}, which is known as the QCD scale and is defined in terms of of μ and $\alpha_s(\mu^2)$ through

$$\log(\Lambda_{QCD}^2) \,=\, \log \mu^2 \,-\, \frac{12\pi}{\alpha_s(\mu^2)(33 - 2 n_f)}\,. \tag{1.16}$$

Eq. (1.14) allows to check the asymptotic freedom of QCD, i.e. its running coupling decreases at high energies, in contrast to the case of QED. If in QED the fact that the coupling constant decreases at long distances is interpreted as the result of the charge screening due to the presence of electron-positron virtual pairs, one thinks of an anti-screening effect in QCD, which is due to the non-Abelian nature of the gluonic interactions.

Although Eq. (1.14) is only valid in the region where α_s is small, since it has been obtained by a perturbative calculation at one-loop level, one expects an increase at low energies, which leads to the confinement of QCD: the asymptotic states of QCD cannot be anymore the free quarks at this regime of energies. The phenomenology supports this idea: the confinement of quarks and gluons inside hadrons can be supposed.

As a consequence we are not able to work perturbatively at low energies by using the QCD lagrangian of Eq. (1.13). An effective field theory approach at long distances turns out to be the appropriate framework.

1.6.2 Chiral Symmetry

In the absence of quark masses, the QCD lagrangian of Eq. (1.13) turns out to be

$$\mathcal{L}^0_{QCD} = i\overline{q}_L \not{D} q_L + i\overline{q}_R \not{D} q_R - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \mathcal{L}_{FP} + \mathcal{L}_{GF}, \tag{1.17}$$

where the '0' index refers to the massless case and the quark fields have been split into their chiral components. This lagrangian is invariant under independent global $G \equiv SU(n_f)_L \otimes SU(n_f)_R$ transformations of the left- and right-handed quarks in flavor space.

Global symmetries have an influence into the spectrum, whereas local ones determine the interaction. Consistently, the global chiral symmetry, which should be approximately good in the light quark sector ($n_f = 3$), should have implications in hadronic spectroscopy. Notwithstanding, it does not mean that it necessarily must be observed in the spectrum, since symmetries have always two possible realizations: either they are manifest, giving rise to a classification within the spectrum, or they are driven by a spontaneous symmetry breaking, with the resulting generation of the Goldstone bosons, according to Goldstone's theorem [5].

Vafa and Witten [6] proved that the lowest energy state has to be necessarily invariant under vector transformations, so that the possible spontaneous chiral symmetry breaking cannot affect the vectorial part of the chiral group.

Phenomenology is the next step. Although hadrons can be nicely classified in $SU(3)_V$ representations, degenerate multiplets with opposite parity do not exist. Moreover, the octet of pseudoscalar mesons happens to be much lighter than all the other hadronic states. This experimental evidence drives to the spontaneous $SU(3)_L \otimes SU(n_f)_R$ symmetry breaking to $SU(3)_{L+R}$. Since there are $n_f^2 - 1 = 8$ broken axial generators of the chiral group, there should be eight lightest hadronic states $J^P = 0^-$ (π^+, π^-, π^0, η, K^+, K^-, K^0 and $\overline{K}^0$). Their small masses are generated by the quark-mass matrix, which explicitly breaks the global chiral symmetry. Taking into account this small explicit breaking, we will refer to the pion multiplet as the pseudo-Goldstone bosons.

1.6.3 The Effective Chiral Lagrangian

The general formalism to build effective lagrangians with spontaneous symmetry breaking was proposed by Callan, Coleman, Wess and Zumino [7], who gave a suitable way to parametrize the Goldstone bosons. In the case of QCD at very low energies, it is possible to use these ideas to construct an effective lagrangian to describe the interaction among the pseudo-Goldstone bosons, the lightest pseudoscalar multiplet. Since there is a mass gap separating the pseudoscalar octet from the rest of the hadronic spectrum, one can imagine an effective field theory containing only these modes.

Thus, the basic assumption is the spontaneous chiral symmetry breaking,

$$G \equiv SU(3)_L \otimes SU(3)_R \longrightarrow H \equiv SU(3)_V \, . \tag{1.18}$$

Denoting by ϕ^a ($a = 1, \dots, n_f^2 - 1 = 8$) the coordinates describing the pseudo-Goldstone fields in the coset space G/H, a coset representative $u_{R,L}(\phi)$ is chosen. The change of coordinates, carrying the pseudo-Goldstone modes, under a chiral transformation $g \equiv (g_L, g_R) \in G$ is ruled by

$$\begin{aligned} u_L(\phi) &\xrightarrow{G} g_L\, u_L(\phi)\, h(g,\phi)^\dagger\,, \\ u_R(\phi) &\xrightarrow{G} g_R\, u_R(\phi)\, h(g,\phi)^\dagger\,, \end{aligned} \tag{1.19}$$

where $h(g, \phi) \in H$. We can take the choice of a coset representative such that $u_R(\phi) = u_L^\dagger(\phi) \equiv u(\phi)$, whose explicit form in the Callan, Coleman, Wess and Zumino parameterization can be written as

$$u(\phi) = e^{\left(\frac{i}{\sqrt{2}F}\phi\right)}\,, \tag{1.20}$$

with ϕ defined through the following expression,

$$\phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\,\phi_i = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}, \tag{1.21}$$

where the normalization of the Gell-Mann matrices is given by $\langle\lambda_i\lambda_j\rangle = 2\delta_{ij}$.

Once the coset space is parameterized, the low-energy effective lagrangian realization of QCD for the light quark sector can be obtained, the so-called Chiral Perturbation Theory (χPT) [8, 9, 10]. One should write the most general lagrangian involving the matrix $u(\phi)$, which is consistent with QCD and its chiral symmetry. It is obvious that this effective approach will be useful until the resonance region, $E \ll M_\rho$, since then new degrees of freedom arise.

χPT is worked out as a perturbative expansion in the momenta and masses of the pseudo-Goldstone bosons and it has proved to be a rigorous and fruitful scheme. Thus, the lagrangian can be organized in terms of increasing powers of momentum or, equivalently, in terms of an increasing number of derivatives,

$$\mathcal{L}_{\chi PT} = \sum_{n=1}\mathcal{L}_{2n}^{\chi PT}\,, \tag{1.22}$$

where the subindex, $2n$, indicates the number of derivatives. Notice that parity conservation requires an even number of these and there is no term without derivatives, since $uu^\dagger = 1$.

As in any quantum field theory, quantum loops with internal lines must be explored. Taking into account the lagrangian expansion of Eq. (1.22) and assuming an arbitrary Feynman diagram with N_d vertices of $\mathcal{O}(p^d)$[1] and L loops, it is easy to check that the chiral dimension of an amplitude is given by [8]

$$D = 2 + 2L + \sum_d N_d(d-2)\,. \tag{1.23}$$

[1]The chiral order, $\mathcal{O}(p^d)$, indicates the number of derivatives

The power suppression of loop diagrams is at the basis of effective field theories. As the chiral lagrangian starts at $\mathcal{O}(p^2)$, so $d \geq 2$, and all terms in Eq. (1.23) are positive. As a result, only a finite number of terms in the lagrangian are needed to work to a fixed chiral order, and the chiral lagrangian acts like a renormalizable field theory. For instance, the leading $D = 2$ contributions are obtained with $L = 0$ and $N_{d>2} = 0$, i.e. tree level graphs with $\mathcal{L}_2^{\chi PT}$. Let us imagine now the calculation of amplitudes to $\mathcal{O}(p^4)$, one only has two possibilities in Eq. (1.23), $L = 0$, $N_4 = 1$ and $N_{d>4} = 0$ or $L = 1$ and $N_{d>2} = 0$; that is, one only needs to consider tree level diagrams with one insertion of $\mathcal{L}_4^{\chi PT}$, or one-loop graphs with the lowest order lagrangian $\mathcal{L}_2^{\chi PT}$ to compute all scattering amplitudes to $\mathcal{O}(p^4)$.

It is clear that the chiral expansion in powers of momenta runs over some typical hadronic scale, the chiral symmetry breaking scale, Λ_χ. In view of different arguments, as the variation of the loop contribution under a rescaling of μ, one has an estimate of the scale, $\Lambda_\chi \sim 4\pi F \sim 1.2$ GeV. Furthermore, one can consider the scale related to the first heavy particles that have been integrated out, the ρ multiplet, $\widetilde{\Lambda}_\chi \sim M_\rho \sim 0.77$ GeV. Notice that $\widetilde{\Lambda}_\chi < \Lambda_\chi$, so that loop contributions tend to be smaller than resonance contributions.

The effective field theory technique becomes much more powerful if couplings to external classical fields are introduced. Considering an extended QCD lagrangian, with quark couplings to external currents v_μ, a_μ, s, p:

$$\mathcal{L}_{QCD} \;=\; \mathcal{L}^0_{QCD} + \overline{q}\gamma^\mu \left(v_\mu + \gamma_5 a_\mu\right) q \,-\, \overline{q}\left(s - i\gamma_5 p\right) q\,, \tag{1.24}$$

the external fields will allow to compute the effective realization of general Green Functions of quark currents in a very straightforward way. Moreover, they can be used to incorporate the electromagnetic and semileptonic weak interactions, and the explicit breaking of chiral symmetry through the quark masses. Taking into account that the lagrangian of Eq. (1.24) is to be chiral invariant, the external fields have the following chiral transformations:

$$s + ip \to g_R(s+ip)g_L^\dagger, \quad \ell_\mu \to g_L \ell_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger, \quad r_\mu \to g_R r_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger, \tag{1.25}$$

where $r_\mu \equiv v_\mu + a_\mu$ and $\ell_\mu \equiv v_\mu - a_\mu$ have been defined.

A very convenient way to construct the chiral invariant operators needed for the effective lagrangian is to consider tensors X transforming as

$$X \xrightarrow{G} h(g,\phi)\, X\, h(g,\phi)^\dagger\,, \tag{1.26}$$

since traces of products of these tensors are chiral invariant. Using the external fields and the matrix $u(\phi)$ of Eq. (1.20), the following tensors, which observe the transformations properties of Eq. (1.26), can be constructed:

$$\begin{aligned}
u_\mu &= i\left\{u^\dagger\left(\partial_\mu - i r_\mu\right)u - u\left(\partial_\mu - i\ell_\mu\right)u^\dagger\right\}\,, \\
\chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,, \\
f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \,\pm\, u^\dagger F_R^{\mu\nu} u\,,
\end{aligned} \tag{1.27}$$

Operator	P	C	h.c.
u_μ	$-u^\mu$	u_μ^T	u_μ
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$

Table 1.1: Transformation properties under C, P and hermitian conjugate of the tensors of Eq. (1.27).

with $\chi = 2B_0(s+ip)$ and the following tensors have been introduced,

$$
\begin{aligned}
F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]\,, \\
F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu]\,.
\end{aligned} \tag{1.28}
$$

B_0 is related to the quark condensate:

$$\langle\, 0|\overline{q}^i q^j|0\,\rangle = -F^2 B_0 \delta^{ij}\,. \tag{1.29}$$

Besides those of Eq. (1.27), one can also construct tensors that follow Eq. (1.26) by using the covariant derivative,

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X]\,, \tag{1.30}$$

which is defined through the chiral connection,

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger\left(\partial_\mu - ir_\mu\right)u + u\left(\partial_\mu - i\ell_\mu\right)u^\dagger\right\}\,, \tag{1.31}$$

so that if X transforms as Eq. (1.26), also does $\nabla_\mu X$.

As it has been been indicated before, the explicit breaking of chiral symmetry through quark masses can be added by using the external currents. Taking into account that the breaking is produced in QCD due to the mass matrix,

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}\,, \tag{1.32}$$

the breaking is introduced in χPT with $s = \mathcal{M}$ and $p = 0$ in χ, see Eq. (1.27). Once the masses have been included, the organization of Eq. (1.22) turns out to be an expansion in derivatives of the pseudo-Goldstone fields and in powers of the light quark masses.

One last remark is convenient in order to understand the construction of the different pieces $\mathcal{L}_{2n}^{\chi PT}$. Taking into account that the pseudo-Goldstone masses are introduced trough χ, one assumes that $\chi_\pm \sim \mathcal{O}(p^2)$, and considering the definitions of u_μ and $f_\pm^{\mu\nu}$ in Eq. (1.27), $u_\mu \sim \mathcal{O}(p)$, $f_\pm^{\mu\nu} \sim \mathcal{O}(p^2)$.

We only have to construct all the operators consisting of the defined tensors observing chiral and QCD symmetries. In Table 1.1 the transformation properties under parity (P), charge conjugation (C) and hermitian conjugate of the tensors of Eq. (1.27) are shown. Employing the organization of Eq. (1.22), one gets that the piece of $\mathcal{O}(p^2)$ reads

$$\mathcal{L}_2^{\chi PT} = \frac{F^2}{4}\langle\, u_\mu u^\mu + \chi_+ \,\rangle\,, \tag{1.33}$$

where the brackets $\langle ... \rangle$ denote a trace of the corresponding flavour matrices. Notice that the coefficient is fixed by considering the canonical form of the kinetic piece. Taking into account the explicit chiral symmetry breaking proposed before, only two constants have been introduced in $\mathcal{L}_2^{\chi PT}$, F and B_0, apart from masses. It is straightforward to check that F is approximately the decay constant of the pion, $F \simeq 92.4$ MeV and B_0 can be related to the hadron masses, once the mass term of the lagrangian is obtained,

$$2\,B_0\,\mathcal{M} = \begin{pmatrix} M_\pi^2 & 0 & 0 \\ 0 & M_\pi^2 & 0 \\ 0 & 0 & 2M_K^2 - M_\pi^2 \end{pmatrix}. \tag{1.34}$$

At $\mathcal{O}(p^4)$, the most general lagrangian, invariant under parity, charge conjugation and the local chiral transformations, is given, in $SU(3)$, by [9]

$$\begin{aligned} \mathcal{L}_4^{\chi PT} = \; & L_1\langle\, u_\mu u^\mu \,\rangle^2 + L_2\langle\, u_\mu u^\nu \,\rangle\langle\, u^\mu u_\nu \,\rangle + L_3\langle\, u_\mu u^\mu u_\nu u^\nu \,\rangle + L_4\langle\, u_\mu u^\mu \,\rangle\langle\, \chi_+ \,\rangle \\ & + L_5\langle\, u_\mu u^\mu \chi_+ \,\rangle + L_6\langle\, \chi_+ \,\rangle^2 + L_7\langle\, \chi_- \,\rangle^2 + L_8/2\,\langle\, \chi_+^2 + \chi_-^2 \,\rangle \\ & - iL_9\langle\, f_+^{\mu\nu} u_\mu u_\nu \,\rangle + L_{10}/4\,\langle\, f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} \,\rangle \\ & + iL_{11}\langle\, \chi_-(\nabla_\mu u^\mu + i/2\,\chi_-) \,\rangle - L_{12}\langle\, (\nabla_\mu u^\mu + i/2\,\chi_-)^2 \,\rangle \\ & + H_1/2\,\langle\, f_{+\mu\nu} f_+^{\mu\nu} + f_{-\mu\nu} f_-^{\mu\nu} \,\rangle + H_2/4\,\langle\, \chi_+^2 - \chi_-^2 \,\rangle\,, \end{aligned} \tag{1.35}$$

where the terms with L_{11} and L_{12} vanish when the equations of motion are used and the ones with H_1 and H_2 are only needed for the renormalization. We have not included here the Wess-Zumino-Witten piece related to the chiral anomaly. In this thesis we do not deal with the odd-intrinsic parity sector of QCD.

1.6.4 Renormalization

Obviously loops are divergent and need to be renormalized. If a regularization which preserves the symmetries of the lagrangian is used, such as dimensional regularization, the needed counterterms will respect necessarily these symmetries. Since Eq. (1.22) contains all possible terms, the divergences can then be absorbed in a renormalization of the coupling constants of the lagrangian. At next-to-leading order, the divergences are of $\mathcal{O}(p^4)$ and are thus renormalized by the low-energy couplings in Eq. (1.35),

$$\begin{aligned} L_i &= L_i^r(\mu) + \Gamma_i \frac{\mu^{D-4}}{32\pi^2}\left\{ \frac{2}{D-4} + C \right\}, \\ H_i &= H_i^r(\mu) + \tilde{\Gamma}_i \frac{\mu^{D-4}}{32\pi^2}\left\{ \frac{2}{D-4} + C \right\}, \end{aligned} \tag{1.36}$$

where D is the space-time dimension and C is the constant that fixes the renormalization scheme; notice that in χPT the modified minimal subtraction -1 scheme ($\overline{\text{MS}}-1$) is used and one has $C=\gamma_E-\log 4\pi-1$, with $\gamma_E\simeq 0.5772$ the Euler's constant. The explicit calculation of the one-loop generating functional gives [9]:

$$\Gamma_1=\tfrac{3}{32},\quad \Gamma_2=\tfrac{3}{16},\quad \Gamma_3=0,\quad \Gamma_4=\tfrac{1}{8},\qquad \Gamma_5=\tfrac{3}{8},\qquad \Gamma_6=\tfrac{11}{144},$$

$$\Gamma_7=0,\qquad \Gamma_8=\tfrac{5}{48},\quad \Gamma_9=\tfrac{1}{4},\quad \Gamma_{10}=-\tfrac{1}{4},\quad \tilde{\Gamma}_1=-\tfrac{1}{8},\quad \tilde{\Gamma}_2=\tfrac{5}{24}.$$

The μ dependence in the renormalized couplings $L_i^r(\mu)$ is canceled by that of the one-loop amplitude in any observable.

Chapter 2

Resonance Chiral Theory

2.1 Improving Phenomenological Lagrangians *à la* Weinberg

Once it is accepted that the study of low-energy hadrodynamics is tampered with by our present inability to implement non-perturbative Quantum Chromodynamics fully in those processes, new ways of dealing with QCD at these regimes are required. As it has been argued in the first chapter, Effective Field Theories are one of the most appealing tool to reach this aim [1, 2]. The success of Chiral Perturbation Theory describing the low-energy dynamics of QCD turns out to be a good proof of these ideas [11]. There are other fruitful effective field theories of QCD that support this statement, think for instance in the Heavy Quark Effective Theory [12] for mesons with one heavy quark or Non-Relativistic Quantum Chromodynamics [13] in the case of mesons with both heavy quarks.

However, in the region of energies we are interested in, $M_\rho \lesssim E \lesssim 2$ GeV, the situation is more involved. Although chiral symmetry still provides stringent dynamical constraints, the usual χPT power counting breaks down in the presence of higher energy scales. Moreover, this regime is populated by many resonances and the absence of a mass gap in the spectrum of states makes difficult to provide a formal Effective Field Theory approach to implement QCD properly, since it is not clear which degrees of freedom are being integrated out and, anyhow, from which energy threshold it would be done the integration of heavy modes. In any case, many of the main features of Effective Field Theories will be very useful in order to carry out our procedure.

The main ingredients of our framework are the following:

1. We should start from the phenomenological lagrangians approach proposed by Weinberg in Ref. [8]. He suggested to construct the most general possible lagrangian, including all terms consistent with assumed symmetry principles, expecting that calculations of matrix elements would give the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the symmetry principles. In the case of low-energy QCD,

one of the highlighted characteristics would be the introduction of the bound states, i.e. the ordinary hadrons, as the degrees of freedom. Notice that the choice of the degrees of freedom is a significant step in order to construct the effective lagrangian.

It is important to stress that such general lagrangians, which we will call phenomenological lagrangians *à la* Weinberg, do not have specific dynamical content beyond the general principles of analyticity, unitarity, cluster decomposition, Lorentz invariance and assumed symmetries. This fact allows to use additional information provided by the strong interaction underlying theory to improve the description.

2. Large-N_C QCD [14, 15] furnishes a practical scenario to work with. The limit of an infinite number of quark colors turns out to be a very useful instrument to understand many features of QCD and supplies an alternative power counting to describe the meson interaction. Assuming confinement, the $N_C \to \infty$ limit strongly constraints meson dynamics by asserting that the Green Functions of the theory are described by the tree diagrams of an effective local lagrangian with local vertices and meson fields, higher corrections in $1/N_C$ being yielded by loops described within the same lagrangian theory. The expansion in $1/N_C$ gives a good quantitative approximation scheme to the hadronic world [16], as it will be reviewed in the next section.

3. Additional progress on our phenomenological approach is carried out by using the short-distance properties of QCD. Most of these asymptotic constraints come from matching Green Functions of QCD currents evaluated within the resonance theory with the results obtained in the leading perturbative OPE expansion. Another source of restrictions arise from form factors.

To summarize, taking into account the difficulties of a formal EFT method in the resonance region, we are going to deal with an effective approach based on the phenomenological lagrangians' ideas of Ref. [8]. This approach can be realized by making use of the $1/N_C$ expansion and the short-distance constraints coming from QCD. All in all, we have advanced the main keys that underline the Resonance Chiral Theory (RχT) [17, 18, 19], the suggested framework in this work to handle Quantum Chromodynamics at intermediate energies, $M_\rho \lesssim E \lesssim 2$ GeV.

2.2 The $1/N_C$ Expansion

Dealing with QCD at intermediate energies would be handier by using an expansion parameter. The ordinary strong coupling α_s cannot be the solution taking into account its renormalization group equations. In the region of resonances the usual chiral counting breaks down. Accordingly, the $SU(3)$ gauge theory with very small quark bare masses has no obvious free parameter that could be used as an expansion parameter.

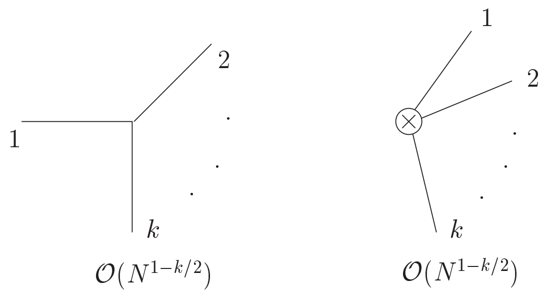


Figure 2.1: $1/N_C$ order of possible vertices.

’t Hooft suggested that one should generalize QCD from three colours and employ an $SU(N_C)$ gauge group [14]. The hope is that it may be possible to solve the theory in the large-N_C limit, and that the physical $N_C = 3$ case may be qualitatively and quantitatively close to the large-N_C limit.

Although one might think that letting $N_C \to \infty$ would make the analysis more complicated because of the larger gauge group and consequent increase in the number of dynamical degrees of freedom, QCD simplifies as N_C becomes large, and there exists a systematic expansion in powers of $1/N_C$.

Choosing the coupling constant g_s to be of $\mathcal{O}(1/\sqrt{N_C})$, i.e. taking the large-$N_C$ limit with $\alpha_s N_C$ fixed, the main results are the following:

1. At $N_C \to \infty$ the mesons and glue states are free, stable and non-interacting. Meson masses have smooth limits and the number of meson states is infinite.

2. Meson decay amplitudes are of $\mathcal{O}(1/\sqrt{N_C})$, and meson-meson elastic scattering amplitudes are of $\mathcal{O}(1/N_C)$. These amplitudes follow the pattern of Figure 2.1.

3. At leading order in the $1/N_C$ expansion, meson dynamics is ruled by a sum of tree diagrams involving the exchange, not of quarks and gluons, but of infinite physical mesons. More generally, meson physics in the large-N_C limit is described by the tree diagrams of an effective local lagrangian, with local vertices and local meson fields. This fact invites us quickly to think about the proper approach of the phenomenological lagrangians *à la* Weinberg, proposed in the last section as the suitable tool for QCD at $M_\rho \lesssim E \lesssim 2$ GeV.

4. Zweig’s rule is exact in the large-N_C limit, that is, mesons should be classified as nonets. The axial anomaly has disappeared and flavour $U(n_f)_L \otimes U(n_f)_R$ has been restored.

5. Mesons are pure $q\overline{q}$ states, that is, one finds a suppression of the $q\overline{q}$ sea at $N_C \to \infty$.

6. In the limit of large number of colours, under reasonable assumptions, $U(n_f)_R \otimes U(n_f)_L$ symmetry must spontaneously break down to $U(n_f)_V$ [20].

The preceding comments can be read in two ways. One may say that one has used the $1/N_C$ expansion to explain certain qualitative facts about the strong interactions. The other possibility is to say that one may use certain qualitative facts about the strong interactions as diagnostic tests showing that large-N_C QCD is probably a good approximation to Nature [15]. Keep in mind that asserting whether the $1/N_C$ expansion is likely to be a good approximation to Nature is a very important matter from a theoretical point of view and very useful for our work, since this expansion can be used in order to justify and improve our effective approach in the resonance region.

Notice that we have only considered the leading order terms in the $1/N_C$ expansion. It is likewise possible to show, by considering unitarity plus the diagrammatic counting rules in large-N_C QCD, that the higher order corrections are sums of loop diagrams of hadrons together with subleading tree-level contributions. Just as in any theory one understands the tree approximations before trying to consider loop diagrams. In fact, the main aim of this work is to make a first step towards the knowledge of the Resonance Chiral Theory at next-to-leading order in the $1/N_C$ expansion, once the tree level contributions are under control.

On the other hand, the idea of the $1/N_C$ expansion is sometimes questioned on the grounds that $1/N_C = 1/3$ is not very small. One cannot really know, theoretically, how large-N_C must be for the expansion to be a good approximation except by calculating the coefficients of some of the terms that are suppressed by powers of $1/N_C$. In other words, the goodness of the expansion depend on the size of the coefficients of the expansion. The best that one can do then is to appeal to phenomenology. As it has been reviewed, there are significant phenomenological reasons to think that $1/N_C = 1/3$ is small enough for the $1/N_C$ expansion to be a good approximation in QCD. In fact, it is interesting to remember why perturbation theory is successful in QED. It is not enough to say that the electric charge is small. Actually, normalized in the usual way the electric charge is approximately $e = 0.302$. Perturbation theory is a good approximation in QED because when one carries out perturbative expansion, one finds that the typical expansion parameter is really $\alpha = e^2/4\pi$. If we had not yet learned how to do perturbative calculations, as in the QCD case, one would have been unable to judge, just from the value of e, whether this expansion would be a good approximation. If, for instance, as it is perfectly possible, the characteristic parameter in the $1/N_C$ expansion would be $1/4\pi N_C$, the next-to-leading corrections would be as tiny as electromagnetic corrections. Although this is only an extreme possibility, we want to justify that there is no reason to reject the $1/N_C$ expansion taking into account the value of N_C, above all considering that phenomenology seems to support the expansion [15].

2.2.1 The $1/N_C$ Expansion in Chiral Perturbation Theory

Let us come back to the very low-energy EFT of QCD, Chiral Perturbation Theory, in order to show how this new tool we have introduced in this section, the $1/N_C$ expansion, turns out to be a useful source of dynamical information [21, 22], in the

i	$L_i^r(M_\rho)$	$\mathcal{O}(N_C)$	source
$2L_1 - L_2$	-0.6 ± 0.6	$\mathcal{O}(1)$	K_{e4}, $\pi\pi \to \pi\pi$
L_2	1.4 ± 0.3	$\mathcal{O}(N_C)$	K_{e4}, $\pi\pi \to \pi\pi$
L_3	-3.5 ± 1.1	$\mathcal{O}(N_C)$	K_{e4}, $\pi\pi \to \pi\pi$
L_4	-0.3 ± 0.5	$\mathcal{O}(1)$	Zweig rule
L_5	1.4 ± 0.5	$\mathcal{O}(N_C)$	$F_K : F_\pi$
L_6	-0.2 ± 0.3	$\mathcal{O}(1)$	Zweig rule
L_7	-0.4 ± 0.2	$\mathcal{O}(1)$	GMO, L_5, L_8
L_8	0.9 ± 0.3	$\mathcal{O}(N_C)$	M_ϕ, L_5
L_9	6.9 ± 0.7	$\mathcal{O}(N_C)$	$\langle r^2 \rangle_V^\pi$
L_{10}	-5.5 ± 0.7	$\mathcal{O}(N_C)$	$\pi \to e\nu\gamma$

Table 2.1: Phenomenological values of the couplings $L_i^r(M_\rho)$ in units of 10^{-3}. The fourth column shows the source used to get this information .

sense that it comes directly from QCD. Keep in mind that in the large-N_C limit the flavour $U(n_f)_L \otimes U(n_f)_R$ has been restored.

Although formally the χPT lagrangian of Eq. (1.22) could be computed from the QCD generating functional, one does not know how to calculate the values of the couplings from QCD because of its non-perturbative nature at low energies. Since it can be proved that the corresponding correlation functions of fermion bilinears are of $\mathcal{O}(N_C)$, the leading-order terms in $1/N_C$ should be of $\mathcal{O}(N_C)$. Moreover, they should have a single flavour trace, as terms with a single trace are of $\mathcal{O}(N_C)$, while the occurrence of each additional trace reduces the order of the term by unity [23].

The leading lagrangian of Eq. (1.33) obeys the correct N_C counting rules: the different fields, the masses and momenta are all of them of $\mathcal{O}(1)$, whereas $F \sim \mathcal{O}(\sqrt{N_C})$. The $u(\phi)$ matrix, defined in Eq. (1.20), generates an expansion in powers of ϕ/F, giving the required $1/\sqrt{N_C}$ suppression for each additional meson field (see Figure 2.1). Clearly, interaction vertices with n mesons scale as $V_n \sim F^{2-n} \sim \mathcal{O}(N_C^{1-n/2})$. Since $\mathcal{L}_2^{\chi PT}$ has an overall factor of N_C and $u(\phi)$ is N_C-independent, the $1/N_C$ expansion is equivalent to a semiclassical expansion. Quantum corrections computed with the chiral lagrangian will have a $1/N_C$ suppression for each loop.

More information from large-N_C QCD can be obtained in the case of $\mathcal{L}_4^{\chi PT}$, shown in Eq. (1.35). As it has been explained in Section 1.6.3, only ten additional couplings L_i ($i = 1, \dots, 10$) are required to determine the low-energy behaviour of the Green Functions at $\mathcal{O}(p^4)$. Large-N_C QCD claims that terms with a single trace are of $\mathcal{O}(N_C)$, while those with two traces should be of $\mathcal{O}(1)$. Therefore one would say that L_3, L_5, L_8, L_9 and L_{10} are of $\mathcal{O}(N_C)$, while L_4, L_6, and L_7 are of $\mathcal{O}(1)$. The case of L_1 and L_2 should be analyzed taking into account the following relation:

$$\langle u_\mu u_\nu u^\mu u^\nu \rangle = -2 \langle u_\mu u^\mu u_\nu u^\nu \rangle + \frac{1}{2} \langle u_\mu u^\mu \rangle \langle u_\nu u^\nu \rangle + \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle . \qquad (2.1)$$

This new operator could have been added in $\mathcal{L}_4^{\chi PT}$, but it is dependent on the terms with couplings L_1, L_2 and L_3. Therefore, the symmetries allow a new operator of $\mathcal{O}(N_C)$ and once one does not include it, one could consider an additional contribution to the couplings L_1, L_2 and L_3, with the result $2\delta L_1 = \delta L_2 = -1/2\delta L_3 \sim \mathcal{O}(N_C)$. In other words, L_1 and L_2 are really of $\mathcal{O}(N_C)$, keeping $2L_1 - L_2$ of $\mathcal{O}(1)$. As shown in Table 2.1, the phenomenologically determined values of those couplings [11] follow the pattern suggested by the $1/N_C$ counting rules.

2.3 The Lagrangian of Resonance Chiral Theory

2.3.1 Introduction

We want to deal with QCD in the resonance region, $M_\rho \lesssim E \lesssim 2$ GeV, by following the phenomenological lagrangians *à la* Weinberg, which will be ruled by the $1/N_C$ expansion. One has to consider the most general lagrangian, that is, including all terms consistent with assumed symmetry principles, and considering the ordinary hadrons as degrees of freedom. The program to construct the lagrangian involves several tasks:

1. In order to be able to recover at very low energies the results of χPT, to consider chiral symmetry seems to be the best choice. On account of large-N_C, the mesons are put together into $U(3)$ multiplets and only operators that have one trace in the flavour space are considered [21, 23].

2. It is a well known fact that, in order to make any effective description meaningful, one needs to properly match the underlying theory (QCD in this case). Notice that the QCD asymptotic behaviour sets in already at energies $E \sim 2$ GeV. Then RχT should recover the short-distance behaviour of QCD. This requirement excludes interactions with large number of derivatives, since they tend to violate the QCD ruled asymptotic behaviour of Green Functions or form factors, explaining the phenomenological success of the usual approximations, where only operators constructed with chiral tensors up to $\mathcal{O}(p^2)$ are kept[1]. Furthermore, this matching provides several relations between the couplings in the lagrangian, reducing the number of unknown parameters. These constraints will be analyzed in Section 2.4.

3. Although large-N_C QCD is a robust instrument to realize QCD at intermediate energies, some approximations are needed to construct the effective lagrangian. As the number of meson states is infinite at large-N_C, the most common one is the cut in the number of resonances, only considering the lightest states. This is known to be a good approximation since contributions from higher states are suppressed by their masses. Phenomenology supports this approximation.

[1]The effective terms will be constructed with resonance fields and tensors which introduce the pseudo-Goldstone bosons and the sources, already introduced in Eqs. (1.27) and (1.30). We will denote as 'chiral tensors' this second group. Accordingly, the operators of the lagrangian will be built by resonances and chiral tensors.

In Ref. [17] only contributions from the lightest resonances with non-exotic quantum numbers were taken into account, the so-called Single Resonance Approximation. Our analysis is also carried under this approximation.

Since there is an infinite number of Green Functions, it is obviously not possible to satisfy all matching conditions with a finite number of resonances and uncertainties due to truncation of the spectrum are introduced in the determination of the parameters. Eventually, one may be driven to inconsistencies in the effective parameter relations. The Minimal Hadronic Approximation (MHA) generalises the Single Resonance Approximation so the effective description includes the minimal number of resonances that allows fulfilling the QCD short-distance constraints in the considered amplitude [24]. Although MHA is an approximation of full large-N_C QCD, it is well supported by the phenomenology of Green Functions that are order-parameter of the chiral symmetry. Deviations from the $N_C \to \infty$ limit are properly understood in some situations [24, 25].

4. It has been shown [18] that $\mathcal{L}_4^{\chi PT}$ is largely saturated[2] by the resonance exchange generated by the linear terms in the resonance field, as it will be explained in Section 2.5. Hence, the explicit introduction of the operators constructed with no resonances and chiral tensors of $\mathcal{O}(p^4)$ would amount to include an overlap between both contributions. An analogous analysis at $\mathcal{O}(p^6)$ has not been systematically performed but it also looks a reasonable assumption. Thus our theory stands for a complete resonance saturation of the χPT lagrangian; in other words, we are assuming that the low-energy couplings of $\mathcal{L}_n^{\chi PT}$ $(n \geq 4)$ are completely determined by the resonance contributions, so one does not have to include these operators when the resonance fields are active degrees of freedom.

5. Besides the kinetic pieces, only linear couplings in the resonance fields were included in Ref. [17], since the aim of the article was to get the leading resonance contributions to the low-energy constants (LEC's) of the $\mathcal{O}(p^4)$ χPT lagrangian[3], see Eq. (1.35). In the next chapter the study of one observable to next-to-leading order in the $1/N_C$ expansion will show that in order to perform the matching with QCD operators constructed with more than one resonance will be needed [26].

Following the path of Ref. [17], the leading resonance contributions to some $\mathcal{O}(p^6)$ χPT LEC's have been studied, by considering different three-point func-

[2] This is much more clear in the case of vector and axial-vector resonances as their phenomenology is better known.

[3] Solving the resonance equations of motion in an expansion in the resonance masses, the resonance fields are expressed as a series of chiral operators times inverse powers of the masses, with chiral tensors starting at $\mathcal{O}(p^2)$ [19]. Therefore, the only possible leading resonance contributions to the LEC's of $\mathcal{L}_4^{\chi PT}$ come from operators constructed with one resonance field and one chiral tensor of $\mathcal{O}(p^2)$ in the chiral counting.

tions [27]. A more systematic and complete approach to this issue can be found in Ref. [19].

Notice that in contrast to many models of the resonance fields that have been widely employed in the literature, RχT only uses basic QCD symmetry features without any additional *ad hoc* assumptions. Its model aspect only comes from the fact that we do not include an infinite spectrum in the theory, which is one of the features of the $N_C \to \infty$ limit of QCD.

2.3.2 Constructing the Lagrangian

As it has been pointed out above, the study is taken under the Single Resonance Approximation, where just the lightest resonances with non-exotic quantum numbers are considered. Taking into account the results at large-N_C, the mesons are put together into $U(3)$ multiplets. Hence, our degrees of freedom are the pseudo-Goldstone boson (the lightest pseudoscalar mesons) along with massive multiplets of the type $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$. With them, one constructs the most general effective action that preserves chiral symmetry invariance and QCD symmetries.

Following the procedure presented in Section 1.6.3 to construct the χPT lagrangian, one considers tensors X transforming as

$$X \xrightarrow{G} h(g,\phi)\, X\, h(g,\phi)^\dagger \,, \tag{2.2}$$

where now $G \equiv U(3)_L \otimes U(3)_R$. The tensors that introduce the pseudo-Goldstone bosons and the sources were already introduced in Eqs. (1.27) and (1.30), which follow the transformation properties under parity (P), charge conjugation (C) and hermitian conjugate (h.c.) of Table 1.1. Notice that the expression of ϕ in Eq. (1.21) changes in the moment one considers nonets instead of octets,

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix} . \tag{2.3}$$

The resonance fields follow the same guide, so that for the vector multiplet one has,

$$V_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 \end{pmatrix}_{\mu\nu} , \tag{2.4}$$

where, as it is explained in Appendix A, the antisymmetric formalism is used for spin-1 fields. The multiplets of the type $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$ are parametrized in an analogous way to Eq. (2.4). The transformation properties under P, C and hermitian conjugate of the resonance fields are shown in Table 2.2.

Operator	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V^T_{\mu\nu}$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A^T_{\mu\nu}$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Table 2.2: Transformation properties under P, C and hermitian conjugate of the resonance fields.

One should now consider the most general lagrangian that preserves chiral symmetry invariance and QCD symmetries, observing the former remarks, i.e. constructed with chiral tensors up to $\mathcal{O}(p^2)$ in the chiral counting and under the Single Resonance Approximation.

In the large-N_C approach, there is no limit to the number of resonances that one may include in the effective operators. One can classify the terms in the lagrangian according to the number of resonances,

$$\mathcal{L}_{R\chi T} \;=\; \mathcal{L}^{(2)}_{pGB} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1,R_2} \mathcal{L}_{R_1R_2} + \sum_{R_1,R_2,R_3} \mathcal{L}_{R_1R_2R_3} + \,...\;, \tag{2.5}$$

where the dots denote operators with four or more resonance fields, and the indexes R_i run over all the different resonance fields, V, A, S and P. However, for the purpose of this work, only operators up to three resonance fields are taken into account.

$\mathcal{L}^{(2)}_{pGB}$ keeps the $\mathcal{O}(p^2)$ terms without resonances, i.e. the lagrangian of Eq. (1.33),

$$\mathcal{L}^{(2)}_{pGB} = \mathcal{L}_2^{\chi PT} \;=\; \frac{F^2}{4}\langle\, u_\mu u^\mu + \chi_+ \,\rangle\,. \tag{2.6}$$

It is important to distinguish between $\mathcal{L}_{\chi PT}$ and $\mathcal{L}_{pGB}$: although both have the same structure and operators, $\mathcal{L}_{pGB}$ differs from $\mathcal{L}_{\chi PT}$ in the value of the couplings as $\mathcal{L}_{pGB}$ belongs to the theory where the resonances are active degrees of freedom. Furthermore, notice that, as mentioned above, once a complete resonance saturation of the χPT lagrangian is supposed, no pieces of $\mathcal{L}^{(4)}_{pGB}$ or higher are added.

The second term of Eq. (2.5) corresponds to the interaction terms with one resonance field [17],

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle\, V_{\mu\nu} f_+^{\mu\nu} \,\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle\, V_{\mu\nu}[u^\mu, u^\nu] \,\rangle\,, \tag{2.7}$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle\, A_{\mu\nu} f_-^{\mu\nu} \,\rangle\,, \tag{2.8}$$

$$\mathcal{L}_S = c_d\langle\, S u_\mu u^\mu \,\rangle + c_m\langle\, S\chi_+ \,\rangle\;, \tag{2.9}$$

$$\mathcal{L}_P = i\, d_m \langle\, P \chi_- \,\rangle \quad . \tag{2.10}$$

The $\mathcal{L}_{R_1R_2}$ contain the kinetic terms and the remaining operators with two resonance fields [17, 19],

$$\mathcal{L}_{\,\mathrm{kin}\,R} = \frac{1}{2}\langle\, \nabla^\mu R \nabla_\mu R \,-\, M_R^2 R^2 \,\rangle\,, \qquad (R = S, P) \tag{2.11}$$

$$\mathcal{L}_{\,\mathrm{kin}\,R} = -\frac{1}{2}\langle\, \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} \,-\, \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \,\rangle\,, \quad (R = V, A) \tag{2.12}$$

$$\mathcal{L}_{RR} = \lambda_1^{RR}\langle\, RR\, u^\mu u_\mu \,\rangle + \lambda_2^{RR}\langle\, R u_\mu R u^\mu \,\rangle + \lambda_3^{RR}\langle\, RR\, \chi_+ \,\rangle\,, \quad (R = S, P) \tag{2.13}$$

$$\mathcal{L}_{SP} = \lambda_1^{SP}\langle\, u_\alpha \{\nabla^\alpha S, P\} \,\rangle + i\lambda_2^{SP}\langle\, \{S, P\} \chi_- \,\rangle\,, \tag{2.14}$$

$$\mathcal{L}_{SV} = i\lambda_1^{SV}\langle\, \{S, V_{\mu\nu}\} u^\mu u^\nu \,\rangle \,+\, i\lambda_2^{SV}\langle\, S u_\mu V^{\mu\nu} u_\nu \,\rangle \,+\, \lambda_3^{SV}\langle\, \{S, V_{\mu\nu}\} f_+^{\mu\nu} \,\rangle\,, \tag{2.15}$$

$$\mathcal{L}_{SA} = \lambda_1^{SA}\langle\, \{\nabla_\mu S, A^{\mu\nu}\} u_\nu \,\rangle \,+\, \lambda_2^{SA}\langle\, \{S, A_{\mu\nu}\} f_-^{\mu\nu} \,\rangle\,, \tag{2.16}$$

$$\mathcal{L}_{PV} = i\lambda_1^{PV}\langle\, [\nabla^\mu P, V_{\mu\nu}] u^\nu \,\rangle \,+\, i\lambda_2^{PV}\langle\, [P, V_{\mu\nu}] f_-^{\mu\nu} \,\rangle\,, \tag{2.17}$$

$$\mathcal{L}_{PA} = i\lambda_1^{PA}\langle\, [P, A_{\mu\nu}] f_+^{\mu\nu} \,\rangle \,+\, \lambda_2^{PA}\langle\, [P, A_{\mu\nu}] u^\mu u^\nu \,\rangle\,, \tag{2.18}$$

$$\begin{aligned}\mathcal{L}_{VA} = {} & \lambda_1^{VA}\langle\, [V^{\mu\nu}, A_{\mu\nu}] \chi_- \,\rangle \,+\, i\lambda_2^{VA}\langle\, [V^{\mu\nu}, A_{\nu\alpha}] h_\mu^\alpha \,\rangle \,+\, i\lambda_3^{VA}\langle\, [\nabla^\mu V_{\mu\nu}, A^{\nu\alpha}] u_\alpha \,\rangle \\ & + i\lambda_4^{VA}\langle\, [\nabla_\alpha V_{\mu\nu}, A^{\alpha\nu}] u^\mu \,\rangle \,+\, i\lambda_5^{VA}\langle\, [\nabla_\alpha V_{\mu\nu}, A^{\mu\nu}] u^\alpha \,\rangle \\ & + i\lambda_6^{VA}\langle\, [V_{\mu\nu}, A^\mu{}_\alpha] f_-^{\alpha\nu} \,\rangle\,,\end{aligned} \tag{2.19}$$

$$\begin{aligned}\mathcal{L}_{RR} = {} & \lambda_1^{RR}\langle\, R_{\mu\nu} R^{\mu\nu} u_\alpha u^\alpha \,\rangle \,+\, \lambda_2^{RR}\langle\, R_{\mu\nu} u^\alpha R^{\mu\nu} u_\alpha \,\rangle \,+\, \lambda_3^{RR}\langle\, R_{\mu\alpha} R^{\nu\alpha} u^\mu u_\nu \,\rangle \\ & + \lambda_4^{RR}\langle\, R_{\mu\alpha} R^{\nu\alpha} u_\nu u^\mu \,\rangle \,+\, \lambda_5^{RR}\langle\, R_{\mu\alpha} \left(u^\alpha R^{\mu\beta} u_\beta + u_\beta R^{\mu\beta} u^\alpha \right) \rangle \\ & + \lambda_6^{RR}\langle\, R_{\mu\nu} R^{\mu\nu} \chi_+ \,\rangle \,+\, i\lambda_7^{RR}\langle\, R_{\mu\alpha} R^\alpha{}_\nu f_+^{\mu\nu} \,\rangle\,. \qquad (R = V, A)\end{aligned} \tag{2.20}$$

In the case of three resonance operators, only terms consisting of resonance fields and the covariant derivative ∇_μ are studied, since they are the only ones that contribute to two-body form factors at tree level, see Chapter 4:

$$\Delta\mathcal{L}_{SRR} = \lambda_0^{SRR}\langle\, SRR \,\rangle \,+\, \lambda_1^{SRR}\langle\, S\, \nabla_\mu R\, \nabla^\mu R \,\rangle\,, \qquad (R = S, P) \tag{2.21}$$

$$\begin{aligned}\Delta\mathcal{L}_{SRR} = {} & \lambda_0^{SRR}\langle\, S R_{\mu\nu} R^{\mu\nu} \,\rangle \,+\, \lambda_1^{SRR}\langle\, S\, \nabla_\mu R^{\mu\alpha}\, \nabla^\nu R_{\nu\alpha} \,\rangle \,+\, \lambda_2^{SRR}\langle\, S\, \nabla^\nu R^{\mu\alpha}\, \nabla_\mu R_{\nu\alpha} \,\rangle \\ & + \lambda_3^{SRR}\langle\, S\, \nabla_\alpha R^{\mu\nu}\, \nabla^\alpha R_{\mu\nu} \,\rangle \,+\, \lambda_4^{SRR}\langle\, S \{R^{\mu\nu}, \nabla^2 R_{\mu\nu} \,\rangle \\ & + \lambda_5^{SRR}\langle\, S \{R_{\mu\alpha}, \nabla^\mu \nabla_\nu R^{\nu\alpha}\} \,\rangle\,, \qquad (R = V, A)\end{aligned} \tag{2.22}$$

$$\Delta\mathcal{L}_{SPA} = \lambda^{SPA}\langle\, A^{\mu\nu} \{\nabla_\mu S, \nabla_\nu P\} \,\rangle\,, \tag{2.23}$$

$$\begin{aligned}
\Delta\mathcal{L}_{PVA} = \; & i\lambda_0^{PVA}\langle\, P[V_{\mu\nu}, A^{\mu\nu}]\,\rangle \,+\, i\lambda_1^{PVA}\langle\, P[\nabla_\mu V^{\mu\alpha}, \nabla^\nu A_{\nu\alpha}]\,\rangle \\
& +\, i\lambda_2^{PVA}\langle\, P[\nabla^\nu V^{\mu\alpha}, \nabla_\mu A_{\nu\alpha}]\,\rangle \,+\, i\lambda_3^{PVA}\langle\, P[\nabla_\alpha V^{\mu\nu}, \nabla^\alpha A_{\mu\nu}]\,\rangle \\
& +\, i\lambda_4^{PVA}\langle\, P[V^{\mu\nu}, \nabla^2 A_{\mu\nu}]\,\rangle + i\lambda_5^{PVA}\langle\, P[V^{\mu\alpha}, \nabla_\mu\nabla^\nu A_{\nu\alpha}]\,\rangle \\
& +\, i\lambda_6^{PVA}\langle\, P\,[\nabla^\nu\nabla_\mu V^{\mu\alpha}, A_{\nu\alpha}]\,\rangle\,, \qquad (2.24)
\end{aligned}$$

$$\Delta\mathcal{L}_{VRR} = i\,\lambda^{VRR}\langle\, V^{\mu\nu}\nabla_\mu R\nabla_\nu R\,\rangle\,, \qquad (R = S, P) \qquad (2.25)$$

$$\begin{aligned}
\Delta\mathcal{L}_{VVV} = \; & i\,\lambda_0^{VVV}\langle\, V^{\mu\nu}V_{\mu\alpha}V_\nu{}^\alpha\,\rangle \,+\, i\,\lambda_1^{VVV}\langle\, V^{\mu\nu}[\nabla_\mu V_{\alpha\beta}, \nabla_\nu V^{\alpha\beta}]\,\rangle \\
& + i\,\lambda_2^{VVV}\langle\, V^{\mu\nu}[\nabla^\beta V_{\mu\alpha}, \nabla_\beta V_\nu^\alpha]\,\rangle + i\,\lambda_3^{VVV}\langle\, V^{\mu\nu}[\nabla_\mu V_{\beta\alpha}, \nabla^\alpha V_\nu^\beta]\,\rangle \\
& + i\,\lambda_4^{VVV}\langle\, V^{\mu\nu}[\nabla_\mu V_{\nu\alpha}, \nabla_\beta V^{\alpha\beta}]\,\rangle + i\,\lambda_5^{VVV}\langle\, V^{\mu\nu}[\nabla^\alpha V_{\mu\nu}, \nabla^\beta V_{\alpha\beta}]\,\rangle \\
& + i\,\lambda_6^{VVV}\langle\, V^{\mu\nu}[\nabla^\alpha V_{\mu\alpha}, \nabla^\beta V_{\nu\beta}]\,\rangle + i\,\lambda_7^{VVV}\langle\, V^{\mu\nu}[\nabla^\alpha V_{\mu\beta}, \nabla^\beta V_{\nu\alpha}]\,\rangle \quad , \qquad (2.26)
\end{aligned}$$

$$\begin{aligned}
\Delta\mathcal{L}_{VAA} = \; & i\,\lambda_0^{VAA}\langle\, V^{\mu\nu}A_{\mu\alpha}A_\nu^{\ \alpha}\,\rangle \,+ i\,\lambda_1^{VAA}\langle\, V^{\mu\nu}[\nabla_\mu A_{\alpha\beta}, \nabla_\nu A^{\alpha\beta}]\,\rangle \\
& + i\,\lambda_2^{VAA}\langle\, V^{\mu\nu}[\nabla^\beta A_{\mu\alpha}, \nabla_\beta A_\nu^\alpha]\,\rangle + i\,\lambda_3^{VAA}\langle\, \nabla^\beta V^{\mu\nu}[A_{\mu\alpha}, \nabla_\beta A_\nu^\alpha]\,\rangle \\
& + i\,\lambda_4^{VAA}\langle\, V^{\mu\nu}[\nabla_\mu A_{\beta\alpha}, \nabla^\alpha A_\nu^\beta]\,\rangle + i\,\lambda_5^{VAA}\langle\, \nabla_\mu V^{\mu\nu}[A_{\beta\alpha}, \nabla^\alpha A_\nu^\beta]\,\rangle \\
& + i\,\lambda_6^{VAA}\langle\, \nabla^\alpha V^{\mu\nu}[\nabla_\mu A_{\beta\alpha}, A_\nu^\beta]\,\rangle + i\,\lambda_7^{VAA}\langle\, V^{\mu\nu}[\nabla_\mu A_{\nu\alpha}, \nabla_\beta A^{\alpha\beta}]\,\rangle \\
& + i\,\lambda_8^{VAA}\langle\, \nabla_\mu V^{\mu\nu}[A_{\nu\alpha}, \nabla_\beta A^{\alpha\beta}]\,\rangle + i\,\lambda_9^{VAA}\langle\, \nabla_\beta V^{\mu\nu}[\nabla_\mu A_{\nu\alpha}, A^{\alpha\beta}]\,\rangle \\
& + i\,\lambda_{10}^{VAA}\langle\, V^{\mu\nu}[\nabla^\alpha A_{\mu\nu}, \nabla^\beta A_{\alpha\beta}]\,\rangle + i\,\lambda_{11}^{VAA}\langle\, V^{\mu\nu}[\nabla^\alpha A_{\mu\alpha}, \nabla^\beta A_{\nu\beta}]\,\rangle \\
& + i\,\lambda_{12}^{VAA}\langle\, \nabla^\alpha V^{\mu\nu}[A_{\mu\alpha}, \nabla^\beta A_{\nu\beta}]\,\rangle + i\,\lambda_{13}^{VAA}\langle\, V^{\mu\nu}[\nabla^\alpha A_{\mu\beta}, \nabla^\beta A_{\nu\alpha}]\,\rangle \\
& + i\,\lambda_{14}^{VAA}\langle\, \nabla^\alpha V^{\mu\nu}[A_{\mu\beta}, \nabla^\beta A_{\nu\alpha}]\,\rangle\,. \qquad (2.27)
\end{aligned}$$

All coupling constants are real, M_R are the corresponding masses of the resonances, the brackets $\langle ...\rangle$ denote a trace of the corresponding flavour matrices, and the notation defined in Ref. [17, 19] is followed.

Keep in mind that as our lagrangian $\mathcal{L}_{R\chi T}$ satisfies the N_C counting rules for an effective theory with $U(3)$ multiplets, only operators that have one trace in the flavour space are considered [21, 23]. The different fields, masses and momenta are of $\mathcal{O}(1)$ in the $1/N_C$ expansion. Taking into account the interaction terms (see Figure 2.1), one is able to check that F, F_V, G_V, F_A, c_d, c_m and d_m are of $\mathcal{O}(\sqrt{N_C})$; $\lambda_i^{R_1R_2}$ of $\mathcal{O}(1)$ and $\lambda_i^{R_1R_2R_3}$ of $\mathcal{O}(1/\sqrt{N_C})$. The mass dimension of these parameters is $[F] = [F_V] = [G_V] = [F_A] = [c_d] = [c_m] = [d_m] = E$, $[\lambda_i^{R_1R_2}] = E^0$ and $[\lambda_i^{R_1R_2R_3}] = E^{-1}$.

Note that the equations of motion have been used in order to reduce the number of operators. For instance, terms like $\langle P \nabla_\mu u^\mu \rangle$ are not present in Eq. (2.10), since using the equations of motion we would generate operators that, either have been already considered, or contain a higher number of resonance fields.

2.4 Matching with QCD

As previously pointed out, a basic ingredient in order to take a step forward in the construction of Resonance Chiral Theory is to consider the short-distance constraints from QCD, i.e. the matching procedure between RχT and the full theory. Actually, without examining the high-energy properties of the underlying strong dynamics there are too many unknown parameters in our effective approach. Take note of the significance of the number of parameters for the predictive power of the lagrangian.

Most of the short-distance constraints used in the literature come from considering the Green Functions of QCD currents obtained in the leading OPE expansion. The other source of information is to consider the Brodsky-Lepage behaviour of the form factors [28], that is, to demand that two-body form factors of hadronic currents vanish at high energies. This behaviour has been experimentally observed for pseudo-Goldstone bosons and photons. The doubt appears when one is considering form factors that involve resonances as asymptotic states. One of the motivations of this work is to clarify this question, relating the two-body form factors with the two-point Green Functions at next-to-leading order in the $1/N_C$ expansion [29, 30]. See Chapter 4 for more information.

Another remark is needed before studying the constraints. Obviously these relations depend on the considered lagrangian. Owing to historical reasons, we start by studying the case in which only the $\mathcal{L}_R$ of Eq. (2.5) together with the kinetic pieces to describe the resonance interactions are included. These are the only required operators to determine the leading resonance contributions to the couplings constants of the $\mathcal{O}(p^4)$ χPT lagrangian. The strong constraints are the following [22]:

1. Vector form factor. At leading order in the $1/N_C$ expansion, the two pseudo-Goldstone boson matrix element of the vector current reads,

$$\mathcal{F}^v_{\pi\pi}(q^2) \;=\; 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2} \,. \tag{2.28}$$

Accepting that the vector form factor should vanish at infinite momentum transfer, the resonance couplings should satisfy

$$F_V G_V \;=\; F^2 \,. \tag{2.29}$$

2. Axial form factor. The matrix element of the axial current between one pseudo-Goldstone and one photon is parameterized by the axial form factor. From the assumed lagrangian one gets

$$\mathcal{F}^a_{\pi\gamma}(q^2) \;=\; \frac{F_A^2}{M_A^2 - q^2} + \frac{2F_V G_V - F_V^2}{M_V^2} \,, \tag{2.30}$$

which vanishes at $q^2 \to \infty$ provided that

$$2F_V G_V - F_V^2 \; = \; 0 \, . \tag{2.31}$$

3. Weinberg sum rules. The two-point function built from a left-handed and a right-handed vector quark current defines the correlator

$$\Pi_{V-A}(q^2) \; = \; \frac{F^2}{q^2} + \frac{F_V^2}{M_V^2 - q^2} - \frac{F_A^2}{M_A^2 - q^2} \, . \tag{2.32}$$

In the chiral limit it vanishes faster than $1/q^4$ at large energies [31]. This implies the conditions [32]:

$$F_V^2 - F_A^2 \; = \; F^2 \, , \qquad M_V^2 F_V^2 - M_A^2 F_A^2 \; = \; 0 \, . \tag{2.33}$$

4. Scalar form factor. The two pseudo-Goldstone bosons matrix element of the scalar quark current contains another dynamical form factor, which for the $K\pi$ case takes the form [33]:

$$\mathcal{F}^s_{K\pi}(q^2) \; = \; 1 + \frac{4c_m}{F^2} \left(c_d + (c_m - c_d) \frac{M_K^2 - M_\pi^2}{M_S^2} \right) \frac{q^2}{M_S^2 - q^2} \, , \tag{2.34}$$

Requiring $\mathcal{F}^s_{K\pi}(q^2)$ to vanish at $q^2 \to \infty$, one finds that [33]:

$$4c_d c_m \; = \; F^2 \, , \qquad c_m - c_d \; = \; 0 \, . \tag{2.35}$$

5. $SS - PP$ sum rules. The difference of the two-point correlation functions of two scalar and two pseudoscalar currents reads

$$\Pi_{S-P}(q^2) \; = \; 16 B_0^2 \left(\frac{c_m^2}{M_S^2 - q^2} - \frac{d_m^2}{M_P^2 - q^2} + \frac{F^2}{8q^2} \right) \, . \tag{2.36}$$

For massless quarks, Π_{S-P} vanishes as $1/q^4$ at large energies, with a small coefficient [34]. Imposing this behaviour [35],

$$8 \left(c_m^2 - d_m^2 \right) \; = \; F^2 \, , \qquad c_m^2 M_S^2 - d_m^2 M_P^2 \simeq 0 \, . \tag{2.37}$$

Finally, assuming Eqs. (2.29), (2.31), (2.33), (2.35) and (2.37) one has that

$$\begin{aligned} F_V \; = \; 2G_V \; = \; \sqrt{2} F_A \; = \; \sqrt{2} F \, , \qquad M_A \; = \; \sqrt{2} M_V \, , \\ c_m \; = \; c_d \; = \; \sqrt{2} d_m \; = \; \frac{F}{2} \, , \qquad M_P \; \simeq \; \sqrt{2} M_S \, , \end{aligned} \tag{2.38}$$

that is, all the parameters of $\mathcal{L}_R$ are given in terms of the pion decay constant F and the two masses of the vector and scalar multiplets, M_V and M_S.

Considering the more general lagrangian of Eq. (2.5) all former constraints are valid except the ones coming from the axial and scalar form factor. In the case of the axial form factor, there are new contributions from Eq. (2.19), see Eq. (D.84) in Appendix D. For the two pseudo-Goldstone bosons matrix element of the scalar quark current there are new contributions when one consider massive quarks. Notice that the required field redefinition of the scalar field, needed to remove the tadpole [36], would generate new contributions to the form factor coming from pieces with two resonances. So that only the first constraint of Eq. (2.35) would be valid in the general case. The couplings of $\mathcal{L}_R$ are fixed now in terms of F and the resonance masses:

$$F_V^2 = F^2 \frac{M_A^2}{M_A^2 - M_V^2}\,, \quad F_A^2 = F^2 \frac{M_V^2}{M_A^2 - M_V^2}\,, \quad G_V^2 = F^2 \frac{M_A^2 - M_V^2}{M_A^2}\,, \quad M_A^2 > M_V^2$$
$$c_m^2 = \frac{F^2}{8} \frac{M_P^2}{M_P^2 - M_S^2}\,, \quad d_m^2 = \frac{F^2}{8} \frac{M_S^2}{M_P^2 - M_S^2}\,, \quad c_d^2 = \frac{F^2}{2} \frac{M_P^2 - M_S^2}{M_P^2}\,, \quad M_P^2 > M_S^2\,. \tag{2.39}$$

2.5 Leading Resonance Contributions to the $\mathcal{O}(p^4)$ χPT Lagrangian

It seems natural to expect that the lowest-mass resonances play an important role on the pseudo-Goldstone bosons dynamics, i.e. Chiral Perturbation Theory. Below the ρ mass scale, the singularities associated with the pole of the resonance propagators can be replaced by the corresponding momentum expansion; the exchange of virtual resonances generates pseudo-Goldstone bosons couplings proportionals to powers of $1/M_R^2$. It can be better understood by using the EFT ideas of Chapter 1. By integrating out the lowest-mass resonances, that is, going from RχT to χPT, one would expect to obtain the largest contributions to the chiral LEC's. The so-called resonance saturation involves considering that the couplings of χPT are largely saturated by the resonance exchange. It can be justified using large-N_C arguments, since tree-level resonance contributions are leading in the $1/N_C$ expansion, to be compared to other contributions related to chiral loops.

In the manner that it has been pointed out in Section 2.3.1, the only possible leading resonance contributions to the χPT LEC's of $\mathcal{L}_4^{\chi PT}$ come from operators constructed with one resonance field and one chiral tensor of $\mathcal{O}(p^2)$ in the chiral counting, $\mathcal{L}_R$ of Eq. (2.5). In Ref. [17] these resonance contributions were studied thoroughly. Under the Single Resonance Approximation and considering nonets for the resonance fields, as large-N_C motivates, one finds the following contributions at leading order in the $1/N_C$ expansion:

$$L_1 = \frac{G_V^2}{8M_V^2}\,, \qquad L_2 = \frac{G_V^2}{4M_V^2}\,, \qquad L_3 = -\frac{3G_V^2}{4M_V^2} + \frac{c_d^2}{2M_S^2}\,,$$
$$L_4 = 0\,, \qquad L_5 = \frac{c_d c_m}{M_S^2}\,, \qquad L_6 = 0\,,$$
$$L_7 = 0\,, \qquad L_8 = \frac{c_m^2}{2M_S^2} - \frac{d_m^2}{2M_P^2}\,, \qquad L_9 = \frac{F_V G_V}{2M_V^2}\,,$$

i	$L_i^r(M_\rho)$	V	A	S	η_1	Total	Total[b)]
1	0.4 ± 0.3	0.6	0.0	0.0	0.0	0.6	0.9
2	1.4 ± 0.3	1.2	0.0	0.0	0.0	1.2	1.8
3	-3.5 ± 1.1	-3.6	0.0	0.6	0.0	-3.0	-4.9
4	-0.3 ± 0.5	0.0	0.0	0.0	0.0	0.0	0.0
5	1.4 ± 0.5	0.0	0.0	1.4[a)]	0.0	1.4	1.4
6	-0.2 ± 0.3	0.0	0.0	0.0	0.0	0.0	0.0
7	-0.4 ± 0.2	0.0	0.0	0.0	-0.3	-0.3	-0.3
8	0.9 ± 0.3	0.0	0.0	0.9[a)]	0.0	0.9	0.9
9	6.9 ± 0.7	6.9[a)]	0.0	0.0	0.0	6.9	7.3
10	-5.5 ± 0.7	-10.0	4.0	0.0	0.0	-6.0	-5.5

Table 2.3: Comparison between the different resonance-exchange contribution with the phenomenologically determined values of $L_i^r(M_\rho)$, in units of 10^{-3} [2]. Motivated by the large-N_C limit we include $U(3)$ multiplets for the resonances. We consider only the contribution from the η_1 in the pseudoscalar channel. The superindex a) refers to an input, whereas in b) the short-distance constraints are taken into account.

$$L_{10} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}\,, \qquad H_1 = -\frac{F_V^2}{8M_V^2} - \frac{F_A^2}{8M_A^2}\,, \qquad H_2 = \frac{c_m}{M_S^2} + \frac{d_m^2}{M_P^2}\,. \tag{2.40}$$

Notice that it is not surprising to miss contributions to L_4, L_6 and L_7 taking into account its subleading order in the $1/N_C$ expansion, see Table 2.1.

η_1 is usually integrated out from the χPT lagrangian. Neglecting then the higher-mass P resonances, the only remaining meson exchange is the one associated with this field, which generates a sizable contribution to L_7,

$$L_7 = -\frac{\widetilde{d}_{\eta_1}^2}{2M_{\eta_1}^2}\,. \tag{2.41}$$

Note that if η_1 is integrated out, L_7 appears naively to be of $\mathcal{O}(N_C^2)$, since $M_{\eta_1}^2 \sim \mathcal{O}(1/N_C)$ in Eq. (2.41). However, the $1/N_C$ counting is not well defined in this case, since N_C cannot be small (M_{η_1} heavy) and big ($1/N_C$ expansion) at the same time.

In Table 2.3 we compare the phenomenological values of these couplings together with the ones predicted by the resonance exchanges. The assumption of resonance saturation has given successful predictions for L_i.

A last remark is suitable. Though the scale at which the results of the integration, μ_0, is known to be of the order of a typical scale of the physical system, let us say $\mu_0 = M_R$, there always remains some ambiguity on the precise value of μ_0 at which the resonance contributions are given. The next-to-leading order predictions would avoid this problem, as the running is under control. See Chapter 4 for more information.

Chapter 3

Vector Form Factor at NLO in the $1/N_C$ Expansion

3.1 Introduction

Quantum loops including virtual resonances are a major technical challenge which still has not been properly addressed in Resonance Chiral Theory. A first step in this direction was the study of resonance loop contributions to the running of the χPT coupling $L_{10}(\mu)$, performed in Ref. [37], which however did not attempt an analysis of the induced ultraviolet divergences and their corresponding renormalization.

Quantum loops involving massive states have been only analysed within explicit models with additional symmetries. For instance, the gauge structure advocated in the so-called "Hidden Local Symmetry" description of vector resonances [38] implies a much simpler ultraviolet behaviour [39]. Loop corrections to some resonance parameters have also been studied [40, 41] within the context of "Heavy Vector Meson χPT" [42], which adopts the $M_R \to \infty$ limit to guarantee a good chiral power counting; and Ref. [43] in the Nambu-Jona-Lasinio model [44].

At the one-loop level the massive states present in RχT generate all kind of ultraviolet problems which start now to be understood. A naive chiral power counting indicates that the renormalization procedure will require higher dimensional counterterms, which presumably could generate a problematic behaviour at large momenta. Therefore, it will be necessary to perform a careful investigation of the constraints implied by the short-distance properties of QCD at the next-to-leading order in $1/N_C$.

A formal renormalization of RχT at the one-loop level appears to be a very involved task, which requires the prior analysis of several technical ingredients, as can be seen in Chapter 5. In order to gain some understanding on the ultraviolet behaviour, it seems worth to perform first some explicit one-loop calculations of well chosen physical amplitudes. In this chapter, we present a detailed investigation of the pion vector form factor (VFF) at next-to-leading order in the $1/N_C$ expansion. This observable is defined through the two pseudo-Goldstone matrix element of the vector current:

$$\langle\, \pi^+(p_1)\, \pi^-(p_2)\, |\, \frac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right) |0\,\rangle \;=\; \mathcal{F}(q^2)\, (p_1 - p_2)^\mu\,, \qquad (3.1)$$

where $q^\mu \equiv (p_1+p_2)^\mu$. At very low energies, the VFF $\mathcal{F}(q^2)$ has been studied within the χPT framework up to $\mathcal{O}(p^6)$ [9, 45]. RχT and the $1/N_C$ expansion have also been used to determine $\mathcal{F}(q^2)$ at the ρ meson peak, including appropriate resummations of subleading infrared logarithms [46, 47].

We will simplify the calculation working in the two flavour theory and taking the massless quark limit. Therefore, we will assume a chiral $U(2)_L \otimes U(2)_R$ symmetry group. The small effects induced by the $U(1)_A$ anomaly will be neglected, because they are not going to be relevant in our discussion. As the isosinglet pseudoscalar can only appear within loops, and the numerical correction generated by its non-zero mass could be taken into account in a straightforward way, together with the finite quark mass effects which we are ignoring.

In the next section we will briefly resume the RχT lagrangian of interest. We will only consider the minimal set of resonance couplings (linear in the resonance fields) introduced in Ref. [17], supplemented with those counterterms required by the renormalization procedure. Notice that one of the main aims of this chapter is to justify the necessity of considering operators with more than one resonance field, in the spirit of the short-distance behaviour of our result. The renormalization of the relevant one-particle-irreducible (1PI) Feynman diagrams will be discussed in Section 3.3 and the final results of our calculation will be collected in Section 3.4. Sections 3.5 and 3.6 analyse the behaviour of the computed vector form factor at low and high energies, respectively. We will finally summarize our findings in Section 3.7. Several technical details and results have been moved to the appendices.

3.2 The Lagrangian

We are going to work within a $U(2)_L \otimes U(2)_R$ chiral theory, containing a multiplet of 4 pseudo-Goldstone bosons,

$$\phi \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_0 \end{pmatrix}\,, \qquad (3.2)$$

to be compared to the $U(3)_L \otimes U(3)_R$ case of Eq. (2.3). Under the Single Resonance Approximation, the pseudo-Goldstone bosons couple to massive $U(2)$ multiplets of the type $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$, with a field content analogous to the one indicated in Eq. (3.2).

Our starting point is the RχT lagrangian introduced in Ref. [17], where, besides the kinetic pieces, only linear couplings in the resonance fields are included, since the intention of Ref. [17] was to obtain the leading resonance contributions to the LEC's of the $\mathcal{O}(p^4)$ χPT lagrangian. Therefore, $\mathcal{L}_{R\chi T}$ reads:

$$\mathcal{L}_{R\chi T}(\phi, \mathrm{V}, \mathrm{A}, \mathrm{S}, \mathrm{P}) \;=\; \mathcal{L}^{(2)}_{pGB} + \sum_R \left(\mathcal{L}_{\mathrm{kin}\, R} + \mathcal{L}_R\right) + \mathcal{L}^{NLO}_{R\chi T}\,, \qquad (3.3)$$

where R runs over all the different resonance fields, V, A, S and P. The notation of Section 2.3.2 is followed: $\mathcal{L}^{(2)}_{pGB}$ is shown in Eq. (2.6); the different kinetic pieces are given in Eqs. (2.11) and (2.12); and the interactive terms are defined in Eqs. (2.7), (2.8), (2.9) and (2.10). $\mathcal{L}^{NLO}_{R\chi T}$ refers to the subleading pieces, which will be defined below.

As it has been explained in the last chapter, taking into account that only the RχT lagrangian of Eq. (3.3) is considered, one should take the usual constraints of Eq. (2.38):

$$\begin{aligned} F_V \;=\; 2G_V \;=\; \sqrt{2}F_A \;=\; \sqrt{2}F\,, &\qquad M_A \;=\; \sqrt{2}M_V\,, \\ c_m \;=\; c_d \;=\; \sqrt{2}d_m \;=\; \frac{F}{2}\,, &\qquad M_P \;\simeq\; \sqrt{2}M_S\,. \end{aligned} \tag{3.4}$$

3.2.1 Subleading Lagrangian

The one loop calculation of the vector form factor with the previous lagrangian generates ultraviolet divergences which require counterterms with a higher number of derivatives. We will only include the minimal set of chiral structures needed to renormalize our calculation. We expect their corresponding couplings to be subleading in the $1/N_C$ expansion, since they are associated with quantum loop corrections.

The following $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ pseudo-Goldstone interactions will be required:

$$\widetilde{\mathcal{L}}^{(4)}_{pGB} \;=\; \frac{i\,\widetilde{\ell}_6}{4}\langle\, f_+^{\mu\nu}\,[u_\mu, u_\nu]\,\rangle - \widetilde{\ell}_{12}\,\langle\, \nabla^\mu u_\mu \nabla^\nu u_\nu\,\rangle\,, \tag{3.5}$$

$$\widetilde{\mathcal{L}}^{(6)}_{pGB} \;=\; i\,\widetilde{c}_{51}\,\langle\, \nabla^\rho f_+^{\mu\nu}[h_{\mu\rho}, u_\nu]\,\rangle + i\,\widetilde{c}_{53}\,\langle\, \nabla_\mu f_+^{\mu\nu}[h_{\nu\rho}, u^\rho]\,\rangle\,. \tag{3.6}$$

Note that the superindex indicates the chiral order of the operator. We use a tilde to denote the RχT couplings in Eqs. (3.5) and (3.6), which are different to the ones with the same names (without tilde) in χPT. For instance, the chiral coupling ℓ_6 (L_9 in the three flavour case) is dominated by a contribution from vector-meson exchange and is of $\mathcal{O}(N_C)$, while the corresponding resonance coupling $\widetilde{\ell}_6$ does not contain this contribution and is of $\mathcal{O}(1)$.

The operator with $\widetilde{\ell}_{12}$ in Eq. (3.5) does not contribute to the tree-level calculation; nevertheless, it is needed to renormalize the pseudo-Goldstone self-energies. At $\mathcal{O}(p^6)$, only the combination of couplings $\widetilde{r}_{V2} \equiv 4F^2\,(\widetilde{c}_{53} - \widetilde{c}_{51})$ is going to be relevant for the VFF [10]. Including the lagrangians of Eqs. (3.5) and (3.6), the tree-level calculation of the vector form factor gives the result:

$$\mathcal{F}(q^2) \;=\; 1 + \frac{F_V\,G_V}{F^2}\,\frac{q^2}{M_V^2 - q^2} - \widetilde{\ell}_6\,\frac{q^2}{F^2} + \widetilde{r}_{V2}\,\frac{q^4}{F^4}\,. \tag{3.7}$$

The Brodsky-Lepage requirement that the form factor should vanish at $q^2 \to \infty$ implies the following conditions at leading order in $1/N_C$:

$$F_V\,G_V \;=\; F^2 \quad , \quad \widetilde{\ell}_6 \;=\; 0 \quad , \quad \widetilde{r}_{V2} \;\equiv\; 4F^2\,(\widetilde{c}_{53} - \widetilde{c}_{51}) \;=\; 0\,. \tag{3.8}$$

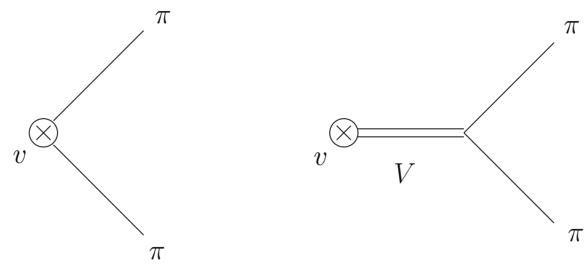


Figure 3.1: Leading-order contributions to the vector form factor of the pion. A single line stands for a pseudo-Goldstone boson while a double line indicates a vector resonance.

Therefore, the couplings $\widetilde{\ell}_6/F^2$ and $\widetilde{r}_{V2}/F^4$ are of subleading order in the $1/N_C$ expansion, i.e. $\mathcal{O}(1/N_C)$, as expected on pure dimensional grounds.

The renormalization of Green Functions including resonance fields forces the presence of the following additional counterterms:

$$\begin{aligned}
\mathcal{L}_Z^{(4)} &= \frac{X_{Z_1}}{2}\langle\, \nabla^2 V^{\mu\nu}\,\{\nabla_\nu,\nabla^\sigma\}\,V_{\mu\sigma}\,\rangle + \frac{X_{Z_2}}{4}\langle\,\{\nabla_\nu,\nabla_\alpha\}\,V^{\mu\nu}\,\{\nabla^\sigma,\nabla^\alpha\}\,V_{\mu\sigma}\,\rangle \\
&\quad + \frac{X_{Z_3}}{4}\langle\,\{\nabla^\sigma,\nabla^\alpha\}\,V^{\mu\nu}\,\{\nabla_\nu,\nabla_\alpha\}\,V_{\mu\sigma}\,\rangle\,, \qquad (3.9)\\
\mathcal{L}_F^{(4)} &= X_{F_1}\langle\, V_{\mu\nu}\nabla^2 f_+^{\mu\nu}\,\rangle + X_{F_2}\langle\, V_{\mu\nu}\,\{\nabla^\mu,\nabla_\alpha\}\,f_+^{\alpha\nu}\,\rangle\,, \qquad (3.10)\\
\mathcal{L}_G^{(4)} &= i\,X_{G_1}\langle\,\{\nabla^\alpha,\nabla_\mu\}\,V^{\mu\nu}\,[u_\nu,u_\alpha]\,\rangle + i\,X_{G_2}\langle\, V^{\mu\nu}\,[h_{\alpha\mu},h_\nu^\alpha]\,\rangle\,. \qquad (3.11)
\end{aligned}$$

The quadratic lagrangian $\mathcal{L}_Z^{(4)}$ is needed to renormalize the vector self-energy. Actually, only the sum of couplings $X_Z \equiv X_{Z_1}+X_{Z_2}+X_{Z_3}$ is relevant for this purpose. The renormalization of the vector matrix element of the vector current involves the sum of $\mathcal{L}_F^{(4)}$ couplings $X_F \equiv X_{F_1}+X_{F_2}$. Finally, the vertex with one external vector resonance and two pseudo-Goldstone legs is renormalized by $\mathcal{L}_G^{(4)}$ through the combination $X_G \equiv X_{G_2} - X_{G_1}/2$. The dimensions of the couplings are $[X_Z] = E^{-2}$ and $[X_F] = [X_G] = E^{-1}$.

Finally, following the notation of Eq. (3.3), one has that

$$\mathcal{L}_{R\chi T}^{NLO} \;=\; \widetilde{\mathcal{L}}_{pGB}^{(4)} + \widetilde{\mathcal{L}}_{pGB}^{(6)} + \mathcal{L}_Z^{(4)} + \mathcal{L}_F^{(4)} + \mathcal{L}_G^{(4)}\,. \qquad (3.12)$$

At next-to-leading order in $1/N_C$, these counterterm lagrangians only contribute through tree-level diagrams. One can then use the leading order equations of motion,

$$\nabla^\mu\nabla_\rho V^{\rho\nu} - \nabla^\nu\nabla_\rho V^{\rho\mu} \;=\; -M_V^2\,V^{\mu\nu} - \frac{F_V}{\sqrt{2}}\,f_+^{\mu\nu} - \frac{iG_V}{\sqrt{2}}\,[u^\mu,u^\nu]\,, \qquad (3.13)$$

to reduce the number of relevant operators. The lagrangians of Eqs. (3.9), (3.10) and (3.11) take then the equivalent forms:

$$\begin{aligned}
\mathcal{L}_Z^{(4)}|_{\rm EOM} &= \frac{X_Z M_V^4}{2}\langle\, V^{\mu\nu}V_{\mu\nu}\,\rangle + \frac{X_Z M_V^2 F_V}{\sqrt{2}}\langle\, V_{\mu\nu} f_+^{\mu\nu}\,\rangle + \frac{i\, X_Z M_V^2 G_V}{\sqrt{2}}\langle\, V_{\mu\nu}\,[u^\mu, u^\nu]\,\rangle \\
&\quad + \frac{i\, X_Z F_V G_V}{2}\langle\, f_+^{\mu\nu}\,[u_\mu, u_\nu]\,\rangle + \cdots\,, \qquad (3.14)\\
\mathcal{L}_F^{(4)}|_{\rm EOM} &= -X_F M_V^2 \langle\, V_{\mu\nu} f_+^{\mu\nu}\,\rangle - \frac{i\, X_F G_V}{\sqrt{2}}\langle\, f_+^{\mu\nu}\,[u_\mu, u_\nu]\,\rangle + \cdots\,, \qquad (3.15)\\
\mathcal{L}_G^{(4)}|_{\rm EOM} &= -2i\, X_G M_V^2 \langle\, V^{\alpha\nu}\,[u_\alpha, u_\nu]\,\rangle - i\,\sqrt{2}\, X_G F_V \langle\, f_+^{\mu\nu}\,[u_\mu, u_\nu]\,\rangle + \cdots\,, \qquad (3.16)
\end{aligned}$$

where the dots denote other terms which are not relevant for the VFF calculation, at this order. The derivatives acting on the vector resonance fields have been traded by the heavy mass scale M_V and/or derivatives acting on the pseudo-Goldstone fields, giving rise to the usual tensor structures of the χPT lagrangian. Therefore, the effect of the counterterm lagrangians $\mathcal{L}_Z^{(4)}$, $\mathcal{L}_F^{(4)}$ and $\mathcal{L}_G^{(4)}$ is just equivalent to the following shift in the couplings at next-to-leading order in $1/N_C$:

$$\begin{aligned}
\widetilde{\ell}_6^{\,\rm eff} &= \widetilde{\ell}_6 + 2\, X_Z F_V G_V - 2\sqrt{2}\, X_F G_V - 4\sqrt{2}\, X_G F_V\,,\\
F_V^{\rm eff} &= F_V + 2\, X_Z M_V^2 F_V - 2\sqrt{2}\, X_F M_V^2\,,\\
G_V^{\rm eff} &= G_V + 2\, X_Z M_V^2 G_V - 4\sqrt{2}\, X_G M_V^2\,,\\
(M_V^2)^{\rm eff} &= M_V^2 + 2\, X_Z M_V^4\,,\\
\widetilde{r}_{V2}^{\,\rm eff} &= \widetilde{r}_{V2}\,. \qquad (3.17)
\end{aligned}$$

Thus, since $\widetilde{\ell}_6^{\,\rm eff} \sim \widetilde{\ell}_6 \sim (M_V^2)^{\rm eff} \sim M_V^2 \sim \mathcal{O}(1)$ and $F_V^{\rm eff} \sim F_V \sim G_V^{\rm eff} \sim G_V \sim \mathcal{O}(\sqrt{N_C})$, a consistent $1/N_C$ counting requires that X_G and X_F are of $\mathcal{O}(1/\sqrt{N_C})$ and X_Z of $\mathcal{O}(1/N_C)$.

3.3 Renormalization

The renormalization procedure follows very systematic and precise steps in any well defined quantum field theory. First of all, the two-point Green Functions must be renormalized. Later the three-point Green Functions and so on. For the vector form factor up to next-to-leading order in the $1/N_C$ expansion only the two- and three-point Green Functions will contribute. The corresponding renormalizations for the one-particle-irreducible diagrams at one-loop level are given in the next subsections.

We will adopt the $\overline{MS}-1$ scheme, usually employed in χPT calculations, where one subtracts the divergent constant

$$\lambda_\infty = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log 4\pi - 1\,, \qquad (3.18)$$

being D the space-time dimension and $\gamma_E \simeq 0.5772$ the Euler's constant. However, we will impose the on-shell condition to renormalize the pion self-energy. This simplifies the calculation of physical amplitudes with external pions. Since we work in

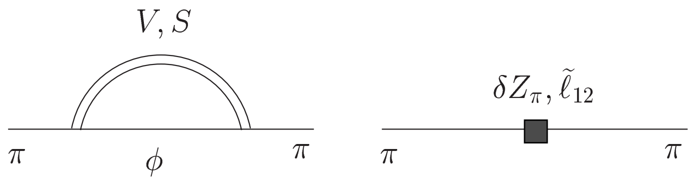


Figure 3.2: One-loop diagrams and local contributions to the pion self-energy.

the massless quark limit, the pseudo-Goldstone tadpoles will not give any contribution. The precise definition of the relevant Feynman integrals with one, two and three propagators are relegated to Appendix B, while the contributions from each diagram are shown in Appendix C.

3.3.1 Pion Self-energy

The diagrams contributing to the pion propagator are shown in Figure 3.2. The kinetic lagrangians of Eqs. (2.11) and (2.12) generate additional tadpole topologies with one resonance propagator, but they are identically zero even with massive pions. The divergences of $\mathcal{O}(p^2)$ are reabsorbed through the wave-function renormalization $\pi^b = (1+\delta Z_\pi)^{\frac{1}{2}}\pi^r$, being π^b and π^r the bare and renormalized pion fields respectively. In the on-shell scheme,

$$\delta Z_\pi = -\frac{2G_V^2}{F^2}\frac{3M_V^2}{16\pi^2F^2}\left\{\lambda_\infty + \log\frac{M_V^2}{\mu^2} + \frac{1}{6}\right\} + \frac{4c_d^2}{F^2}\frac{M_S^2}{16\pi^2F^2}\left\{\lambda_\infty + \log\frac{M_S^2}{\mu^2} - \frac{1}{2}\right\} . \tag{3.19}$$

There are also divergences of $\mathcal{O}(p^4)$ which renormalize one of the couplings in $\widetilde{\mathcal{L}}_\chi^{(4)}$:

$$\widetilde{\ell}_{12} \equiv \widetilde{\ell}^r_{12}(\mu) + \delta\widetilde{\ell}_{12}(\mu)\,, \qquad \delta\widetilde{\ell}_{12}(\mu) = -\frac{G_V^2+2c_d^2}{F^2}\frac{\lambda_\infty}{32\pi^2}\,. \tag{3.20}$$

The renormalized pion self-energy takes the form

$$-i\,\Sigma_\pi^r(p^2) = -i\,\frac{p^4}{16\pi^2F^2}\left\{64\pi^2\widetilde{\ell}^r_{12}(\mu) + \frac{2G_V^2}{F^2}\left[\log\frac{M_V^2}{\mu^2} + \phi\left(\frac{p^2}{M_V^2}\right)\right] + \frac{4c_d^2}{F^2}\left[\log\frac{M_S^2}{\mu^2} + \phi\left(\frac{p^2}{M_S^2}\right)\right]\right\}, \tag{3.21}$$

where the function $\phi(p^2/M_V^2)$,

$$\phi(x) = \left(1-\frac{1}{x}\right)^2\left[\left(1-\frac{1}{x}\right)\log\left(1-x\right) - 1 + \frac{x}{2}\right] = -(1-x)^2\sum_{n=0}^{\infty}\frac{x^n}{(n+2)(n+3)}, \tag{3.22}$$

contains finite and scale-independent contributions.

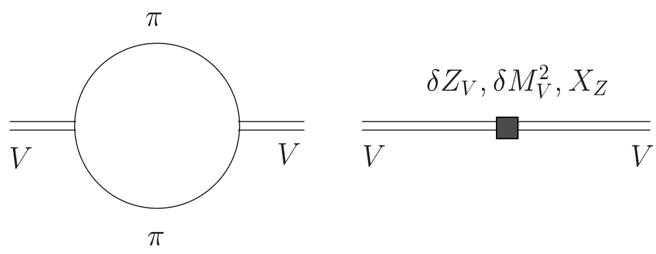


Figure 3.3: One-loop diagrams and local contributions to the ρ self-energy.

3.3.2 Rho Self-energy

The one-loop ρ self-energy contains only an $\mathcal{O}(p^4)$ divergence, which renormalizes the coupling X_Z of the subleading resonance lagrangian:

$$X_Z \equiv X_Z^r(\mu) + \delta X_Z(\mu)\,, \qquad \delta X_Z(\mu) = -\frac{2G_V^2}{F^2}\frac{\lambda_\infty}{192\pi^2 F^2}\,. \tag{3.23}$$

Thus, the vector mass and wave-function are not renormalized:

$$\delta M_V^2 = 0\,, \qquad \delta Z_V = 0\,. \tag{3.24}$$

The renormalized ρ self-energy then becomes:

$$-i\,\Sigma_V^r(q)^{\mu\nu,\rho\sigma} = -\frac{i}{2}\Omega^{L\,\mu\nu,\,\rho\sigma}(q)\,\Sigma_V^r(q^2)\,, \tag{3.25}$$

where the antisymmetric tensor structure $\Omega^{L\,\mu\nu,\rho\sigma}(q)$ is defined in Appendix A and

$$\Sigma_V^r(q^2) = -q^4\left\{2X_Z^r(\mu) - \frac{2G_V^2}{F^2}\frac{1}{F^2}\left[\frac{1}{6}\hat{B}_0(q^2/\mu^2) + \frac{1}{144\pi^2}\right]\right\}\,, \tag{3.26}$$

with $\hat{B}_0(q^2/\mu^2)$ defined in Appendix B.

3.3.3 $\langle v^\mu\, V^{\rho\sigma}\rangle$ One-particle-irreducible Vertex

The one-particle-irreducible amputated diagrams connecting an external vector quark current to an outgoing vector resonance are shown in Figure 3.4. The one-loop contribution brings an $\mathcal{O}(p^4)$ divergence which gets reabsorbed through the following renormalization of the coupling X_F:

$$X_F \equiv X_F^r(\mu) + \delta X_F(\mu)\,, \qquad \delta X_F(\mu) = -\frac{\sqrt{2}G_V}{F}\frac{\lambda_\infty}{192\pi^2 F}\,. \tag{3.27}$$

Since there are no divergences of $\mathcal{O}(p^2)$, the lowest-order coupling F_V remains unchanged:

$$\delta F_V = 0\,. \tag{3.28}$$

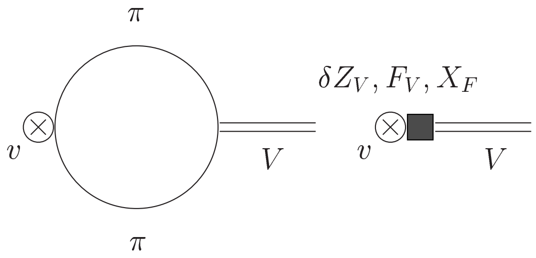


Figure 3.4: Diagrams contributing to the $\langle v^\mu \, V^{\rho\sigma}\rangle$ Green Function at NLO in $1/N_C$.

The renormalized vertex function takes the form

$$i\,\Phi(q)^{\mu,\rho\sigma} \;=\; -i\,\mathcal{I}^{\rho\sigma}_{\alpha\beta}\,q^\alpha\,g^{\mu\beta}\left\{F_V - 2\sqrt{2}X_F^r(\mu)q^2 + \frac{2G_V}{F^2}q^2\left[\frac{1}{6}\hat{B}_0(q^2/\mu^2) + \frac{1}{144\pi^2}\right]\right\}\,, \qquad (3.29)$$

where the first term is the leading order contribution. The antisymmetric tensor structure $\mathcal{I}^{\rho\sigma}_{\alpha\beta}$ is defined in Appendix A and the massless two-point function $\hat{B}_0(q^2/\mu^2)$ in Appendix B.

3.3.4 $\langle V_{\mu\nu}\pi\pi\rangle$ One-particle-irreducible Vertex

The one-particle-irreducible amputated diagrams connecting a vector resonance with two outgoing pseudo-Goldstone bosons at next-to-leading order in $1/N_C$ are shown in Figure 3.5. The loop diagrams generate $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ divergences, which renormalize the couplings G_V and X_G, respectively:

$$G_V \equiv G_V^r(\mu) + \delta G_V(\mu),\;\; \delta G_V = G_V\left[3M_V^2\left(\frac{2G_V^2}{F^2} - \frac{1}{2}\right) - M_S^2\frac{4c_d^2}{F^2}\right]\frac{\lambda_\infty}{16\pi^2F^2}, \qquad (3.30)$$

$$X_G \equiv X_G^r(\mu) + \delta X_G(\mu),\;\; \delta X_G = \frac{\sqrt{2}G_V}{F}\left[\frac{2G_V^2}{F^2} + \frac{4c_d^2}{F^2} - 2\right]\frac{\lambda_\infty}{1536\pi^2F}. \qquad (3.31)$$

The wave-function renormalization of the external vector and pion legs amounts to a global factor $\left(\delta Z_\pi + \frac{1}{2}\delta Z_V\right)$ multiplying the lowest-order contribution (keep in mind that $\delta Z_V = 0$). Taking this into account, one finally gets the finite vertex function

$$i\,\Gamma^r_{\mu\nu}(p_1,p_2) = \mathcal{I}^{\alpha\beta}_{\mu\nu}\,q_\alpha\,(p_1-p_2)_\beta\frac{1}{F^2}\left\{G_V^r(\mu)\left[1 - \Delta\Gamma(q^2,\mu^2)\right] - 4\sqrt{2}X_G^r(\mu)q^2\right\}\,, \qquad (3.32)$$

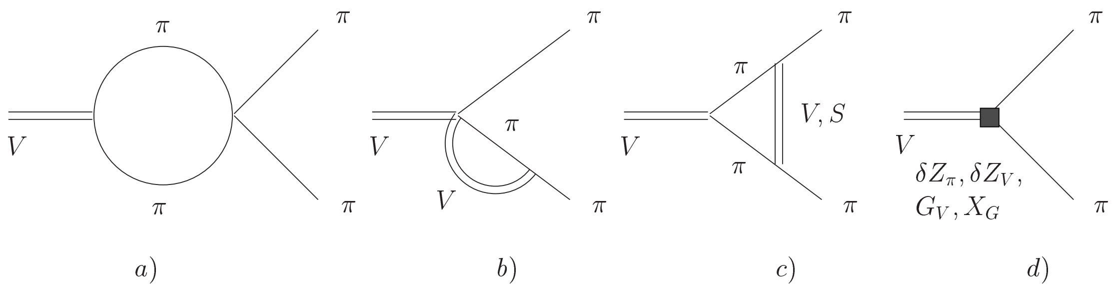


Figure 3.5: NLO diagrams contributing to the three-point Green Function $V^{\mu\nu} \to \pi\pi$.

where

$$
\begin{aligned}
\Delta\Gamma(q^2,\mu^2) = \frac{1}{F^2}\Bigg\{ & \hat{B}_0(q^2/\mu^2)\left[\frac{2G_V^2}{F^2}\left(\frac{M_V^4}{q^2}+2M_V^2+\frac{q^2}{12}\right)+\frac{4c_d^2}{F^2}\left(\frac{M_S^4}{q^2}+\frac{q^2}{12}\right)-\frac{q^2}{6}\right] \\
&+\frac{M_V^2}{16\pi^2}\log\frac{M_V^2}{\mu^2}\left[\frac{2G_V^2}{F^2}\left(\frac{M_V^2}{q^2}+5\right)-\frac{3}{2}\right]+\frac{M_S^2}{16\pi^2}\log\frac{M_S^2}{\mu^2}\frac{4c_d^2}{F^2}\left(\frac{M_S^2}{q^2}-1\right) \\
&+\frac{M_V^2}{64\pi^2}\left[3\frac{2G_V^2}{F^2}-1\right]+\frac{3M_S^2}{64\pi^2}\frac{4c_d^2}{F^2}+\frac{q^2}{288\pi^2}\left[\frac{2G_V^2}{F^2}+\frac{4c_d^2}{F^2}-2\right] \\
&+\frac{2G_V^2}{F^2}C_0(q^2,0,0,M_V^2)\left[\frac{M_V^6}{q^2}+\frac{5M_V^4}{2}+q^2M_V^2\right] \\
&+\frac{4c_d^2}{F^2}C_0(q^2,0,0,M_S^2)\left[\frac{M_S^6}{q^2}+\frac{M_S^4}{2}\right]\Bigg\}\,. \qquad (3.33)
\end{aligned}
$$

The three-propagator integral $C_0(q^2, M_a^2, M_b^2, M_c^2)$ is defined in Appendix B.

3.3.5 $\langle v_\mu \pi\pi \rangle$ One-particle-irreducible Vertex

The divergences generated by the one-particle-irreducible loop diagrams shown in Figure 3.6 get reabsorbed through the renormalization of the pion wave function δZ_π and the $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ couplings $\widetilde{\ell}_6$ and $\widetilde{r}_{V2}$:

$$
\widetilde{\ell}_6 \equiv \widetilde{\ell}_6^r(\mu)+\delta\widetilde{\ell}_6(\mu)\,, \quad \delta\widetilde{\ell}_6(\mu) = \left\{3-2\frac{2G_V^2}{F^2}+\frac{4c_d^2}{F^2}\right\}\frac{\lambda_\infty}{96\pi^2}\,, \qquad (3.34)
$$

$$
\widetilde{r}_{V2} \equiv \widetilde{r}_{V2}^r(\mu)+\delta\widetilde{r}_{V2}(\mu)\,, \quad \delta\widetilde{r}_{V2}(\mu) = \frac{F^2\lambda_\infty}{96\pi^2}\left\{\frac{1}{M_V^2}+\frac{1}{M_A^2}\right\}\,. \qquad (3.35)
$$

The resulting finite correction to the lowest-order pion form factor,

$$
\Delta\mathcal{F}(q^2)_{1\mathrm{PI}} = \Delta\mathcal{F}^{\,\mathrm{ct}}+\Delta\mathcal{F}^{\chi}+\Delta\mathcal{F}^{V}+\Delta\mathcal{F}^{A}+\Delta\mathcal{F}^{S}+\Delta\mathcal{F}^{P}\,, \qquad (3.36)
$$

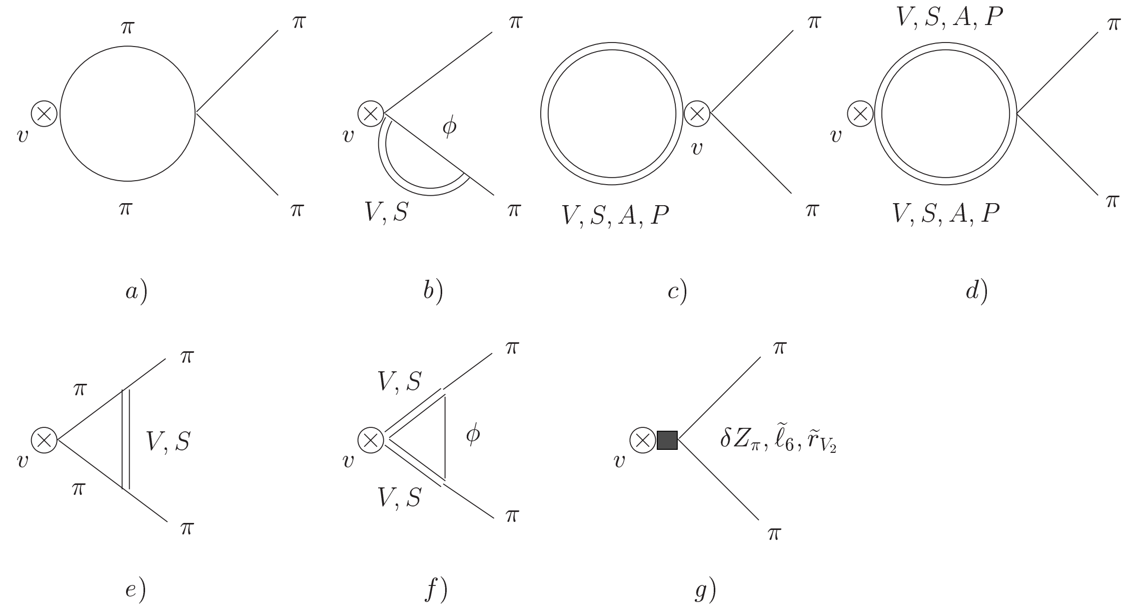


Figure 3.6: 1PI diagrams connecting an external vector current and two outgoing pions, at next-to-leading order in the $1/N_C$ expansion.

contains contributions from tree-level counterterms,

$$\begin{aligned}\Delta\mathcal{F}^{\,\mathrm{ct}} &= -\frac{2G_V^2}{F^2}\frac{M_V^2}{16\pi^2F^2}\left\{3\log\frac{M_V^2}{\mu^2}+\frac{1}{2}\right\}+\frac{4c_d^2}{F^2}\frac{M_S^2}{16\pi^2F^2}\left\{\log\frac{M_S^2}{\mu^2}-\frac{1}{2}\right\}\\ &\quad -\widetilde{\ell}_6^{\,r}(\mu)\frac{q^2}{F^2}+\widetilde{r}_{V2}^{\,r}(\mu)\frac{q^4}{F^4}\,, \end{aligned}\tag{3.37}$$

and loop diagrams with internal pseudo-Goldstone bosons (first diagram in Figure 3.6),

$$\Delta\mathcal{F}^{\chi} = \frac{q^2}{F^2}\left\{\frac{1}{6}\hat{B}_0(q^2/\mu^2)+\frac{1}{144\pi^2}\right\}\,,\tag{3.38}$$

and vector,

$$\begin{aligned}\Delta\mathcal{F}^{V} &= \frac{2G_V^2}{F^2}\frac{1}{F^2}\left\{-C_0(q^2,0,0,M_V^2)\left[\frac{M_V^6}{q^2}+\frac{5M_V^4}{2}+q^2M_V^2\right]\right.\\ &\quad\left.+C_0(q^2,M_V^2,M_V^2,0)\left[\frac{M_V^6}{q^2}+\frac{M_V^4}{2}\right]-\hat{B}_0(q^2/\mu^2)\left[\frac{M_V^4}{q^2}+2M_V^2+\frac{q^2}{12}\right]\right\}\\ &\quad-\frac{\overline{B}_0(q^2,M_V^2)}{F^2}\left[\left(2M_V^2+\frac{q^2}{6}-\frac{q^4}{6M_V^2}\right)+\frac{2G_V^2}{F^2}\left(\frac{M_V^4}{q^2}+\frac{2M_V^2}{3}-\frac{5q^2}{12}\right)\right]\\ &\quad+\frac{M_V^2}{16\pi^2F^2}\log\frac{M_V^2}{\mu^2}\left[\left(\frac{q^2}{2M_V^2}-\frac{q^4}{6M_V^4}\right)-\frac{2G_V^2}{F^2}\left(\frac{M_V^2}{q^2}-1+\frac{5q^2}{12M_V^2}\right)\right]\\ &\quad+\frac{M_V^2}{16\pi^2F^2}\left[\left(\frac{q^2}{2M_V^2}-\frac{2q^4}{9M_V^4}\right)+\frac{2G_V^2}{F^2}\left(\frac{M_V^2}{q^2}+1-\frac{19q^2}{36M_V^2}\right)\right]\,,\end{aligned}\tag{3.39}$$

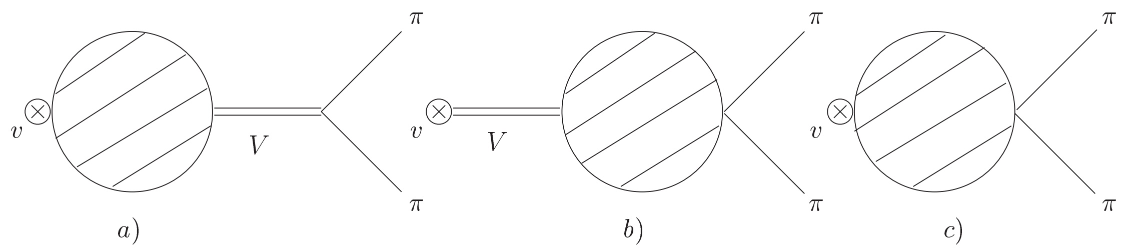


Figure 3.7: Basic topologies contributing to the Vector Form Factor at NLO.

axial-vector,

$$\begin{aligned}\Delta\mathcal{F}^A &= -\frac{\overline{B}_0(q^2,M_A^2)}{F^2}\left[2M_A^2+\frac{q^2}{6}-\frac{q^4}{6M_A^2}\right]+\frac{M_A^2}{16\pi^2F^2}\log\frac{M_A^2}{\mu^2}\left[\frac{q^2}{2M_A^2}-\frac{q^4}{6M_A^4}\right]\\ &\quad+\frac{q^2}{32\pi^2F^2}-\frac{q^4}{72\pi^2F^2M_A^2}\,, \qquad (3.40)\end{aligned}$$

scalar,

$$\begin{aligned}\Delta\mathcal{F}^S &= \frac{4c_d^2}{F^2}\frac{1}{F^2}\left\{-C_0(q^2,0,0,M_S^2)\left[\frac{M_S^6}{q^2}+\frac{M_S^4}{2}\right]+C_0(q^2,M_S^2,M_S^2,0)\left[\frac{M_S^6}{q^2}-\frac{M_S^4}{2}\right]\right.\\ &\quad\left.-\hat{B}_0(q^2/\mu^2)\left[\frac{M_S^4}{q^2}+\frac{q^2}{12}\right]+\frac{M_S^4}{16\pi^2q^2}\right\}-\frac{q^2}{288\pi^2F^2}\left[1+\frac{1}{2}\frac{4c_d^2}{F^2}\right]\\ &\quad-\frac{\overline{B}_0(q^2,M_S^2)}{F^2}\left[\left(\frac{2M_S^2}{3}-\frac{q^2}{6}\right)+\frac{4c_d^2}{F^2}\left(\frac{M_S^4}{q^2}-\frac{M_S^2}{3}+\frac{q^2}{12}\right)\right]\\ &\quad-\frac{M_S^2}{16\pi^2F^2}\log\frac{M_S^2}{\mu^2}\left[\frac{4c_d^2}{F^2}\left(1+\frac{M_S^2}{q^2}-\frac{q^2}{12M_S^2}\right)+\frac{q^2}{6M_S^2}\right]\,, \qquad (3.41)\end{aligned}$$

and pseudoscalar resonances,

$$\Delta\mathcal{F}^P = \frac{\overline{B}_0(q^2,M_P^2)}{F^2}\left[-\frac{2M_P^2}{3}+\frac{q^2}{6}\right]-\frac{q^2}{96\pi^2F^2}\left[\log\frac{M_P^2}{\mu^2}+\frac{1}{3}\right]. \qquad (3.42)$$

All the Feynman integrals are shown in Appendix B.

3.4 Vector Form Factor

The basic topologies contributing to the vector form factor are shown in Figure 3.7, in terms of the one-loop level 1PI diagrams computed in the previous section. The internal ρ line denotes the dressed vector propagator, including the self-energy correction of Eq. (3.26), which regulates the ρ pole. Taking this self-energy and the subleading running of G_V into account, the leading order contribution takes the form:

$$\mathcal{F}(q^2)_{\rm LO} = 1+\frac{F_VG_V^r(\mu)}{F^2}\frac{q^2}{M_V^2-q^2-\Sigma_V^r(q^2)}\,. \qquad (3.43)$$

The topology in Figure 3.7.a generates the following subleading correction:

$$\Delta\mathcal{F}(q^2)_F = \frac{q^2}{M_V^2 - q^2 - \Sigma_V^r(q^2)}\frac{q^2}{F^2}\left\{\frac{2G_V^2}{F^2}\left[\frac{1}{6}\hat{B}_0(q^2/\mu^2) + \frac{1}{144\pi^2}\right] - 2\sqrt{2}G_V X_F^r(\mu)\right\}. \tag{3.44}$$

Figure 3.7.b brings the contribution:

$$\Delta\mathcal{F}(q^2)_G = -\frac{q^2}{M_V^2 - q^2 - \Sigma_V^r(q^2)}\frac{F_V}{\sqrt{2}F}\left\{\frac{\sqrt{2}G_V}{F}\Delta\Gamma(q^2,\mu^2) + \frac{8X_G^r(\mu)}{F}q^2\right\}\,, \tag{3.45}$$

where $\Delta\Gamma(q^2,\mu^2)$ is given in Eq. (3.33). Finally, Figure 3.7.c denotes the 1PI correction $\Delta\mathcal{F}(q^2)_{\rm 1PI}$ in Eq. (3.36). Adding all contributions together, one gets the VFF at NLO:

$$\mathcal{F}(q^2) \;=\; \mathcal{F}(q^2)_{\rm LO} + \Delta\mathcal{F}(q^2)_F + \Delta\mathcal{F}(q^2)_G + \Delta\mathcal{F}(q^2)_{\rm 1PI}\,. \tag{3.46}$$

Using the large-N_C relations of Eq. (3.4) in this result, it can be written in the form:

$$\mathcal{F}(q^2) \;=\; A(q^2)\frac{M_V^2}{M_V^2 - q^2 - \Sigma_V^r(q^2)} + B(q^2)\,, \tag{3.47}$$

where

$$\begin{aligned} A(q^2) &= 1 + \hat{\delta}_V + 2M_V^2\hat{X} - \Delta\tilde{\Gamma}(q^2)\,,\\ B(q^2) &= \mathcal{G}(q^2) - \hat{\delta}_V - 2(M_V^2 + q^2)\hat{X}\,. \end{aligned} \tag{3.48}$$

The constants

$$\begin{aligned} \hat{\delta}_V &\equiv \frac{F_V G_V^r(\mu)}{F^2} - 1 - \Delta\Gamma(0,\mu^2)\,,\\ \hat{X} &\equiv X_Z^r(\mu) - \frac{1}{F}\left[X_F^r(\mu) + 4X_G^r(\mu)\right]\,, \end{aligned} \tag{3.49}$$

and the functions $\Sigma_V^r(q^2)$,

$$\Delta\tilde{\Gamma}(q^2) \;\equiv\; \Delta\Gamma(q^2,\mu^2) - \Delta\Gamma(0,\mu^2)\,, \tag{3.50}$$

and

$$\mathcal{G}(q^2) \;\equiv\; \Delta\mathcal{F}(q^2)_{\rm 1PI} + \Delta\tilde{\Gamma}(q^2) \;\equiv\; G(q^2,\mu^2) - \Delta\Gamma(0,\mu^2)\,, \tag{3.51}$$

are independent of the renormalization scale μ. The subleading RχT couplings $X_F^r(\mu)$ and $X_G^r(\mu)$ only appear through the constant $\hat{X}$, while $X_Z^r(\mu)$ is also present in the function $\Sigma_V^r(q^2)$. At $q^2 = 0$, $\Delta\tilde{\Gamma}(0) = \mathcal{G}(0) = \Sigma_V^r(0) = 0$. Therefore $\mathcal{F}(0) = 1$, as it should.

Some 1PI diagrams (Figures 3.6.a and 3.6.e and the vector terms in Figures 3.6.b and 3.6.c) have a corresponding reducible counterpart involving a vector propagator.

The combination of both contributions can be then incorporated in $A(q^2)$. The function $G(q^2, \mu^2)$ contains the corrections generated by the other 1PI diagrams (Figures 3.6.d, 3.6.f, the S term in Figure 3.6.b, the S, A and P terms in Figure 3.6.c and the $\widetilde{\ell}_6$ and $\widetilde{r}_{V2}$ pieces in Figure 3.6.g). Subtracting their contribution at $q^2 = 0$, which contains the dependence on the renormalization scale μ,

$$G(0,\mu^2) = \Delta\Gamma(0,\mu^2) = \frac{1}{16\pi^2 F^2}\left\{ M_V^2\left[\frac{3}{2}\log\frac{M_V^2}{\mu^2} + \frac{1}{4}\right] + M_S^2\left[-\log\frac{M_S^2}{\mu^2} + \frac{1}{2}\right]\right\}, \tag{3.52}$$

one gets:

$$\begin{aligned}
\mathcal{G}(q^2) = {} & \frac{C_0(q^2, M_V^2, M_V^2, 0)}{F^2}\left[\frac{M_V^6}{q^2} + \frac{M_V^4}{2}\right] + \frac{C_0(q^2, M_S^2, M_S^2, 0)}{F^2}\left[\frac{M_S^6}{q^2} - \frac{M_S^4}{2}\right] \\
& + \frac{\overline{B}_0(q^2, M_V^2)}{F^2}\left[-\frac{M_V^4}{q^2} - \frac{8M_V^2}{3} + \frac{q^2}{4} + \frac{q^4}{6M_V^2}\right] + \frac{\overline{B}_0(q^2, M_P^2)}{F^2}\left[-\frac{2M_P^2}{3} + \frac{q^2}{6}\right] \\
& + \frac{\overline{B}_0(q^2, M_A^2)}{F^2}\left[-2M_A^2 - \frac{q^2}{6} + \frac{q^4}{6M_A^2}\right] + \frac{\overline{B}_0(q^2, M_S^2)}{F^2}\left[-\frac{M_S^4}{q^2} - \frac{M_S^2}{3} + \frac{q^2}{12}\right] \\
& + \frac{1}{16\pi^2F^2}\left\{\frac{M_V^4 + M_S^4}{q^2} + \frac{3}{4}M_V^2 - \frac{1}{4}M_S^2 + q^2\left[\frac{1}{12}\log\frac{M_V^2}{\mu^2} + \frac{1}{2}\log\frac{M_A^2}{\mu^2}\right.\right. \\
& \left. -\frac{1}{12}\log\frac{M_S^2}{\mu^2} - \frac{1}{6}\log\frac{M_P^2}{\mu^2} + \frac{4}{9} - 16\pi^2\widetilde{\ell}_6^r(\mu)\right] - \frac{q^4}{6}\left[\frac{1}{M_V^2}\log\frac{M_V^2}{\mu^2}\right. \\
& \left.\left. + \frac{1}{M_A^2}\log\frac{M_A^2}{\mu^2} + \frac{4}{3}\left(\frac{1}{M_V^2} + \frac{1}{M_A^2}\right) - \frac{96\pi^2}{F^2}\widetilde{r}_{V2}^r(\mu)\right]\right\} .
\end{aligned} \tag{3.53}$$

3.5 Low-Energy Limit

As it has been reviewed in Section 2.5, at very low energies, $q^2 \ll M_R^2$, the resonance fields can be integrated out from the effective theory. One recovers then the standard χPT lagrangian, which leads to the following result for the vector form factor of the pion [9, 10]:

$$\begin{aligned}
\mathcal{F}_{\chi PT}(q^2) = {} & 1 - \frac{q^2}{F^2}\left\{\ell_6^r(\mu) + \frac{1}{96\pi^2}\left[\log\left(-\frac{q^2}{\mu^2}\right) - \frac{5}{3}\right]\right\} + \frac{q^4}{F^4}\left\{r_{V2}^r(\mu) + \frac{1}{96\pi^2}\times\right. \\
& \left. \times\left[\log\left(-\frac{q^2}{\mu^2}\right) - \frac{5}{3}\right](2\ell_1^r - \ell_2^r + \ell_6^r)(\mu) + \mathcal{O}\left(N_C^0\right)\right\} + \mathcal{O}\left(\frac{q^6}{F^6}\right).
\end{aligned} \tag{3.54}$$

The Taylor expansion in powers of q^2 of the RχT prediction of Eq. (3.47) reproduces the χPT formula, as it should. The coefficient of the $\mathcal{O}\left[q^4 \log\left(-q^2/\mu^2\right)\right]$ term satisfies the known large-N_C equality [17, 22]

$$2\ell_1^r(\mu) - \ell_2^r(\mu) + \ell_6^r(\mu) \;=\; F^2\left(\frac{1}{2M_S^2} - \frac{5}{2M_V^2}\right) . \tag{3.55}$$

The non-logarithmic $\mathcal{O}(q^4)$ and $\mathcal{O}(q^6)$ terms relate the low-energy chiral couplings ℓ_6 and r_{V2} with their RχT counterparts $\widetilde{\ell}_6$ and $\widetilde{r}_{V2}$:

$$\begin{aligned}\ell_6^r(\mu) &= -\frac{F^2}{M_V^2}(1+\hat{\delta}_V) + \widetilde{\ell}_6^r(\mu) - \frac{1}{96\pi^2}\left[\log\frac{M_V^2}{\mu^2} - \log\frac{M_P^2}{\mu^2} + 3\log\frac{M_A^2}{\mu^2} - \frac{13}{6}\right] \\ &= -\frac{F_V G_V^r(\mu)}{M_V^2} + \widetilde{\ell}_6^r(\mu) + \frac{1}{16\pi^2}\left[\frac{4}{3}\log\frac{M_V^2}{\mu^2} - \frac{1}{2}\log\frac{M_A^2}{\mu^2} + \frac{1}{6}\log\frac{M_P^2}{\mu^2}\right. \\ &\qquad \left. -\frac{M_S^2}{M_V^2}\log\frac{M_S^2}{\mu^2} + \frac{11}{18} + \frac{M_S^2}{2M_V^2}\right], \qquad (3.56)\end{aligned}$$

$$\begin{aligned}r_{V2}^r(\mu) &= \frac{F^2 F_V G_V^r(\mu)}{M_V^4} + \widetilde{r}_{V2}^r(\mu) + \frac{2F^4}{M_V^2}\left[\hat{X} - X_Z^r(\mu)\right] \\ &+ \frac{F^2}{96\pi^2}\left\{\left(6\frac{M_S^2}{M_V^4} + \frac{1}{2M_V^2} - \frac{1}{2M_S^2}\right)\log\frac{M_S^2}{\mu^2} - \frac{9}{M_V^2}\log\frac{M_V^2}{\mu^2} - \frac{1}{M_A^2}\log\frac{M_A^2}{\mu^2}\right. \\ &\qquad \left. -\frac{167}{60M_V^2} - \frac{17}{10M_A^2} - \frac{3M_S^2}{M_V^4} + \frac{17}{20M_S^2} + \frac{1}{10M_P^2}\right\}. \qquad (3.57)\end{aligned}$$

Notice that the combination of subleading RχT couplings $\hat{X}$ does not appear at $\mathcal{O}(p^4)$. Therefore, the relation of Eq. (3.56) adopts the same form in terms of the effective couplings defined in Eq. (3.17), i.e.

$$\widetilde{\ell}_6^{\,\mathrm{eff},r}(\mu) - \frac{F_V^{\mathrm{eff}} G_V^{\mathrm{eff},r}(\mu)}{(M_V^2)^{\mathrm{eff},r}(\mu)} \;=\; \widetilde{\ell}_6^r(\mu) - \frac{F_V G_V^r(\mu)}{M_V^2}\,. \qquad (3.58)$$

As shown in Eq. (3.57), this is no longer true at $\mathcal{O}(p^6)$; nevertheless, the explicit dependence on $\hat{X} - X_Z^r(\mu)$ present in $r_{V2}^r(\mu)$ can be reabsorbed into the leading term, through the use of the effective couplings, i.e.

$$r_{V2}^r(\mu) \;=\; F^2\frac{F_V^{\mathrm{eff}} G_V^{\mathrm{eff},r}(\mu)}{(M_V^4)^{\mathrm{eff},r}(\mu)} + \widetilde{r}_{V2}^{\,\mathrm{eff},r} + \cdots\,. \qquad (3.59)$$

Eqs. (3.56) and (3.57) contain the well known lowest-order predictions for the two χPT couplings: $\ell_6 = -M_V^2 r_{V2}^r/F^2 = -F^2/M_V^2$. Moreover, they give their dependence on the renormalization scale at the next-to-leading order. The running of the renormalized couplings $\ell_6^r(\mu)$, $r_{V2}^r(\mu)$ and $\widetilde{\ell}_6^r(\mu)$, $\widetilde{r}_{V2}^r(\mu)$ is different, because their corresponding effective theories have a very different particle content.

The μ dependence of a given coupling "g" can be characterized through the logarithmic derivative

$$\mu\,\frac{dg}{d\mu} \;=\; -\frac{\gamma_g}{16\pi^2}\,. \qquad (3.60)$$

From Eqs. (3.34) and (3.35) one gets the running of the RχT couplings:

$$\gamma_{\widetilde{\ell}_6} = \frac{2}{3}\,, \qquad \gamma_{\widetilde{r}_{V2}} = \frac{F^2}{3}\left(\frac{1}{M_V^2} + \frac{1}{M_A^2}\right) = \frac{F^2}{2M_V^2}\,. \qquad (3.61)$$

Eqs. (3.56) and (3.57) give then the dependence on the renormalization scale of the corresponding χPT couplings:

$$\gamma_{\ell_6} = -\frac{1}{3}, \qquad \gamma_{r_{V2}} = \frac{F^2}{6}\left(\frac{5}{M_V^2} - \frac{1}{M_S^2}\right). \tag{3.62}$$

These values are in perfect agreement with the low-energy results of Refs. [9, 10, 45]. The running of the $\mathcal{O}(p^6)$ coupling $r_{V2}(\mu)/F^4$ receives of course additional 2-loop contributions which are of $\mathcal{O}(1/N_C^2)$.

The rigorous control of the renormalization scale dependences allows us to investigate the successful resonance saturation approximation at subleading order. The χPT couplings ℓ_6 and r_{V2} have been phenomenologically extracted from a fit to the VFF data at low momenta. This determines the scale-invariant combination [45]:

$$\bar{\ell}_6 \equiv \frac{32\pi^2}{\gamma_{\ell_6}}\ell_6^r(\mu) - \log\frac{m_\pi^2}{\mu^2} = 16.0 \pm 0.5 \pm 0.7, \tag{3.63}$$

$$r_{V2}^r(M_\rho) = (1.6 \pm 0.5)\cdot 10^{-4}. \tag{3.64}$$

Inserting these numbers in Eqs. (3.56) and (3.57), one can estimate the corresponding scale-invariant combinations of NLO couplings in RχT:

$$\hat{\ell}_6 \equiv \widetilde{\ell}_6^r(\mu) - \frac{\gamma_{\widetilde{\ell}_6}}{32\pi^2}\log\frac{M_V^2}{\mu^2} - \frac{F^2}{M_V^2}\hat{\delta}_V, \tag{3.65}$$

$$\hat{r}_{V2} \equiv \widetilde{r}_{V2}^r(\mu) + \frac{F^4}{M_V^4}\left(\hat{\delta}_V + 2M_V^2\left[\hat{X} - X_Z^r(\mu)\right]\right) - \frac{\gamma_{\widetilde{r}_{V2}} - \frac{2F^4}{M_V^2}\gamma_{X_Z}}{32\pi^2}\log\frac{M_V^2}{\mu^2}, \tag{3.66}$$

where $\gamma_{X_Z} = -1/(6F^2)$. Taking $F = 92.4$ MeV, $M_V = 770$ MeV and $M_S = 1$ GeV, one gets $\hat{\ell}_6 = (-0.2 \pm 0.9)\cdot 10^{-3}$ and $\hat{r}_{V2} = (-0.2 \pm 0.5)\cdot 10^{-4}$, while a larger value of the scalar resonance mass $M_S = 1.4$ GeV shifts the $\mathcal{O}(p^4)$ coupling to $\hat{l}_6 = (-0.9 \pm 0.9)\cdot 10^{-3}$, without affecting $\hat{r}_{V2}$ at the quoted level of accuracy. These numbers should be compared with the large-N_C predictions for the χPT couplings $\ell_6|_{N_C\to\infty} = -F^2/M_V^2 = -0.014$ and $r_{V2}|_{N_C\to\infty} = F^4/M_V^4 = 2.1\cdot 10^{-4}$. Put in a different way, the hypothesis $\hat{\ell}_6 = \hat{r}_{V2} = 0$ generates excellent predictions for $\ell_6^r(\mu)$ and $r_{V2}^r(\mu)$ at any scale μ.

3.6 Behaviour at Large Energies

At large momentum transfer, the relevant renormalization scale invariant functions take the forms:

$$\mathcal{G}(q^2) = \frac{1}{16\pi^2F^2}\left\{-q^4\left[\frac{1}{6}\left(\frac{1}{M_V^2} + \frac{1}{M_A^2}\right)\left(\log\frac{-q^2}{\mu^2} - \frac{2}{3}\right) - \frac{16\pi^2}{F^2}\widetilde{r}_{V2}^r(\mu)\right]\right.$$
$$\left. + q^2\left[\frac{1}{3}\log\frac{-q^2}{\mu^2} + \frac{16}{9} - 16\pi^2\widetilde{\ell}_6^r(\mu)\right] + \mathcal{O}\left(q^0\right)\right\},$$

$$\Delta\tilde{\Gamma}(q^2) = \frac{M_V^2}{16\pi^2F^2}\left\{\log\frac{-q^2}{M_V^2}\left[\log\frac{q^2}{M_V^2}-2\right]-\frac{1}{2}\log^2\frac{q^2}{M_V^2}-\frac{\pi^2}{6}+\frac{9}{4}+\frac{M_S^2}{4M_V^2}\right\}+\mathcal{O}\left(\frac{1}{q^2}\right),$$
$$\Sigma_V^r(q^2) = \frac{-q^4}{96\pi^2F^2}\left\{\log\frac{-q^2}{\mu^2}-\frac{5}{3}+192\pi^2F^2X_Z^r(\mu)\right\}\,. \qquad (3.67)$$

The ρ propagator makes the $A(q^2)$ piece of the VFF well behaved when $q^2 \to \infty$. However, the 1PI contributions generate a wrong behaviour $\mathcal{G}(q^2) \sim q^4 \log(-q^2/\mu^2)$ in the $B(q^2)$ term, which cannot be eliminated with a local contribution. The problem originates in the two-resonance cut which has an unphysical growing with momenta.

Although our leading RχT lagrangian of Eq. (3.3) only incorporates couplings linear in the resonance fields, the kinetic resonance lagrangian introduces some bilinear interactions through the chiral connection included in the covariant derivatives. Their couplings are fixed by chiral symmetry and give rise to the diagrams in Figures 3.5.b, 3.6.c, 3.6.d and 3.6.f. Obviously, these are not the only interactions bilinear in the resonance fields even at large-N_C [19, 29, 30, 54]. Therefore, it is not surprising that our calculation is unable to find the correct behaviour at large energies for those contributions with two intermediate resonances.

The contributions with an internal vector propagator in diagrams 3.6.b and 3.6.c give us some hint about which pieces could be missing in our calculation. These two diagrams combine with a reducible contribution of the type 3.7.b: the 1PI $\langle V_{\mu\nu}\pi\pi\rangle$ vertex in Figure 3.5.b. The three contributions contain identical loop functions and their sum generates a global factor $M_V^2/(M_V^2-q^2)$, which suppresses the large-q^2 behaviour. Thus, these corrections have been included in the term $A(q^2)$.

It seems natural to conjecture that the remaining 1PI contributions with two-resonance cuts should combine with the corresponding reducible topologies, including $\langle VRR\rangle$ and $\langle v^\mu RR\rangle$ vertices, to generate the final propagator suppression:

$$G(q^2) \quad\longrightarrow\quad \frac{M_V^2}{M_V^2-q^2-\Sigma_V^r(q^2)}\,G(q^2)\,. \qquad (3.68)$$

The needed lagrangian takes the form

$$\Delta\mathcal{L}_{VRR} \;=\; i\,\lambda^{VSS}\,\langle\, V^{\mu\nu}\,\nabla_\mu S\,\nabla_\nu S\,\rangle + i\,\lambda^{VPP}\,\langle\, V^{\mu\nu}\,\nabla_\mu P\,\nabla_\nu P\,\rangle\,. \qquad (3.69)$$

Our conjecture fixes the new chiral couplings in the large-N_C limit. In fact, the main aim of the next chapter is to follow these ideas: once it is accepted the necessity of new terms with more than one resonance field by studying the asymptotic behaviour at large energies, we are going to analyse all the two-body form factors that can be found in the even-intrinsic-parity sector of Resonance Chiral Theory in the Single Resonance Approximation. This will be done in the spirit of correlators at next-to-leading order in the $1/N_C$ expansion.

3.7 Conclusions

The one-loop analysis of the vector form factor of the pion has shown a series of interesting features:

1. As expected, loop diagrams with massive resonance states in the internal lines generate ultraviolet divergences, which require additional higher-dimensional counterterms in the RχT lagrangian. Since these counterterms give rise to tree-level contributions which grow too fast at large momenta, their corresponding couplings should be zero at leading order in the large-N_C expansion. Thus, one can establish a well defined counting in powers of $1/N_C$ to organize the calculation.

   The formal renormalization is completely straightforward at one loop. One can easily determine the μ dependence of all relevant renormalized couplings. Moreover, the final result is only sensitive to some combinations of the chiral couplings. In fact, using the lowest-order equations of motion, one can eliminate most of the higher-order couplings. Their effects get then reabsorbed into redefinitions of the lowest-order parameters.

2. Expanding the result in powers of q^2/M_R^2, one recovers the usual χPT expression at low momenta. This relates the low-energy chiral couplings ℓ_6 and r_{V2} with their corresponding RχT counterparts $\widetilde{\ell}_6$ and $\widetilde{r}_{V2}$.

   The rigorous control of the renormalization scale dependences has allowed us to investigate the successful resonance saturation approximation at the next-to-leading order in $1/N_C$. The assumption $\hat{\ell}_6 = \hat{r}_{V2} = 0$ generates excellent predictions for $\ell_6^r(\mu)$ and $r_{V2}^r(\mu)$ at any scale μ.

   We stress again the importance of determining the resonance contributions to the chiral LEC's at next-to-leading order in $1/N_C$, since one keeps a full control of their renormalization scale dependence. Notice how the uncertainty related to the running disappears. This chapter represents a first step towards a systematic procedure to evaluate next-to-leading order contributions in the $1/N_C$ counting: in the next chapter we will present a NLO prediction of L_8.

3. At higher energies, we have identified an unphysical behaviour which originates in the two-resonance cuts: they generate an increase of the form factor at large values of momentum transfer. This is not surprising, since there are additional contributions generated by interaction terms with several resonances, which have not been included in the minimal RχT lagrangian. These new chiral structures should be taken into account to achieve a physical description of the VFF above the two-resonance thresholds. The short-distance QCD constraints can be used to determine their corresponding couplings.

   In the next chapter we will check with several form factors the requirement of these new terms in order to fulfill a good behaviour at large energies.

Chapter 4

Two-body Hadronic Form Factors From QCD

4.1 Introduction

Once it is accepted the importance of matching the effective results evaluated within the Resonance Chiral Theory with the ones obtained with QCD, one has to study how to carry out this procedure. There are two ways of getting short-distance constraints: either to consider the Green Functions of QCD currents calculated in the leading OPE expansion or to demand that two-body form factors of hadronic currents vanish at high energies [28]. Although in the first case there is no doubt about the necessity of fulfilling the asymptotic constraints in the considered amplitude, the second one is more controversial. Actually, this behaviour has only been experimentally observed for pseudo-Goldstone bosons and photons. The question appears when one is studying form factors that involve resonances as "asymptotic states". In this chapter we present an analysis of all two-body form factors that can be found in the even-intrinsic-parity sector of RχT in the Single Resonance Approximation [29, 30]. In the spirit of correlators at next-to-leading order in the $1/N_C$ expansion, the requirement of considering the short-distance behaviour of these form factors is justified.

As a continuation of the ideas proposed in the last chapter, once these new constraints are incorporated, we expect to avoid the non-vanishing behaviour at large momentum transfer for those contributions in the vector form factor at one-loop level coming from diagrams with resonances as intermediate states. In Section 3.6 we showed the need of new operators, that is, operators with more than one resonance field, in order to generate this suppression. Notice that we propose a relation between well-behaved form factors with resonances in the final state and observables at NLO.

As soon as one is dealing with well-behaved amplitudes at large energies, a one-loop calculation provides a clear NLO prediction of the related χPT LEC's, where the scale dependence is under control. Following this path, we present a subleading prediction of L_8 [29]. A first step in this direction was the study of resonance loop contributions to the chiral coupling L_{10} [37]. In Ref. [37] it was suggested the importance of considering well-behaved amplitudes before studying these contributions. Sections 3.6 and 3.7 [26] are a good example of these ideas in the case of L_9 (or ℓ_6 in the two flavour case).

In Section 4.2, the lagrangian needed to describe all the possible two-body form factors within the Single Resonance Approximation is reviewed. Section 4.3 is devoted to clarify how to get the short-distance constraints for the form factors by relating them to one-loop correlators through the optical theorem; the relation between quantum loops in RχT and form factors with resonances in the final state is explained. A phenomenological example of these results is developed in Section 4.4, where a prediction of $L_8^r(\mu)$ is given, making use of dispersive relations. The study of possible inconsistencies between constraints due to the truncation of the large-N_C spectrum, already suggested in former works [25, 48], is relegated to Section 4.5. The main conclusions are summarised in Section 4.6. Some technical details and the full list of results for the form factors are collected in the Appendices D and E.

4.2 The Effective Lagrangian

As pointed out in the introduction, the study is taken under the Single Resonance Approximation, where just the lightest resonances with non-exotic quantum numbers are considered. On account of large-N_C, the mesons are put together into $U(3)$ multiplets. Since we will be interested on the structure of the interaction at short distances, we will work under the chiral limit.

As the Resonance Chiral Theory should get the high-energy behaviour of QCD, only operators constructed with chiral tensors of $\mathcal{O}(p^2)$ will be allowed; interactions with higher order chiral tensors tend to violate the asymptotic behaviour ruled by QCD.

In the large-N_C approach, there is no limit to the number of resonances that one may include in the effective operators. However, as we are interested just in the two-body form factors at tree level, only operators up to three resonance fields are considered. Moreover, in the case of three resonance operators, only terms consisting of resonance fields and the covariant derivative ∇_μ will be required.

Following these remarks the terms in the lagrangian can be classified as:

$$\mathcal{L}_{R\chi T} \;=\; \mathcal{L}_{pGB}^{(2)} + \sum_{R_1} \mathcal{L}_{R_1} + \sum_{R_1,R_2} \mathcal{L}_{R_1R_2} + \sum_{R_1,R_2,R_3} \Delta\mathcal{L}_{R_1R_2R_3}\,, \qquad (4.1)$$

where the indexs R_i run over all the different resonance fields, V, A, S and P. We use Δ in the last term to stress that only some terms with three resonances are added to the lagrangian. The different pieces are shown and explained in Section 2.3.2, Eqs. (2.6) - (2.27).

4.3 Form Factors and Short-distance Constraints

Let us consider the two-point correlation function of two QCD currents in the chiral limit:

$$\begin{aligned}\Pi^{\mu\nu}_{XX}(q) &\equiv i\int \mathrm{d}^4x\,\mathrm{e}^{iqx}\,\langle 0|T\left(X^\mu(x)X^\nu(0)^\dagger\right)|0\rangle \;=\; \left(-g^{\mu\nu}q^2+q^\mu q^\nu\right)\,\Pi_{XX}(q^2)\,,\\ \Pi_{YY}(q) &\equiv i\int \mathrm{d}^4x\,\mathrm{e}^{iqx}\,\langle 0|T\left(Y(x)Y(0)^\dagger\right)|0\rangle\,,\end{aligned}\tag{4.2}$$

where $X^\mu(x)$ can denote the vector or axial-vector current ($X = V, A$) and $Y(x)$ the scalar or pseudo-scalar density ($Y = S, P$),

$$\begin{aligned} V_i^\mu &= \bar\psi\,\gamma^\mu\,\frac{\lambda_i}{2}\,\psi\,, & S_i &= \bar\psi\,\frac{\lambda_i}{2}\,\psi\,,\\ A_i^\mu &= \bar\psi\,\gamma^\mu\gamma_5\,\frac{\lambda_i}{2}\,\psi\,, & P_i &= i\,\bar\psi\,\gamma_5\,\frac{\lambda_i}{2}\,\psi\,.\end{aligned}\tag{4.3}$$

The associated spectral functions are a sum of positive contributions corresponding to the different intermediate states. At large q^2, $\mathrm{Im}\,\Pi_{XX}$ tends to a constant whereas $\mathrm{Im}\,\Pi_{YY}$ grows as q^2 [31, 34]. Therefore, since there is an infinite number of possible states, we assume a similar supression for all the absorptive contributions in the spin-1 correlators coming from each intermediate state in the $q^2\to\infty$ limit. The high energy behaviour in the spin-0 $\mathrm{Im}\,\Pi_{YY}$ is not so clear as, *a priori*, one could think of a constant behaviour for each intermediate cut. However, the fact that $\Pi_{SS}-\Pi_{PP}$ vanishes as $1/q^4$ in the chiral limit [34], the Brodsky-Lepage rules for the form factors [28] and the $1/q^2$ behaviour of each one-particle intermediate cut (tree-level exchanges) seems to point out that every absorptive contribution to $\mathrm{Im}\,\Pi_{YY}$ must also vanish at large momentum transfer.

The spectral functions of the correlators at next-to-leading order can be easily obtained from form factors by making use of the optical theorem. Thence, all possible two-body form factors have been calculated in order to get the imaginary part of the two-point function. In the simplest cases with just one form-factor $\mathcal{F}_{m_1,m_2}(q^2)$, one finds the relation

$$\mathrm{Im}\,\Pi(q^2)\big|_{m_1,m_2} \;=\; \xi(q^2)\;|\mathcal{F}_{m_1,m_2}(q^2)|^2\,,\tag{4.4}$$

with $\xi(q^2)$ a kinematic factor that depends on the considered channel. Imposing that the spectral function must vanish as $1/q^2$ at $q^2\to\infty$ yields a specific behaviour for $\mathcal{F}_{m_1,m_2}(q^2)$, depending on $\xi(q^2)$. Thus, some constraints on the effective parameters will be needed. In Appendix D, we give the whole list of form factors in the even-intrinsic-parity sector of RχT in the Single Resonance Approximation, the exact relations between them and the spectral functions, the constraints which are derived from the high energy analysis and the structure of the form factors after imposing the proper short-distance behaviour. Some of them can be found in former literature [22, 26].

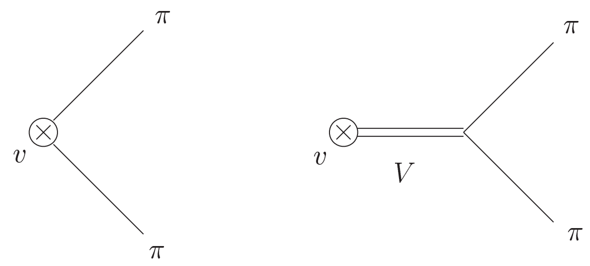


Figure 4.1: Tree-level contributions to the vector form factor of the pion. A single line stands for a pseudo-Goldstone boson while a double line indicates a vector resonance.

As an example, we show here the case of the two pseudo-Goldstones matrix element of the vector current. The diagrams that contribute at leading order in $1/N_C$ are those depicted in Figure 4.1. The form factor is defined through the corresponding matrix element,

$$\langle\, \pi^0(p_1)\pi^-(p_2)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \sqrt{2}\,\mathcal{F}^v_{\pi\pi}(q^2)\,(p_2-p_1)^\mu\,, \tag{4.5}$$

where $\mathcal{F}^v_{\pi\pi}$ reads

$$\mathcal{F}^v_{\pi\pi}(q^2) \;=\; 1+\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\,, \tag{4.6}$$

and it is the same form factor than the one of Eq. (3.1) in Chapter 3. Using the optical theorem, the imaginary part of the correlator is found to be

$$\mathrm{Im}\Pi_{VV}(q^2)|_{\pi\pi} \;=\; \frac{\theta(q^2)}{24\pi}|\mathcal{F}^v_{\pi\pi}(q^2)|^2\,. \tag{4.7}$$

Imposing that $\mathrm{Im}\Pi_{VV}(q^2)|_{\pi\pi}$ vanishes in the $q^2\to\infty$ limit leads to demanding that the form factor also does, so we find the constraint

$$F_V\,G_V \;=\; F^2\,. \tag{4.8}$$

Taking into account this constraint, the form factor follows now the right asymptotic behaviour and reads as

$$\widetilde{\mathcal{F}}^v_{\pi\pi}(q^2) \;=\; \frac{M_V^2}{M_V^2-q^2}\,, \tag{4.9}$$

as we would have obtained imposing the Brodsky-Lepage behaviour in Eq. (4.6). In this work, the tilde over a form factors denotes that the QCD short-distance constraints have already been imposed.

4.4 A next-to-leading order prediction of $L_8^r(\mu)$

As an application of our results and because of its phenomenological importance, the observable $\Pi_{S-P}(q^2) \equiv \Pi_{SS}(q^2) - \Pi_{PP}(q^2)$ is studied in this section in order to predict $L_8^r(\mu)$ at next-to-leading order.

The one-loop χPT result, in the chiral limit, is

$$\Pi_{S-P}(q^2)|_{\chi PT} = \frac{2F^2 B_0^2}{q^2} + 32 B_0^2 \, L_8^r(\mu) + \frac{n_f}{2} \frac{B_0^2}{8\pi^2} \left(1 - \log \frac{-q^2}{\mu^2}\right) + \mathcal{O}\left(q^2\right) \, , \quad (4.10)$$

for the $U(n_f)$ case. The running in $L_8^r(\mu)$,

$$L_8^r(\mu_2) \;\; = \;\; L_8^r(\mu_1) + \frac{\Gamma_8}{16\pi^2} \log \frac{\mu_1}{\mu_2}, \quad (4.11)$$

with $\Gamma_8 = 3/16$ for the $U(3)$ case [49], makes $\Pi_{S-P}(q^2)$ scale independent.

A leading order prediction of the χPT coupling can be obtained easily by considering the tree-level contributions in our hadronic effective approach,

$$\Pi_{S-P}(q^2)|_{R\chi T}^{N_C\to\infty} \;\; = \;\; B_0^2 \left(\frac{16\, c_m^2}{M_S^2 - q^2} - \frac{16\, d_m^2}{M_P^2 - q^2} + \frac{2\, F^2}{q^2} \right) . \quad (4.12)$$

Demanding the right high-energy behaviour ($\sim 1/q^4$) in $\Pi_{S-P}(q^2)|_{R\chi T}^{N_C\to\infty}$ constraints the resonance parameters to obey the relations:

$$F^2 \, - \, 8\, c_m^2 \, + \, 8\, d_m^2 \;=\; 0\,, \quad c_m^2\, M_S^2 \, - \, d_m^2\, M_P^2 \;=\; \widetilde{\delta}\,, \quad (4.13)$$

where $\widetilde{\delta} \equiv 3\pi\alpha_s F^4/4 \approx 0.08\alpha_s F^2 \times (1\text{GeV})^2$ is negligible.

In Section 2.4 it is reviewed how to fix all the low-energy couplings of $\mathcal{L}_{R\chi T}$ of Eq. (4.1) linear in the resonance fields, by using different short-distance constraints,

$$\begin{aligned} F_V^2 &= F^2 \frac{M_A^2}{M_A^2 - M_V^2}\,, \quad F_A^2 = F^2 \frac{M_V^2}{M_A^2 - M_V^2}\,, \quad G_V^2 = F^2 \frac{M_A^2 - M_V^2}{M_A^2}\,, \quad M_A^2 > M_V^2 \\ c_m^2 &= \frac{F^2}{8} \frac{M_P^2}{M_P^2 - M_S^2}\,, \quad d_m^2 = \frac{F^2}{8} \frac{M_S^2}{M_P^2 - M_S^2}\,, \quad c_d^2 = \frac{F^2}{2} \frac{M_P^2 - M_S^2}{M_P^2}\,, \quad M_P^2 > M_S^2\,. \end{aligned} \quad (4.14)$$

where, at LO in $1/N_C$, the couplings are fixed in terms of the decay constant F and the resonance masses in the chiral and large-N_C limit, M_V, M_A, M_S, M_P.

The low-energy expansion of Eq. (4.12) fixes the leading-order prediction of $L_8^r(\mu)$ [17],

$$L_8 \;\; = \;\; \frac{c_m^2}{2\, M_S^2} - \frac{d_m^2}{2\, M_P^2} \;\; = \;\; \frac{F^2}{16\, M_S^2} + \frac{F^2}{16\, M_P^2}\,, \quad (4.15)$$

where the constraints in Eq. (4.14) have been considered to produce the final result. It is expected that Eq. (4.15) provides the coupling at scales of the order of the momenta involved in the processes ($\mu_0 \sim M_R$), though until now there was no information about the scale of saturation. Therefore, at LO in $1/N_C$, the uncertainty on μ_0 induces an error, which for the coupling $L_8^r(\mu)$ is sizable and competes with the leading contributions.

In the large-N_C limit a correlator that accepts an unsubtracted dispersive relation is determined by the position of the poles and the value of their residues. Hence, within the Single Resonance Approximation, Eq. (4.12) shows the general structure for Π_{S-P}. This corresponds to the leading order saturation of the χPT $\mathcal{O}(p^4)$ lagrangian by the resonance exchange.

4.4.1 Dispersive Calculation of Π_{S-P}

In this section, $\Pi_{S-P}(q^2) \equiv \Pi_{SS}(q^2) - \Pi_{PP}(q^2)$ is computed at next-to-leading order within the Resonance Chiral Theory in the Single Resonance Approximation. By using the dispersive relations (Appendix E), it is possible to prove that the amplitude at NLO in $1/N_C$ shows the structure

$$\Pi_{S-P}(q^2) = \frac{2F^2B_0^2}{q^2} + \frac{16c_m^{r\,2}B_0^2}{M_S^{r\,2}-q^2} - \frac{16d_m^{r\,2}B_0^2}{M_P^{r\,2}-q^2} + \sum_{m_1,m_2} \Delta\Pi_{S-P}(q^2)|_{m_1,m_2}\,, \tag{4.16}$$

where the contributions $\Delta\Pi_{S-P}(q^2)|_{m_1,m_2}$ are given by the two meson absorptive cut m_1, m_2. Their imaginary part is related to the corresponding two-meson form factors through the optical theorem (the precise relations are given in Appendix E), so the functions are given by the dispersive integral

$$\begin{aligned}\Delta\Pi_{S-P}(q^2)|_{m_1,m_2} = \lim_{\epsilon\to 0}\Bigg[\int_0^{M_R^2-\epsilon} dt\,\frac{1}{\pi}\frac{\mathrm{Im}\Pi_{S-P}(t)|_{m_1,m_2}}{t\,-\,q^2} + \int_{M_R^2+\epsilon}^{\infty} dt\,\frac{1}{\pi}\frac{\mathrm{Im}\Pi_{S-P}(t)|_{m_1,m_2}}{t\,-\,q^2}\\ -\frac{2}{\pi\epsilon}\lim_{t\to M_R^2}\left\{(M_R^2-t)^2\,\frac{\mathrm{Im}\Pi_{S-P}(t)|_{m_1,m_2}}{t\,-\,q^2}\right\}\Bigg]\,,\end{aligned} \tag{4.17}$$

where M_R is the mass of the intermediate resonance produced in the m_1, m_2 form-factor. It obeys the properties

$$\lim_{t\to M_R^2} \mathrm{Re}\overline{D}(t)|_{m_1,m_2} = 0\,, \qquad \lim_{t\to M_R^2} \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Re}\overline{D}(t)|_{m_1,m_2} = 0\,, \tag{4.18}$$

with $\overline{D}(t)|_{m_1,m_2} \equiv (M_R^2-t)^2\Delta\Pi_{S-P}(t)|_{m_1,m_2}$.

Notice that the dispersive integrals are convergent because the form-factors are well behaved at infinite momentum. This ensures the absence of non-vanishing contributions in the part of the amplitude that comes from unitarity. The remaining terms in the correlator do not contain cuts and are analytical. These polynomial terms must vanish, remaining only the pole+unitarity structure in Eq. (4.16). This fixes any possible $\widetilde{L}_8$ arising at NLO, since the full polynomial must be zero. Furthermore, we will impose the $1/q^4$ behaviour prescribed by the OPE for $\Pi_{S-P}(q^2)$ up to NLO in $1/N_C$.

For the first absorptive cut one gets the contributions

$$\Pi_{S-P}(q^2)|_{tree} = B_0^2 \left\{ \frac{2F^2}{q^2} + \frac{16c_m^{r\,2}}{M_S^{r\,2}-q^2} - \frac{16d_m^{r\,2}}{M_P^{r\,2}-q^2} \right\}, \tag{4.19}$$

$$\Delta\Pi_{S-P}(q^2)|_{\eta\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\left(\frac{M_S^2}{M_S^2-q^2}\right)^2 \left[-1+\frac{q^2}{M_S^2} - \log\left(\frac{-q^2}{M_S^2}\right)\right], \tag{4.20}$$

$$\begin{aligned}\Delta\Pi_{S-P}(q^2)|_{V\pi} = {}& \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{2G_V^2}{F^2}\left(\frac{M_P^2}{M_P^2-q^2}\right)^2 \left[\left(1-\frac{q^2}{M_P^2}\right)\left(-\frac{M_V^4}{q^4}-\frac{M_V^4}{q^2M_P^2}\right.\right. \\ &\left. +\frac{5M_V^2}{2q^2}+1-\frac{9M_V^2}{2M_P^2}+\frac{3M_V^4}{M_P^4}\right) - \left(1-\frac{4M_V^2}{M_P^2}+\frac{3M_V^2q^2}{M_P^4}\right)\times \\ &\left.\times\left(1-\frac{M_V^2}{M_P^2}\right)^2\log\frac{M_P^2-M_V^2}{M_V^2}+\left(1-\frac{M_V^2}{q^2}\right)^3\log\left(1-\frac{q^2}{M_V^2}\right)\right],\end{aligned} \tag{4.21}$$

$$\Delta\Pi_{S-P}(q^2)|_{A\pi} = 0\,, \tag{4.22}$$

$$\begin{aligned}\Delta\Pi_{S-P}(q^2)|_{S\pi} = {}& \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{4c_d^2}{F^2}\left(\frac{M_P^2}{M_P^2-q^2}\right)^2\left\{\left(\frac{F^2}{2c_d^2}-1\right)^2\left(1-\frac{M_S^2}{M_P^2}\right)^2\times\right. \\ &\times\left[1-\frac{q^2}{M_P^2}+\left(1-\frac{2M_S^2}{M_P^2}+\frac{M_S^2q^2}{M_P^4}\right)\log\frac{M_S^2}{M_P^2-M_S^2}+\left(1-\frac{M_S^2}{q^2}\right)\times\right. \\ &\left.\times\log\left(1-\frac{q^2}{M_S^2}\right)\right]+\left(\frac{F^2}{2c_d^2}-1\right)\left(1-\frac{M_S^2}{M_P^2}\right)\left[\frac{4M_S^2}{M_P^2}-\frac{2M_S^2}{q^2}-\frac{2M_S^2q^2}{M_P^4}\right. \\ &+\left(\frac{2M_S^2}{M_P^2}-\frac{2M_S^4}{M_P^4}-\frac{2M_S^2q^2}{M_P^4}+\frac{2M_S^4q^2}{M_P^6}\right)\log\frac{M_S^2}{M_P^2-M_S^2}-2\left(\frac{M_S^2}{M_P^2}\right. \\ &\left.\left.-\frac{M_S^2}{q^2}\right)\left(1-\frac{M_S^2}{q^2}\right)\log\left(1-\frac{q^2}{M_S^2}\right)\right]-\frac{M_S^2}{M_P^2}-\frac{M_S^4}{M_P^4}-\frac{M_S^4}{q^4}+\frac{M_S^2}{2q^2} \\ &\left.+\frac{2M_S^4}{M_P^2q^2}+\frac{M_S^2q^2}{2M_P^4}+\left(\frac{M_S^2}{M_P^2}-\frac{M_S^2}{q^2}\right)^2\left(1-\frac{M_S^2}{q^2}\right)\log\left(1-\frac{q^2}{M_S^2}\right)\right\},\end{aligned} \tag{4.23}$$

$$\begin{aligned}\Delta\Pi_{S-P}(q^2)|_{P\pi} = {}& \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{16d_m^2}{F^2}\left(\frac{M_P^2-M_S^2}{M_S^2-q^2}\right)^2\left[-1+\frac{q^2}{M_S^2}+\left(1-\frac{2M_P^2}{M_S^2}+\frac{M_P^2q^2}{M_S^4}\right)\times\right. \\ &\left.\times\log\frac{M_P^2-M_S^2}{M_P^2}-\left(1-\frac{M_P^2}{q^2}\right)\log\left(1-\frac{q^2}{M_P^2}\right)\right],\end{aligned} \tag{4.24}$$

It is possible to show that states with higher thresholds turn out to be more an more suppressed (Appendix E.2). Only contributions from cuts that contain up to one resonance field are taking into account: the $\pi\eta$, the $A\pi$ and the $P\pi$ cut of the scalar correlator (Sections D.3.1, D.3.2 and D.3.3 respectively) and the $V\pi$ and the $S\pi$ cut of the pseudoscalar correlator (Sections D.4.1 and D.4.2). All the results from Appendix D have been multiplied by a factor $n_f/2$ in order to go from 2 to n_f light flavours. The results in Eqs. (4.21)-(4.24) include also a factor 2 that accounts the two possible absorptive structure, e.g., in the case of Eq. (4.21) it is possible $\rho^0\pi^-$ and $\rho^-\pi^0$. The pion scalar form factor constraint from Eq. (D.99) has been used in Eq. (4.23).

4.4.2 Short-distance Constraints at One-loop

At high q^2 the first absorptive contribution vanishes as

$$\Pi_{S-P}(q^2)|_{tree} = \frac{B_0^2}{q^2}\left\{2F^2-16c_m^{r\,2}+16d_m^{r\,2}+\frac{16}{q^2}\left[d_m^{r\,2}M_P^2-c_m^{r\,2}M_S^2\right]\right\}+\mathcal{O}\left(\frac{1}{q^6}\right), \quad (4.25)$$

$$\Delta\Pi_{S-P}(q^2)|_{\eta\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2q^2}M_S^2\left\{1+\frac{M_S^2}{q^2}\left[1-\log\frac{-q^2}{M_S^2}\right]\right\}+\mathcal{O}\left(\frac{1}{q^6}\right), \quad (4.26)$$

$$\Delta\Pi_{S-P}(q^2)|_{V\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2q^2}\frac{2G_V^2}{F^2}M_P^2\left\{-1+\frac{9M_V^2}{2M_P^2}-\frac{3M_V^4}{M_P^4}-\frac{3M_V^2}{M_P^2}\left(1-\frac{M_V^2}{M_P^2}\right)^2\times\right.$$
$$\times\log\frac{M_P^2-M_V^2}{M_V^2}+\frac{M_P^2}{q^2}\left[-1-\frac{2M_V^4}{M_P^4}+\frac{2M_V^2}{M_P^2}+\log\frac{-q^2}{M_V^2}\right.$$
$$\left.\left.-\left(1+\frac{2M_V^2}{M_P^2}\right)\left(1-\frac{M_V^2}{M_P^2}\right)^2\log\left(\frac{M_V^2}{M_V^2}-1\right)\right]\right\}+\mathcal{O}\left(\frac{1}{q^6}\right), \quad (4.27)$$

$$\Delta\Pi_{S-P}(q^2)|_{A\pi} = 0\,, \quad (4.28)$$

$$\Delta\Pi_{S-P}(q^2)|_{S\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2q^2}\frac{4c_d^2}{F^2}M_P^2\left\{\left(\frac{F^2}{2c_d^2}-1\right)^2\left(1-\frac{M_S^2}{M_P^2}\right)^2\left(-1+\frac{M_S^2}{M_P^2}\times\right.\right.$$
$$\left.\times\log\frac{M_S^2}{M_P^2-M_S^2}\right)+\frac{M_S^2}{2M_P^2}+\frac{2M_S^2}{M_P^2}\left(\frac{F^2}{2\,c_d^2}-1\right)\left(1-\frac{M_S^2}{M_P^2}\right)\left(-1\right.$$
$$\left.+\left(-1+\frac{M_S^2}{M_P^2}\right)\log\frac{M_S^2}{M_P^2-M_S^2}\right)+\frac{M_P^2}{q^2}\left[\left(\frac{F^2}{2c_d^2}-1\right)^2\left(1-\frac{M_S^2}{M_P^2}\right)^2\times\right.$$
$$\times\left(-1+\log\frac{-q^2}{M_P^2-M_S^2}\right)+\frac{M_S^4}{M_P^4}\left(-1+\log\frac{-q^2}{M_S^2}\right)+\frac{2M_S^2}{M_P^2}\left(\frac{F^2}{2c_d^2}-1\right)\times$$
$$\left.\left.\times\left(1-\frac{M_S^2}{M_P^2}\right)\left(\frac{M_S^2}{M_P^2}\log\frac{M_S^2}{M_P^2-M_S^2}-\log\frac{-q^2}{M_P^2-M_S^2}\right)\right]\right\}+\mathcal{O}\left(\frac{1}{q^6}\right), \quad (4.29)$$

$$\Delta\Pi_{S-P}(q^2)|_{P\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2q^2}\frac{16d_m^2}{F^2}\left(1-\frac{M_S^2}{M_P^2}\right)^2M_P^2\left\{\frac{M_P^2}{M_S^2}+\frac{M_P^4}{M_S^4}\log\frac{M_P^2-M_S^2}{M_P^2}+\right.$$
$$\left.+\frac{M_P^2}{q^2}\left[1-\log\frac{-q^2}{M_P^2-M_S^2}\right]\right\}+\mathcal{O}\left(\frac{1}{q^6}\right), \quad (4.30)$$

Once the leading-order relations in Eq. (4.14) have been used, imposing the vanishing of the logarithm $\ln(-q^2)/q^4$ gives the constraint

$$\left(1-\frac{M_V^2}{M_A^2}\right) = \frac{M_S^2}{M_P^2}\left(1-\frac{M_S^2}{2\,M_P^2}\right), \quad (4.31)$$

which requires $M_A \leq \sqrt{2}M_V$. Imposing the right short-distance behaviour ($\sim 1/q^4$) in $\Pi(t)$, one gets

$$F^2\,(1+\delta^{(2)}_{\rm NLO}) \;-\; 8c_m^{r\,2} \;+\; 8\,d_m^{r\,2} \;=\; 0\,, \tag{4.32}$$

$$F^2\,M_S^2\,\delta^{(4)}_{\rm NLO} \;-\; 8c_m^{r\,2}\,M_S^{r\,2} \;+\; 8\,d_m^{r\,2}\,M_P^{r\,2} \;=\; -8\,\widetilde{\delta}\,, \tag{4.33}$$

where the corrections

$$\delta^{(m)}_{\rm NLO} = \frac{3M_S^2}{32\pi^2F^2}\left\{1+\left(1-\frac{M_S^2}{M_P^2}\right)\xi^{(m)}_{S\pi}+2\left(\frac{M_P^2}{M_S^2}-1\right)\xi^{(m)}_{P\pi}-\frac{2M_P^2}{M_S^2}\left(1-\frac{M_V^2}{M_A^2}\right)\xi^{(m)}_{V\pi}\right\} \tag{4.34}$$

are known functions of the resonance masses:

$$\begin{aligned}
\xi^{(2)}_{S\pi} &= 1-\frac{6M_S^2}{M_P^2}+\left(\frac{4M_S^2}{M_P^2}-\frac{6M_S^4}{M_P^4}\right)\ln\left(\frac{M_P^2}{M_S^2}-1\right),\\
\xi^{(2)}_{P\pi} &= 1+\frac{M_P^2}{M_S^2}\ln\left(1-\frac{M_S^2}{M_P^2}\right),\\
\xi^{(2)}_{V\pi} &= 1+\frac{3M_V^2}{M_P^2}\left[\frac{M_V^2}{M_P^2}-\frac{3}{2}+\left(1-\frac{M_V^2}{M_P^2}\right)^2\ln\left(\frac{M_P^2}{M_V^2}-1\right)\right],\\
\xi^{(4)}_{S\pi} &= -4+\left(2-\frac{4M_S^2}{M_P^2}\right)\ln\left(\frac{M_P^2}{M_S^2}-1\right),\\
\xi^{(4)}_{P\pi} &= 1+\ln\left(\frac{M_P^2}{M_S^2}-1\right),\\
\xi^{(4)}_{V\pi} &= \frac{M_P^2}{M_S^2}\left(1-\ln\frac{M_S^2}{M_V^2}\right)-\frac{2M_V^2}{M_S^2}\left(1-\frac{M_V^2}{M_P^2}\right)\\
&\quad+\left(\frac{M_P^2}{M_S^2}+\frac{2M_V^2}{M_S^2}\right)\left(1-\frac{M_V^2}{M_P^2}\right)^2\ln\left(\frac{M_P^2}{M_V^2}-1\right).
\end{aligned} \tag{4.35}$$

Note that from Eqs. (4.32) and (4.33) one determines the effective couplings c_m^r and d_m^r:

$$c_m^{r\,2} \;=\; \frac{F^2}{8}\frac{M_P^{r\,2}}{M_P^{r\,2}-M_S^{r\,2}}\left(1+\delta^{(2)}_{\rm NLO}-\frac{M_S^2}{M_P^2}\delta^{(4)}_{\rm NLO}-\frac{8}{M_P^2F^2}\widetilde{\delta}\right)\,, \tag{4.36}$$

$$d_m^{r\,2} \;=\; \frac{F^2}{8}\frac{M_S^{r\,2}}{M_P^{r\,2}-M_S^{r\,2}}\left(1+\delta^{(2)}_{\rm NLO}-\delta^{(4)}_{\rm NLO}-\frac{8}{M_S^2F^2}\widetilde{\delta}\right)\,. \tag{4.37}$$

4.4.3 Saturation of $L_8^r(\mu)$ at Next-to-leading Order in $1/N_C$

Once we have extracted information from short distance QCD, we are ready to study the low energy limit of the theory. One finds the contributions:

$$\Pi_{S-P}(q^2)|_{tree} = B_0^2\left(\frac{2\,F^2}{q^2}\;+\;\frac{16\,c_m^{r\,2}}{M_S^{r\,2}}\;-\;\frac{16\,d_m^{r\,2}}{M_P^{r\,2}}\right)+\mathcal{O}\left(q^2\right)\,, \tag{4.38}$$

$$\Delta\Pi_{S-P}(q^2)|_{\eta\pi} = \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\left[-1-\log\frac{-q^2}{M_S^2}\right]+\mathcal{O}\left(q^2\right)\,, \tag{4.39}$$

$$\begin{aligned}\Delta\Pi_{S-P}(q^2)|_{V\pi} =& -\frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{2G_V^2}{F^2}\left\{-\frac{17}{6}+7\frac{M_V^2}{M_P^2}-4\frac{M_V^4}{M_P^4}+\left(1-\frac{4M_V^2}{M_P^2}\right)\times\right.\\ &\left.\times\left(1-\frac{M_V^2}{M_P^2}\right)^2\log\frac{M_P^2-M_V^2}{M_V^2}\right\}+\mathcal{O}\left(q^2\right), \qquad (4.40)\\ \Delta\Pi_{S-P}(q^2)|_{A\pi} =& \, 0\,, \qquad (4.41)\\ \Delta\Pi_{S-P}(q^2)|_{S\pi} =& -\frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{4c_d^2}{F^2}\left\{\left(\frac{F^2}{2c_d^2}-1\right)^2\left(1-\frac{M_S^2}{M_P^2}\right)^2\left[-2+\left(1-\frac{2M_S^2}{M_P^2}\right)\times\right.\right.\\ &\left.\times\log\frac{M_P^2-M_S^2}{M_S^2}\right]+\frac{1}{6}+\left(\frac{F^2}{2\,c_d^2}-1\right)\left(1-\frac{M_S^2}{M_P^2}\right)\times\\ &\left.\times\left[1-\frac{2M_S^2}{M_P^2}+\left(\frac{2M_S^2}{M_P^2}-\frac{2M_S^4}{M_P^4}\right)\log\frac{M_P^2-M_S^2}{M_S^2}\right]\right\}+\mathcal{O}\left(q^2\right), \qquad (4.42)\\ \Delta\Pi_{S-P}(q^2)|_{P\pi} =& \frac{n_f}{2}\frac{B_0^2}{8\pi^2}\frac{16d_m^2}{F^2}\left(\frac{M_P^2-M_S^2}{M_S^2}\right)^2\left[-2+\frac{2M_P^2-M_S^2}{M_S^2}\times\right.\\ &\left.\times\log\frac{M_P^2}{M_P^2-M_S^2}\right]+\mathcal{O}\left(q^2\right). \qquad (4.43)\end{aligned}$$

It is interesting to remark that the non-analytic $\log(-q^2)$ structure that arises in χPT from the $\pi\eta$ loop is exactly reproduced at low energies by the $\pi\eta$ cut within the resonance theory; working within a chiral invariant framework ensures the proper low energy behaviour. The remaining cuts with resonances are absent in χPT and they only produce analytical contributions that go to the low-energy constants.

This produces for $L_8^r(\mu)$ within $U(n_f)$ at any renormalization scale μ,

$$\begin{aligned}L_8^r(\mu) =& \frac{F^2}{16}\left(\frac{1}{{M_S^r}^2}+\frac{1}{{M_P^r}^2}\right)\left\{1+\delta^{(2)}_{\rm NLO}-\frac{{M_S^r}^2\delta^{(4)}_{\rm NLO}+8\widetilde{\delta}/F^2}{{M_S^r}^2+{M_P^r}^2}\right\}\\ &+\frac{n_f}{2}\frac{1}{256\,\pi^2}\left[-2-\log\frac{\mu^2}{M_S^2}\right]+\\ &+\frac{n_f}{2}\frac{1}{128\,\pi^2}\left(\frac{M_A^2-M_V^2}{M_A^2}\right)\left[\frac{17}{6}-7\frac{M_V^2}{M_P^2}+4\frac{M_V^4}{M_P^4}\right.\\ &\left.-\left(1-\frac{4M_V^2}{M_P^2}\right)\left(1-\frac{M_V^2}{M_P^2}\right)^2\log\frac{M_P^2-M_V^2}{M_V^2}\right]+\\ &+\frac{n_f}{2}\frac{1}{128\,\pi^2}\left(\frac{M_P^2-M_S^2}{M_P^2}\right)\left[-\frac{1}{6}-\frac{M_S^2}{M_P^2}+4\frac{M_S^4}{M_P^4}\right.\\ &\left.+\frac{M_S^4}{M_P^4}\left(-3+\frac{4M_S^2}{M_P^2}\right)\log\frac{M_P^2-M_S^2}{M_S^2}\right]+\\ &+\frac{n_f}{2}\frac{1}{128\,\pi^2}\left(\frac{M_P^2-M_S^2}{M_S^2}\right)\left[-2+\frac{2M_P^2-M_S^2}{M_S^2}\log\frac{M_P^2}{M_P^2-M_S^2}\right]. \qquad (4.44)\end{aligned}$$

In the first line we have the tree-level contribution, where the NLO relation from Eqs. (4.36) and (4.37) have been used. The next lines contain the one-loop contributions, respectively from $\pi\eta$, $V\pi$, $S\pi$ and $P\pi$, and where the LO constraints from Eq. (4.14) have been employed.

A last remark is required: the calculation has been done within the $U(3)$ case, whereas the usual χPT results are obtained in the $SU(3)$ framework. Therefore, we have to take into account the matching between the $U(3)$ and $SU(3)$ Chiral Perturbation Theories [21]. The difference between the value of L_8 in the two versions of the effective theory is related to the difference between the corresponding coefficients Γ_8, that is, the different running. Accordingly, the leading order prediction of L_8 is the same in both cases [23], since the running is a next-to-leading order effect. One gets [21]

$$L_8^{SU(3)}(\mu) \;=\; L_8^{U(3)}(\mu) + \frac{\Gamma_8^{SU(3)} - \Gamma_8^{U(3)}}{16\pi^2} \log\frac{M_0}{\mu}\,, \tag{4.45}$$

where $\Gamma_8^{U(3)} = 3/16$ [49], $\Gamma_8^{SU(3)} = 5/48$ [9], and $M_0 = 850 \pm 50$ MeV [50] is the mass of the η' in the chiral limit.

4.4.4 Phenomenology

At this point we have the chiral coupling $L_8^r(\mu)$ expressed in terms of the resonance masses M_V, M_A, $M_S \simeq M_S^r$, $M_P \simeq M_P^r$, the decay constant F and the $U(3)-SU(3)$ matching contribution, given by M_0, the mass of the η' in the chiral limit.

The different input parameters are defined in the chiral limit. We take the ranges [9, 22, 50, 51, 52] $M_V = (770 \pm 5)$ MeV, $M_S^r = (1.14 \pm 0.16)$ GeV, $M_P^r = (1.3 \pm 0.1)$ GeV, $M_0 = (0.85 \pm 0.05)$ GeV and $F = (89 \pm 2)$ MeV, and use the relation of Eq. (4.31) to fix M_A, keeping the constraint $M_P \geq M_S$ from Eq. (4.14) and imposing $M_A \geq 1$ GeV. The correction $\widetilde{\delta}$ turns out to be negligible. For the renormalization scale $\mu_0 = 770$ MeV, one obtains the following contributions

$$10^3 \cdot L_8^r(\mu_0) \;=\; \underbrace{0.33}_{tree} - \underbrace{0.05}_{U(3)\to SU(3)} - \underbrace{0.72}_{\pi\pi} + \underbrace{0.55}_{V\pi} + \underbrace{0.38}_{S\pi} + \underbrace{0.00}_{A\pi} + \underbrace{0.12}_{P\pi} \pm 0.4\,, \tag{4.46}$$

where one finds the expected suppression of heavier thresholds.

The largest uncertainties originate in the badly known values of M_S^r and M_P^r, which already appear in the leading order prediction. The keypoint is the fact that the rest are purely NLO errors in $1/N_C$ and they remain small, validating the perturbative expansion in $1/N_C$. To account for the higher-mass intermediate states which have been neglected, we have added an additional truncation error equal to $0.12\cdot 10^{-3}$, the size of the heaviest included channel ($P\pi$). Note that the smallness of the truncation error ensures that the Single Resonance Approximation is fair within this framework. All errors have been added in quadrature. Therefore we arribe to

$$L_8^r(\mu_0) = (0.6 \pm 0.4) \cdot 10^{-3}\,, \tag{4.47}$$

to be compared with the value $L_8^r(\mu_0) = (0.9 \pm 0.3) \cdot 10^{-3}$, usually adopted in phenomenological analyses.

It is interesting to recall that for the considered scale $\mu_0 = 770$ MeV, our NLO prediction for $L_8^r(\mu_0)$ suffers small deviations with respect to its value at leading order, $L_8^{N_C\to\infty} = 0.8\cdot 10^{-3}$, given by Eq. (4.15). Through a simple χPT analysis one finds that varying the renormalization scale between $\mu_1 = 0.5$ GeV and $\mu_2 = 1$ GeV produces a variation on the renormalized coupling of the order of $|L_8^r(\mu_2)-L_8^r(\mu_1)| \sim 0.5\cdot 10^{-3}$. The outcome of our $1/N_C$ calculation shows a perfect agreement with these considerations, being the possible deviations between LO and NLO in $1/N_C$ of the order of the expected renormalization scale uncertainties in L_8.

4.5 Conflict between High-energy Constraints

The Resonance Chiral Theory is an effective approach of QCD that models large-N_C by cutting the tower of resonances, that is, an infinite number of meson fields is not considered. However, it is known that really an infinite tower of resonances is needed to recover the large-N_C behaviour within QCD. Therefore, it is not surprising to find some conflicts between the constraints of Appendix D. In fact, it should have been expected, since our approach does not fully recover QCD and, eventually, it may lead to inconsistencies: not all behaviours of QCD can be satisfied at the same time within the MHA.

In Ref. [48] it was claimed that there exists in general a problem between QCD short-distance constraints for Green Functions and those coming from form factors and cross-sections following from the quark counting rule [28]. However, from the general analysis of two-body form factors, developed in detail in Appendix D, we find that the spin-0 sector does not lead to contradictions. On the other hand, the form factors related to spin-1 mesons drive us in some cases to constraints which do not agree with those coming from other form factors.

This incompatibility can be solved by including a second multiplet, being this idea supported by large-N_C. Note that we follow the Minimal Hadronic Approximation [24], therefore a second multiplet should be incorporated if there exists a conflict between the short-distance constraints of the problem at hand.

For instance, in the left-right correlator, if the analysis is taken up to next-to-leading order in the $1/N_C$ expansion, all the constraints related to the vector and axial form factors (Appendix D) should be considered. However, the restrictions for λ_i^{VA} in Eq. (D.9) from the vector form factor to an axial resonance field and a pion, and those in Eq. (D.54) from the axial form factor to a vector resonance field and a pion are incompatible. The proposed solution is the inclusion of a second multiplet for the $V(1^{--})$ and $A(1^{++})$ resonances, but only for internal lines. Then one should add new pieces to the lagrangian of Eq. (4.1):

$$\mathcal{L}_{V'} = \frac{F_V'}{2\sqrt{2}}\langle\, V'_{\mu\nu} f_+^{\mu\nu}\,\rangle + \frac{iG_V'}{2\sqrt{2}}\langle\, V'_{\mu\nu}[u^\mu, u^\nu]\,\rangle\,, \tag{4.48}$$

$$\mathcal{L}_{A'} = \frac{F_A'}{2\sqrt{2}}\langle\, A'_{\mu\nu} f_-^{\mu\nu}\,\rangle\,, \tag{4.49}$$

$$\begin{aligned}\mathcal{L}_{V'A} =\;& \lambda_1^{V'A}\langle\, [V'_{\mu\nu}, A^{\mu\nu}]\chi_- \,\rangle + i\,\lambda_2^{V'A}\langle\, [V'^{\mu\nu}, A_{\nu\alpha}]h^\alpha_\mu \,\rangle \\ &+ i\,\lambda_3^{V'A}\langle\, [\nabla^\mu V'_{\mu\nu}, A^{\nu\alpha}]u_\alpha \,\rangle + i\,\lambda_4^{V'A}\langle\, [\nabla_\alpha V'_{\mu\nu}, A^{\alpha\nu}]u^\mu \,\rangle \\ &+ i\,\lambda_5^{V'A}\langle\, [\nabla_\alpha V'_{\mu\nu}, A^{\mu\nu}]u^\alpha \,\rangle + i\,\lambda_6^{V'A}\langle\, [V'_{\mu\nu}, A^\mu_\alpha]f_-^{\alpha\nu} \,\rangle\,, \qquad (4.50)\\ \mathcal{L}_{VA'} =\;& \lambda_1^{VA'}\langle\, [V_{\mu\nu}, A'^{\mu\nu}]\chi_- \,\rangle + i\,\lambda_2^{VA'}\langle\, [V^{\mu\nu}, A'_{\nu\alpha}]h^\alpha_\mu \,\rangle \\ &+ i\,\lambda_3^{VA'}\langle\, [\nabla^\mu V_{\mu\nu}, A'^{\nu\alpha}]u_\alpha \,\rangle + i\,\lambda_4^{VA'}\langle\, [\nabla_\alpha V_{\mu\nu}, A'^{\alpha\nu}]u^\mu \,\rangle \\ &+ i\,\lambda_5^{V'A}\langle\, [\nabla_\alpha V_{\mu\nu}, A'^{\mu\nu}]u^\alpha \,\rangle + i\,\lambda_6^{VA'}\langle\, [V_{\mu\nu}, A'^\mu_\alpha]f_-^{\alpha\nu} \,\rangle\,. \qquad (4.51)\end{aligned}$$

From the results from Eq. (D.9) and Eq. (D.54) is now obvious that the new constraints are, respectively:

$$\begin{aligned}F_V(2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA}) + F'_V(2\lambda_2^{V'A} - 2\lambda_3^{V'A} + \lambda_4^{V'A} + 2\lambda_5^{V'A}) &= F_A\,,\\ F_V(-2\lambda_2^{VA} + \lambda_3^{VA}) + F'_V(-2\lambda_2^{V'A} + \lambda_3^{V'A}) &= 0\,,\\ F_A(2\lambda_2^{VA} - \lambda_4^{VA} - 2\lambda_5^{VA}) + F'_A(2\lambda_2^{VA'} - \lambda_4^{VA'} - 2\lambda_5^{VA'}) &= -F_V + 2G_V\,,\\ F_A(-2\lambda_2^{VA} + \lambda_3^{VA}) + F'_A(-2\lambda_2^{VA'} + \lambda_3^{VA'}) &= -G_V\,,\end{aligned} \qquad (4.52)$$

so the incompatibility is not present any longer. In this way, the incompatibilities in the lightest resonances couplings can be carried to the couplings of higher states that produce mild effects on the region of validity of our effective description.

4.6 Conclusions

Resonance Chiral Theory is an effective framework to handle QCD at energies where one has hadronic resonances and pseudo-Goldstones from the chiral symmetry breaking. The expansion in powers of $1/N_C$ provides a key in order to construct the effective action. In addition to embed χPT at low energies, this theory must recover perturbative QCD and the OPE at short distances.

Several constraints on the RχT couplings are derived from the study of Green Functions of QCD currents at large-N_C. The other source of information is the consideration of the Brodsky-Lepage behaviour of the form factors, e.g. the pion vector form factor. This work shows the necessity of also taking into account the form factors with resonances in the final state. They are related to two-point Green Functions at next-to-leading order in the $1/N_C$ expansion and rule their asymptotic behaviour at one-loop. All two-body form factors that can be found in the even-intrinsic-parity sector of Resonance Chiral Theory (Single Resonance Approximation) have been analysed, producing the constraints and form factor structures shown in Appendix D.

It is important to remark that there are no new constraints coming from the short-distance analysis of form factors with one photon and one meson in the final state, that is, two-meson form factor analysis provides the most stringent set of constraints. This is not a surprise taking into account the relation between the vector resonances and the photon because of their quantum numbers.

The need of taming the resonance form factors at high energies was hinted in the last chapter, in order to improve the short distance behaviour of the pion vector form factor at the one-loop level [26]. This immediately leads to demand operators with more than one resonance field. Thus, one must study amplitudes with resonances as external states at LO in $1/N_C$ whenever a calculation is carried at the loop level. In our case, the optical theorem tells us that the relevant amplitudes are just the two-body form factors.

We have illustrated the analysis showing the case of the $\Pi_{S-P}(q^2)$ correlator and the RχT prediction of the corresponding low energy coupling $L_8^r(\mu)$ at NLO in $1/N_C$. From the Weinberg sum rules for $\Pi_{S-P}(q^2)$ and the pion scalar form factor one gets its expression at leading order. Dispersive integrals show that the correlator up to NLO in $1/N_C$ is just given by terms proportional to the squared modulus of form factors and renormalized resonance parameters. Furthermore, the local χPT operators are shown to be absent within our present realization of the resonance lagrangian. The modified parameters can be partially fixed by taking the Weinberg sum rule analysis of $\Pi_{S-P}(q^2)$ up to NLO. This produces a slight modification to the leading relation, which can be read as:

$$F^2\,(1+\delta^{(2)}_{\rm NLO}) \,-\, 8c_m^{r\,2} \,+\, 8\,d_m^{r\,2} \,=\, 0\,,$$
$$F^2\,M_S^2\,\delta^{(4)}_{\rm NLO} \,-\, 8c_m^{r\,2}\,M_S^{r\,2} \,+\, 8\,d_m^{r\,2}\,M_P^{r\,2} \,\simeq\, 0\,.$$

The chiral invariance in RχT leads to the recovering of the χPT structure at low energies. The $\pi\eta$ cut in RχT reproduces the long distance non-analytic term $\log(-q^2)$ from one-loop χPT. This keeps the control on the renormalization scale μ appearing in χPT within $\log(-q^2/\mu^2)$ and $L_8^r(\mu)$. The remaining absorptive cuts generate analytic terms in q^2 and they only contribute to the LEC's. All this provides the determination for $\mu_0 = 770$ MeV,

$$L_8^r(\mu_0) \quad = \quad (\,0.6\,\pm 0.4\,)\,\cdot\,10^{-3}\,.$$

which can be compared to χPT value $L_8(\mu_0)|_{\rm exp} = (0.9 \pm 0.3)\cdot 10^{-3}$ [17]. Since the dependence on the scale is always exactly controlled, this problematic uncertainty disappears in our picture. On the other hand, the bulk of the error is due to the current ignorance on the values of the masses of the scalar and pseudoscalar multiplets in the chiral limit. The reduction of their relative uncertainties below the 5% level would drastically improve the result. Until then, purely NLO errors in $1/N_C$ and the Single Resonance Approximation produce just a subdominant contribution to the global error. This validates the perturbative expansion in $1/N_C$ and points out the way to proceed in order in increase the accuracy of the determination.

To end with, we have commented some problems that appear in the spin-1 sector due to the truncation of the large–N_C spectrum of infinite resonances. Because of this cut in the tower of resonances, it is clear that QCD cannot be exactly recovered through our effective approach and that some conflicts between constraints may eventually arise. In our case, it is shown that this incompatibility can be solved by including a second multiplet.

Chapter 5

One-loop Renormalization: Scalar and Pseudoscalar Resonances

5.1 Introduction

Since its inception Resonance Chiral Theory has been applied both to the study of resonance contributions in weak interaction processes (radiative and non–leptonic kaon decays) [53] and to the study of form factors of mesons [46], where only the RχT lagrangian at tree level has been used and, accordingly, the leading contribution in the large-N_C approach we are describing has been obtained.

The next-to-leading order in the $1/N_C$ expansion arises from one loop calculations within the theory and its control starts to be necessary both on grounds of the convergence of the predictions and to straighten our knowledge of non-perturbative QCD. A Dyson-Schwinger resummation of subleading orders is required to describe the amplitudes near the resonance peak [47], leading, eventually, to systematic one-loop calculations [26, 29, 30, 37, 54]. Improving the phenomenological determinations of non-perturbative QCD quantities is needed in order to distinguish new physics effects. As it has been pointed out in the previous chapter, it also allows getting the resonance contributions to the χPT LEC's at next-to-leading order, keeping the dependence on the renormalization scale under control. Furthermore, quantum loops are essential to find the quantum field theory description and to properly understand the hadronic interactions beyond *ad hoc* modelings.

RχT is non-renormalizable. Moreover the lack of an expansion parameter in the lagrangian does not make feasible the application of a perturbative renormalization program based on a well defined power-counting scheme analogous to the one in χPT. Nevertheless from a practical point of view the situation is similar to the χPT case [55]. As shown in Chapter 3 [26], where the vector form factor of the pion was calculated at one-loop level in RχT, it is possible to construct a finite number of operators, within the theory, whose couplings can absorb the divergences coming from one loop diagrams. The only requirement is, of course, that the regularization procedure of the loop divergences respects the symmetries of the lagrangian.

In the present chapter we have studied the full one-loop generating functional that arises from RχT when one multiplet of scalar and pseudoscalar resonances are considered and only up to bilinear couplings in the resonances are included. The divergent contributions have been evaluated and, consequently, the full set of operators needed to renormalize the theory properly has been obtained. The conceptual differences with the χPT renormalization program will also be stressed.

In Section 5.2 we describe shortly the content of RχT that is of interest in our case and its main features. Section 5.3 is devoted to explain the procedure and hints which are followed to perform the evaluation of the generating functional, whose results are given in Section 5.4 and commented in Section 5.5. In Section 5.6 we point out the conclusions and summarize. Some technical details are relegated to Appendix F and most of the results to Appendix G.

5.2 RχT with Scalars and Pseudoscalars Resonances

We consider the RχT lagrangian constituted by pseudo-Goldstone bosons and one multiplet of both scalar and pseudoscalar resonances. Motivated by the large-N_C limit we include $U(3)$ multiplets for the spectrum though we limit ourselves to $SU(3)$ external currents as we are not interested in anomaly related issues. Our lagrangian reads:

$$\mathcal{L}_{R\chi T}(\phi, \mathrm{S}, \mathrm{P}) = \mathcal{L}_{pGB}^{(2)} + \mathcal{L}_{\mathrm{kin}\, S} + \mathcal{L}_{\mathrm{kin}\, P} + \mathcal{L}_S + \mathcal{L}_P + \mathcal{L}_{SS} + \mathcal{L}_{PP} + \mathcal{L}_{SP} \,, \quad (5.1)$$

where the notation of Section 2.3.2 is followed. The different pieces of Eq. (5.1) are given in Eqs. (2.6), (2.11), (2.11), (2.9), (2.10), (2.13), (2.13) and (2.14) respectively. In other words, we have considered all terms observing chiral and QCD symmetries which are constructed with scalar and pseudoscalar resonances together with chiral tensors of $\mathcal{O}(p^2)$, up to bilinear couplings in the resonance fields and under the Single Resonance Approximation.

Several comments on our lagrangian theory are suitable here:

- The RχT lagrangian satisfies, by construction, the structures of chiral dynamics at very low-energies ($E \ll M_R$). Notwithstanding, it is clear that there is no small coupling or kinematical parameter that could allow us to perform a perturbative expansion in order to solve the effective action of the theory, as it happens in χPT. We stress again that the large-N_C limit guides a loop perturbative expansion, not in the lagrangian, but in the observables evaluated with it.

  It has also been proposed [56] that, due to the fact that the chiral counting is spoiled when resonances are included in loops, it could be possible to keep the chiral counting by disentangling the "*hard*" modes that could be absorbed in the renormalization program. In this way one gets a chiral expansion even if resonance contributions in the loop are considered. This procedure can be useful but only if one is interested in the application at very low energies out of the resonance region.

- Short-distance constraints on the asymptotic behaviour of form factors and Green Functions provide, in the $1/N_C$ expansion, different relations between the couplings. Assuming the usual constraints of Eq. (2.38) [22], one has for the $\mathcal{L}_S$, $\mathcal{L}_P$, $\mathcal{L}_{\mathrm{kin}\,S}$ and $\mathcal{L}_{\mathrm{kin}\,P}$ couplings:

$$c_m = c_d = \sqrt{2}d_m = \frac{F}{2}, \qquad M_P \simeq \sqrt{2}M_S, \tag{5.2}$$

as it has been explained in Section 2.4. High-energy constraints on the λ_i^{RR} couplings in the $N_C \to \infty$ are shown in Appendix D, see Chapter 4 for more information. Taking into account that no terms with three resonance fields are considered, the following relations are found [54]:

$$\begin{aligned} \lambda_3^{SS} = \lambda_3^{PP} &= 0, \\ \lambda_1^{SP} = 4\,\lambda_2^{SP} = -\frac{d_m}{c_m} = \frac{-2c_m + c_d}{2d_m} &= -\frac{1}{\sqrt{2}}, \end{aligned} \tag{5.3}$$

where we have used Eq. (5.2). From Appendix D these results can be obtained easily, by neglecting the couplings with three resonances:

(a) From the scalar form factor $\langle\, P^i|s^j|\pi^k\,\rangle$, see Eq. (D.109), it is obtained that

$$\lambda_1^{SP} = -\frac{d_m}{c_m}. \tag{5.4}$$

(b) The asymptotic behaviour of the scalar form factor $\langle\, S^i|s^j|S^k\,\rangle$ gives, see Eq. (D.119),

$$\lambda_3^{SS} = 0. \tag{5.5}$$

(c) Studying the high-energy behaviour of the scalar form factor $\langle\, P^i|s^j|P^k\,\rangle$, see Eq. (D.124), one gets

$$\lambda_3^{PP} = 0. \tag{5.6}$$

(d) From the ultraviolet limit of the pseudoscalar form factor $\langle\, S^i|p^j|\pi^k\,\rangle$, see Eq. (D.152), it is found that

$$\lambda_1^{SP} = \frac{-2c_m + c_d}{2d_m}. \tag{5.7}$$

(e) The pseudoscalar form factor $\langle\, S^i|p^j|P^k\,\rangle$, see Eq. (D.172), relates λ_1^{SP} and λ_2^{SP}:

$$\lambda_1^{SP} = 4\,\lambda_2^{SP}. \tag{5.8}$$

Though the relations shown in Eqs. (5.2) and (5.3) could be used to simplify the outcome of the calculations presented in this chapter, we will give the full results without short-distance constraints built-in so as not to lose generality.

- From the RχT lagrangian in Eq. (5.1), the equations of motion for the pseudo-Goldstone and resonance fields are obtained as the system of coupled equations:

$$\begin{aligned}\nabla^\mu u_\mu =& \frac{i}{2}\chi_- - \frac{2c_d}{F^2}\nabla^\mu\{u_\mu, S\} + \frac{i\,c_m}{F^2}\{\chi_-, S\} - \frac{1}{2F^2}\big[u_\mu, [\nabla^\mu S, S]\big] \\ & - \frac{2\lambda_1^{\rm SS}}{F^2}\nabla_\mu\{u^\mu, SS\} - \frac{4\lambda_2^{\rm SS}}{F^2}\nabla_\mu(S\,u^\mu S) + \frac{i\,\lambda_3^{\rm SS}}{F^2}\{\chi_-, SS\} \\ & - \frac{d_m}{F^2}\{\chi_+, P\} - \frac{1}{2F^2}\big[u_\mu, [\nabla^\mu P, P]\big] - \frac{2\lambda_1^{\rm PP}}{F^2}\nabla_\mu\{u^\mu, PP\} \\ & - \frac{4\lambda_2^{\rm PP}}{F^2}\nabla_\mu(P\,u^\mu P) + \frac{i\,\lambda_3^{\rm PP}}{F^2}\{\chi_-, PP\} - \frac{2\lambda_1^{\rm SP}}{F^2}\nabla^\mu\{\nabla_\mu S, P\} \\ & + \frac{\lambda_1^{\rm SP}}{2F^2}\Big[u_\mu, \big[S, \{P, u^\mu\}\big]\Big] - \frac{\lambda_2^{\rm SP}}{F^2}\{\chi_+, \{S, P\}\}\,, \end{aligned} \tag{5.9}$$

$$\begin{aligned}\nabla^\mu\nabla_\mu S =& -M_S^2\,S + c_m\,\chi_+ + c_d\,u_\mu u^\mu + \lambda_1^{\rm SS}\{S, u_\mu u^\mu\} + 2\lambda_2^{\rm SS}u_\mu S u^\mu \\ & + \lambda_3^{\rm SS}\{S, \chi_+\} - \lambda_1^{\rm SP}\nabla_\mu\{P, u^\mu\} + i\lambda_2^{\rm SP}\{P, \chi_-\}\,, \end{aligned} \tag{5.10}$$

$$\begin{aligned}\nabla^\mu\nabla_\mu P =& -M_P^2\,P + i\,d_m\,\chi_- + \lambda_1^{\rm PP}\{P, u_\mu u^\mu\} + 2\lambda_2^{\rm PP}u_\mu P u^\mu \\ & + \lambda_3^{\rm PP}\{P, \chi_+\} + \lambda_1^{\rm SP}\{\nabla_\mu S, u^\mu\} + i\lambda_2^{\rm SP}\{S, \chi_-\}\,. \end{aligned} \tag{5.11}$$

Like it has been stressed previously, the lack of an expansion coupling or parameter in RχT hinders a perturbative renormalization like the one applied in χPT. By studying the vector form factor of the pion at next-to-leading order, in Chapter 3 it was shown that, using dimensional regularization, all the divergences could be absorbed by the introduction of local operators fulfilling the symmetry requirements. This is a particular case of the well known fact that all divergences are local in a quantum field theory [57], and are given by a polynomial in the external momenta or masses. Hence it is reasonable to consider the construction of the full set of operators that renders our $\mathcal{L}_{R\chi T}(\phi, S, P)$ theory finite up to one-loop. Accordingly we perform the one-loop generating functional of our lagrangian theory to evaluate the full set of divergences that arise. This we pursue in the rest of the chapter.

5.3 Generating Functional at One Loop

The generating functional of the connected Green Functions, $W[J]$, is the logarithm of the vacuum-to-vacuum transition amplitude in the presence of external sources

$J(x)$ coupled to bilinear quark currents:

$$e^{i\,W[J]} \;=\; \frac{1}{\mathcal{N}} \int [\,\mathrm{d}\psi\,]\; e^{i\,S_0[\psi,J]}\;, \tag{5.12}$$

where the normalization is such that $W[0] = 0$ and the field ψ is, in our case, short for the pseudo-Goldstone and resonance mesons. The evaluation of the generating functional of our lagrangian theory $\mathcal{L}_{R\chi T}(\phi, S, P)$, is readily done with the background field method [58, 59], where the action is expanded around the classical fields ψ_{cl}. By defining the quantum field as $\Delta\psi = \psi - \psi_{cl}$, the expansion up to one loop ($L = 1$) is given by:

$$\begin{aligned} W[J]_{L=1} \;=\; & S_0[\psi_{cl}, J] \;-\; i\,\log\Bigg[\int [\,\mathrm{d}\Delta\psi\,]\,\exp\Bigg(i\int \mathrm{d}^4x_1\,\frac{\delta\, S_0[\psi,J]}{\delta\psi_i(x_1)}\Big|_{\psi_{cl}}\,\Delta\psi_i(x_1) \\ & +\frac{i}{2!}\int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\;\Delta\psi_i(x_1)\,\frac{\delta^2\, S_0[\psi,J]}{\delta\psi_i(x_1)\,\delta\psi_j(x_2)}\Big|_{\psi_{cl}}\,\Delta\psi_j(x_2)\Bigg)\Bigg]\,, \end{aligned} \tag{5.13}$$

but for an irrelevant constant. The i, j indices run over all the different fields and are summed over. The classical field ψ_{cl} is, by definition, the solution of:

$$\left.\frac{\delta S_0[\psi,J]}{\delta\psi_i(x)}\right|_{\psi_{cl}} \;=\; 0\,, \tag{5.14}$$

that provides the implicit relation $\psi_{cl} = \psi_{cl}[J]$ and the equations of motion for the classical fields. Solving the remaining gaussian integral in the Euclidean spacetime and coming back to Minkowsky we have finally:

$$W[J]_{L=1} \;=\; S_0[\psi_{cl}, J] \,+\, S_1[\psi_{cl}, J]\,, \tag{5.15}$$

$$S_1[\psi_{cl}, J] \;=\; \frac{i}{2}\,\log\,\det\mathcal{D}(\psi_{cl}, J)\;\;, \tag{5.16}$$

where $\mathcal{D}(\psi_{cl}, J)$ is the quadratic differential operator specified by:

$$\langle\, x\,|\,\mathcal{D}(\psi_{cl}, J)\,|\,y\,\rangle_{ij} \;=\; \left.\frac{\delta^2\, S_0[\psi,J]}{\delta\psi_i(x)\,\delta\psi_j(y)}\right|_{\psi_{cl}}\;. \tag{5.17}$$

The action at one loop needs regularization and, following the use within χPT, we will proceed by working in D spacetime dimensions, a procedure that preserves the relevant symmetries of our theory. Divergences in the functional integration are local and, within dimensional regularization, can be absorbed through local operators that satisfy the same symmetries than the original theory [57]. The one-loop renormalized lagrangian is thus defined by:

$$\mathcal{L}_1[\psi, J] \;=\; \mu^{D-4}\,\left(\mathcal{L}_1^{\mathrm{ren}}[\psi, J;\mu] \,+\, \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\mathcal{L}_1^{\mathrm{div}}[\psi, J]\right)\;. \tag{5.18}$$

In Eq. (5.18) we have split the one-loop bare lagrangian into a renormalized and a divergent part, and the scale μ is introduced in order to restore the correct dimensions in the renormalized lagrangian for $D \neq 4$. The divergent part $\mathcal{L}_1^{\rm div}$ contains the counterterms which exactly cancel the divergences found in the result for the one-loop generating functional of Eq. (5.15).

Up to one loop $\mathcal{L}_1[\psi, J]$ can be written in terms of a minimal basis of N operators $\mathcal{O}_i[\psi, J]$. For a non-renormalizable theory, such as RχT, N grows with the number of loops. Accordingly we expect to find in our evaluation of $S_1[\psi, J]$ many more operators that those in the original tree level theory $S_0[\psi, J]$. The structure of these obeys the same construction principles (symmetries) that gave $\mathcal{L}_{R\chi T}(\phi, S, P)$ in Eq. (5.1), though we foresee that higher-order chiral tensors may be involved. A detailed study of the functional integration shows that the new terms have the structure $\chi^{(4)}$, $R\,\chi^{(4)}$ or $R\,R\,\chi^{(4)}$ (with a single or multiple traces) and $\chi^{(2)}$, $R\,\chi^{(2)}$ and $R\,R\,\chi^{(2)}$ (with multiple traces)[1].

5.3.1 Expansion Around the Classical Solutions

Following the aforementioned procedure we expand the action associated to our lagrangian $\mathcal{L}_{R\chi T}(\phi, S, P)$ in Eq. (5.1) around the solutions of the classical equations of motion: $u_{cl}(\phi)$, S_{cl} and P_{cl}. The fluctuations of the pseudoscalar Goldstone fields Δ_i $(i = 0, ..., 8)$, and of the scalar and pseudoscalar resonances ε_{S_i} and ε_{P_i}, are parameterized as[2]:

$$\begin{aligned} u_R &= u_{cl}\, e^{i\Delta/2}\,, & u_L &= u_{cl}^\dagger\, e^{-i\Delta/2}\,, \\ S &= S_{cl} + \frac{1}{\sqrt{2}}\varepsilon_{\rm S}\,, & P &= P_{cl} + \frac{1}{\sqrt{2}}\varepsilon_{\rm P}\,, \end{aligned} \tag{5.19}$$

with

$$\Delta = \Delta_i\lambda_i/F\,, \qquad \varepsilon_{\rm S} = \varepsilon_{{\rm S}i}\,\lambda_i\,, \qquad \varepsilon_{\rm P} = \varepsilon_{{\rm P}i}\,\lambda_i\,. \tag{5.20}$$

In the following we will drop the subindex "*cl*" for simplicity.

Expanding the lagrangian using Eqs. (5.19) and (5.20) up to terms quadratic in the fields (Δ_i, ε_{S_i}, ε_{P_i}) and using the EOM of Eqs. (5.9), (5.10) and (5.11), we

[1] As it will be emphasized later, in the procedure and due to a necessary field redefinition, terms with more than two resonances will be generated. We attach to our initial scheme and only will keep terms with up to two resonances.

[2] This is a convenient choice for the pseudoscalar fluctuation variables in order to simplify several cumbersome expressions. Notice that, once the "gauge" $u_R = u_L^\dagger \equiv u$ is enforced, it implies that the classical and the quantum pseudo-Goldstone fields commute: $u_{cl}\,\exp(i\Delta/2) = \exp(i\Delta/2)\,u_{cl}$.

obtain the second-order fluctuation lagrangian, that takes the form[3]:

$$\begin{aligned}\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = & -\frac{1}{2}\Delta_i\left(d'_\mu d'^\mu+\sigma\right)_{ij}\Delta_j-\frac{1}{2}\varepsilon_{\mathrm{S}i}\left(d^\mu d_\mu+k^{\mathrm{S}}\right)_{ij}\varepsilon_{\mathrm{S}j}-\frac{1}{2}\varepsilon_{\mathrm{P}i}\left(d^\mu d_\mu+k^{\mathrm{P}}\right)_{ij}\varepsilon_{\mathrm{P}j}\\ & +\varepsilon_{\mathrm{S}i}\,{a^{\mathrm{S}}}_{ij}\,\Delta_j+\varepsilon_{\mathrm{P}i}\,{a^{\mathrm{P}}}_{ij}\,\Delta_j+\varepsilon_{\mathrm{P}i}\,{a^{\mathrm{SP}}}_{ij}\,\varepsilon_{\mathrm{S}j}\\ & +\varepsilon_{\mathrm{S}k}\,b^{\mathrm{S}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j+\varepsilon_{\mathrm{P}k}\,b^{\mathrm{P}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j+\varepsilon_{\mathrm{P}k}\,b^{\mathrm{SP}}_{\mu\,ki}\,d^\mu_{ij}\varepsilon_{\mathrm{S}j}\ .\end{aligned} \tag{5.21}$$

Derivatives and matrices are defined in Appendix F where it is also shown that in order to write $\Delta\mathcal{L}_{R\chi T}$ in the form displayed above we need to perform two field redefinitions. This procedure generates operators with multiple resonance fields. However our theory, as specified in Section 5.2, does not include operators with more than two resonances and, for consistency, we shall keep this structure in the fluctuation lagrangian, thus disregarding operators with three or more resonance fields in the following. We will comment later on the consequences of this feature. It is customary to write the second-order fluctuation lagrangian as:

$$\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = -\frac{1}{2}\eta\,(\,\Sigma_\mu\,\Sigma^\mu\,+\,\Lambda\,)\,\eta^\top\ , \tag{5.22}$$

where η collects the fluctuation fields, $\eta=\left(\Delta_i,\varepsilon_{\mathrm{S}j},\varepsilon_{\mathrm{P}k}\right)$, $i,j,k=0,...,8$, $\eta^\top$ is its transposed and the rest of definitions are given in Appendix F.

5.3.2 Divergent Part of the Generating Functional at One Loop

After we have performed the second-order fluctuation on our lagrangian theory we come back to our discussion at the beginning of this section in order to identify the one-loop generating functional, specified now by the action:

$$S_1 = \frac{i}{2}\log\det\,(\,\Sigma_\mu\,\Sigma^\mu\,+\,\Lambda\,)\ . \tag{5.23}$$

We use dimensional regularization to extract the divergence of this expression. As emphasized in the literature [60] it is convenient to employ the Schwinger-DeWitt proper-time representation, embedded in the heat-kernel formalism, in order to extract the residue at the $D-4$ pole. Ref. [58] shows that, in fact, symmetry considerations can also provide this information (at least up to one loop).

Hence we get:

$$S_1 = -\frac{1}{(4\pi)^2}\frac{1}{D-4}\int \mathrm{d}^4x\ \mathrm{Tr}\left(\frac{1}{12}\,Y_{\mu\nu}\,Y^{\mu\nu}\,+\,\frac{1}{2}\Lambda^2\right)\,+\,S_1^{\mathrm{finite}}\,, \tag{5.24}$$

where Tr is short for the trace in the flavour space, $Y_{\mu\nu}$ denotes the field strength tensor of Y_μ in Eq. (F.27):

$$Y_{\mu\nu} = \partial_\mu Y_\nu-\partial_\nu Y_\mu+[Y_\mu,Y_\nu]\,. \tag{5.25}$$

[3]The intricacies of this evaluation are explained in detail in Appendix F

The finite remainder S_1^{finite} cannot be simply expressed as a local lagrangian, but can be worked out for a given transition [9, 61].

Finally we get the one-loop divergence as:

$$S_1^{\mathrm{div}} \;=\; -\frac{1}{(4\,\pi)^2}\,\frac{1}{D-4}\int \mathrm{d}^4x\,\mathcal{L}_1^{\mathrm{div}}\;, \tag{5.26}$$

where

$$\begin{aligned}
\mathcal{L}_1^{\mathrm{div}} \;=\;& \frac{1}{12}\langle \gamma'_{\mu\nu}\gamma'^{\,\mu\nu} + 2\gamma_{\mu\nu}\gamma^{\mu\nu}\rangle + \frac{1}{2}\langle \sigma^2 + k^{\mathrm{P}\,2} + k^{\mathrm{S}\,2}\rangle + \langle a^{\mathrm{S}} a^{\mathrm{S}\,\top} + a^{\mathrm{P}} a^{\mathrm{P}\,\top} + a^{\mathrm{SP}} a^{\mathrm{SP}\,\top}\rangle \\
& -\frac{1}{12}\langle \gamma'^{\,\mu\nu}\left(b_\mu^{\mathrm{S}\,\top} b_\nu^{\mathrm{S}} + b_\mu^{\mathrm{P}\,\top} b_\nu^{\mathrm{P}}\right)\rangle - \frac{1}{12}\langle \gamma^{\mu\nu}\left(b_\mu^{\mathrm{S}} b_\nu^{\mathrm{S}\,\top} + b_\mu^{\mathrm{P}} b_\nu^{\mathrm{P}\,\top} + b_\mu^{\mathrm{SP}} b_\nu^{\mathrm{SP}\,\top} + b_\mu^{\mathrm{SP}\,\top} b_\nu^{\mathrm{SP}}\right)\rangle \\
& -\langle a^{\mathrm{S}\,\top}\big(\bar{d}_+^{\mu} b_\mu^{\mathrm{S}} + \frac{1}{2} b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{P}\,\mu}\big) + a^{\mathrm{P}\,\top}\big(\bar{d}_+^{\mu} b_\mu^{\mathrm{P}} - \frac{1}{2} b_\mu^{\mathrm{SP}} b^{\mathrm{S}\,\mu}\big) + a^{\mathrm{SP}\,\top}\big(\hat{d}^{\mu} b_\mu^{\mathrm{SP}} + \frac{1}{2} b^{\mathrm{P}\,\mu} b_\mu^{\mathrm{S}\,\top}\big)\rangle \\
& +\frac{1}{4}\langle \sigma\big(b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\mu} + b_\mu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\mu}\big) + k^{\mathrm{S}}\big(b^{\mathrm{S}\,\mu} b_\mu^{\mathrm{S}\,\top} + b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\mu}\big) + k^{\mathrm{P}}\big(b^{\mathrm{P}\,\mu} b_\mu^{\mathrm{P}\,\top} + b^{\mathrm{SP}\,\mu} b_\mu^{\mathrm{SP}\,\top}\big)\rangle \\
& +\frac{1}{4}\langle \tilde{d}_-^{\mu} b_\mu^{\mathrm{S}\,\top} \bar{d}_+^{\nu} b_\nu^{\mathrm{S}} + \tilde{d}_-^{\mu} b_\mu^{\mathrm{P}\,\top} \bar{d}_+^{\nu} b_\nu^{\mathrm{P}} + \hat{d}^{\mu} b_\mu^{\mathrm{SP}\,\top} \hat{d}^{\nu} b_\nu^{\mathrm{SP}}\rangle \\
& -\frac{1}{12}\langle \tilde{d}_{+\mu} b_\nu^{\mathrm{S}\,\top} \bar{d}_-^{[\mu} b^{\mathrm{S}\,\nu]} + \tilde{d}_{+\mu} b_\nu^{\mathrm{P}\,\top} \bar{d}_-^{[\mu} b^{\mathrm{P}\,\nu]} + \hat{d}_{\mu} b_\nu^{\mathrm{SP}\,\top} \hat{d}^{[\mu} b^{\mathrm{SP}\,\nu]}\rangle \\
& +\frac{1}{4}\langle \tilde{d}_-^{\mu} b_\mu^{\mathrm{S}\,\top} b_\nu^{\mathrm{SP}\,\top} b^{\mathrm{P}\,\nu} - \tilde{d}_-^{\mu} b_\mu^{\mathrm{P}\,\top} b_\nu^{\mathrm{SP}} b^{\mathrm{S}\,\nu} + \hat{d}^{\mu} b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{P}\,\nu} b_\nu^{\mathrm{S}\,\top}\rangle \\
& -\frac{1}{12}\langle \tilde{d}_+^{\mu} b^{\mathrm{S}\,\nu\,\top} b_{[\mu}^{\mathrm{SP}\,\top} b_{\nu]}^{\mathrm{P}} - \tilde{d}_+^{\mu} b^{\mathrm{P}\,\nu\,\top} b_{[\mu}^{\mathrm{SP}} b_{\nu]}^{\mathrm{S}} + \hat{d}^{\mu} b^{\mathrm{SP}\,\nu\,\top} b_{[\mu}^{\mathrm{P}} b_{\nu]}^{\mathrm{S}\;\top}\rangle \\
& +\frac{1}{48}\langle \big(b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\mu} b_\nu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\nu} + b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\nu} b_\nu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\mu} + b_\mu^{\mathrm{S}\,\top} b_\nu^{\mathrm{S}} b^{\mathrm{S}\,\mu\,\top} b^{\mathrm{S}\,\nu}\big) \\
& \qquad +\big(b_\mu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\mu} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\nu} + b_\mu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\nu} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\mu} + b_\mu^{\mathrm{P}\,\top} b_\nu^{\mathrm{P}} b^{\mathrm{P}\,\mu\,\top} b^{\mathrm{P}\,\nu}\big) \\
& \qquad +\big(b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\mu} b_\nu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\nu} + b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\nu} b_\nu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\mu} + b_\mu^{\mathrm{SP}\,\top} b_\nu^{\mathrm{SP}} b^{\mathrm{SP}\,\mu\,\top} b^{\mathrm{SP}\,\nu}\big)\rangle \\
& +\frac{1}{24}\langle \big(b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\mu} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\nu} + b_\mu^{\mathrm{S}\,\top} b^{\mathrm{S}\,\nu} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{P}\,\mu} + b_\mu^{\mathrm{S}\,\top} b_\nu^{\mathrm{S}} b^{\mathrm{P}\,\mu\,\top} b^{\mathrm{P}\,\nu}\big) \\
& \qquad +\big(b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\mu} b^{\mathrm{S}\,\nu} b_\nu^{\mathrm{S}\,\top} + b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\nu} b_\nu^{\mathrm{S}} b^{\mathrm{S}\,\mu\,\top} + b_\mu^{\mathrm{SP}\,\top} b^{\mathrm{SP}\,\nu} b^{\mathrm{S}\,\mu} b_\nu^{\mathrm{S}\,\top}\big) \\
& \qquad +\big(b^{\mathrm{P}\,\mu} b_\mu^{\mathrm{P}\,\top} b^{\mathrm{SP}\,\nu} b_\nu^{\mathrm{SP}\,\top} + b^{\mathrm{P}\,\mu} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{SP}\,\nu} b_\mu^{\mathrm{SP}\,\top} + b_\mu^{\mathrm{P}} b_\nu^{\mathrm{P}\,\top} b^{\mathrm{SP}\,\mu} b^{\mathrm{SP}\,\nu\,\top}\big)\rangle\,,
\end{aligned} \tag{5.27}$$

where derivatives and matrices are defined in Appendix F and $\gamma_{\mu\nu} = \partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu + [\gamma_\mu, \gamma_\nu]$ (correspondingly for $\gamma'_{\mu\nu}$). Moreover for two vectors A_μ, B_μ we write $A_{[\mu}B_{\nu]} = A_\mu B_\nu - A_\nu B_\mu$. This result is completely general for the second-order fluctuation lagrangian in Eq. (5.21). However, and as explained in Appendix F, the expressions given there are valid only for operators with up to two resonances as we limit ourselves in this article.

5.3.3 Result

When worked out, S_1^{div} in Eq. (5.26) can be expressed in a basis of operators that satisfy the same symmetry requirements than our starting lagrangian $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}(\phi, S, P)$.

A minimal basis of $R\chi T$ operators that, upon integration of the resonances, contributes to the $\mathcal{O}(p^6)$ χPT lagrangian, in $SU(3)$, can be found in Ref. [19]. However, up to now, a basis for the one-loop $R\chi T$ has still not been worked out. This is precisely our result generated by $S_1^{\rm div}$. Hence, at one loop, the $R\chi T$ lagrangian needed to renormalize our theory reads:

$$\mathcal{L}_1 = \sum_{i=1}^{18} \alpha_i \, \mathcal{O}_i + \sum_{i=1}^{66} \beta_i^R \, \mathcal{O}_i^R + \sum_{i=1}^{379} \beta_i^{RR} \, \mathcal{O}_i^{RR} \, . \tag{5.28}$$

The $\mathcal{O}_i$ operators correspond to those up to $\mathcal{O}(p^4)$ in $U(3)_L \otimes U(3)_R$ χPT [49]. $\mathcal{O}_i^R$ and $\mathcal{O}_i^{RR}$ involve one and two resonance fields, respectively, together with $\chi^{(2)}$ and $\chi^{(4)}$ chiral tensors. The couplings in the bare lagrangian $\mathcal{L}_1$ read, in accordance with Eq. (5.18):

$$\begin{aligned}
\alpha_i &= \mu^{D-4} \left(\alpha_i^r(\mu) + \frac{1}{(4\pi)^2} \frac{1}{D-4} \gamma_i \right) , \\
\beta_i^R &= \mu^{D-4} \left(\beta_i^{R,r}(\mu) + \frac{1}{(4\pi)^2} \frac{1}{D-4} \gamma_i^R \right) , \\
\beta_i^{RR} &= \mu^{D-4} \left(\beta_i^{RR,r}(\mu) + \frac{1}{(4\pi)^2} \frac{1}{D-4} \gamma_i^{RR} \right) ,
\end{aligned} \tag{5.29}$$

where γ_i, γ_i^R and γ_i^{RR} are the divergent coefficients given by $S_1^{\rm div}$ that constitute the β-function of our lagrangian (we use the terminology of Ref. [55]). The determination of the latter though straightforward involves a long calculation. In order to diminish the possibility of errors we have performed two independent evaluations. One of them has been carried out with the help of the FORM 3 program [62] and the other with Mathematica [63]. In Table 5.1 we show the $\mathcal{O}_i$ operators, together with their β-function. The operators in this table constitute a minimal basis. The rest of the result is rather lengthy and is relegated to Appendix G.

5.4 Features and Use of the Renormalized RχT Lagrangian

In order to understand the aspects and use of the renormalized $R\chi T$ lagrangian that we have obtained above, we would like to emphasize here several of its features:

1. In Table 5.1 we have collected the full basis of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ $U(3)_L \otimes U(3)_R$ χPT operators generated in the functional integration at one loop. We should recover the result first obtained in Ref. [49]. After the comparison is made[4] we agree indeed with their results. Notice though that in order to disentangle the resonances, it is not enough to withdraw all the resonance couplings. This is

[4] Notice that the notation of Ref. [49] is different to ours though, to ease the comparison, the order chosen is the same. We always quote our notation for the operators.

i	$\mathcal{O}_i$	γ_i
1	$\langle u \cdot u \rangle$	$-2N\lambda_1^{\mathrm{PP}}M_P^2 + 1/2NM_P^2(\lambda_1^{\mathrm{SP}})^2 - 2N\lambda_1^{\mathrm{SS}}M_S^2 + 1/2NM_S^2(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}c_d^2M_S^2$
2	$\langle \chi_+ \rangle$	$-2N\lambda_3^{\mathrm{PP}}M_P^2 - 2N\lambda_3^{\mathrm{SS}}M_S^2$
3	$-\langle u_\mu \rangle^2$	$2\lambda_2^{\mathrm{PP}}M_P^2 - 1/2M_P^2(\lambda_1^{\mathrm{SP}})^2 + 2\lambda_2^{\mathrm{SS}}M_S^2 - 1/2M_S^2(\lambda_1^{\mathrm{SP}})^2 - c_d^2F^{-2}M_S^2$
4	$\langle u_\mu u_\nu u^\mu u^\nu \rangle$	$1/6NF^{-4}c_d^4 - 1/12N(\lambda_1^{\mathrm{SP}})^2 + 1/24N(\lambda_1^{\mathrm{SP}})^4 + 1/16N + 1/6NF^{-2}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/6NF^{-2}c_d^2$
5	$\langle u \cdot u \rangle^2$	$1/2F^{-4}c_d^4 - 1/2\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + (\lambda_1^{\mathrm{PP}})^2 - 1/2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + (\lambda_1^{\mathrm{SS}})^2 + 1/8(\lambda_1^{\mathrm{SP}})^4 + 1/16 - F^{-2}c_d^2\lambda_1^{\mathrm{SS}} + 1/2F^{-2}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}c_d^2$
6	$\langle u_\mu u_\nu \rangle^2$	$F^{-4}c_d^4 - \lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 2(\lambda_2^{\mathrm{PP}})^2 - \lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2(\lambda_2^{\mathrm{SS}})^2 + 1/4(\lambda_1^{\mathrm{SP}})^4 + 1/8 - 2F^{-2}c_d^2\lambda_2^{\mathrm{SS}} + F^{-2}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-2}c_d^2$
7	$\langle u \cdot u u \cdot u \rangle$	$1/3NF^{-4}c_d^4 - 1/2N\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + N(\lambda_1^{\mathrm{PP}})^2 - 1/2N\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + N(\lambda_1^{\mathrm{SS}})^2 + 1/12N(\lambda_1^{\mathrm{SP}})^2 + 1/12N(\lambda_1^{\mathrm{SP}})^4 - NF^{-2}c_d^2\lambda_1^{\mathrm{SS}} + 1/3NF^{-2}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/12NF^{-2}c_d^2$
8	$\langle \chi_+ \rangle\langle u \cdot u \rangle$	$F^{-4}c_d^3c_m + 2\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 2\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - 1/2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/8 + F^{-2}d_mc_d\lambda_1^{\mathrm{SP}} + F^{-2}c_dc_m(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-2}c_dc_m - F^{-2}c_d^2\lambda_3^{\mathrm{SS}} - 1/4F^{-2}c_d^2$
9	$\langle \chi_+ u \cdot u \rangle$	$NF^{-4}c_d^3c_m + 2N\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2N\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 2N\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - 1/2N\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/8N + NF^{-2}d_mc_d\lambda_1^{\mathrm{SP}} + NF^{-2}c_dc_m(\lambda_1^{\mathrm{SP}})^2 - 1/2NF^{-2}c_dc_m - NF^{-2}c_d^2\lambda_3^{\mathrm{SS}} - 1/4NF^{-2}c_d^2$
10	$\langle \chi_+ \rangle^2$	$F^{-4}c_d^2c_m^2 + (\lambda_3^{\mathrm{PP}})^2 + (\lambda_3^{\mathrm{SS}})^2 + 1/16 + 2F^{-2}d_mc_m\lambda_1^{\mathrm{SP}} + F^{-2}d_m^2 - 1/2F^{-2}c_dc_m + F^{-2}c_m^2(\lambda_1^{\mathrm{SP}})^2$
11	$\langle \chi_- \rangle^2$	$\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 1/8(\lambda_1^{\mathrm{SP}})^2 - 2(\lambda_2^{\mathrm{SP}})^2 + F^{-2}d_mc_d\lambda_1^{\mathrm{SP}} - 2F^{-2}d_mc_m\lambda_1^{\mathrm{SP}} - F^{-2}d_m^2(\lambda_1^{\mathrm{SP}})^2 + F^{-2}c_dc_m - 1/4F^{-2}c_d^2 - F^{-2}c_m^2$
12	$1/2\langle \chi_+^2 + \chi_-^2 \rangle$	$NF^{-4}c_d^2c_m^2 + N(\lambda_3^{\mathrm{PP}})^2 + N(\lambda_3^{\mathrm{SS}})^2 + 1/16N + NF^{-2}d_m^2 + 1/2NF^{-2}c_dc_m + NF^{-2}c_m^2(\lambda_1^{\mathrm{SP}})^2 + N\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 1/8N(\lambda_1^{\mathrm{SP}})^2 - 2N(\lambda_2^{\mathrm{SP}})^2 + NF^{-2}d_mc_d\lambda_1^{\mathrm{SP}} - NF^{-2}d_m^2(\lambda_1^{\mathrm{SP}})^2 - 1/4NF^{-2}c_d^2 - NF^{-2}c_m^2$
13	$-i\langle f_+^{\mu\nu}u_\mu u_\nu \rangle$	$-1/6N(\lambda_1^{\mathrm{SP}})^2 + 1/4N - 1/3NF^{-2}c_d^2$
14	$1/4\langle f_{\mu\nu}^{+2} - f_{\mu\nu}^{-2} \rangle$	$-1/4N + 1/6N(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}c_d^2$
15	$1/2\langle f_{\mu\nu}^{+2} + f_{\mu\nu}^{-2} \rangle$	$-1/8N - 1/12N(\lambda_1^{\mathrm{SP}})^2 - 1/6NF^{-2}c_d^2$
16	$1/4\langle \chi_+^2 - \chi_-^2 \rangle$	$2NF^{-4}c_d^2c_m^2 + 2N(\lambda_3^{\mathrm{PP}})^2 + 2N(\lambda_3^{\mathrm{SS}})^2 + 1/8N + 8F^{-2}Nd_mc_m\lambda_1^{\mathrm{SP}} + 2NF^{-2}d_m^2 - 3NF^{-2}c_dc_m + 2NF^{-2}c_m^2(\lambda_1^{\mathrm{SP}})^2 - 2N\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/4N(\lambda_1^{\mathrm{SP}})^2 + 4N(\lambda_2^{\mathrm{SP}})^2 - 2NF^{-2}d_mc_d\lambda_1^{\mathrm{SP}} + 2NF^{-2}d_m^2(\lambda_1^{\mathrm{SP}})^2 + 1/2NF^{-2}c_d^2 + 2NF^{-2}c_m^2$
17	$-\langle u_\mu \rangle\langle u^\mu u \cdot u \rangle$	$-4\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + \lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + \lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} + \lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + \lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/2(\lambda_1^{\mathrm{SP}})^4 + 1/4 + 2F^{-2}c_d^2\lambda_1^{\mathrm{SS}} + 2F^{-2}c_d^2\lambda_2^{\mathrm{SS}} - F^{-2}c_d^2$
18	$\langle u_\mu \rangle\langle u^\mu \chi_+ \rangle$	$-2F^{-4}c_d^3c_m + 4\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - \lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} - \lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/4 - 2F^{-2}d_mc_d\lambda_1^{\mathrm{SP}} - 2F^{-2}c_dc_m(\lambda_1^{\mathrm{SP}})^2 + F^{-2}c_dc_m - 2F^{-2}c_d^2\lambda_3^{\mathrm{SS}} + 1/2F^{-2}c_d^2$

Table 5.1: Operators involving only pseudo-Goldstone bosons and external currents and their β-function coefficients at one loop, when both scalar and pseudoscalar resonances are included.

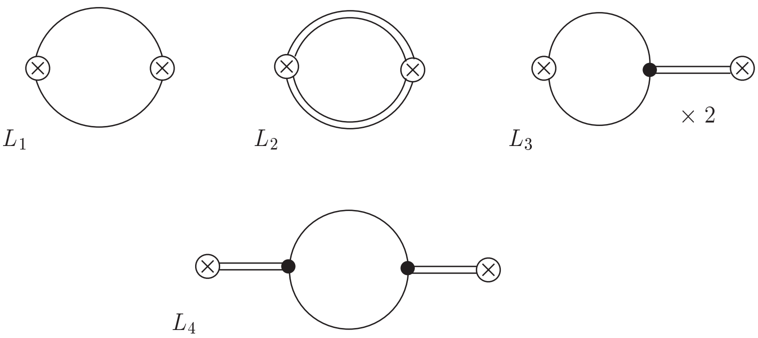


Figure 5.1: One-loop contributions to the $\Pi^{ij}_{SS}(q^2)$ correlator in the chiral limit when only scalar resonances are included. A single line stands for a pseudo-Goldstone boson while a double line indicates a scalar resonance. Their result is divergent.

because the derivative terms in $\mathcal{L}_{\rm kin}(S,P)$, which do not carry any resonance coupling, also contribute through the functional integration to several of the operators, namely $\mathcal{O}_4$, $\mathcal{O}_7$, $\mathcal{O}_{13}$, $\mathcal{O}_{14}$ and $\mathcal{O}_{15}$ in Table 5.1. We have confirmed that $\mathcal{L}_{\rm kin}(S,P)$ gives precisely the difference between our coefficients γ_4, γ_7, γ_{13}, γ_{14}, γ_{15} and those of Ref. [49] once the resonance couplings have been switched off.

2. In the procedure we have employed to evaluate the functional integration of $\mathcal{L}_{\rm R\chi T}$ up to one loop we have withdrawn those operators with three or more resonance fields and kept up to two resonances. A cut in the number of resonances is necessary because to reach the Gaussian expression in Eq. (5.22) we need to perform several field transformations (see Appendix F) that generate operators with more resonance fields which in turn require additional field transformations and so on. One of the differences of RχT with respect to χPT (in the strong [8, 9, 10] or electroweak interaction [64] form of the latter) is that we do not have an expansion parameter into the lagrangian that can provide a natural cut for higher order terms in these field transformations. Notice that the cut in the number of resonances seems to hinder our result, as it does not allow us to renormalize divergent one loop diagrams with three or more resonance fields as external legs. However we would not expect to treat these loops as we are not including, in our leading order lagrangian, interacting terms with three or more resonance fields.

To end this section we would like to show a simple example of the application of our result. We consider the one-loop renormalization of the two-point function of scalar currents:

$$\Pi^{ij}_{SS}(q^2) = i\int {\rm d}^4x\, e^{iq\cdot x}\, \langle 0|T\{S^i(x)S^j(0)\}|0\rangle\,, \qquad S^i(x) = \overline{q}(x)\lambda^i q(x)\,, \quad (5.30)$$

in the chiral limit and when only scalar resonances are considered. The divergent loop diagrams contributing are those depicted in Fig. 5.1. In order to cancel the

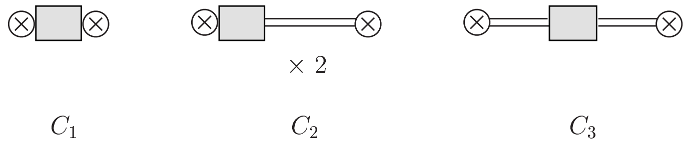


Figure 5.2: Counterterm contributions that renormalize the one-loop result of 5.1. A double line stands for a scalar resonance.

divergences one needs to add the counterterm contributions in Figure 5.2, where diagram C_1 is given by $\mathcal{O}_{12} + 2\mathcal{O}_{16} = \langle \chi_+^2 \rangle$ in Table 5.1, C_2 by $\mathcal{O}_4^R = \langle S\chi_+ \rangle$ in Table G.1 of Appendix G and C_3 by $\mathcal{O}_1^{RR} = \langle SS \rangle$ in Table G.2 of Appendix G, once the pseudoscalar resonance couplings are disconnected. The cancellation works as follows: one part of the contribution of C_1 cancels completely the divergence in the loops L_1+L_2. Another piece of C_1 together with C_2 eliminates the divergence coming from L_3 and, finally, all remaining contributions of C_1 and C_2 add to C_3 in order to render L_4 finite. Notice that, as there are no nonlocal divergences, the contributions of one-particle-reducible diagrams are brought finite once one-particle-irreducible diagrams have been properly renormalized.

5.5 Running of the couplings and short-distance behaviour

5.5.1 Running of the couplings

Our result provides the running of the α_i, β_i^R and β_i^{RR} couplings through the renormalization group equations. From Eq. (5.29) we get:

$$\mu \frac{d}{d\mu} \alpha_i^r(\mu) = -\frac{\gamma_i}{16\,\pi^2}\,, \tag{5.31}$$

and, analogously, for β_i^R and β_i^{RR}. This result can be potentially useful if we are interested in the evaluation of the resonance couplings at this order. Though μ is known to be of the order of a typical scale of the physical system, let us say $\mu_0 = M_S$ or $\mu_0 = M_P$, there always remains some ambiguity on the precise value of μ_0 at which the low-energy couplings are determined at leading-order in the $1/N_C$ expansion. The running provides an estimate of the reliance of such determinations. If the coupling under request varies drastically with the scale it is clear that the value obtained has a large uncertainty, while if it has a smooth dependence on the scale the determination is more reliable. Note that the running is a next-to-leading order effect.

In the case of RχT with only scalar and pseudoscalar resonance fields the Weinberg's dimensional analysis of Eq. (1.23) holds[5], once one assumes that the masses of the resonances are of $\mathcal{O}(p)$ in the chiral counting and they appears explicitly in the lagrangian, unlike the pseudo-Goldstone bosons, whose masses are taking into account trough the chiral tensors $\chi_\pm$. Therefore, as it has been pointed out in Section 5.3, the terms needed to renormalize the theory at subleading order in the $1/N_C$ expansion are constructed with up to two resonances and chiral tensors up to $\mathcal{O}(p^4)$. Notice that in the case of chiral tensors of $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0)$ a M_R^2 and $M_{R_1}^2 M_{R_2}^2$ factor respectively are required in the divergent pieces in order to fulfill the "generalized" chiral counting.

This has an *a priori* surprising consequence: there are counterterms associated with the operators of $\mathcal{L}_{pGB}^{(2)}$. In other words, the structure of $\mathcal{L}_{pGB}^{(2)}$ changes when one goes beyond the leading order:

$$\mathcal{L}_{pGB}^{(2)} \;=\; \alpha_1 \langle\, u_\mu u^\mu \,\rangle + \alpha_2 \langle\, \chi_+ \,\rangle \,, \tag{5.32}$$

where we have followed the notation of Eq. (5.28). Therefore,

$$\begin{aligned}
\alpha_1 &= \frac{F^2}{4} \sum_{n=0} \alpha_1^{(n)} \left(\frac{M_R}{F}\right)^{2n} , \\
\alpha_2 &= \frac{F^2}{4} \sum_{n=0} \alpha_2^{(n)} \left(\frac{M_R}{F}\right)^{2n} ,
\end{aligned} \tag{5.33}$$

where the coefficients have been defined in such a way that $\alpha_1^{(0)} = \alpha_2^{(0)} = 1$. Notice that the suppression of higher terms in the $1/N_C$ expansion is explicitly shown, since $F \sim \mathcal{O}(\sqrt{N_C})$ and $M_R \sim \mathcal{O}(1)$. At next-to-leading order only the coefficients until $n = 1$ must be considered.

5.5.2 Vanishing β-functions and short-distance behaviour

Within this issue it is interesting to take a closer look to the running of the couplings in $\mathcal{L}_{pGB}^{(2)}$ and $\mathcal{L}_{pGB}^{(4)}$, corresponding to the $\mathcal{O}_i$ operators involving only pseudo-Goldstones. The corresponding β-function coefficients are γ_i, see Table 5.1.

An interesting aspect is the interval over which μ runs. It is well known [57] that the couplings are only relevant at the scale of the momenta involved in the processes (in order to diminish the role of the logarithms). In our case $\mu \sim M_S,\, M_P$. Thus we do not expect a large running for the scale, namely a few hundreds of MeV. This last conclusion brings us to the next point. At next to leading order in $1/N_C$ we can ignore the running on the couplings appearing in γ_i, since the running, as expected,

[5] This is not longer true in the case of vector and axial-vector resonances due to the propagator structure. Keep in mind the necessity of $\widetilde{\mathcal{L}}_{pGB}^{(6)}$ in order to renormalize the vector form factor at next-to-leading order in Chapter 3. In Eq. (3.35) one can see that only contributions from spin-1 resonances are responsible for these kind of divergences.

is a next-to-leading order effect in the $1/N_C$ expansion[6]. Hence we can input the leading order values for the couplings, given by Eqs. (5.2) and (5.3), to obtain the leading logarithm in the evolution of these couplings. It is remarkable that, at this order, Eq. (5.31) predict a vanishing β-function for $\mathcal{O}_2$, $\mathcal{O}_8$, $\mathcal{O}_9$, $\mathcal{O}_{10}$, $\mathcal{O}_{11}$, $\mathcal{O}_{12}$, $\mathcal{O}_{16}$ and $\mathcal{O}_{18}$, i.e. all those operators involving χ_+ and/or χ_-. If the Large-N_C estimates for the couplings are to be reliable we come to the conclusion that the predictions for these couplings are rather robust.

This feature must be explained. Notice that this behaviour was predicted for $\widetilde{L}_8$ in Chapter 4, relating it to short-distance constraints.

This result can be understood by following the optical theorem and taking into account the short-distance constraints that have been used. The imaginary part of any Feynman diagram can be obtained by cutting through the diagrams in all possible ways such that the cut propagators can simultaneously be put on shell, that is, by replacing $1/(p^2 - m^2 + i\varepsilon) \to -2\pi i\delta(p^2 - m^2)$ in each cut propagator. Then one should perform the loop integrals and finally sum the contributions of all possible cuts.

To understand it we can start by considering again the one-loop renormalization of the two-point function of scalar currents, defined in Eq. (5.30). We can use the cutting rules to calculate the spectral function of the correlator, as it was explained in Chapter 4. There are only four possible cuts: two pseudo-Goldstone, one pseudo-Goldstone and one pseudoscalar resonance, and two scalar or pseudoscalar resonance fields. The optical theorem allows to use the constrained form factors reviewed in Section 5.2 and analyzed in great detail in Section D.3 of Appendix D. Taking into account that for these contributions the highest behaviour at large energies could be $\mathcal{O}(q^0)$, the suppression ruled by the constrained form factors leads to an $\mathcal{O}(q^{-4})$ behaviour at large energies.

The following step consists of relating the spectral function to the divergent part of the contributions, which is responsible of the μ dependence on our couplings. We see that the relevant discontinuities can come only from two-point Feynman integrals. In Appendix B it can be seen that the divergent piece and the imaginary part have always the same asymptotic behaviour. In other words, the same suppression must happen for the divergent piece. The one-point Feynman integral is not important for this purpuse, because although it has not discontinuities, its behaviour at large energies is always lower than the two-point functions, as it does not depend on q^2.

The needed counterterms contributions that renormalize the one-loop result are depicted in Figure 5.2. As it was explained at the end of Section 5.4, diagram C_1 is given by $\mathcal{O}_{12} + 2\mathcal{O}_{16} = \langle \chi_+^2 \rangle$, C_2 by $\mathcal{O}_4^R = \langle S\chi_+ \rangle$ and C_3 by $\mathcal{O}_1^{RR} = \langle SS \rangle$. In Table 5.1 and in Appendix G the β-function coefficients of the corresponding vertices are available. C_1, C_2 and C_3 give a behaviour at large energies of $\mathcal{O}(q^0)$, $\mathcal{O}(q^{-2})$ and $\mathcal{O}(q^{-4})$ respectively. Considering the suppression explained before, C_1 is not needed to renormalize the process. So, as we have obtained and following our

[6] This fact is clear taking into account that F, F_V, G_V, F_A, c_d, c_m and d_m are of $\mathcal{O}(\sqrt{N_C})$ and $\lambda_i^{R_1R_2}$ and M_R of $\mathcal{O}(1)$.

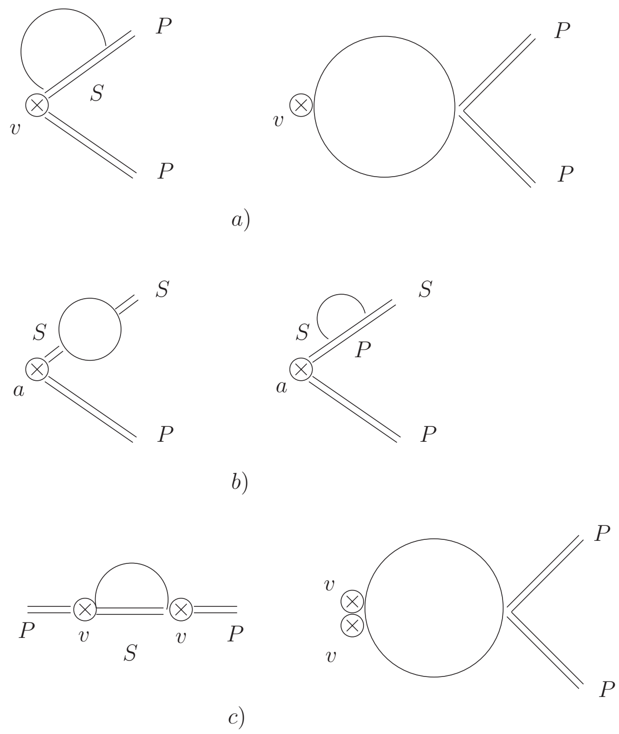


Figure 5.3: One-loop diagrams which are renormalized with $\mathcal{O}^{RR}_{180}$ (first line), $\mathcal{O}^{RR}_{316-318}$ (second line) and $\mathcal{O}^{RR}_{190-191}$ (third line).

notation, $\gamma_{12}+2\gamma_{16}$ vanish once the short-distance contrainsts of Eqs.(5.2) and (5.3) are implemented.

Therefore, the process is very easy. The commented suppression will be observed in those processes where the scalar and/or pseudoscalar form factor play a role, understanding now why the operators involving χ_+ and/or χ_- of Table 5.1 do not run at one loop.

Following these ideas one can understand most of the found suppressions:

1. Two-point function of scalar current. The affected counterterms by the suppression are the following: $\mathcal{O}_{10} = \langle\, \chi_+ \,\rangle^2$ and $\mathcal{O}_{12} + 2\mathcal{O}_{16} = \langle \chi_+^2 \rangle$. Then, one gets $\gamma_{10} = \gamma_{12} + 2\gamma_{16} = 0$.

2. Two-point function of pseudoscalar current. Affected counterterms by the suppression: $\mathcal{O}_{11} = \langle\, \chi_- \,\rangle^2$ and $\mathcal{O}_{12} - 2\mathcal{O}_{16} = \langle\, \chi_-^2 \,\rangle$. Then, $\gamma_{11} = \gamma_{12} - 2\gamma_{16} = 0$.

3. Scalar Form Factor $\langle\, \pi^i | s^j | \pi^k \,\rangle$. Affected counterterms by the suppression: $\mathcal{O}_8 = \langle\, \chi_+ \,\rangle\langle\, u_\mu u^\mu \,\rangle$, $\mathcal{O}_9 = \langle\, \chi_+ u_\mu u^\mu \,\rangle$ and $\mathcal{O}_{18} = \langle\, u_\mu \,\rangle\langle\, u^\mu \chi_+ \,\rangle$. Then, $\gamma_8 = \gamma_9 = \gamma_{18} = 0$.

4. Scalar Form Factor $\langle\, P^i | s^j | \pi^k \,\rangle$. Affected counterterms by the suppression: $\mathcal{O}^R_{52} = \langle\, \chi_+ \{u_\mu, \nabla^\mu P\} \,\rangle$, $\mathcal{O}^R_{53}\langle\, u^\mu \chi_+ \,\rangle\langle\, \nabla_\mu P \,\rangle$, $\mathcal{O}^R_{54} = \langle\, \chi_+ \,\rangle\langle\, u_\mu \nabla^\mu P \,\rangle$ and $\mathcal{O}^R_{55} = \langle\, \chi_+ \nabla^\mu P \,\rangle\langle\, u_\mu \,\rangle$. Then, $\gamma^R_{52} = \gamma^R_{53} = \gamma^R_{54} = \gamma^R_{55} = 0$.

5. Pseudoscalar Form Factor $\langle\, S^i|p^j|\pi^k\,\rangle$. Affected counterterms by the suppression: $\mathcal{O}^R_{32} = i\langle\, \chi_-\{u_\mu, \nabla^\mu S\}\,\rangle$, $\mathcal{O}^R_{33} = i\langle\, u_\mu\chi_-\,\rangle\langle\, \nabla^\mu S\,\rangle$, $\mathcal{O}^R_{34} = i\langle\, \chi_-\,\rangle\langle\, u_\mu\nabla^\mu S\,\rangle$ and $\mathcal{O}^R_{35} = i\langle\, \chi_-\nabla^\mu S\,\rangle\langle\, u_\mu\,\rangle$. Then, $\gamma^R_{32} = \gamma^R_{33} = \gamma^R_{34} = \gamma^R_{35} = 0$.

We cannot consider the form factors to two resonance fields because they involve other cuts that have been not analyzed asymptotically.

Following these ideas we have been able to explain 15 vanishing β-functions. In total 28 vanishing ones have been found, so 13 are not so clear. In any case, in 6 of these 13 we can conjecture that an accidental suppression happens because the structure of the loops is very similar to the loops that appeared in the constrained form factors, so that the high-energy constraints can lead to the found suppression: $\mathcal{O}^{RR}_{180} = i\langle\, f_+^{\mu\nu}\nabla_\mu P\nabla_\nu P\,\rangle$ is a counterterm for $\langle\, P^i|v^{\mu\,j}|P^k\,\rangle$; $\mathcal{O}^{RR}_{316} = \langle\, f_-^{\mu\nu}\{\nabla_\mu P, \nabla_\nu S\}\,\rangle$, $\mathcal{O}^{RR}_{317} = \langle\, \nabla_\mu P\,\rangle\langle\, f_-^{\mu\nu}\nabla_\nu S\,\rangle$ and $\mathcal{O}^{RR}_{318} = \langle\, \nabla_\mu S\,\rangle\langle\, f_-^{\mu\nu}\nabla_\nu P\,\rangle$ are counterterm for $\langle\, S^i|a^{\mu\,j}|P^k\,\rangle$; $\mathcal{O}^{RR}_{190} = \langle\, Pf_+^{\mu\nu}Pf_{+\,\mu\nu}\,\rangle$ and $\mathcal{O}^{RR}_{191} = \langle\, PPf_+^{\mu\nu}f_{\mu\nu\,+}\,\rangle$ are counterterms of $\langle\, P^i|v^{\mu\,j}v^{\nu\,k}|P^l\,\rangle$. The corresponding loop contibutions are shown in Figure 5.3 and the cancellation between these pairs of loops can be checked in Table G.2 of Appendix G, taking into account the relevant couplings for each diagram.

In the case of 6 of the other 7 vanishing β-function we can conclude nothing following the same procedure, since not all the operators with the same structure have a vanishing β-function. For instance, the coupling related to the operator $\mathcal{O}^R_{26} = \langle\, u_\nu S u^\nu \chi_+\,\rangle$ has not running, while it does not happen the same with $\mathcal{O}^R_{25}$ and $\mathcal{O}^R_{27-31}$, all of them constructed with the same operators.

The case of $\mathcal{O}_2 = \langle\, \chi_+\,\rangle$ is a different question. As a consequence of imposing the correct short-distance behaviour of the scalar form factors $\langle\, S^i|s^j|S^k\,\rangle$ and $\langle\, P^i|s^j|P^k\,\rangle$, one has $\lambda_3^{SS} = \lambda_3^{PP} = 0$, so that, for instance, there are not divergences of $\langle\, 0|s^i|0\,\rangle$ that can be renormalized by the counterm $\mathcal{O}_2$. Therefore, following the notation of Eq.(5.33), α_2 does not run at one-loop level.

As we have pointed out above, the case of vector and axial-vector resonances is different because the structure of the propagators breaks down the Weinberg chiral counting. In any case, if the procedure of Chapter 4 is followed, there is no problem because the interest is directly in the imaginary part and not in the form factors, so the needed suppression is obtained. Therefore, extrapolating this behaviour to all the resonance fields and studying all form factors, these vanishing β-functions will be obtained in many more cases. Furthermore, if the behaviour of different scatterings were studied at large energies, the same could happen with operators that are not related to external currents. Eventually one could expect to obtain $\gamma_i = 0$ for all i, that is, all operators involving only pseudo-Goldstone bosons and external currents would have vanishing β-function, what would be very interesting in order to understand the saturation, since $\widetilde{L}_i$ could vanish without problems, allowing an easy resonance saturation of the couplings of $\mathcal{L}_4^{\chi PT}$ at one loop, as it was suggested in Ref. [37].

5.6 Conclusions

RχT provides a consistent framework to study the energy region of the hadronic resonances, $M_V \lesssim E \lesssim 2$ GeV. It embodies a phenomenological lagrangian where pseudo-Goldstone bosons and resonances fields are kept as active degrees of freedom; this is the key ingredient for the application of the large-N_C expansion. Recently, and after its multiple explorations at tree level, it has emerged some interest in the application of RχT at one loop level mainly to understand how the features of QCD are implemented into the theory.

In this chapter we have systematically obtained, by using the background field method and for the first time, both the full basis of operators and the β-function coefficients that render finite, up to one loop, our initial lagrangian $\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ in Eq. (5.1). This would correspond to the next-to-leading order in the $1/N_C$ expansion but including one multiplet of scalar and pseudoscalar resonances only. Our main result is given by Eq. (5.28) and the γ_i, γ_i^R, γ_i^{RR} parameters in Eq. (5.29). The outcome is relevant for the study of those diagrams involving a loop with up to two resonances and any number of pseudo-Goldstone bosons in the legs.

The β-function coefficients are crucial in order to study the running of the couplings, see Eq. (5.31). In Section 5.5.2 we have studied this running once the short-distance constraints have been implemented. We have found 28 coefficients without running and we have been able to explain this result taking into account the considered short-distance constraints coming from the form factors. Specially important it is the fact that all the couplings $\widetilde{L}_i$ of $\mathcal{L}_{pGB}^{(4)}$ that contain χ_+ and/or χ_- do not run. This is very interesting in order to understand an easy resonance saturation of the couplings of $\mathcal{L}_4^{\chi PT}$, since the running of $\widetilde{L}_i$ would make difficult to consider a saturation of these couplings at next-to-leading order in the $1/N_C$ expansion.

Conclusions

Una vegada hom accepta que la Teoria Quiral de Ressonàncies és una modelització bona de QCD a energies intermèdies, $M_\rho \lesssim E \lesssim 2$ GeV, i la necessitat de considerar correccions quàntiques, l'objectiu d'aquesta tesi ha estat fer un pas endavant en aquesta direcció. És important subratllar que, a diferència del que ocorre amb molts altres models de QCD que inclouen ressonàncies i han estat emprats en la bibliografia, RχT sols utilitza informació provinent de QCD, sense més suposicions *ad hoc*.

Com s'ha justificat en la introducció, ens semblava una bona idea començar aquest camí fent un càlcul d'una amplitud física ben determinada a nivell subdominant en l'expansió en $1/N_C$ [26]. Per la seua senzillesa i importància fenomenològica, vàrem triar el factor de forma vectorial del pió. Com a punt de partida també hem considerat adient emprar el lagrangià "històric", és a dir, el lagrangià de la Ref. [17], on únicament es consideren les interaccions lineals en els camps de les ressonàncies. Aquesta tasca es desenvolupa en el capítol 3; els principals resultats del qual són:

1. Com era esperat, els diagrames de Feynman amb bucles que inclouen ressonàncies massives en les línies internes generen divergències ultraviolades, que requereixen contratermes addicionals per tal de fer la teoria finita. Com que aquests contratermes, construïts amb tensors quirals d'$\mathcal{O}(p^4)$, donen lloc a contribucions a ordre arbre que creixen massa ràpidament amb el moment, els seus acoblaments han de ser zero a nivell dominant en l'expansió en $1/N_C$. Llavors, hom pot establir un bon contatge en potències d'$1/N_C$ per tal d'organitzar el càlcul.

   La renormalització a un bucle segueix un procediment clar. D'aquesta manera es pot trobar la dependència en l'escala μ dels acoblaments rellevants. A més a més, és interessant ressaltar que solament algunes combinacions d'acoblaments quirals apareixen en el resultat final. En realitat, utilitzant les equacions de moviment, hom pot eliminar la majoria dels nous acoblaments subdominants: llurs efectes són reabsorvits mitjançant redefinicions dels acoblaments del lagrangià $\mathcal{L}_{pGB}$, i.e. amb peces que no tenen ressonàncies.

2. Expandint el resultat en potències de q^2/M_R^2, es recupera el resultat de χPT a baixes energies. Açò permet relacionar els acoblaments quirals ℓ_6 i r_{V2} amb les seues correspondències en RχT $\widetilde{\ell}_6$ i $\widetilde{r}_{V2}$.

   El control rigorós de la dependència d'escala permet investigar la saturació a nivell subdominant en l'expansió en $1/N_C$. Es comprova que suposar que $\widetilde{\ell}_6 = \widetilde{r}_{V2} = 0$ genera excel·lents prediccions per a $\ell_6^r(\mu)$ i $r_{V2}^r(\mu)$ a qualsevol escala μ. És important assenyalar la importància d'aquest resultat, ja que malgrat que s'ha analitzada la saturació del lagrangià $\mathcal{L}_4^{\chi PT}$ per les contribucions amb ressonàncies a ordre arbre [17, 18], aquest és el primer treball que estudia aquest procés incloent diagrames amb un bucle, alhora que analitzant les divergències ultraviolades i la seua renormalització corresponent.

De més a més, la importància de determinar les contribucions de les ressonàncies als acoblaments quirals de χPT a nivell subdominant en l'expansió en $1/N_C$ és clara, ja que es manté un control de la dependència en l'escala de renormalització, i s'eviten les incerteses que apareixen quan no s'han considerat bucles i que es deuen al corriment dels acoblaments analitzats. Aquest primer treball és un primer pas cap a la determinació sistemàtica de les contribucions subdominants en els acoblaments de la Teoria de Pertorbacions Quirals.

3. A altes energies hem trobat un comportament problemàtic, originat en el tall amb dues ressonàncies: es genera un increment del factor de forma en el límit d'un moment transferit gran que viola l'empalmament amb QCD.

   Més important que haver trobat el problema ha estat haver identificat la causa i haver-ne donat la solució. No és sorprenent aquest resultat, tenint en compte que hi ha contribucions addicionals generades per termes d'interacció amb més d'una ressonància, que no estaven en el lagrangià de partida. Dit d'altra manera, mentre que no es consideren totes les interaccions permeses per les simetries, no podrem assegurar que es trobe un resultat físic del factor de forma del pió. A més a més, en analitzar el comportament a altes energies, en efecte, es troben certes combinacions d'aquests nous acoblaments.

Fet i fet, el capítol 4 [29, 30] és en definitiva una continuació pel camí que deixa obert els resultats del capítol 3. Una vegada s'accepta la necessitat de considerar totes les interaccions amb més d'un camp de ressonància si es volen empalmar els nostres resultats efectius amb els de QCD, es volen estudiar els lligams que han de satisfer aquests nous acoblaments introduïts. Com s'ha explicat repetidament al llarg d'aquest treball, és fonamental tenir clar quins són els lligams que s'han d'acomplir: l'empalmament és un punt clau en el nostre enfocament efectiu.

Molts del lligams dels acoblaments de RχT vénen d'estudiar les funcions de Green dels corrents de QCD a gran N_C, mitjançant l'expansió pertorbativa OPE. Els altres lligams es troben estudiant els factors de forma dels corrents hadrònics: s'exigeix que s'anul·len en el límit $q^2 \to \infty$ [28]. Encara que aquest comportament és clar en el cas de pseudo-bosons de Goldstone i fotons, en el cas de ressonàncies com a estats asimptòtics no és tan evident. El capítol 4 intenta justificar la necessitat d'estudiar el comportament a altes energies de tots aquests factors de forma. L'anàlisi es fa mitjançant les funcions de Green de dos punts a nivell subdominant en l'expansió en $1/N_C$. Així, s'estudien a nivell arbre tots els factors de forma a dos cossos que poden trobar-se en el sector de paritat intrínseca parella de la Teoria Quiral de Ressonàncies sota l'aproximació d'una única ressonància. Els lligams i les estructures dels diferents factors de forma es mostren en l'apèndix D.

La idea és molt senzilla i es pot entendre fàcilment mitjançant un exemple: el correlador $\Pi_{VV}(q^2)$ té un mal comportament en el tall de dues ressonàncies escalars o pseudoescalars si únicament es considera el lagrangià de la Ref. [17]. En aquest cas existeix una contribució a conseqüència dels termes cinètics, deguda a la connexió de la derivada covariant. Aquest mal comportament pot ser solucionat si afegim al lagrangià una nova peça,

$$\Delta\mathcal{L} \;=\; i\,\lambda^{VRR}\langle\, V^{\mu\nu}\,\nabla_\mu R\,\nabla_\nu R\,\rangle\,, \qquad (R = S, P)$$

de tal manera que si $\lambda^{VRR} = \sqrt{2}/F$, es recupera un bon comportament a altes energies. Aquest lligam és el trobat en analitzar el factor de forma vectorial corresponent.

Dels resultats de l'apèndix D cal subratllar que no hi apareixen nous lligams de l'anàlisi dels factors de forma que inclouen un fotó com a estat final, és a dir, l'anàlisi dels factors de forma amb dos mesons en les potes externes subministra el conjunt més reduït de lligams. Això no és estrany tenint en compte la relació entre les ressonàncies vectorials i els fotons deguda als seus nombres quàntics. També és important fer notar que es troben certes inconsistències entre els lligams en el cas del sector d'spin 1, conseqüència del tall en el nombre de ressonàncies. Aquest tall impedeix una recuperació total dels resultats de QCD. En aquest cas les inconsistències poden ser salvades incloent-hi un segon multiplet, com suggeriria l'aproximació hadrònica mínima.

De la mateixa manera que haver considerat els factors de forma amb ressonàncies en les potes externes a nivell arbre soluciona el comportament asimptòtic de les funcions de Green a dos punts a un bucle, aquests lligams són necessaris per càlculs d'altres observables a nivell subdominant, com ara el factor de forma vectorial. És a dir, s'han de tenir amplituds amb ressonàncies com a estats externs ben comportades a nivell arbre si es volen portar a terme càlculs a un bucle.

Com a aplicació fenomenològica d'aquests resultats, i seguint les idees del capítol 3, hem estudiat el correlador $\Pi_{S-P}(q^2)$ per tal de donar la predicció subdominant de $L_8^r(\mu)$. Cal emfatitzar les següents característiques:

1. Les diferents funcions espectrals de cada tall són obtingudes mitjançant el teorema òptic. Per una altra banda, a partir de la part imaginària podem trobar el resultat total fent ús de les regles de dispersió que s'expliquen en l'apèndix E.

2. De les regles de suma de Weinberg per al correlador $\Pi_{S-P}(q^2)$ sabem que aquest és d'$\mathcal{O}(1/q^4)$ a altes energies. Els lligams obtesos a partir de l'anàlisi dels factors de forma ens donen un correlador que s'anul·la a curtes distàncies, però ho fa a $\mathcal{O}(1/q^2)$. Això no és cap problema, tenint en compte que els nostres lligams ho són en el límit $N_C \to \infty$, o siga, encara tenim llibertat en la part subdominant d'aquells acoblaments que apareixen a ordre arbre

en aquest càlcul, c_m i d_m. Així, exigint un comportament d'$\mathcal{O}(1/q^4)$ a altes energies, s'arriba a la següent relació:

$$
\begin{aligned}
F^2\,(1+\delta^{(2)}_{\rm NLO}) \;-\; 8c_m^{r\,2} \;+\; 8\,d_m^{r\,2} \;=\; 0\,,\\
F^2\,M_S^2\,\delta^{(4)}_{\rm NLO} \;-\; 8c_m^{r\,2}\,M_S^{r\,2} \;+\; 8\,d_m^{r\,2}\,M_P^{r\,2} \;\simeq\; 0\,,
\end{aligned}
$$

que són paral·leles a les relacions que s'obtenen en el límit $N_C \to \infty$: $F^2 - 8c_m^2 + 8d_m^2 = 0$ i $8d_m^2 M_P^2 - 8c_m^2 M_S^2 \simeq 0$.

3. Les integrals de dispersió mostren que no són necessàries les peces locals sense ressonàncies una vegada s'han considerat factors de forma ben comportats a altes energies. Amb altres paraules, $\widetilde{L}_8^r = 0$, la qual cosa condueix a una saturació a nivell subdominant ben entesa.

4. Tenint en compte els resultats de QCD a gran N_C, el càlcul està fet en el cas de $U(3)$, per la qual cosa s'ha de considerar l'empalmamaent entre L_8^r en $U(3)$ i $SU(3)$ respectivament.

5. Gràcies a la invariància quiral en RχT, l'estructura de χPT es recupera en el límit de baixes energies del nostre resultat. Així, el tall amb un π i una η dóna el terme quiral no analític que va amb un $\log(-q^2)$, reproduint la dependència en l'escala de renormalització que apareix en el cas de χPT entre el $\log(-q^2/\mu^2)$ i $L_8^r(\mu)$. La resta dels talls absortius genera termes analítics en q^2 i per tant únicament contribucions als acoblaments quirals de baixes energies.

   Per poder donar una predicció numèrica ens quedem únicament amb els talls fins a una ressonància, perquè en el talls amb dues, tot i haver fixat moltes relacions entre acoblaments, resten encara paràmetres per determinar. Com s'explica al llarg del treball, les contribucions que vénen de talls amb llindars més alts són cada vegada menors.

   Amb tots aquests ingredients, es troben les contribucions amb ressonàncies a L_8 a nivell subdominant,

$$
L_8^r(\mu_0) \quad = \quad (0,6 \pm 0,4)\cdot 10^{-3}\,,
$$

   on $\mu_0 = 770\,\mathrm{MeV}$, que pot ser comparat amb el resultat experimental $L_8^r(\mu_0) = (0,9 \pm 0,3)\cdot 10^{-3}$ [17]. Com que la dependència en l'escala de renormalització està sota control en la nostra predicció, la incertesa que genera aquesta ha desaparegut. La major part de l'error vé de la ignorància dels valors de les masses de les ressonàncies escalar i pseudoescalar en el límit quiral. La reducció de l'error d'aquestes masses milloraria enormement la precisió de la nostra predicció.

Arribats a aquest punt, creiem que ja cal abordar la renormalització de la Teoria Quiral de Ressonàncies de manera completa, de la mateixa manera que va ser duta a terme per Gasser i Leutwyler en el cas de la Teoria de Pertorbacions Quiral [9].

És a dir, emprant el mètode del camp de fons [58, 59], hom pot trobar tots els operadors necessaris per tal de renormalitzar la teoria efectiva, juntament amb el corriment dels coeficients de cada nou operador. Ara bé, les dificultats tècniques són evidents en aquest cas: el nombre de graus de llibertat és molt més gran una vegada les ressonàncies estan incloses, al mateix temps que el nombre de termes del lagrangià dominant també és molt gran. Conseqüentment, el nombre d'operadors nous és molt gran, a banda de les dificultats tècniques per assolir el resultat.

Per tot això, en el capítol 5 [54], com a primer càlcul en aquesta direcció, es considera el cas en què únicament tenim ressonàncies escalars i pseudoescalars, a més de considerar solament fins acoblaments bilineals en el nombre de camps de ressonàncies. D'aquesta manera, es troba la llista dels termes necessaris per fer la teoria finita, juntament amb les funcions beta corresponents. Cal destacar les següents característiques del resultat:

1. A partir del resultat trobat pot recuperar-se el corriment dels acoblaments de $\mathcal{L}_4^{\chi PT}$ en el cas de χPT en $U(3)$.

2. És molt important conéixer el corriment dels acoblaments a l'hora d'extraure el seu valor de la fenomenologia. Un dels principals profits del nostre càlcul és en realitat això.

3. Com que, a diferència del cas de χPT, les masses de les ressonàncies estan incloses directament com a paràmetres, hi ha divergències que seran renormalitzades amb el lagrangià dominant de partida.

4. Una vegada han estat considerats els lligams asimptòtics que vénen d'estudiar a curtes distàncies els possibles factors de forma escalars i pseudoescalars, no s'han trobat divergències que vagen amb operadors $\mathcal{O}_i$, que impliquen solament els pseudo-bosons de Goldstone i els corrents externs, que inclouen χ_+ o χ_-. Això no és estrany tenint en compte el mal comportament a altes energies que donen en general els acoblaments de $\mathcal{L}_{pGB}^{(4)}$. És a dir, una vegada hom treballa amb factors de forma ben comportats, no hi ha divergències "mal comportades" i, per tant, no hi calen aquest tipus de contratermes.

Appendix A

The Antisymmetric Tensor Formalism

Although the antisymmetric tensor formalism for spin 1 massive fields was already proposed at the end of 60's [65], its use was not regular until it was rediscovered in Ref. [9] in order to introduce the ρ resonance field in the chiral lagrangian, Ecker *et al.* turned it into the usual way to work with spin-1 resonances in RχT [17].

In Ref. [66] it was proved that for antisymmetric tensor fields with mass there are (up to multiplicative factors and a total four divergence) only two possible lagrangians of second order in derivatives, if one assumes the existence of a Klein-Gordon divisor. They correspond to having either the Lorentz condition or else a Bianchi identity satisfied by the fields. In the case of describing spin 1 particles, one has these two possibilities, where $\phi_{\mu\nu} = -\phi_{\nu\mu}$,

1. The subsidiary condition is the Bianchi identity, i.e. $\epsilon^{\mu\lambda\rho\sigma}\partial_\lambda\phi_{\rho\sigma} = 0$, and ϕ_{ik} are frozen, so the dynamical degrees of freedom are ϕ_{i0}, where i runs over $i = 1, 2, 3$. Notice that there are 3 degrees of freedom, as it should be.

2. The subsidiary condition is now the Lorentz condition, that is, $\partial^\rho\phi_{\rho\nu} = 0$, and ϕ_{i0} are frozen, so the three degrees of freedom are ϕ_{ij}.

Because of historical reasons, the first option has been chosen. In this case the free lagrangian is proved to be

$$\mathcal{L} \;=\; -\frac{1}{2}\partial^\mu\phi_{\mu\nu}\,\partial_\rho\phi^{\rho\nu} + \frac{1}{4}M^2\phi_{\mu\nu}\phi^{\mu\nu}\,, \tag{A.1}$$

from where the equations of motion are

$$\partial^\mu\partial_\sigma\phi^{\sigma\nu} - \partial^\nu\partial_\sigma\phi^{\sigma\mu} + M^2\phi^{\mu\nu} \;=\; 0\,. \tag{A.2}$$

With the definition $\phi_\mu = \partial^\nu\phi_{\nu\mu}/M$ one obtains from Eq. (A.2) the familiar Proca equation

$$\partial_\rho\left(\partial^\rho\phi^\mu - \partial^\mu\phi^\rho\right) + M^2\phi^\mu \;=\; 0\,. \tag{A.3}$$

From the lagrangian of Eq. (A.1) one derives the free propagator

$$\langle 0|T\left\{\phi_{\mu\nu}(x),\phi_{\rho\sigma}(y)\right\}|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \left\{ \frac{2i}{M^2-q^2}\Omega^L_{\mu\nu,\rho\sigma} + \frac{2i}{M^2}\Omega^T_{\mu\nu,\rho\sigma} \right\}, \tag{A.4}$$

where the following antisymmetric tensors have been defined

$$\begin{aligned} \Omega^L_{\mu\nu,\rho\sigma}(q) &= \frac{1}{2q^2}\left(g_{\mu\rho}q_\nu q_\sigma - g_{\rho\nu}q_\mu q_\sigma - (\rho\leftrightarrow\sigma)\right), \\ \Omega^T_{\mu\nu,\rho\sigma}(q) &= -\frac{1}{2q^2}\left(g_{\mu\rho}q_\nu q_\sigma - g_{\rho\nu}q_\mu q_\sigma - q^2 g_{\mu\rho}g_{\nu\sigma} - (\rho\leftrightarrow\sigma)\right), \end{aligned} \tag{A.5}$$

and superindexs L and T refer to longitudinal and transversal respectively. Let us consider as "generalized identity" the $\mathcal{I}_{\mu\nu,\rho\sigma}$ tensor,

$$\mathcal{I}_{\mu\nu,\rho\sigma} = \frac{1}{2}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right), \tag{A.6}$$

since any antisymmetric tensor $\mathcal{A}_{\mu\nu} = -\mathcal{A}_{\nu\mu}$ satisfies that

$$\mathcal{A}\cdot\mathcal{I} = \mathcal{I}\cdot\mathcal{A} = \mathcal{A}. \tag{A.7}$$

$\Omega^T_{\mu\nu}(q)$ and $\Omega^L_{\mu\nu}(q)$ satisfy the properties of the projectors:

$$\begin{aligned} \Omega^T + \Omega^L &= \mathcal{I} \qquad & \Omega^T\cdot\Omega^L &= \Omega^L\cdot\Omega^T = 0, \\ \Omega^T\cdot\Omega^T &= \Omega^T, & \Omega^L\cdot\Omega^L &= \Omega^L. \end{aligned} \tag{A.8}$$

The propagator of Eq. (A.4) has the normalization

$$\langle 0|\,\phi_{\mu\nu}\,|\phi,p\rangle = \frac{i}{M}\left[p_\mu\epsilon_\nu(p) - p_\nu\epsilon_\mu(p)\right], \tag{A.9}$$

where $\epsilon_\mu(p)$ is the polarization vector.

Advantages Using the Antisymmetric Formalism

There are different ways to include massive spin-1 fields in effective lagrangians, mainly the Proca and the antisymmetric tensor formalisms. In Ref. [18] it was analysed this ambiguity in the context of χPT to $\mathcal{O}(p^4)$. It was shown that, provided the consistency with QCD asymptotic behaviour is incorporated, the structure of the effective couplings induced by vector and axial-vector exchange is model independent.

In the case of the antisymmetric tensor formalism no local terms, of the $\mathcal{L}_{pGB}$, constructed with chiral tensors of $\mathcal{O}(p^4)$ or higher are required to fulfill the short-distance behaviour of QCD, that is, $\widetilde{L}_i = 0$, where $\widetilde{L}_i$ are the couplings in Resonance Chiral Theory, while L_i are used for the χPT case, when resonances have been integrated out.

Notwithstanding, for the Proca formalism business is not so easy. Considering the Proca lagrangian,

$$\mathcal{L}^{\rm Proca} \;=\; \sum_{R=V,A} \left(\mathcal{L}^{\rm Proca}_{\rm kin}(R) + \mathcal{L}^{\rm Proca}_{\rm int}(R)\right) \,, \tag{A.10}$$

where the lagrangian has been split in a kinetic and an interaction piece,

$$\begin{aligned} \mathcal{L}^{\rm Proca}_{\rm kin}(R) &= -\frac{1}{4}\langle\, \hat{R}_{\mu\nu}\hat{R}^{\mu\nu} \,-\, 2M_R^2\hat{R}_\mu\hat{R}^\mu \,\rangle \qquad (R=V,A)\,, \\ \mathcal{L}^{\rm Proca}_{\rm int}(V) &= -\frac{1}{2\sqrt{2}}\Big(f_V\langle\, \hat{V}^{\mu\nu} f^+_{\mu\nu}\,\rangle \,+\, i g_V\langle\, \hat{V}_{\mu\nu}[u^\mu,u^\nu]\,\rangle\Big) \,+\,\ldots\,, \\ \mathcal{L}^{\rm Proca}_{\rm int}(A) &= f_A\langle\, \hat{A}^{\mu\nu} f^-_{\mu\nu}\,\rangle \,+\,\ldots\,, \end{aligned} \tag{A.11}$$

and the dots refer to terms not relevant for the Green Functions that are analysed at large energies. Finally the following definition has been used,

$$\hat{R}_{\mu\nu} \;=\; \nabla_\mu\hat{R}_\nu \,-\, \nabla_\nu\hat{R}_\mu\,. \tag{A.12}$$

Imposing a reasonable short-distance behaviour for the two-point function built from a left- and a right-handed vector quark current, the pion form factor and the elastic meson-meson scattering, one finds the following constraints:

$$\begin{aligned} &\widetilde{L}_1^{\rm Proca} = \frac{1}{8}g_V^2\,, \qquad \widetilde{L}_2^{\rm Proca} = \frac{1}{4}g_V^2\,, \qquad \widetilde{L}_3^{\rm Proca} = -\frac{3}{4}g_V^2\,, \\ &\widetilde{L}_9^{\rm Proca} = \frac{1}{2}f_V g_V\,, \qquad \widetilde{L}_{10}^{\rm Proca} = -\frac{1}{4}f_V^2 + \frac{1}{4}f_A^2\,, \end{aligned} \tag{A.13}$$

while the rest vanish. Therefore the convenience of the antisymmetric formalism is manifest.

Appendix B

Feynman Integrals

The calculation of Chapter 3 involves the following Feynman Integrals:

$$A_0(M^2) \equiv \int \frac{\mathrm{d}k^D}{i(2\pi)^D} \frac{1}{k^2 + i\epsilon - M^2} = -\frac{M^2}{16\pi^2}\left[\lambda_\infty + \log\frac{M^2}{\mu^2}\right], \tag{B.1}$$

$$\begin{aligned} B_0(q^2, M_a^2, M_b^2) &\equiv \int \frac{\mathrm{d}k^D}{i(2\pi)^D} \frac{1}{(k^2 + i\epsilon - M_a^2)[(q-k)^2 + i\epsilon - M_b^2]} \\ &= -\frac{1}{16\pi^2}\left[\lambda_\infty + \frac{M_a^2}{M_a^2 - M_b^2}\log\frac{M_a^2}{\mu^2} - \frac{M_b^2}{M_a^2 - M_b^2}\log\frac{M_b^2}{\mu^2}\right] + \bar{J}(q^2, M_a^2, M_b^2), \end{aligned} \tag{B.2}$$

and the finite function

$$\begin{aligned} &C_0(q^2, M_a^2, M_b^2, M_c^2) \equiv \\ &\quad \int \frac{\mathrm{d}k^D}{i(2\pi)^D} \frac{1}{[(p_1-k)^2 + i\epsilon - M_a^2][(p_2+k)^2 + i\epsilon - M_b^2](k^2 + i\epsilon - M_c^2)}, \end{aligned} \tag{B.3}$$

with D the space-time dimension, $q \equiv p_1 + p_2$ and, with massless outgoing pions, $p_1^2 = p_2^2 = 0$. The divergences are collected in the factor

$$\lambda_\infty \equiv \frac{2\mu^{D-4}}{D-4} + \gamma_E - \log 4\pi - 1\,, \tag{B.4}$$

being $\gamma_E \simeq 0.5772$ the Euler's constant and μ the renormalization scale.

The two-propagator integral contains the finite function

$$\begin{aligned} \bar{J}(q^2, M_a^2, M_b^2) &= \frac{1}{32\pi^2}\left\{2 + \left[\frac{M_a^2 - M_b^2}{q^2} - \frac{M_a^2 + M_b^2}{M_a^2 - M_b^2}\right]\log\frac{M_b^2}{M_a^2}\right. \\ &\left. -\frac{\lambda^{1/2}(q^2, M_a^2, M_b^2)}{q^2}\log\left(\frac{[q^2 + \lambda^{1/2}(q^2, M_a^2, M_b^2)]^2 - (M_a^2 - M_b^2)^2}{[q^2 - \lambda^{1/2}(q^2, M_a^2, M_b^2)]^2 - (M_a^2 - M_b^2)^2}\right)\right\}, \end{aligned} \tag{B.5}$$

with $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

Some useful particular cases are:

$$B_0(q^2,0,0) = -\frac{\lambda_\infty}{16\pi^2} + \hat{B}_0(q^2/\mu^2)\,, \tag{B.6}$$

$$B_0(q^2,M^2,M^2) = -\frac{1}{16\pi^2}\left\{\lambda_\infty + \log\frac{M^2}{\mu^2} + 1\right\} + \overline{B}_0(q^2,M^2)\,, \tag{B.7}$$

$$B_0(q^2,0,M^2) = -\frac{1}{16\pi^2}\left\{\lambda_\infty + \log\frac{M^2}{\mu^2}\right\} + \bar{J}(q^2,0,M^2)\,, \tag{B.8}$$

with the finite parts

$$\hat{B}_0(q^2/\mu^2) = \frac{1}{16\pi^2}\left\{1 - \log\left(-\frac{q^2}{\mu^2}\right)\right\}\,, \tag{B.9}$$

$$\overline{B}_0(q^2,M^2) = \bar{J}(q^2,M^2,M^2) = \frac{1}{16\pi^2}\left\{2 - \sigma_M\log\left(\frac{\sigma_M+1}{\sigma_M-1}\right)\right\}\,, \tag{B.10}$$

$$\bar{J}(q^2,0,M^2) = \frac{1}{16\pi^2}\left\{1 - \left(1-\frac{M^2}{q^2}\right)\log\left(1-\frac{q^2}{M^2}\right)\right\}\,, \tag{B.11}$$

where $\sigma_M = \sqrt{1-4M^2/q^2}$.

The relevant three-propagator integrals are:

$$C_0(q^2,0,0,M^2) = -\frac{1}{16\pi^2 q^2}\left\{\mathrm{Li}_2\left(1+\frac{q^2}{M^2}\right) - \mathrm{Li}_2(1)\right\}\,, \tag{B.12}$$

$$C_0(q^2,M^2,M^2,0) = \frac{1}{16\pi^2 q^2}\log^2\left(\frac{\sigma_M-1}{\sigma_M+1}\right), \tag{B.13}$$

where

$$\mathrm{Li}_2(y) \equiv -\int_0^1 \frac{\mathrm{d}x}{x}\log(1-xy) = -\int_0^y \frac{\mathrm{d}x}{x}\log(1-x) \tag{B.14}$$

is the usual dilogarithmic function.

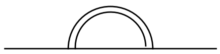
Appendix C

Feynman Diagrams for the Vector Form Factor

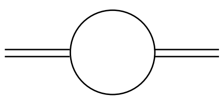
We show the contributions from the different Feynman diagrams to the vector form factor of the pion at next-to-leading order in the $1/N_C$ expansion. As it has been pointed out in Chapter 3, a $U(2)_L \otimes U(2)_R$ chiral theory is used and we work in the massless limit. Keep in mind that only the lagrangian of Section 3.2 is employed.

In the following figures a single line stands for a pseudo-Goldstone boson while a double line indicates a resonance field; notice that the resonance in the s-channel is always a ρ^0.

Wave-function Renormalization

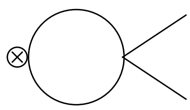


$$= i\frac{2G_V^2}{F^4}\left\{-(p^2+M_V^2)A_0(M_V^2)+(p^2-M_V^2)^2B_0(p^2,0,M_V^2)\right\}$$
$$+i\frac{4c_d^2}{F^4}\left\{(3p^2-M_S^2)A_0(M_S^2)+(p^2-M_S^2)^2B_0(p^2,0,M_S^2)\right\}\ ,$$



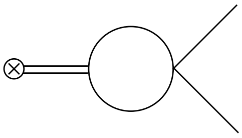
$$= -i\frac{2G_V^2}{F^4}\frac{(q^2)^2}{2}\Omega^L_{\mu\nu,\rho\sigma}(q)\left\{\frac{1}{6}B_0(q^2,0,0)+\frac{1}{144\pi^2}\right\}.$$

Contributions without Resonance Fields

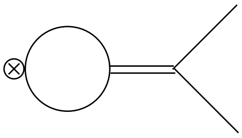


$$\rightarrow\ \frac{q^2}{F^2}\left\{\frac{1}{6}B_0(q^2,0,0)+\frac{1}{144\pi^2}\right\}\ .$$

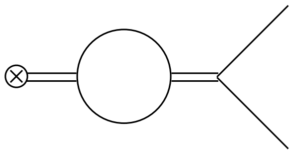
Contributions with Vector Resonance Fields



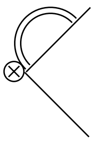
$$\to \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2} \frac{q^2}{F^2} \left\{ \frac{1}{6} B_0(q^2, 0, 0) + \frac{1}{144\pi^2} \right\} ,$$



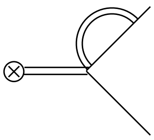
$$\to \frac{2G_V^2}{F^2} \frac{q^2}{M_V^2 - q^2} \frac{q^2}{F^2} \left\{ \frac{1}{6} B_0(q^2, 0, 0) + \frac{1}{144\pi^2} \right\} ,$$



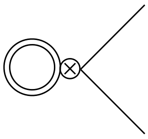
$$\to \frac{F_V G_V}{F^2} \frac{2G_V^2}{F^2} \left(\frac{q^2}{M_V^2 - q^2} \right)^2 \frac{q^2}{F^2} \left\{ \frac{1}{6} B_0(q^2, 0, 0) + \frac{1}{144\pi^2} \right\} ,$$



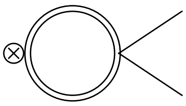
$$\to \frac{2G_V^2}{F^4} \left\{ -3A_0(M_V^2) + \frac{M_V^2}{32\pi^2} \right\} ,$$



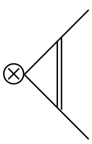
$$\to \frac{F_V G_V}{F^4} \frac{q^2}{M_V^2 - q^2} \left\{ -\frac{3}{2} A_0(M_V^2) + \frac{M_V^2}{64\pi^2} \right\} ,$$



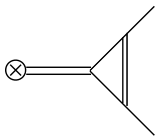
$$\to \frac{1}{F^2} \left\{ \frac{3}{2} A_0(M_V^2) - \frac{M_V^2}{64\pi^2} \right\} ,$$



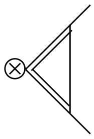
$$\to \frac{1}{F^2} \left\{ B_0(q^2, M_V^2, M_V^2, 0) \left[-2M_V^2 - \frac{q^2}{6} + \frac{q^4}{6M_V^2} \right] \right.$$
$$\left. + A_0(M_V^2) \left[\frac{1}{2} - \frac{q^2}{3M_V^2} \right] - \frac{7M_V^2}{64\pi^2} + \frac{q^2}{48\pi^2} - \frac{q^4}{288\pi^2 M_V^2} \right\} ,$$



$$\to \frac{2G_V^2}{F^4} \left\{ C_0(q^2, 0, 0, M_V^2) \left[-\frac{M_V^6}{q^2} - \frac{5M_V^4}{2} - q^2 M_V^2 \right] \right.$$
$$+ B_0(q^2, 0, 0) \left[-\frac{M_V^4}{q^2} - 2M_V^2 - \frac{q^2}{12} \right]$$
$$\left. + A_0(M_V^2) \left[\frac{M_V^2}{q^2} + 2 \right] - \frac{M_V^2}{64\pi^2} - \frac{q^2}{288\pi^2} \right\} ,$$

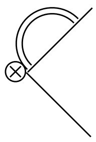


$$\rightarrow \frac{2G_V^2}{F^4}\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\left\{C_0(q^2,0,0,M_V^2)\left[-\frac{M_V^6}{q^2}-q^2M_V^2 -\frac{5M_V^4}{2}\right] + B_0(q^2,0,0)\left[-\frac{M_V^4}{q^2}-2M_V^2-\frac{q^2}{12}\right] +A_0(M_V^2)\left[\frac{M_V^2}{q^2}+2\right]-\frac{M_V^2}{64\pi^2}-\frac{q^2}{288\pi^2}\right\},$$

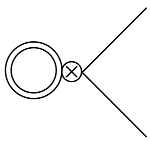


$$\rightarrow \frac{2G_V^2}{F^4}\left\{C_0(q^2,M_V^2,M_V^2,0)\left[\frac{M_V^6}{q^2}+\frac{M_V^4}{2}\right] +B_0(q^2,M_V^2,M_V^2)\left[-\frac{M_V^4}{q^2}-\frac{2M_V^2}{3}+\frac{5q^2}{12}\right] +A_0(M_V^2)\left[\frac{M_V^2}{q^2}+\frac{2}{3}\right]+\frac{M_V^2}{192\pi^2}-\frac{q^2}{288\pi^2}\right\}.$$

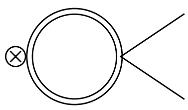
Contributions with Scalar Resonance Fields



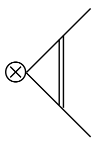
$$\rightarrow \frac{4c_d^2}{F^4}\left\{A_0(M_S^2)+\frac{M_S^2}{32\pi^2}\right\},$$



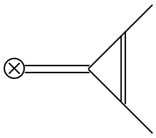
$$\rightarrow \frac{1}{F^2}A_0(M_S^2)\,,$$



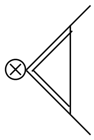
$$\rightarrow \frac{1}{F^2}\left\{B_0(q^2,M_S^2,M_S^2)\left[-\frac{2M_S^2}{3}+\frac{q^2}{6}\right] -\frac{1}{3}A_0(M_S^2)-\frac{M_S^2}{24\pi^2}+\frac{q^2}{144\pi^2}\right\},$$



$$\rightarrow \frac{4c_d^2}{F^4}\left\{C_0(q^2,0,0,M_S^2)\left[-\frac{M_S^6}{q^2}-\frac{M_S^4}{2}\right]+B_0(q^2,0,0)\left[-\frac{M_S^4}{q^2} -\frac{q^2}{12}\right]+\frac{M_S^2}{q^2}A_0(M_S^2)-\frac{M_S^2}{64\pi^2}-\frac{q^2}{288\pi^2}\right\},$$

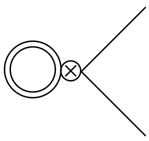


$$\rightarrow \frac{4c_d^2}{F^4}\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\left\{C_0(q^2,0,0,M_S^2)\left[-\frac{M_S^6}{q^2}-\frac{M_S^4}{2}\right]-\frac{M_S^2}{64\pi^2}\right.$$
$$\left.+B_0(q^2,0,0)\left[-\frac{M_S^4}{q^2}-\frac{q^2}{12}\right]+\frac{M_S^2}{q^2}A_0(M_V^2)-\frac{q^2}{288\pi^2}\right\},$$

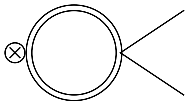


$$\rightarrow \frac{4c_d^2}{F^4}\left\{C_0(q^2,M_S^2,M_S^2,0)\left[\frac{M_S^6}{q^2}-\frac{M_S^4}{2}\right]\right.$$
$$+B_0(q^2,M_S^2,M_S^2)\left[-\frac{M_S^4}{q^2}+\frac{M_S^2}{3}-\frac{q^2}{12}\right]$$
$$\left.+A_0(M_S^2)\left[\frac{M_S^2}{q^2}-\frac{1}{3}\right]+\frac{M_S^2}{192\pi^2}-\frac{q^2}{288\pi^2}\right\}.$$

Contributions with Axial Resonance Fields

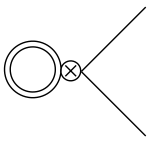


$$\rightarrow \frac{1}{F^2}\left\{\frac{3}{2}A_0(M_A^2)-\frac{M_A^2}{64\pi^2}\right\},$$

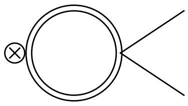


$$\rightarrow \frac{1}{F^2}\left\{B_0(q^2,M_A^2,M_A^2,0)\left[-2M_A^2-\frac{q^2}{6}+\frac{q^4}{6M_A^2}\right]\right.$$
$$\left.+A_0(M_A^2)\left[\frac{1}{2}-\frac{q^2}{3M_A^2}\right]-\frac{7M_A^2}{64\pi^2}+\frac{q^2}{48\pi^2}-\frac{q^4}{288\pi^2M_A^2}\right\}.$$

Contributions with Pseudoscalar Resonance Fields



$$\rightarrow \frac{1}{F^2}A_0(M_P^2),$$



$$\rightarrow \frac{1}{F^2}\left\{B_0(q^2,M_P^2,M_P^2)\left[-\frac{2M_P^2}{3}+\frac{q^2}{6}\right]\right.$$
$$\left.-\frac{1}{3}A_0(M_P^2)-\frac{M_P^2}{24\pi^2}+\frac{q^2}{144\pi^2}\right\}.$$

Appendix D

Form Factors and Constraints

In this appendix all two-body form factors that can be found in the even-intrinsic-parity sector of the Resonance Chiral Theory in the Single Resonance Approximation are analysed, following the ideas of Section 4.3.

The following items are presented for each form factor:

1. The form factor(s) is(are) defined through the corresponding matrix element.
2. The expression of the form factor(s) is(are) shown.
3. Using the optical theorem, the spectral function is given in terms of the form factors.
4. The constraints found by imposing a good high-energy behaviour of the spectral function.
5. Once the constraints are imposed, the well behaved form factor(s) is(are) presented again and quoted with a tilde.

Notice that when $R^0_{I=0}$ or η is written, we refer to the singlet in the $U(2)$ case. The following usual notation is employed throughout the section :
$\lambda\,(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc\,,\ \sigma_M = \lambda^{1/2}(q^2,M^2,M^2)/q^2 = \sqrt{1-4M^2/q^2}\,.$

D.1 Vector Form Factors

Vector Form Factor to $\pi\pi$ (Figure D.1)

$$\langle\, \pi^0(p_1)\pi^-(p_2)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \sqrt{2}\,\mathcal{F}^v_{\pi\pi}(q^2)\,(p_2-p_1)^\mu\,, \tag{D.1}$$

$$\mathcal{F}^v_{\pi\pi}(q^2) \;=\; 1+\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\,, \tag{D.2}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{\pi\pi} \;=\; \frac{\theta(q^2)}{24\pi}|\mathcal{F}^v_{\pi\pi}(q^2)|^2\,, \tag{D.3}$$

$$F_V\,G_V \;=\; F^2\,, \tag{D.4}$$

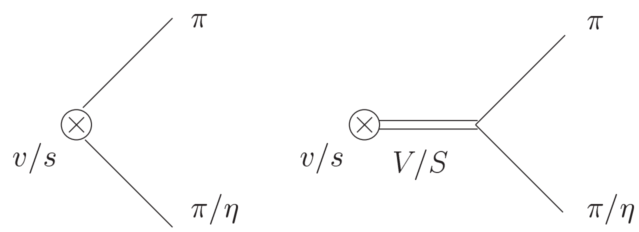


Figure D.1: Tree-level contributions to the vector/scalar form factor to two pseudo-Goldstones.

$$\tilde{\mathcal{F}}^{v}_{\pi\pi}(q^2) \;=\; \frac{M_V^2}{M_V^2-q^2}\,. \tag{D.5}$$

Vector Form Factor to Aπ (Figure D.2)

$$\begin{aligned}\langle\, A^0_{I=1}(p_A,\varepsilon)\pi^-(p_\pi)|\bar{d}\gamma^\mu u|0\,\rangle \;&=\; \frac{i\sqrt{2}}{M_A}\Big\{(q\varepsilon^*\,p_A^\mu \,-\, qp_A\,\varepsilon^{*\mu})\,\mathcal{F}^v_{A\pi}(q^2)\\ &\quad +(q\varepsilon^*\,p_\pi^\mu \,-\, qp_\pi\varepsilon^{*\mu})\,\mathcal{G}^v_{A\pi}(q^2)\Big\}\,,\end{aligned} \tag{D.6}$$

$$\begin{aligned}\mathcal{F}^v_{A\pi}(q^2) \;&=\; \frac{F_A}{F} + \frac{F_V}{F}\frac{M_A^2-q^2}{M_V^2-q^2}\Big[-2\lambda_2^{VA}+2\lambda_3^{VA}-\lambda_4^{VA}-2\lambda_5^{VA}\Big]\,,\\ \mathcal{G}^v_{A\pi}(q^2) \;&=\; \frac{2F_V}{F}\frac{M_A^2}{M_V^2-q^2}\Big[-2\lambda_2^{VA}+\lambda_3^{VA}\Big]\,,\end{aligned} \tag{D.7}$$

$$\begin{aligned}\mathrm{Im}\Pi_{VV}(q^2)|_{A\pi} = \theta(q^2-M_A^2)\frac{1-M_A^2/q^2}{48\pi}\Bigg\{\left(\frac{M_A^2}{q^2}+4+\frac{q^2}{M_A^2}\right)|\mathcal{F}^v_{A\pi}|^2+(1-M_A^2/q^2)^2\times\\ \times\left(\frac{q^2}{M_A^2}+\frac{q^4}{2M_A^4}\right)|\mathcal{G}^v_{A\pi}|^2+2(1-M_A^2/q^2)\left(1+\frac{2q^2}{M_A^2}\right)\mathrm{Re}\{\mathcal{F}^v_{A\pi}\mathcal{G}^{v\,*}_{A\pi}\}\Bigg\}\,,\end{aligned} \tag{D.8}$$

$$2\lambda_2^{VA}-2\lambda_3^{VA}+\lambda_4^{VA}+2\lambda_5^{VA} \;=\; F_A/F_V\,, \qquad -2\lambda_2^{VA}+\lambda_3^{VA} \;=\; 0\,, \tag{D.9}$$

$$\tilde{\mathcal{F}}^v_{A\pi}(q^2) \;=\; \frac{F_A}{F}\frac{M_V^2-M_A^2}{M_V^2-q^2}\,, \qquad \tilde{\mathcal{G}}^v_{A\pi}(q^2) \;=\; 0\,. \tag{D.10}$$

Vector Form Factor to Pπ (Figure D.2)

$$\langle\, P^-(p_P)\pi^0(p_\pi)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \sqrt{2}\,(qp_\pi\,p_P^\mu - qp_P\,p_\pi^\mu)\;\mathcal{F}^v_{P\pi}(q^2)\,, \tag{D.11}$$

$$\mathcal{F}^v_{P\pi}(q^2) \;=\; \frac{2\lambda_1^{PV}F_V}{F}\frac{1}{M_V^2-q^2}\,, \tag{D.12}$$

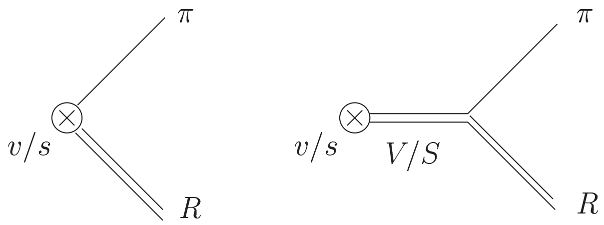


Figure D.2: Tree-level contributions to the vector/scalar form factor to one resonance field and one pseudo-Goldstone.

$$\begin{aligned} \mathrm{Im}\Pi_{VV}(q^2)|_{P\pi} &= \theta(q^2-M_P^2)\,\frac{(1-M_P^2/q^2)^3}{96\pi} q^4 |\mathcal{F}^v_{P\pi}|^2\,, && \text{(D.13)} \\ \lambda_1^{PV} &= 0\,, && \text{(D.14)} \\ \tilde{\mathcal{F}}^v_{P\pi}(q^2) &= 0\,. && \text{(D.15)} \end{aligned}$$

Vector Form Factor to VV (Figure D.3)

$$\begin{aligned} &\langle V^0_{I=1}(p_1,\varepsilon_1)V^-(p_2,\varepsilon_2)|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}\Big(\varepsilon_1^*\varepsilon_2^*\,(p_2-p_1)^\mu - (q\varepsilon_1^*\,\varepsilon_2^{*\mu} - q\varepsilon_2^*\,\varepsilon_1^{*\mu})\Big)\mathcal{F}^v_{VV}(q^2) \\ &+\sqrt{2}(q\varepsilon_1^*\,\varepsilon_2^{*\mu} - q\varepsilon_2^*\,\varepsilon_1^{*\mu})\mathcal{G}^v_{VV}(q^2) + \sqrt{2}\frac{(p_2-p_1)^\mu}{M_V^2}(q\varepsilon_1^*\,q\varepsilon_2^* \,-\, p_1p_2\,\varepsilon_1^*\varepsilon_2^*)\mathcal{H}^v_{VV}(q^2), \end{aligned} \qquad \text{(D.16)}$$

$$\begin{aligned} \mathcal{F}^v_{VV}(q^2) &= -1 \,+\, 2\lambda_7^{VV} \,+\, \frac{F_V}{\sqrt{2}(M_V^2-q^2)}\Big[6\lambda_0^{VVV} + (4M_V^2+2q^2)\lambda_2^{VVV} \\ &\quad + (4M_V^2-2q^2)\left(-2\lambda_1^{VVV}+\lambda_3^{VVV}+\lambda_4^{VVV}-2\lambda_5^{VVV}\right) + 4q^2\lambda_6^{VVV} \\ &\quad + 8M_V^2\lambda_7^{VVV}\Big]\,, \\ \mathcal{G}^v_{VV}(q^2) &= \frac{4\,F_V\,M_V^2}{\sqrt{2}(M_V^2-q^2)}\Big[-2\lambda_1^{VVV}+\lambda_3^{VVV}+\lambda_4^{VVV}-2\lambda_5^{VVV}-\lambda_6^{VVV}+\lambda_7^{VVV}\Big]\,, \\ \mathcal{H}^v_{VV}(q^2) &= -2\lambda_7^{VV} \,+\, \frac{F_V}{\sqrt{2}(M_V^2-q^2)}\Big[-6\lambda_0^{VVV} + (4M_V^2+2q^2)\big(2\lambda_1^{VVV}-\lambda_2^{VVV} \\ &\quad -\lambda_3^{VVV}-\lambda_4^{VVV}+2\lambda_5^{VVV}-2\lambda_7^{VVV}\big)\Big]\,, \end{aligned} \qquad \text{(D.17)}$$

$$\begin{aligned} \mathrm{Im}\Pi_{VV}(q^2)|_{VV} = \theta(q^2-4M_V^2)\frac{\sigma^3_{M_V}}{24\pi}\Bigg\{ &\left(3+\frac{q^2}{M_V^2}\right)|\mathcal{F}^v_{VV}|^2 + \left(\frac{q^2}{M_V^2}+\frac{q^4}{4M_V^4}\right)|\mathcal{G}^v_{VV}|^2 \\ &+\left(3-\frac{2q^2}{M_V^2}+\frac{q^4}{2M_V^4}\right)|\mathcal{H}^v_{VV}|^2 - \frac{3q^2}{M_V^2}\mathrm{Re}\{\mathcal{F}^v_{VV}\mathcal{G}^v_{VV}{}^*\} \end{aligned}$$

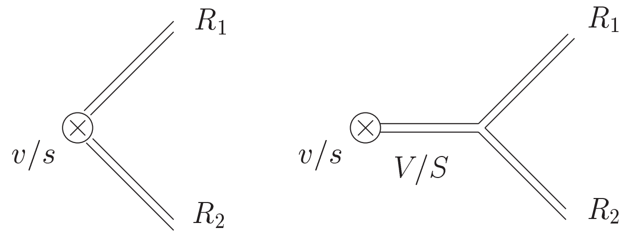


Figure D.3: Tree-level contributions to the vector/scalar form factor to two resonance fields.

$$+\left(6-\frac{2q^2}{M_V^2}\right)\mathrm{Re}\{\mathcal{F}^v_{VV}\mathcal{H}^{v\,*}_{VV}\}-\frac{q^2}{M_V^2}\mathrm{Re}\{\mathcal{G}^v_{VV}\mathcal{H}^{v\,*}_{VV}\}\Bigg\}\,, \qquad (D.18)$$

$$\begin{aligned}
2\lambda_1^{VVV}+\lambda_2^{VVV}-\lambda_3^{VVV}-\lambda_4^{VVV}+2\lambda_5^{VVV}+2\lambda_6^{VVV} &= -\frac{1}{\sqrt{2}F_V}+\frac{\sqrt{2}}{F_V}\lambda_7^{VV}\,,\\
-2\lambda_1^{VVV}+\lambda_3^{VVV}+\lambda_4^{VVV}-2\lambda_5^{VVV}-\lambda_6^{VVV}+\lambda_7^{VVV} &= 0\,,\\
2\lambda_1^{VVV}-\lambda_2^{VVV}-\lambda_3^{VVV}-\lambda_4^{VVV}+2\lambda_5^{VVV}-2\lambda_7^{VVV} &= -\frac{\sqrt{2}}{F_V}\lambda_7^{VV}\,,\\
-\frac{3}{2M_V^2}\lambda_0^{VVV}+2\lambda_1^{VVV}-\lambda_2^{VVV}-\lambda_3^{VVV}-\lambda_4^{VVV}+2\lambda_5^{VVV}-2\lambda_7^{VVV} &= \frac{\lambda_7^{VV}}{\sqrt{2}F_V}\,, \qquad (D.19)
\end{aligned}$$

$$\tilde{\mathcal{F}}^v_{VV}(q^2)\;=\;-\frac{M_V^2}{M_V^2-q^2}\,, \qquad \tilde{\mathcal{G}}^v_{VV}(q^2)\;=\;\tilde{\mathcal{H}}^v_{VV}(q^2)\;=\;0\,. \qquad (D.20)$$

Vector Form Factor to AA (Figure D.3)

$$\langle A^0_{I=1}(p_1,\varepsilon_1)A^-(p_2,\varepsilon_2)|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}\Big(\varepsilon_1^*\varepsilon_2^*\,(p_2-p_1)^\mu-(q\varepsilon_1^*\,\varepsilon_2^{*\mu}-q\varepsilon_2^*\,\varepsilon_1^{*\mu})\Big)\mathcal{F}^v_{AA}(q^2)$$
$$+\sqrt{2}\,(q\varepsilon_1^*\,\varepsilon_2^{*\mu}-q\varepsilon_2^*\,\varepsilon_1^{*\mu})\mathcal{G}^v_{AA}(q^2)+\sqrt{2}\frac{(p_2-p_1)^\mu}{M_V^2}(q\varepsilon_1^*\,q\varepsilon_2^*\;-\;p_1p_2\,\varepsilon_1^*\varepsilon_2^*)\mathcal{H}^v_{AA}(q^2)\,, \qquad (D.21)$$

$$\begin{aligned}
\mathcal{F}^v_{AA}(q^2) = {} & -1\;+\;2\lambda_7^{AA}\;+\;\frac{F_V}{\sqrt{2}(M_V^2-q^2)}\Big[2\lambda_0^{VAA}+2q^2(\lambda_3^{VAA}+\lambda_8^{VAA})\\
& +(2M_A^2-q^2)\left(2\lambda_2^{VAA}+\lambda_7^{VAA}-\lambda_9^{VAA}-2\lambda_{10}^{VAA}+\lambda_{12}^{VAA}+2\lambda_{13}^{VAA}-\lambda_{14}^{VAA}\right)\\
& +(-q^2-2M_A^2)\lambda_6^{VAA}\Big]\,,
\end{aligned}$$

$$\mathcal{G}^v_{AA}(q^2) = \frac{\sqrt{2}F_VM_A^2}{M_V^2-q^2}\Big[-\lambda_6^{VAA}+\lambda_7^{VAA}-\lambda_9^{VAA}-2(\lambda_{10}^{VAA}+\lambda_{11}^{VAA})+\lambda_{12}^{VAA}-\lambda_{14}^{VAA}\Big]\,,$$

$$\mathcal{H}^v_{AA}(q^2) = -2\lambda_7^{AA} + \frac{F_V}{\sqrt{2}(M_V^2-q^2)}\Big[-2\lambda_0^{VAA}+4q^2\lambda_1^{VAA}+(-4M_A^2+2q^2)\lambda_2^{VAA} - 2q^2(\lambda_3^{VAA}+\lambda_4^{VAA}-\lambda_5^{VAA})+(2M_A^2+2q^2)\lambda_6^{VAA} + 2M_A^2(-\lambda_7^{VAA}+\lambda_9^{VAA}+2\lambda_{10}^{VAA}-\lambda_{12}^{VAA}-2\lambda_{13}^{VAA}+\lambda_{14}^{VAA})\Big], \tag{D.22}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{AA} = \theta(q^2-4M_A^2)\frac{\sigma^3_{M_A}}{24\pi}\Bigg\{\left(3+\frac{q^2}{M_A^2}\right)|\mathcal{F}^v_{AA}|^2+\left(\frac{q^2}{M_A^2}+\frac{q^4}{4M_A^4}\right)|\mathcal{G}^v_{AA}|^2 + \left(3-\frac{2q^2}{M_A^2}+\frac{q^4}{2M_A^4}\right)|\mathcal{H}^v_{AA}|^2-\frac{3q^2}{M_A^2}\mathrm{Re}\{\mathcal{F}^v_{AA}\mathcal{G}^{v\,*}_{AA}\} + \left(6-\frac{2q^2}{M_A^2}\right)\mathrm{Re}\{\mathcal{F}^v_{AA}\mathcal{H}^{v\,*}_{AA}\}-\frac{q^2}{M_A^2}\mathrm{Re}\{\mathcal{G}^v_{AA}\mathcal{H}^{v\,*}_{AA}\}\Bigg\}, \tag{D.23}$$

$$\begin{aligned}
-2\lambda_2^{VAA}+2\lambda_3^{VAA}-\lambda_6^{VAA}-\lambda_7^{VAA}+2\lambda_8^{VAA}+ & \\
+\lambda_9^{VAA}+2\lambda_{10}^{VAA}-\lambda_{12}^{VAA}-2\lambda_{13}^{VAA}+\lambda_{14}^{VAA} &= -\frac{\sqrt{2}}{F_V}+\frac{2\sqrt{2}}{F_V}\lambda_7^{AA}, \\
-\lambda_6^{VAA}+\lambda_7^{VAA}-\lambda_9^{VAA}-2\lambda_{10}^{VAA}-2\lambda_{11}^{VAA}+\lambda_{12}^{VAA}-\lambda_{14}^{VAA} &= 0, \\
2\lambda_1^{VAA}+\lambda_2^{VAA}-\lambda_3^{VAA}-\lambda_4^{VAA}+\lambda_5^{VAA}+\lambda_6^{VAA} &= -\frac{\sqrt{2}}{F_V}\lambda_7^{AA}, \\
-\frac{1}{M_A^2}\lambda_0^{VAA}-2\lambda_2^{VAA}+\lambda_6^{VAA}-\lambda_7^{VAA}+\lambda_9^{VAA}+ & \\
+2\lambda_{10}^{VAA}-\lambda_{12}^{VAA}-2\lambda_{13}^{VAA}+\lambda_{14}^{VAA} &= \frac{\sqrt{2}M_V^2\lambda_7^{AA}}{F_VM_A^2},
\end{aligned} \tag{D.24}$$

$$\tilde{\mathcal{F}}^v_{AA}(q^2) = -\frac{M_V^2}{M_V^2-q^2}, \qquad \tilde{\mathcal{G}}^v_{AA}(q^2) = \tilde{\mathcal{H}}^v_{AA}(q^2) = 0. \tag{D.25}$$

Vector Form Factor to RR (R=S,P) (Figure D.3)

$$\langle R^0_{I=1}(p_1)R^-(p_2)|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}\,\mathcal{F}^v_{RR}(q^2)\,(p_2-p_1)^\mu, \tag{D.26}$$

$$\mathcal{F}^v_{RR}(q^2) = 1+\frac{F_V}{\sqrt{2}}\lambda^{VRR}\frac{q^2}{M_V^2-q^2}, \tag{D.27}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{RR} = \theta(q^2-4M_R^2)\frac{\sigma^3_{M_R}}{24\pi}|\mathcal{F}^v_{RR}(q^2)|^2, \tag{D.28}$$

$$\lambda^{VRR} = \frac{\sqrt{2}}{F_V}, \tag{D.29}$$

$$\tilde{\mathcal{F}}^v_{RR}(q^2) = \frac{M_V^2}{M_V^2-q^2}. \tag{D.30}$$

Vector Form Factor to SV (Figure D.3)

$$\langle\, S^0_{I=0}(p_S)V^-(p_V,\varepsilon)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \frac{\sqrt{2}}{M_V}\Big\{(q\varepsilon^*\,p_V^\mu \;-\; qp_V\,\varepsilon^{*\mu})\,\mathcal{F}^v_{SV}(q^2) + (q\varepsilon^*\,p_S^\mu \;-\; qp_S\varepsilon^{*\mu})\,\mathcal{G}^v_{SV}(q^2)\Big\}\,, \qquad \text{(D.31)}$$

$$\mathcal{F}^v_{SV}(q^2) \;=\; 4\lambda_3^{SV} + \frac{\sqrt{2}F_V}{M_V^2-q^2}\Big[-2\lambda_0^{SVV} - M_V^2\lambda_1^{SVV} - \frac{q^2+M_V^2-M_S^2}{2}\times \times(\lambda_2^{SVV}+2\lambda_3^{SVV}) + (M_V^2+q^2)(2\lambda_4^{SVV}+\lambda_5^{SVV})\Big]\,,$$

$$\mathcal{G}^v_{SV}(q^2) \;=\; -\frac{\sqrt{2}F_V M_V^2}{M_V^2-q^2}\lambda_1^{SVV}\,, \qquad \text{(D.32)}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{SV} \;=\; \theta(q^2-(M_S+M_V)^2)\,\frac{\lambda^{1/2}(q^2,M_S^2,M_V^2)}{48\pi q^2}\;\Bigg\{\frac{1}{M_V^2q^2}\Big[(M_V^2-M_S^2)^2 - 2q^2(M_S^2-2M_V^2)+q^4\Big]|\mathcal{F}^v_{SV}|^2 + \frac{1}{2M_V^4q^2}\Big[2M_V^2(M_V^2-M_S^2)^2 + q^2(-3M_V^4+6M_S^2M_V^2+M_S^4) - 2M_S^2q^4+q^6\Big]|\mathcal{G}^v_{SV}|^2 + \frac{1}{M_V^2q^2}\Big[-2(M_V^2-M_S^2)^2-2q^2(M_V^2+M_S^2)+4q^4\Big]\mathrm{Re}\{\mathcal{F}^v_{SV}\mathcal{G}^{v\,*}_{SV}\}\Bigg\}, \qquad \text{(D.33)}$$

$$\lambda_2^{SVV}+2\lambda_3^{SVV}-4\lambda_4^{SVV}-2\lambda_5^{SVV} \;=\; -\frac{4\sqrt{2}}{F_V}\lambda_3^{SV}\,, \qquad \lambda_1^{SVV} \;=\; 0\,, \qquad \text{(D.34)}$$

$$\tilde{\mathcal{F}}^v_{SV}(q^2) = \frac{\sqrt{2}F_V}{M_V^2-q^2}\left[\frac{8M_V^2}{\sqrt{2}F_V}\lambda_3^{SV} - 2\lambda_0^{SVV} + \frac{M_S^2}{2}\left(\lambda_2^{SVV}+2\lambda_3^{SVV}\right)\right]\,,\; \tilde{\mathcal{G}}^v_{SV}(q^2) = 0\,. \qquad \text{(D.35)}$$

Vector Form Factor to PA (Figure D.3)

$$\langle\, P^0_{I=1}(p_P)A^-(p_A,\varepsilon)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \frac{i\sqrt{2}}{M_A}\Big\{(q\varepsilon^*\,p_A^\mu \;-\; qp_A\,\varepsilon^{*\mu})\,\mathcal{F}^v_{PA}(q^2) + (q\varepsilon^*\,p_P^\mu \;-\; qp_P\varepsilon^{*\mu})\,\mathcal{G}^v_{PA}(q^2)\Big\}\,, \qquad \text{(D.36)}$$

$$\mathcal{F}^v_{PA}(q^2) \;=\; 4\lambda_1^{PA} + \frac{\sqrt{2}F_V}{M_V^2-q^2}\Big[2\lambda_0^{PVA} + M_A^2\lambda_1^{PVA} + \frac{q^2+M_A^2-M_P^2}{2}(\lambda_2^{PVA}+2\lambda_3^{PVA}) - M_A^2(2\lambda_4^{PVA}+\lambda_5^{PVA}) - q^2\lambda_6^{PVA}\Big]\,,$$

$$\mathcal{G}^v_{PA}(q^2) \;=\; \frac{\sqrt{2}F_V M_A^2}{M_V^2-q^2}\lambda_1^{PVA}\,, \qquad \text{(D.37)}$$

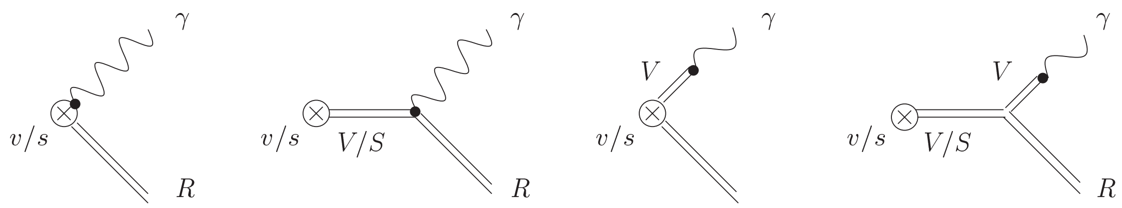


Figure D.4: Tree-level contributions to the vector/scalar form factor to one resonance field and one photon.

$$\begin{aligned}\mathrm{Im}\Pi_{VV}(q^2)|_{PA} \;=\; & \theta(q^2-(M_P+M_A)^2)\,\frac{\lambda^{1/2}(q^2,M_P^2,M_A^2)}{48\pi q^2}\left\{\frac{1}{M_A^2q^2}\Big[(M_A^2-M_P^2)^2\right.\\ & -2q^2(M_P^2-2M_A^2)+q^4\Big]|\mathcal{F}^v_{PA}|^2+\frac{1}{2M_A^4q^2}\Big[2M_A^2(M_A^2-M_P^2)^2\\ & +q^2(-3M_A^4+6M_P^2M_A^2+M_P^4)-2M_P^2q^4+q^6\Big]|\mathcal{G}^v_{PA}|^2\\ & \left.+\frac{1}{M_A^2q^2}\Big[-2(M_A^2-M_P^2)^2-2q^2(M_A^2+M_P^2)+4q^4\Big]\mathrm{Re}\{\mathcal{F}^v_{PA}\mathcal{G}^v_{PA}{}^*\}\right\}, \quad \text{(D.38)}\end{aligned}$$

$$\lambda_2^{PVA}+2\lambda_3^{PVA}-2\lambda_6^{PVA} \;=\; \frac{4\sqrt{2}}{F_V}\lambda_1^{PA}\,, \qquad \lambda_1^{PVA}\;=\;0\,, \qquad \text{(D.39)}$$

$$\begin{aligned}\tilde{\mathcal{F}}^v_{PA}(q^2) \;=\; & \frac{\sqrt{2}F_V}{M_V^2-q^2}\left[\frac{4M_V^2}{\sqrt{2}F_V}\lambda_1^{PA}+2\lambda_0^{PVA}+\frac{M_A^2-M_P^2}{2}\left(\lambda_2^{PVA}+2\lambda_3^{PVA}\right)\right.\\ & \left.-M_A^2\left(2\lambda_4^{PVA}+\lambda_5^{PVA}\right)\right], \qquad \tilde{\mathcal{G}}^v_{PA}(q^2)\;=\;0\,. \qquad \text{(D.40)}\end{aligned}$$

Vector Form Factor to Vγ (Figure D.4)

$$\begin{aligned}&\langle\,\gamma(p_\gamma,\varepsilon_\gamma)V^-(p_V,\varepsilon_V)|\bar{d}\gamma^\mu u|0\,\rangle \;=\; \frac{\sqrt{2}\,e\,F_V}{M_V}\times\\ &\times\left[\frac{1}{q^2}\Big\{M_V^2\,q\varepsilon_V^*(qp_\gamma\,\varepsilon_\gamma^{*\mu}-q\varepsilon_\gamma^*\,p_\gamma^\mu)+(qp_V\,p_\gamma^\mu-qp_\gamma\,p_V^\mu)(qp_\gamma\,\varepsilon_V^*\varepsilon_\gamma^*-q\varepsilon_\gamma^*\,q\varepsilon_V^*)\Big\}\mathcal{F}^v_{V\gamma}(q^2)\right.\\ &+\Big\{M_V^2\,q\varepsilon_V^*\,\varepsilon_\gamma^{*\mu}-\varepsilon_\gamma^*\varepsilon_V^*(qp_V\,p_\gamma^\mu-qp_\gamma\,p_V^\mu)+q\varepsilon_V^*\,q\varepsilon_\gamma^*(p_\gamma-p_V)^\mu\Big\}\mathcal{G}^v_{V\gamma}(q^2)+\Big\{q\varepsilon_\gamma^*\,\varepsilon_V^{*\,\mu}\\ &\left.-q\varepsilon_V^*\,\varepsilon_\gamma^{*\mu}+\varepsilon_V^*\varepsilon_\gamma^*\,(p_\gamma^\mu-p_V^\mu)+\frac{2}{M_V^2-q^2}\left(q\varepsilon_\gamma^*\,qp_V\,\varepsilon_V^{*\,\mu}-q\varepsilon_V^*\,q\varepsilon_\gamma^*\,p_V^\mu\right)\Big\}\right], \qquad \text{(D.41)}\end{aligned}$$

$$\mathcal{F}^v_{V\gamma}(q^2)\;=\;\frac{2\sqrt{2}\,F_V\,q^2}{(M_V^2-q^2)M_V^2}\left[2\lambda_1^{VVV}-\lambda_3^{VVV}-\lambda_4^{VVV}+2\lambda_5^{VVV}+\lambda_6^{VVV}-\lambda_7^{VVV}\right],$$

$$\mathcal{G}^v_{V\gamma}(q^2) = \frac{\sqrt{2}\,F_V}{(M_V^2-q^2)M_V^2}\left[3\lambda_0^{VVV} + 2qp_V\left(\lambda_2^{VVV}+\lambda_6^{VVV}+\lambda_7^{VVV}\right)\right] + \frac{2\lambda_7^{VV}}{M_V^2} + \frac{1}{M_V^2-q^2}\left[2\lambda_7^{VV}-1\right], \tag{D.42}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{V\gamma} \propto \Bigg[\left(\frac{q^4}{8}-\frac{3M_V^2q^2}{8}+\frac{M_V^4}{4}\right)|\mathcal{F}^v_{V\gamma}|^2 + \left(\frac{q^4}{2}-\frac{M_V^2q^2}{2}-\frac{M_V^4}{2}\right)|\mathcal{G}^v_{V\gamma}|^2 + \left(-\frac{q^4}{2}+2M_V^2q^2-3M_V^4\right)\mathrm{Re}\{\mathcal{F}^v_{V\gamma}\mathcal{G}^{v\,*}_{V\gamma}\} + \left(\frac{q^2}{2}-\frac{3M_V^2}{2}\right)\mathrm{Re}\{\mathcal{F}^v_{V\gamma}\} + \left(-3q^2+6M_V^2\right)\mathrm{Re}\{\mathcal{G}^v_{V\gamma}\} + \left(\frac{q^2}{2M_V^2}+1\right) + \mathcal{O}\left(\frac{1}{q^2}\right)\Bigg], \tag{D.43}$$

$$\begin{aligned} -2\lambda_1^{VVV}+\lambda_3^{VVV}+\lambda_4^{VVV}-2\lambda_5^{VVV}-\lambda_6^{VVV}+\lambda_7^{VVV} &= \frac{1}{2\sqrt{2}F_V}\,, \text{ [cf D.19]} \\ \lambda_2^{VVV}+\lambda_6^{VVV}+\lambda_7^{VVV} &= \frac{\sqrt{2}}{F_V}\lambda_7^{VV}-\frac{1}{2\sqrt{2}F_V}\,, \text{ [cf D.19]} \\ \lambda_7^{VV} &= -\frac{F_V\,\lambda_0^{VVV}}{\sqrt{2}\,M_V^2}\,, \text{ [cf D.19]} \end{aligned} \tag{D.44}$$

$$\tilde{\mathcal{F}}^v_{V\gamma} = -\frac{q^2}{(M_V^2-q^2)M_V^2}\,, \qquad \tilde{\mathcal{G}}^v_{V\gamma} = -\frac{3M_V^2+q^2}{2(M_V^2-q^2)M_V^2}\,. \tag{D.45}$$

Vector Form Factor to Sγ (Figure D.4)

$$\langle\,\gamma(p_\gamma,\varepsilon)S^-(p_S)|\bar{d}\gamma^\mu u|0\,\rangle = \frac{\sqrt{2}\,e\,F_V}{3}\left(q\varepsilon^*\,p_\gamma^\mu - qp_\gamma\,\varepsilon^{*\mu}\right)\mathcal{F}^v_{S\gamma}(q^2)\,, \tag{D.46}$$

$$\mathcal{F}^v_{S\gamma}(q^2) = 4\lambda_3^{SV}\left(\frac{1}{M_V^2-q^2}+\frac{1}{M_V^2}\right) + \frac{\sqrt{2}F_V}{M_V^2(M_V^2-q^2)}\left[-2\lambda_0^{SVV} - \frac{q^2-M_S^2}{2}(\lambda_2^{SVV}+2\lambda_3^{SVV}) + q^2(2\lambda_4^{SVV}+\lambda_5^{SVV})\right], \tag{D.47}$$

$$\mathrm{Im}\Pi_{VV}(q^2)|_{S\gamma} = \theta(q^2-M_S^2)\,F_V^2\,e^2\,\frac{(1-M_S^2/q^2)^3}{432\pi}\,q^2\,|\mathcal{F}^v_{S\gamma}|^2, \tag{D.48}$$

$$\lambda_2^{SVV}+2\lambda_3^{SVV}-4\lambda_4^{SVV}-2\lambda_5^{SVV} = -\frac{4\sqrt{2}}{F_V}\lambda_3^{SV}\,, \qquad \text{[cf D.34]} \tag{D.49}$$

$$\tilde{\mathcal{F}}^v_{S\gamma}(q^2) = \frac{\sqrt{2}F_V}{(M_V^2-q^2)M_V^2}\left[\frac{8\,M_V^2}{\sqrt{2}\,F_V}\lambda_3^{SV} - 2\lambda_0^{SVV} + \frac{M_S^2}{2}\left(\lambda_2^{SVV}+2\lambda_3^{SVV}\right)\right]. \tag{D.50}$$

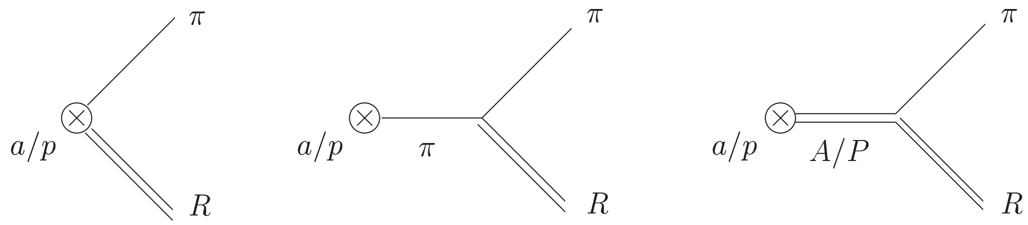


Figure D.5: Tree-level contributions to the axial/pseudoscalar form factor to one resonance field and one pseudo-Goldstone.

D.2 Axial Form Factors

Axial Form Factor to Vπ (Figure D.5)

$$\begin{aligned}\langle\, V^0_{I=1}(p_V,\varepsilon)\pi^-(p_\pi)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;&=\; \frac{i\sqrt{2}}{M_V}\Big\{(q\varepsilon^*\, p_V^\mu \,-\, qp_V\,\varepsilon^{*\mu})\,\mathcal{F}^a_{V\pi}(q^2)\\ &+(q\varepsilon^*\, p_\pi^\mu \,-\, qp_\pi\varepsilon^{*\mu})\,\mathcal{G}^a_{V\pi}(q^2)\Big\}\,, \qquad (D.51)\end{aligned}$$

$$\begin{aligned}\mathcal{F}^a_{V\pi}(q^2) \;&=\; -\frac{F_V}{F}+\frac{2G_V}{F}-\frac{2G_V}{F}\frac{M_V^2}{q^2}+\frac{F_A}{F}\frac{q^2}{M_A^2-q^2}\Big[(-\frac{2M_V^2}{q^2}+2)\lambda_2^{VA}\\ &+(\frac{M_V^2}{q^2}-1)\lambda_4^{VA}+(\frac{2M_V^2}{q^2}-2)\lambda_5^{VA}\Big]\,,\\ \mathcal{G}^a_{V\pi}(q^2) \;&=\; -\frac{2G_V}{F}\frac{M_V^2}{q^2}+\frac{2F_A}{F}\frac{M_V^2}{M_A^2-q^2}\Big[-2\lambda_2^{VA}+\lambda_3^{VA}\Big]\,, \qquad (D.52)\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\Pi_{AA}(q^2)|_{V\pi} \;&=\; \theta(q^2-M_V^2)\frac{1-M_V^2/q^2}{48\pi}\Bigg\{\left(\frac{M_V^2}{q^2}+4+\frac{q^2}{M_V^2}\right)|\mathcal{F}^a_{V\pi}|^2\\ &+(1-M_V^2/q^2)^2\left(\frac{q^2}{M_V^2}+\frac{q^4}{2M_V^4}\right)|\mathcal{G}^a_{V\pi}|^2\\ &+2(1-M_V^2/q^2)\left(1+\frac{2q^2}{M_V^2}\right)\mathrm{Re}\{\mathcal{F}^a_{V\pi}\mathcal{G}^{a}_{V\pi}{}^*\}\Bigg\}\,, \qquad (D.53)\end{aligned}$$

$$2\lambda_2^{VA}-\lambda_4^{VA}-2\lambda_5^{VA} \;=\; -\frac{F_V}{F_A}+\frac{2G_V}{F_A}\,, \qquad -2\lambda_2^{VA}+\lambda_3^{VA} \;=\; -\frac{G_V}{F_A}\,, \qquad (D.54)$$

$$\begin{aligned}\tilde{\mathcal{F}}^a_{V\pi}(q^2) \;&=\; \left(\frac{F_V}{F}-\frac{2G_V}{F}\right)\frac{M_V^2-M_A^2}{M_A^2-q^2}-\frac{2G_V}{F}\frac{M_V^2}{q^2}\,,\\ \tilde{\mathcal{G}}^a_{V\pi}(q^2) \;&=\; -\frac{2G_V}{F}\frac{M_V^2M_A^2}{(M_A^2-q^2)q^2}\,. \qquad (D.55)\end{aligned}$$

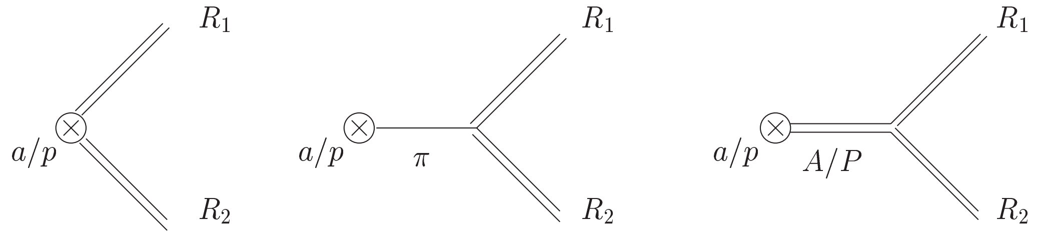


Figure D.6: Tree-level contributions to the axial/pseudoscalar form factor to two resonances.

Axial Form Factor to Sπ (Figure D.5)

$$\langle\, S^0_{I=0}(p_S)\pi^-(p_\pi)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;=\; -2\,i\,\mathcal{F}^a_{S\pi}(q^2)\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)p_{\pi\nu}\,, \qquad \text{(D.56)}$$

$$\mathcal{F}^a_{S\pi}(q^2) \;=\; \frac{2c_d}{F}-\frac{\sqrt{2}\,F_A}{F}\frac{q^2}{M_A^2-q^2}\lambda_1^{SA}\,, \qquad \text{(D.57)}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{S\pi} \;=\; \theta(q^2-M_S^2)\frac{(1-M_S^2/q^2)^3}{48\pi}|\mathcal{F}^a_{S\pi}(q^2)|^2\,, \qquad \text{(D.58)}$$

$$\lambda_1^{SA} \;=\; -\frac{\sqrt{2}c_d}{F_A}\,, \qquad \text{(D.59)}$$

$$\tilde{\mathcal{F}}^a_{S\pi}(q^2) \;=\; \frac{2c_d}{F}\frac{M_A^2}{M_A^2-q^2}\,. \qquad \text{(D.60)}$$

Axial Form Factor to VA (Figure D.6)

$$\begin{aligned}
\langle\, V^0_{I=1}(p_V,\varepsilon_V)A^-(p_A,\varepsilon_A)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;=\; \frac{\sqrt{2}}{M_V M_A}\frac{1}{2q^2}\times \\
\times\Big\{2\,(qp_Ap_V^\mu - qp_Vp_A^\mu)\Big[p_Ap_V\varepsilon_A^*\varepsilon_V^* - q\varepsilon_A^* q\varepsilon_V^*\Big]\mathcal{F}^a_{VA}(q^2) \\
+\,2M_V^2\Big[(qp_Ap_V^\mu - qp_Vp_A^\mu)\,\varepsilon_A^*\varepsilon_V^* - (p_V^\mu + p_A^\mu)\,q\varepsilon_A^* q\varepsilon_V^* + q^2 q\varepsilon_V^*\varepsilon_A^{*\,\mu}\Big]\mathcal{G}^a_{VA}(q^2) \\
+\,2M_A^2\Big[(qp_Ap_V^\mu - qp_Vp_A^\mu)\,\varepsilon_A^*\varepsilon_V^* + (p_V^\mu + p_A^\mu)\,q\varepsilon_A^* q\varepsilon_V^* - q^2 q\varepsilon_A^*\varepsilon_V^{*\,\mu}\Big]\mathcal{H}^a_{VA}(q^2) \\
+\Big[\big(M_V^2+M_A^2\big)(qp_Ap_V^\mu - qp_Vp_A^\mu)\,\varepsilon_A^*\varepsilon_V^* + \big(M_V^2+M_A^2\big)(p_A^\mu - p_V^\mu)\,q\varepsilon_A^* q\varepsilon_V^* \\
+\big(M_V^2-M_A^2\big)\big(M_A^2\,q\varepsilon_A^*\varepsilon_V^{*\,\mu} + M_V^2\,q\varepsilon_V^*\varepsilon_A^{*\,\mu}\big)\Big]\mathcal{I}^a_{VA}(q^2)\Big\}\,, \qquad \text{(D.61)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}^a_{VA}(q^2) = 2\lambda_4^{VA} + 4\lambda_5^{VA} + 4\lambda_6^{VA} - \frac{\sqrt{2}F_A}{M_A^2-q^2}\Big[(M_A^2-M_V^2)(-2\lambda_1^{VAA}+\lambda_4^{VAA}-\lambda_5^{VAA}) \\
-\,2\lambda_0^{VAA} - 4qp_A\lambda_2^{VAA} - 2M_V^2\lambda_3^{VAA} + (q^2+M_A^2/2+3M_V^2/2)\lambda_6^{VAA} - (M_A^2+M_V^2)\lambda_8^{VAA} \\
+\,(q^2+M_A^2/2-M_V^2/2)\big(-\lambda_7^{VAA}+\lambda_9^{VAA}+2\lambda_{10}^{VAA}-\lambda_{12}^{VAA}-2\lambda_{13}^{VAA}+\lambda_{14}^{VAA}\big)\Big]\,,
\end{aligned}$$

$$\mathcal{G}^a_{VA}(q^2) = -2\lambda_2^{VA} + 2\lambda_3^{VA} + 2\lambda_6^{VA} - \frac{\sqrt{2}F_A}{M_A^2 - q^2}\Big[-\lambda_0^{VAA} - 2qp_A\lambda_2^{VAA} + (M_A^2 - M_V^2)/4\times$$
$$\big(-\lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}\big) - M_V^2\lambda_3^{VAA} + (M_A^2/4 + 3M_V^2/4)\lambda_6^{VAA}$$
$$+ (-q^2 - M_A^2/2 - M_V^2/2)\lambda_8^{VAA} + (q^2 + M_A^2/2 - M_V^2/2)\left(-2\lambda_1^{VAA} + \lambda_4^{VAA} - \lambda_5^{VAA}\right)\Big],$$

$$\mathcal{H}^a_{VA}(q^2) = 2\lambda_2^{VA} + 2\lambda_6^{VA} - \frac{\sqrt{2}F_A}{M_A^2 - q^2}\Big[-\lambda_0^{VAA} - 2qp_A\lambda_2^{VAA} - M_V^2\lambda_3^{VAA}$$
$$+ (M_A^2 - M_V^2)/2\left(-2\lambda_1^{VAA} + \lambda_4^{VAA} - \lambda_5^{VAA}\right) + (-2q^2 + 3M_V^2 + M_A^2)\lambda_6^{VAA}/4$$
$$- 2q^2\lambda_{11}^{VAA} + (2q^2 + M_V^2 - M_A^2)/4\left(\lambda_7^{VAA} - \lambda_9^{VAA} - 2\lambda_{10}^{VAA} + \lambda_{12}^{VAA} - \lambda_{14}^{VAA}\right)$$
$$- (M_V^2 + M_A^2)\lambda_8^{VAA}/2 - (q^2 + M_A^2/2 - M_V^2/2)\lambda_{13}^{VAA}\Big],$$

$$\mathcal{I}^a_{VA}(q^2) = -\frac{F_A\, q^2}{\sqrt{2}(M_A^2 - q^2)}\Big[+4\lambda_1^{VAA} - 2\lambda_4^{VAA} + 2\lambda_5^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + 2\lambda_8^{VAA}$$
$$+ \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}\Big], \tag{D.62}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{VA} \propto \Big[\left(q^4/8 + \mathcal{O}(q^2)\right)|\mathcal{F}^a_{VA}|^2 + \mathcal{O}(q^2)\,|\mathcal{G}^a_{VA}|^2 + \mathcal{O}(q^2)\,|\mathcal{H}^a_{VA}|^2$$
$$+ \left((M_A^4 + 4M_V^2M_A^2 + M_V^4)/8 + \mathcal{O}(q^{-2})\right)|\mathcal{I}^a_{VA}|^2 + \mathcal{O}(q^2)\,\mathrm{Re}\{\mathcal{F}^a_{VA}\mathcal{G}^{a\,*}_{VA}\}$$
$$+ \mathcal{O}(q^2)\,\mathrm{Re}\{\mathcal{F}^a_{VA}\mathcal{H}^{a\,*}_{VA}\} + \left(q^2(M_A^2 + M_V^2)/4 + \mathcal{O}(q^0)\right)\mathrm{Re}\{\mathcal{F}^a_{VA}\mathcal{I}^{a\,*}_{VA}\}$$
$$+ \mathcal{O}(q^0)\,\mathrm{Re}\{\mathcal{G}^a_{VA}\mathcal{H}^{a\,*}_{VA}\} + \mathcal{O}(q^0)\,\mathrm{Re}\{\mathcal{G}^a_{VA}\mathcal{I}^{a\,*}_{VA}\} + \mathcal{O}(q^0)\,\mathrm{Re}\{\mathcal{H}^a_{VA}\mathcal{I}^{a\,*}_{VA}\}\Big], \tag{D.63}$$

$$- 2\lambda_2^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}$$
$$= \frac{1}{\sqrt{2}F_A}\Big\{ -2\lambda_4^{VA} - 4\lambda_5^{VA} - 4\lambda_6^{VA}\Big\},$$

$$M_V^2\Big\{4\lambda_1^{VAA} + 4\lambda_2^{VAA} - 4\lambda_3^{VAA} - 2\lambda_4^{VAA} + 2\lambda_5^{VAA} + 3\lambda_6^{VAA} + \lambda_7^{VAA} - 2\lambda_8^{VAA} - \lambda_9^{VAA}$$
$$- 2\lambda_{10}^{VAA} + \lambda_{12}^{VAA} + 2\lambda_{13}^{VAA} - \lambda_{14}^{VAA}\Big\} + M_A^2\Big\{ -4\lambda_1^{VAA} - 4\lambda_2^{VAA} + 2\lambda_4^{VAA}$$
$$- 2\lambda_5^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} - 2\lambda_8^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}\Big\}$$
$$- 4\lambda_0^{VAA} = \frac{2\sqrt{2}M_A^2}{F_A}\Big\{\lambda_4^{VA} + 2\lambda_5^{VA} + 2\lambda_6^{VA}\Big\},$$

$$- 2\lambda_1^{VAA} - \lambda_2^{VAA} + \lambda_4^{VAA} - \lambda_5^{VAA} - \lambda_8^{VAA} = \frac{\sqrt{2}}{F_A}\Big\{\lambda_2^{VA} - \lambda_3^{VA} - \lambda_6^{VA}\Big\},$$

$$- 2\lambda_2^{VAA} - \lambda_6^{VAA} + \lambda_7^{VAA} - \lambda_9^{VAA} - 2\lambda_{10}^{VAA}$$
$$- 4\lambda_{11}^{VAA} + \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} - \lambda_{14}^{VAA} = -2\sqrt{2}/F_A\left\{\lambda_2^{VA} + \lambda_6^{VA}\right\},$$

$$4\lambda_1^{VAA} - 2\lambda_4^{VAA} + 2\lambda_5^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + 2\lambda_8^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA} = 0 \,, \tag{D.64}$$

$$\begin{aligned}
\tilde{\mathcal{F}}_{VA}^a(q^2) &= \tilde{\mathcal{I}}_{VA}^a(q^2) = 0 \,, \\
\tilde{\mathcal{G}}_{VA}^a(q^2) &= -\frac{\sqrt{2}F_A}{M_A^2 - q^2}\Bigg\{ \frac{\sqrt{2}M_A^2}{F_A}\big(\lambda_2^{VA} - \lambda_3^{VA} - \lambda_6^{VA}\big) - \lambda_0^{VAA} + (M_A^2 - M_V^2)/4 \times \\
&\times \big(-4\lambda_1^{VAA} + 2\lambda_4^{VAA} - 2\lambda_5^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}\big) \\
&- M_A^2\lambda_2^{VAA} - M_V^2\lambda_3^{VAA} + (M_A^2/4 + 3M_V^2/4)\lambda_6^{VAA} - (M_A^2/2 + M_V^2/2)\lambda_8^{VAA} \Bigg\} \,, \\
\tilde{\mathcal{H}}_{VA}^a(q^2) &= \tilde{\mathcal{G}}_{VA}^a(q^2) \quad + \quad \frac{2M_A^2}{M_A^2 - q^2}\left[2\lambda_2^{VA} - \lambda_3^{VA}\right] .
\end{aligned} \tag{D.65}$$

Axial Form Factor to PV (Figure D.6)

$$\begin{aligned}
\langle\, P_{I=1}^0(p_P) V^-(p_V,\varepsilon)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;&=\; \frac{\sqrt{2}\,i}{M_V}\Big\{ (q\varepsilon^*\, p_V^\mu \,-\, qp_V\,\varepsilon^{*\mu})\,\mathcal{F}_{PV}^a(q^2) \\
&+(q\varepsilon^*\, p_P^\mu \,-\, qp_P\varepsilon^{*\mu})\,\mathcal{G}_{PV}^a(q^2)\Big\} ,
\end{aligned} \tag{D.66}$$

$$\begin{aligned}
\mathcal{F}_{PV}^a(q^2) &= 2\lambda_1^{PV}\left(\frac{M_V^2}{q^2} - 1\right) - 4\lambda_2^{PV} + \frac{\sqrt{2}F_A}{M_A^2 - q^2}\Bigg[2\lambda_0^{PVA} + M_V^2\lambda_1^{PVA} \\
&+ \frac{q^2 + M_V^2 - M_P^2}{2}(\lambda_2^{PVA} + 2\lambda_3^{PVA}) - q^2(2\lambda_4^{PVA} + \lambda_5^{PVA}) - M_V^2\lambda_6^{PVA}\Bigg] , \\
\mathcal{G}_{PV}^a(q^2) &= \frac{2M_V^2}{q^2}\lambda_1^{PV} + \frac{\sqrt{2}F_A}{M_A^2 - q^2}\left(M_V^2\lambda_1^{PVA}\right) ,
\end{aligned} \tag{D.67}$$

$$\begin{aligned}
\mathrm{Im}\Pi_{AA}(q^2)|_{PV} &= \theta(q^2 - (M_P + M_V)^2)\frac{\lambda^{1/2}(q^2, M_P^2, M_V^2)}{48\pi q^2}\Bigg\{ \frac{1}{M_V^2 q^2}\Big[(M_V^2 - M_P^2)^2 \\
&- 2q^2(M_P^2 - 2M_V^2) + q^4\Big]|\mathcal{F}_{PV}^a|^2 + \frac{1}{2M_V^4 q^2}\Big[2M_V^2(M_V^2 - M_P^2)^2 \\
&+ q^2(-3M_V^4 + 6M_P^2M_V^2 + M_P^4) - 2M_P^2q^4 + q^6\Big]|\mathcal{G}_{PV}^a|^2 \\
&+ \frac{1}{M_V^2 q^2}\Big[- 2(M_V^2 - M_P^2)^2 - 2q^2(M_V^2 + M_P^2) + 4\,q^4\Big]\mathrm{Re}\{\mathcal{F}_{PV}^a\mathcal{G}_{PV}^{a\,*}\}\Bigg\},
\end{aligned} \tag{D.68}$$

$$\lambda_2^{PVA} + 2\lambda_3^{PVA} - 4\lambda_4^{PVA} - 2\lambda_5^{PVA} = -\frac{2\sqrt{2}}{F_A}\left(\lambda_1^{PV} + 2\lambda_2^{PV}\right) , \; \lambda_1^{PVA} = \frac{\sqrt{2}\lambda_1^{PV}}{F_A} , \tag{D.69}$$

$$\tilde{\mathcal{F}}^a_{PV}(q^2) = \frac{\sqrt{2}F_A}{M_A^2-q^2}\left[\frac{\sqrt{2}}{F_A}\left(\frac{M_A^2M_V^2}{q^2}-M_A^2\right)\lambda_1^{PV}-\frac{2\sqrt{2}M_A^2}{F_A}\lambda_2^{PV}+2\lambda_0^{PVA}+\right.$$
$$\left.\frac{M_V^2-M_P^2}{2}\left(\lambda_2^{PVA}+2\lambda_3^{PVA}\right)-M_V^2\lambda_6^{PVA}\right],$$
$$\tilde{\mathcal{G}}^a_{PV}(q^2) = \frac{2M_V^2M_A^2}{(M_A^2-q^2)q^2}\lambda_1^{PV}. \qquad \text{(D.70)}$$

Axial Form Factor to SA (Figure D.6)

$$\langle\, S^0_{I=0}(p_S)A^-(p_A,\varepsilon)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle = \frac{\sqrt{2}}{M_A}\Big\{(q\varepsilon^*\,p_A^\mu\,-\,qp_A\,\varepsilon^{*\mu})\,\mathcal{F}^a_{SA}(q^2)$$
$$+(q\varepsilon^*\,p_S^\mu\,-\,qp_S\varepsilon^{*\mu})\,\mathcal{G}^a_{SA}(q^2)\Big\}, \qquad \text{(D.71)}$$

$$\mathcal{F}^a_{SA}(q^2) = 2\lambda_1^{SA}\left(\frac{M_A^2}{q^2}-1\right)-4\lambda_2^{SA}+\frac{\sqrt{2}F_A}{M_A^2-q^2}\Big[2\lambda_0^{SAA}+M_A^2\lambda_1^{SAA}$$
$$+\frac{q^2+M_A^2-M_S^2}{2}(\lambda_2^{SAA}+2\lambda_3^{SAA})-(M_A^2+q^2)(2\lambda_4^{SAA}+\lambda_5^{SAA})\Big],$$
$$\mathcal{G}^a_{SA}(q^2) = \frac{2M_A^2}{q^2}\lambda_1^{SA}+\frac{\sqrt{2}F_A}{M_A^2-q^2}\left(M_A^2\lambda_1^{SAA}\right), \qquad \text{(D.72)}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{SA} = \theta(q^2-(M_S+M_A)^2)\,\frac{\lambda^{1/2}(q^2,M_S^2,M_A^2)}{48\pi q^2}\,\Big\{\frac{1}{M_A^2q^2}\Big[(M_A^2-M_S^2)^2$$
$$-2q^2(M_S^2-2M_A^2)+q^4\Big]|\mathcal{F}^a_{SA}|^2+\frac{1}{2M_A^4q^2}\Big[2M_A^2(M_A^2-M_S^2)^2$$
$$+q^2(-3M_A^4+6M_S^2M_A^2+M_S^4)-2M_S^2q^4+q^6\Big]|\mathcal{G}^a_{SA}|^2$$
$$+\frac{1}{M_A^2q^2}\Big[-2(M_A^2-M_S^2)^2-2q^2(M_A^2+M_S^2)+4\,q^4\Big]\mathrm{Re}\{\mathcal{F}^a_{SA}\mathcal{G}^{a\,*}_{SA}\}\Big\}, \qquad \text{(D.73)}$$

$$\lambda_2^{SAA}+2\lambda_3^{SAA}-4\lambda_4^{SAA}-2\lambda_5^{SAA} = -\frac{2\sqrt{2}}{F_A}\left(\lambda_1^{SA}+2\lambda_2^{SA}\right),\ \lambda_1^{SAA} = \frac{\sqrt{2}\lambda_1^{SA}}{F_A}, \qquad \text{(D.74)}$$

$$\tilde{\mathcal{F}}^a_{SA}(q^2) = \frac{\sqrt{2}F_A}{M_A^2-q^2}\left[-\frac{\sqrt{2}}{F_A}\left(-\frac{M_A^4}{q^2}+3M_A^2\right)\lambda_1^{SA}-\frac{4\sqrt{2}M_A^2}{F_A}\lambda_2^{SA}\right.$$
$$\left.+2\lambda_0^{SAA}+M_A^2\lambda_1^{SAA}-\frac{M_S^2}{2}\left(\lambda_2^{SAA}+2\lambda_3^{SAA}\right)\right],$$
$$\tilde{\mathcal{G}}^a_{SA}(q^2) = \frac{2M_A^4}{(M_A^2-q^2)q^2}\lambda_1^{SA}. \qquad \text{(D.75)}$$

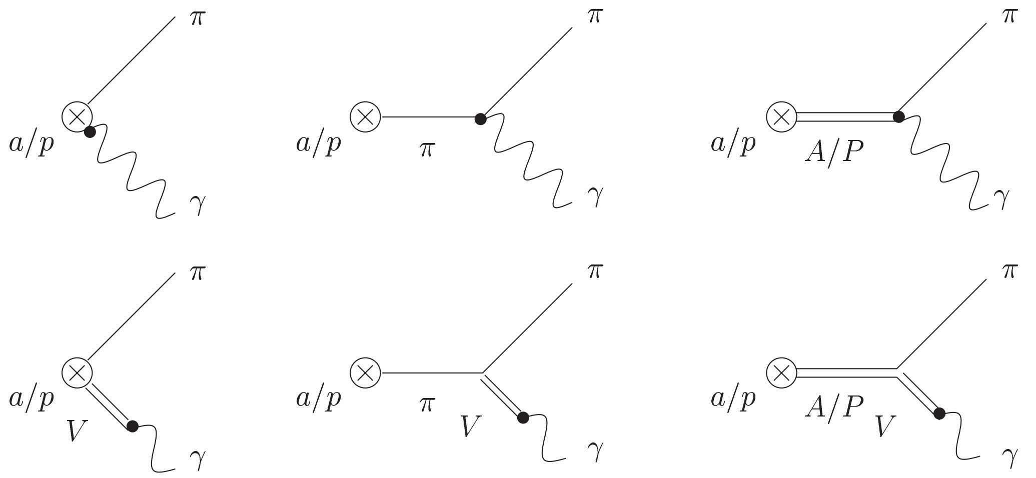


Figure D.7: Tree-level contributions to the axial/pseudoscalar form factor to one pseudo-Goldstone and one photon.

Axial Form Factor to SP (Figure D.6)

$$\langle\, S^0_{I=0}(p_S)P^-(p_P)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle = -2\,i\,\mathcal{F}^a_{SP}(q^2)\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)p_{P_\nu}\,, \quad \text{(D.76)}$$

$$\mathcal{F}^a_{SP}(q^2) = \sqrt{2}\lambda_1^{SP} - \frac{q^2}{M_A^2-q^2}F_A\lambda^{SPA}\,, \quad \text{(D.77)}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{SP} = \theta(q^2-(M_S+M_P)^2)\frac{\lambda^{3/2}(q^2,M_S^2,M_P^2)}{48\pi q^6}|\mathcal{F}^a_{SP}(q^2)|^2\,, \quad \text{(D.78)}$$

$$\lambda^{SPA} = -\frac{\sqrt{2}\lambda_1^{SP}}{F_A}\,, \quad \text{(D.79)}$$

$$\tilde{\mathcal{F}}^a_{SP}(q^2) = \frac{\sqrt{2}M_A^2}{M_A^2-q^2}\lambda_1^{SP}\,. \quad \text{(D.80)}$$

Axial Form Factor to $\pi\gamma$ (Figure D.7)

$$\langle\gamma(p_\gamma,\varepsilon)\pi^-(p_\pi)|\bar{d}\gamma^\mu\gamma_5 u|0\rangle = i\sqrt{2}eF\left(\varepsilon^{*\mu}-2q\varepsilon^*\frac{q^\mu}{q^2}\right)+\frac{i\sqrt{2}e}{F}(q\varepsilon^*\,p_\gamma^\mu - qp_\gamma\,\varepsilon^{*\mu})\mathcal{F}^a_{\pi\gamma}(q^2), \quad \text{(D.81)}$$

$$\mathcal{F}^a_{\pi\gamma}(q^2) = \frac{F_A^2}{M_A^2-q^2}+\frac{2F_VG_V-F_V^2}{M_V^2}+\frac{F_AF_V}{M_V^2}\frac{q^2}{M_A^2-q^2}\left(2\lambda_2^{VA}-\lambda_4^{VA}-2\lambda_5^{VA}\right), \quad \text{(D.82)}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{\pi\gamma} = \frac{e^2}{F^2}\frac{q^2}{48\pi}|\mathcal{F}^a_{\pi\gamma}|^2 - \frac{e^2}{12\pi}\mathrm{Re}\{\mathcal{F}^a_{\pi\gamma}\} + \frac{e^2\,F^2}{12\pi q^2}\,, \quad \text{(D.83)}$$

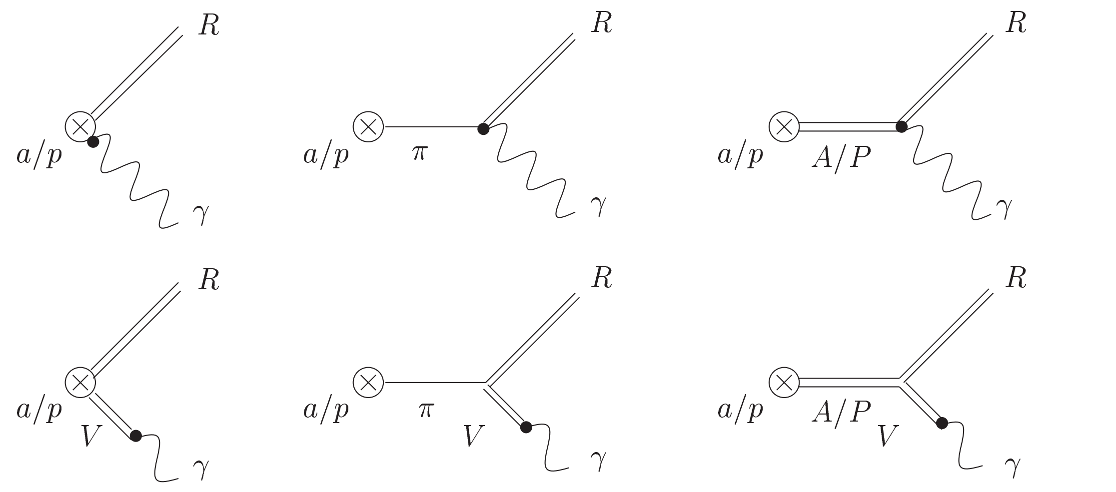


Figure D.8: Tree-level contributions to the axial/pseudoscalar form factor to one resonance field and one photon.

$$2\lambda_2^{VA} - \lambda_4^{VA} - 2\lambda_5^{VA} \;=\; -\frac{F_V}{F_A} + \frac{2G_V}{F_A}\,, \qquad \text{[cf D.54]} \tag{D.84}$$

$$\tilde{\mathcal{F}}^{a}_{\pi\gamma}(q^2) \;=\; \frac{1}{M_A^2 - q^2}\left[F_A^2 + \frac{M_A^2}{M_V^2}\left(2F_V G_V - F_V^2\right)\right]. \tag{D.85}$$

Axial Form Factor to Aγ (Figure D.8)

$$\begin{aligned}
\langle\, \gamma(p_\gamma,\varepsilon_\gamma) A^-(p_A,\varepsilon_A)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;=\; \frac{e}{\sqrt{2}M_A}\frac{1}{q^2}\times \\
\times\Big\{ 2/M_V^2\left(qp_A p_\gamma^\mu - qp_\gamma p_A^\mu\right)\Big[p_A p_\gamma \varepsilon_A^*\varepsilon_\gamma^* - q\varepsilon_A^* q\varepsilon_\gamma^*\Big]\mathcal{F}^a_{A\gamma}(q^2) \\
+\, 2M_A^2/M_V^2\Big[\left(qp_A p_\gamma^\mu - qp_\gamma p_A^\mu\right)\varepsilon_A^*\varepsilon_\gamma^* + \left(p_\gamma^\mu + p_A^\mu\right) q\varepsilon_A^* q\varepsilon_\gamma^* - q^2 q\varepsilon_A^*\varepsilon_\gamma^{*\mu}\Big]\mathcal{G}^a_{A\gamma}(q^2) \\
+\, 2M_A^2\, F_A\Big[\varepsilon_A^*\varepsilon_\gamma^*\left(p_\gamma^\mu + p_A^\mu\right) + \frac{2}{M_A^2 - q^2}\left(\left(p_\gamma^\mu + p_A^\mu\right) q\varepsilon_A^* q\varepsilon_\gamma^* - q^2 q\varepsilon_\gamma^*\varepsilon_A^{*\,\mu}\right)\Big]\Big\},
\end{aligned} \tag{D.86}$$

$$\begin{aligned}
\mathcal{F}^a_{A\gamma}(q^2) = 2F_V\left(\lambda_4^{VA} + 2\lambda_5^{VA} + 2\lambda_6^{VA}\right) + \frac{4F_A}{M_A^2 - q^2}\Big\{ M_V^2\lambda_7^{AA} - \frac{F_V}{\sqrt{2}}\Big[-\lambda_0^{VAA} \\
+\, qp_A\left(-2\lambda_2^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA}\right)\Big]\Big\}, \\
\mathcal{G}^a_{A\gamma}(q^2) = 2F_V\left(\lambda_2^{VA} + \lambda_6^{VA}\right) + \frac{F_A}{M_A^2 - q^2}\Big\{ -M_V^2 + 2M_V^2\lambda_7^{AA} + \sqrt{2}F_V\Big[\lambda_0^{VAA} + 2q^2\lambda_{11}^{VAA} \\
+\, qp_A\left(2\lambda_2^{VAA} + 2\lambda_{13}^{VAA}\right) + qp_\gamma\left(\lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} + \lambda_{14}^{VAA}\right)\Big]\Big\},
\end{aligned} \tag{D.87}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{VA} \propto \Big[\mathcal{O}(q^4)\,|\mathcal{F}^a_{A\gamma}|^2 + \mathcal{O}(q^2)\,|\mathcal{G}^a_{A\gamma}|^2 + \mathcal{O}(q^2)\,\mathrm{Re}\{\mathcal{F}^a_{A\gamma}\mathcal{G}^{a\;*}_{A\gamma}\} + \mathcal{O}(q^{-4})\Big], \tag{D.88}$$

$$\begin{aligned} &-2\lambda_2^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA} \\ &\quad = \frac{1}{\sqrt{2}F_A}\Big\{-2\lambda_4^{VA} - 4\lambda_5^{VA} - 4\lambda_6^{VA}\Big\}\,, \text{ [cf D.64]} \end{aligned}$$

$$\begin{aligned} &-2\lambda_2^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} - \lambda_{12}^{VAA} - 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA} - \frac{2}{M_A^2}\lambda_0^{VAA} \\ &\quad = \frac{\sqrt{2}}{F_A}\Big\{\lambda_4^{VA} + 2\lambda_5^{VA} + 2\lambda_6^{VA}\Big\} + \frac{2\sqrt{2}M_V^2}{F_V M_A^2}\lambda_7^{AA}\,, \text{ [cf D.24, D.64]} \end{aligned}$$

$$\begin{aligned} &2\lambda_2^{VAA} + \lambda_6^{VAA} - \lambda_7^{VAA} + \lambda_9^{VAA} + 2\lambda_{10}^{VAA} \\ &\quad + 4\lambda_{11}^{VAA} - \lambda_{12}^{VAA} + 2\lambda_{13}^{VAA} + \lambda_{14}^{VAA} = \frac{2\sqrt{2}}{F_A}\Big\{\lambda_2^{VA} + \lambda_6^{VA}\Big\}\,, \text{ [cf D.64]} \end{aligned} \tag{D.89}$$

$$\tilde{\mathcal{F}}^a_{A\gamma}(q^2) = 0, \quad \tilde{\mathcal{G}}^a_{A\gamma}(q^2) = \frac{F_A}{M_A^2 - q^2}\Big\{-M_V^2 + \frac{M_A^2 F_V}{F_A}\left(2\lambda_2^{VA} - \lambda_4^{VA} - 2\lambda_5^{VA}\right)\Big\}, \tag{D.90}$$

Axial Form Factor to Pγ (Figure D.8)

$$\langle\,\gamma(p_\gamma,\varepsilon)P^-(p_P)|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;=\; i\,\sqrt{2}\,e(q\varepsilon^*\,p_\gamma^\mu \;-\; qp_\gamma\,\varepsilon^{*\mu})\,\mathcal{F}^a_{P\gamma}(q^2)\,, \tag{D.91}$$

$$\begin{aligned} \mathcal{F}^a_{P\gamma}(q^2) \;=\; &-\frac{4F_A\lambda_1^{PA}}{M_A^2 - q^2} + \frac{2F_V\lambda_1^{PV}}{M_V^2} + \frac{4F_V\lambda_2^{PV}}{M_V^2} + \frac{\sqrt{2}F_A F_V}{(M_A^2 - q^2)M_V^2}\times \\ &\times\Big[-2\lambda_0^{PVA} - \frac{q^2 - M_P^2}{2}(\lambda_2^{PVA} + 2\lambda_3^{PVA}) + q^2(2\lambda_4^{PVA} + \lambda_5^{PVA})\Big]\,, \end{aligned} \tag{D.92}$$

$$\mathrm{Im}\Pi_{AA}(q^2)|_{P\gamma} = \theta(q^2 - M_P^2)\,e^2\,\frac{\left(1 - M_P^2/q^2\right)^3}{48\pi}\,q^2\,|\mathcal{F}^a_{P\gamma}|^2\,, \tag{D.93}$$

$$\lambda_2^{PVA} + 2\lambda_3^{PVA} - 4\lambda_4^{PVA} - 2\lambda_5^{PVA} = -\frac{2\sqrt{2}}{F_A}\left(\lambda_1^{PV} + 2\lambda_2^{PV}\right), \quad \text{[cf D.69]} \tag{D.94}$$

$$\begin{aligned} \tilde{\mathcal{F}}^a_{P\gamma}(q^2) \;=\; &-\frac{\sqrt{2}F_A F_V}{(M_A^2 - q^2)M_V^2}\Big\{-\frac{\sqrt{2}M_A^2}{F_A}\left(\lambda_1^{PV} + 2\lambda_2^{PV}\right) + \frac{2\sqrt{2}M_V^2}{F_V}\lambda_1^{PA} \\ &+2\lambda_0^{PVA} - \frac{M_P^2}{2}\left(\lambda_2^{PVA} + 2\lambda_3^{PVA}\right)\Big\}. \end{aligned} \tag{D.95}$$

D.3 Scalar Form Factors

Scalar Form Factor to $\pi\eta$ (Figure D.1)

$$\langle\, \eta(p_\eta)\pi^-(p_\pi)|\bar{d}u|0\,\rangle = \mathcal{F}^s_{\pi\eta}(q^2)\,, \tag{D.96}$$

$$\mathcal{F}^s_{\pi\eta}(q^2) = \sqrt{2}\,B_0\left(1+4\,\frac{c_m c_d}{F^2}\,\frac{q^2}{M_S^2-q^2}\right), \tag{D.97}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{\pi\eta} = \theta(q^2)\frac{1}{16\pi}|\mathcal{F}^s_{\pi\eta}(q^2)|^2\,, \tag{D.98}$$

$$4\,c_d\,c_m = F^2\,, \tag{D.99}$$

$$\tilde{\mathcal{F}}^s_{\pi\eta}(q^2) = \sqrt{2}B_0\frac{M_S^2}{M_S^2-q^2}\,. \tag{D.100}$$

Scalar Form Factor to Aπ (Figure D.2)

$$\langle\, A^0_{I=0}(p_A,\varepsilon)\pi^-(p_S)|\bar{d}u|0\,\rangle = \frac{i}{M_A}\,q\varepsilon^*\,\mathcal{F}^s_{A\pi}(q^2)\,, \tag{D.101}$$

$$\mathcal{F}^s_{A\pi}(q^2) = -\frac{8\,B_0\,c_m\,\lambda_1^{SA}}{F}\,\frac{M_A^2}{M_S^2-q^2}\,, \tag{D.102}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{A\pi} = \theta(q^2-M_A^2)\,\frac{(q^2-M_A^2)^3}{64\pi M_A^4 q^2}\,|\mathcal{F}^s_{A\pi}|^2\,, \tag{D.103}$$

$$\lambda_1^{SA} = 0\,, \tag{D.104}$$

$$\tilde{\mathcal{F}}^s_{A\pi}(q^2) = 0\,. \tag{D.105}$$

Scalar Form Factor to Pπ (Figure D.2)

$$\langle\, P^0_{I=0}(p_P)\pi^-(p_\pi)|\bar{d}u|0\,\rangle = \mathcal{F}^s_{P\pi}(q^2)\,, \tag{D.106}$$

$$\mathcal{F}^s_{P\pi}(q^2) = -\frac{4B_0\,d_m}{F} + \frac{4B_0\,c_m}{F}\frac{q^2-M_P^2}{M_S^2-q^2}\lambda_1^{SP}\,, \tag{D.107}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{P\pi} = \theta(q^2-M_P^2)\frac{1-M_P^2/q^2}{16\pi}|\mathcal{F}^s_{P\pi}(q^2)|^2\,, \tag{D.108}$$

$$\lambda_1^{SP} = -\frac{d_m}{c_m}\,, \tag{D.109}$$

$$\tilde{\mathcal{F}}^s_{P\pi}(q^2) = \frac{4B_0 d_m}{F}\,\frac{M_P^2-M_S^2}{M_S^2-q^2}\,. \tag{D.110}$$

Scalar Form Factor to RR (R=V,A) (Figure D.3)

$$\langle\, R^0_{I=0}(p_1,\varepsilon_1)R^-(p_2,\varepsilon_2)|\bar{d}u|0\,\rangle \,=\, \frac{1}{M_R^2}(q\varepsilon_1^*\, q\varepsilon_2^* - p_1p_2\,\varepsilon_1^*\varepsilon_2^*)\mathcal{F}^s_{RR}(q^2) \,+\, \varepsilon_1^*\,\varepsilon_2^*\,\mathcal{G}^s_{RR}(q^2)\,, \tag{D.111}$$

$$\begin{aligned}\mathcal{F}^s_{RR}(q^2) &= -8\sqrt{2}B_0\Big[\lambda_6^{RR} + \frac{c_m}{M_S^2-q^2}\big(\lambda_0^{SRR} - \frac{p_1p_2}{2}\lambda_2^{SRR} - p_1p_2\lambda_3^{SRR} \\ &\qquad - 2M_R^2\lambda_4^{SRR} - M_R^2\lambda_5^{SRR}\big)\Big]\,, \\ \mathcal{G}^s_{RR}(q^2) &= -8\sqrt{2}B_0\frac{c_m\lambda_1^{SRR}}{2}\frac{M_R^2}{M_S^2-q^2}\,,\end{aligned} \tag{D.112}$$

$$\begin{aligned}\mathrm{Im}\Pi_{SS}(q^2)|_{RR} &= \theta(q^2-4M_R^2)\frac{\sigma^2_{M_R}}{16\pi}\Bigg\{\left(3-\frac{2q^2}{M_R^2}+\frac{q^4}{2M_R^4}\right)|\mathcal{F}^s_{RR}|^2 \\ &+ \left(3-\frac{q^2}{M_R^2}+\frac{q^4}{4M_R^4}\right)|\mathcal{G}^s_{RR}|^2 + \left(6-\frac{3q^2}{M_R^2}\right)\mathrm{Re}\{\mathcal{F}^s_{RR}\mathcal{G}^{s\;*}_{RR}\}\Bigg\}\,,\end{aligned} \tag{D.113}$$

$$\begin{aligned}\lambda_2^{SRR} + 2\lambda_3^{SRR} &= -\frac{4\lambda_6^{RR}}{c_m}\,, \\ \frac{\lambda_0^{SRR}}{M_R^2} + \frac{\lambda_2^{SRR}}{2} + \lambda_3^{SRR} - 2\lambda_4^{SRR} - \lambda_5^{SRR} &= -\frac{\lambda_6^{RR}}{c_m}\frac{M_S^2}{M_R^2}\,, \\ \lambda_1^{SRR} &= 0\,,\end{aligned} \tag{D.114}$$

$$\tilde{\mathcal{F}}^s_{RR}(q^2) \,=\, \tilde{\mathcal{G}}^s_{RR}(q^2) \,=\, 0\,. \tag{D.115}$$

Scalar Form Factor to SS (Figure D.3)

$$\langle\, S^0_{I=0}(p_1)S^-(p_2)|\bar{d}u|0\,\rangle \;=\; \mathcal{F}^s_{SS}(q^2)\,, \tag{D.116}$$

$$\mathcal{F}^s_{SS}(q^2) = -4\sqrt{2}B_0\left[\lambda_3^{SS} + \frac{3\,c_m\,\lambda_0^{SSS}}{M_S^2-q^2} + \frac{c_m\lambda_1^{SSS}}{2}\frac{q^2+2M_S^2}{M_S^2-q^2}\right]\,, \tag{D.117}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{SS} = \theta(q^2-4M_S^2)\frac{\sigma_{M_S}}{16\pi}|\mathcal{F}^s_{SS}(q^2)|^2\,, \tag{D.118}$$

$$\lambda_1^{SSS} = \frac{2\,\lambda_3^{SS}}{c_m}\,, \tag{D.119}$$

$$\tilde{\mathcal{F}}^s_{SS}(q^2) = -\frac{4\sqrt{2}B_0}{M_S^2-q^2}\left[3M_S^2\lambda_3^{SS} + 3c_m\lambda_0^{SSS}\right]\,. \tag{D.120}$$

Scalar Form Factor to PP (Figure D.3)

$$\langle\, P^0_{I=0}(p_1) P^-(p_2) | \bar{d} u | 0 \,\rangle \;=\; \mathcal{F}^s_{PP}(q^2)\,, \tag{D.121}$$

$$\mathcal{F}^s_{PP}(q^2) = -4\sqrt{2} B_0 \left[\lambda_3^{PP} + \frac{c_m\, \lambda_0^{SPP}}{M_S^2 - q^2} + \frac{c_m \lambda_1^{SPP}}{2} \frac{-q^2 + 2M_P^2}{M_S^2 - q^2} \right], \tag{D.122}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{PP} = \theta(q^2 - 4M_P^2) \frac{\sigma_{M_P}}{16\pi} |\mathcal{F}^s_{PP}(q^2)|^2\,, \tag{D.123}$$

$$\lambda_1^{SPP} = -\frac{2\,\lambda_3^{PP}}{c_m}\,, \tag{D.124}$$

$$\tilde{\mathcal{F}}^s_{PP}(q^2) = -\frac{4\sqrt{2} B_0}{M_S^2 - q^2} \left[(M_S^2 - 2M_P^2) \lambda_3^{PP} + c_m \lambda_0^{SPP} \right]. \tag{D.125}$$

Scalar Form Factor to SV (Figure D.3)

$$\langle\, S^0_{I=1}(p_S) V^-(p_V, \varepsilon) | \bar{d} u | 0 \,\rangle \;=\; \frac{1}{M_V}\, q\varepsilon^*\, \mathcal{F}^s_{SV}(q^2)\,, \tag{D.126}$$

$$\mathcal{F}^s_{SV}(q^2) = -4\sqrt{2}\, B_0\, c_m\, \lambda^{VSS}\, \frac{M_V^2}{M_S^2 - q^2}\,, \tag{D.127}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{SV} = \theta(q^2 - (M_S + M_V)^2)\, \frac{\lambda^{3/2}\,(q^2, M_S^2, M_V^2)}{64\pi M_V^4 q^2}\, |\mathcal{F}^s_{SV}|^2\,, \tag{D.128}$$

$$\lambda^{VSS} = 0\,, \tag{D.129}$$

$$\tilde{\mathcal{F}}^s_{SV}(q^2) = 0\,. \tag{D.130}$$

Scalar Form Factor to PA (Figure D.3)

$$\langle\, P^0_{I=0}(p_P) A^-(p_A, \varepsilon) | \bar{d} u | 0 \,\rangle \;=\; \frac{i}{M_A}\, q\varepsilon^*\, \mathcal{F}^s_{PA}(q^2)\,, \tag{D.131}$$

$$\mathcal{F}^s_{PA}(q^2) = 4\sqrt{2}\, B_0\, c_m\, \lambda^{SPA}\, \frac{M_A^2}{M_S^2 - q^2}\,, \tag{D.132}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{PA} = \theta(q^2 - (M_P + M_A)^2)\, \frac{\lambda^{3/2}\,(q^2, M_P^2, M_A^2)}{64\pi M_A^4 q^2}\, |\mathcal{F}^s_{PA}|^2\,, \tag{D.133}$$

$$\lambda^{SPA} = 0\,, \tag{D.134}$$

$$\tilde{\mathcal{F}}^s_{PA}(q^2) = 0\,. \tag{D.135}$$

Scalar Form Factor to Vγ (Figure D.4)

$$\langle\, \gamma(p_\gamma,\varepsilon_\gamma) V^-(p_V,\varepsilon_V)|\bar{d}u|0\,\rangle \;=\; \frac{e}{3M_V}(q\varepsilon_V^*\, q\varepsilon_\gamma^* - p_V p_\gamma\, \varepsilon_V^*\varepsilon_\gamma^*)\mathcal{F}^s_{V\gamma}(q^2)\,, \tag{D.136}$$

$$\begin{aligned}\mathcal{F}^s_{V\gamma}(q^2) \;&=\; \frac{16B_0\, c_m}{M_S^2-q^2}\lambda_3^{SV} - \frac{8\sqrt{2}B_0\, F_V}{M_V^2}\lambda_6^{VV} - \frac{4\sqrt{2}B_0\, c_m\, F_V}{M_V^2(M_S^2-q^2)}\times \\ &\times\left[2\lambda_0^{SVV} - p_V p_\gamma(\lambda_2^{SVV}+2\lambda_3^{SVV}) - M_V^2(2\lambda_4^{SVV}+\lambda_5^{SVV})\right]\,,\end{aligned} \tag{D.137}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{V\gamma} \;=\; \theta(q^2-M_V^2)\frac{(1-M_V^2/q^2)^3}{288\pi\, M_V^2}e^2\, q^4|\mathcal{F}^s_{V\gamma}|^2\,, \tag{D.138}$$

$$\begin{aligned}\lambda_2^{SVV}+2\lambda_3^{SVV} \;&=\; -\frac{4\lambda_6^{VV}}{c_m}\,,\ [\text{cf D.114}]\\ \frac{4\lambda_0^{SVV}}{M_V^2}+\lambda_2^{SVV}+2\lambda_3^{SVV}-4\lambda_4^{SVV}-2\lambda_5^{SVV} \;&=\; -\frac{4\,\lambda_6^{VV}}{c_m}\frac{M_S^2}{M_V^2}+\frac{4\sqrt{2}\lambda_3^{SV}}{F_V},\ [\text{cf D.34, D.114}]\end{aligned} \tag{D.139}$$

$$\tilde{\mathcal{F}}^s_{V\gamma}(q^2) \;=\; 0\,. \tag{D.140}$$

Scalar Form Factor to Sγ (Figure D.4)

$$\langle\, \gamma\,(p_\gamma,\varepsilon) S^-(p_S)|\bar{d}u|0\,\rangle \;=\; e\, q\varepsilon^*\, \mathcal{F}^s_{S\gamma}(q^2)\,, \tag{D.141}$$

$$\mathcal{F}^s_{S\gamma}(q^2) \;=\; \frac{8\,B_0\, c_m}{M_S^2-q^2}\,, \tag{D.142}$$

$$\mathrm{Im}\Pi_{SS}(q^2)|_{S\gamma} \;=\; 0\,, \tag{D.143}$$

D.4 Pseudoscalar Form Factors

Pseudoscalar Form Factor to Vπ (Figure D.5)

$$\langle\, \pi^0(p_\pi) V^-(p_V,\varepsilon)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \frac{1}{M_V}\, q\varepsilon^*\, \mathcal{F}^p_{V\pi}(q^2)\,, \tag{D.144}$$

$$\mathcal{F}^p_{V\pi}(q^2) \;=\; -\frac{2\,B_0}{F}\left(\sqrt{2}G_V\frac{M_V^2}{q^2}+4d_m\,\lambda_1^{PV}\frac{M_V^2}{M_P^2-q^2}\right)\,, \tag{D.145}$$

$$\mathrm{Im}\Pi_{PP}(q^2)|_{V\pi} \;=\; \theta(q^2-M_V^2)\,\frac{(q^2-M_V^2)^3}{64\pi M_V^4 q^2}\,|\mathcal{F}^p_{V\pi}|^2\,, \tag{D.146}$$

$$-\sqrt{2}G_V+4d_m\,\lambda_1^{PV} \;=\; 0\,, \tag{D.147}$$

$$\tilde{\mathcal{F}}^p_{V\pi}(q^2) \;=\; -\frac{2\sqrt{2}B_0\, G_V}{F}\frac{M_V^2 M_P^2}{(M_P^2-q^2)q^2}\,. \tag{D.148}$$

Pseudoscalar Form Factor to Sπ (Figure D.5)

$$\langle\, S^0_{I=0}(p_S)\pi^-(p_\pi)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \mathcal{F}^p_{S\pi}(q^2)\,, \tag{D.149}$$

$$\mathcal{F}^p_{S\pi}(q^2) = \frac{4B_0\,c_m}{F} - \frac{2B_0\,c_d}{F}\frac{q^2-M_S^2}{q^2} + \frac{4B_0\,d_m}{F}\frac{M_S^2-q^2}{M_P^2-q^2}\lambda_1^{SP}\,, \tag{D.150}$$

$$\mathrm{Im}\Pi_{PP}(q^2)|_{S\pi} = \theta(q^2-M_S^2)\frac{1-M_S^2/q^2}{16\pi}|\mathcal{F}^p_{S\pi}(q^2)|^2\,, \tag{D.151}$$

$$\lambda_1^{SP} = \frac{-2c_m+c_d}{2d_m}\,, \tag{D.152}$$

$$\tilde{\mathcal{F}}^p_{S\pi}(q^2) = \frac{4B_0c_m}{F}\frac{M_P^2-M_S^2}{M_P^2-q^2} + \frac{2B_0c_d}{F}\frac{M_P^2}{M_P^2-q^2}\left(\frac{M_S^2}{q^2}-1\right)\,. \tag{D.153}$$

Pseudoscalar Form Factor to VA (Figure D.6)

$$\langle\, V^0_{I=1}(p_V,\varepsilon_V)A^-(p_A,\varepsilon_A)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \frac{i}{M_VM_A}(q\varepsilon_V^*\,q\varepsilon_A^* - p_Vp_A\,\varepsilon_V^*\varepsilon_A^*)\mathcal{F}^p_{VA}(q^2) + i\,\varepsilon_V^*\,\varepsilon_A^*\,\mathcal{G}^p_{VA}(q^2)\,, \tag{D.154}$$

$$\begin{aligned}\mathcal{F}^p_{VA}(q^2) =& -4\sqrt{2}B_0\Big[-2\lambda_1^{VA} + \frac{1}{4\,q^2}\big(-2(q^2+M_V^2+M_A^2)\lambda_2^{VA} + 2M_V^2\lambda_3^{VA} \\ &- (q^2+M_V^2-M_A^2)(\lambda_4^{VA}+2\lambda_5^{VA})\big) + \frac{d_m}{M_P^2-q^2}\big(2\lambda_0^{PVA} \\ &- p_Vp_A(\lambda_2^{PVA}+2\lambda_3^{PVA}) - M_A^2(2\lambda_4^{PVA}+\lambda_5^{PVA}) - M_V^2\lambda_6^{PVA}\big)\Big]\,,\\ \mathcal{G}^p_{VA}(q^2) =& -4\sqrt{2}B_0M_AM_V\left[\frac{1}{2q^2}\big(2\lambda_2^{VA}+\lambda_3^{VA}\big) + \frac{d_m}{M_P^2-q^2}\lambda_1^{PVA}\right]\,,\end{aligned} \tag{D.155}$$

$$\begin{aligned}\mathrm{Im}\Pi_{PP}(q^2)|_{VA} =& \,\theta(q^2-(M_V+M_A)^2)\frac{\lambda^{1/2}(q^2,M_V^2,M_A^2)}{16\pi q^2}\Bigg\{-\frac{6p_Ap_V}{M_AM_V}\mathrm{Re}\{\mathcal{F}^s_{RR}\mathcal{G}^s_{RR}{}^*\} \\ &+ \frac{4M_A^2M_V^2-q^4+(q^2-M_V^2)^2+(q^2-M_A^2)^2}{2M_A^2M_V^2}|\mathcal{F}^s_{RR}|^2 \\ &+ \frac{10M_A^2M_V^2-q^4+(q^2-M_V^2)^2+(q^2-M_A^2)^2}{4M_A^2M_V^2}|\mathcal{G}^s_{RR}|^2\Bigg\}\,,\end{aligned} \tag{D.156}$$

$$\begin{aligned}\lambda_2^{PVA}+2\lambda_3^{PVA} \;=&\; \frac{1}{2d_m}\left(8\lambda_1^{VA}+2\lambda_2^{VA}+\lambda_4^{VA}+2\lambda_5^{VA}\right)\,,\\ 4\lambda_0^{PVA}+(M_V^2+M_A^2)(\lambda_2^{PVA}+2\lambda_3^{PVA}) &- M_A^2(4\lambda_4^{PVA}+2\lambda_5^{PVA}) - 2M_V^2\lambda_6^{PVA} \;=\\ \frac{1}{d_m}\Bigg(4M_P^2\lambda_1^{VA}+(M_P^2-M_V^2-M_A^2)\lambda_2^{VA}&+M_V^2\lambda_3^{VA}+\frac{1}{2}(M_P^2-M_V^2+M_A^2)(\lambda_4^{VA}+2\lambda_5^{VA})\Bigg),\\ 2\lambda_1^{PVA} \;=&\; \frac{1}{d_m}\left(2\lambda_2^{VA}+\lambda_3^{VA}\right)\,,\end{aligned} \tag{D.157}$$

$$\tilde{\mathcal{F}}^{p}_{VA}(q^2) = \frac{\sqrt{2}B_0\, M_P^2}{(M_P^2-q^2)q^2}\left[2(M_V^2+M_A^2)\lambda_2^{VA}-2M_V^2\lambda_3^{VA}+(M_V^2-M_A^2)(\lambda_4^{VA}+2\lambda_5^{VA})\right],$$
$$\tilde{\mathcal{G}}^{p}_{VA}(q^2) = -2\sqrt{2}B_0\frac{M_A M_V M_P^2}{(M_P^2-q^2)q^2}\left(2\lambda_2^{VA}+\lambda_3^{VA}\right)\,. \tag{D.158}$$

Pseudoscalar Form Factor to PV (Figure D.6)

$$\langle\, P^0_{I=1}(p_P)V^-(p_V,\varepsilon)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \frac{1}{M_V}\, q\varepsilon^*\, \mathcal{F}^p_{PV}(q^2)\,, \tag{D.159}$$

$$\mathcal{F}^p_{PV}(q^2) = 2\sqrt{2}\,B_0\left(-\frac{M_V^2}{q^2}\lambda_1^{PV}-\frac{2\,d_m\,M_V^2}{M_P^2-q^2}\lambda^{VPP}\right), \tag{D.160}$$
$$\mathrm{Im}\Pi_{PP}(q^2)|_{PV} = \theta(q^2-(M_P+M_V)^2)\,\frac{\lambda^{3/2}\,(q^2,M_P^2,M_V^2)}{64\pi M_V^4 q^2}\,|\mathcal{F}^p_{PV}|^2\,, \tag{D.161}$$
$$\lambda^{VPP} = \frac{1}{2d_m}\lambda_1^{PV}\,, \tag{D.162}$$
$$\tilde{\mathcal{F}}^p_{V\pi}(q^2) = -2\sqrt{2}B_0\frac{M_V^2 M_P^2}{(M_P^2-q^2)q^2}\lambda_1^{PV}\,. \tag{D.163}$$

Pseudoscalar Form Factor to SA (Figure D.6)

$$\langle\, S^0_{I=0}(p_S)A^-(p_A,\varepsilon)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \frac{i}{M_A}\, q\varepsilon^*\, \mathcal{F}^p_{SA}(q^2)\,, \tag{D.164}$$

$$\mathcal{F}^p_{SA}(q^2) = 2\sqrt{2}\,B_0\left(\frac{M_A^2}{q^2}\lambda_1^{SA}-\frac{2\,d_m\,M_A^2}{M_P^2-q^2}\lambda^{SPA}\right), \tag{D.165}$$
$$\mathrm{Im}\Pi_{PP}(q^2)|_{SA} = \theta(q^2-(M_S+M_A)^2)\,\frac{\lambda^{3/2}\,(q^2,M_S^2,M_A^2)}{64\pi M_A^4 q^2}\,|\mathcal{F}^p_{SA}|^2\,, \tag{D.166}$$
$$\lambda^{SPA} = -\frac{1}{2d_m}\lambda_1^{SA}\,, \tag{D.167}$$
$$\tilde{\mathcal{F}}^p_{SA}(q^2) = 2\sqrt{2}B_0\frac{M_A^2 M_P^2}{(M_P^2-q^2)q^2}\lambda_1^{SA}\,. \tag{D.168}$$

Pseudoscalar Form Factor to SP (Figure D.6)

$$\langle\, S^0_{I=0}(p_S)P^-(p_P)|i\bar{d}\gamma_5 u|0\,\rangle \;=\; \mathcal{F}^p_{SP}(q^2)\,, \tag{D.169}$$

$$\mathcal{F}^p_{SP}(q^2) = -4\sqrt{2}B_0\Big[\lambda_2^{SP}-\frac{q^2+M_S^2-M_P^2}{4\,q^2}\lambda_1^{SP}$$
$$+\frac{d_m}{2\,(M_P^2-q^2)}\left(2\lambda_0^{SPP}+(q^2+M_P^2-M_S^2)\lambda_1^{SPP}\right)\Big]\,, \tag{D.170}$$

$$\mathrm{Im}\Pi_{_{PP}}(q^2)|_{SP} = \theta(q^2-(M_S+M_P)^2)\frac{\lambda^{1/2}(q^2,M_S^2,M_P^2)}{16\pi q^2}|\mathcal{F}^p_{SP}(q^2)|^2\,, \qquad \text{(D.171)}$$

$$\lambda_1^{SPP} = -\frac{1}{2d_m}\lambda_1^{SP}+\frac{2}{d_m}\lambda_2^{SP}\,, \qquad \text{(D.172)}$$

$$\tilde{\mathcal{F}}^p_{SP}(q^2) = -\frac{4\sqrt{2}B_0}{M_P^2-q^2}\Bigg[\left(-\frac{M_S^2M_P^2}{4q^2}+\frac{M_P^4}{4q^2}-\frac{3M_P^2}{4}+\frac{M_S^2}{2}\right)\lambda_1^{SP}+ + \left(2M_P^2-M_S^2\right)\lambda_2^{SP}+d_m\lambda_0^{SPP}\Bigg]\,. \qquad \text{(D.173)}$$

Pseudoscalar Form Factor to $\pi\gamma$ (Figure D.7)

$$\langle\,\gamma\,(p_\gamma,\varepsilon)\pi^-(p_\pi)|i\bar{d}\gamma_5 u|0\,\rangle = e\,q\varepsilon^*\,\mathcal{F}^p_{\pi\gamma}(q^2)\,, \qquad \text{(D.174)}$$

$$\mathcal{F}^p_{\pi\gamma}(q^2) = \frac{2\sqrt{2}\,B_0\,F}{q^2}\,, \qquad \text{(D.175)}$$

$$\mathrm{Im}\Pi_{_{PP}}(q^2)|_{\pi\gamma} = 0\,. \qquad \text{(D.176)}$$

Pseudoscalar Form Factor to Aγ (Figure D.8)

$$\langle\,\gamma(p_\gamma,\varepsilon_\gamma)A^-(p_A,\varepsilon_A)|i\bar{d}\gamma_5 u|0\,\rangle = \frac{i\,e}{M_A}(q\varepsilon_\gamma^*\,q\varepsilon_A^*-p_\gamma p_A\,\varepsilon_\gamma^*\varepsilon_A^*)\mathcal{F}^p_{A\gamma}(q^2)\,, \qquad \text{(D.177)}$$

$$\mathcal{F}^p_{A\gamma}(q^2) = \frac{\sqrt{2}F_A\,B_0}{q^2}-\frac{16B_0\,d_m}{M_P^2-q^2}\lambda_1^{PA}-\frac{4\sqrt{2}B_0\,F_V}{M_V^2}\Bigg\{-2\lambda_1^{VA}+\frac{1}{4\,q^2}\Big[-2(q^2+M_A^2)\lambda_2^{VA} - (q^2-M_A^2)(\lambda_4^{VA}+2\lambda_5^{VA})\Big]+\frac{d_m}{M_P^2-q^2}\Big[2\lambda_0^{PVA}-p_\gamma p_A(\lambda_2^{PVA}+2\lambda_3^{PVA}) - M_A^2(2\lambda_4^{PVA}+\lambda_5^{PVA})\Big]\Bigg\}\,, \qquad \text{(D.178)}$$

$$\mathrm{Im}\Pi_{_{PP}}(q^2)|_{A\gamma} = \theta(q^2-M_A^2)\frac{(1-M_A^2/q^2)^3}{32\pi\,M_A^2}e^2\,q^4|\mathcal{F}^p_{A\gamma}|^2\,, \qquad \text{(D.179)}$$

$$\lambda_2^{PVA}+2\lambda_3^{PVA} = \frac{1}{2d_m}\left(8\lambda_1^{VA}+2\lambda_2^{VA}+\lambda_4^{VA}+2\lambda_5^{VA}\right)\,, \qquad \text{[cf D.157]}$$

$$4\lambda_0^{PVA}+M_A^2(\lambda_2^{PVA}+2\lambda_3^{PVA}-4\lambda_4^{PVA}-2\lambda_5^{PVA}) = \frac{1}{d_m}\Big(4M_P^2\lambda_1^{VA}+(M_P^2-M_A^2)\lambda_2^{VA} +\frac{1}{2}(M_P^2+M_A^2)(\lambda_4^{VA}+2\lambda_5^{VA})\Big)-\frac{M_V^2}{2\sqrt{2}F_V d_m}\left(\sqrt{2}F_A+16d_m\lambda_1^{PA}\right)\,,$$

$$\text{[cf D.9, D.39, D.157]} \qquad \text{(D.180)}$$

$$\tilde{\mathcal{F}}^p_{A\gamma}(q^2) = \frac{\sqrt{2}B_0\,M_P^2}{(M_P^2-q^2)q^2}\left[F_A-\frac{F_V}{M_V^2}\left(-2M_A^2\lambda_2^{VA}+M_A^2(\lambda_4^{VA}+2\lambda_5^{VA})\right)\right]\,. \qquad \text{(D.181)}$$

Pseudoscalar Form Factor to Pγ (Figure D.8)

$$\langle\, \gamma\,(p_\gamma, \varepsilon^*) P^-(p_P)| i\bar{d}\gamma_5 u|0\,\rangle \;=\; e\, q\varepsilon\, \mathcal{F}^p_{P\gamma}(q^2)\,, \tag{D.182}$$

$$\mathcal{F}^p_{P\gamma}(q^2) \;=\; \frac{8\,B_0\,d_m}{M_P^2 - q^2}\,, \tag{D.183}$$

$$\mathrm{Im}\Pi_{_{PP}}(q^2)|_{P\gamma} \;=\; 0\,. \tag{D.184}$$

D.5 Form Factors with a Photon

As pointed out in Section 4.3, no new constraints have been obtained from the analysis of form factors with a photon:

1. $\mathcal{F}^v_{V\gamma}$ and $\mathcal{G}^v_{V\gamma}$ (Eq. (D.44)): the 1st constraint is got by adding the 1st and the 3rd constraint of Eq. (D.19) ($\mathcal{F}^v_{VV}$, $\mathcal{G}^v_{VV}$ and $\mathcal{H}^v_{VV}$); the 2nd one subtracting the 1st constraint to the 3rd one of Eq. (D.19); and the 3rd one subtracting the 3rd constraint to the 4th one of Eq. (D.19).

2. $\mathcal{F}^v_{S\gamma}$ (Eq. (D.49)): same constraint than the 1st one of Eq. (D.34) ($\mathcal{F}^v_{SV}$ and $\mathcal{G}^v_{SV}$).

3. $\mathcal{F}^a_{\pi\gamma}$ (Eq. (D.84)): same constraint than the 1st one of Eq. (D.54) ($\mathcal{F}^a_{V\pi}$ and $\mathcal{G}^a_{V\pi}$).

4. $\mathcal{F}^a_{A\gamma}$ and $\mathcal{G}^a_{A\gamma}$ (Eq. (D.89)): the 1st constraint is the same than the 1st one of Eq. (D.64) ($\mathcal{F}^a_{VA}$, $\mathcal{G}^a_{VA}$, $\mathcal{H}^a_{VA}$ and $\mathcal{I}^a_{VA}$); the 2nd one is got subtracting two times the 4th one of Eq. (D.24) ($\mathcal{F}^v_{AA}$, $\mathcal{G}^v_{AA}$ and $\mathcal{H}^v_{AA}$) to the 1st one of Eq. (D.64); and the 3rd one is the same than the 4th one of Eq. (D.64).

5. $\mathcal{F}^a_{P\gamma}$ (Eq. (D.94)): same constraint than the 1st one of Eq. (D.69) ($\mathcal{F}^a_{PV}$ and $\mathcal{G}^a_{PV}$).

6. $\mathcal{F}^s_{V\gamma}$ (Eq. (D.139)): the 1st constraint is the same than the 1st one of Eq. (D.114) ($\mathcal{F}^s_{VV}$ and $\mathcal{G}^s_{VV}$); and the 2nd one can be obtained subtracting the 1st one of Eq. (D.34) ($\mathcal{F}^v_{SV}$ and $\mathcal{G}^v_{SV}$) to four times the 2nd one of Eq. (D.114).

7. $\mathcal{F}^s_{S\gamma}$: no constraints.

8. $\mathcal{F}^p_{\pi\gamma}$: no constraints.

9. $\mathcal{F}^p_{A\gamma}$ (Eq. (D.180)): the 1st one is the same than the 1st one of Eq. (D.157) ($\mathcal{F}^p_{VA}$ and $\mathcal{G}^p_{VA}$); and the 2nd one is got summing $-M_V^2/(2d_m)$ times the 1st one of Eq. (D.9) ($\mathcal{F}^v_{A\pi}$ and $\mathcal{G}^v_{A\pi}$), $-M_V^2$ times the 1st one of Eq. (D.39) ($\mathcal{F}^v_{PA}$ and $\mathcal{G}^v_{PA}$) and the 2nd one of Eq. (D.157).

10. $\mathcal{F}^p_{P\gamma}$: no constraints.

Appendix E

Dispersive Relations

In the purely perturbative calculation (without Dyson resummations) and under the Single Resonance Approximation, the two-point function at next-to-leading order in the $1/N_C$ expansion reads as:

$$\Pi(t) \quad = \quad \frac{D(t)}{\left(M_R^2 - t\right)^2} \,, \tag{E.1}$$

where M_R is the mass of the corresponding resonance in the s–channel, and $D(t)$ is an analytical function except for the unitarity logarithmic branch (without poles).

In order to recover the correlator, the complex integration in the circuit of Figure E.1 is performed:

$$\Pi(q^2) \quad = \quad \frac{1}{2\pi i} \oint \; \mathrm{d}t \, \frac{\Pi(t)}{t - q^2} \,. \tag{E.2}$$

If it is assumed that $|\Pi(t)| \to 0$ when $|t| \to \infty$, the contribution from the external circle of the circuit is zero and it is found that:

$$\Pi(q^2) \quad = \quad \frac{\overline{D}(q^2)}{(M_R^2 - q^2)^2} - \frac{\mathrm{Re}D'(M_R^2)}{M_R^2 - q^2} + \frac{\mathrm{Re}D(M_R^2)}{\left(M_R^2 - q^2\right)^2} \,, \tag{E.3}$$

with $D'(t) \equiv \frac{d}{dt} D(t)$ and being

$$\frac{\overline{D}(q^2)}{(M_R^2 - q^2)^2} = \lim_{\epsilon \to 0} \left[\int_0^{M_R^2 - \epsilon} \mathrm{d}t \, \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t - q^2} + \int_{M_R^2 + \epsilon}^{\infty} \mathrm{d}t \, \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t - q^2} - \frac{2}{\pi \epsilon} \lim_{t \to M_R^2} \left(\frac{\mathrm{Im}D(t)}{t - q^2} \right) \right] \,, \tag{E.4}$$

which obeys $\overline{D}(M_R^2) = 0$ and $\overline{D}\,'(M_R^2) = 0$. Notice that in order to recover $\overline{D}(q^2)$ it is not necessary to know $\mathrm{Im}D(t)$ at $t = M_R^2$, but just the amplitude in the region $[0, +\infty) - \{M_R^2\}$, where $\Pi(q^2)$ is well defined.

It is important to remark that, in order to recover the proper asymptotic behaviour of $\Pi(t)$, one must have a spectral function that vanishes at high energies, so the form factors must follow the proper asymptotic behaviours.

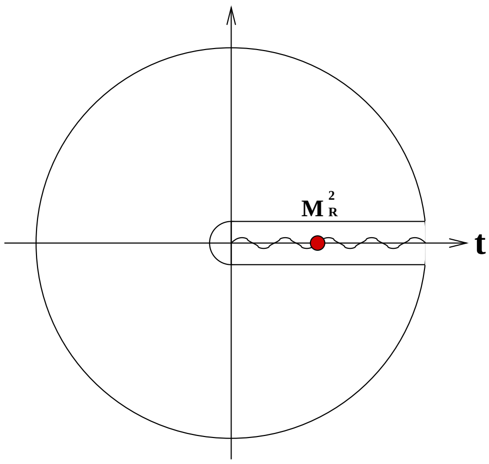


Figure E.1: Integration circuit.

From Eq. (E.3), one notices that, as soon as the value of the real part of $D(t)$ and its first derivative are fixed at M_R^2, the whole correlator becomes fixed. This corresponds to providing a renormalization prescription for the corresponding coupling and resonance mass.

The fact that the spectral function vanishes at infinite momentum ensures that there are no terms of the form $\Pi(t) \sim t^m \ln(-t)$, with $m \geq 0$. Furthermore, the polynomial terms $\Pi(t) \sim t^m$ with $m \geq 0$ must be also identically zero in order to keep $\Pi(t) \to 0$ at $|t| \to \infty$. Hence, the expression in Eq. (E.3), is the general expression for the correlator within the Single Resonance Approximation. The inclusion of higher resonances can be performed in a straightforward way.

This means that although the presence of $\mathcal{O}(p^4)$ χPT operators with NLO couplings in $1/N_C$, $\widetilde{L}_i$, is not forbidden by the symmetry, the QCD short-distance behaviour imposes that, in our realization, they do not get renormalized, as suggested in Ref. [37], and they do not contribute to the observable at the end of the day (the polynomial terms $\Pi^(t) \sim t^m$ are identically zero). This lack of running in the $\widetilde{L}_i$ related to the analysed currents arisen in the one-loop analysis of the RχT generating functional with only pseudo-Goldstones, scalar and pseudoscalar resonances after imposing the high energy constraints [54].

E.1 Diagrammatic Calculation

For sake of simplicity we will refer now just to the scalar correlator although the extension to other channels is straight-forward. At tree-level order it is found that

$$\Pi_{SS}(q^2) \;=\; \frac{16\, B_0^2\, c_m^2}{M_S^2 - q^2}\,. \qquad \text{(E.5)}$$

The resonance parameters c_m and M_S get renormalized at next-to-leading order $1/N_C$ ($c_m = c_m^r + \delta c_m$ and $M_S^2 = M_S^{r\,2} + \delta M_S^2$) in order to cancel the ultraviolet divergences from the one-loop diagrams:

$$\Pi_{SS}(q^2)|_{\rm tree} = \frac{16\, B_0^2\, c_m^{r\,2}}{M_S^{r\,2} - q^2} + \frac{32\, B_0^2\, c_m^r\, \delta c_m}{M_S^{r\,2} - q^2} - \frac{16\, B_0^2\, c_m^{r\,2}\, \delta M_S^2}{(M_S^{2\,r} - q^2)^2} + \mathcal{O}\left(\frac{1}{N_C}\right), \tag{E.6}$$

$$\Pi_{SS}(q^2)|_{\rm 1-loop} = \frac{D(q^2)|_{\rm 1-loop}}{(M_S^{r\,2} - q^2)^2} = \frac{\overline{D}(q^2)}{(M_S^{r\,2} - q^2)^2} + \frac{c_1 + \gamma_1\, \lambda_\infty}{M_S^{r\,2} - q^2} + \frac{c_2 + \gamma_2\, \lambda_\infty}{(M_S^{r\,2} - q^2)^2}, \tag{E.7}$$

where $\overline{D}(t)$ is provided in terms of the spectral function in Eq. (E.4) and $c_{1,2}$ and $\gamma_{1,2}$ are constants determined by the one-loop calculation. Taking into account Eq. (E.3), one gets

$$\begin{aligned} c_1 + \gamma_1\, \lambda_\infty &= -\mathrm{Re}\left\{ D'(q^2 = M_S^{r\,2})|_{\rm 1-loop} \right\} , \\ c_2 + \gamma_2\, \lambda_\infty &= \mathrm{Re}\left\{ D(q^2 = M_S^{r\,2})|_{\rm 1-loop} \right\} . \end{aligned} \tag{E.8}$$

All the relevant ultraviolet divergences are shown in Eq. (E.7). As mentioned before, the polynomial divergences $\Pi_{SS}(t) \sim \gamma_{-m}\, t^m\, \lambda_\infty$ cannot produce any contribution at the end of the day, so they exactly cancel at any energy. Once again, considering well behaved correlators –and therefore form factors– at large energies is crucial.

The renormalization procedure through the c_m and M_S counterterms gives

$$\begin{aligned} 32\, B_0^2\, c_m^r \delta c_m + \gamma_1\, \lambda_\infty &= 0\,, \\ -16\, B_0\, c_m^{r\,2}\, \delta M_S^{r\,2} + \gamma_2\, \lambda_\infty &= 0\,. \end{aligned} \tag{E.9}$$

The renormalized amplitude up to next-to-leading order in the $1/N_C$ expansion shows the general structure

$$\Pi_{SS}(q^2) = \frac{\overline{D}(q^2)}{(M_S^{r\,2} - q^2)^2} + \frac{16\, B_0\, c_m^{r\,2} + c_1}{M_S^{r\,2} - q^2} + \frac{c_2}{(M_S^{r\,2} - q^2)^2}\,. \tag{E.10}$$

The unknown subraction constants c_1 and c_2 can be absorved by a redefinition of c_m^r and M_S^r, so

$$\Pi_{SS}(q^2) = \frac{\overline{D}(q^2)}{(M_S^{r\,2} - q^2)^2} + \frac{16\, B_0\, c_m^{r\,2}}{M_S^{r\,2} - q^2}\,, \tag{E.11}$$

where c_m^r and M_S^r are now renormalization scale independent.

E.2 Contribution from High Mass Absorptive Cuts

Because of the approximation of neglecting intermediate states with two resonances, made in Section 4.4, it is convenient to analyse the effect on the χPT couplings of absorptive cuts with higher and higher production thresholds. When the threshold Λ_{th}^2 is above the resonance mass M_R^2, one finds for the low energy limit $q^2 \ll \Lambda_{th}^2$,

$$\frac{\overline{D}(q^2)}{(M_R^2 - q^2)^2} = \int_{\Lambda_{th}^2}^{\infty} \mathrm{d}t \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t - q^2} = \sum_{n=0}^{\infty} \left(\frac{q^2}{\Lambda_{th}^2} \right)^n \int_1^{\infty} \mathrm{d}x \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(x \cdot \Lambda_{th}^2)}{x^{n+1}} . \tag{E.12}$$

The contributions become smaller and smaller as the value of the production threshold Λ_{th}^2 is increased, supporting the approximation in Section 4.4.

On the other hand, in the deep euclidean region $Q^2 = -q^2 \gg \Lambda_{th}^2$, one gets

$$\left| \frac{\overline{D}(q^2)}{(M_R^2 - q^2)^2} \right| \le \frac{1}{Q^2} \int_{\Lambda_{th}^2}^{\infty} \mathrm{d}t \frac{1}{\pi} \left| \mathrm{Im}\Pi(t) \right| , \tag{E.13}$$

which becomes smaller and smaller as Λ_{th}^2 is increased.

Appendix F

Second-order Fluctuation of the Lagrangian

The expansion around the classical solution of the fields in our lagrangian of Eq. (5.1) up to second order (as required for the one loop evaluation) gives:

$$\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} \;=\; \Delta\mathcal{L}^{(2)}_{pGB} + \Delta\mathcal{L}_{\mathrm{kin}}(\mathrm{S},\mathrm{P}) + \Delta\mathcal{L}_2(\mathrm{S}) + \Delta\mathcal{L}_2(\mathrm{P}) + \Delta\mathcal{L}_2(\mathrm{S},\mathrm{P})\,, \quad \text{(F.1)}$$

where

$$\begin{aligned}
\Delta\mathcal{L}^{(2)}_{pGB} &= -\frac{F^2}{8}\langle\, \chi_+\Delta^2\,\rangle + \frac{F^2}{4}\langle\, \nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{4}\,[u_\mu,\Delta]\,[u^\mu,\Delta]\,\rangle\,, \qquad \text{(F.2)}\\
\Delta\mathcal{L}_{\mathrm{kin}}(\mathrm{S},\mathrm{P}) &= \frac{1}{4}\langle\, \nabla^\mu\varepsilon_{\mathrm{S}}\nabla_\mu\varepsilon_{\mathrm{S}}\,\rangle - \frac{M_S^2}{4}\langle\, \varepsilon_{\mathrm{S}}\,\varepsilon_{\mathrm{S}}\,\rangle + \frac{1}{32}\langle\, \big[[u^\mu,\Delta],S\big]\big[[u_\mu,\Delta],S\big]\,\rangle\\
&- \frac{1}{8}\langle\, [\nabla_\mu\Delta,\Delta][S,\nabla^\mu S]\,\rangle + \frac{1}{4\sqrt{2}}\langle\, [u_\mu,\Delta]\Big([S,\nabla^\mu\varepsilon_{\mathrm{S}}]-[\nabla^\mu S,\varepsilon_{\mathrm{S}}]\Big)\,\rangle\\
&+ \frac{1}{4}\langle\, \nabla^\mu\varepsilon_{\mathrm{P}}\nabla_\mu\varepsilon_{\mathrm{P}}\,\rangle - \frac{M_P^2}{4}\langle\, \varepsilon_{\mathrm{P}}\,\varepsilon_{\mathrm{P}}\,\rangle + \frac{1}{32}\langle\, \big[[u^\mu,\Delta],P\big]\big[[u_\mu,\Delta],P\big]\,\rangle\\
&- \frac{1}{8}\langle\, [\nabla_\mu\Delta,\Delta][P,\nabla^\mu P]\,\rangle + \frac{1}{4\sqrt{2}}\langle\, [u_\mu,\Delta]\Big([P,\nabla^\mu\varepsilon_{\mathrm{P}}]-[\nabla^\mu P,\varepsilon_{\mathrm{P}}]\Big)\rangle, \qquad \text{(F.3)}\\
\Delta\mathcal{L}_2(\mathrm{S}) &= -\frac{i\,c_m}{2\sqrt{2}}\langle\, \varepsilon_{\mathrm{S}}\{\Delta,\chi_-\}\,\rangle - \frac{c_m}{8}\langle\, \{S,\Delta\}\{\chi_+,\Delta\}\,\rangle - \frac{c_d}{\sqrt{2}}\langle\, \varepsilon_{\mathrm{S}}\{\nabla_\mu\Delta,u^\mu\}\,\rangle\\
&+ \langle\big(c_d S + \lambda_1^{\mathrm{SS}} SS\big)\Big(\nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{8}\Big\{\,[\Delta,[u_\mu,\Delta]]\,,u^\mu\Big\}\Big)\rangle + \frac{\lambda_1^{\mathrm{SS}}}{2}\langle\, \varepsilon_{\mathrm{S}}^2\,u^\mu u_\mu\,\rangle\\
&- \frac{\lambda_1^{\mathrm{SS}}}{\sqrt{2}}\langle\, \{S,\varepsilon_{\mathrm{S}}\}\,\{u_\mu,\nabla^\mu\Delta\}\,\rangle + \lambda_2^{\mathrm{SS}}\langle\, S\,\nabla_\mu\Delta\,S\,\nabla^\mu\Delta\,\rangle + \frac{\lambda_2^{\mathrm{SS}}}{2}\langle\, \varepsilon_{\mathrm{S}}u_\mu\varepsilon_{\mathrm{S}}u^\mu\,\rangle\\
&- \sqrt{2}\lambda_2^{\mathrm{SS}}\langle\, \varepsilon_{\mathrm{S}}\big(\nabla_\mu\Delta\,S\,u^\mu + u_\mu S\,\nabla^\mu\Delta\big)\,\rangle + \frac{\lambda_2^{\mathrm{SS}}}{4}\langle\, \big[[\Delta,u_\mu],\Delta\big]\,S\,u^\mu S\,\rangle\\
&- \frac{\lambda_3^{\mathrm{SS}}}{8}\langle\, \{SS,\Delta\}\{\chi_+,\Delta\}\,\rangle - \frac{i\,\lambda_3^{\mathrm{SS}}}{2\sqrt{2}}\langle\, \{S,\varepsilon_{\mathrm{S}}\}\{\chi_-,\Delta\}\,\rangle + \frac{\lambda_3^{\mathrm{SS}}}{2}\langle\, \varepsilon_{\mathrm{S}}^2\,\chi_+\,\rangle, \qquad \text{(F.4)}
\end{aligned}$$

$$
\begin{aligned}
\Delta\mathcal{L}_2(\mathrm{P}) = \; & \frac{d_m}{2\sqrt{2}}\langle\, \varepsilon_{\mathrm{P}}\{\Delta,\chi_+\}\,\rangle \;-\; \frac{i\,d_m}{8}\langle\, \{P,\Delta\}\{\chi_-,\Delta\}\,\rangle \\
& + \lambda_1^{\mathrm{PP}}\langle\, PP\left(\nabla^\mu\Delta\nabla_\mu\Delta + \frac{1}{8}\Big\{\big[\Delta,[u_\mu,\Delta]\big],u^\mu\Big\}\right)\rangle \;+\; \frac{\lambda_1^{\mathrm{PP}}}{2}\langle\, \varepsilon_{\mathrm{P}}^2\, u^\mu u_\mu\,\rangle \\
& - \;\frac{\lambda_1^{\mathrm{PP}}}{\sqrt{2}}\langle\, \{P,\varepsilon_{\mathrm{P}}\}\,\{u_\mu,\nabla^\mu\Delta\}\,\rangle + \lambda_2^{\mathrm{PP}}\langle\, P\,\nabla_\mu\Delta\, P\,\nabla^\mu\Delta\,\rangle + \frac{\lambda_2^{\mathrm{PP}}}{2}\langle\, \varepsilon_{\mathrm{P}}u_\mu\varepsilon_{\mathrm{P}}u^\mu\,\rangle \\
& - \sqrt{2}\lambda_2^{\mathrm{PP}}\langle\, \varepsilon_{\mathrm{P}}\big(\nabla_\mu\Delta\,P\,u^\mu + u_\mu P\,\nabla^\mu\Delta\big)\,\rangle\;\rangle \;+\; \frac{\lambda_2^{\mathrm{PP}}}{4}\langle\, \big[[\Delta,u_\mu],\Delta\big]P\,u^\mu P\,\rangle \\
& -\frac{\lambda_3^{\mathrm{PP}}}{8}\langle\, \{PP,\Delta\}\{\chi_+,\Delta\}\,\rangle - \frac{i\,\lambda_3^{\mathrm{PP}}}{2\sqrt{2}}\langle\, \{P,\varepsilon_{\mathrm{P}}\}\{\chi_-,\Delta\}\,\rangle + \frac{\lambda_3^{\mathrm{PP}}}{2}\langle\, \varepsilon_{\mathrm{P}}^2\,\chi_+\,\rangle,
\end{aligned} \tag{F.5}
$$

$$
\begin{aligned}
\Delta\mathcal{L}_2(\mathrm{S},\mathrm{P}) = \; & \frac{\lambda_1^{\mathrm{SP}}}{8}\langle\, \{\nabla_\mu S,P\}\big[[\Delta,u^\mu],\Delta\big]\,\rangle - \frac{\lambda_1^{\mathrm{SP}}}{\sqrt{2}}\langle\, \nabla^\mu\Delta\big(\{\nabla_\mu\varepsilon_{\mathrm{S}},P\} + \{\nabla_\mu S,\varepsilon_{\mathrm{P}}\}\big)\,\rangle \\
& + \frac{\lambda_1^{\mathrm{SP}}}{4\sqrt{2}}\langle\, \big[[u_\mu,\Delta],S\big]\big(\{\varepsilon_{\mathrm{P}},u^\mu\} - \sqrt{2}\{P,\nabla^\mu\Delta\}\big)\,\rangle + \frac{\lambda_1^{\mathrm{SP}}}{2}\langle\, \{\nabla_\mu\varepsilon_{\mathrm{S}},\varepsilon_{\mathrm{P}}\}u^\mu\,\rangle \\
& + \frac{\lambda_1^{\mathrm{SP}}}{4\sqrt{2}}\langle\, \big[[u_\mu,\Delta],\varepsilon_{\mathrm{S}}\big]\{P,u^\mu\}\,\rangle + \frac{\lambda_1^{\mathrm{SP}}}{8}\langle\, \big[[\Delta,\nabla_\mu\Delta],S\big]\{P,u^\mu\}\,\rangle \\
& - \frac{i\,\lambda_2^{\mathrm{SP}}}{8}\langle\, \{S,P\}\big\{\Delta,\{\chi_-,\Delta\}\big\}\,\rangle + \frac{\lambda_2^{\mathrm{SP}}}{2\sqrt{2}}\langle\, \{\Delta,\chi_+\}\Big(\{\varepsilon_{\mathrm{S}},P\} + \{S,\varepsilon_{\mathrm{P}}\}\Big)\,\rangle \\
& + \frac{i\,\lambda_2^{\mathrm{SP}}}{2}\langle\, \chi_-\{\varepsilon_{\mathrm{S}},\varepsilon_{\mathrm{P}}\}\,\rangle\,.
\end{aligned} \tag{F.6}
$$

The evaluation of the path integral requires a Gaussian rearrangement of the integration variables. However the second-order fluctuation $\Delta\mathcal{L}_{R\chi T}$ does not have this structure due to the terms $\langle\, PP\,\nabla_\mu\Delta\nabla^\mu\Delta\,\rangle$, $\langle\, P\,\nabla_\mu\Delta\,P\,\nabla^\mu\Delta\,\rangle$, $\langle\, S\nabla_\mu\Delta\nabla^\mu\Delta\,\rangle$, $\langle\, SS\,\nabla_\mu\Delta\nabla^\mu\Delta\,\rangle$, $\langle\, S\,\nabla_\mu\Delta\,S\,\nabla^\mu\Delta\,\rangle$ and $\langle\, \{\nabla_\mu\varepsilon_{\mathrm{S}},P\}\nabla^\mu\Delta\,\rangle$ in Eqs. (F.4), (F.5) and (F.6). A way out is provided by a redefinition of the fields that eliminates the unwanted terms:

$$
\begin{aligned}
\Delta \;&\to\; \Delta - \frac{c_d}{F^2}\{\Delta,S\} - \frac{\tilde\lambda_1^{\mathrm{SS}}}{F^2}\{\Delta,SS\} - \frac{2\tilde\lambda_2^{\mathrm{SS}}}{F^2}S\,\Delta\,S - \frac{\tilde\lambda_1^{\mathrm{PP}}}{F^2}\{\Delta,PP\} - \frac{2\tilde\lambda_2^{\mathrm{PP}}}{F^2}P\,\Delta\,P\,, \\
\varepsilon_{\mathrm{S}} \;&\to\; \varepsilon_{\mathrm{S}} + \sqrt{2}\lambda_1^{\mathrm{SP}}\{P,\Delta\} - \frac{\sqrt{2}\lambda_1^{\mathrm{SP}}c_d}{F^2}\big\{P,\{\Delta,S\}\big\}\,,
\end{aligned} \tag{F.7}
$$

where the following constants have been defined:

$$
\begin{aligned}
\tilde\lambda_1^{\mathrm{SS}} &\equiv \lambda_1^{\mathrm{SS}} - \frac{3}{2}\frac{c_d^2}{F^2}\,, & \tilde\lambda_2^{\mathrm{SS}} &\equiv \lambda_2^{\mathrm{SS}} - \frac{3}{2}\frac{c_d^2}{F^2}\,, \\
\tilde\lambda_1^{\mathrm{PP}} &\equiv \lambda_1^{\mathrm{PP}} - (\lambda_1^{\mathrm{SP}})^2\,, & \tilde\lambda_2^{\mathrm{PP}} &\equiv \lambda_2^{\mathrm{PP}} - (\lambda_1^{\mathrm{SP}})^2\,.
\end{aligned} \tag{F.8}
$$

The transformation of the integration measure only yields $\delta^4(0)$ terms which have no effect on the theory [67] [7].

[7] In dimensional regularization the later result is immediate, as $\delta^d(0) = 0$.

Performing the transformations given by Eq. (F.7) on $\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}$ and keeping only terms with up to two resonances we finally obtain:

$$\begin{aligned}\Delta\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = &-\frac{1}{2}\,\Delta_i\left(d'_\mu d'^\mu + \sigma\right)_{ij}\Delta_j - \frac{1}{2}\,\varepsilon_{\mathrm{S}i}\left(d^\mu d_\mu + k^{\mathrm{S}}\right)_{ij}\varepsilon_{\mathrm{S}j} - \frac{1}{2}\,\varepsilon_{\mathrm{P}i}\left(d^\mu d_\mu + k^{\mathrm{P}}\right)_{ij}\varepsilon_{\mathrm{P}j}\\ &+ \varepsilon_{\mathrm{S}i}\,{a^{\mathrm{S}}}_{ij}\,\Delta_j + \varepsilon_{\mathrm{P}i}\,{a^{\mathrm{P}}}_{ij}\,\Delta_j + \varepsilon_{\mathrm{P}i}\,{a^{\mathrm{SP}}}_{ij}\,\varepsilon_{\mathrm{S}j}\\ &+ \varepsilon_{\mathrm{S}k}\,b^{\mathrm{S}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j + \varepsilon_{\mathrm{P}k}\,b^{\mathrm{P}}_{\mu\,ki}\,d^\mu_{ij}\Delta_j + \varepsilon_{\mathrm{P}k}\,b^{\mathrm{SP}}_{\mu\,ki}\,d^\mu_{ij}\varepsilon_{\mathrm{S}j}\;, \qquad \text{(F.9)}\end{aligned}$$

that has the proper Gaussian structure and where the following definitions have been introduced:

$$d^\mu_{ij} = \delta_{ij}\,\partial^\mu + \gamma^\mu_{ij}\big|_\chi\,, \qquad \text{(F.10)}$$

$$d'^\mu_{ij} = d^\mu_{ij} + \gamma^\mu_{ij}\big|_{\mathrm{R}}\,, \qquad \text{(F.11)}$$

$$\gamma^\mu_{ij}\big|_\chi = -\frac{1}{2}\langle\,\Gamma^\mu[\lambda_i,\lambda_j]\,\rangle\,, \qquad \text{(F.12)}$$

$$\begin{aligned}\gamma^\mu_{ij}\big|_{\mathrm{R}} = &\;\frac{c_d\lambda_1^{\mathrm{SP}}}{2F^2}\langle\,\{P,\lambda_i\}\{u^\mu,\lambda_j\}\,\rangle + (-\frac{1}{16F^2}+\frac{c_d^2}{8F^4})\langle\,[S,\nabla^\mu S]\,[\lambda_i,\lambda_j]\,\rangle\\ &-\frac{1}{16F^2}\langle\,[P,\nabla^\mu P]\,[\lambda_i,\lambda_j]\,\rangle - \frac{\lambda_1^{\mathrm{SP}}}{16F^2}\langle\,\big[S,\{P,u^\mu\}\big][\lambda_i,\lambda_j]\,\rangle\\ &+\frac{\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}}{F^2}\langle\,\{P,\lambda_i\}\big(\lambda_j S\,u^\mu + u^\mu S\,\lambda_j\big)\,\rangle + \frac{\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}}{2F^2}\langle\,\big\{S,\{P,\lambda_i\}\big\}\{u^\mu,\lambda_j\}\,\rangle\\ &-\frac{c_d^2\lambda_1^{\mathrm{SP}}}{2F^4}\langle\,\{S,\lambda_i\}\big[[P,u^\mu],\lambda_j\big]\,\rangle - \Big\{i\leftrightarrow j\Big\}\,, \qquad \text{(F.13)}\end{aligned}$$

$${k^{\mathrm{S}}}_{ij} = \delta_{ij}M_S^2 - \frac{\lambda_1^{\mathrm{SS}}}{2}\langle\,u^\mu u_\mu\{\lambda_i,\lambda_j\}\,\rangle - \lambda_2^{\mathrm{SS}}\langle\,\lambda_i u_\mu\lambda_j u^\mu\,\rangle - \frac{\lambda_3^{\mathrm{SS}}}{2}\langle\,\chi_+\{\lambda_i,\lambda_j\}\,\rangle\,, \qquad \text{(F.14)}$$

$${k^{\mathrm{P}}}_{ij} = \delta_{ij}M_P^2 - \frac{\lambda_1^{\mathrm{PP}}}{2}\langle\,u^\mu u_\mu\{\lambda_i,\lambda_j\}\,\rangle - \lambda_2^{\mathrm{PP}}\langle\,\lambda_i u_\mu\lambda_j u^\mu\,\rangle - \frac{\lambda_3^{\mathrm{PP}}}{2}\langle\,\chi_+\{\lambda_i,\lambda_j\}\,\rangle, \qquad \text{(F.15)}$$

$$\begin{aligned}\sigma_{ij} = &\;\frac{1}{16}\langle\,\chi_+\{\hat\lambda_i,\hat\lambda_j\}\,\rangle - \frac{1}{16}\langle\,[u_\mu,\hat\lambda_i][u^\mu,\hat\lambda_j]\,\rangle\\ &-\frac{c_d}{4F^2}\langle\,\nabla^2 S\{\lambda_i,\lambda_j\}\,\rangle + \frac{c_m}{8F^2}\langle\,\{S,\hat\lambda_i\}\{\chi_+,\hat\lambda_j\}\,\rangle + \frac{c_d}{8F^2}\langle\,\{S,u^\mu\}\big[[u_\mu,\hat\lambda_i],\hat\lambda_j\big]\,\rangle\\ &-\frac{c_d\lambda_1^{\mathrm{SP}}}{2F^2}\big(\langle\,\{\nabla_\mu P,\hat\lambda_i\}\{u^\mu,\hat\lambda_j\}\,\rangle + \langle\,\{P,\hat\lambda_i\}\{\nabla_\mu u^\mu,\hat\lambda_j\}\,\rangle\big)\\ &+\frac{i}{2F^2}\big(\frac{d_m}{4}+c_m\lambda_1^{\mathrm{SP}}\big)\langle\,\{P,\hat\lambda_i\}\{\chi_-,\hat\lambda_j\}\,\rangle - \frac{1}{32F^2}\langle\,\big[S,[u^\mu,\lambda_i]\big]\big[S,[u_\mu,\lambda_j]\big]\,\rangle\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{8F^2}\langle\, [u^\mu,\lambda_i]\big[u_\mu, \tilde\lambda_1^{\rm SS}\{SS,\lambda_j\} + 2\tilde\lambda_2^{\rm SS} S\lambda_j S)\big]\,\rangle \\
&+ \frac{1}{8F^2}\langle\, [u^\mu,\lambda_i]\big[\lambda_j, \big(\lambda_1^{\rm SS}\{SS,u_\mu\} + 2\lambda_2^{\rm SS} S u_\mu S)\big]\Big)\,\rangle \\
&- \frac{1}{8F^2}\langle\, \{\chi_+,\lambda_i\}\Big(\big(\tilde\lambda_1^{\rm SS} - \lambda_3^{\rm SS}\big)\{SS,\lambda_j\} + 2\tilde\lambda_2^{\rm SS} S\lambda_j S\Big)\,\rangle \\
&- \frac{c_d^2}{2F^4}\langle\, \{\nabla^\mu S,\lambda_i\}\{\nabla_\mu S,\lambda_j\}\,\rangle - \frac{c_d^2}{4F^4}\langle\, \{S,\lambda_i\}\{\nabla^2 S,\lambda_j\}\,\rangle - \frac{\tilde\lambda_1^{\rm SS}}{4F^2}\langle\, \nabla^2 S^2\{\lambda_i,\lambda_j\}\,\rangle \\
&- \frac{\tilde\lambda_2^{\rm SS}}{F^2}\langle\, \lambda_i\nabla_\mu\big(\nabla^\mu S\lambda_j S\big)\,\rangle + \frac{M_S^2(\lambda_1^{\rm SP})^2}{2F^2}\langle\, \{P,\lambda_i\}\{P,\lambda_j\}\,\rangle \\
&- \frac{\tilde\lambda_1^{\rm PP}}{8F^2}\langle\, \{\chi_+,\lambda_i\}\{PP,\lambda_j\}\,\rangle - \frac{\tilde\lambda_2^{\rm PP}}{4F^2}\langle\, \{\chi_+,\lambda_i\}P\,\lambda_j P\,\rangle \\
&- \frac{\lambda_1^{\rm SP}\lambda_2^{\rm SP}}{2F^2}\langle\, \big\{P,\{P,\lambda_i\}\big\}\{\chi_+,\lambda_j\}\,\rangle + \frac{\lambda_3^{\rm PP}}{8F^2}\langle\, \{PP,\lambda_i\}\{\chi_+,\lambda_j\}\,\rangle \\
&- \frac{\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2}{F^2}\langle\, \{P,\lambda_i\}\{P,\lambda_j\}\chi_+\,\rangle - \frac{1}{32F^2}\langle\, \big[P,[u^\mu,\lambda_i]\big]\big[P,[u_\mu,\lambda_j]\big]\,\rangle \\
&+ \frac{c_d^2(\lambda_1^{\rm SP})^2}{4F^4}\langle\, \big[[u^\mu,P],\lambda_i\big]\big[[u_\mu,P],\lambda_j\big]\,\rangle - \frac{(\lambda_1^{\rm SP})^2}{4F^2}\langle\, \big[[u^\mu,\lambda_i],\{P,\lambda_j\}\big]\{P,u_\mu\}\,\rangle \\
&- \frac{1}{8F^2}\langle\, \Big(\lambda_1^{\rm PP}\{u_\mu,PP\} + 2\lambda_2^{\rm PP} P\,u_\mu P\Big)\big[[\lambda_i,u^\mu],\lambda_j\big]\,\rangle \\
&- \frac{(\lambda_1^{\rm SP})^2}{F^2}\langle\, u^\mu\{P,\lambda_i\}\Big(\lambda_1^{\rm SS}\{P,\lambda_j\}u_\mu + \lambda_2^{\rm SS}u_\mu\{P,\lambda_j\}\Big)\,\rangle \\
&+ \frac{\tilde\lambda_1^{\rm PP}}{8F^2}\langle\, \big[u_\mu,\{PP,\lambda_i\}\big][u^\mu,\lambda_j]\,\rangle + \frac{\tilde\lambda_2^{\rm PP}}{4F^2}\langle\, [u_\mu,P\,\lambda_i P\,][u^\mu,\lambda_j]\,\rangle \\
&- \frac{\tilde\lambda_1^{\rm PP}}{4F^2}\langle\, \nabla^2 P^2\{\lambda_i,\lambda_j\}\rangle - \frac{\tilde\lambda_2^{\rm PP}}{F^2}\langle\, \lambda_i\nabla_\mu\big(\nabla^\mu P\lambda_j P\big)\rangle - \frac{(\lambda_1^{\rm SP})^2}{2F^2}\langle\{\nabla_\mu P,\lambda_i\}\{\nabla^\mu P,\lambda_j\}\rangle \\
&+ \frac{i\lambda_2^{\rm SP}}{8F^2}\langle\, \{S,P\}\big\{\lambda_i,\{\chi_-,\lambda_j\}\big\}\,\rangle + \frac{i\,\lambda_3^{\rm SS}\lambda_1^{\rm SP}}{2F^2}\langle\, \big\{S,\{P,\lambda_i\}\big\}\{\chi_-,\lambda_j\}\,\rangle \\
&- \frac{c_d^2\lambda_1^{\rm SP}}{2F^4}\langle\, \{P,\lambda_i\}\big\{u^\mu,\{\nabla_\mu S,\lambda_j\}\big\}\,\rangle - \frac{\lambda_2^{\rm SS}\lambda_1^{\rm SP}}{F^2}\langle\, \nabla_\mu\Big(\{P,\lambda_i\}\big(\lambda_j S u^\mu + u^\mu S\lambda_j\big)\Big)\,\rangle \\
&- \frac{\lambda_1^{\rm SS}\lambda_1^{\rm SP}}{2F^2}\langle\, \{P,\lambda_i\}\nabla_\mu\big\{S,\{u^\mu,\lambda_j\}\big\}\,\rangle \\
&+ \frac{\lambda_1^{\rm SP}}{8F^2}\langle\, [u^\mu,\lambda_i]\Big(\big[\lambda_j,\{\nabla_\mu S,P\}\big] + 2\big[\nabla^\mu S,\{P,\lambda_j\}\big] - 2\big[S,\{\nabla^\mu P,\lambda_j\}\big]\Big)\,\rangle \\
&+ \frac{\lambda_1^{\rm SP}}{2F^2}\langle\, \{u^\mu,\lambda_i\}\Big(\frac{c_d^2}{F^2}\big\{P,\{\nabla_\mu S,\lambda_j\}\big\} - \lambda_1^{\rm SS}\big\{S,\{\nabla_\mu P,\lambda_j\}\big\}\Big)\rangle + \Big\{i\leftrightarrow j\Big\}, \quad \text{(F.16)}
\end{aligned}
$$

$$
\begin{aligned}
a^{\rm S}{}_{ij} = &-\frac{i\,c_m}{2\sqrt{2}F}\langle\, \chi_-\{\lambda_i,\hat\lambda_j\}\rangle - \frac{1}{2\sqrt{2}F}\langle[\nabla^\mu S,\lambda_i][u_\mu,\hat\lambda_j]\rangle - \frac{1}{4\sqrt{2}F}\langle[S,\lambda_i][\nabla^\mu u_\mu,\hat\lambda_j]\rangle \\
&+ \frac{c_d^2}{\sqrt{2}F^3}\langle\{u^\mu,\lambda_i\}\{\nabla_\mu S,\lambda_j\}\rangle - \frac{i\,\lambda_3^{\rm SS}}{2\sqrt{2}F}\langle\{S,\lambda_i\}\{\chi_-,\hat\lambda_j\}\rangle - \frac{M_S^2\lambda_1^{\rm SP}}{\sqrt{2}F}\langle\, P\{\lambda_i,\hat\lambda_j\}\rangle
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\lambda_2^{\rm SP}}{2\sqrt{2}F}\langle\{P,\lambda_i\}\{\chi_+,\hat\lambda_j\}\rangle+\frac{\lambda_1^{\rm SP}\lambda_3^{\rm SS}}{\sqrt{2}F}\langle\{\chi_+,\lambda_i\}\{P,\hat\lambda_j\}\rangle-\frac{\lambda_1^{\rm SP}}{\sqrt{2}F}\langle\,\nabla^2P\{\lambda_i,\hat\lambda_j\}\rangle\\
&-\frac{\lambda_1^{\rm SP}}{4\sqrt{2}F}\langle\{P,u^\mu\}\big[\lambda_i,[u_\mu,\hat\lambda_j]\big]\rangle+\frac{\lambda_1^{\rm SP}}{\sqrt{2}F}\langle\{P,\hat\lambda_j\}\Big(\lambda_1^{\rm SS}\{u_\mu u^\mu,\lambda_i\}+2\lambda_2^{\rm SS}u_\mu\lambda_i u^\mu\Big)\rangle\\
&+\frac{c_d}{\sqrt{2}F^3}\langle\,\{u^\mu,\lambda_i\}\,\Big(\tilde\lambda_1^{\rm SS}\{\nabla_\mu(SS),\lambda_j\}+2\tilde\lambda_2^{\rm SS}\nabla_\mu(S\lambda_jS)\Big)\,\rangle\\
&+\frac{\sqrt{2}\lambda_2^{\rm SS}\,c_d}{F^3}\langle\,\{\nabla^\mu S,\lambda_j\}\Big(Su_\mu\lambda_i+\lambda_iu_\mu S\Big)\,\rangle+\frac{\lambda_1^{\rm SS}\,c_d}{\sqrt{2}F^3}\langle\,\{S,\lambda_i\}\big\{u_\mu,\{\nabla^\mu S,\lambda_j\}\big\}\,\rangle\\
&+\frac{c_d}{4\sqrt{2}F^3}\langle[S,\lambda_i]\big[u_\mu,\{\nabla^\mu S,\lambda_j\}\big]\rangle+\frac{i\,c_m}{2\sqrt{2}F^3}\langle\{\chi_-,\lambda_i\}\Big(\tilde\lambda_1^{\rm SS}\{SS,\lambda_j\}+2\tilde\lambda_2^{\rm SS}S\lambda_jS\Big)\rangle\\
&+\frac{i\,c_m}{2\sqrt{2}F^3}\langle\,\{\chi_-,\lambda_i\}\Big(\tilde\lambda_1^{\rm PP}\{PP,\lambda_j\}+2\tilde\lambda_2^{\rm PP}P\,\lambda_jP\Big)\,\rangle\\
&+\frac{c_d}{\sqrt{2}F^3}\langle\,\{u^\mu,\lambda_i\}\Big(\tilde\lambda_1^{\rm PP}\{\nabla_\mu(PP),\lambda_j\}+2\tilde\lambda_2^{\rm PP}\nabla_\mu(P\,\lambda_jP)\Big)\,\rangle\\
&+\frac{c_d\lambda_1^{\rm SP}}{\sqrt{2}F^3}\langle\,\{\nabla_\mu P,\lambda_i\}\{\nabla^\mu S,\lambda_j\}\,\rangle\,,
\end{aligned}
\tag{F.17}
$$

$$
\begin{aligned}
a^{\rm P}{}_{ij}=&\ \frac{d_m}{2\sqrt{2}F}\langle\,\chi_+\{\lambda_i,\hat\lambda_j\}\rangle+\frac{\lambda_2^{\rm SP}}{2\sqrt{2}F}\langle\{S,\lambda_i\}\{\chi_+,\hat\lambda_j\}\rangle-\frac{\lambda_1^{\rm SP}}{4\sqrt{2}F}\langle\{u_\mu,\lambda_i\}\big[S,[u^\mu,\hat\lambda_j]\big]\rangle\\
&-\frac{1}{4\sqrt{2}F}\langle\,\Big([P,\lambda_i][\nabla^\mu u_\mu,\hat\lambda_j]+2[\nabla^\mu P,\lambda_i][u_\mu,\hat\lambda_j]\Big)\,\rangle\\
&+\frac{i\,\lambda_1^{\rm SP}}{\sqrt{2}F}\lambda_2^{\rm SP}\langle\,\{\chi_-,\lambda_i\}\{P,\hat\lambda_j\}\,\rangle-\frac{i\,\lambda_3^{\rm PP}}{2\sqrt{2}F}\langle\,\{P,\lambda_i\}\{\chi_-,\hat\lambda_j\}\,\rangle\\
&+\frac{(\lambda_1^{\rm SP})^2}{\sqrt{2}F}\langle\,\{u^\mu,\lambda_i\}\{\nabla_\mu P,\hat\lambda_j\}\,\rangle+\frac{c_d\lambda_1^{\rm SP}}{\sqrt{2}F^3}\langle\,\{\nabla_\mu S,\lambda_i\}\{\nabla^\mu S,\lambda_j\}\,\rangle\\
&-\frac{d_m}{2\sqrt{2}F^3}\langle\{\chi_+,\lambda_i\}\Big(\tilde\lambda_1^{\rm SS}\{SS,\lambda_j\}+2\tilde\lambda_2^{\rm SS}S\,\lambda_jS+\tilde\lambda_1^{\rm PP}\{PP,\lambda_j\}+2\tilde\lambda_2^{\rm PP}P\,\lambda_jP\Big)\rangle\\
&-\frac{c_d(\lambda_1^{\rm SP})^2}{\sqrt{2}F^3}\langle\,\{u^\mu,\lambda_i\}\big\{P,\{\nabla_\mu S,\lambda_j\}\big\}\,\rangle+\frac{c_d}{4\sqrt{2}F^3}\langle\,[P,\lambda_i]\big[u_\mu,\{\nabla^\mu S,\lambda_j\}\big]\,\rangle\\
&+\frac{c_d}{\sqrt{2}F^3}\langle\,\{\nabla^\mu S,\lambda_j\}\Big(\lambda_1^{\rm PP}\big\{u_\mu,\{P,\lambda_i\}\big\}+2\lambda_2^{\rm PP}\left(P\,u_\mu\lambda_i+\lambda_iu_\mu P\right)\Big)\,\rangle\,,
\end{aligned}
\tag{F.18}
$$

$$
a^{\rm SP}{}_{ij}=\ \frac{i\,\lambda_2^{\rm SP}}{2}\langle\,\chi_-\{\lambda_i,\lambda_j\}\,\rangle\,,
\tag{F.19}
$$

$$
\begin{aligned}
b^{{\rm S}\mu}_{\ ij}=&\ -\frac{c_d}{\sqrt{2}F}\langle\,u^\mu\{\lambda_i,\hat\lambda_j\}\,\rangle-\frac{1}{4\sqrt{2}F}\langle\,[S,\lambda_i][u^\mu,\hat\lambda_j]\,\rangle-\frac{\lambda_1^{\rm SS}}{\sqrt{2}F}\langle\,\{S,\lambda_i\}\{u^\mu,\hat\lambda_j\}\,\rangle\\
&-\frac{\sqrt{2}\lambda_2^{\rm SS}}{F}\langle\,S\Big(u^\mu\lambda_i\hat\lambda_j+\hat\lambda_j\lambda_iu^\mu\Big)\,\rangle-\frac{\lambda_1^{\rm SP}}{\sqrt{2}F}\langle\,\nabla^\mu P\{\lambda_i,\hat\lambda_j\}\rangle
\end{aligned}
$$

$$+\frac{c_d}{\sqrt{2}F^3}\langle\{u^\mu,\lambda_i\}\Big(\tilde{\lambda}_1^{\rm SS}\{SS,\lambda_j\}+2\tilde{\lambda}_2^{\rm SS}S\lambda_j S+\tilde{\lambda}_1^{\rm PP}\{PP,\lambda_j\}+2\tilde{\lambda}_2^{\rm PP}P\lambda_j P\Big)\rangle, \quad \text{(F.20)}$$

$$b^{{\rm P}\mu}_{\ \ ij} = -\frac{\lambda_1^{\rm SP}}{\sqrt{2}F}\langle\,\nabla^\mu S\{\lambda_i,\hat{\lambda}_j\}\,\rangle - \frac{1}{4\sqrt{2}F}\langle\,[P,\lambda_i][u^\mu,\hat{\lambda}_j]\,\rangle - \frac{\lambda_1^{\rm PP}}{\sqrt{2}F}\langle\,\{P,\lambda_i\}\{u^\mu,\hat{\lambda}_j\}\,\rangle$$
$$- \frac{\sqrt{2}\lambda_2^{\rm PP}}{F}\langle\, P\Big(u^\mu\lambda_i\hat{\lambda}_j+\hat{\lambda}_j\lambda_i u^\mu\Big)\,\rangle + \frac{(\lambda_1^{\rm SP})^2}{\sqrt{2}F}\langle\,\{u^\mu,\lambda_i\}\{P,\hat{\lambda}_j\}\,\rangle\,, \quad \text{(F.21)}$$

$$b^{{\rm SP}\mu}_{\ \ ij} = \frac{\lambda_1^{\rm SP}}{2}\langle\, u^\mu\{\lambda_i,\lambda_j\}\,\rangle\,, \quad \text{(F.22)}$$

and the following definitions have been used,

$$\hat{\lambda}_i \equiv \lambda_i - \frac{c_d}{F^2}\{\lambda_i,S\}\,, \qquad \nabla_\mu\,(A\,\lambda_i\,B) \equiv \nabla_\mu A\,\lambda_i\,B + A\,\lambda_i\,\nabla_\mu B\,, \quad \text{(F.23)}$$

where A and B are any chiral tensor or resonance field.

As commented in the text we can write Eq. (F.9) as:

$$\Delta\mathcal{L}_{{\rm R}\chi{\rm T}} = -\frac{1}{2}\,\eta\,(\,\Sigma_\mu\,\Sigma^\mu + \Lambda\,)\,\eta^\top\,, \quad \text{(F.24)}$$

where η collects the fluctuation fields, $\eta=\Big(\Delta_i,\varepsilon_{{\rm S}j},\varepsilon_{{\rm P}k}\Big)$, $i,j,k=0,...,8$, $\eta^\top$ is its transposed and Λ and Σ_μ are defined as:

$$(\Lambda)_{ij} = \begin{pmatrix} \sigma+\frac14 b^{{\rm S}\,\top}_\mu b^{{\rm S}\,\mu} & -a^{{\rm S}\,\top}+\frac12\tilde{d}^\mu_- b^{{\rm S}\,\top}_\mu & -a^{{\rm P}\,\top}+\frac12\tilde{d}^\mu_- b^{{\rm P}\,\top}_\mu \\ +\frac14 b^{{\rm P}\,\top}_\mu b^{{\rm P}\,\mu} & +\frac14 b^{{\rm P}\,\top}_\mu b^{{\rm SP}\,\mu} & -\frac14 b^{{\rm S}\,\top}_\mu b^{{\rm SP}\,\mu\,\top} \\ & & \\ -a^{\rm S}+\frac12\bar{d}^\mu_+ b^{\rm S}_\mu & k^{\rm S}+\frac14 b^{{\rm S}\,\mu}b^{{\rm S}\,\top}_\mu & -a^{{\rm SP}\,\top}+\frac12\hat{d}^\mu b^{{\rm SP}\,\top}_\mu \\ +\frac14 b^{{\rm SP}\,\top}_\mu b^{{\rm P}\,\mu} & +\frac14 b^{{\rm SP}\,\top}_\mu b^{{\rm SP}\,\mu} & +\frac14 b^{{\rm S}\,\mu}b^{{\rm P}\,\top}_\mu \\ & & \\ -a^{\rm P}+\frac12\bar{d}^\mu_+ b^{\rm P}_\mu & -a^{\rm SP}+\frac12\hat{d}^\mu b^{\rm SP}_\mu & k^{\rm P}+\frac14 b^{{\rm P}\,\mu}b^{{\rm P}\,\top}_\mu \\ -\frac14 b^{{\rm SP}\,\mu}b^{\rm S}_\mu & +\frac14 b^{{\rm P}\,\mu}b^{{\rm S}\,\top}_\mu & +\frac14 b^{{\rm SP}\,\mu}b^{{\rm SP}\,\top}_\mu \end{pmatrix}_{ij}. \quad \text{(F.25)}$$

$$(\Sigma_\mu)_{ij} \;=\; \delta_{ij}\,\partial_\mu \,+\, (Y_\mu)_{ij}\;, \tag{F.26}$$

$$(Y_\mu)_{ij} \;=\; \begin{pmatrix} \gamma'_\mu & \frac{1}{2}b_\mu^{\mathrm{S}\,\top} & \frac{1}{2}b_\mu^{\mathrm{P}\,\top} \\ -\frac{1}{2}b_\mu^{\mathrm{S}} & \gamma_\mu & \frac{1}{2}b_\mu^{\mathrm{SP}\,\top} \\ -\frac{1}{2}b_\mu^{\mathrm{P}} & -\frac{1}{2}b_\mu^{\mathrm{SP}} & \gamma_\mu \end{pmatrix}_{ij}\;, \tag{F.27}$$

Here some new expressions have been defined:

$$\begin{aligned} \gamma^\mu &= \gamma^\mu\big|_\chi\,, \\ \gamma'^\mu &= \gamma^\mu\big|_\chi + \gamma^\mu\big|_{\mathrm{R}}\,, \\ \hat{d}^\mu\,X &= \partial^\mu\,X \,+\, [\,\gamma^\mu\,,X\,]\;, \\ \tilde{d}^\mu_\pm X &= \hat{d}^\mu\,X \,\pm\, (\gamma'^\mu - \gamma^\mu)\;X\,, \\ \bar{d}^\mu_\pm X &= \hat{d}^\mu\,X \,\pm\, X\;(\gamma'^\mu - \gamma^\mu)\;. \end{aligned} \tag{F.28}$$

Appendix G

β-function Coefficients

The divergent part of the RχT lagrangian shown in Chapter 5, at one loop, can be expressed in a basis of operators that satisfy the same symmetry requirements that our starting lagrangian of Eq. (5.1). At one loop our bare lagrangian reads:

$$\mathcal{L}_1 \;=\; \sum_{i=1}^{18} \alpha_i \, \mathcal{O}_i \;+\; \sum_{i=1}^{66} \beta_i^R \, \mathcal{O}_i^R \;+\; \sum_{i=1}^{379} \beta_i^{RR} \, \mathcal{O}_i^{RR} \;. \tag{G.1}$$

The notation of Section 5.3.3 is followed. The couplings in the lagrangian $\mathcal{L}_{L=1}$ read:

$$\begin{aligned} \alpha_i &= \mu^{D-4} \left(\alpha_i^r(\mu) \,+\, \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i \right) \,, \\ \beta_i^R &= \mu^{D-4} \left(\beta_i^{R,r}(\mu) \,+\, \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i^R \right) \,, \\ \beta_i^{RR} &= \mu^{D-4} \left(\beta_i^{RR,r}(\mu) \,+\, \frac{1}{(4\pi)^2}\,\frac{1}{D-4}\,\gamma_i^{RR} \right) \,, \end{aligned} \tag{G.2}$$

where γ_i, γ_i^R and γ_i^{RR} are the divergent coefficients that constitute the β-function of our lagrangian. γ_i^R and γ_i^{RR} are given in Tables G.1 and G.2, while γ_i were shown in Table 5.1.

We indicate with an asterisk all the operators whose β-function coefficient vanishes once the short-distance constraints of Eqs. (5.2) and (5.3) are considered.

Table G.1: Operators with one resonance and their β-function coefficients.

i	$\mathcal{O}_i^R$	γ_i^R
1	$\langle\, Su\cdot u \,\rangle$	$-3NF^{-4}c_d^3M_S^2 + 2NF^{-2}c_d\lambda_1^{\mathrm{SS}}M_S^2 + 4NF^{-2}c_d\lambda_2^{\mathrm{SS}}M_S^2 - NF^{-2}c_dM_S^2(\lambda_1^{\mathrm{SP}})^2 - NF^{-2}c_dM_P^2(\lambda_1^{\mathrm{SP}})^2$

2	$\langle S \rangle\langle u\cdot u \rangle$	$-3F^{-4}c_d^3M_S^2 + 2F^{-2}c_d\lambda_1^{SS}M_S^2 - F^{-2}c_dM_S^2(\lambda_1^{SP})^2 + F^{-2}c_dM_S^2 - F^{-2}c_dM_P^2(\lambda_1^{SP})^2$
3	$\langle u_\mu \rangle\langle u^\mu S \rangle$	$-2F^{-4}c_d^3M_S^2 + 4F^{-2}c_d\lambda_1^{SS}M_S^2 + 4F^{-2}c_d\lambda_2^{SS}M_S^2 + 2F^{-2}c_dM_S^2(\lambda_1^{SP})^2 - F^{-2}c_dM_S^2$
4	$\langle S\chi_+ \rangle$	$-2NF^{-4}c_d^2c_mM_S^2 - 2NF^{-2}d_mM_S^2\lambda_1^{SP} + 1/2NF^{-2}c_dM_S^2 - 2NF^{-2}c_mM_S^2(\lambda_1^{SP})^2 - NF^{-2}c_mM_P^2(\lambda_1^{SP})^2$
5	$\langle S \rangle\langle \chi_+ \rangle$	$-2F^{-4}c_d^2c_mM_S^2 - 2F^{-2}d_mM_S^2\lambda_1^{SP} + 1/2F^{-2}c_dM_S^2 - 2F^{-2}c_mM_S^2(\lambda_1^{SP})^2 - F^{-2}c_mM_P^2(\lambda_1^{SP})^2$
6*	$\langle Su_\mu u_\nu u^\mu u^\nu \rangle$	$-2/3NF^{-6}c_d^5 + 2/3NF^{-4}c_d^3\lambda_1^{SS} + 4/3NF^{-4}c_d^3\lambda_2^{SS} - 1/3NF^{-4}c_d^3(\lambda_1^{SP})^2 + 1/6NF^{-4}c_d^3 + 1/3NF^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/3NF^{-2}c_d\lambda_1^{SS} + 2/3NF^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2 - 2/3NF^{-2}c_d\lambda_2^{SS} - 1/12NF^{-2}c_d(\lambda_1^{SP})^2 + 1/12NF^{-2}c_d$
7	$\langle Su\cdot uu\cdot u \rangle$	$10/3NF^{-6}c_d^5 + 2/3NF^{-4}c_d^3\lambda_1^{SS} - 8/3NF^{-4}c_d^3\lambda_2^{SS} + 5/3NF^{-4}c_d^3(\lambda_1^{SP})^2 + 1/6NF^{-4}c_d^3 + 7/3NF^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 4/3NF^{-2}c_d\lambda_1^{SS} - 2NF^{-2}c_d(\lambda_1^{SS})^2 - 4/3NF^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2 + 4/3NF^{-2}c_d\lambda_2^{SS} - 1/12NF^{-2}c_d(\lambda_1^{SP})^2 - 1/24NF^{-2}c_d$
8	$\langle u_\nu Su^\nu u\cdot u \rangle$	$-2/3NF^{-6}c_d^5 + 2/3NF^{-4}c_d^3\lambda_1^{SS} + 10/3NF^{-4}c_d^3\lambda_2^{SS} - 1/3NF^{-4}c_d^3(\lambda_1^{SP})^2 - 1/3NF^{-4}c_d^3 - 4NF^{-2}c_d\lambda_1^{SS}\lambda_2^{SS} - 2/3NF^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 + 2/3NF^{-2}c_d\lambda_1^{SS} + 8/3NF^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2 - 2/3NF^{-2}c_d\lambda_2^{SS} - 1/3NF^{-2}c_d(\lambda_1^{SP})^2 + 1/12NF^{-2}c_d$
9	$\langle S \rangle\langle u\cdot uu\cdot u \rangle$	$8/3F^{-6}c_d^5 + 4/3F^{-4}c_d^3\lambda_1^{SS} - 2F^{-4}c_d^3\lambda_2^{SS} + 4/3F^{-4}c_d^3(\lambda_1^{SP})^2 + 1/2F^{-4}c_d^3 - 1/3F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 + 1/3F^{-2}c_d\lambda_1^{SS} - 2F^{-2}c_d(\lambda_1^{SS})^2 - F^{-2}c_d\lambda_2^{SS} + 5/12F^{-2}c_d(\lambda_1^{SP})^2 - 1/24F^{-2}c_d$
10	$\langle u_\mu \rangle\langle u^\mu \{S, u\cdot u\} \rangle$	$4/3F^{-6}c_d^5 - 4/3F^{-4}c_d^3\lambda_1^{SS} + 2F^{-4}c_d^3\lambda_2^{SS} + 2/3F^{-4}c_d^3(\lambda_1^{SP})^2 - 3/2F^{-4}c_d^3 - 4F^{-2}c_d\lambda_1^{SS}\lambda_2^{SS} - 5/3F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 + 8/3F^{-2}c_d\lambda_1^{SS} - 2F^{-2}c_d(\lambda_1^{SS})^2 + c_dF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/6F^{-2}c_d(\lambda_1^{SP})^2 + 1/24F^{-2}c_d$
11	$\langle u_\mu S \rangle\langle u^\mu u\cdot u \rangle$	$8F^{-4}c_d^3\lambda_1^{SS} - 2F^{-4}c_d^3 - 8F^{-2}c_d\lambda_1^{SS}\lambda_2^{SS} + 2F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 4F^{-2}c_d(\lambda_1^{SS})^2 + 2F^{-2}c_d\lambda_2^{SS} - 1/2F^{-2}c_d(\lambda_1^{SP})^2 + 1/4F^{-2}c_d$
12*	$\langle S \rangle\langle u_\mu u_\nu u^\mu u^\nu \rangle$	$-2/3F^{-6}c_d^5 + 2/3F^{-4}c_d^3\lambda_1^{SS} - 1/3F^{-4}c_d^3(\lambda_1^{SP})^2 + 1/2F^{-4}c_d^3 + 1/3F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/3F^{-2}c_d\lambda_1^{SS} + 1/12F^{-2}c_d(\lambda_1^{SP})^2 - 1/12F^{-2}c_d$
13	$\langle Su_\nu u^\mu u^\nu \rangle\langle u_\mu \rangle$	$-8/3F^{-6}c_d^5 + 8/3F^{-4}c_d^3\lambda_1^{SS} + 4F^{-4}c_d^3\lambda_2^{SS} - 4/3F^{-4}c_d^3(\lambda_1^{SP})^2 + F^{-4}c_d^3 + 4/3F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 4/3F^{-2}c_d\lambda_1^{SS} - 2F^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2 + 2F^{-2}c_d\lambda_2^{SS} - 8F^{-2}c_d(\lambda_2^{SS})^2 + 5/6F^{-2}c_d(\lambda_1^{SP})^2 - 1/3F^{-2}c_d$

14	$\langle Su_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$	$-8F^{-6}c_d^5+8F^{-4}c_d^3\lambda_1^{SS}+16F^{-4}c_d^3\lambda_2^{SS}-4F^{-4}c_d^3(\lambda_1^{SP})^2+2F^{-4}c_d^3-8F^{-2}c_d\lambda_1^{SS}\lambda_2^{SS}+4F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2-2F^{-2}c_d\lambda_1^{SS}+4F^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2-2F^{-2}c_d\lambda_2^{SS}-8F^{-2}c_d(\lambda_2^{SS})^2$
15	$\langle Su\cdot u\rangle\langle u\cdot u\rangle$	$4F^{-6}c_d^5+4F^{-4}c_d^3\lambda_1^{SS}+2F^{-4}c_d^3(\lambda_1^{SP})^2+F^{-4}c_d^3-4F^{-2}c_d\lambda_1^{SS}\lambda_2^{SS}+2F^{-2}c_d\lambda_1^{SS}(\lambda_1^{SP})^2-F^{-2}c_d\lambda_1^{SS}-4F^{-2}c_d(\lambda_1^{SS})^2+2F^{-2}c_d\lambda_2^{SS}(\lambda_1^{SP})^2-F^{-2}c_d\lambda_2^{SS}$
16	$i\langle u_\mu f_+^{\mu\nu}u_\nu S\rangle$	$1/3NF^{-4}c_d^3-1/3NF^{-2}c_d\lambda_1^{SS}-2/3NF^{-2}c_d\lambda_2^{SS}+1/12NF^{-2}c_d$
17	$i\langle\{S,u_\mu u_\nu\}f_+^{\mu\nu}\rangle$	$-1/6NF^{-4}c_d^3+1/6NF^{-2}c_d\lambda_1^{SS}+1/3NF^{-2}c_d\lambda_2^{SS}-1/24NF^{-2}c_d$
18	$i\langle u_\mu\rangle\langle f_+^{\mu\nu}[S,u_\nu]\rangle$	$-1/3F^{-4}c_d^3+1/3F^{-2}c_d\lambda_1^{SS}+1/12F^{-2}c_d$
19	$i\langle S\rangle\langle f_+^{\mu\nu}u_\mu u_\nu\rangle$	$-2/3F^{-4}c_d^3+2/3F^{-2}c_d\lambda_1^{SS}+1/6F^{-2}c_d$
20	$\langle f_-^{\mu\nu}\{u_\mu,\nabla_\nu S\}\rangle$	$1/3NF^{-4}c_d^3-1/3NF^{-2}c_d\lambda_1^{SS}-2/3NF^{-2}c_d\lambda_2^{SS}+1/12NF^{-2}c_d$
21	$\langle u_\mu\rangle\langle f_-^{\mu\nu}\nabla_\nu S\rangle$	$2/3F^{-4}c_d^3-2/3F^{-2}c_d\lambda_1^{SS}-1/6F^{-2}c_d$
22	$\langle u_\mu f_-^{\mu\nu}\rangle\langle\nabla_\nu S\rangle$	$2/3F^{-4}c_d^3-2/3F^{-2}c_d\lambda_1^{SS}-1/6F^{-2}c_d$
23	$\langle Sf_-^{\mu\nu}f_{-\mu\nu}\rangle$	$1/3NF^{-4}c_d^3-1/3NF^{-2}c_d\lambda_1^{SS}-2/3NF^{-2}c_d\lambda_2^{SS}+1/12NF^{-2}c_d$
24	$\langle S\rangle\langle f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$1/3F^{-4}c_d^3-1/3F^{-2}c_d\lambda_1^{SS}-1/12F^{-2}c_d$
25	$\langle S\{\chi_+,u\cdot u\}\rangle$	$2NF^{-6}c_d^4c_m-NF^{-4}d_mc_d^2\lambda_1^{SP}+NF^{-4}c_d^2c_m\lambda_1^{SS}-2NF^{-4}c_d^2c_m\lambda_2^{SS}+NF^{-4}c_d^2c_m(\lambda_1^{SP})^2+1/4NF^{-4}c_d^2c_m+NF^{-4}c_d^3\lambda_3^{SS}+1/2NF^{-4}c_d^3+2NF^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}-NF^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-NF^{-2}c_d\lambda_1^{SS}\lambda_3^{SS}-NF^{-2}c_d\lambda_1^{SS}+1/2NF^{-2}c_d\lambda_2^{SS}+NF^{-2}c_d\lambda_3^{SS}(\lambda_1^{SP})^2-1/4NF^{-2}c_d\lambda_3^{SS}+1/2NF^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}-1/8NF^{-2}c_d+2NF^{-2}c_m\lambda_1^{SS}(\lambda_1^{SP})^2-1/2NF^{-2}c_m\lambda_1^{SS}-NF^{-2}c_m\lambda_2^{SS}(\lambda_1^{SP})^2+1/8NF^{-2}c_m$
26*	$\langle u_\nu Su^\nu\chi_+\rangle$	$-2NF^{-4}c_d^2c_m\lambda_1^{SS}+4NF^{-4}c_d^2c_m\lambda_2^{SS}-1/2NF^{-4}c_d^2c_m+2NF^{-4}c_d^3\lambda_3^{SS}-NF^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}+4NF^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-3/4NF^{-2}d_m\lambda_1^{SP}+1/2NF^{-2}c_d\lambda_1^{SS}-4NF^{-2}c_d\lambda_2^{SS}\lambda_3^{SS}-NF^{-2}c_d\lambda_2^{SS}+1/8NF^{-2}c_d-NF^{-2}c_m\lambda_1^{SS}(\lambda_1^{SP})^2+4F^{-2}Nc_m\lambda_2^{SS}(\lambda_1^{SP})^2-3/4NF^{-2}c_m(\lambda_1^{SP})^2$
27	$\langle Su\cdot u\rangle\langle\chi_+\rangle$	$4F^{-6}c_d^4c_m-2F^{-4}d_mc_d^2\lambda_1^{SP}+2F^{-4}c_d^2c_m\lambda_1^{SS}-2F^{-4}c_d^2c_m\lambda_2^{SS}+2F^{-4}c_d^2c_m(\lambda_1^{SP})^2+2F^{-4}c_d^3\lambda_3^{SS}+F^{-4}c_d^3+3F^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}+2F^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-3/4F^{-2}d_m\lambda_1^{SP}-2F^{-2}c_d\lambda_1^{SS}\lambda_3^{SS}-3/2F^{-2}c_d\lambda_1^{SS}-4F^{-2}c_d\lambda_2^{SS}\lambda_3^{SS}+1/2F^{-2}c_d\lambda_3^{SS}+F^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}-1/8F^{-2}c_d+3F^{-2}c_m\lambda_1^{SS}(\lambda_1^{SP})^2+2F^{-2}c_m\lambda_2^{SS}(\lambda_1^{SP})^2-F^{-2}c_m\lambda_2^{SS}-3/4F^{-2}c_m(\lambda_1^{SP})^2+1/4F^{-2}c_m$

28	$\langle S\chi_+\rangle\langle u\cdot u\rangle$	$4F^{-6}c_d^4c_m - 2F^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} + 2F^{-4}c_d^2c_m(\lambda_1^{\mathrm{SP}})^2 + F^{-4}c_d^2c_m + 4F^{-4}c_d^3\lambda_3^{\mathrm{SS}} + F^{-4}c_d^3 - F^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/4F^{-2}d_m\lambda_1^{\mathrm{SP}} - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2F^{-2}c_d\lambda_1^{\mathrm{SS}} - F^{-2}c_d\lambda_2^{\mathrm{SS}} + 2F^{-2}c_d\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 3/2F^{-2}c_d\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 3/8F^{-2}c_d - F^{-2}c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}c_m\lambda_1^{\mathrm{SS}} + 3/4F^{-2}c_m(\lambda_1^{\mathrm{SP}})^2 + 1/4F^{-2}c_m$
29	$\langle u_\mu\rangle\langle u^\mu\{S,\chi_+\}\rangle$	$4F^{-6}c_d^4c_m + 2F^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + 2F^{-4}c_d^2c_m(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-4}c_d^2c_m - F^{-4}c_d^3 - F^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}d_m\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2F^{-2}c_d\lambda_1^{\mathrm{SS}} - 2F^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2F^{-2}c_d\lambda_2^{\mathrm{SS}} - 2F^{-2}c_d\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}c_d\lambda_3^{\mathrm{SS}} - F^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/4F^{-2}c_d - F^{-2}c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}c_m\lambda_1^{\mathrm{SS}} - F^{-2}c_m\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}c_m$
30	$\langle u_\mu S\rangle\langle u^\mu\chi_+\rangle$	$8F^{-6}c_d^4c_m + 4F^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 8F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + 4F^{-4}c_d^2c_m(\lambda_1^{\mathrm{SP}})^2 - F^{-4}c_d^2c_m + 4F^{-4}c_d^3\lambda_3^{\mathrm{SS}} - 2F^{-4}c_d^3 - 2F^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}d_m\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_1^{\mathrm{SS}} - 4F^{-2}c_d\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_2^{\mathrm{SS}} - 2F^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}c_d - 2F^{-2}c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}c_m\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}c_m\lambda_2^{\mathrm{SS}} - 1/2F^{-2}c_m$
31	$\langle S\rangle\langle\chi_+u\cdot u\rangle$	$4F^{-6}c_d^4c_m - 2F^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d^2c_m\lambda_1^{\mathrm{SS}} - 2F^{-4}c_d^2c_m\lambda_2^{\mathrm{SS}} + 2F^{-4}c_d^2c_m(\lambda_1^{\mathrm{SP}})^2 + F^{-4}c_d^2c_m + 2F^{-4}c_d^3\lambda_3^{\mathrm{SS}} + F^{-4}c_d^3 - F^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/4F^{-2}d_m\lambda_1^{\mathrm{SP}} - 2F^{-2}c_d\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}} + 1/2F^{-2}c_d\lambda_1^{\mathrm{SS}} - F^{-2}c_d\lambda_2^{\mathrm{SS}} - 1/2F^{-2}c_d\lambda_3^{\mathrm{SS}} + F^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 3/8F^{-2}c_d - F^{-2}c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}c_m\lambda_2^{\mathrm{SS}} + 3/4F^{-2}c_m(\lambda_1^{\mathrm{SP}})^2 + 1/4F^{-2}c_m$
32$*$	$i\langle\chi_-\{u_\mu,\nabla^\mu S\}\rangle$	$3NF^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} + 3NF^{-4}c_d^2c_m - 3/2NF^{-4}c_d^3 - NF^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2NF^{-2}d_m\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/4NF^{-2}d_m\lambda_1^{\mathrm{SP}} + 3/2NF^{-2}d_m(\lambda_1^{\mathrm{SP}})^3 + 1/2NF^{-2}c_d\lambda_1^{\mathrm{SS}} + NF^{-2}c_d\lambda_2^{\mathrm{SS}} + 3NF^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 3/2NF^{-2}c_d(\lambda_1^{\mathrm{SP}})^2 + 3/8NF^{-2}c_d - NF^{-2}c_m\lambda_1^{\mathrm{SS}} - 2NF^{-2}c_m\lambda_2^{\mathrm{SS}} + 3/2NF^{-2}c_m(\lambda_1^{\mathrm{SP}})^2 - 3/4NF^{-2}c_m$
33$*$	$i\langle u_\mu\chi_-\rangle\langle\nabla^\mu S\rangle$	$-2F^{-4}d_mc_d^2\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d^2c_m + F^{-4}c_d^3 - 2F^{-2}d_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/2F^{-2}d_m\lambda_1^{\mathrm{SP}} - F^{-2}d_m(\lambda_1^{\mathrm{SP}})^3 + F^{-2}c_d\lambda_1^{\mathrm{SS}} - 2F^{-2}c_d\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + F^{-2}c_d(\lambda_1^{\mathrm{SP}})^2 - 3/4F^{-2}c_d - 2F^{-2}c_m\lambda_1^{\mathrm{SS}} - F^{-2}c_m(\lambda_1^{\mathrm{SP}})^2 + 3/2F^{-2}c_m$

34*	$i\langle\chi_-\rangle\langle u_\mu\nabla^\mu S\rangle$	$6F^{-4}d_mc_d^2\lambda_1^{SP}+6F^{-4}c_d^2c_m-3F^{-4}c_d^3-2F^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}-4F^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-3/2F^{-2}d_m\lambda_1^{SP}+3F^{-2}d_m(\lambda_1^{SP})^3+F^{-2}c_d\lambda_1^{SS}+2F^{-2}c_d\lambda_2^{SS}+6F^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}-3F^{-2}c_d(\lambda_1^{SP})^2+3/4F^{-2}c_d-2F^{-2}c_m\lambda_1^{SS}-4F^{-2}c_m\lambda_2^{SS}+3F^{-2}c_m(\lambda_1^{SP})^2-3/2F^{-2}c_m$
35*	$i\langle\chi_-\nabla^\mu S\rangle\langle u_\mu\rangle$	$-2F^{-4}d_mc_d^2\lambda_1^{SP}-2F^{-4}c_d^2c_m+F^{-4}c_d^3-2F^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}+3/2F^{-2}d_m\lambda_1^{SP}-F^{-2}d_m(\lambda_1^{SP})^3+F^{-2}c_d\lambda_1^{SS}-2F^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}+F^{-2}c_d(\lambda_1^{SP})^2-3/4F^{-2}c_d-2F^{-2}c_m\lambda_1^{SS}-F^{-2}c_m(\lambda_1^{SP})^2+3/2F^{-2}c_m$
36	$\langle S\chi_+\chi_+\rangle$	$-4NF^{-4}d_mc_dc_m\lambda_1^{SP}-2NF^{-4}d_m^2c_d-NF^{-4}c_dc_m^2+4NF^{-4}c_d^2c_m\lambda_3^{SS}+2NF^{-4}c_d^2c_m+4NF^{-2}d_m\lambda_3^{SS}\lambda_1^{SP}+2NF^{-2}d_m\lambda_2^{SP}-NF^{-2}c_d\lambda_3^{SS}-1/4NF^{-2}c_d-NF^{-2}c_m\lambda_1^{SS}+4NF^{-2}c_m\lambda_3^{SS}(\lambda_1^{SP})^2+2NF^{-2}c_m\lambda_1^{SP}\lambda_2^{SP}+1/4NF^{-2}c_m$
37	$\langle S\rangle\langle\chi_+\chi_+\rangle$	$-4F^{-4}d_mc_dc_m\lambda_1^{SP}-2F^{-4}d_m^2c_d-F^{-4}c_dc_m^2+2F^{-4}c_d^2c_m+2F^{-2}d_m\lambda_2^{SP}-1/4F^{-2}c_d-F^{-2}c_m\lambda_2^{SS}+2F^{-2}c_m\lambda_1^{SP}\lambda_2^{SP}+1/4F^{-2}c_m$
38	$\langle S\chi_+\rangle\langle\chi_+\rangle$	$-8F^{-4}d_mc_dc_m\lambda_1^{SP}-4F^{-4}d_m^2c_d-2F^{-4}c_dc_m^2+4F^{-4}c_d^2c_m\lambda_3^{SS}+4F^{-4}c_d^2c_m+4F^{-2}d_m\lambda_3^{SS}\lambda_1^{SP}+4F^{-2}d_m\lambda_2^{SP}-F^{-2}c_d\lambda_3^{SS}-1/2F^{-2}c_d-F^{-2}c_m\lambda_1^{SS}-F^{-2}c_m\lambda_2^{SS}+4F^{-2}c_m\lambda_3^{SS}(\lambda_1^{SP})^2+4F^{-2}c_m\lambda_1^{SP}\lambda_2^{SP}+1/2F^{-2}c_m$
39	$\langle S\chi_-\chi_-\rangle$	$8NF^{-4}d_mc_dc_m\lambda_1^{SP}-6NF^{-4}d_mc_d^2\lambda_1^{SP}+6NF^{-4}c_dc_m^2-8NF^{-4}c_d^2c_m+5/2NF^{-4}c_d^3+NF^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}+2NF^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-2NF^{-2}d_m\lambda_3^{SS}\lambda_1^{SP}+1/4NF^{-2}d_m\lambda_1^{SP}-4NF^{-2}d_m(\lambda_1^{SP})^2\lambda_2^{SP}-1/2NF^{-2}c_d\lambda_1^{SS}-NF^{-2}c_d\lambda_2^{SS}+NF^{-2}c_d\lambda_3^{SS}-2NF^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}+NF^{-2}c_d(\lambda_1^{SP})^2-1/8NF^{-2}c_d+NF^{-2}c_m\lambda_1^{SS}+2NF^{-2}c_m\lambda_2^{SS}-2NF^{-2}c_m\lambda_3^{SS}-NF^{-2}c_m(\lambda_1^{SP})^2+1/4NF^{-2}c_m$
40	$\langle S\rangle\langle\chi_-\chi_-\rangle$	$4F^{-4}d_mc_dc_m\lambda_1^{SP}-2F^{-4}d_mc_d^2\lambda_1^{SP}+2F^{-4}c_dc_m^2-2F^{-4}c_d^2c_m+1/2F^{-4}c_d^3+F^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}-2F^{-2}d_m\lambda_3^{SS}\lambda_1^{SP}-1/4F^{-2}d_m\lambda_1^{SP}-1/2F^{-2}c_d\lambda_1^{SS}+F^{-2}c_d\lambda_3^{SS}+1/8F^{-2}c_d+F^{-2}c_m\lambda_1^{SS}-2F^{-2}c_m\lambda_3^{SS}-1/4F^{-2}c_m$
41	$\langle S\chi_-\rangle\langle\chi_-\rangle$	$12F^{-4}d_mc_dc_m\lambda_1^{SP}-8F^{-4}d_mc_d^2\lambda_1^{SP}+8F^{-4}c_dc_m^2-10F^{-4}c_d^2c_m+3F^{-4}c_d^3+2F^{-2}d_m\lambda_1^{SS}\lambda_1^{SP}+2F^{-2}d_m\lambda_2^{SS}\lambda_1^{SP}-4F^{-2}d_m\lambda_3^{SS}\lambda_1^{SP}-4F^{-2}d_m(\lambda_1^{SP})^2\lambda_2^{SP}-F^{-2}c_d\lambda_1^{SS}-F^{-2}c_d\lambda_2^{SS}+2F^{-2}c_d\lambda_3^{SS}-2F^{-2}c_d\lambda_1^{SP}\lambda_2^{SP}+F^{-2}c_d(\lambda_1^{SP})^2+2F^{-2}c_m\lambda_1^{SS}+2F^{-2}c_m\lambda_2^{SS}-4F^{-2}c_m\lambda_3^{SS}-F^{-2}c_m(\lambda_1^{SP})^2$

42	$i\langle P\chi_-\rangle$	$-2NF^{-2}d_mM_P^2(\lambda_1^{SP})^2 + 4NF^{-2}d_mM_S^2(\lambda_1^{SP})^2 + NF^{-2}M_P^2c_d\lambda_1^{SP} - 2NF^{-2}M_P^2c_m\lambda_1^{SP} - 2NF^{-2}c_dM_S^2\lambda_1^{SP} + 4NF^{-2}c_mM_S^2\lambda_1^{SP} - NF^{-2}d_mM_S^2(\lambda_1^{SP})^2 - NF^{-2}c_dM_S^2\lambda_1^{SP}$
43	$i\langle P\rangle\langle\chi_-\rangle$	$-2F^{-2}d_mM_P^2(\lambda_1^{SP})^2+4F^{-2}d_mM_S^2(\lambda_1^{SP})^2+F^{-2}M_P^2c_d\lambda_1^{SP} - 2F^{-2}M_P^2c_m\lambda_1^{SP} - 2F^{-2}c_dM_S^2\lambda_1^{SP} + 4F^{-2}c_mM_S^2\lambda_1^{SP} - F^{-2}d_mM_S^2(\lambda_1^{SP})^2 - F^{-2}c_dM_S^2\lambda_1^{SP}$
44	$\langle\nabla_\mu Pu_\nu u^\mu u^\nu\rangle$	$-1/3NF^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} + 2/3NF^{-2}\lambda_2^{PP}c_d\lambda_1^{SP} + 1/12NF^{-2}c_d\lambda_1^{SP}$
45	$\langle u\cdot u\{u_\mu,\nabla^\mu P\}\rangle$	$2/3NF^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} + 2/3NF^{-2}\lambda_2^{PP}c_d\lambda_1^{SP} - NF^{-2}c_d\lambda_1^{SS}\lambda_1^{SP} + 7/12NF^{-2}c_d\lambda_1^{SP} - 1/2NF^{-2}c_d(\lambda_1^{SP})^3$
46	$\langle u_\mu\rangle\langle\nabla^\mu Pu\cdot u\rangle$	$F^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} + 2F^{-4}c_d^3\lambda_1^{SP} - 5/4F^{-2}c_d\lambda_1^{SP} + F^{-2}c_d(\lambda_1^{SP})^3 - 2F^{-2}c_d\lambda_1^{PP}\lambda_1^{SP}$
47	$\langle u_\mu\rangle\langle\nabla_\nu P\{u^\mu,u^\nu\}\rangle$	$2F^{-4}c_d^3\lambda_1^{SP} - F^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} - 2F^{-2}\lambda_2^{PP}c_d\lambda_1^{SP} - 2F^{-2}c_d\lambda_2^{SS}\lambda_1^{SP} - 5/4F^{-2}c_d\lambda_1^{SP} + 2F^{-2}c_d(\lambda_1^{SP})^3$
48	$\langle u_\mu u_\nu\rangle\langle u^\nu\nabla^\mu P\rangle$	$-2F^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} - 4F^{-2}c_d\lambda_2^{SS}\lambda_1^{SP} + 5/2F^{-2}c_d\lambda_1^{SP}$
49	$\langle u_\mu u\cdot u\rangle\langle\nabla^\mu P\rangle$	$2F^{-4}c_d^3\lambda_1^{SP} + F^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} - 2F^{-2}c_d\lambda_1^{SS}\lambda_1^{SP} - 5/4F^{-2}c_d\lambda_1^{SP} + F^{-2}c_d(\lambda_1^{SP})^3$
50	$\langle u\cdot u\rangle\langle u_\mu\nabla^\mu P\rangle$	$F^{-2}\lambda_1^{PP}c_d\lambda_1^{SP} + 2F^{-2}\lambda_2^{PP}c_d\lambda_1^{SP} - 2F^{-2}c_d\lambda_1^{SS}\lambda_1^{SP} + 5/4F^{-2}c_d\lambda_1^{SP} - F^{-2}c_d(\lambda_1^{SP})^3$
51	$\langle[P,u_\mu u_\nu]f_-^{\mu\nu}\rangle$	$-1/3NF^{-4}c_d^3\lambda_1^{SP}$
52*	$\langle\chi_+\{u_\mu,\nabla^\mu P\}\rangle$	$-NF^{-4}c_d^2c_m\lambda_1^{SP} + NF^{-2}d_m\lambda_1^{PP} + 2NF^{-2}d_m\lambda_2^{PP} - 3/2NF^{-2}d_m(\lambda_1^{SP})^2 + 3/4NF^{-2}d_m + NF^{-2}\lambda_1^{PP}c_m\lambda_1^{SP} + 2NF^{-2}\lambda_2^{PP}c_m\lambda_1^{SP} - NF^{-2}c_d\lambda_3^{SS}\lambda_1^{SP} + 1/4NF^{-2}c_d\lambda_1^{SP} + 3/4NF^{-2}c_m\lambda_1^{SP} - 3/2NF^{-2}c_m(\lambda_1^{SP})^3$
53*	$\langle u_\mu\chi_+\rangle\langle\nabla^\mu P\rangle$	$2F^{-4}c_d^2c_m\lambda_1^{SP}+2F^{-2}d_m\lambda_1^{PP}+F^{-2}d_m(\lambda_1^{SP})^2-3/2F^{-2}d_m+ 2F^{-2}\lambda_1^{PP}c_m\lambda_1^{SP} - 2F^{-2}c_d\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}c_d\lambda_1^{SP} - 3/2F^{-2}c_m\lambda_1^{SP} + F^{-2}c_m(\lambda_1^{SP})^3$
54*	$\langle\chi_+\rangle\langle u_\mu\nabla^\mu P\rangle$	$-2F^{-4}c_d^2c_m\lambda_1^{SP} + 2F^{-2}d_m\lambda_1^{PP} + 4F^{-2}d_m\lambda_2^{PP} - 3F^{-2}d_m(\lambda_1^{SP})^2 + 3/2F^{-2}d_m + 2F^{-2}\lambda_1^{PP}c_m\lambda_1^{SP} + 4F^{-2}\lambda_2^{PP}c_m\lambda_1^{SP} - 2F^{-2}c_d\lambda_3^{SS}\lambda_1^{SP} + 1/2F^{-2}c_d\lambda_1^{SP} + 3/2F^{-2}c_m\lambda_1^{SP} - 3F^{-2}c_m(\lambda_1^{SP})^3$
55*	$\langle\chi_+\nabla^\mu P\rangle\langle u_\mu\rangle$	$2F^{-4}c_d^2c_m\lambda_1^{SP}+2F^{-2}d_m\lambda_1^{PP}+F^{-2}d_m(\lambda_1^{SP})^2-3/2F^{-2}d_m+ 2F^{-2}\lambda_1^{PP}c_m\lambda_1^{SP} - 2F^{-2}c_d\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}c_d\lambda_1^{SP} - 3/2F^{-2}c_m\lambda_1^{SP} + F^{-2}c_m(\lambda_1^{SP})^3$
56	$i\langle P\{\chi_-,u\cdot u\}\rangle$	$-NF^{-4}d_m\lambda_1^{PP}c_d^2+6NF^{-4}d_mc_d^2(\lambda_1^{SP})^2-1/4NF^{-4}d_mc_d^2+ 4NF^{-4}c_d^2c_m\lambda_1^{SP} - 5/2NF^{-4}c_d^3\lambda_1^{SP} + NF^{-4}c_d^3\lambda_2^{SP} + 2NF^{-2}d_m\lambda_1^{PP}(\lambda_1^{SP})^2 - 1/2NF^{-2}d_m\lambda_1^{PP} - NF^{-2}d_m\lambda_2^{PP}(\lambda_1^{SP})^2 - 2NF^{-2}d_m\lambda_1^{SS}(\lambda_1^{SP})^2 + 1/2NF^{-2}d_m(\lambda_1^{SP})^4+1/8NF^{-2}d_m-1/2NF^{-2}\lambda_1^{PP}c_d\lambda_1^{SP}- NF^{-2}\lambda_1^{PP}c_d\lambda_2^{SP} + 2NF^{-2}\lambda_1^{PP}c_m\lambda_1^{SP} + NF^{-2}\lambda_2^{PP}c_d\lambda_1^{SP} - NF^{-2}\lambda_2^{PP}c_m\lambda_1^{SP}-1/2NF^{-2}\lambda_3^{PP}c_d\lambda_1^{SP}+NF^{-2}c_d\lambda_1^{SS}\lambda_1^{SP}+ 1/4NF^{-2}c_d\lambda_1^{SP}+6NF^{-2}c_d(\lambda_1^{SP})^2\lambda_2^{SP}-2NF^{-2}c_d(\lambda_1^{SP})^3- 1/4NF^{-2}c_d\lambda_2^{SP} - 2NF^{-2}c_m\lambda_1^{SS}\lambda_1^{SP} + 1/2NF^{-2}c_m(\lambda_1^{SP})^3$

57∗	$i\langle u_\nu P u^\nu \chi_-\rangle$	$-2NF^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 - 2NF^{-4}c_d^2 c_m\lambda_1^{\rm SP} + NF^{-4}c_d^3\lambda_1^{\rm SP} - NF^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 4NF^{-2}d_m\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4NF^{-2}d_m(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 4NF^{-2}\lambda_2^{\rm PP}c_d\lambda_2^{\rm SP} + 4NF^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} + 3/8NF^{-2}c_d\lambda_1^{\rm SP} + 2NF^{-2}c_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2NF^{-2}c_d(\lambda_1^{\rm SP})^3 - 3/4NF^{-2}c_m\lambda_1^{\rm SP}$
58	$i\langle u_\mu\rangle\langle u^\mu\{P,\chi_-\}\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d^2 + 2F^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-4}d_m c_d^2 + 2F^{-4}c_d^2 c_m\lambda_1^{\rm SP} - 2F^{-4}c_d^3\lambda_2^{\rm SP} - F^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + F^{-2}d_m\lambda_1^{\rm PP} - F^{-2}d_m\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4F^{-2}d_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-2}d_m(\lambda_1^{\rm SP})^4 - 1/4F^{-2}d_m + 1/2F^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}c_d\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}c_d\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/8F^{-2}c_d\lambda_1^{\rm SP} - 2F^{-2}c_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2F^{-2}c_d(\lambda_1^{\rm SP})^3 + F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} + F^{-2}c_m(\lambda_1^{\rm SP})^3$
59	$i\langle u_\mu P\rangle\langle u^\mu\chi_-\rangle$	$-4F^{-4}d_m\lambda_2^{\rm PP}c_d^2 + 4F^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 + F^{-4}d_m c_d^2 + 4F^{-4}c_d^2 c_m\lambda_1^{\rm SP} - 2F^{-4}c_d^3\lambda_1^{\rm SP} - 2F^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}d_m\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}d_m\lambda_2^{\rm PP} - 8F^{-2}d_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}d_m(\lambda_1^{\rm SP})^4 - 1/2F^{-2}d_m + F^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 4F^{-2}\lambda_1^{\rm PP}c_d\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm PP}c_d\lambda_2^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 4F^{-2}c_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}c_d\lambda_1^{\rm SP} - F^{-2}c_d(\lambda_1^{\rm SP})^3 - 8F^{-2}c_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}c_m(\lambda_1^{\rm SP})^3$
60	$i\langle Pu\cdot u\rangle\langle\chi_-\rangle$	$-2F^{-4}d_m\lambda_2^{\rm PP}c_d^2 + 10F^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 - 1/2F^{-4}d_m c_d^2 + 6F^{-4}c_d^2 c_m\lambda_1^{\rm SP} - 3F^{-4}c_d^3\lambda_1^{\rm SP} + 3F^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}d_m\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}d_m\lambda_2^{\rm PP} - 4F^{-2}d_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}d_m(\lambda_1^{\rm SP})^2 + F^{-2}d_m(\lambda_1^{\rm SP})^4 + 1/4F^{-2}d_m - 1/2F^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}c_d\lambda_2^{\rm SP} + 3F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm PP}c_d\lambda_2^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} - F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/8F^{-2}c_d\lambda_1^{\rm SP} + 12F^{-2}c_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 7/2F^{-2}c_d(\lambda_1^{\rm SP})^3 + 1/2F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-2}c_m\lambda_1^{\rm SP} + F^{-2}c_m(\lambda_1^{\rm SP})^3$

61	$i\langle P\rangle\langle \chi_- u\cdot u\rangle$	$-2F^{-4}d_m\lambda_2^{\rm PP}c_d^2 + 2F^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 - 1/2F^{-4}d_m c_d^2 - 2F^{-4}c_d^2c_m\lambda_1^{\rm SP} + F^{-4}c_d^3\lambda_1^{\rm SP} - F^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}d_m\lambda_2^{\rm PP} - 4F^{-2}d_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/4F^{-2}d_m(\lambda_1^{\rm SP})^2 + F^{-2}d_m(\lambda_1^{\rm SP})^4 + 1/4F^{-2}d_m + 3/2F^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}c_d\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/8F^{-2}c_d\lambda_1^{\rm SP} + 4F^{-2}c_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3/2F^{-2}c_d(\lambda_1^{\rm SP})^3 - 1/2F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}c_m\lambda_1^{\rm SP} + F^{-2}c_m(\lambda_1^{\rm SP})^3$
62	$i\langle P\chi_-\rangle\langle u\cdot u\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d^2 + 2F^{-4}d_m c_d^2(\lambda_1^{\rm SP})^2 - 1/2F^{-4}d_m c_d^2 - 2F^{-4}c_d^2c_m\lambda_1^{\rm SP} + 2F^{-4}c_d^3\lambda_2^{\rm SP} - F^{-2}d_m\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}d_m\lambda_1^{\rm PP} - 4F^{-2}d_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/4F^{-2}d_m(\lambda_1^{\rm SP})^2 + F^{-2}d_m(\lambda_1^{\rm SP})^4 + 1/4F^{-2}d_m + 3/2F^{-2}\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}c_d\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} + F^{-2}\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/8F^{-2}c_d\lambda_1^{\rm SP} + 6F^{-2}c_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 5/2F^{-2}c_d(\lambda_1^{\rm SP})^3 - 3/2F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}c_m\lambda_1^{\rm SP} + F^{-2}c_m(\lambda_1^{\rm SP})^3$
63	$i\langle P\{\chi_-,\chi_+\}\rangle$	$4NF^{-4}d_mc_dc_m(\lambda_1^{\rm SP})^2 + 3/2NF^{-4}d_mc_dc_m - NF^{-4}d_mc_d^2 + 2NF^{-4}d_m^2c_d\lambda_1^{\rm SP} + 2NF^{-4}c_dc_m^2\lambda_1^{\rm SP} - 2NF^{-4}c_d^2c_m\lambda_1^{\rm SP} + 2NF^{-4}c_d^2c_m\lambda_2^{\rm SP} + NF^{-2}d_m\lambda_2^{\rm PP} + 2NF^{-2}d_m\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - NF^{-2}d_m\lambda_3^{\rm PP} - 2NF^{-2}d_m\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 5NF^{-2}d_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/2NF^{-2}d_m(\lambda_1^{\rm SP})^2 + 1/4NF^{-2}d_m + 1/2NF^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} + NF^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} - NF^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + NF^{-2}\lambda_3^{\rm PP}c_m\lambda_1^{\rm SP} + NF^{-2}c_d\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}c_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 5/8NF^{-2}c_m\lambda_1^{\rm SP} + 8NF^{-2}c_m(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 5/2NF^{-2}c_m(\lambda_1^{\rm SP})^3 - NF^{-2}c_m\lambda_2^{\rm SP}$
64	$i\langle P\rangle\langle \chi_-\chi_+\rangle$	$-F^{-4}d_mc_dc_m - 4F^{-4}c_dc_m^2\lambda_1^{\rm SP} + 2F^{-4}c_d^2c_m\lambda_1^{\rm SP} + F^{-2}d_m\lambda_1^{\rm PP} - F^{-2}d_m\lambda_2^{\rm PP} - 2F^{-2}d_m\lambda_3^{\rm PP} - 4F^{-2}d_m\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}d_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - 2F^{-2}\lambda_3^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}c_d\lambda_1^{\rm SP} + F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}c_m\lambda_1^{\rm SP} + 4F^{-2}c_m(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - F^{-2}c_m(\lambda_1^{\rm SP})^3 - 2F^{-2}c_m\lambda_2^{\rm SP}$
65	$i\langle P\chi_+\rangle\langle \chi_-\rangle$	$8F^{-4}d_mc_dc_m(\lambda_1^{\rm SP})^2 + 3F^{-4}d_mc_dc_m - 2F^{-4}d_mc_d^2 + 4F^{-4}d_m^2c_d\lambda_1^{\rm SP} + 4F^{-4}c_dc_m^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2c_m\lambda_1^{\rm SP} + F^{-2}d_m\lambda_1^{\rm PP} - F^{-2}d_m\lambda_2^{\rm PP} + 4F^{-2}d_m\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}d_m\lambda_3^{\rm PP} - 4F^{-2}d_m\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 6F^{-2}d_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}d_m(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - 2F^{-2}\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} + 2F^{-2}\lambda_3^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}c_d\lambda_1^{\rm SP} + F^{-2}c_d\lambda_2^{\rm SP} - 4F^{-2}c_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}c_m\lambda_1^{\rm SP} + 12F^{-2}c_m(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3F^{-2}c_m(\lambda_1^{\rm SP})^3 - 2F^{-2}c_m\lambda_2^{\rm SP}$

66	$i\langle P\chi_-\rangle\langle\chi_+\rangle$	$-F^{-4}d_m c_d c_m - 4F^{-4}c_d c_m^2\lambda_1^{\rm SP} + 4F^{-4}c_d^2 c_m\lambda_2^{\rm SP} + 2F^{-2}d_m\lambda_2^{\rm PP} - 2F^{-2}d_m\lambda_3^{\rm PP} - 4F^{-2}d_m\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 6F^{-2}d_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-2}d_m(\lambda_1^{\rm SP})^2 + 1/2F^{-2}d_m + F^{-2}\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} - 2F^{-2}\lambda_3^{\rm PP}c_m\lambda_1^{\rm SP} + 2F^{-2}c_d\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}c_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 5/4F^{-2}c_m\lambda_1^{\rm SP} + 8F^{-2}c_m(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3F^{-2}c_m(\lambda_1^{\rm SP})^3 - 2F^{-2}c_m\lambda_2^{\rm SP}$

Table G.2: Operators with two resonances, scalars and pseudoscalars, and their β-function coefficients.

i	$\mathcal{O}_i^{RR}$	γ_i^{RR}
1	$\langle SS\rangle$	$NF^{-4}c_d^2M_S^4 + NF^{-2}M_S^4(\lambda_1^{\rm SP})^2 + NF^{-2}M_P^2M_S^2(\lambda_1^{\rm SP})^2$
2	$\langle S\rangle^2$	$F^{-4}c_d^2M_S^4 + F^{-2}M_S^4(\lambda_1^{\rm SP})^2 + F^{-2}M_P^2M_S^2(\lambda_1^{\rm SP})^2$
3	$\langle SSu\cdot u\rangle$	$-4NF^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 4NF^{-4}c_d^2\lambda_2^{\rm SS}M_S^2 - 2NF^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 - NF^{-4}c_d^2M_S^2 - 4NF^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}\lambda_1^{\rm SS}M_S^2 + NF^{-2}(\lambda_1^{\rm SS})^2M_S^2 + 2NF^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 4NF^{-2}(\lambda_2^{\rm SS})^2M_S^2 + 1/16NF^{-2}M_S^2 - 2NF^{-2}\lambda_1^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2$
4	$\langle u_\nu Su^\nu S\rangle$	$-2NF^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 4NF^{-4}c_d^2\lambda_2^{\rm SS}M_S^2 + 3/2NF^{-4}c_d^2M_S^2 + 4NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}M_S^2 + NF^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - 4NF^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_2^{\rm SS}M_S^2 + 3/4NF^{-2}M_S^2(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2$
5	$\langle u_\mu\rangle\langle u^\mu SS\rangle$	$-4F^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 4F^{-4}c_d^2\lambda_2^{\rm SS}M_S^2 - 4F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-4}c_d^2M_S^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}M_S^2 + 2F^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm SS}M_S^2 + 2F^{-2}(\lambda_1^{\rm SS})^2M_S^2 + 2F^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_2^{\rm SS}M_S^2 - 1/8F^{-2}M_S^2$
6	$\langle u_\mu S\rangle^2$	$-4F^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 4F^{-4}c_d^2\lambda_2^{\rm SS}M_S^2 - 4F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-4}c_d^2M_S^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}M_S^2 + 2F^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_1^{\rm SS})^2M_S^2 + 2F^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - 3F^{-2}\lambda_2^{\rm SS}M_S^2 + 4F^{-2}(\lambda_2^{\rm SS})^2M_S^2 + 1/8F^{-2}M_S^2$
7	$\langle S\rangle\langle Su\cdot u\rangle$	$-8F^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 8F^{-4}c_d^2\lambda_2^{\rm SS}M_S^2 - 4F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2M_S^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}M_S^2 - 2F^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_1^{\rm SS})^2M_S^2 - 2F^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm SS}M_S^2 - 1/8F^{-2}M_S^2 - 2F^{-2}\lambda_1^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_2^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2$
8	$\langle SS\rangle\langle u\cdot u\rangle$	$-2F^{-4}c_d^2\lambda_1^{\rm SS}M_S^2 - 2F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 - 5/2F^{-4}c_d^2M_S^2 + F^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 3/2F^{-2}\lambda_1^{\rm SS}M_S^2 + F^{-2}(\lambda_1^{\rm SS})^2M_S^2 - 3/4F^{-2}M_S^2(\lambda_1^{\rm SP})^2 + 1/16F^{-2}M_S^2$

9	$\langle SS\chi_+ \rangle$	$4NF^{-4}d_mc_dM_S^2\lambda_1^{\rm SP}+NF^{-4}c_dc_mM_S^2-4NF^{-4}c_d^2\lambda_3^{\rm SS}M_S^2-2NF^{-4}c_d^2M_S^2+NF^{-2}\lambda_1^{\rm SS}M_S^2-4NF^{-2}\lambda_3^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2-2NF^{-2}M_S^2\lambda_1^{\rm SP}\lambda_2^{\rm SP}-2NF^{-2}\lambda_3^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2$
10	$\langle S\rangle\langle S\chi_+ \rangle$	$8F^{-4}d_mc_dM_S^2\lambda_1^{\rm SP}+2F^{-4}c_dc_mM_S^2-4F^{-4}c_d^2\lambda_3^{\rm SS}M_S^2-4F^{-4}c_d^2M_S^2+2F^{-2}\lambda_2^{\rm SS}M_S^2-4F^{-2}\lambda_3^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2-4F^{-2}M_S^2\lambda_1^{\rm SP}\lambda_2^{\rm SP}-2F^{-2}\lambda_3^{\rm SS}M_P^2(\lambda_1^{\rm SP})^2$
11	$\langle SS\rangle\langle \chi_+ \rangle$	$4F^{-4}d_mc_dM_S^2\lambda_1^{\rm SP}+F^{-4}c_dc_mM_S^2-2F^{-4}c_d^2M_S^2+F^{-2}\lambda_1^{\rm SS}M_S^2-2F^{-2}M_S^2\lambda_1^{\rm SP}\lambda_2^{\rm SP}$
12	$\langle u_\mu u_\nu S u^\mu u^\nu S\rangle$	$2/3NF^{-8}c_d^6-8/3NF^{-6}c_d^4\lambda_2^{\rm SS}-1/3NF^{-4}c_d^2\lambda_1^{\rm SS}+2/3NF^{-4}c_d^2(\lambda_1^{\rm SS})^2+1/3NF^{-4}c_d^2\lambda_2^{\rm SS}+8/3NF^{-4}c_d^2(\lambda_2^{\rm SS})^2-1/6NF^{-4}c_d^2(\lambda_1^{\rm SP})^2-1/24NF^{-4}c_d^2-1/12NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+1/24NF^{-2}\lambda_1^{\rm SS}+1/6NF^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2-1/12NF^{-2}(\lambda_1^{\rm SS})^2+1/6NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2-1/3NF^{-2}(\lambda_2^{\rm SS})^2-1/32NF^{-2}(\lambda_1^{\rm SP})^2+1/64NF^{-2}$
13	$\langle Su_\mu Su_\nu u^\mu u^\nu\rangle$	$-8/3NF^{-6}c_d^4\lambda_1^{\rm SS}+2/3NF^{-6}c_d^4+4NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS}-2/3NF^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+2/3NF^{-4}c_d^2\lambda_1^{\rm SS}-NF^{-4}c_d^2\lambda_2^{\rm SS}+1/6NF^{-4}c_d^2(\lambda_1^{\rm SP})^2-1/6NF^{-4}c_d^2+2/3NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2-2/3NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}-1/6NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2+1/6NF^{-2}\lambda_2^{\rm SS}$
14	$\langle Su\cdot uSu\cdot u\rangle$	$2/3NF^{-8}c_d^6+4NF^{-6}c_d^4\lambda_1^{\rm SS}-8/3NF^{-6}c_d^4\lambda_2^{\rm SS}-4NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS}+2NF^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+2/3NF^{-4}c_d^2\lambda_1^{\rm SS}+2/3NF^{-4}c_d^2(\lambda_1^{\rm SS})^2-NF^{-4}c_d^2\lambda_2^{\rm SS}+8/3NF^{-4}c_d^2(\lambda_2^{\rm SS})^2-1/12NF^{-4}c_d^2-2NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2-1/12NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2-1/16NF^{-2}\lambda_1^{\rm SS}+13/6NF^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2-1/2NF^{-2}(\lambda_1^{\rm SS})^2-NF^{-2}(\lambda_1^{\rm SS})^3+NF^{-2}(\lambda_2^{\rm SS})^2+1/96NF^{-2}(\lambda_1^{\rm SP})^2$
15	$\langle SSu_\mu u_\nu u^\mu u^\nu\rangle$	$4/3NF^{-8}c_d^6-2/3NF^{-6}c_d^4\lambda_1^{\rm SS}-8/3NF^{-6}c_d^4\lambda_2^{\rm SS}+2/3NF^{-6}c_d^4(\lambda_1^{\rm SP})^2-2/3NF^{-6}c_d^4-1/3NF^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2+1/6NF^{-4}c_d^2\lambda_1^{\rm SS}+1/3NF^{-4}c_d^2(\lambda_1^{\rm SS})^2-4/3NF^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2+NF^{-4}c_d^2\lambda_2^{\rm SS}+4/3NF^{-4}c_d^2(\lambda_2^{\rm SS})^2+1/6NF^{-4}c_d^2(\lambda_1^{\rm SP})^2+5/48NF^{-4}c_d^2+1/24NF^{-2}\lambda_1^{\rm SS}-1/12NF^{-2}(\lambda_1^{\rm SS})^2-1/6NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2+2/3NF^{-2}(\lambda_2^{\rm SS})^2(\lambda_1^{\rm SP})^2-1/3NF^{-2}(\lambda_2^{\rm SS})^2+1/24NF^{-2}(\lambda_1^{\rm SP})^2-5/192NF^{-2}$

16	$\langle\, SSu\cdot uu\cdot u\,\rangle$	$4/3NF^{-8}c_d^6 + 10/3NF^{-6}c_d^4\lambda_1^{\mathrm{SS}} - 8/3NF^{-6}c_d^4\lambda_2^{\mathrm{SS}} + 2/3NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 4/3NF^{-6}c_d^4 - 4NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} + 5/3NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/6NF^{-4}c_d^2\lambda_1^{\mathrm{SS}} + 1/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SS}})^2 - 4/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 5/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}} + 4/3NF^{-4}c_d^2(\lambda_2^{\mathrm{SS}})^2 + 7/48NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-2}\lambda_1^{\mathrm{SS}} + 2NF^{-2}(\lambda_1^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 11/12NF^{-2}(\lambda_1^{\mathrm{SS}})^2 + 2/3NF^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}(\lambda_2^{\mathrm{SS}})^2 + 1/192NF^{-2}$
17	$\langle\, u_\nu S u^\nu \{S, u\cdot u\}\,\rangle$	$-8/3NF^{-6}c_d^4\lambda_1^{\mathrm{SS}} + 4NF^{-6}c_d^4\lambda_2^{\mathrm{SS}} - 1/3NF^{-6}c_d^4 + 6NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} - 2/3NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 11/6NF^{-4}c_d^2\lambda_1^{\mathrm{SS}} + 2NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 3/2NF^{-4}c_d^2\lambda_2^{\mathrm{SS}} - 4NF^{-4}c_d^2(\lambda_2^{\mathrm{SS}})^2 - 1/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-4}c_d^2 + 14/3NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} - 3/4NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}(\lambda_1^{\mathrm{SS}})^2\lambda_2^{\mathrm{SS}} - NF^{-2}(\lambda_1^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/24NF^{-2}\lambda_2^{\mathrm{SS}} - 2NF^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2$
18	$\langle\, u_\mu S u^\mu u_\nu S u^\nu\,\rangle$	$2/3NF^{-8}c_d^6 - 8/3NF^{-6}c_d^4\lambda_2^{\mathrm{SS}} - 4NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} + 2/3NF^{-4}c_d^2\lambda_1^{\mathrm{SS}} + 2/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SS}})^2 - 4/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}} + 20/3NF^{-4}c_d^2(\lambda_2^{\mathrm{SS}})^2 + 1/6NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/8NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 5/12NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-2}\lambda_1^{\mathrm{SS}} + 1/6NF^{-2}(\lambda_1^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/12NF^{-2}(\lambda_1^{\mathrm{SS}})^2 - 5/3NF^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}NF^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}(\lambda_2^{\mathrm{SS}})^2 + 17/96NF^{-2}(\lambda_1^{\mathrm{SP}})^2 - 1/64NF^{-2}$
19	$\langle\, u_\nu SS u^\nu u\cdot u\,\rangle$	$4/3NF^{-8}c_d^6 - 14/3NF^{-6}c_d^4\lambda_1^{\mathrm{SS}} - 8/3NF^{-6}c_d^4\lambda_2^{\mathrm{SS}} + 2/3NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-6}c_d^4 + 8NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}} - 1/3NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/6NF^{-4}c_d^2\lambda_1^{\mathrm{SS}} + 7/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SS}})^2 - 4/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}} + 4/3NF^{-4}c_d^2(\lambda_2^{\mathrm{SS}})^2 - 1/6NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 3/16NF^{-4}c_d^2 - 4NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_2^{\mathrm{SS}})^2 + 1/8NF^{-2}\lambda_1^{\mathrm{SS}} + 1/4NF^{-2}(\lambda_1^{\mathrm{SS}})^2 + 1/6NF^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2/3NF^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-2}(\lambda_1^{\mathrm{SP}})^2 - 5/192NF^{-2}$

20	$\langle u_\mu \rangle \langle u^\mu \{SS, u\cdot u\} \rangle$	$1/24F^{-2} + 16/3F^{-8}c_d^6 - 16/3F^{-6}c_d^4\lambda_1^{SS} - 8F^{-6}c_d^4\lambda_2^{SS} + 8/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 1/3F^{-6}c_d^4 + 14/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS} - 11/6F^{-4}c_d^2\lambda_1^{SS} + 4/3F^{-4}c_d^2(\lambda_1^{SS})^2 - 4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/3F^{-4}c_d^2\lambda_2^{SS} + 8/3F^{-4}c_d^2(\lambda_2^{SS})^2 + 1/3F^{-4}c_d^2(\lambda_1^{SP})^2 - 1/6F^{-4}c_d^2 - 1/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 2F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS} - F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 + 4/3F^{-2}(\lambda_1^{SS})^2 - 5/12F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/24F^{-2}\lambda_2^{SS} + 4/3F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2 + 1/48F^{-2}(\lambda_1^{SP})^2$
21	$\langle u_\mu \rangle \langle u^\mu \{u_\nu S u^\nu, S\} \rangle$	$8/3F^{-8}c_d^6 - 8F^{-6}c_d^4\lambda_1^{SS} - 2/3F^{-6}c_d^4 + 16/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS} - 2F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 + 4/3F^{-4}c_d^2\lambda_1^{SS} + 8/3F^{-4}c_d^2(\lambda_1^{SS})^2 + 4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 - 2F^{-4}c_d^2\lambda_2^{SS} + 4/3F^{-4}c_d^2(\lambda_2^{SS})^2 - 1/2F^{-4}c_d^2(\lambda_1^{SP})^2 + 1/4F^{-4}c_d^2 - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 4F^{-2}\lambda_1^{SS}(\lambda_2^{SS})^2 + 1/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 2/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 + 1/6F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}\lambda_2^{SS} - 2F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2 + F^{-2}(\lambda_2^{SS})^2$
22	$\langle u_\mu S \rangle \langle u^\mu \{S, u\cdot u\} \rangle$	$-1/12F^{-2} + 16F^{-8}c_d^6 - 64/3F^{-6}c_d^4\lambda_1^{SS} - 80/3F^{-6}c_d^4\lambda_2^{SS} + 16/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 8/3F^{-6}c_d^4 + 24F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS} - 4/3F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 - 14/3F^{-4}c_d^2\lambda_1^{SS} + 12F^{-4}c_d^2(\lambda_1^{SS})^2 - 8F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 - 8/3F^{-4}c_d^2\lambda_2^{SS} + 8F^{-4}c_d^2(\lambda_2^{SS})^2 + F^{-4}c_d^2(\lambda_1^{SP})^2 - 7/12F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 + 3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 4F^{-2}\lambda_1^{SS}(\lambda_2^{SS})^2 - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/24F^{-2}\lambda_1^{SS} - 6F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS} - 2/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 + 2/3F^{-2}(\lambda_1^{SS})^2 - 2F^{-2}(\lambda_1^{SS})^3 - 1/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 5/8F^{-2}\lambda_2^{SS} + 4/3F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2 + F^{-2}(\lambda_2^{SS})^2 - 1/24F^{-2}(\lambda_1^{SP})^2$
23	$\langle u_\mu S u^\nu S \rangle \langle u^\mu u_\nu \rangle$	$16/3F^{-8}c_d^6 - 16F^{-6}c_d^4\lambda_1^{SS} - 32/3F^{-6}c_d^4\lambda_2^{SS} + 4/3F^{-6}c_d^4 + 64/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS} - 4F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 + 2F^{-4}c_d^2\lambda_1^{SS} + 16/3F^{-4}c_d^2(\lambda_1^{SS})^2 - 4/3F^{-4}c_d^2\lambda_2^{SS} + 8/3F^{-4}c_d^2(\lambda_2^{SS})^2 - F^{-4}c_d^2(\lambda_1^{SP})^2 - 2/3F^{-4}c_d^2 + 4/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 4F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS} + 4/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 + 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 3/4F^{-2}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SP})^2$
24	$\langle u_\mu SS \rangle \langle u^\mu u\cdot u \rangle$	$3/16F^{-2} + 24F^{-8}c_d^6 - 48F^{-6}c_d^4\lambda_1^{SS} - 24F^{-6}c_d^4\lambda_2^{SS} + 8F^{-6}c_d^4(\lambda_1^{SP})^2 + 32F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS} - 12F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 + 24F^{-4}c_d^2(\lambda_1^{SS})^2 - 4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 + F^{-4}c_d^2(\lambda_1^{SP})^2 - 1/2F^{-4}c_d^2 + 2F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 3/8F^{-2}\lambda_1^{SS} - 6F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS} + 2F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_1^{SS})^3 - 1/2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 3/8F^{-2}\lambda_2^{SS}$

25	$\langle S \rangle \langle S u_\mu u_\nu u^\mu u^\nu \rangle$	$1/48F^{-2} + 4F^{-8}c_d^6 - 16/3F^{-6}c_d^4\lambda_1^{\rm SS} - 20/3F^{-6}c_d^4\lambda_2^{\rm SS} + 4/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 4/3F^{-6}c_d^4 + 4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 4/3F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 4/3F^{-4}c_d^2\lambda_1^{\rm SS} + 2F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 2F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 3F^{-4}c_d^2\lambda_2^{\rm SS} - 1/8F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 1/3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 1/3F^{-2}(\lambda_1^{\rm SS})^2 + 1/6F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}\lambda_2^{\rm SS} - 1/48F^{-2}(\lambda_1^{\rm SP})^2$
26	$\langle S \rangle \langle S u\cdot u u \cdot u \rangle$	$-1/96F^{-2} + 4F^{-8}c_d^6 + 8/3F^{-6}c_d^4\lambda_1^{\rm SS} - 20/3F^{-6}c_d^4\lambda_2^{\rm SS} + 4/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 14/3F^{-6}c_d^4 + 8/3F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/3F^{-4}c_d^2\lambda_1^{\rm SS} + 2F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 2F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 6F^{-4}c_d^2\lambda_2^{\rm SS} + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/8F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 3/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/8F^{-2}\lambda_1^{\rm SS} - 5/3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 + 1/6F^{-2}(\lambda_1^{\rm SS})^2 - 2F^{-2}(\lambda_1^{\rm SS})^3 - 5/6F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}\lambda_2^{\rm SS} - 1/48F^{-2}(\lambda_1^{\rm SP})^2$
27	$\langle u_\mu \rangle \langle u^\mu S u\cdot u S \rangle$	$8/3F^{-8}c_d^6 - 8F^{-6}c_d^4\lambda_2^{\rm SS} + 4/3F^{-6}c_d^4 + 16/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 2F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 14/3F^{-4}c_d^2\lambda_1^{\rm SS} - 4/3F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - F^{-4}c_d^2\lambda_2^{\rm SS} + 16/3F^{-4}c_d^2(\lambda_2^{\rm SS})^2 - 1/2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-4}c_d^2 - 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 1/6F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/8F^{-2}\lambda_1^{\rm SS} - 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_2^{\rm SS} - 4/3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_1^{\rm SS})^3 + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/8F^{-2}\lambda_2^{\rm SS} - 2F^{-2}(\lambda_2^{\rm SS})^2 + 2F^{-2}(\lambda_1^{\rm SS})^2$
28	$\langle S u_\nu u^\mu u^\nu \rangle \langle u_\mu S \rangle$	$-5/24F^{-2} + 16F^{-8}c_d^6 - 64/3F^{-6}c_d^4\lambda_1^{\rm SS} - 80/3F^{-6}c_d^4\lambda_2^{\rm SS} + 16/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 16/3F^{-6}c_d^4 + 16F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 16/3F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 16/3F^{-4}c_d^2\lambda_1^{\rm SS} + 8F^{-4}c_d^2(\lambda_1^{\rm SS})^2 + 4/3F^{-4}c_d^2\lambda_2^{\rm SS} + 16F^{-4}c_d^2(\lambda_2^{\rm SS})^2 - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 7/6F^{-4}c_d^2 - 4/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 8F^{-2}\lambda_1^{\rm SS}(\lambda_2^{\rm SS})^2 + F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}\lambda_1^{\rm SS} + 4/3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 4/3F^{-2}(\lambda_1^{\rm SS})^2 + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 8/3F^{-2}(\lambda_2^{\rm SS})^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_2^{\rm SS})^2 + 1/12F^{-2}(\lambda_1^{\rm SP})^2$
29	$\langle S\,[u_\mu, u_\nu] \rangle^2$	$3/64F^{-2} - 4F^{-6}c_d^4\lambda_2^{\rm SS} + F^{-6}c_d^4 + 6F^{-4}c_d^2(\lambda_2^{\rm SS})^2 - F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 3/8F^{-4}c_d^2 + 1/2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 2F^{-2}\lambda_1^{\rm SS}(\lambda_2^{\rm SS})^2 - F^{-2}(\lambda_1^{\rm SS})^2\lambda_2^{\rm SS} - 1/4F^{-2}(\lambda_1^{\rm SS})^2 + 1/2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/16F^{-2}\lambda_2^{\rm SS} - F^{-2}(\lambda_2^{\rm SS})^2 - 2F^{-2}(\lambda_2^{\rm SS})^3 - 1/8F^{-2}(\lambda_1^{\rm SP})^2$

30	$\langle\, S\,\{u_\mu, u_\nu\}\,\rangle^2$	$3/64F^{-2} + 6F^{-8}c_d^6 - 8F^{-6}c_d^4\lambda_1^{\rm SS} - 14F^{-6}c_d^4\lambda_2^{\rm SS} + 2F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - F^{-6}c_d^4 + 10F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-4}c_d^2\lambda_1^{\rm SS} + 3F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 3F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/2F^{-4}c_d^2\lambda_2^{\rm SS} + 10F^{-4}c_d^2(\lambda_2^{\rm SS})^2 - 1/16F^{-4}c_d^2 + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2F^{-2}\lambda_1^{\rm SS}(\lambda_2^{\rm SS})^2 - F^{-2}(\lambda_1^{\rm SS})^2\lambda_2^{\rm SS} + 1/2F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 1/4F^{-2}(\lambda_1^{\rm SS})^2 - 1/12F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/16F^{-2}\lambda_2^{\rm SS} + 2/3F^{-2}(\lambda_2^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_2^{\rm SS})^3 + 1/96F^{-2}(\lambda_1^{\rm SP})^2$
31	$\langle\, Su\cdot u\,\rangle^2$	$3/32F^{-2} + 12F^{-8}c_d^6 - 16F^{-6}c_d^4\lambda_1^{\rm SS} - 12F^{-6}c_d^4\lambda_2^{\rm SS} + 4F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 2F^{-6}c_d^4 + 16F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} - F^{-4}c_d^2\lambda_1^{\rm SS} + 14F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 2F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2\lambda_2^{\rm SS} + 3/8F^{-4}c_d^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 3/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/8F^{-2}\lambda_1^{\rm SS} - 4F^{-2}(\lambda_1^{\rm SS})^2\lambda_2^{\rm SS} + 3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 + 1/2F^{-2}(\lambda_1^{\rm SS})^2 - 2F^{-2}(\lambda_1^{\rm SS})^3 - 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm SS} + 4/3F^{-2}(\lambda_2^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_2^{\rm SS})^2 + 13/48F^{-2}(\lambda_1^{\rm SP})^2$
32	$\langle\, SS\,\rangle\langle\, u_\mu u_\nu u^\mu u^\nu\,\rangle$	$-1/96F^{-2} + 2F^{-8}c_d^6 - 10/3F^{-6}c_d^4\lambda_1^{\rm SS} + 2/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 4/3F^{-6}c_d^4 - F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/2F^{-4}c_d^2\lambda_1^{\rm SS} + F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 1/6F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 11/48F^{-4}c_d^2 + 1/12F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/12F^{-2}\lambda_1^{\rm SS} + 1/6F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 1/6F^{-2}(\lambda_1^{\rm SS})^2 + 1/96F^{-2}(\lambda_1^{\rm SP})^2$
33	$\langle\, SS\,\rangle\langle\, u\cdot uu\cdot u\,\rangle$	$-7/192F^{-2} + 4F^{-8}c_d^6 - 32/3F^{-6}c_d^4\lambda_1^{\rm SS} + 4/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 7/3F^{-6}c_d^4 - 2F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2\lambda_1^{\rm SS} + 8F^{-4}c_d^2(\lambda_1^{\rm SS})^2 + 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/12F^{-4}c_d^2 - 1/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 5/48F^{-2}\lambda_1^{\rm SS} + 1/3F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 1/12F^{-2}(\lambda_1^{\rm SS})^2 - F^{-2}(\lambda_1^{\rm SS})^3 + 7/48F^{-2}(\lambda_1^{\rm SP})^2$
34	$\langle\, SSu_\nu u^\mu u^\nu\,\rangle\langle\, u_\mu\,\rangle$	$5/48F^{-2} + 16/3F^{-8}c_d^6 - 16/3F^{-6}c_d^4\lambda_1^{\rm SS} - 16F^{-6}c_d^4\lambda_2^{\rm SS} + 8/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 2/3F^{-6}c_d^4 + 20/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 5/3F^{-4}c_d^2\lambda_1^{\rm SS} + 4/3F^{-4}c_d^2(\lambda_1^{\rm SS})^2 - 4F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/3F^{-4}c_d^2\lambda_2^{\rm SS} + 56/3F^{-4}c_d^2(\lambda_2^{\rm SS})^2 - 1/6F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 1/6F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 1/4F^{-2}\lambda_1^{\rm SS} - 2/3F^{-2}(\lambda_1^{\rm SS})^2 + 1/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/12F^{-2}\lambda_2^{\rm SS} + 4/3F^{-2}(\lambda_2^{\rm SS})^2(\lambda_1^{\rm SP})^2 - 8F^{-2}(\lambda_2^{\rm SS})^3 - 1/24F^{-2}(\lambda_1^{\rm SP})^2$

35	$\langle SSu_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$	$-3/16F^{-2}+56/3F^{-8}c_d^6-24F^{-6}c_d^4\lambda_1^{SS}-112/3F^{-6}c_d^4\lambda_2^{SS}+8F^{-6}c_d^4(\lambda_1^{SP})^2-16/3F^{-6}c_d^4+80/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS}-8F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2+4F^{-4}c_d^2\lambda_1^{SS}+20/3F^{-4}c_d^2(\lambda_1^{SS})^2-8F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2+16/3F^{-4}c_d^2\lambda_2^{SS}+64/3F^{-4}c_d^2(\lambda_2^{SS})^2+F^{-4}c_d^2(\lambda_1^{SP})^2+22/24F^{-4}c_d^2+8/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2-8F^{-2}\lambda_1^{SS}(\lambda_2^{SS})^2+2/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2-F^{-2}(\lambda_1^{SS})^2-1/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+4/3F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2-2F^{-2}(\lambda_2^{SS})^2+14/48F^{-2}(\lambda_1^{SP})^2$
36	$\langle SSu\cdot u \rangle \langle u\cdot u \rangle$	$-3/32F^{-2}+28/3F^{-8}c_d^6-12F^{-6}c_d^4\lambda_1^{SS}-32/3F^{-6}c_d^4\lambda_2^{SS}+4F^{-6}c_d^4(\lambda_1^{SP})^2+10/3F^{-6}c_d^4+40/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS}+2F^{-4}c_d^2\lambda_1^{SS}+10/3F^{-4}c_d^2(\lambda_1^{SS})^2-4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2-10/3F^{-4}c_d^2\lambda_2^{SS}+8/3F^{-4}c_d^2(\lambda_2^{SS})^2+1/2F^{-4}c_d^2(\lambda_1^{SP})^2-1/24F^{-4}c_d^2+4/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2-4F^{-2}\lambda_1^{SS}(\lambda_2^{SS})^2+3/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2-1/8F^{-2}\lambda_1^{SS}-5/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2-3/2F^{-2}(\lambda_1^{SS})^2-2F^{-2}(\lambda_1^{SS})^3-2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+2/3F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2+F^{-2}(\lambda_2^{SS})^2+1/48F^{-2}(\lambda_1^{SP})^2$
37	$\langle u_\nu Su^\nu u\cdot u \rangle \langle S \rangle$	$1/12F^{-2}+4F^{-8}c_d^6-40/3F^{-6}c_d^4\lambda_1^{SS}+4/3F^{-6}c_d^4\lambda_2^{SS}+4/3F^{-6}c_d^4(\lambda_1^{SP})^2-4/3F^{-6}c_d^4+12F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS}-4/3F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2-2/3F^{-4}c_d^2\lambda_1^{SS}+6F^{-4}c_d^2(\lambda_1^{SS})^2+2F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2+3F^{-4}c_d^2\lambda_2^{SS}-4F^{-4}c_d^2(\lambda_2^{SS})^2-F^{-4}c_d^2(\lambda_1^{SP})^2-1/8F^{-4}c_d^2-4/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2-2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}-4F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS}+1/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2+2/3F^{-2}(\lambda_1^{SS})^2+5/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+1/12F^{-2}\lambda_2^{SS}-2F^{-2}(\lambda_2^{SS})^2-13/48F^{-2}(\lambda_1^{SP})^2$
38	$\langle u_\nu Su^\nu S \rangle \langle u\cdot u \rangle$	$8/3F^{-8}c_d^6-8F^{-6}c_d^4\lambda_1^{SS}+8/3F^{-6}c_d^4\lambda_2^{SS}-4/3F^{-6}c_d^4-4/3F^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SS}-2F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2-2F^{-4}c_d^2\lambda_1^{SS}+20/3F^{-4}c_d^2(\lambda_1^{SS})^2+4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2+13/3F^{-4}c_d^2\lambda_2^{SS}+4/3F^{-4}c_d^2(\lambda_2^{SS})^2-1/2F^{-4}c_d^2(\lambda_1^{SP})^2-1/3F^{-4}c_d^2-4/3F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2-4F^{-2}(\lambda_1^{SS})^2\lambda_2^{SS}+2/3F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2+4/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+1/4F^{-2}\lambda_2^{SS}-7/24F^{-2}(\lambda_1^{SP})^2$
39	$\langle u_\mu u_\nu \nabla^\mu S\nabla^\nu S \rangle$	$2/3NF^{-6}c_d^4-4/3NF^{-4}c_d^2\lambda_2^{SS}+1/3NF^{-4}c_d^2(\lambda_1^{SP})^2-1/3NF^{-4}c_d^2-1/6NF^{-2}\lambda_1^{SS}+1/3NF^{-2}(\lambda_1^{SS})^2-2/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+4/3NF^{-2}(\lambda_2^{SS})^2-1/6NF^{-2}(\lambda_1^{SP})^2+1/6NF^{-2}(\lambda_1^{SP})^4+5/48NF^{-2}$
40	$\langle u_\mu u_\nu \nabla^\nu S\nabla^\mu S \rangle$	$14/3NF^{-6}c_d^4-4NF^{-4}c_d^2\lambda_1^{SS}-4NF^{-4}c_d^2\lambda_2^{SS}+23/3NF^{-4}c_d^2(\lambda_1^{SP})^2-8/3NF^{-4}c_d^2-2NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2+3/2NF^{-2}\lambda_1^{SS}+NF^{-2}(\lambda_1^{SS})^2-2NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2+4NF^{-2}(\lambda_2^{SS})^2-4/3NF^{-2}(\lambda_1^{SP})^2+7/6NF^{-2}(\lambda_1^{SP})^4+23/48NF^{-2}$

41	$\langle u_\mu \nabla^\mu S u_\nu \nabla^\nu S \rangle + \text{h.c.}$	$2NF^{-6}c_d^4 - 4/3NF^{-4}c_d^2\lambda_1^{\rm SS} - 4NF^{-4}c_d^2\lambda_2^{\rm SS} + 3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3NF^{-4}c_d^2 + 8/3NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2/3NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 4/3NF^{-2}\lambda_2^{\rm SS} - 1/3NF^{-2}(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}(\lambda_1^{\rm SP})^4$
42	$\langle \nabla_\mu S u_\nu \nabla^\mu S u^\nu \rangle$	$2/3NF^{-4}c_d^2\lambda_1^{\rm SS} - 1/6NF^{-4}c_d^2 - 4/3NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 1/3NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/3NF^{-2}\lambda_2^{\rm SS} - 1/12NF^{-2}(\lambda_1^{\rm SP})^2$
43	$\langle u \cdot u \nabla_\mu S \nabla^\mu S \rangle$	$2/3NF^{-6}c_d^4 - 2NF^{-4}c_d^2\lambda_1^{\rm SS} + 4/3NF^{-4}c_d^2\lambda_2^{\rm SS} - NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/2NF^{-4}c_d^2 - NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 5/6NF^{-2}\lambda_1^{\rm SS} - 1/3NF^{-2}(\lambda_1^{\rm SS})^2 + 2/3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 4/3NF^{-2}(\lambda_2^{\rm SS})^2 + 1/4NF^{-2}(\lambda_1^{\rm SP})^2 + 1/6NF^{-2}(\lambda_1^{\rm SP})^4 - 1/48NF^{-2}$
44	$\langle u_\mu \rangle \langle u^\mu \nabla_\nu S \nabla^\nu S \rangle$	$1/24F^{-2} + 4/3F^{-6}c_d^4 - 8/3F^{-4}c_d^2\lambda_1^{\rm SS} + 4/3F^{-4}c_d^2\lambda_2^{\rm SS} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - F^{-4}c_d^2 - 2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm SS} - 2/3F^{-2}(\lambda_1^{\rm SS})^2 + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_2^{\rm SS} - 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
45	$\langle u_\mu u_\nu \rangle \langle \nabla^\mu S \nabla^\nu S \rangle$	$7/12F^{-2} + 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{\rm SS} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 5/3F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm SS} + 4/3F^{-2}(\lambda_1^{\rm SS})^2 - 5/6F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
46	$\langle u_\mu \nabla^\mu S \rangle \langle u_\nu \nabla^\nu S \rangle$	$7/12F^{-2} + 28/3F^{-6}c_d^4 - 20/3F^{-4}c_d^2\lambda_1^{\rm SS} - 12F^{-4}c_d^2\lambda_2^{\rm SS} + 14F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 14/3F^{-4}c_d^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 10/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 5/3F^{-2}\lambda_1^{\rm SS} + 4/3F^{-2}(\lambda_1^{\rm SS})^2 - 6F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm SS} + 4F^{-2}(\lambda_2^{\rm SS})^2 - 7/3F^{-2}(\lambda_1^{\rm SP})^2 + 7/3F^{-2}(\lambda_1^{\rm SP})^4$
47	$\langle u_\mu \nabla_\nu S \rangle^2$	$-1/24F^{-2} + 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{\rm SS} - 8/3F^{-4}c_d^2\lambda_2^{\rm SS} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - F^{-4}c_d^2 - 2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm SS})^2 + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 7/3F^{-2}\lambda_2^{\rm SS} - 4/3F^{-2}(\lambda_2^{\rm SS})^2 - 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
48	$\langle u_\mu \nabla_\nu S \rangle \langle u^\nu \nabla^\mu S \rangle$	$7/12F^{-2} + 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{\rm SS} - 4/3F^{-4}c_d^2\lambda_2^{\rm SS} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4/3F^{-4}c_d^2 + 4/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 5/3F^{-2}\lambda_1^{\rm SS} + 4/3F^{-2}(\lambda_1^{\rm SS})^2 - 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_2^{\rm SS} + 4/3F^{-2}(\lambda_2^{\rm SS})^2 - 2/3F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
49	$\langle u_\mu \{\nabla^\mu S, \nabla^\nu S\} \rangle \langle u_\nu \rangle$	$-7/12F^{-2} - 8/3F^{-6}c_d^4 - 8/3F^{-4}c_d^2\lambda_1^{\rm SS} + 4/3F^{-4}c_d^2\lambda_2^{\rm SS} - 4F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 3F^{-4}c_d^2 + 8/3F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 4/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 4/3F^{-2}(\lambda_1^{\rm SS})^2 + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_2^{\rm SS} + 3/2F^{-2}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm SP})^4$

50	$\langle \nabla_\mu S \rangle \langle \nabla^\mu S u \cdot u \rangle$	$1/24F^{-2} + 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{SS} - 8/3F^{-4}c_d^2\lambda_2^{SS} - 2F^{-4}c_d^2(\lambda_1^{SP})^2 + F^{-4}c_d^2 - 2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - 4/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 2/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 2/3F^{-2}(\lambda_1^{SS})^2 + 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 7/3F^{-2}\lambda_2^{SS} + 1/2F^{-2}(\lambda_1^{SP})^2 + 1/3F^{-2}(\lambda_1^{SP})^4$
51	$\langle \nabla^\mu S \{u_\mu, u_\nu\} \rangle \langle \nabla^\nu S \rangle$	$-7/12F^{-2} - 8/3F^{-6}c_d^4 - 8/3F^{-4}c_d^2\lambda_1^{SS} + 4/3F^{-4}c_d^2\lambda_2^{SS} - 4F^{-4}c_d^2(\lambda_1^{SP})^2 + 3F^{-4}c_d^2 + 8/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 4/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 4/3F^{-2}(\lambda_1^{SS})^2 + 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 4/3F^{-2}\lambda_2^{SS} + 3/2F^{-2}(\lambda_1^{SP})^2 - 2/3F^{-2}(\lambda_1^{SP})^4$
52	$\langle u \cdot u \rangle \langle \nabla_\mu S \nabla^\mu S \rangle$	$-1/48F^{-2} + 2/3F^{-6}c_d^4 - 4/3F^{-4}c_d^2\lambda_1^{SS} - F^{-4}c_d^2(\lambda_1^{SP})^2 + 2/3F^{-4}c_d^2 - F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + 1/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 7/6F^{-2}\lambda_1^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 + 1/3F^{-2}(\lambda_1^{SP})^2 + 1/6F^{-2}(\lambda_1^{SP})^4$
53	$i\langle u_\mu u_\nu S S f_+^{\mu\nu} \rangle + \text{h.c.}$	$1/3NF^{-6}c_d^4 - 1/6NF^{-4}c_d^2\lambda_1^{SS} - 1/3NF^{-4}c_d^2\lambda_2^{SS} - 1/6NF^{-4}c_d^2(\lambda_1^{SP})^2 - 1/8NF^{-4}c_d^2 + 1/6NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/12NF^{-2}(\lambda_1^{SP})^2 + 1/24NF^{-2}$
54	$i\langle u_\mu S u_\nu S f_+^{\mu\nu} \rangle + \text{h.c.}$	$-2/3NF^{-4}c_d^2\lambda_1^{SS} + 1/6NF^{-4}c_d^2 + 2/3NF^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/6NF^{-2}\lambda_2^{SS}$
55	$i\langle S u_\mu u_\nu S f_+^{\mu\nu} \rangle$	$-2/3NF^{-4}c_d^2\lambda_2^{SS} + 1/3NF^{-4}c_d^2(\lambda_1^{SP})^2 + 1/4NF^{-4}c_d^2 - 1/12NF^{-2}\lambda_1^{SS} + 1/6NF^{-2}(\lambda_1^{SS})^2 - 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 2/3NF^{-2}(\lambda_2^{SS})^2 + 1/6NF^{-2}(\lambda_1^{SP})^2 - 7/96NF^{-2}$
56	$i\langle u_\mu f_+^{\mu\nu} u_\nu S S \rangle$	$-2/3NF^{-6}c_d^4 + 1/3NF^{-4}c_d^2\lambda_1^{SS} + 4/3NF^{-4}c_d^2\lambda_2^{SS} + 1/12NF^{-2}\lambda_1^{SS} - 1/6NF^{-2}(\lambda_1^{SS})^2 - 2/3NF^{-2}(\lambda_2^{SS})^2 - 1/96NF^{-2}$
57	$i\langle u_\mu \rangle \langle f_+^{\mu\nu} [SS, u_\nu] \rangle$	$2/3F^{-6}c_d^4 - F^{-4}c_d^2\lambda_1^{SS} - 1/3F^{-4}c_d^2\lambda_2^{SS} - 1/12F^{-4}c_d^2 + 1/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 1/6F^{-2}(\lambda_1^{SS})^2 - 1/6F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/12F^{-2}\lambda_2^{SS} + 1/24F^{-2}(\lambda_1^{SP})^2 - 1/96F^{-2}$
58	$i\langle u_\mu S \rangle \langle f_+^{\mu\nu} [S, u_\nu] \rangle$	$4/3F^{-6}c_d^4 - 4/3F^{-4}c_d^2\lambda_1^{SS} - 4/3F^{-4}c_d^2\lambda_2^{SS} - 1/6F^{-4}c_d^2 + 1/6F^{-2}\lambda_1^{SS} + 1/3F^{-2}(\lambda_1^{SS})^2 + 1/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/12F^{-2}(\lambda_1^{SP})^2 + 1/48F^{-2}$
59	$i\langle S \rangle \langle u_\mu S f_+^{\mu\nu} u_\nu \rangle + \text{h.c.}$	$-2/3F^{-6}c_d^4 + 2/3F^{-4}c_d^2\lambda_1^{SS} + F^{-4}c_d^2\lambda_2^{SS} - 1/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/6F^{-2}(\lambda_1^{SS})^2 - 1/12F^{-2}\lambda_2^{SS} + 1/96F^{-2}$
60	$i\langle SS \rangle \langle f_+^{\mu\nu} u_\mu u_\nu \rangle$	$4/3F^{-6}c_d^4 - 2F^{-4}c_d^2\lambda_1^{SS} - 1/3F^{-4}c_d^2 + 1/6F^{-2}\lambda_1^{SS} + 1/3F^{-2}(\lambda_1^{SS})^2 + 1/48F^{-2}$
61	$i\langle u_\mu f_+^{\mu\nu} u_\nu S \rangle \langle S \rangle$	$-4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{SS} + 2F^{-4}c_d^2\lambda_2^{SS} - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 - 1/6F^{-2}\lambda_2^{SS} + 1/48F^{-2}$
62	$i\langle f_+^{\mu\nu} \nabla_\mu S \nabla_\nu S \rangle$	$1/3NF^{-4}c_d^2 + 1/3NF^{-2}(\lambda_1^{SP})^2 - 1/6NF^{-2}$
63	$\langle u_\mu f_-^{\mu\nu} \nabla_\nu S S \rangle + \text{h.c.}$	$-1/2NF^{-6}c_d^4 + 1/3NF^{-4}c_d^2\lambda_1^{SS} + 2/3NF^{-4}c_d^2\lambda_2^{SS} + 1/3NF^{-4}c_d^2(\lambda_1^{SP})^2 - 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2$

64	$\langle u_\mu \nabla_\nu S S f_-^{\mu\nu} \rangle + \text{h.c.}$	$-5/6NF^{-6}c_d^4 + 1/3NF^{-4}c_d^2\lambda_1^{SS} + 2NF^{-4}c_d^2\lambda_2^{SS} - 1/3NF^{-4}c_d^2(\lambda_1^{SP})^2 + 1/6NF^{-2}\lambda_1^{SS} - 1/3NF^{-2}(\lambda_1^{SS})^2 + 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 4/3NF^{-2}(\lambda_2^{SS})^2 - 1/48NF^{-2}$
65	$\langle u_\mu S f_-^{\mu\nu} \nabla_\nu S \rangle + \text{h.c.}$	$4/3NF^{-4}c_d^2\lambda_1^{SS} - 1/3NF^{-4}c_d^2 - 4/3NF^{-2}\lambda_1^{SS}\lambda_2^{SS} + 1/3NF^{-2}\lambda_2^{SS}$
66	$\langle u_\mu \rangle \langle f_-^{\mu\nu} \{S, \nabla_\nu S\} \rangle$	$1/48F^{-2} - 4/3F^{-6}c_d^4 + 2F^{-4}c_d^2\lambda_1^{SS} + 2/3F^{-4}c_d^2\lambda_2^{SS} + 1/6F^{-4}c_d^2 - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 + 1/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/6F^{-2}\lambda_2^{SS} - 1/12F^{-2}(\lambda_1^{SP})^2$
67	$\langle u_\mu S \rangle \langle f_-^{\mu\nu} \nabla_\nu S \rangle$	$-1/24F^{-2} - 8/3F^{-6}c_d^4 + 8/3F^{-4}c_d^2\lambda_1^{SS} + 8/3F^{-4}c_d^2\lambda_2^{SS} + 1/3F^{-4}c_d^2 - 1/3F^{-2}\lambda_1^{SS} - 2/3F^{-2}(\lambda_1^{SS})^2 - 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/6F^{-2}(\lambda_1^{SP})^2$
68	$\langle \nabla_\mu S \rangle \langle f_-^{\mu\nu} \{S, u_\nu\} \rangle$	$-1/48F^{-2} + 4/3F^{-6}c_d^4 - 4/3F^{-4}c_d^2\lambda_1^{SS} - 2F^{-4}c_d^2\lambda_2^{SS} + 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 1/3F^{-2}(\lambda_1^{SS})^2 + 1/6F^{-2}\lambda_2^{SS}$
69	$\langle S \rangle \langle f_-^{\mu\nu} \{u_\mu, \nabla_\nu S\} \rangle$	$1/48F^{-2} - 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{SS} + 2F^{-4}c_d^2\lambda_2^{SS} - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 - 1/6F^{-2}\lambda_2^{SS}$
70	$\langle S f_-^{\mu\nu} \rangle \langle u_\mu \nabla_\nu S \rangle$	$-1/24F^{-2} - 8/3F^{-6}c_d^4 + 8/3F^{-4}c_d^2\lambda_1^{SS} + 16/3F^{-4}c_d^2\lambda_2^{SS} - 1/3F^{-4}c_d^2 - 8/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 1/3F^{-2}\lambda_1^{SS} - 2/3F^{-2}(\lambda_1^{SS})^2 + 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 2/3F^{-2}\lambda_2^{SS} - 8/3F^{-2}(\lambda_2^{SS})^2 - 1/6F^{-2}(\lambda_1^{SP})^2$
71	$\langle S \nabla_\mu S \rangle \langle u_\nu f_-^{\mu\nu} \rangle$	$1/24F^{-2} + 8/3F^{-6}c_d^4 - 4F^{-4}c_d^2\lambda_1^{SS} - 2/3F^{-4}c_d^2 + 1/3F^{-2}\lambda_1^{SS} + 2/3F^{-2}(\lambda_1^{SS})^2$
72	$\langle S f_+^{\mu\nu} S f_{+\mu\nu} \rangle$	$1/12NF^{-4}c_d^2 + 1/12NF^{-2}(\lambda_1^{SP})^2 - 1/24NF^{-2}$
73	$\langle S S f_+^{\mu\nu} f_{+\mu\nu} \rangle$	$-1/12NF^{-4}c_d^2 - 1/12NF^{-2}(\lambda_1^{SP})^2 + 1/24NF^{-2}$
74	$\langle S f_-^{\mu\nu} S f_{-\mu\nu} \rangle$	$2/3NF^{-4}c_d^2\lambda_1^{SS} - 1/6NF^{-4}c_d^2 - 2/3NF^{-2}\lambda_1^{SS}\lambda_2^{SS} + 1/6NF^{-2}\lambda_2^{SS}$
75	$\langle S S f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$-2/3NF^{-6}c_d^4 + 1/3NF^{-4}c_d^2\lambda_1^{SS} + 4/3NF^{-4}c_d^2\lambda_2^{SS} + 1/12NF^{-2}\lambda_1^{SS} - 1/6NF^{-2}(\lambda_1^{SS})^2 - 2/3NF^{-2}(\lambda_2^{SS})^2 - 1/96NF^{-2}$
76	$\langle S \rangle \langle S f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$1/48F^{-2} - 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{SS} + 2F^{-4}c_d^2\lambda_2^{SS} - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 - 1/6F^{-2}\lambda_2^{SS}$
77	$\langle S f_-^{\mu\nu} \rangle^2$	$-1/48F^{-2} - 4/3F^{-6}c_d^4 + 4/3F^{-4}c_d^2\lambda_1^{SS} + 2F^{-4}c_d^2\lambda_2^{SS} - 2/3F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 1/3F^{-2}(\lambda_1^{SS})^2 + 1/6F^{-2}\lambda_2^{SS} - 2/3F^{-2}(\lambda_2^{SS})^2$
78	$\langle S S \rangle \langle f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$-1/96F^{-2} - 2/3F^{-6}c_d^4 + F^{-4}c_d^2\lambda_1^{SS} + 1/6F^{-4}c_d^2 - 1/12F^{-2}\lambda_1^{SS} - 1/6F^{-2}(\lambda_1^{SS})^2$
79	$\langle S \chi_+ S u \cdot u \rangle$	$-NF^{-6}c_d^3c_m + 4NF^{-6}c_d^4\lambda_3^{SS} + NF^{-6}c_d^4 - 4NF^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - NF^{-4}c_dc_m\lambda_1^{SS} + NF^{-4}c_dc_m\lambda_2^{SS} - 1/4NF^{-4}c_dc_m + 2NF^{-4}c_d^2\lambda_1^{SS}\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_1^{SS} - 4NF^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} - 2NF^{-4}c_d^2\lambda_2^{SS} + 2NF^{-4}c_d^2\lambda_3^{SS}(\lambda_1^{SP})^2 + NF^{-4}c_d^2\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + 4NF^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 1/2NF^{-2}\lambda_1^{SS}\lambda_3^{SS} + 2NF^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - NF^{-2}(\lambda_1^{SS})^2\lambda_3^{SS} - NF^{-2}(\lambda_1^{SS})^2 - 2NF^{-2}\lambda_2^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 2NF^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + NF^{-2}(\lambda_2^{SS})^2 - 1/16NF^{-2}\lambda_3^{SS}$

80	$\langle SS\{\chi_+, u\cdot u\}\rangle$	$-2NF^{-6}d_m c_d^3\lambda_1^{SP} - 1/2NF^{-6}c_d^3c_m + 2NF^{-6}c_d^4\lambda_3^{SS} + NF^{-6}c_d^4 - NF^{-4}d_m c_d\lambda_1^{SS}\lambda_1^{SP} + 2NF^{-4}d_m c_d\lambda_2^{SS}\lambda_1^{SP} - 1/2NF^{-4}c_dc_m\lambda_1^{SS} + 1/2NF^{-4}c_dc_m\lambda_2^{SS} - 1/8NF^{-4}c_dc_m + NF^{-4}c_d^2\lambda_1^{SS}\lambda_3^{SS} + 1/2NF^{-4}c_d^2\lambda_1^{SS} - 2NF^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} - NF^{-4}c_d^2\lambda_2^{SS} + NF^{-4}c_d^2\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/4NF^{-4}c_d^2\lambda_3^{SS} + 1/4NF^{-4}c_d^2 + 2NF^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 1/2NF^{-2}\lambda_1^{SS}\lambda_3^{SS} + NF^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/8NF^{-2}\lambda_1^{SS} - 1/2NF^{-2}(\lambda_1^{SS})^2 - NF^{-2}\lambda_2^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/8NF^{-2}\lambda_3^{SS}$
81	$\langle u_\nu S u^\nu\{S,\chi_+\}\rangle$	$NF^{-4}d_m c_d\lambda_1^{SS}\lambda_1^{SP} - 4NF^{-4}d_m c_d\lambda_2^{SS}\lambda_1^{SP} + 3/4NF^{-4}d_m c_d\lambda_1^{SP} + 1/2NF^{-4}c_dc_m\lambda_1^{SS} - NF^{-4}c_dc_m\lambda_2^{SS} + 1/8NF^{-4}c_dc_m - NF^{-4}c_d^2\lambda_1^{SS} + 4NF^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_2^{SS} - NF^{-4}c_d^2\lambda_3^{SS} - 1/4NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_3^{SS} - 1/2NF^{-2}\lambda_1^{SS}\lambda_2^{SS} - NF^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 1/2NF^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 4NF^{-2}\lambda_2^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/2NF^{-2}\lambda_2^{SS}\lambda_3^{SS} + 2NF^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/8NF^{-2}\lambda_2^{SS} - 3/4NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 - 3/8NF^{-2}\lambda_1^{SP}\lambda_2^{SP}$
82	$\langle u_\nu SSu^\nu\chi_+\rangle$	$-4NF^{-6}c_d^4\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_1^{SS}\lambda_3^{SS} + 8NF^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} + 1/8NF^{-2}\lambda_1^{SS} + 1/4NF^{-2}(\lambda_1^{SS})^2 - 4NF^{-2}(\lambda_2^{SS})^2\lambda_3^{SS} - 3/64NF^{-2}$
83	$\langle u_\mu\rangle\langle u^\mu\{SS,\chi_+\}\rangle$	$3/64F^{-2} - 4F^{-6}d_m c_d^3\lambda_1^{SP} - F^{-6}c_d^3c_m + 2F^{-6}c_d^4 + 3F^{-4}d_m c_d\lambda_1^{SS}\lambda_1^{SP} + 2F^{-4}d_m c_d\lambda_2^{SS}\lambda_1^{SP} - 3/4F^{-4}d_m c_d\lambda_1^{SP} + 1/2F^{-4}c_dc_m\lambda_1^{SS} + 1/2F^{-4}c_dc_m\lambda_2^{SS} + 1/4F^{-4}c_dc_m - 2F^{-4}c_d^2\lambda_1^{SS} + 2F^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} - F^{-4}c_d^2\lambda_2^{SS} + 2F^{-4}c_d^2\lambda_3^{SS}(\lambda_1^{SP})^2 - 1/2F^{-4}c_d^2 - 2F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 1/2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/4F^{-2}\lambda_1^{SS} + 1/4F^{-2}(\lambda_1^{SS})^2 - F^{-2}\lambda_2^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 1/2F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 1/4F^{-2}\lambda_3^{SS} - 3/8F^{-2}\lambda_1^{SP}\lambda_2^{SP}$
84	$\langle u_\mu\rangle\langle u^\mu S\chi_+S\rangle$	$-2F^{-6}c_d^3c_m + 8F^{-6}c_d^4\lambda_3^{SS} + 2F^{-6}c_d^4 + 2F^{-4}d_m c_d\lambda_1^{SS}\lambda_1^{SP} + 3/2F^{-4}d_m c_d\lambda_1^{SP} + F^{-4}c_dc_m\lambda_1^{SS} + F^{-4}c_dc_m\lambda_2^{SS} + 1/2F^{-4}c_dc_m - 4F^{-4}c_d^2\lambda_1^{SS}\lambda_3^{SS} - 2F^{-4}c_d^2\lambda_1^{SS} - 4F^{-4}c_d^2\lambda_2^{SS}\lambda_3^{SS} - 2F^{-4}c_d^2\lambda_2^{SS} + 4F^{-4}c_d^2\lambda_3^{SS}(\lambda_1^{SP})^2 - 3F^{-4}c_d^2\lambda_3^{SS} + 4F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 2F^{-2}(\lambda_1^{SS})^2\lambda_3^{SS} - 2F^{-2}\lambda_2^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/4F^{-2}\lambda_2^{SS} + 1/8F^{-2}\lambda_3^{SS} + 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP}$

85	$\langle u_\mu S\rangle\langle u^\mu\{S,\chi_+\}\rangle$	$-3/32F^{-2} - 8F^{-6}d_m c_d^3\lambda_1^{\rm SP} - 4F^{-6}c_d^3c_m + 6F^{-6}c_d^4 + 4F^{-4}d_m c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8F^{-4}d_m c_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d c_m\lambda_1^{\rm SS} + 2F^{-4}c_d c_m\lambda_2^{\rm SS} + F^{-4}c_d c_m + 4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_3^{\rm SS} - 4F^{-4}c_d^2\lambda_1^{\rm SS} + 4F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 6F^{-4}c_d^2\lambda_2^{\rm SS} + 4F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2\lambda_3^{\rm SS} + 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-4}c_d^2 - 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_3^{\rm SS} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_3^{\rm SS} + 1/2F^{-2}(\lambda_1^{\rm SS})^2 - 2F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm SS} - 4F^{-2}(\lambda_2^{\rm SS})^2\lambda_3^{\rm SS} + F^{-2}(\lambda_2^{\rm SS})^2 - 1/8F^{-2}\lambda_3^{\rm SS}$
86	$\langle u_\mu SS\rangle\langle u^\mu\chi_+\rangle$	$3/32F^{-2} - 8F^{-6}d_m c_d^3\lambda_1^{\rm SP} - 4F^{-6}c_d^3c_m - 8F^{-6}c_d^4\lambda_3^{\rm SS} + 6F^{-6}c_d^4 + 8F^{-4}d_m c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}d_m c_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d c_m\lambda_1^{\rm SS} + 2F^{-4}c_d c_m\lambda_2^{\rm SS} + F^{-4}c_d c_m + 12F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_3^{\rm SS} - 6F^{-4}c_d^2\lambda_1^{\rm SS} + 8F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 4F^{-4}c_d^2\lambda_2^{\rm SS} + F^{-4}c_d^2\lambda_3^{\rm SS} + 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-4}c_d^2 - 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_3^{\rm SS} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm SS} - 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_3^{\rm SS} + 1/2F^{-2}(\lambda_1^{\rm SS})^2 - F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/4F^{-2}\lambda_2^{\rm SS} - 3/8F^{-2}\lambda_3^{\rm SS}$
87	$\langle S\rangle\langle S\{\chi_+, u\cdot u\}\rangle$	$-4F^{-6}d_m c_d^3\lambda_1^{\rm SP} - 2F^{-6}c_d^3c_m + 4F^{-6}c_d^4\lambda_3^{\rm SS} + 3F^{-6}c_d^4 - 3F^{-4}d_m c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}d_m c_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-4}d_m c_d\lambda_1^{\rm SP} - 1/2F^{-4}c_d c_m\lambda_1^{\rm SS} + F^{-4}c_d c_m\lambda_2^{\rm SS} - 5/8F^{-4}c_d c_m + F^{-4}c_d^2\lambda_1^{\rm SS} - 2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 3F^{-4}c_d^2\lambda_2^{\rm SS} + 2F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-4}c_d^2\lambda_3^{\rm SS} + 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/4F^{-4}c_d^2 - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}(\lambda_1^{\rm SS})^2\lambda_3^{\rm SS} - F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS} - F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/8F^{-2}\lambda_2^{\rm SS} + 3/4F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/16F^{-2}\lambda_3^{\rm SS} + 3/8F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP}$
88	$\langle S\chi_+\rangle\langle Su\cdot u\rangle$	$3/32F^{-2} - 8F^{-6}d_m c_d^3\lambda_1^{\rm SP} - 4F^{-6}c_d^3c_m + 6F^{-6}c_d^4 - 4F^{-4}d_m c_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-4}c_d c_m + 12F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_3^{\rm SS} + 8F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_3^{\rm SS} - 2F^{-4}c_d^2\lambda_2^{\rm SS} + 4F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-4}c_d^2\lambda_3^{\rm SS} + 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-4}c_d^2 - 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_3^{\rm SS} - F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} + 6F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_3^{\rm SS} + 1/2F^{-2}(\lambda_1^{\rm SS})^2 + 4F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 3F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS} + 2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/4F^{-2}\lambda_2^{\rm SS} - 2F^{-2}(\lambda_2^{\rm SS})^2 - 3/2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/8F^{-2}\lambda_3^{\rm SS}$

89	$\langle SS \rangle \langle \chi_+ u \cdot u \rangle$	$-3/64F^{-2} - 4F^{-6}d_m c_d^3 \lambda_1^{SP} - 2F^{-6}c_d^3 c_m - 4F^{-6}c_d^4 \lambda_3^{SS} + 3F^{-6}c_d^4 + 4F^{-4}d_m c_d \lambda_1^{SS}\lambda_1^{SP} - 3/2F^{-4}d_m c_d \lambda_1^{SP} + F^{-4}c_d c_m \lambda_1^{SS} - 3/4F^{-4}c_d c_m + 6F^{-4}c_d^2 \lambda_1^{SS}\lambda_3^{SS} - 3F^{-4}c_d^2 \lambda_1^{SS} + 3/2F^{-4}c_d^2 \lambda_3^{SS} + 2F^{-4}c_d^2 \lambda_1^{SP}\lambda_2^{SP} + F^{-4}c_d^2 - 1/2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 3/8F^{-2}\lambda_1^{SS} - F^{-2}(\lambda_1^{SS})^2\lambda_3^{SS} + 1/4F^{-2}(\lambda_1^{SS})^2 + 3/16F^{-2}\lambda_3^{SS} + 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP}$
90	$\langle SSu \cdot u \rangle \langle \chi_+ \rangle$	$-3/64F^{-2} - 4F^{-6}d_m c_d^3 \lambda_1^{SP} - 2F^{-6}c_d^3 c_m - 4F^{-6}c_d^4 \lambda_3^{SS} + 3F^{-6}c_d^4 - 6F^{-4}d_m c_d \lambda_1^{SS}\lambda_1^{SP} + 4F^{-4}d_m c_d \lambda_2^{SS}\lambda_1^{SP} - 2F^{-4}c_d c_m \lambda_1^{SS} + 2F^{-4}c_d c_m \lambda_2^{SS} - 1/2F^{-4}c_d c_m + 2F^{-4}c_d^2 \lambda_1^{SS}\lambda_3^{SS} + 3F^{-4}c_d^2 \lambda_1^{SS} + 8F^{-4}c_d^2 \lambda_2^{SS}\lambda_3^{SS} - 4F^{-4}c_d^2 \lambda_2^{SS} + 1/2F^{-4}c_d^2 \lambda_3^{SS} + 2F^{-4}c_d^2 \lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-4}c_d^2 + 1/2F^{-2}\lambda_1^{SS}\lambda_3^{SS} + 4F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/8F^{-2}\lambda_1^{SS} - F^{-2}(\lambda_1^{SS})^2\lambda_3^{SS} - 7/4F^{-2}(\lambda_1^{SS})^2 - 2F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 4F^{-2}(\lambda_2^{SS})^2\lambda_3^{SS} + F^{-2}(\lambda_2^{SS})^2 + 3/16F^{-2}\lambda_3^{SS}$
91	$\langle SS\chi_+ \rangle \langle u \cdot u \rangle$	$-3/64F^{-2} - 4F^{-6}d_m c_d^3 \lambda_1^{SP} - 2F^{-6}c_d^3 c_m + 4F^{-6}c_d^4 \lambda_3^{SS} + 3F^{-6}c_d^4 + 4F^{-4}d_m c_d \lambda_1^{SS}\lambda_1^{SP} - 3/2F^{-4}d_m c_d \lambda_1^{SP} + F^{-4}c_d c_m \lambda_1^{SS} - 3/4F^{-4}c_d c_m - 2F^{-4}c_d^2 \lambda_1^{SS}\lambda_3^{SS} - 3F^{-4}c_d^2 \lambda_1^{SS} + 4F^{-4}c_d^2 \lambda_3^{SS}(\lambda_1^{SP})^2 + 9/2F^{-4}c_d^2 \lambda_3^{SS} + 2F^{-4}c_d^2 \lambda_1^{SP}\lambda_2^{SP} + F^{-4}c_d^2 - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS}(\lambda_1^{SP})^2 - 5/2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 3/8F^{-2}\lambda_1^{SS} - F^{-2}(\lambda_1^{SS})^2\lambda_3^{SS} + 1/4F^{-2}(\lambda_1^{SS})^2 + 3/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/16F^{-2}\lambda_3^{SS} + 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP}$
92	$\langle u_\nu S u^\nu \chi_+ \rangle \langle S \rangle$	$3/32F^{-2} - 8F^{-6}c_d^4 \lambda_3^{SS} + 2F^{-4}d_m c_d \lambda_1^{SS}\lambda_1^{SP} - 8F^{-4}d_m c_d \lambda_2^{SS}\lambda_1^{SP} + 3/2F^{-4}d_m c_d \lambda_1^{SP} + F^{-4}c_d c_m \lambda_1^{SS} - 2F^{-4}c_d c_m \lambda_2^{SS} + 1/4F^{-4}c_d c_m + 4F^{-4}c_d^2 \lambda_1^{SS}\lambda_3^{SS} - 2F^{-4}c_d^2 \lambda_1^{SS} + 12F^{-4}c_d^2 \lambda_2^{SS}\lambda_3^{SS} + 4F^{-4}c_d^2 \lambda_2^{SS} + F^{-4}c_d^2 \lambda_3^{SS} - 1/2F^{-4}c_d^2 - 4F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-2}(\lambda_1^{SS})^2 - F^{-2}\lambda_2^{SS}\lambda_3^{SS} + 4F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 2F^{-2}(\lambda_2^{SS})^2 - 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP}$
93	$\langle u_\nu S u^\nu S \rangle \langle \chi_+ \rangle$	$2F^{-4}d_m c_d \lambda_1^{SS}\lambda_1^{SP} - 8F^{-4}d_m c_d \lambda_2^{SS}\lambda_1^{SP} + 3/2F^{-4}d_m c_d \lambda_1^{SP} + F^{-4}c_d c_m \lambda_1^{SS} - 2F^{-4}c_d c_m \lambda_2^{SS} + 1/4F^{-4}c_d c_m + 4F^{-4}c_d^2 \lambda_1^{SS}\lambda_3^{SS} - 2F^{-4}c_d^2 \lambda_1^{SS} + 4F^{-4}c_d^2 \lambda_2^{SS} - F^{-4}c_d^2 \lambda_3^{SS} - 1/2F^{-4}c_d^2 - 4F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_2^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_2^{SS}\lambda_3^{SS} + 4F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/4F^{-2}\lambda_2^{SS} - 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP}$

94	$\langle\,\chi_+\nabla^\mu S\nabla_\mu S\,\rangle$	$2NF^{-4}d_mc_d\lambda_1^{\rm SP} + 1/2NF^{-4}c_d^2 - NF^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_1^{\rm SS} + 1/4NF^{-2}(\lambda_1^{\rm SP})^2$
95	$\langle\,\chi_+\,\rangle\langle\,\nabla_\mu S\nabla^\mu S\,\rangle$	$2F^{-4}d_mc_d\lambda_1^{\rm SP} + 1/2F^{-4}c_d^2 - F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_1^{\rm SS} + 1/4F^{-2}(\lambda_1^{\rm SP})^2$
96	$\langle\,\chi_+\nabla_\mu S\,\rangle\langle\,\nabla^\mu S\,\rangle$	$4F^{-4}d_mc_d\lambda_1^{\rm SP} + F^{-4}c_d^2 - 2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_2^{\rm SS} + 1/2F^{-2}(\lambda_1^{\rm SP})^2$
97	$i\,\langle\,u_\mu\nabla^\mu S\chi_- S\,\rangle + {\rm h.c.}$	$-12NF^{-6}d_mc_d^3\lambda_1^{\rm SP} - 15NF^{-6}c_d^3c_m + 9NF^{-6}c_d^4 + 2NF^{-4}d_mc_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 12NF^{-4}d_mc_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/2NF^{-4}d_mc_d\lambda_1^{\rm SP} + 4NF^{-4}c_dc_m\lambda_1^{\rm SS} + 14NF^{-4}c_dc_m\lambda_2^{\rm SS} - 5NF^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 3NF^{-4}c_dc_m - 3NF^{-4}c_d^2\lambda_1^{\rm SS} - 10NF^{-4}c_d^2\lambda_2^{\rm SS} + NF^{-4}c_d^2\lambda_3^{\rm SS} - 5NF^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 6NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2NF^{-4}c_d^2 + 2NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS} - 2NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2NF^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS} + 6NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + NF^{-2}\lambda_2^{\rm SS} + 1/2NF^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2NF^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/8NF^{-2}(\lambda_1^{\rm SP})^2 + 3/2NF^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP}$
98	$i\,\langle\,u_\mu\nabla^\mu SS\chi_-\,\rangle + {\rm h.c.}$	$-12NF^{-6}d_mc_d^3\lambda_1^{\rm SP} - 13NF^{-6}c_d^3c_m + 8NF^{-6}c_d^4 + 6NF^{-4}d_mc_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4NF^{-4}d_mc_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 3/2NF^{-4}d_mc_d\lambda_1^{\rm SP} + 6NF^{-4}c_dc_m\lambda_1^{\rm SS} + 10NF^{-4}c_dc_m\lambda_2^{\rm SS} - 4NF^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 5/2NF^{-4}c_dc_m - 9/2NF^{-4}c_d^2\lambda_1^{\rm SS} - 7NF^{-4}c_d^2\lambda_2^{\rm SS} + 2NF^{-4}c_d^2\lambda_3^{\rm SS} - 5NF^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 11/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 15/8NF^{-4}c_d^2 - NF^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS} + 3NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/2NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}\lambda_1^{\rm SS} + 1/2NF^{-2}(\lambda_1^{\rm SS})^2 - 2NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 2NF^{-2}(\lambda_2^{\rm SS})^2 + NF^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/4NF^{-2}\lambda_3^{\rm SS} + 1/4NF^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/8NF^{-2}(\lambda_1^{\rm SP})^2 + 3/2NF^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 3/32NF^{-2}$
99	$i\,\langle\,u_\mu\chi_-\nabla^\mu SS\,\rangle + {\rm h.c.}$	$2NF^{-6}c_d^3c_m - NF^{-6}c_d^4 + 4NF^{-4}d_mc_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} - NF^{-4}d_mc_d\lambda_1^{\rm SP} + 3NF^{-4}c_dc_m\lambda_1^{\rm SS} - 2NF^{-4}c_dc_m\lambda_2^{\rm SS} - 5/4NF^{-4}c_dc_m - 3/2NF^{-4}c_d^2\lambda_1^{\rm SS} + NF^{-4}c_d^2\lambda_2^{\rm SS} + 2NF^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 5/8NF^{-4}c_d^2 + 4NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/4NF^{-2}(\lambda_1^{\rm SP})^2$

100	$i\langle u_\mu\rangle\langle \nabla^\mu S\{S,\chi_-\}\rangle$	$6F^{-6}c_d^3c_m - 4F^{-6}c_d^4 + F^{-4}c_dc_m\lambda_1^{SS} + 3F^{-4}c_dc_m(\lambda_1^{SP})^2 - 13/4F^{-4}c_dc_m - F^{-4}c_d^2\lambda_1^{SS} + F^{-4}c_d^2\lambda_2^{SS} - F^{-4}c_d^2\lambda_3^{SS} - 3F^{-4}c_d^2(\lambda_1^{SP})^2 + 5/2F^{-4}c_d^2 + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 3F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}\lambda_1^{SS} + 1/2F^{-2}(\lambda_1^{SS})^2 + 1/2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 3/4F^{-2}\lambda_2^{SS} - 1/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/4F^{-2}\lambda_3^{SS} + 5/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} + 1/8F^{-2}(\lambda_1^{SP})^2 - F^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} - 3/32F^{-2}$
101	$i\langle u_\mu\{S,\chi_-\}\rangle\langle \nabla^\mu S\rangle$	$6F^{-6}c_d^3c_m - 4F^{-6}c_d^4 + 3F^{-4}c_dc_m\lambda_1^{SS} - 4F^{-4}c_dc_m\lambda_2^{SS} + 3F^{-4}c_dc_m(\lambda_1^{SP})^2 - 11/4F^{-4}c_dc_m - 2F^{-4}c_d^2\lambda_1^{SS} + 3F^{-4}c_d^2\lambda_2^{SS} - F^{-4}c_d^2\lambda_3^{SS} - 3F^{-4}c_d^2(\lambda_1^{SP})^2 + 9/4F^{-4}c_d^2 + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 3F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}\lambda_1^{SS} + 1/2F^{-2}(\lambda_1^{SS})^2 - 2F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 3/4F^{-2}\lambda_2^{SS} - 1/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/4F^{-2}\lambda_3^{SS} + 7/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} - F^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} - 3/32F^{-2}$
102	$i\langle \chi_-\nabla^\mu S\rangle\langle u_\mu S\rangle$	$8F^{-6}c_d^3c_m - 4F^{-6}c_d^4 + 2F^{-4}c_dc_m\lambda_1^{SS} - 8F^{-4}c_dc_m\lambda_2^{SS} + 2F^{-4}c_dc_m(\lambda_1^{SP})^2 - 5/2F^{-4}c_dc_m + 4F^{-4}c_d^2\lambda_2^{SS} - 2F^{-4}c_d^2\lambda_3^{SS} + 4F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 2F^{-4}c_d^2(\lambda_1^{SP})^2 + F^{-4}c_d^2 - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{SS} + F^{-2}(\lambda_1^{SS})^2 - 4F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/2F^{-2}\lambda_3^{SS} + 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 1/4F^{-2}(\lambda_1^{SP})^2 + 3/16F^{-2}$
103	$i\langle S\rangle\langle \chi_-\{u_\mu,\nabla^\mu S\}\rangle$	$-8F^{-6}d_mc_d^3\lambda_1^{SP} - 4F^{-6}c_d^3c_m + 2F^{-6}c_d^4 + 4F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - F^{-4}d_mc_d\lambda_1^{SP} + 5F^{-4}c_dc_m\lambda_1^{SS} + 2F^{-4}c_dc_m\lambda_2^{SS} - 3F^{-4}c_dc_m(\lambda_1^{SP})^2 + 1/4F^{-4}c_dc_m - 4F^{-4}c_d^2\lambda_1^{SS} - F^{-4}c_d^2\lambda_2^{SS} + 3F^{-4}c_d^2\lambda_3^{SS} + 2F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + F^{-4}c_d^2(\lambda_1^{SP})^2 + 1/4F^{-4}c_d^2 + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - F^{-2}\lambda_1^{SS}\lambda_3^{SS} - F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 1/4F^{-2}\lambda_1^{SS} + 1/2F^{-2}(\lambda_1^{SS})^2 - 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 1/4F^{-2}\lambda_2^{SS} + 3/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 - 3/4F^{-2}\lambda_3^{SS} - 1/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} + 1/4F^{-2}(\lambda_1^{SP})^2 - 3/32F^{-2}$

104	$i\langle S\nabla^\mu S\rangle\langle u_\mu\chi_-\rangle$	$8F^{-6}c_d^3c_m - 4F^{-6}c_d^4 - 2F^{-4}c_dc_m\lambda_1^{SS} + 2F^{-4}c_dc_m(\lambda_1^{SP})^2 - 7/2F^{-4}c_dc_m + 2F^{-4}c_d^2\lambda_1^{SS} - 2F^{-4}c_d^2\lambda_3^{SS} + 4F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 2F^{-4}c_d^2(\lambda_1^{SP})^2 + 3/2F^{-4}c_d^2 - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{SS} + F^{-2}(\lambda_1^{SS})^2 - F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/2F^{-2}\lambda_3^{SS} - 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} + 3/16F^{-2}$
105	$i\langle S\{u_\mu, \nabla^\mu S\}\rangle\langle\chi_-\rangle$	$-24F^{-6}d_mc_d^3\lambda_1^{SP} - 20F^{-6}c_d^3c_m + 10F^{-6}c_d^4 + 12F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} + 16F^{-4}d_mc_d\lambda_2^{SS}\lambda_1^{SP} + F^{-4}d_mc_d\lambda_1^{SP} + 11F^{-4}c_dc_m\lambda_1^{SS} + 18F^{-4}c_dc_m\lambda_2^{SS} - 3F^{-4}c_dc_m(\lambda_1^{SP})^2 + 11/4F^{-4}c_dc_m - 7F^{-4}c_d^2\lambda_1^{SS} - 9F^{-4}c_d^2\lambda_2^{SS} + 3F^{-4}c_d^2\lambda_3^{SS} - 14F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + 5F^{-4}c_d^2(\lambda_1^{SP})^2 - F^{-4}c_d^2 + F^{-2}\lambda_1^{SS}\lambda_2^{SS} - F^{-2}\lambda_1^{SS}\lambda_3^{SS} + 7F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 5/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 1/4F^{-2}\lambda_1^{SS} + 1/2F^{-2}(\lambda_1^{SS})^2 - 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} + 6F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 3/2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}\lambda_2^{SS} + 3/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 - 3/4F^{-2}\lambda_3^{SS} + 1/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} + 1/8F^{-2}(\lambda_1^{SP})^2 - 3/32F^{-2}$
106	$i\langle S\chi_-\rangle\langle u_\mu\nabla^\mu S\rangle$	$-16F^{-6}d_mc_d^3\lambda_1^{SP} - 20F^{-6}c_d^3c_m + 16F^{-6}c_d^4 + 8F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - 2F^{-4}d_mc_d\lambda_1^{SP} + 14F^{-4}c_dc_m\lambda_1^{SS} + 12F^{-4}c_dc_m\lambda_2^{SS} - 18F^{-4}c_dc_m(\lambda_1^{SP})^2 + 7/2F^{-4}c_dc_m - 12F^{-4}c_d^2\lambda_1^{SS} - 16F^{-4}c_d^2\lambda_2^{SS} + 6F^{-4}c_d^2\lambda_3^{SS} + 16F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + 14F^{-4}c_d^2(\lambda_1^{SP})^2 - 4F^{-4}c_d^2 + 4F^{-2}\lambda_1^{SS}\lambda_2^{SS} - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 6F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{SS} + F^{-2}(\lambda_1^{SS})^2 - 4F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 8F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_2^{SS} + 4F^{-2}(\lambda_2^{SS})^2 + 3F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 - 3/2F^{-2}\lambda_3^{SS} - 7/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 1/4F^{-2}(\lambda_1^{SP})^2 + 6F^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} + 3/16F^{-2}$
107	$\langle SS\chi_+\chi_+\rangle$	$4NF^{-6}d_m^2c_d^2 - 4NF^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 2NF^{-4}d_m^2\lambda_1^{SS} - NF^{-4}c_dc_m\lambda_3^{SS} - 1/2NF^{-4}c_dc_m + 2NF^{-4}c_d^2\lambda_3^{SS} + 2NF^{-4}c_d^2(\lambda_3^{SS})^2 + 1/2NF^{-4}c_d^2 + 1/8NF^{-4}c_m^2 - NF^{-2}\lambda_1^{SS}\lambda_3^{SS} - 1/4NF^{-2}\lambda_1^{SS} + 2NF^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/4NF^{-2}\lambda_3^{SS} + 2NF^{-2}(\lambda_3^{SS})^2(\lambda_1^{SP})^2 + NF^{-2}(\lambda_2^{SP})^2$
108	$\langle S\chi_+S\chi_+\rangle$	$-4NF^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 4NF^{-4}d_mc_d\lambda_2^{SP} - NF^{-4}c_dc_m\lambda_3^{SS} - 1/2NF^{-4}c_dc_m + 2NF^{-4}c_d^2\lambda_3^{SS} + 2NF^{-4}c_d^2(\lambda_3^{SS})^2 + 1/4NF^{-4}c_d^2 + 1/8NF^{-4}c_m^2 - NF^{-2}\lambda_1^{SS}\lambda_3^{SS} + 2NF^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 2NF^{-2}(\lambda_3^{SS})^2(\lambda_1^{SP})^2$
109	$\langle S\rangle\langle S\chi_+\chi_+\rangle$	$8F^{-6}d_m^2c_d^2 - 8F^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 8F^{-4}d_mc_d\lambda_2^{SP} - 4F^{-4}d_m^2\lambda_2^{SS} - 2F^{-4}c_dc_m\lambda_3^{SS} - 2F^{-4}c_dc_m + 4F^{-4}c_d^2\lambda_3^{SS} + 3/2F^{-4}c_d^2 + 1/2F^{-4}c_m^2 - 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 1/2F^{-2}\lambda_2^{SS} + 4F^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 2F^{-2}(\lambda_2^{SP})^2$

110	$\langle S\chi_+\rangle^2$	$8F^{-6}d_m^2c_d^2 - 8F^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 8F^{-4}d_mc_d\lambda_2^{SP} - 4F^{-4}d_m^2\lambda_2^{SS} - 2F^{-4}c_dc_m\lambda_3^{SS} - 2F^{-4}c_dc_m + 4F^{-4}c_d^2\lambda_3^{SS} + 4F^{-4}c_d^2(\lambda_3^{SS})^2 + 3/2F^{-4}c_d^2 + 1/2F^{-4}c_m^2 - 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 1/2F^{-2}\lambda_2^{SS} + 4F^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 4F^{-2}(\lambda_3^{SS})^2(\lambda_1^{SP})^2 + 2F^{-2}(\lambda_2^{SP})^2$
111	$\langle SS\rangle\langle\chi_+\chi_+\rangle$	$4F^{-6}d_m^2c_d^2 - 4F^{-4}d_mc_d\lambda_2^{SP} - 2F^{-4}d_m^2\lambda_1^{SS} - F^{-4}c_dc_m + 3/4F^{-4}c_d^2 + 1/4F^{-4}c_m^2 - 1/4F^{-2}\lambda_1^{SS} + 1/4F^{-2}\lambda_3^{SS} + F^{-2}(\lambda_2^{SP})^2$
112	$\langle SS\chi_+\rangle\langle\chi_+\rangle$	$8F^{-6}d_m^2c_d^2 - 8F^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 8F^{-4}d_mc_d\lambda_2^{SP} - 4F^{-4}d_m^2\lambda_1^{SS} - 2F^{-4}c_dc_m\lambda_3^{SS} - 2F^{-4}c_dc_m + 4F^{-4}c_d^2\lambda_3^{SS} + 3/2F^{-4}c_d^2 + 1/2F^{-4}c_m^2 - 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 1/2F^{-2}\lambda_1^{SS} + 4F^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-2}\lambda_3^{SS} + 2F^{-2}(\lambda_2^{SP})^2$
113	$\langle S\chi_-S\chi_-\rangle$	$-12NF^{-6}d_mc_d^2c_m\lambda_1^{SP} + 12NF^{-6}d_mc_d^3\lambda_1^{SP} - 14NF^{-6}c_d^2c_m^2 + 22NF^{-6}c_d^3c_m - 8NF^{-6}c_d^4 - 2NF^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - 8NF^{-4}d_mc_d\lambda_2^{SS}\lambda_1^{SP} - 1/2NF^{-4}d_mc_d\lambda_1^{SP} + 2NF^{-4}d_mc_m\lambda_1^{SS}\lambda_1^{SP} + 4NF^{-4}d_mc_m\lambda_2^{SS}\lambda_1^{SP} + 1/2NF^{-4}d_mc_m\lambda_1^{SP} - 6NF^{-4}c_dc_m\lambda_1^{SS} - 12NF^{-4}c_dc_m\lambda_2^{SS} + 6NF^{-4}c_dc_m\lambda_3^{SS} + 4NF^{-4}c_dc_m(\lambda_1^{SP})^2 - 3/2NF^{-4}c_dc_m + 3NF^{-4}c_d^2\lambda_1^{SS} + 6NF^{-4}c_d^2\lambda_2^{SS} - 4NF^{-4}c_d^2\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 3NF^{-4}c_d^2(\lambda_1^{SP})^2 + 3/4NF^{-4}c_d^2 + 2NF^{-4}c_m^2\lambda_1^{SS} + 4NF^{-4}c_m^2\lambda_2^{SS} - NF^{-4}c_m^2(\lambda_1^{SP})^2 + 1/2NF^{-4}c_m^2 - NF^{-2}\lambda_1^{SS}\lambda_2^{SS} + NF^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + 2NF^{-2}\lambda_2^{SS}\lambda_3^{SS} - 2NF^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/4NF^{-2}\lambda_2^{SS} - 2NF^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} + 1/4NF^{-2}\lambda_1^{SP}\lambda_2^{SP} - 2NF^{-2}(\lambda_1^{SP})^2(\lambda_2^{SP})^2$
114	$\langle SS\chi_-\chi_-\rangle$	$-12NF^{-6}d_mc_d^2c_m\lambda_1^{SP} + 12NF^{-6}d_mc_d^3\lambda_1^{SP} - 18NF^{-6}c_d^2c_m^2 + 26NF^{-6}c_d^3c_m - 9NF^{-6}c_d^4 - 6NF^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - 4NF^{-4}d_mc_d\lambda_2^{SS}\lambda_1^{SP} + 4NF^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - 1/2NF^{-4}d_mc_d\lambda_1^{SP} + 2NF^{-4}d_mc_m\lambda_1^{SS}\lambda_1^{SP} + 4NF^{-4}d_mc_m\lambda_2^{SS}\lambda_1^{SP} + 1/2NF^{-4}d_mc_m\lambda_1^{SP} - 10NF^{-4}c_dc_m\lambda_1^{SS} - 12NF^{-4}c_dc_m\lambda_2^{SS} + 6NF^{-4}c_dc_m\lambda_3^{SS} + 4NF^{-4}c_dc_m(\lambda_1^{SP})^2 - NF^{-4}c_dc_m + 9/2NF^{-4}c_d^2\lambda_1^{SS} + 6NF^{-4}c_d^2\lambda_2^{SS} - 4NF^{-4}c_d^2\lambda_3^{SS} + 2NF^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 3NF^{-4}c_d^2(\lambda_1^{SP})^2 + 1/2NF^{-4}c_d^2 + 4NF^{-4}c_m^2\lambda_1^{SS} + 4NF^{-4}c_m^2\lambda_2^{SS} - NF^{-4}c_m^2(\lambda_1^{SP})^2 + 1/2NF^{-4}c_m^2 + NF^{-2}\lambda_1^{SS}\lambda_3^{SS} - 3NF^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/8NF^{-2}\lambda_1^{SS} - 1/4NF^{-2}(\lambda_1^{SS})^2 + 2NF^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - NF^{-2}(\lambda_2^{SS})^2 + 2NF^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} - NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/4NF^{-2}\lambda_3^{SS} - NF^{-2}(\lambda_3^{SS})^2 + 1/4NF^{-2}\lambda_1^{SP}\lambda_2^{SP} - 2NF^{-2}(\lambda_1^{SP})^2(\lambda_2^{SP})^2 - 1/64NF^{-2}$

115	$\langle S\rangle\langle S\chi_-\chi_-\rangle$	$1/32F^{-2} - 8F^{-6}d_mc_d^2c_m\lambda_1^{SP} + 8F^{-6}d_mc_d^3\lambda_1^{SP} - 16F^{-6}c_d^2c_m^2 + 20F^{-6}c_d^3c_m - 6F^{-6}c_d^4 - 4F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} + F^{-4}d_mc_d\lambda_1^{SP} + 4F^{-4}d_mc_m\lambda_1^{SS}\lambda_1^{SP} - F^{-4}d_mc_m\lambda_1^{SP} - 12F^{-4}c_dc_m\lambda_1^{SS} - 8F^{-4}c_dc_m\lambda_2^{SS} + 12F^{-4}c_dc_m\lambda_3^{SS} + 8F^{-4}c_dc_m\lambda_1^{SP}\lambda_2^{SP} + 2F^{-4}c_dc_m + 6F^{-4}c_d^2\lambda_1^{SS} + 3F^{-4}c_d^2\lambda_2^{SS} - 8F^{-4}c_d^2\lambda_3^{SS} - 4F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - F^{-4}c_d^2 + 4F^{-4}c_m^2\lambda_1^{SS} + 4F^{-4}c_m^2\lambda_2^{SS} - F^{-4}c_m^2 - F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} + 2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/2F^{-2}(\lambda_1^{SS})^2 + 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} + 1/4F^{-2}\lambda_2^{SS} - 4F^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} - 2F^{-2}(\lambda_3^{SS})^2 - 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP}$
116	$\langle SS\rangle\langle\chi_-\chi_-\rangle$	$-1/64F^{-2} - 4F^{-6}c_d^2c_m^2 + 4F^{-6}c_d^3c_m - F^{-6}c_d^4 - 4F^{-4}c_dc_m\lambda_1^{SS} + 4F^{-4}c_dc_m\lambda_3^{SS} + 1/2F^{-4}c_dc_m + 3/2F^{-4}c_d^2\lambda_1^{SS} - 2F^{-4}c_d^2\lambda_3^{SS} - 1/4F^{-4}c_d^2 + 2F^{-4}c_m^2\lambda_1^{SS} + F^{-2}\lambda_1^{SS}\lambda_3^{SS} + 1/8F^{-2}\lambda_1^{SS} - 1/4F^{-2}(\lambda_1^{SS})^2 - 1/4F^{-2}\lambda_3^{SS} - F^{-2}(\lambda_3^{SS})^2$
117	$\langle S\chi_-\rangle^2$	$-1/32F^{-2} - 8F^{-6}d_mc_d^2c_m\lambda_1^{SP} + 8F^{-6}d_mc_d^3\lambda_1^{SP} - 20F^{-6}c_d^2c_m^2 + 28F^{-6}c_d^3c_m - 10F^{-6}c_d^4 - 4F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} + F^{-4}d_mc_d\lambda_1^{SP} + 4F^{-4}d_mc_m\lambda_1^{SS}\lambda_1^{SP} - F^{-4}d_mc_m\lambda_1^{SP} - 12F^{-4}c_dc_m\lambda_1^{SS} - 12F^{-4}c_dc_m\lambda_2^{SS} + 12F^{-4}c_dc_m\lambda_3^{SS} + 16F^{-4}c_dc_m\lambda_1^{SP}\lambda_2^{SP} + 4F^{-4}c_dc_m(\lambda_1^{SP})^2 + F^{-4}c_dc_m + 6F^{-4}c_d^2\lambda_1^{SS} + 7F^{-4}c_d^2\lambda_2^{SS} - 8F^{-4}c_d^2\lambda_3^{SS} - 12F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 2F^{-4}c_d^2(\lambda_1^{SP})^2 + 4F^{-4}c_m^2\lambda_1^{SS} + 4F^{-4}c_m^2\lambda_2^{SS} - 2F^{-4}c_m^2(\lambda_1^{SP})^2 - F^{-4}c_m^2 - F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} + 2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/2F^{-2}(\lambda_1^{SS})^2 + 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} + 4F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} - 1/4F^{-2}\lambda_2^{SS} - F^{-2}(\lambda_2^{SS})^2 - 4F^{-2}\lambda_3^{SS}\lambda_1^{SP}\lambda_2^{SP} - 2F^{-2}(\lambda_3^{SS})^2 + 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 4F^{-2}(\lambda_1^{SP})^2(\lambda_2^{SP})^2$
118	$\langle SS\chi_-\rangle\langle\chi_-\rangle$	$1/32F^{-2} - 24F^{-6}d_mc_d^2c_m\lambda_1^{SP} + 24F^{-6}d_mc_d^3\lambda_1^{SP} - 32F^{-6}c_d^2c_m^2 + 44F^{-6}c_d^3c_m - 14F^{-6}c_d^4 - 8F^{-4}d_mc_d\lambda_1^{SS}\lambda_1^{SP} - 12F^{-4}d_mc_d\lambda_2^{SS}\lambda_1^{SP} + 4F^{-4}d_mc_d\lambda_3^{SS}\lambda_1^{SP} - F^{-4}d_mc_d\lambda_1^{SP} + 4F^{-4}d_mc_m\lambda_1^{SS}\lambda_1^{SP} + 8F^{-4}d_mc_m\lambda_2^{SS}\lambda_1^{SP} + F^{-4}d_mc_m\lambda_1^{SP} - 20F^{-4}c_dc_m\lambda_1^{SS} - 20F^{-4}c_dc_m\lambda_2^{SS} + 16F^{-4}c_dc_m\lambda_3^{SS} - 8F^{-4}c_dc_m\lambda_1^{SP}\lambda_2^{SP} + 4F^{-4}c_dc_m(\lambda_1^{SP})^2 - F^{-4}c_dc_m + 9F^{-4}c_d^2\lambda_1^{SS} + 8F^{-4}c_d^2\lambda_2^{SS} - 10F^{-4}c_d^2\lambda_3^{SS} + 12F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 4F^{-4}c_d^2(\lambda_1^{SP})^2 + 8F^{-4}c_m^2\lambda_1^{SS} + 8F^{-4}c_m^2\lambda_2^{SS} + F^{-4}c_m^2 - F^{-2}\lambda_1^{SS}\lambda_2^{SS} + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS} - 2F^{-2}\lambda_1^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/2F^{-2}(\lambda_1^{SS})^2 + 2F^{-2}\lambda_2^{SS}\lambda_3^{SS} - 4F^{-2}\lambda_2^{SS}\lambda_1^{SP}\lambda_2^{SP} + F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/4F^{-2}\lambda_2^{SS} - F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_3^{SS})^2 - 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP}$

119	$\langle PP \rangle$	$-4NF^{-2}M_P^2M_S^2(\lambda_1^{\rm SP})^2 + NF^{-2}M_P^4(\lambda_1^{\rm SP})^2 + 4NF^{-2}M_S^4(\lambda_1^{\rm SP})^2 + NF^{-2}M_S^2M_P^2(\lambda_1^{\rm SP})^2$
120	$\langle P \rangle^2$	$-4F^{-2}M_P^2M_S^2(\lambda_1^{\rm SP})^2 + F^{-2}M_P^4(\lambda_1^{\rm SP})^2 + 4F^{-2}M_S^4(\lambda_1^{\rm SP})^2 + F^{-2}M_S^2M_P^2(\lambda_1^{\rm SP})^2$
121	$\langle u_\nu P u^\nu P \rangle$	$2NF^{-4}M_P^2c_d^2(\lambda_1^{\rm SP})^2 - 4NF^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 4NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}M_P^2 - NF^{-2}\lambda_1^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 - 4NF^{-2}\lambda_2^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_2^{\rm PP}M_P^2 + 8NF^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 5/4NF^{-2}M_P^2(\lambda_1^{\rm SP})^2 - 3/2NF^{-2}M_S^2(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2$
122	$\langle PPu\cdot u \rangle$	$2NF^{-4}\lambda_1^{\rm PP}M_P^2c_d^2 - 2NF^{-4}\lambda_1^{\rm PP}c_d^2M_S^2 - 12NF^{-4}M_P^2c_d^2(\lambda_1^{\rm SP})^2 + 24NF^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 - 4NF^{-2}\lambda_1^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}\lambda_1^{\rm PP}M_P^2 + 8NF^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + NF^{-2}(\lambda_1^{\rm PP})^2M_P^2 - 2NF^{-2}\lambda_2^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - 4NF^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 4NF^{-2}(\lambda_2^{\rm PP})^2M_P^2 + 4NF^{-2}M_P^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/16NF^{-2}M_P^2 - 8NF^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - NF^{-2}M_S^2(\lambda_1^{\rm SP})^2 + 2NF^{-2}M_S^2(\lambda_1^{\rm SP})^4 - 2NF^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2$
123	$\langle u_\mu \rangle\langle u^\mu PP \rangle$	$4F^{-4}\lambda_1^{\rm PP}M_P^2c_d^2 - 4F^{-4}\lambda_1^{\rm PP}c_d^2M_S^2 - 4F^{-4}M_P^2c_d^2(\lambda_1^{\rm SP})^2 + 8F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}M_P^2 - 2F^{-2}\lambda_1^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm PP}M_P^2 - 4F^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_1^{\rm PP})^2M_P^2 - 2F^{-2}\lambda_2^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_2^{\rm PP}M_P^2 - 4F^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 8F^{-2}M_P^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/8F^{-2}M_P^2 - 16F^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}M_S^2(\lambda_1^{\rm SP})^4$
124	$\langle u_\mu P \rangle^2$	$4F^{-4}\lambda_2^{\rm PP}M_P^2c_d^2 - 4F^{-4}\lambda_2^{\rm PP}c_d^2M_S^2 - 4F^{-4}M_P^2c_d^2(\lambda_1^{\rm SP})^2 + 8F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}M_P^2 - 2F^{-2}\lambda_1^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_1^{\rm PP})^2M_P^2 - 2F^{-2}\lambda_2^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 - 3F^{-2}\lambda_2^{\rm PP}M_P^2 - 4F^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}(\lambda_2^{\rm PP})^2M_P^2 + 8F^{-2}M_P^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/8F^{-2}M_P^2 - 16F^{-2}\lambda_2^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}M_S^2(\lambda_1^{\rm SP})^4$
125	$\langle P \rangle\langle Pu\cdot u \rangle$	$4F^{-4}\lambda_2^{\rm PP}M_P^2c_d^2 - 4F^{-4}\lambda_2^{\rm PP}c_d^2M_S^2 - 12F^{-4}M_P^2c_d^2(\lambda_1^{\rm SP})^2 + 24F^{-4}c_d^2M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}M_P^2 - 6F^{-2}\lambda_1^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 2F^{-2}(\lambda_1^{\rm PP})^2M_P^2 - 6F^{-2}\lambda_2^{\rm PP}M_P^2(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm PP}M_P^2 + 4F^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 + 8F^{-2}M_P^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/8F^{-2}M_P^2 - 16F^{-2}\lambda_1^{\rm SS}M_S^2(\lambda_1^{\rm SP})^2 - 2F^{-2}M_S^2(\lambda_1^{\rm SP})^2 + 4F^{-2}M_S^2(\lambda_1^{\rm SP})^4 - 2F^{-2}\lambda_1^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_2^{\rm PP}M_S^2(\lambda_1^{\rm SP})^2$

126	$\langle PP\rangle\langle u\cdot u\rangle$	$2F^{-4}\lambda_1^{\mathrm{PP}}M_P^2c_d^2 - 2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2M_S^2 - 2F^{-4}M_P^2c_d^2(\lambda_1^{\mathrm{SP}})^2 + 4F^{-4}c_d^2M_S^2(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_1^{\mathrm{PP}}M_P^2(\lambda_1^{\mathrm{SP}})^2 + 3/2F^{-2}\lambda_1^{\mathrm{PP}}M_P^2 - 2F^{-2}\lambda_1^{\mathrm{PP}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + F^{-2}(\lambda_1^{\mathrm{PP}})^2M_P^2 + 4F^{-2}M_P^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 5/4F^{-2}M_P^2(\lambda_1^{\mathrm{SP}})^2 + 1/16F^{-2}M_P^2 - 8F^{-2}\lambda_1^{\mathrm{SS}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}M_S^2(\lambda_1^{\mathrm{SP}})^4$
127	$\langle PP\chi_+\rangle$	$-4NF^{-4}d_mM_P^2c_d\lambda_1^{\mathrm{SP}} + 8NF^{-4}d_mc_dM_S^2\lambda_1^{\mathrm{SP}} - 8NF^{-4}M_P^2c_dc_m(\lambda_1^{\mathrm{SP}})^2 + 16NF^{-4}c_dc_mM_S^2(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}\lambda_1^{\mathrm{PP}}M_P^2 - 4NF^{-2}\lambda_3^{\mathrm{PP}}M_P^2(\lambda_1^{\mathrm{SP}})^2 + 8NF^{-2}\lambda_3^{\mathrm{PP}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-2}M_P^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2NF^{-2}M_P^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - NF^{-2}M_P^2(\lambda_1^{\mathrm{SP}})^2 - 8NF^{-2}\lambda_3^{\mathrm{SS}}M_S^2(\lambda_1^{\mathrm{SP}})^2 - 4NF^{-2}M_S^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + NF^{-2}M_S^2(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}\lambda_3^{\mathrm{PP}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-4}c_dd_mM_S^2\lambda_1^{\mathrm{SP}}$
128	$\langle P\rangle\langle P\chi_+\rangle$	$-4F^{-4}d_mM_P^2c_d\lambda_1^{\mathrm{SP}} + 8F^{-4}d_mc_dM_S^2\lambda_1^{\mathrm{SP}} - 8F^{-4}M_P^2c_dc_m(\lambda_1^{\mathrm{SP}})^2 + 16F^{-4}c_dc_mM_S^2(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}\lambda_2^{\mathrm{PP}}M_P^2 - 4F^{-2}\lambda_3^{\mathrm{PP}}M_P^2(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}\lambda_3^{\mathrm{PP}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}M_P^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}M_P^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}M_P^2(\lambda_1^{\mathrm{SP}})^2 - 16F^{-2}\lambda_3^{\mathrm{SS}}M_S^2(\lambda_1^{\mathrm{SP}})^2 - 8F^{-2}M_S^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2F^{-2}M_S^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_3^{\mathrm{PP}}M_S^2(\lambda_1^{\mathrm{SP}})^2 + 4F^{-4}c_dd_mM_S^2\lambda_1^{\mathrm{SP}}$
129	$\langle PP\rangle\langle\chi_+\rangle$	$F^{-2}\lambda_1^{\mathrm{PP}}M_P^2 + 4F^{-2}M_P^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}M_P^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - F^{-2}M_P^2(\lambda_1^{\mathrm{SP}})^2 - 8F^{-2}\lambda_3^{\mathrm{SS}}M_S^2(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}M_S^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + F^{-2}M_S^2(\lambda_1^{\mathrm{SP}})^2$
130	$\langle u_\nu PPu^\nu u\cdot u\rangle$	$-2/3NF^{-6}\lambda_1^{\mathrm{PP}}c_d^4 + 2NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 2NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SS}} - 1/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/6NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2 + 1/3NF^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 - 4/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 - 7/48NF^{-4}c_d^2 + 4NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_2^{\mathrm{PP}})^2 - NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/8NF^{-2}\lambda_1^{\mathrm{PP}} + 1/4NF^{-2}(\lambda_1^{\mathrm{PP}})^2 - 1/6NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 2/3NF^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-2}(\lambda_1^{\mathrm{SP}})^2 + 1/12NF^{-2}(\lambda_1^{\mathrm{SP}})^4 + 1/6NF^{-2}(\lambda_1^{\mathrm{SP}})^6 - 5/192NF^{-2}$

131	$\langle u_\mu u_\nu P u^\mu u^\nu P \rangle$	$2/3NF^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 4/3NF^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 4/3NF^{-4}(\lambda_2^{\rm PP})^2c_d^2 - 1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^4 - 1/24NF^{-4}c_d^2 - 1/12NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/24NF^{-2}\lambda_1^{\rm PP} + 1/6NF^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 1/12NF^{-2}(\lambda_1^{\rm PP})^2 + 1/6NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3NF^{-2}(\lambda_2^{\rm PP})^2 - 1/32NF^{-2}(\lambda_1^{\rm SP})^2 + 1/64NF^{-2}$
132	$\langle Pu\cdot uPu\cdot u \rangle$	$40/3NF^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 26/3NF^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 2NF^{-4}(\lambda_1^{\rm PP})^2c_d^2 - 28/3NF^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 4/3NF^{-4}(\lambda_2^{\rm PP})^2c_d^2 - 10NF^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 8/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 37/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^4 - 2NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/12NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/16NF^{-2}\lambda_1^{\rm PP} + 13/6NF^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 1/2NF^{-2}(\lambda_1^{\rm PP})^2 - NF^{-2}(\lambda_1^{\rm PP})^3 + NF^{-2}(\lambda_2^{\rm PP})^2 + NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/96NF^{-2}(\lambda_1^{\rm SP})^2 - 1/4NF^{-2}(\lambda_1^{\rm SP})^4$
133	$\langle Pu_\mu Pu_\nu u^\mu u^\nu \rangle$	$4/3NF^{-4}\lambda_1^{\rm PP}\lambda_2^{\rm PP}c_d^2 - 4/3NF^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 4/3NF^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 1/3NF^{-4}\lambda_2^{\rm PP}c_d^2 + 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^4 + 2/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 1/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/6NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/6NF^{-2}\lambda_2^{\rm PP} - 1/12NF^{-2}(\lambda_1^{\rm SP})^2 + 1/12NF^{-2}(\lambda_1^{\rm SP})^4$
134	$\langle PPu_\mu u_\nu u^\mu u^\nu \rangle$	$-2/3NF^{-6}\lambda_1^{\rm PP}c_d^4 + 2/3NF^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 1/3NF^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 1/6NF^{-4}\lambda_1^{\rm PP}c_d^2 + 1/3NF^{-4}(\lambda_1^{\rm PP})^2c_d^2 - 4/3NF^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 1/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + NF^{-4}c_d^2(\lambda_1^{\rm SP})^4 + 1/16NF^{-4}c_d^2 + 1/24NF^{-2}\lambda_1^{\rm PP} - 1/12NF^{-2}(\lambda_1^{\rm PP})^2 + 1/2NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^4 + 2/3NF^{-2}(\lambda_2^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 1/3NF^{-2}(\lambda_2^{\rm PP})^2 + 1/24NF^{-2}(\lambda_1^{\rm SP})^2 - 1/6NF^{-2}(\lambda_1^{\rm SP})^4 + 1/6NF^{-2}(\lambda_1^{\rm SP})^6 - 5/192NF^{-2}$

135	$\langle PPu\cdot uu\cdot u\rangle$	$-2/3NF^{-6}\lambda_1^{\mathrm{PP}}c_d^4 + 40/3NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 23/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 5/6NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2 - 5/3NF^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 - 28/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 12NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 5/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 13NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 1/48NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 1/24NF^{-2}\lambda_1^{\mathrm{PP}} + 2NF^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 11/12NF^{-2}(\lambda_1^{\mathrm{PP}})^2 + 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 2/3NF^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}(\lambda_2^{\mathrm{PP}})^2 - 2NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 + 4NF^{-2}(\lambda_1^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/12NF^{-2}(\lambda_1^{\mathrm{SP}})^4 + 1/6NF^{-2}(\lambda_1^{\mathrm{SP}})^6 + 1/192NF^{-2}$
136	$\langle u_\nu P u^\nu \{P,u\cdot u\}\rangle$	$-6NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 - 2/3NF^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 16/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 28/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/6NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 2NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 7/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 4/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 14/3NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 7/6NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/3NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 2NF^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} - 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/24NF^{-2}\lambda_2^{\mathrm{PP}} - 2NF^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 3/4NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/24NF^{-2}(\lambda_1^{\mathrm{SP}})^2 - 1/6NF^{-2}(\lambda_1^{\mathrm{SP}})^4$
137	$\langle u_\mu P u^\mu u_\nu P u^\nu\rangle$	$2/3NF^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 2/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 16/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 4/3NF^{-4}(\lambda_2^{\mathrm{PP}})^2c_d^2 + NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 1/24NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 5/12NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/24NF^{-2}\lambda_1^{\mathrm{PP}} + 1/6NF^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/12NF^{-2}(\lambda_1^{\mathrm{PP}})^2 - 5/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/3NF^{-2}(\lambda_2^{\mathrm{PP}})^2 + 17/96NF^{-2}(\lambda_1^{\mathrm{SP}})^2 - 1/64NF^{-2}$
138	$\langle u_\mu PP\rangle\langle u^\mu u\cdot u\rangle$	$3/16F^{-2} - 8F^{-6}\lambda_1^{\mathrm{PP}}c_d^4 + 8F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 4F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SS}} + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SS}} - 12F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 4F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 - 4F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 8F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 10F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 6F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 3/8F^{-2}\lambda_1^{\mathrm{PP}} - 6F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} + 6F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}(\lambda_1^{\mathrm{PP}})^3 + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 3/8F^{-2}\lambda_2^{\mathrm{PP}} + 16F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 2F^{-2}(\lambda_1^{\mathrm{SP}})^6$

139	$\langle u_\mu \rangle \langle u^\mu \{PP, u\cdot u\} \rangle$	$1/24F^{-2} - 8/3F^{-6}\lambda_1^{PP}c_d^4 + 10/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 2F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SS} - 8/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 - 1/2F^{-4}\lambda_1^{PP}c_d^2 - 2/3F^{-4}(\lambda_1^{PP})^2c_d^2 - 8/3F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 - 4F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 - 12F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 + 19/6F^{-4}c_d^2(\lambda_1^{SP})^2 + 4/3F^{-4}c_d^2(\lambda_1^{SP})^4 + 1/8F^{-4}c_d^2 + 5/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 - 1/3F^{-2}\lambda_1^{PP}\lambda_2^{PP} + F^{-2}\lambda_1^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 - 4F^{-2}\lambda_1^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/12F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 - 2F^{-2}(\lambda_1^{PP})^2\lambda_2^{PP} + 4/3F^{-2}(\lambda_1^{PP})^2 + 2F^{-2}\lambda_2^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 + 4F^{-2}\lambda_2^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 - 5/12F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^4 + 1/24F^{-2}\lambda_2^{PP} + 4/3F^{-2}(\lambda_2^{PP})^2(\lambda_1^{SP})^2 + 8F^{-2}\lambda_1^{SS}\lambda_2^{SS}(\lambda_1^{SP})^2 - 3/4F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^4 - 2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^4 - 1/48F^{-2}(\lambda_1^{SP})^2 + 1/4F^{-2}(\lambda_1^{SP})^4 + 2/3F^{-2}(\lambda_1^{SP})^6$
140	$\langle u_\mu \rangle \langle u^\mu \{u_\nu P u^\nu, P\} \rangle$	$-8/3F^{-6}c_d^4(\lambda_1^{SP})^2 - 4/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 + 8/3F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 + 2/3F^{-4}\lambda_2^{PP}c_d^2 + 8/3F^{-4}(\lambda_2^{PP})^2c_d^2 + 4F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 - 2/3F^{-4}c_d^2(\lambda_1^{SP})^2 - 2/3F^{-4}c_d^2(\lambda_1^{SP})^4 + 4/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{PP}\lambda_2^{PP} - 4F^{-2}\lambda_1^{PP}(\lambda_2^{PP})^2 + 2F^{-2}\lambda_1^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 + 2/3F^{-2}(\lambda_1^{PP})^2(\lambda_1^{SP})^2 - 8F^{-2}\lambda_2^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^4 - 1/4F^{-2}\lambda_2^{PP} - 2F^{-2}(\lambda_2^{PP})^2(\lambda_1^{SP})^2 + F^{-2}(\lambda_2^{PP})^2 + 3/2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}(\lambda_1^{SP})^4$
141	$\langle u_\mu \rangle \langle P u^\mu P u\cdot u \rangle$	$4/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 4F^{-4}\lambda_1^{PP}\lambda_2^{PP}c_d^2 - 4/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 - 4/3F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 - 4/3F^{-4}\lambda_2^{PP}c_d^2 - 4F^{-4}(\lambda_1^{PP})^2c_d^2 + 8/3F^{-4}(\lambda_2^{PP})^2c_d^2 - 20F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 + 16/3F^{-4}c_d^2(\lambda_1^{SP})^2 - 14/3F^{-4}c_d^2(\lambda_1^{SP})^4 - 2/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_1^{PP}\lambda_2^{PP} + 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 - 8F^{-2}\lambda_1^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 - 2/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 + 1/8F^{-2}\lambda_1^{PP} - 2F^{-2}(\lambda_1^{PP})^2\lambda_2^{PP} + 2/3F^{-2}(\lambda_1^{PP})^2(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_1^{PP})^3 + 2/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 - 3/8F^{-2}\lambda_2^{PP} + 2F^{-2}(\lambda_1^{PP})^2 - 2F^{-2}(\lambda_2^{PP})^2 - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 1/4F^{-2}(\lambda_1^{SP})^4$

142	$\langle u_\mu P \rangle \langle u^\mu \{P, u\cdot u\} \rangle$	$-1/12F^{-2} - 16/3F^{-6}\lambda_2^{\mathrm{PP}}c_d^4 + 40/3F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 4/3F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 16/3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 8/3F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 + 4F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SS}} - 16F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/3F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 8/3F^{-4}(\lambda_2^{\mathrm{PP}})^2c_d^2 - 8F^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 24F^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 28/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 - 1/4F^{-4}c_d^2 + 20/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} - 4F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_2^{\mathrm{PP}})^2 + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 6F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 5/6F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 1/24F^{-2}\lambda_1^{\mathrm{PP}} - 6F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} + 10/3F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 2/3F^{-2}(\lambda_1^{\mathrm{PP}})^2 - 2F^{-2}(\lambda_1^{\mathrm{PP}})^3 + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 7/6F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 5/8F^{-2}\lambda_2^{\mathrm{PP}} + 4/3F^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + F^{-2}(\lambda_2^{\mathrm{PP}})^2 + 16F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 + 1/2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - 1/24F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 2/3F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 4/3F^{-2}(\lambda_1^{\mathrm{SP}})^6$
143	$\langle u_\mu P u_\nu P \rangle \langle u^\mu u^\nu \rangle$	$-4/3F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 8F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 8/3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 4/3F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 8/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 4/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 16/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 4F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} + 4/3F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 3/4F^{-2}\lambda_2^{\mathrm{PP}} - F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/3F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^4$
144*	$\langle P \rangle \langle P u_\mu u_\nu u^\mu u^\nu \rangle$	$1/48F^{-2} - 4/3F^{-6}\lambda_2^{\mathrm{PP}}c_d^4 + 4/3F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 4/3F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 4/3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 2/3F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 - 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 - 2/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 4/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 - 1/24F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 2/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/3F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/3F^{-2}(\lambda_1^{\mathrm{PP}})^2 + 5/6F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 1/6F^{-2}\lambda_2^{\mathrm{PP}} - 1/48F^{-2}(\lambda_1^{\mathrm{SP}})^2 - 1/3F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 1/3F^{-2}(\lambda_1^{\mathrm{SP}})^6$

145	$\langle P \rangle \langle Pu \cdot uu \cdot u \rangle$	$-1/96F^{-2} - 4/3F^{-6}\lambda_2^{\mathrm{PP}}c_d^4 - 20/3F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 28/3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 2/3F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 - 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 - 24F^{-4}c_d^2\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 13/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 40/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 - 1/24F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} - 6F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/8F^{-2}\lambda_1^{\mathrm{PP}} + 1/3F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/6F^{-2}(\lambda_1^{\mathrm{PP}})^2 - 2F^{-2}(\lambda_1^{\mathrm{PP}})^3 + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 7/6F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 1/6F^{-2}\lambda_2^{\mathrm{PP}} + 1/2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 + 8F^{-2}(\lambda_1^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/48F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 1/6F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 1/3F^{-2}(\lambda_1^{\mathrm{SP}})^6$
146	$\langle Pu_\nu u^\mu u^\nu \rangle \langle u_\mu P \rangle$	$-5/24F^{-2} - 16/3F^{-6}\lambda_2^{\mathrm{PP}}c_d^4 + 16/3F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 16/3F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 16/3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 8/3F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 + 8F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SS}} + 2/3F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 - 16/3F^{-4}(\lambda_2^{\mathrm{PP}})^2c_d^2 - 14/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 16/3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 + 1/2F^{-4}c_d^2 + 8/3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 8F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_2^{\mathrm{PP}})^2 + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 5/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 1/6F^{-2}\lambda_1^{\mathrm{PP}} + 4/3F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 4/3F^{-2}(\lambda_1^{\mathrm{PP}})^2 - 16F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 7/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 8/3F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 16/3F^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}(\lambda_2^{\mathrm{PP}})^2 + 3F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/12F^{-2}(\lambda_1^{\mathrm{SP}})^2 - 4/3F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 4/3F^{-2}(\lambda_1^{\mathrm{SP}})^6$
147	$\langle P\,[u_\mu, u_\nu] \rangle^2$	$3/64F^{-2} + 2F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SS}} - F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 - 1/8F^{-4}c_d^2 + 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 2F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_2^{\mathrm{PP}})^2 - F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} - 1/4F^{-2}(\lambda_1^{\mathrm{PP}})^2 - 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/16F^{-2}\lambda_2^{\mathrm{PP}} + 2F^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - F^{-2}(\lambda_2^{\mathrm{PP}})^2 - 2F^{-2}(\lambda_2^{\mathrm{PP}})^3 + 1/2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - 4F^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/8F^{-2}(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}(\lambda_1^{\mathrm{SP}})^4$
148	$\langle P\,\{u_\mu, u_\nu\} \rangle^2$	$3/64F^{-2} - 2F^{-6}\lambda_2^{\mathrm{PP}}c_d^4 + 2F^{-6}c_d^4(\lambda_1^{\mathrm{SP}})^2 + 2F^{-4}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}c_d^2 - 2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + F^{-4}(\lambda_1^{\mathrm{PP}})^2c_d^2 + 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SS}} - 3F^{-4}\lambda_2^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 5/6F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 4/3F^{-4}(\lambda_2^{\mathrm{PP}})^2c_d^2 - 4F^{-4}c_d^2\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^4 - 5/48F^{-4}c_d^2 + 3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} - 2F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_2^{\mathrm{PP}})^2 + 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_2^{\mathrm{PP}} + 1/2F^{-2}(\lambda_1^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}(\lambda_1^{\mathrm{PP}})^2 + 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 1/12F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 3/16F^{-2}\lambda_2^{\mathrm{PP}} + 8/3F^{-2}(\lambda_2^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}(\lambda_2^{\mathrm{PP}})^3 - F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 + 4F^{-2}(\lambda_2^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 + 1/96F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 1/4F^{-2}(\lambda_1^{\mathrm{SP}})^4 + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^6$

149	$\langle Pu\cdot u\rangle^2$	$3/32F^{-2} - 4F^{-6}\lambda_2^{\rm PP}c_d^4 + 12F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 4F^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 2F^{-4}(\lambda_1^{\rm PP})^2c_d^2 + 4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SS} - 2F^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 7/3F^{-4}\lambda_2^{\rm PP}c_d^2 - 4/3F^{-4}(\lambda_2^{\rm PP})^2c_d^2 - 24F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 24F^{-4}c_d^2(\lambda_1^{\rm SP})^4 + 7/24F^{-4}c_d^2 + 8F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 + 1/8F^{-2}\lambda_1^{\rm PP} - 4F^{-2}(\lambda_1^{\rm PP})^2\lambda_2^{\rm PP} + 7F^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 + 1/2F^{-2}(\lambda_1^{\rm PP})^2 - 2F^{-2}(\lambda_1^{\rm PP})^3 - 4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm PP} + 4/3F^{-2}(\lambda_2^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_2^{\rm PP})^2 + 2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^4 + 8F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 + 13/48F^{-2}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm SP})^4 + F^{-2}(\lambda_1^{\rm SP})^6$
150*	$\langle PP\rangle\langle u_\mu u_\nu u^\mu u^\nu\rangle$	$-1/96F^{-2} - 2/3F^{-6}\lambda_1^{\rm PP}c_d^4 + 2/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - F^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-4}\lambda_1^{\rm PP}c_d^2 + 1/3F^{-4}(\lambda_1^{\rm PP})^2c_d^2 - 1/2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^4 + 1/48F^{-4}c_d^2 + 5/12F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/12F^{-2}\lambda_1^{\rm PP} + 1/6F^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 1/6F^{-2}(\lambda_1^{\rm PP})^2 + 3/32F^{-2}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}(\lambda_1^{\rm SP})^4 + 1/6F^{-2}(\lambda_1^{\rm SP})^6$
151	$\langle PP\rangle\langle u\cdot uu\cdot u\rangle$	$-7/192F^{-2} - 4/3F^{-6}\lambda_1^{\rm PP}c_d^4 + 4/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SS} - 2F^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - F^{-4}\lambda_1^{\rm PP}c_d^2 + 2/3F^{-4}(\lambda_1^{\rm PP})^2c_d^2 - 4F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^4 - 1/12F^{-4}c_d^2 + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 5/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 5/48F^{-2}\lambda_1^{\rm PP} + 7/3F^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 1/12F^{-2}(\lambda_1^{\rm PP})^2 - F^{-2}(\lambda_1^{\rm PP})^3 - 1/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^4 + 4F^{-2}(\lambda_1^{\rm SS})^2(\lambda_1^{\rm SP})^2 + 1/16F^{-2}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}(\lambda_1^{\rm SP})^4 + 1/3F^{-2}(\lambda_1^{\rm SP})^6$
152	$\langle PPu_\nu u^\mu u^\nu\rangle\langle u_\mu\rangle$	$5/48F^{-2} - 8/3F^{-6}\lambda_1^{\rm PP}c_d^4 + 16/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 + 4F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_2^{\rm SS} - 8/3F^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + F^{-4}\lambda_1^{\rm PP}c_d^2 + 4/3F^{-4}(\lambda_1^{\rm PP})^2c_d^2 - 8/3F^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 4F^{-4}c_d^2\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 10/3F^{-4}c_d^2(\lambda_1^{\rm SP})^4 - 1/4F^{-4}c_d^2 + 2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/4F^{-2}\lambda_1^{\rm PP} - 2/3F^{-2}(\lambda_1^{\rm PP})^2 + 1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/12F^{-2}\lambda_2^{\rm PP} + 28/3F^{-2}(\lambda_2^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 8F^{-2}(\lambda_2^{\rm PP})^3 + 1/24F^{-2}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}(\lambda_1^{\rm SP})^4 + 2/3F^{-2}(\lambda_1^{\rm SP})^6$

153	$\langle PPu_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$	$-3/16F^{-2} - 8F^{-6}\lambda_1^{PP}c_d^4 + 28/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 8F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SS} - 28/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 + 2F^{-4}\lambda_1^{PP}c_d^2 + 4F^{-4}(\lambda_1^{PP})^2c_d^2 - 16/3F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 + 8/3F^{-4}(\lambda_2^{PP})^2c_d^2 - 16F^{-4}c_d^2\lambda_2^{SS}(\lambda_1^{SP})^2 - 8/3F^{-4}c_d^2(\lambda_1^{SP})^2 + 28/3F^{-4}c_d^2(\lambda_1^{SP})^4 + 5/12F^{-4}c_d^2 + 20/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 - 8F^{-2}\lambda_1^{PP}(\lambda_2^{PP})^2 + 4F^{-2}\lambda_1^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 + 2/3F^{-2}(\lambda_1^{PP})^2(\lambda_1^{SP})^2 - F^{-2}(\lambda_1^{PP})^2 + 8F^{-2}\lambda_2^{PP}\lambda_2^{SS}(\lambda_1^{SP})^2 + 2/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 - 8F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^4 + 28/3F^{-2}(\lambda_2^{PP})^2(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_2^{PP})^2 - 3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2 - 8F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^4 + 16F^{-2}(\lambda_2^{SS})^2(\lambda_1^{SP})^2 + 7/24F^{-2}(\lambda_1^{SP})^2 + 1/2F^{-2}(\lambda_1^{SP})^4 + 2F^{-2}(\lambda_1^{SP})^6$
154	$\langle PPu\cdot u\rangle\langle u\cdot u\rangle$	$-3/32F^{-2} - 4F^{-6}\lambda_1^{PP}c_d^4 + 20/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SS} - 38/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 - F^{-4}\lambda_1^{PP}c_d^2 - 2F^{-4}(\lambda_1^{PP})^2c_d^2 - 32/3F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 + 4/3F^{-4}(\lambda_2^{PP})^2c_d^2 - 28F^{-4}c_d^2\lambda_1^{SS}(\lambda_1^{SP})^2 - 4/3F^{-4}c_d^2(\lambda_1^{SP})^2 + 74/3F^{-4}c_d^2(\lambda_1^{SP})^4 - 7/24F^{-4}c_d^2 + 16/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 - 4F^{-2}\lambda_1^{PP}(\lambda_2^{PP})^2 - 6F^{-2}\lambda_1^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 - 1/8F^{-2}\lambda_1^{PP} + 1/3F^{-2}(\lambda_1^{PP})^2(\lambda_1^{SP})^2 - 3/2F^{-2}(\lambda_1^{PP})^2 - 2F^{-2}(\lambda_1^{PP})^3 + 4F^{-2}\lambda_2^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 - 2/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^4 + 2/3F^{-2}(\lambda_2^{PP})^2(\lambda_1^{SP})^2 + F^{-2}(\lambda_2^{PP})^2 + 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 4F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^4 + 8F^{-2}(\lambda_1^{SS})^2(\lambda_1^{SP})^2 + 1/48F^{-2}(\lambda_1^{SP})^2 - 1/4F^{-2}(\lambda_1^{SP})^4 + F^{-2}(\lambda_1^{SP})^6$
155	$\langle u_\nu Pu^\nu u\cdot u\rangle\langle P\rangle$	$1/12F^{-2} - 4/3F^{-6}\lambda_2^{PP}c_d^4 + 4/3F^{-6}c_d^4(\lambda_1^{SP})^2 + 4/3F^{-4}\lambda_1^{PP}\lambda_2^{PP}c_d^2 - 4/3F^{-4}\lambda_1^{PP}c_d^2(\lambda_1^{SP})^2 + 2/3F^{-4}(\lambda_1^{PP})^2c_d^2 + 4F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SS} + 2F^{-4}\lambda_2^{PP}c_d^2(\lambda_1^{SP})^2 - 4F^{-4}(\lambda_2^{PP})^2c_d^2 - 2/3F^{-4}c_d^2(\lambda_1^{SP})^2 + 4/3F^{-4}c_d^2(\lambda_1^{SP})^4 + 5/24F^{-4}c_d^2 + 8/3F^{-2}\lambda_1^{PP}\lambda_2^{PP}(\lambda_1^{SP})^2 - 2/3F^{-2}\lambda_1^{PP}\lambda_2^{PP} + 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 + 1/6F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - 8/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^4 - 4F^{-2}(\lambda_1^{PP})^2\lambda_2^{PP} + 7/3F^{-2}(\lambda_1^{PP})^2(\lambda_1^{SP})^2 + 2/3F^{-2}(\lambda_1^{PP})^2 - 8F^{-2}\lambda_2^{PP}\lambda_1^{SS}(\lambda_1^{SP})^2 + 4/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + 4/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^4 + 1/12F^{-2}\lambda_2^{PP} - 2F^{-2}(\lambda_2^{PP})^2 + 3/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2 - 13/48F^{-2}(\lambda_1^{SP})^2 - 1/3F^{-2}(\lambda_1^{SP})^4 + 1/3F^{-2}(\lambda_1^{SP})^6$

156	$\langle u_\nu P u^\nu P \rangle \langle u \cdot u \rangle$	$-8/3F^{-6}c_d^4(\lambda_1^{\rm SP})^2 - 4/3F^{-4}\lambda_1^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 8/3F^{-4}\lambda_2^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 4/3F^{-4}\lambda_2^{\rm PP}c_d^2 + 4F^{-4}c_d^2\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 5/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^4 - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^4 - 4F^{-2}(\lambda_1^{\rm PP})^2\lambda_2^{\rm PP} + 8/3F^{-2}(\lambda_1^{\rm PP})^2(\lambda_1^{\rm SP})^2 - 8F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 4/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^4 + 1/4F^{-2}\lambda_2^{\rm PP} + 3/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 7/24F^{-2}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}(\lambda_1^{\rm SP})^4$
157	$\langle \nabla_\mu P u_\nu \nabla^\mu P u^\nu \rangle$	$1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 1/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/3NF^{-2}\lambda_2^{\rm PP} - 1/12NF^{-2}(\lambda_1^{\rm SP})^2$
158	$\langle u_\mu u_\nu \nabla^\mu P \nabla^\nu P \rangle$	$2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 1/6NF^{-4}c_d^2 - 1/6NF^{-2}\lambda_1^{\rm PP} + 1/3NF^{-2}(\lambda_1^{\rm PP})^2 - 2/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 4/3NF^{-2}(\lambda_2^{\rm PP})^2 - 1/6NF^{-2}(\lambda_1^{\rm SP})^2 + 1/6NF^{-2}(\lambda_1^{\rm SP})^4 + 5/48NF^{-2}$
159	$\langle u_\mu u_\nu \nabla^\nu P \nabla^\mu P \rangle$	$2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/6NF^{-4}c_d^2 - 2NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/2NF^{-2}\lambda_1^{\rm PP} + NF^{-2}(\lambda_1^{\rm PP})^2 - 2NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 4NF^{-2}(\lambda_2^{\rm PP})^2 - 4/3NF^{-2}(\lambda_1^{\rm SP})^2 + 7/6NF^{-2}(\lambda_1^{\rm SP})^4 + 23/48NF^{-2}$
160	$\langle u_\mu \nabla^\mu P u_\nu \nabla^\nu P \rangle + \text{h.c.}$	$1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 8/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 2/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 4/3NF^{-2}\lambda_2^{\rm PP} - 1/3NF^{-2}(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}(\lambda_1^{\rm SP})^4$
161	$\langle u \cdot u \nabla_\mu P \nabla^\mu P \rangle$	$-2NF^{-4}\lambda_1^{\rm PP}c_d^2 + 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 5/6NF^{-2}\lambda_1^{\rm PP} - 1/3NF^{-2}(\lambda_1^{\rm PP})^2 + 2/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3NF^{-2}(\lambda_2^{\rm PP})^2 - NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/4NF^{-2}(\lambda_1^{\rm SP})^2 + 1/6NF^{-2}(\lambda_1^{\rm SP})^4 - 1/48NF^{-2}$
162	$\langle u_\mu \rangle \langle u^\mu \nabla_\nu P \nabla^\nu P \rangle$	$1/24F^{-2} - 4F^{-4}\lambda_1^{\rm PP}c_d^2 + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm PP} - 2/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_2^{\rm PP} - 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
163	$\langle u_\mu u_\nu \rangle \langle \nabla^\mu P \nabla^\nu P \rangle$	$7/12F^{-2} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm PP} + 4/3F^{-2}(\lambda_1^{\rm PP})^2 - 5/6F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
164	$\langle u_\mu \{\nabla^\mu P, \nabla^\nu P\} \rangle \langle u_\nu \rangle$	$-7/12F^{-2} + 8/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 4/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 4/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_2^{\rm PP} + 3/2F^{-2}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm SP})^4$
165	$\langle u_\mu \nabla^\mu P \rangle \langle u_\nu \nabla^\nu P \rangle$	$7/12F^{-2} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 10/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 5/3F^{-2}\lambda_1^{\rm PP} + 4/3F^{-2}(\lambda_1^{\rm PP})^2 - 6F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm PP} + 4F^{-2}(\lambda_2^{\rm PP})^2 - 7/3F^{-2}(\lambda_1^{\rm SP})^2 + 7/3F^{-2}(\lambda_1^{\rm SP})^4$

166	$\langle u_\mu \nabla_\nu P \rangle^2$	$-1/24F^{-2} - 4F^{-4}\lambda_2^{\rm PP}c_d^2 + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 7/3F^{-2}\lambda_2^{\rm PP} - 4/3F^{-2}(\lambda_2^{\rm PP})^2 - 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
167	$\langle u_\mu \nabla^\nu P \rangle \langle u_\nu \nabla^\mu P \rangle$	$7/12F^{-2} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 5/3F^{-2}\lambda_1^{\rm PP} + 4/3F^{-2}(\lambda_1^{\rm PP})^2 - 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}\lambda_2^{\rm PP} + 4/3F^{-2}(\lambda_2^{\rm PP})^2 - 2/3F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
168	$\langle \nabla_\mu P \rangle \langle \nabla^\mu P u \cdot u \rangle$	$1/24F^{-2} - 4F^{-4}\lambda_2^{\rm PP}c_d^2 + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 7/3F^{-2}\lambda_2^{\rm PP} - 2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4$
169	$\langle \nabla^\mu P \{u_\mu, u_\nu\} \rangle \langle \nabla^\nu P \rangle$	$-7/12F^{-2} + 8/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 4/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 4/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4/3F^{-2}\lambda_2^{\rm PP} + 3/2F^{-2}(\lambda_1^{\rm SP})^2 - 2/3F^{-2}(\lambda_1^{\rm SP})^4$
170	$\langle u \cdot u \rangle \langle \nabla_\mu P \nabla^\mu P \rangle$	$-1/48F^{-2} - 2F^{-4}\lambda_1^{\rm PP}c_d^2 + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 7/6F^{-2}\lambda_1^{\rm PP} - 1/3F^{-2}(\lambda_1^{\rm PP})^2 - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}(\lambda_1^{\rm SP})^4$
171	$i \langle u_\mu u_\nu P P f_+^{\mu\nu} \rangle + \text{h.c.}$	$-1/6NF^{-4}\lambda_1^{\rm PP}c_d^2 + 1/6NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 1/24NF^{-4}c_d^2 - 1/6NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/12NF^{-2}(\lambda_1^{\rm SP})^2 + 1/12NF^{-2}(\lambda_1^{\rm SP})^4 + 1/24NF^{-2}$
172	$i \langle u_\mu P u_\nu P f_+^{\mu\nu} \rangle + \text{h.c.}$	$-1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 2/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 1/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/6NF^{-2}\lambda_2^{\rm PP} + 1/12NF^{-2}(\lambda_1^{\rm SP})^2$
173	$i \langle P u_\mu u_\nu P f_+^{\mu\nu} \rangle$	$1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/12NF^{-4}c_d^2 - 1/12NF^{-2}\lambda_1^{\rm PP} + 1/6NF^{-2}(\lambda_1^{\rm PP})^2 - 1/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 2/3NF^{-2}(\lambda_2^{\rm PP})^2 + 1/6NF^{-2}(\lambda_1^{\rm SP})^2 - 7/96NF^{-2}$
174	$i \langle u_\mu f_+^{\mu\nu} u_\nu P P \rangle$	$1/3NF^{-4}\lambda_1^{\rm PP}c_d^2 - 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/12NF^{-2}\lambda_1^{\rm PP} - 1/6NF^{-2}(\lambda_1^{\rm PP})^2 + 2/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3NF^{-2}(\lambda_2^{\rm PP})^2 - 1/6NF^{-2}(\lambda_1^{\rm SP})^4 - 1/96NF^{-2}$
175	$i \langle u_\mu \rangle \langle f_+^{\mu\nu} [PP, u_\nu] \rangle$	$-1/3F^{-4}\lambda_1^{\rm PP}c_d^2 + 1/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}(\lambda_1^{\rm PP})^2 - 1/6F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/12F^{-2}\lambda_2^{\rm PP} - 1/24F^{-2}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}(\lambda_1^{\rm SP})^4 - 1/96F^{-2}$
176	$i \langle u_\mu P \rangle \langle f_+^{\mu\nu} [P, u_\nu] \rangle$	$-2/3F^{-4}\lambda_2^{\rm PP}c_d^2 + 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}\lambda_1^{\rm PP} + 1/3F^{-2}(\lambda_1^{\rm PP})^2 - 1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/12F^{-2}(\lambda_1^{\rm SP})^2 + 1/3F^{-2}(\lambda_1^{\rm SP})^4 + 1/48F^{-2}$
177	$i \langle P \rangle \langle u_\mu u_\nu P f_+^{\mu\nu} \rangle + \text{h.c.}$	$-1/3F^{-4}\lambda_2^{\rm PP}c_d^2 + 1/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}(\lambda_1^{\rm PP})^2 - 1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/12F^{-2}\lambda_2^{\rm PP} + 1/6F^{-2}(\lambda_1^{\rm SP})^4 - 1/96F^{-2}$

178	$i\langle PP\rangle\langle f_+^{\mu\nu}u_\mu u_\nu\rangle$	$-2/3F^{-4}\lambda_1^{\rm PP}c_d^2+2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2-2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+1/6F^{-2}\lambda_1^{\rm PP}+1/3F^{-2}(\lambda_1^{\rm PP})^2-1/6F^{-2}(\lambda_1^{\rm SP})^2+1/3F^{-2}(\lambda_1^{\rm SP})^4+1/48F^{-2}$
179	$i\langle u_\mu f_+^{\mu\nu}u_\nu P\rangle\langle P\rangle$	$2/3F^{-4}\lambda_2^{\rm PP}c_d^2-2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2-2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}+2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2-1/3F^{-2}(\lambda_1^{\rm PP})^2+2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-1/6F^{-2}\lambda_2^{\rm PP}-1/3F^{-2}(\lambda_1^{\rm SP})^4+1/48F^{-2}$
180*	$i\langle f_+^{\mu\nu}\nabla_\mu P\nabla_\nu P\rangle$	$1/3NF^{-2}(\lambda_1^{\rm SP})^2-1/6NF^{-2}$
181	$\langle u_\mu f_-^{\mu\nu}\nabla_\nu PP\rangle+\text{h.c.}$	$1/3NF^{-4}\lambda_1^{\rm PP}c_d^2-1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2+1/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-1/6NF^{-2}(\lambda_1^{\rm SP})^4$
182	$\langle u_\mu\nabla_\nu PPf_-^{\mu\nu}\rangle+\text{h.c.}$	$1/3NF^{-4}\lambda_1^{\rm PP}c_d^2-2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2+1/6NF^{-2}\lambda_1^{\rm PP}-1/3NF^{-2}(\lambda_1^{\rm PP})^2+NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-4/3NF^{-2}(\lambda_2^{\rm PP})^2-1/6NF^{-2}(\lambda_1^{\rm SP})^4-1/48NF^{-2}$
183	$\langle u_\mu Pf_-^{\mu\nu}\nabla_\nu P\rangle+\text{h.c.}$	$1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2-4/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}+2/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+1/3NF^{-2}\lambda_2^{\rm PP}-1/6NF^{-2}(\lambda_1^{\rm SP})^2$
184	$\langle u_\mu\rangle\langle f_-^{\mu\nu}\{P,\nabla_\nu P\}\rangle$	$1/48F^{-2}+2/3F^{-4}\lambda_1^{\rm PP}c_d^2-2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2-2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}+2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2-1/3F^{-2}(\lambda_1^{\rm PP})^2+1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-1/6F^{-2}\lambda_2^{\rm PP}+1/12F^{-2}(\lambda_1^{\rm SP})^2-1/3F^{-2}(\lambda_1^{\rm SP})^4$
185	$\langle u_\mu P\rangle\langle f_-^{\mu\nu}\nabla_\nu P\rangle$	$-1/24F^{-2}+4/3F^{-4}\lambda_2^{\rm PP}c_d^2-4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2+4/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2-1/3F^{-2}\lambda_1^{\rm PP}-2/3F^{-2}(\lambda_1^{\rm PP})^2+2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2+1/6F^{-2}(\lambda_1^{\rm SP})^2-2/3F^{-2}(\lambda_1^{\rm SP})^4$
186	$\langle\nabla_\mu P\rangle\langle f_-^{\mu\nu}\{P,u_\nu\}\rangle$	$-1/48F^{-2}-2/3F^{-4}\lambda_2^{\rm PP}c_d^2+2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2+2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}-2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+1/3F^{-2}(\lambda_1^{\rm PP})^2-2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2+1/6F^{-2}\lambda_2^{\rm PP}+1/3F^{-2}(\lambda_1^{\rm SP})^4$
187	$\langle P\rangle\langle f_-^{\mu\nu}\{u_\mu,\nabla_\nu P\}\rangle$	$1/48F^{-2}+2/3F^{-4}\lambda_2^{\rm PP}c_d^2-2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2-2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}+2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2-1/3F^{-2}(\lambda_1^{\rm PP})^2+2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-1/6F^{-2}\lambda_2^{\rm PP}-1/3F^{-2}(\lambda_1^{\rm SP})^4$
188	$\langle Pf_-^{\mu\nu}\rangle\langle u_\nu\nabla_\mu P\rangle$	$1/24F^{-2}-4/3F^{-4}\lambda_2^{\rm PP}c_d^2+4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2+8/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}-4/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2-1/3F^{-2}\lambda_1^{\rm PP}+2/3F^{-2}(\lambda_1^{\rm PP})^2-2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2-2/3F^{-2}\lambda_2^{\rm PP}+8/3F^{-2}(\lambda_2^{\rm PP})^2+1/6F^{-2}(\lambda_1^{\rm SP})^2+2/3F^{-2}(\lambda_1^{\rm SP})^4$
189	$\langle P\nabla_\mu P\rangle\langle u_\nu f_-^{\mu\nu}\rangle$	$1/24F^{-2}-4/3F^{-4}\lambda_1^{\rm PP}c_d^2+4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2-4/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+1/3F^{-2}\lambda_1^{\rm PP}+2/3F^{-2}(\lambda_1^{\rm PP})^2-1/3F^{-2}(\lambda_1^{\rm SP})^2+2/3F^{-2}(\lambda_1^{\rm SP})^4$
190*	$\langle Pf_+^{\mu\nu}Pf_{+\mu\nu}\rangle$	$1/12NF^{-2}(\lambda_1^{\rm SP})^2-1/24NF^{-2}$
191*	$\langle PPf_+^{\mu\nu}f_{+\mu\nu}\rangle$	$-1/12NF^{-2}(\lambda_1^{\rm SP})^2+1/24NF^{-2}$
192	$\langle Pf_-^{\mu\nu}Pf_{-\mu\nu}\rangle$	$1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2-2/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}+1/3NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2+1/6NF^{-2}\lambda_2^{\rm PP}-1/12NF^{-2}(\lambda_1^{\rm SP})^2$

193	$\langle PPf_-^{\mu\nu}f_{-\mu\nu}\rangle$	$1/3NF^{-4}\lambda_1^{\rm PP}c_d^2 - 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/12NF^{-2}\lambda_1^{\rm PP} - 1/6NF^{-2}(\lambda_1^{\rm PP})^2 + 2/3NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 2/3NF^{-2}(\lambda_2^{\rm PP})^2 - 1/6NF^{-2}(\lambda_1^{\rm SP})^4 - 1/96NF^{-2}$
194	$\langle Pf_-^{\mu\nu}\rangle^2$	$-1/48F^{-2} + 2/3F^{-4}\lambda_2^{\rm PP}c_d^2 - 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/6F^{-2}\lambda_2^{\rm PP} - 2/3F^{-2}(\lambda_2^{\rm PP})^2 - 1/3F^{-2}(\lambda_1^{\rm SP})^4$
195	$\langle P\rangle\langle Pf_-^{\mu\nu}f_{-\mu\nu}\rangle$	$1/48F^{-2} + 2/3F^{-4}\lambda_2^{\rm PP}c_d^2 - 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/3F^{-2}(\lambda_1^{\rm PP})^2 + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}\lambda_2^{\rm PP} - 1/3F^{-2}(\lambda_1^{\rm SP})^4$
196	$\langle PP\rangle\langle f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$-1/96F^{-2} + 1/3F^{-4}\lambda_1^{\rm PP}c_d^2 - 1/3F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/12F^{-2}\lambda_1^{\rm PP} - 1/6F^{-2}(\lambda_1^{\rm PP})^2 + 1/12F^{-2}(\lambda_1^{\rm SP})^2 - 1/6F^{-2}(\lambda_1^{\rm SP})^4$
197	$\langle P\chi_+Pu\cdot u\rangle$	$10NF^{-6}d_mc_d^3\lambda_1^{\rm SP} + 20NF^{-6}c_d^3c_m(\lambda_1^{\rm SP})^2 - 4NF^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 12NF^{-4}d_m\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 4NF^{-4}d_mc_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/2NF^{-4}d_mc_d\lambda_1^{\rm SP} + 20NF^{-4}d_mc_d(\lambda_1^{\rm SP})^3 - 2NF^{-4}\lambda_1^{\rm PP}\lambda_3^{\rm PP}c_d^2 - 16NF^{-4}\lambda_2^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 12NF^{-4}\lambda_3^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 8NF^{-4}c_dc_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 3NF^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 28NF^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 10NF^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 6NF^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 4NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} - 4NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - NF^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} - NF^{-2}(\lambda_1^{\rm PP})^2 - 2NF^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + NF^{-2}(\lambda_2^{\rm PP})^2 - 4NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + NF^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^4 - 1/16NF^{-2}\lambda_3^{\rm PP} + 4NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + NF^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - NF^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/4NF^{-2}(\lambda_1^{\rm SP})^4$

198	$\langle PP\{\chi_+, u\cdot u\}\rangle$	$5NF^{-6}d_m c_d^3\lambda_1^{\mathrm{SP}} + 10NF^{-6}c_d^3 c_m(\lambda_1^{\mathrm{SP}})^2 - 6NF^{-4}d_m\lambda_2^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 2NF^{-4}d_m c_d\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - NF^{-4}d_m c_d\lambda_1^{\mathrm{SP}} + 9NF^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 - NF^{-4}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}c_d^2 - 1/2NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2 - 8NF^{-4}\lambda_2^{\mathrm{PP}}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 6NF^{-4}\lambda_3^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/4NF^{-4}\lambda_3^{\mathrm{PP}}c_d^2 - 4NF^{-4}c_d c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 3/2NF^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 14NF^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^4 - 6NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 3NF^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/8NF^{-2}\lambda_1^{\mathrm{PP}} - 1/2NF^{-2}(\lambda_1^{\mathrm{PP}})^2 - NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2NF^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/8NF^{-2}\lambda_3^{\mathrm{PP}} + 4NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - NF^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4$
199	$\langle u_\nu PPu^\nu\chi_+\rangle$	$4NF^{-4}d_m\lambda_2^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 2NF^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 + 2NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_3^{\mathrm{SS}} - 2NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 + 1/8NF^{-2}\lambda_1^{\mathrm{PP}} + 1/4NF^{-2}(\lambda_1^{\mathrm{PP}})^2 + 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4NF^{-2}(\lambda_2^{\mathrm{PP}})^2\lambda_3^{\mathrm{PP}} - NF^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 3/64NF^{-2}$
200	$\langle u_\nu Pu^\nu\{P,\chi_+\}\rangle$	$-2NF^{-6}d_m c_d^3\lambda_1^{\mathrm{SP}} - 4NF^{-6}c_d^3 c_m(\lambda_1^{\mathrm{SP}})^2 - NF^{-4}d_m c_d\lambda_1^{\mathrm{SP}} + 2NF^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 - 2NF^{-4}\lambda_1^{\mathrm{PP}}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-4}\lambda_2^{\mathrm{PP}}c_d c_m(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-4}\lambda_3^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 3/2NF^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 2NF^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^4 + 2NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + NF^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 1/2NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 1/4NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/8NF^{-2}\lambda_2^{\mathrm{PP}} - NF^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 3/4NF^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 3/8NF^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 3/16NF^{-2}(\lambda_1^{\mathrm{SP}})^2$

201	$\langle u_\mu \rangle \langle u^\mu \{PP, \chi_+\} \rangle$	$3/64F^{-2} + 2F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 4F^{-4}d_mc_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 3/2F^{-4}d_mc_d\lambda_1^{\rm SP} - 4F^{-4}d_mc_d(\lambda_1^{\rm SP})^3 - 2F^{-4}\lambda_1^{\rm PP}\lambda_3^{\rm PP}c_d^2 - 2F^{-4}\lambda_1^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_3^{\rm SS} - F^{-4}\lambda_1^{\rm PP}c_d^2 + 2F^{-4}\lambda_3^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-4}\lambda_3^{\rm PP}c_d^2 - 8F^{-4}c_dc_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 5/2F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 - 2F^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 4F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/4F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}\lambda_1^{\rm PP} + 1/4F^{-2}(\lambda_1^{\rm PP})^2 + F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_3^{\rm PP} + 8F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^4 + 3/8F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3/16F^{-2}(\lambda_1^{\rm SP})^2$
202	$\langle u_\mu \rangle \langle u^\mu P\chi_+P \rangle$	$-4F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 8F^{-4}d_mc_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}d_mc_d\lambda_1^{\rm SP} - 4F^{-4}\lambda_1^{\rm PP}\lambda_3^{\rm PP}c_d^2 - 4F^{-4}\lambda_1^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 4F^{-4}\lambda_3^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 16F^{-4}c_dc_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 5F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 - 4F^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} - 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm PP} - 8F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^4 + 1/8F^{-2}\lambda_3^{\rm PP} + 8F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/8F^{-2}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^4$
203	$\langle u_\mu P \rangle \langle u^\mu \{P, \chi_+\} \rangle$	$-3/32F^{-2} + 4F^{-6}d_mc_d^3\lambda_1^{\rm SP} + 8F^{-6}c_d^3c_m(\lambda_1^{\rm SP})^2 - 2F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 8F^{-4}d_mc_d\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-4}d_mc_d\lambda_1^{\rm SP} - 2F^{-4}d_mc_d(\lambda_1^{\rm SP})^3 - 4F^{-4}\lambda_1^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 - 4F^{-4}\lambda_2^{\rm PP}\lambda_3^{\rm PP}c_d^2 - 8F^{-4}\lambda_2^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_3^{\rm SS} - 2F^{-4}\lambda_2^{\rm PP}c_d^2 + 4F^{-4}\lambda_3^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 - 16F^{-4}c_dc_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 4F^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 8F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} + 1/2F^{-2}(\lambda_1^{\rm PP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 3F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + 4F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/4F^{-2}\lambda_2^{\rm PP} - 4F^{-2}(\lambda_2^{\rm PP})^2\lambda_3^{\rm PP} + F^{-2}(\lambda_2^{\rm PP})^2 - 8F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/8F^{-2}\lambda_3^{\rm PP} + 16F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 8F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^4 - 2F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^4$

204	$\langle\, u_\mu PP\,\rangle\langle\, u^\mu \chi_+\,\rangle$	$3/32F^{-2} + 4F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 4F^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_3^{\mathrm{SS}} - 2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2 + F^{-4}\lambda_3^{\mathrm{PP}}c_d^2 - 8F^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} + F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}\lambda_1^{\mathrm{PP}} - 2F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_3^{\mathrm{PP}} + 1/2F^{-2}(\lambda_1^{\mathrm{PP}})^2 + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} + 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}\lambda_2^{\mathrm{PP}} - 2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 - 3/8F^{-2}\lambda_3^{\mathrm{PP}} + 16F^{-2}\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - 2F^{-2}(\lambda_1^{\mathrm{SP}})^3\lambda_2^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^4$
205	$\langle\, P\,\rangle\langle\, P\{\chi_+, u\cdot u\}\,\rangle$	$-2F^{-6}d_m c_d^3\lambda_1^{\mathrm{SP}} - 12F^{-6}c_d^3c_m(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} + 2F^{-4}d_m\lambda_2^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 4F^{-4}d_m c_d\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/2F^{-4}d_m c_d\lambda_1^{\mathrm{SP}} + 6F^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 - 10F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m(\lambda_1^{\mathrm{SP}})^2 - 2F^{-4}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}c_d^2 - F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 2F^{-4}\lambda_3^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 5/2F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 10F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^4 - 12F^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 7F^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} - 3F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 3/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 3/4F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_3^{\mathrm{PP}} - F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} + 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 1/2F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 3/8F^{-2}\lambda_2^{\mathrm{PP}} - 4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^4 + 1/16F^{-2}\lambda_3^{\mathrm{PP}} + 8F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/4F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - 3/8F^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 3/16F^{-2}(\lambda_1^{\mathrm{SP}})^2 - F^{-2}(\lambda_1^{\mathrm{SP}})^3\lambda_2^{\mathrm{SP}} + 1/4F^{-2}(\lambda_1^{\mathrm{SP}})^4$
206	$\langle\, P\chi_+\,\rangle\langle\, Pu\cdot u\,\rangle$	$3/32F^{-2} + 12F^{-6}d_m c_d^3\lambda_1^{\mathrm{SP}} + 8F^{-6}c_d^3c_m(\lambda_1^{\mathrm{SP}})^2 - 6F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} + 4F^{-4}d_m\lambda_2^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 8F^{-4}d_m c_d\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/2F^{-4}d_m c_d\lambda_1^{\mathrm{SP}} + 18F^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^3 - 4F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m(\lambda_1^{\mathrm{SP}})^2 - 4F^{-4}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}c_d^2 + 4F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_3^{\mathrm{SS}} - 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2 + 20F^{-4}\lambda_3^{\mathrm{PP}}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 16F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^2 + 36F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^4 - 24F^{-4}c_d^2\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 12F^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 6F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{PP}} + 10F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}(\lambda_1^{\mathrm{PP}})^2\lambda_3^{\mathrm{PP}} + 1/2F^{-2}(\lambda_1^{\mathrm{PP}})^2 + 8F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 3F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + F^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 3/4F^{-2}\lambda_2^{\mathrm{PP}} - 2F^{-2}(\lambda_2^{\mathrm{PP}})^2 - 8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 3/2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/8F^{-2}\lambda_3^{\mathrm{PP}} + 16F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^4 - 2F^{-2}(\lambda_1^{\mathrm{SP}})^3\lambda_2^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^4$

207	$\langle PP\rangle\langle\chi_+u\cdot u\rangle$	$-3/64F^{-2} + 2F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 2F^{-4}d_mc_d(\lambda_1^{\rm SP})^3 + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_3^{\rm SS} - F^{-4}\lambda_1^{\rm PP}c_d^2 + 1/2F^{-4}\lambda_3^{\rm PP}c_d^2 - 4F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/8F^{-2}\lambda_1^{\rm PP} - F^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} + 1/4F^{-2}(\lambda_1^{\rm PP})^2 + 1/2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}\lambda_3^{\rm PP} + 8F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^4 - 3/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/8F^{-2}(\lambda_1^{\rm SP})^2 - F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/4F^{-2}(\lambda_1^{\rm SP})^4$
208	$\langle PPu\cdot u\rangle\langle\chi_+\rangle$	$-3/64F^{-2} - 8F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 12F^{-4}d_m\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 3/2F^{-4}d_mc_d\lambda_1^{\rm SP} + 20F^{-4}d_mc_d(\lambda_1^{\rm SP})^3 - 8F^{-4}\lambda_1^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_3^{\rm SS} - F^{-4}\lambda_1^{\rm PP}c_d^2 - 16F^{-4}\lambda_2^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 1/2F^{-4}\lambda_3^{\rm PP}c_d^2 - 2F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 24F^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 24F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 12F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} - 8F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/8F^{-2}\lambda_1^{\rm PP} - F^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} - 7/4F^{-2}(\lambda_1^{\rm PP})^2 + 4F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 4F^{-2}(\lambda_2^{\rm PP})^2\lambda_3^{\rm PP} + F^{-2}(\lambda_2^{\rm PP})^2 - F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}\lambda_3^{\rm PP} + 8F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^4 - F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/4F^{-2}(\lambda_1^{\rm SP})^4$
209	$\langle PP\chi_+\rangle\langle u\cdot u\rangle$	$-3/64F^{-2} - 4F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - 4F^{-4}d_m\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 8F^{-4}d_mc_d\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/2F^{-4}d_mc_d\lambda_1^{\rm SP} + 8F^{-4}d_mc_d(\lambda_1^{\rm SP})^3 - 4F^{-4}\lambda_1^{\rm PP}\lambda_3^{\rm PP}c_d^2 - 12F^{-4}\lambda_1^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_3^{\rm SS} - F^{-4}\lambda_1^{\rm PP}c_d^2 - 8F^{-4}\lambda_2^{\rm PP}c_dc_m(\lambda_1^{\rm SP})^2 + 4F^{-4}\lambda_3^{\rm PP}c_d^2(\lambda_1^{\rm SP})^2 + 1/2F^{-4}\lambda_3^{\rm PP}c_d^2 - 16F^{-4}c_dc_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 3F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 20F^{-4}c_dc_m(\lambda_1^{\rm SP})^4 - 4F^{-4}c_d^2\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 5/2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/8F^{-2}\lambda_1^{\rm PP} - F^{-2}(\lambda_1^{\rm PP})^2\lambda_3^{\rm PP} + 1/4F^{-2}(\lambda_1^{\rm PP})^2 - 8F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}\lambda_3^{\rm PP} + 8F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^4 - 3/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/8F^{-2}(\lambda_1^{\rm SP})^2 - F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/4F^{-2}(\lambda_1^{\rm SP})^4$

210	$\langle u_\nu P u^\nu P \rangle \langle \chi_+ \rangle$	$2F^{-4}d_m\lambda_1^{PP}c_d\lambda_1^{SP} - 4F^{-4}d_m\lambda_2^{PP}c_d\lambda_1^{SP} - 1/2F^{-4}d_m c_d\lambda_1^{SP} + 2F^{-4}d_m c_d(\lambda_1^{SP})^3 + 4F^{-4}c_d^2\lambda_3^{SS}(\lambda_1^{SP})^2 + 2F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - F^{-4}c_d^2(\lambda_1^{SP})^2 - 4F^{-2}\lambda_1^{PP}\lambda_2^{PP}\lambda_3^{PP} - F^{-2}\lambda_1^{PP}\lambda_2^{PP} + 2F^{-2}\lambda_1^{PP}\lambda_3^{PP}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_1^{PP}\lambda_3^{SS}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + F^{-2}\lambda_2^{PP}\lambda_3^{PP} - 8F^{-2}\lambda_2^{PP}\lambda_3^{SS}(\lambda_1^{SP})^2 - 4F^{-2}\lambda_2^{PP}\lambda_1^{SP}\lambda_2^{SP} + 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + 1/4F^{-2}\lambda_2^{PP} - 1/2F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 + 3/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 3/8F^{-2}(\lambda_1^{SP})^2$
211	$\langle u_\nu P u^\nu \chi_+ \rangle \langle P \rangle$	$3/32F^{-2} + 2F^{-4}d_m\lambda_1^{PP}c_d\lambda_1^{SP} + 1/2F^{-4}d_m c_d\lambda_1^{SP} - 2F^{-4}d_m c_d(\lambda_1^{SP})^3 + 4F^{-4}\lambda_2^{PP}c_d^2\lambda_3^{SS} + 2F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 4F^{-2}\lambda_1^{PP}\lambda_2^{PP}\lambda_3^{PP} + 2F^{-2}\lambda_1^{PP}\lambda_3^{PP}(\lambda_1^{SP})^2 + 2F^{-2}\lambda_1^{PP}\lambda_3^{SS}(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + 1/2F^{-2}(\lambda_1^{PP})^2 + 4F^{-2}\lambda_2^{PP}\lambda_3^{PP}(\lambda_1^{SP})^2 - F^{-2}\lambda_2^{PP}\lambda_3^{PP} - 8F^{-2}\lambda_2^{PP}\lambda_3^{SS}(\lambda_1^{SP})^2 - 4F^{-2}\lambda_2^{PP}\lambda_1^{SP}\lambda_2^{SP} + 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_2^{PP})^2 + 1/2F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 - 2F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^4 + 3/2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 3/4F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 3/8F^{-2}(\lambda_1^{SP})^2$
212	$\langle \chi_+ \nabla_\mu P \nabla^\mu P \rangle$	$-NF^{-2}\lambda_1^{PP} - NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/4NF^{-2}(\lambda_1^{SP})^2$
213	$\langle \chi_+ \rangle \langle \nabla_\mu P \nabla^\mu P \rangle$	$-F^{-2}\lambda_1^{PP} - F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/4F^{-2}(\lambda_1^{SP})^2$
214	$\langle \chi_+ \nabla_\mu P \rangle \langle \nabla^\mu P \rangle$	$-2F^{-2}\lambda_2^{PP} - 2F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2 + 1/2F^{-2}(\lambda_1^{SP})^2$
215	$i \langle u_\mu \nabla^\mu P \chi_- P \rangle + \text{h.c.}$	$8NF^{-4}d_m\lambda_2^{PP}c_d\lambda_1^{SP} + 1/4NF^{-4}d_m c_d\lambda_1^{SP} - 4NF^{-4}d_m c_d(\lambda_1^{SP})^3 + 8NF^{-4}\lambda_2^{PP}c_d c_m - 4NF^{-4}\lambda_2^{PP}c_d^2 - 2NF^{-4}c_d c_m(\lambda_1^{SP})^2 - NF^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + 3/2NF^{-4}c_d^2(\lambda_1^{SP})^2 + 2NF^{-2}\lambda_1^{PP}\lambda_2^{PP} + 2NF^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} - NF^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - 2NF^{-2}\lambda_2^{PP}\lambda_3^{PP} + 10NF^{-2}\lambda_2^{PP}\lambda_1^{SP}\lambda_2^{SP} - 4NF^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + NF^{-2}\lambda_2^{PP} + 1/2NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 + 5/2NF^{-2}\lambda_1^{SP}\lambda_2^{SP} - 7/8NF^{-2}(\lambda_1^{SP})^2 - 15/2NF^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} + 9/4NF^{-2}(\lambda_1^{SP})^4$
216	$i \langle u_\mu \nabla^\mu P P \chi_- \rangle + \text{h.c.}$	$4NF^{-4}d_m\lambda_1^{PP}c_d\lambda_1^{SP} + 5/4NF^{-4}d_m c_d\lambda_1^{SP} - 4NF^{-4}d_m c_d(\lambda_1^{SP})^3 + 3NF^{-4}\lambda_1^{PP}c_d c_m - 3/2NF^{-4}\lambda_1^{PP}c_d^2 - 2NF^{-4}c_d c_m(\lambda_1^{SP})^2 + 5/4NF^{-4}c_d c_m - NF^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} + 3/2NF^{-4}c_d^2(\lambda_1^{SP})^2 - 5/8NF^{-4}c_d^2 - NF^{-2}\lambda_1^{PP}\lambda_3^{PP} + 5NF^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} - 2NF^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + 1/2NF^{-2}\lambda_1^{PP} + 1/2NF^{-2}(\lambda_1^{PP})^2 + 6NF^{-2}\lambda_2^{PP}\lambda_1^{SP}\lambda_2^{SP} - 5/2NF^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + 2NF^{-2}(\lambda_2^{PP})^2 + NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 - 3/4NF^{-2}\lambda_3^{PP} + 7/4NF^{-2}\lambda_1^{SP}\lambda_2^{SP} - 3/4NF^{-2}(\lambda_1^{SP})^2 - 13/2NF^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} + 2NF^{-2}(\lambda_1^{SP})^4 + 3/32NF^{-2}$

217	$i\langle u_\mu \chi_- \nabla^\mu P P\rangle + \text{h.c.}$	$4NF^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} - NF^{-4}d_mc_d\lambda_1^{\rm SP} + 3NF^{-4}\lambda_1^{\rm PP}c_dc_m - 3/2NF^{-4}\lambda_1^{\rm PP}c_d^2 - 5/4NF^{-4}c_dc_m + 5/8NF^{-4}c_d^2 + 4NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/2NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/4NF^{-2}(\lambda_1^{\rm SP})^2 + NF^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - 1/4NF^{-2}(\lambda_1^{\rm SP})^4$
218	$i\langle u_\mu\rangle\langle \nabla^\mu P\{P,\chi_-\}\rangle$	$-1/2F^{-4}d_mc_d\lambda_1^{\rm SP} - 2F^{-4}\lambda_1^{\rm PP}c_dc_m + F^{-4}\lambda_1^{\rm PP}c_d^2 + 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_1^{\rm PP} + 1/2F^{-2}(\lambda_1^{\rm PP})^2 + 1/2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_2^{\rm PP} - 1/2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/4F^{-2}\lambda_3^{\rm PP} - 13/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 5/4F^{-2}(\lambda_1^{\rm SP})^2 + 3F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - F^{-2}(\lambda_1^{\rm SP})^4 - 3/32F^{-2}$
219	$i\langle u_\mu\{P,\chi_-\}\rangle\langle \nabla^\mu P\rangle$	$-1/2F^{-4}d_mc_d\lambda_1^{\rm SP} - 2F^{-4}\lambda_2^{\rm PP}c_dc_m + F^{-4}\lambda_2^{\rm PP}c_d^2 + 2F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_1^{\rm PP} + 1/2F^{-2}(\lambda_1^{\rm PP})^2 - 2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_2^{\rm PP} - 1/2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/4F^{-2}\lambda_3^{\rm PP} - 11/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 9/8F^{-2}(\lambda_1^{\rm SP})^2 + 3F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - F^{-2}(\lambda_1^{\rm SP})^4 - 3/32F^{-2}$
220	$i\langle \chi_-\nabla^\mu P\rangle\langle u_\mu P\rangle$	$-F^{-4}d_mc_d\lambda_1^{\rm SP} - 4F^{-4}\lambda_2^{\rm PP}c_dc_m + 2F^{-4}\lambda_2^{\rm PP}c_d^2 - 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP} + F^{-2}(\lambda_1^{\rm PP})^2 - 4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/2F^{-2}\lambda_3^{\rm PP} - 5/2F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^2 + 4F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - F^{-2}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}$
221	$i\langle P\rangle\langle \chi_-\{u_\mu,\nabla^\mu P\}\rangle$	$1/2F^{-4}d_mc_d\lambda_1^{\rm SP} - 2F^{-4}\lambda_2^{\rm PP}c_dc_m + F^{-4}\lambda_2^{\rm PP}c_d^2 + 4F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}\lambda_1^{\rm PP} + 1/2F^{-2}(\lambda_1^{\rm PP})^2 - 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + 4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm PP} + F^{-2}3/2\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_3^{\rm PP} + 5/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/8F^{-2}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^4 - 3/32F^{-2}$

222	$i\langle P\nabla^\mu P\rangle\langle u_\mu\chi_-\rangle$	$-F^{-4}d_m c_d\lambda_1^{\rm SP} - 4F^{-4}\lambda_1^{\rm PP}c_d c_m + 2F^{-4}\lambda_1^{\rm PP}c_d^2 - 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP} + F^{-2}(\lambda_1^{\rm PP})^2 - F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 3/2F^{-2}\lambda_3^{\rm PP} - 7/2F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3/4F^{-2}(\lambda_1^{\rm SP})^2 + 4F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - F^{-2}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}$
223	$i\langle P\{u_\mu, \nabla^\mu P\}\rangle\langle\chi_-\rangle$	$8F^{-4}d_m\lambda_1^{\rm PP}c_d\lambda_1^{\rm SP} + 8F^{-4}d_m\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} + 1/2F^{-4}d_m c_d\lambda_1^{\rm SP} - 8F^{-4}d_m c_d(\lambda_1^{\rm SP})^3 + 6F^{-4}\lambda_1^{\rm PP}c_d c_m - 3F^{-4}\lambda_1^{\rm PP}c_d^2 + 8F^{-4}\lambda_2^{\rm PP}c_d c_m - 4F^{-4}\lambda_2^{\rm PP}c_d^2 - 4F^{-4}c_d c_m(\lambda_1^{\rm SP})^2 + 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 9F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}\lambda_1^{\rm PP} + 1/2F^{-2}(\lambda_1^{\rm PP})^2 - 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + 10F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 5/2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm PP} + 3/2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4F^{-2}\lambda_3^{\rm PP} + 7/4F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 1/4F^{-2}(\lambda_1^{\rm SP})^2 - 10F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 5/2F^{-2}(\lambda_1^{\rm SP})^4 - 3/32F^{-2}$
224	$i\langle P\chi_-\rangle\langle u_\mu\nabla^\mu P\rangle$	$F^{-4}d_m c_d\lambda_1^{\rm SP} - 4F^{-4}\lambda_2^{\rm PP}c_d c_m + 2F^{-4}\lambda_2^{\rm PP}c_d^2 + 8F^{-4}c_d c_m(\lambda_1^{\rm SP})^2 - 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 + 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} + 6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 4F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + F^{-2}\lambda_1^{\rm PP} + F^{-2}(\lambda_1^{\rm PP})^2 - 4F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} + 16F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 9F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 2F^{-2}\lambda_2^{\rm PP} + 4F^{-2}(\lambda_2^{\rm PP})^2 + 3F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/2F^{-2}\lambda_3^{\rm PP} + 11/2F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 5/2F^{-2}(\lambda_1^{\rm SP})^2 - 10F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} + 4F^{-2}(\lambda_1^{\rm SP})^4 + 3/16F^{-2}$
225	$\langle P\chi_+ P\chi_+\rangle$	$8NF^{-6}d_m c_d^2 c_m\lambda_1^{\rm SP} + 2NF^{-6}d_m^2 c_d^2 + 8NF^{-6}c_d^2 c_m^2(\lambda_1^{\rm SP})^2 - 6NF^{-4}d_m\lambda_1^{\rm PP}c_m\lambda_1^{\rm SP} - 12NF^{-4}d_m\lambda_2^{\rm PP}c_m\lambda_1^{\rm SP} + 4NF^{-4}d_m\lambda_3^{\rm PP}c_d\lambda_1^{\rm SP} - 4NF^{-4}d_m c_d\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-4}d_m c_d\lambda_2^{\rm SP} - 3/2NF^{-4}d_m c_m\lambda_1^{\rm SP} + 22NF^{-4}d_m c_m(\lambda_1^{\rm SP})^3 - 2NF^{-4}d_m^2\lambda_1^{\rm PP} - 4NF^{-4}d_m^2\lambda_2^{\rm PP} + 7NF^{-4}d_m^2(\lambda_1^{\rm SP})^2 - 1/2NF^{-4}d_m^2 - 4NF^{-4}\lambda_1^{\rm PP}c_m^2(\lambda_1^{\rm SP})^2 - 8NF^{-4}\lambda_2^{\rm PP}c_m^2(\lambda_1^{\rm SP})^2 + 8NF^{-4}\lambda_3^{\rm PP}c_d c_m(\lambda_1^{\rm SP})^2 - 8NF^{-4}c_d c_m\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 4NF^{-4}c_d c_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-4}c_m^2(\lambda_1^{\rm SP})^2 + 16NF^{-4}c_m^2(\lambda_1^{\rm SP})^4 - NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} - 4NF^{-2}\lambda_3^{\rm PP}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 - 2NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + NF^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 + 2NF^{-2}(\lambda_3^{\rm PP})^2(\lambda_1^{\rm SP})^2 + 4NF^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - NF^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2$

226	$\langle PP\chi_+\chi_+\rangle$	$8NF^{-6}d_mc_d^2c_m\lambda_1^{\mathrm{SP}} + 2NF^{-6}d_m^2c_d^2 + 8NF^{-6}c_d^2c_m^2(\lambda_1^{\mathrm{SP}})^2 - 6NF^{-4}d_m\lambda_1^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} - 12NF^{-4}d_m\lambda_2^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} + 4NF^{-4}d_m\lambda_3^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 4NF^{-4}d_mc_d\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2NF^{-4}d_mc_d\lambda_2^{\mathrm{SP}} - 3/2NF^{-4}d_mc_m\lambda_1^{\mathrm{SP}} + 22NF^{-4}d_mc_m(\lambda_1^{\mathrm{SP}})^3 - 4NF^{-4}d_m^2\lambda_1^{\mathrm{PP}} - 4NF^{-4}d_m^2\lambda_2^{\mathrm{PP}} + 9NF^{-4}d_m^2(\lambda_1^{\mathrm{SP}})^2 - 1/2NF^{-4}d_m^2 - 4NF^{-4}\lambda_1^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 - 8NF^{-4}\lambda_2^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 8NF^{-4}\lambda_3^{\mathrm{PP}}c_dc_m(\lambda_1^{\mathrm{SP}})^2 - 8NF^{-4}c_dc_m\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4NF^{-4}c_dc_m\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - NF^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 16NF^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^4 - NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/4NF^{-2}\lambda_1^{\mathrm{PP}} - 4NF^{-2}\lambda_3^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + NF^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/4NF^{-2}\lambda_3^{\mathrm{PP}} + 2NF^{-2}(\lambda_3^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-2}(\lambda_3^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - NF^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/4NF^{-2}(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}(\lambda_2^{\mathrm{SP}})^2$
227	$\langle P\rangle\langle P\chi_+\chi_+\rangle$	$-8F^{-6}d_mc_d^2c_m\lambda_1^{\mathrm{SP}} - 16F^{-6}c_d^2c_m^2(\lambda_1^{\mathrm{SP}})^2 - 12F^{-4}d_m\lambda_1^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} - 8F^{-4}d_mc_d\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-4}d_mc_d\lambda_1^{\mathrm{SP}} - 4F^{-4}d_mc_d\lambda_2^{\mathrm{SP}} + 3F^{-4}d_mc_m\lambda_1^{\mathrm{SP}} + 12F^{-4}d_mc_m(\lambda_1^{\mathrm{SP}})^3 - 4F^{-4}d_m^2\lambda_1^{\mathrm{PP}} - 4F^{-4}d_m^2\lambda_2^{\mathrm{PP}} + 8F^{-4}d_m^2(\lambda_1^{\mathrm{SP}})^2 + F^{-4}d_m^2 - 8F^{-4}\lambda_1^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 - 16F^{-4}c_dc_m\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_dc_m\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 4F^{-4}c_dc_m(\lambda_1^{\mathrm{SP}})^2 + 2F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 8F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^4 - 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2F^{-2}\lambda_2^{\mathrm{PP}} - 8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}(\lambda_3^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}(\lambda_2^{\mathrm{SP}})^2$
228	$\langle P\chi_+\rangle^2$	$8F^{-6}d_mc_d^2c_m\lambda_1^{\mathrm{SP}} + 4F^{-6}d_m^2c_d^2 - 12F^{-4}d_m\lambda_1^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} + 8F^{-4}d_m\lambda_3^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 8F^{-4}d_mc_d\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-4}d_mc_d\lambda_1^{\mathrm{SP}} - 4F^{-4}d_mc_d\lambda_2^{\mathrm{SP}} + 3F^{-4}d_mc_m\lambda_1^{\mathrm{SP}} + 20F^{-4}d_mc_m(\lambda_1^{\mathrm{SP}})^3 - 4F^{-4}d_m^2\lambda_1^{\mathrm{PP}} - 4F^{-4}d_m^2\lambda_2^{\mathrm{PP}} + 10F^{-4}d_m^2(\lambda_1^{\mathrm{SP}})^2 + F^{-4}d_m^2 - 8F^{-4}\lambda_1^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 16F^{-4}\lambda_3^{\mathrm{PP}}c_dc_m(\lambda_1^{\mathrm{SP}})^2 - 16F^{-4}c_dc_m\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_dc_m\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 4F^{-4}c_dc_m(\lambda_1^{\mathrm{SP}})^2 + 2F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 16F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^4 - 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2F^{-2}\lambda_2^{\mathrm{PP}} - 8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}(\lambda_3^{\mathrm{PP}})^2(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}(\lambda_3^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}(\lambda_2^{\mathrm{SP}})^2$

229	$\langle PP\rangle\langle\chi_+\chi_+\rangle$	$-2F^{-4}d_m^2\lambda_1^{\mathrm{PP}} + 2F^{-4}d_m^2(\lambda_1^{\mathrm{SP}})^2 - 1/4F^{-2}\lambda_1^{\mathrm{PP}} + 1/4F^{-2}\lambda_3^{\mathrm{PP}} + 4F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 4F^{-2}(\lambda_3^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - F^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/4F^{-2}(\lambda_1^{\mathrm{SP}})^2 + F^{-2}(\lambda_2^{\mathrm{SP}})^2$
230	$\langle PP\chi_+\rangle\langle\chi_+\rangle$	$-12F^{-4}d_m\lambda_1^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} - 24F^{-4}d_m\lambda_2^{\mathrm{PP}}c_m\lambda_1^{\mathrm{SP}} - 8F^{-4}d_mc_d\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4F^{-4}d_mc_d\lambda_2^{\mathrm{SP}} - 3F^{-4}d_mc_m\lambda_1^{\mathrm{SP}} + 36F^{-4}d_mc_m(\lambda_1^{\mathrm{SP}})^3 - 8F^{-4}d_m^2\lambda_1^{\mathrm{PP}} - 8F^{-4}d_m^2\lambda_2^{\mathrm{PP}} + 16F^{-4}d_m^2(\lambda_1^{\mathrm{SP}})^2 - F^{-4}d_m^2 - 8F^{-4}\lambda_1^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 - 16F^{-4}\lambda_2^{\mathrm{PP}}c_m^2(\lambda_1^{\mathrm{SP}})^2 - 16F^{-4}c_dc_m\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 8F^{-4}c_dc_m\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^2 + 24F^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^4 - 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 1/2F^{-2}\lambda_1^{\mathrm{PP}} - 8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 - 4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-2}\lambda_3^{\mathrm{PP}} + 8F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 8F^{-2}(\lambda_3^{\mathrm{SS}})^2(\lambda_1^{\mathrm{SP}})^2 - 2F^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^2 + 2F^{-2}(\lambda_2^{\mathrm{SP}})^2$
231	$\langle PP\chi_-\chi_-\rangle$	$-4NF^{-4}d_m\lambda_1^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} + 4NF^{-4}d_m\lambda_3^{\mathrm{PP}}c_d\lambda_1^{\mathrm{SP}} - 8NF^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 4NF^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^3 + NF^{-4}d_mc_d\lambda_2^{\mathrm{SP}} - NF^{-4}d_mc_m\lambda_1^{\mathrm{SP}} - 1/8NF^{-4}d_m^2 - 6NF^{-4}\lambda_1^{\mathrm{PP}}c_dc_m + 5/2NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2 + 2NF^{-4}\lambda_1^{\mathrm{PP}}c_m^2 + 4NF^{-4}\lambda_3^{\mathrm{PP}}c_dc_m - 2NF^{-4}\lambda_3^{\mathrm{PP}}c_d^2 - 4NF^{-4}c_dc_m\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 6NF^{-4}c_dc_m(\lambda_1^{\mathrm{SP}})^2 + 4NF^{-4}c_d^2\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 5/2NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2 - 2NF^{-4}c_d^2(\lambda_2^{\mathrm{SP}})^2 - 4NF^{-4}c_m^2(\lambda_1^{\mathrm{SP}})^2 + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}} - 5NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 3/2NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - 1/8NF^{-2}\lambda_1^{\mathrm{PP}} - 1/4NF^{-2}(\lambda_1^{\mathrm{PP}})^2 - 6NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} + 2NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 - NF^{-2}(\lambda_2^{\mathrm{PP}})^2 + 6NF^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 2NF^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2 + 1/4NF^{-2}\lambda_3^{\mathrm{PP}} - NF^{-2}(\lambda_3^{\mathrm{PP}})^2 - 1/4NF^{-2}\lambda_1^{\mathrm{SP}}\lambda_2^{\mathrm{SP}} - 18NF^{-2}(\lambda_1^{\mathrm{SP}})^2(\lambda_2^{\mathrm{SP}})^2 + 1/8NF^{-2}(\lambda_1^{\mathrm{SP}})^2 + 13NF^{-2}(\lambda_1^{\mathrm{SP}})^3\lambda_2^{\mathrm{SP}} - 9/4NF^{-2}(\lambda_1^{\mathrm{SP}})^4 - 1/64NF^{-2}$

232	$\langle P\chi_-P\chi_-\rangle$	$-4NF^{-4}d_m\lambda_2^{\rm PP}c_d\lambda_1^{\rm SP} - 8NF^{-4}d_mc_d(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 4NF^{-4}d_mc_d(\lambda_1^{\rm SP})^3 + NF^{-4}d_mc_d\lambda_2^{\rm SP} - NF^{-4}d_mc_m\lambda_1^{\rm SP} - 1/8NF^{-4}d_m^2 - 4NF^{-4}\lambda_2^{\rm PP}c_dc_m + 2NF^{-4}\lambda_2^{\rm PP}c_d^2 - 4NF^{-4}c_dc_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 4NF^{-4}c_dc_m(\lambda_1^{\rm SP})^2 + 4NF^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 2NF^{-4}c_d^2(\lambda_2^{\rm SP})^2 - 2NF^{-4}c_m^2(\lambda_1^{\rm SP})^2 - NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} - 3NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 + 2NF^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} - 6NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4NF^{-2}\lambda_2^{\rm PP} + 6NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 2NF^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 3/4NF^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 14NF^{-2}(\lambda_1^{\rm SP})^2(\lambda_2^{\rm SP})^2 + 1/4NF^{-2}(\lambda_1^{\rm SP})^2 + 11NF^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - 2NF^{-2}(\lambda_1^{\rm SP})^4$
233	$\langle P\rangle\langle P\chi_-\chi_-\rangle$	$1/32F^{-2}+F^{-4}d_mc_d\lambda_1^{\rm SP}+2F^{-4}d_mc_d\lambda_2^{\rm SP}-4F^{-4}d_mc_m\lambda_1^{\rm SP} - 1/2F^{-4}d_m^2 - 4F^{-4}\lambda_2^{\rm PP}c_dc_m + F^{-4}\lambda_2^{\rm PP}c_d^2 + 4F^{-4}\lambda_2^{\rm PP}c_m^2 + 8F^{-4}c_dc_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 8F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 - 4F^{-4}c_d^2\lambda_1^{\rm SP}\lambda_2^{\rm SP} - F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 12F^{-4}c_m^2(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP} - 6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm PP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} - 4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 + 1/4F^{-2}\lambda_2^{\rm PP} + 12F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 4F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_3^{\rm PP})^2 + 1/2F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 16F^{-2}(\lambda_1^{\rm SP})^2(\lambda_2^{\rm SP})^2 - 1/4F^{-2}(\lambda_1^{\rm SP})^2 + 10F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - 3/2F^{-2}(\lambda_1^{\rm SP})^4$
234	$\langle P\chi_-\rangle^2$	$-1/32F^{-2} + F^{-4}d_mc_d\lambda_1^{\rm SP} + 2F^{-4}d_mc_d\lambda_2^{\rm SP} - 4F^{-4}d_mc_m\lambda_1^{\rm SP} - 1/2F^{-4}d_m^2 - 4F^{-4}\lambda_2^{\rm PP}c_dc_m + F^{-4}\lambda_2^{\rm PP}c_d^2 + 4F^{-4}\lambda_2^{\rm PP}c_m^2 + 8F^{-4}c_dc_m\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 8F^{-4}c_dc_m(\lambda_1^{\rm SP})^2 - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^2 - 4F^{-4}c_d^2(\lambda_2^{\rm SP})^2 - 12F^{-4}c_m^2(\lambda_1^{\rm SP})^2 - F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP} + 2\lambda_1^{\rm PP}\lambda_3^{\rm PP} - 6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm PP})^2 + 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP} - 8F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2 - 1/4F^{-2}\lambda_2^{\rm PP} - F^{-2}(\lambda_2^{\rm PP})^2 + 12F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 4F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^2 - 2F^{-2}(\lambda_3^{\rm PP})^2 - 1/2F^{-2}\lambda_1^{\rm SP}\lambda_2^{\rm SP} - 20F^{-2}(\lambda_1^{\rm SP})^2(\lambda_2^{\rm SP})^2 + 1/4F^{-2}(\lambda_1^{\rm SP})^2 + 14F^{-2}(\lambda_1^{\rm SP})^3\lambda_2^{\rm SP} - 5/2F^{-2}(\lambda_1^{\rm SP})^4$

235	$\langle PP\rangle\langle\chi_-\chi_-\rangle$	$-1/64F^{-2} + F^{-4}d_mc_d\lambda_1^{SP} - 2F^{-4}d_mc_m\lambda_1^{SP} - 1/4F^{-4}d_m^2 - 2F^{-4}\lambda_1^{PP}c_dc_m + 1/2F^{-4}\lambda_1^{PP}c_d^2 + 2F^{-4}\lambda_1^{PP}c_m^2 + 6F^{-4}c_dc_m(\lambda_1^{SP})^2 - 3/2F^{-4}c_d^2(\lambda_1^{SP})^2 - 6F^{-4}c_m^2(\lambda_1^{SP})^2 + F^{-2}\lambda_1^{PP}\lambda_3^{PP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 + 1/8F^{-2}\lambda_1^{PP} - 1/4F^{-2}(\lambda_1^{PP})^2 + 4F^{-2}\lambda_3^{PP}\lambda_1^{SP}\lambda_2^{SP} - F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 - 1/4F^{-2}\lambda_3^{PP} - F^{-2}(\lambda_3^{PP})^2 + 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 4F^{-2}(\lambda_1^{SP})^2(\lambda_2^{SP})^2 - 1/8F^{-2}(\lambda_1^{SP})^2 + 2F^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} - 1/4F^{-2}(\lambda_1^{SP})^4$
236	$\langle PP\chi_-\rangle\langle\chi_-\rangle$	$1/32F^{-2} - 4F^{-4}d_m\lambda_1^{PP}c_d\lambda_1^{SP} - 4F^{-4}d_m\lambda_2^{PP}c_d\lambda_1^{SP} + 4F^{-4}d_m\lambda_3^{PP}c_d\lambda_1^{SP} + F^{-4}d_mc_d\lambda_1^{SP} - 16F^{-4}d_mc_d(\lambda_1^{SP})^2\lambda_2^{SP} + 8F^{-4}d_mc_d(\lambda_1^{SP})^3 + 2F^{-4}d_mc_d\lambda_2^{SP} - 4F^{-4}d_mc_m\lambda_1^{SP} - 1/2F^{-4}d_m^2 - 8F^{-4}\lambda_1^{PP}c_dc_m + 3F^{-4}\lambda_1^{PP}c_d^2 + 4F^{-4}\lambda_1^{PP}c_m^2 - 4F^{-4}\lambda_2^{PP}c_dc_m + 2F^{-4}\lambda_2^{PP}c_d^2 + 4F^{-4}\lambda_3^{PP}c_dc_m - 2F^{-4}\lambda_3^{PP}c_d^2 - 8F^{-4}c_dc_m\lambda_1^{SP}\lambda_2^{SP} + 16F^{-4}c_dc_m(\lambda_1^{SP})^2 + 4F^{-4}c_d^2\lambda_1^{SP}\lambda_2^{SP} - 5F^{-4}c_d^2(\lambda_1^{SP})^2 - 12F^{-4}c_m^2(\lambda_1^{SP})^2 - F^{-2}\lambda_1^{PP}\lambda_2^{PP} + 2F^{-2}\lambda_1^{PP}\lambda_3^{PP} - 10F^{-2}\lambda_1^{PP}\lambda_1^{SP}\lambda_2^{SP} + 3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2 - 1/2F^{-2}(\lambda_1^{PP})^2 + 2F^{-2}\lambda_2^{PP}\lambda_3^{PP} - 8F^{-2}\lambda_2^{PP}\lambda_1^{SP}\lambda_2^{SP} + 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2 + 1/4F^{-2}\lambda_2^{PP} + 16F^{-2}\lambda_3^{PP}\lambda_1^{SP}\lambda_2^{SP} - 5F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2 - 2F^{-2}(\lambda_3^{PP})^2 + 1/2F^{-2}\lambda_1^{SP}\lambda_2^{SP} - 32F^{-2}(\lambda_1^{SP})^2(\lambda_2^{SP})^2 - 1/4F^{-2}(\lambda_1^{SP})^2 + 22F^{-2}(\lambda_1^{SP})^3\lambda_2^{SP} - 7/2F^{-2}(\lambda_1^{SP})^4$
237	$\langle P\{u_\mu,\nabla^\mu S\}\rangle$	$-3NF^{-4}M_P^2c_d^2\lambda_1^{SP} + 6NF^{-4}c_d^2M_S^2\lambda_1^{SP} + NF^{-2}\lambda_1^{PP}M_P^2\lambda_1^{SP} + 2NF^{-2}\lambda_2^{PP}M_P^2\lambda_1^{SP} + NF^{-2}M_P^2\lambda_1^{SS}\lambda_1^{SP} + 2NF^{-2}M_P^2\lambda_2^{SS}\lambda_1^{SP} + 1/2NF^{-2}M_P^2\lambda_1^{SP} - 5/2NF^{-2}M_P^2(\lambda_1^{SP})^3 - 2NF^{-2}\lambda_1^{SS}M_S^2\lambda_1^{SP} - 4NF^{-2}\lambda_2^{SS}M_S^2\lambda_1^{SP} - 3/2NF^{-2}M_S^2\lambda_1^{SP} + 3NF^{-2}M_S^2(\lambda_1^{SP})^3 - NF^{-2}M_S^2(\lambda_1^{SP})^3 + 4NF^{-4}c_d^2M_S^2\lambda_1^{SP}$
238	$\langle u_\mu P\rangle\langle\nabla^\mu S\rangle$	$2F^{-4}M_P^2c_d^2\lambda_1^{SP} - 4F^{-4}c_d^2M_S^2\lambda_1^{SP} + 2F^{-2}\lambda_1^{PP}M_P^2\lambda_1^{SP} + 4F^{-2}\lambda_2^{PP}M_P^2\lambda_1^{SP} + 2F^{-2}M_P^2\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}M_P^2\lambda_1^{SP} - F^{-2}M_P^2(\lambda_1^{SP})^3 - 4F^{-2}\lambda_1^{SS}M_S^2\lambda_1^{SP} + 3F^{-2}M_S^2\lambda_1^{SP} - 2F^{-2}M_S^2(\lambda_1^{SP})^3$
239	$\langle P\rangle\langle u_\mu\nabla^\mu S\rangle$	$-6F^{-4}M_P^2c_d^2\lambda_1^{SP} + 12F^{-4}c_d^2M_S^2\lambda_1^{SP} + 2F^{-2}\lambda_1^{PP}M_P^2\lambda_1^{SP} + 2F^{-2}M_P^2\lambda_1^{SS}\lambda_1^{SP} + 4F^{-2}M_P^2\lambda_2^{SS}\lambda_1^{SP} + 2F^{-2}M_P^2\lambda_1^{SP} - 5F^{-2}M_P^2(\lambda_1^{SP})^3 - 4F^{-2}\lambda_1^{SS}M_S^2\lambda_1^{SP} - 8F^{-2}\lambda_2^{SS}M_S^2\lambda_1^{SP} - 3F^{-2}M_S^2\lambda_1^{SP} + 6F^{-2}M_S^2(\lambda_1^{SP})^3 - 2F^{-2}M_S^2(\lambda_1^{SP})^3 + 4F^{-4}c_d^2M_S^2\lambda_1^{SP}$

240	$\langle P\nabla_\mu S\rangle\langle u^\mu\rangle$	$2F^{-4}M_P^2c_d^2\lambda_1^{\rm SP} - 4F^{-4}c_d^2M_S^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}M_P^2\lambda_1^{\rm SP} + 2F^{-2}M_P^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}M_P^2\lambda_1^{\rm SP} - F^{-2}M_P^2(\lambda_1^{\rm SP})^3 - 4F^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} + 3F^{-2}M_S^2\lambda_1^{\rm SP} - 2F^{-2}M_S^2(\lambda_1^{\rm SP})^3$
241	$\langle S\{u_\mu,\nabla^\mu P\}\rangle$	$-NF^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}M_S^2\lambda_1^{\rm SP} - 2NF^{-2}\lambda_2^{\rm PP}M_S^2\lambda_1^{\rm SP} + NF^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} + 2NF^{-2}\lambda_2^{\rm SS}M_S^2\lambda_1^{\rm SP} - NF^{-2}M_S^2\lambda_1^{\rm SP} + 3/2NF^{-2}M_S^2(\lambda_1^{\rm SP})^3 + NF^{-2}M_P^2(\lambda_1^{\rm SP})^3$
242	$\langle u_\mu S\rangle\langle\nabla^\mu P\rangle$	$-6F^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}M_S^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} + 4F^{-2}\lambda_2^{\rm SS}M_S^2\lambda_1^{\rm SP} + F^{-2}M_S^2\lambda_1^{\rm SP} - F^{-2}M_S^2(\lambda_1^{\rm SP})^3$
243	$\langle u_\mu\rangle\langle\nabla^\mu PS\rangle$	$-6F^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}M_S^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} + 2F^{-2}M_S^2\lambda_1^{\rm SP} - F^{-2}M_S^2(\lambda_1^{\rm SP})^3$
244	$\langle S\rangle\langle u_\mu\nabla^\mu P\rangle$	$-2F^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}M_S^2\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm PP}M_S^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} - F^{-2}M_S^2\lambda_1^{\rm SP} + 3F^{-2}M_S^2(\lambda_1^{\rm SP})^3 + 2F^{-2}M_P^2(\lambda_1^{\rm SP})^3$
245	$i\langle\chi_-\{S,P\}\rangle$	$-4NF^{-4}d_mc_dM_S^2(\lambda_1^{\rm SP})^2 + 1/2NF^{-4}d_mc_dM_S^2 + 4NF^{-4}M_P^2c_dc_m\lambda_1^{\rm SP} - 3NF^{-4}M_P^2c_d^2\lambda_1^{\rm SP} - 10NF^{-4}c_dc_mM_S^2\lambda_1^{\rm SP} + 8NF^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - 2NF^{-4}c_d^2M_S^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_1^{\rm PP}M_S^2\lambda_1^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}M_S^2\lambda_1^{\rm SP} + NF^{-2}\lambda_3^{\rm PP}M_S^2\lambda_1^{\rm SP} + 1/2NF^{-2}M_P^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + NF^{-2}M_P^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}M_P^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 1/8NF^{-2}M_P^2\lambda_1^{\rm SP} - 2NF^{-2}M_P^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - NF^{-2}\lambda_1^{\rm SS}M_S^2\lambda_1^{\rm SP} - 2NF^{-2}\lambda_2^{\rm SS}M_S^2\lambda_1^{\rm SP} + 2NF^{-2}\lambda_3^{\rm SS}M_S^2\lambda_1^{\rm SP} - 3/8NF^{-2}M_S^2\lambda_1^{\rm SP} - 4NF^{-2}M_S^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 5/2NF^{-2}M_S^2(\lambda_1^{\rm SP})^3 - NF^{-2}\lambda_2^{\rm SP}M_P^2(\lambda_1^{\rm SP})^2 + 1/2NF^{-2}M_P^2(\lambda_1^{\rm SP})^3 - NF^{-2}\lambda_2^{\rm SP}M_S^2(\lambda_1^{\rm SP})^2 + 2NF^{-4}c_d^2M_S^2\lambda_1^{\rm SP} - 2NF^{-4}c_mc_dM_S^2\lambda_1^{\rm SP}$

246	$i\langle S\rangle\langle P\chi_-\rangle$	$F^{-4}d_m c_d M_S^2 + 4F^{-4}M_P^2 c_d c_m \lambda_1^{SP} - 2F^{-4}M_P^2 c_d^2 \lambda_1^{SP} - 4F^{-4}c_d c_m M_S^2 \lambda_1^{SP} + 4F^{-4}c_d^2 M_S^2 \lambda_1^{SP} - 4F^{-4}c_d^2 M_S^2 \lambda_2^{SP} - F^{-2}\lambda_1^{PP} M_S^2 \lambda_1^{SP} - 2F^{-2}\lambda_2^{PP} M_S^2 \lambda_1^{SP} + 2F^{-2}\lambda_3^{PP} M_S^2 \lambda_1^{SP} + F^{-2}M_P^2 \lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}M_P^2 \lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-2}M_P^2 \lambda_1^{SP} - 2F^{-2}\lambda_1^{SS} M_S^2 \lambda_1^{SP} + 4F^{-2}\lambda_3^{SS} M_S^2 \lambda_1^{SP} + 1/4F^{-2}M_S^2 \lambda_1^{SP} - 8F^{-2}M_S^2(\lambda_1^{SP})^2\lambda_2^{SP} + 3F^{-2}M_S^2(\lambda_1^{SP})^3 + F^{-2}M_P^2(\lambda_1^{SP})^3 - 2F^{-2}\lambda_2^{SP} M_P^2(\lambda_1^{SP})^2$
247	$i\langle S\chi_-\rangle\langle P\rangle$	$F^{-4}d_m c_d M_S^2 + 8F^{-4}M_P^2 c_d c_m \lambda_1^{SP} - 6F^{-4}M_P^2 c_d^2 \lambda_1^{SP} - 12F^{-4}c_d c_m M_S^2 \lambda_1^{SP} + 10F^{-4}c_d^2 M_S^2 \lambda_1^{SP} - F^{-2}\lambda_1^{PP} M_S^2 \lambda_1^{SP} + 2F^{-2}\lambda_3^{PP} M_S^2 \lambda_1^{SP} + F^{-2}M_P^2 \lambda_1^{SS}\lambda_1^{SP} + 2F^{-2}M_P^2 \lambda_2^{SS}\lambda_1^{SP} - 2F^{-2}M_P^2 \lambda_3^{SS}\lambda_1^{SP} + 1/4F^{-2}M_P^2 \lambda_1^{SP} - 4F^{-2}M_P^2(\lambda_1^{SP})^2\lambda_2^{SP} - 2F^{-2}\lambda_1^{SS} M_S^2 \lambda_1^{SP} - 4F^{-2}\lambda_2^{SS} M_S^2 \lambda_1^{SP} + 4F^{-2}\lambda_3^{SS} M_S^2 \lambda_1^{SP} - 1/4F^{-2}M_S^2 \lambda_1^{SP} + 4F^{-2}M_S^2(\lambda_1^{SP})^2\lambda_2^{SP} + F^{-2}M_S^2(\lambda_1^{SP})^3 - 2F^{-2}\lambda_2^{SP} M_S^2(\lambda_1^{SP})^2 + 4NF^{-4}c_d^2 M_S^2 \lambda_1^{SP} - 4F^{-4}c_d c_m M_S^2 \lambda_1^{SP}$
248	$i\langle SP\rangle\langle\chi_-\rangle$	$-8F^{-4}d_m c_d M_S^2(\lambda_1^{SP})^2 + F^{-4}d_m c_d M_S^2 + 4F^{-4}M_P^2 c_d c_m \lambda_1^{SP} - 2F^{-4}M_P^2 c_d^2 \lambda_1^{SP} - 12F^{-4}c_d c_m M_S^2 \lambda_1^{SP} + 6F^{-4}c_d^2 M_S^2 \lambda_1^{SP} - F^{-2}\lambda_1^{PP} M_S^2 \lambda_1^{SP} + 2F^{-2}\lambda_3^{PP} M_S^2 \lambda_1^{SP} + F^{-2}M_P^2 \lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}M_P^2 \lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-2}M_P^2 \lambda_1^{SP} - 2F^{-2}\lambda_1^{SS} M_S^2 \lambda_1^{SP} + 4F^{-2}\lambda_3^{SS} M_S^2 \lambda_1^{SP} + 3/4F^{-2}M_S^2 \lambda_1^{SP} - 12F^{-2}M_S^2(\lambda_1^{SP})^2\lambda_2^{SP} + 3F^{-2}M_S^2(\lambda_1^{SP})^3$
249	$\langle u_\mu u\cdot u\nabla^\mu SP\rangle + \text{h.c.}$	$NF^{-6}c_d^4\lambda_1^{SP} + 1/3NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 4/3NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 1/3NF^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 7/12NF^{-4}c_d^2\lambda_1^{SP} + 1/3NF^{-4}c_d^2(\lambda_1^{SP})^3 - 2NF^{-2}\lambda_1^{PP}\lambda_2^{PP}\lambda_1^{SP} + 1/4NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + NF^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 + 2/3NF^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{PP}(\lambda_1^{SP})^3 - 1/12NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 + 1/24NF^{-2}\lambda_1^{SP} - 1/24NF^{-2}(\lambda_1^{SP})^3 - 1/6NF^{-2}(\lambda_1^{SP})^5$
250	$\langle u_\mu\nabla^\mu Su\cdot uP\rangle + \text{h.c.}$	$7NF^{-6}c_d^4\lambda_1^{SP} - 2/3NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 10/3NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - 7NF^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 6NF^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 37/12NF^{-4}c_d^2\lambda_1^{SP} + 12NF^{-4}c_d^2(\lambda_1^{SP})^3 - 7/6F^{-2}N\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 5/3NF^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 23/24NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 5/3NF^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 - NF^{-2}(\lambda_1^{PP})^2\lambda_1^{SP} + NF^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} + 5/4NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - NF^{-2}\lambda_2^{PP}(\lambda_1^{SP})^3 + 37/24NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 5/2NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 2NF^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} - 1/12NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/96NF^{-2}\lambda_1^{SP} - 5/12NF^{-2}(\lambda_1^{SP})^3 + 1/2NF^{-2}(\lambda_1^{SP})^5$

251	$\langle\, u_\mu \nabla^\mu S P u \cdot u \,\rangle + \mathrm{h.c.}$	$20/3NF^{-6}c_d^4\lambda_1^{\mathrm{SP}} + 1/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 8/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 5NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 6NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 31/12NF^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 11NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 - NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 7/6NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 3/2NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/6NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 2/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 4NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - NF^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 3/16NF^{-2}\lambda_1^{\mathrm{SP}} - 1/12NF^{-2}(\lambda_1^{\mathrm{SP}})^3 + 1/3NF^{-2}(\lambda_1^{\mathrm{SP}})^5$
252	$\langle\, P\,\{u_\mu, u_\nu \nabla^\mu S u^\nu\}\,\rangle$	$-1/3NF^{-6}c_d^4\lambda_1^{\mathrm{SP}} + 2/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 2/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/6NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/3NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/8NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/6NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 1/3NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/24NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/6NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 1/12NF^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 5/96NF^{-2}\lambda_1^{\mathrm{SP}} + 1/24NF^{-2}(\lambda_1^{\mathrm{SP}})^3$
253	$\langle\, P\,\{\nabla_\mu S, u_\nu u^\mu u^\nu\}\,\rangle$	$1/3NF^{-6}c_d^4\lambda_1^{\mathrm{SP}} + 1/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 1/3NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/4NF^{-4}c_d^2\lambda_1^{\mathrm{SP}} - 1/3NF^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 - 1/12NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 2/3NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/6NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 1/12NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/6NF^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/3NF^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 1/12NF^{-2}\lambda_1^{\mathrm{SP}} + 1/8NF^{-2}(\lambda_1^{\mathrm{SP}})^3 - 1/6NF^{-2}(\lambda_1^{\mathrm{SP}})^5$
254	$\langle\, u_\nu P u^\nu\,\{u_\mu, \nabla^\mu S\}\,\rangle$	$-8/3NF^{-6}c_d^4\lambda_1^{\mathrm{SP}} - 4/3NF^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 8/3NF^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + NF^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 4/3NF^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 5/12NF^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 1/2NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 11/24NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 5/6NF^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 - 7/3NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 5/4NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 3NF^{-2}\lambda_2^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 7/24NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/6NF^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 11/12NF^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 23/96NF^{-2}\lambda_1^{\mathrm{SP}} - 5/8NF^{-2}(\lambda_1^{\mathrm{SP}})^3$

255	$\langle u_\mu u\cdot u\nabla^\mu PS\rangle+\text{h.c.}$	$-4/3NF^{-6}c_d^4\lambda_1^{\rm SP} + 2/3NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4/3NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 7/12NF^{-4}c_d^2\lambda_1^{\rm SP} - 2/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 1/12NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2/3NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/6NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/4NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2/3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/8NF^{-2}\lambda_1^{\rm SP} - 1/24NF^{-2}(\lambda_1^{\rm SP})^3$
256	$\langle u_\mu\nabla^\mu Pu\cdot uS\rangle+\text{h.c.}$	$-2/3NF^{-6}c_d^4\lambda_1^{\rm SP} + NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/6NF^{-4}c_d^2\lambda_1^{\rm SP} - NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 7/6NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/24NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 5/3NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/12NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 35/24NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 7/6NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - NF^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} - 5/4NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/96NF^{-2}\lambda_1^{\rm SP} - 1/12NF^{-2}(\lambda_1^{\rm SP})^3$
257	$\langle u_\mu\nabla^\mu PSu\cdot u\rangle+\text{h.c.}$	$-2/3NF^{-6}c_d^4\lambda_1^{\rm SP} + 2NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2/3NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2/3NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/2NF^{-4}c_d^2\lambda_1^{\rm SP} - 1/3NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 7/6NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/2NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/6NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 1/48NF^{-2}\lambda_1^{\rm SP}$
258	$\langle S\{u_\mu,u_\nu\nabla^\mu Pu^\nu\}\rangle$	$1/3NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 1/12NF^{-4}c_d^2\lambda_1^{\rm SP} - 1/6NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/8NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 1/3NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/12NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 1/24NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/4NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/96NF^{-2}\lambda_1^{\rm SP} - 1/24NF^{-2}(\lambda_1^{\rm SP})^3$
259	$\langle S\{\nabla_\mu P,u_\nu u^\mu u^\nu\}\rangle$	$-2/3NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 1/12NF^{-4}c_d^2\lambda_1^{\rm SP} - 1/12NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 2/3NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/6NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 1/12NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/6NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/12NF^{-2}\lambda_1^{\rm SP} + 1/24NF^{-2}(\lambda_1^{\rm SP})^3$
260	$\langle u_\nu Su^\nu\{u_\mu,\nabla^\mu P\}\rangle$	$-4/3NF^{-6}c_d^4\lambda_1^{\rm SP} - 1/3NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2/3NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2/3NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/12NF^{-4}c_d^2\lambda_1^{\rm SP} - 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 7/3NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 11/24NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 11/12NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 7/24NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2/3NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 5/4NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 23/96NF^{-2}\lambda_1^{\rm SP} + 5/8NF^{-2}(\lambda_1^{\rm SP})^3$

261	$\langle u_\mu u_\nu \nabla^\mu SP \rangle\langle u^\nu \rangle + \text{h.c.}$	$2F^{-6}c_d^4\lambda_1^{SP} + F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 8/3F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 1/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 3/4F^{-4}c_d^2\lambda_1^{SP} + 1/3F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/6F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 17/24F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/6F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 + 1/3F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/12F^{-2}\lambda_2^{PP}\lambda_1^{SP} + 8/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^3 - 4F^{-2}(\lambda_2^{PP})^2\lambda_1^{SP} - 1/24F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/6F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/12F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 11/96F^{-2}\lambda_1^{SP} + 1/12F^{-2}(\lambda_1^{SP})^3 - 1/2F^{-2}(\lambda_1^{SP})^5$
262	$\langle u_\mu \{P, u\cdot u\}\rangle\langle \nabla^\mu S\rangle$	$-17/3F^{-6}c_d^4\lambda_1^{SP} - 2F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4/3F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - 11/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 59/12F^{-4}c_d^2\lambda_1^{SP} - 14/3F^{-4}c_d^2(\lambda_1^{SP})^3 - 2F^{-2}\lambda_1^{PP}\lambda_2^{PP}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 - F^{-2}(\lambda_1^{PP})^2\lambda_1^{SP} + 4/3F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/6F^{-2}\lambda_2^{PP}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^3 - 3/2F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 2F^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} - 7/48F^{-2}\lambda_1^{SP} + 1/6F^{-2}(\lambda_1^{SP})^3 - 1/3F^{-2}(\lambda_1^{SP})^5$
263	$\langle u_\mu \nabla^\mu S u_\nu P\rangle\langle u^\nu\rangle + \text{h.c.}$	$-F^{-6}c_d^4\lambda_1^{SP} - 14/3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 8/3F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 10F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 29/12F^{-4}c_d^2\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_2^{PP}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 4/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 17/12F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 + 4/3F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} + 1/6F^{-2}\lambda_2^{PP}\lambda_1^{SP} - F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^3 + 4F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_1^{SP} - 5/12F^{-2}\lambda_1^{SS}\lambda_1^{SP} - F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 11/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 13/48F^{-2}\lambda_1^{SP} - 2/3F^{-2}(\lambda_1^{SP})^3 + F^{-2}(\lambda_1^{SP})^5$
264	$\langle P\rangle\langle \nabla^\mu S u_\nu u^\mu u^\nu\rangle$	$2/3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 2/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 4/3F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 1/6F^{-4}c_d^2\lambda_1^{SP} - 2/3F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 5/12F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 + 5/12F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 5/6F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 5/48F^{-2}\lambda_1^{SP} + 1/2F^{-2}(\lambda_1^{SP})^3 - 1/3F^{-2}(\lambda_1^{SP})^5$
265	$\langle P\rangle\langle u\cdot u\{u_\mu, \nabla^\mu S\}\rangle$	$-4F^{-6}c_d^4\lambda_1^{SP} - 16/3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 2F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - 20/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 8/3F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 7/6F^{-4}c_d^2\lambda_1^{SP} + 22/3F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 4/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 7/12F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^3 - F^{-2}(\lambda_1^{PP})^2\lambda_1^{SP} - F^{-2}\lambda_2^{PP}\lambda_1^{SP} + 4F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_1^{SP} + 11/12F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 10/3F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 2F^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} - 1/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 4/3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 1/24F^{-2}\lambda_1^{SP} + 2/3F^{-2}(\lambda_1^{SP})^5$

266	$\langle u_\mu \rangle \langle \nabla^\mu S \{P, u\cdot u\} \rangle$	$-5F^{-6}c_d^4\lambda_1^{\rm SP} - 5/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_1^{\rm SP} - 5F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 13/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 37/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 1/6F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm PP})^2\lambda_1^{\rm SP} + F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 - 35/24F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} + 1/12F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 31/96F^{-2}\lambda_1^{\rm SP} + 5/12F^{-2}(\lambda_1^{\rm SP})^3 - 1/2F^{-2}(\lambda_1^{\rm SP})^5$
267	$\langle u_\mu u_\nu P \nabla^\mu S \rangle \langle u^\nu \rangle + \text{h.c.}$	$-F^{-6}c_d^4\lambda_1^{\rm SP} - 7/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 6F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/2F^{-4}c_d^2\lambda_1^{\rm SP} - 7/3F^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 37/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 5/6F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 1/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/12F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 3/8F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/6F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 3/4F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 8F^{-2}(\lambda_2^{\rm SS})^2\lambda_1^{\rm SP} - 7/32F^{-2}\lambda_1^{\rm SP} - 1/4F^{-2}(\lambda_1^{\rm SP})^3 + 1/2F^{-2}(\lambda_1^{\rm SP})^5$
268	$\langle u_\mu P \rangle \langle \nabla^\mu S u\cdot u \rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 11/6F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 2F^{-2}(\lambda_1^{\rm PP})^2\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 - 47/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 4F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} - 1/3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 11/48F^{-2}\lambda_1^{\rm SP} + 1/3F^{-2}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SP})^5$
269	$\langle u_\mu P \rangle \langle \nabla_\nu S \{u^\mu, u^\nu\} \rangle$	$8F^{-6}c_d^4\lambda_1^{\rm SP} - 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 28/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 16F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 19/6F^{-4}c_d^2\lambda_1^{\rm SP} + 6F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 + 2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 7/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 19/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 8F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 8F^{-2}(\lambda_2^{\rm SS})^2\lambda_1^{\rm SP} - 19/48F^{-2}\lambda_1^{\rm SP} - 2/3F^{-2}(\lambda_1^{\rm SP})^3 + F^{-2}(\lambda_1^{\rm SP})^5$

270	$\langle P\nabla^\mu S\rangle\langle u_\mu u\cdot u\rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 2F^{-2}(\lambda_1^{\rm PP})^2\lambda_1^{\rm SP} - 15/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 4F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} - 1/16F^{-2}\lambda_1^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SP})^5$
271	$\langle Pu_\nu u^\mu u^\nu\rangle\langle\nabla_\mu S\rangle$	$10/3F^{-6}c_d^4\lambda_1^{\rm SP} + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 8/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4/3F^{-4}c_d^2\lambda_1^{\rm SP} - 2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^3 + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 14/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 17/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 4/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 3/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 31/48F^{-2}\lambda_1^{\rm SP} + 2/3F^{-2}(\lambda_1^{\rm SP})^3 - 1/3F^{-2}(\lambda_1^{\rm SP})^5$
272	$\langle Pu_\nu\nabla^\mu Su^\nu\rangle\langle u_\mu\rangle$	$2F^{-6}c_d^4\lambda_1^{\rm SP} + 4/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 5/6F^{-4}c_d^2\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 14/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 19/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 5/6F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 7/12F^{-2}\lambda_1^{\rm SP} + 1/3F^{-2}(\lambda_1^{\rm SP})^3$
273	$\langle P\{u_\mu,u_\nu\}\rangle\langle u^\mu\nabla^\nu S\rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/6F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 5/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 + 4/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 7/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 10/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 - 4F^{-2}(\lambda_2^{\rm PP})^2\lambda_1^{\rm SP} + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 17/48F^{-2}\lambda_1^{\rm SP} + 2/3F^{-2}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SP})^5$
274	$\langle P\{u_\mu,\nabla^\mu S\}\rangle\langle u\cdot u\rangle$	$-7F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 14/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 7F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 13/12F^{-4}c_d^2\lambda_1^{\rm SP} + 11F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/8F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm PP})^2\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 7/12F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 25/24F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 7/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/96F^{-2}\lambda_1^{\rm SP} + 1/6F^{-2}(\lambda_1^{\rm SP})^3 + 1/2F^{-2}(\lambda_1^{\rm SP})^5$

275	$\langle P\{u_\mu,\nabla_\nu S\}\rangle\langle u^\mu u^\nu\rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 8/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 7/6F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 5/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 + 2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 8/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 31/48F^{-2}\lambda_1^{\rm SP} + 5/6F^{-2}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SP})^5$
276	$\langle Pu\cdot u\rangle\langle u_\mu\nabla^\mu S\rangle$	$16F^{-6}c_d^4\lambda_1^{\rm SP} + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 8/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - 22F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 20F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 41/6F^{-4}c_d^2\lambda_1^{\rm SP} + 38F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 6F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 9/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 7F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^3 - 2F^{-2}(\lambda_1^{\rm PP})^2\lambda_1^{\rm SP} - 8/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 11/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 10/3F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^3 + 8F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 47/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 7F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 4F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} + 3/2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 47/48F^{-2}\lambda_1^{\rm SP} - 11/6F^{-2}(\lambda_1^{\rm SP})^3 + F^{-2}(\lambda_1^{\rm SP})^5$
277	$\langle u_\mu\rangle\langle\nabla^\mu P\{S,u\cdot u\}\rangle$	$4/3F^{-6}c_d^4\lambda_1^{\rm SP} + F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 16/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 8/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/6F^{-4}c_d^2\lambda_1^{\rm SP} + 13/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 1/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/12F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 49/24F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 17/96F^{-2}\lambda_1^{\rm SP} + 1/12F^{-2}(\lambda_1^{\rm SP})^3$
278	$\langle u_\mu u_\nu\nabla^\mu PS\rangle\langle u^\nu\rangle + {\rm h.c.}$	$-8/3F^{-6}c_d^4\lambda_1^{\rm SP} + 1/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 20/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 7/6F^{-4}c_d^2\lambda_1^{\rm SP} - 4/3F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 1/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/12F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 13/24F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/12F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 4F^{-2}(\lambda_2^{\rm SS})^2\lambda_1^{\rm SP} - 37/96F^{-2}\lambda_1^{\rm SP} - 1/12F^{-2}(\lambda_1^{\rm SP})^3$
279	$\langle u_\mu\{S,u\cdot u\}\rangle\langle\nabla^\mu P\rangle$	$4/3F^{-6}c_d^4\lambda_1^{\rm SP} + 4/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 16/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 8/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/3F^{-4}c_d^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/3F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 11/48F^{-2}\lambda_1^{\rm SP} - 1/6F^{-2}(\lambda_1^{\rm SP})^3$

280	$\langle u_\mu u\cdot u\rangle\langle\nabla^\mu PS\rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}-F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+3/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP}-15/16F^{-2}\lambda_1^{\rm SP}+1/2F^{-2}(\lambda_1^{\rm SP})^3$
281	$\langle u_\mu u_\nu S\nabla^\mu P\rangle\langle u^\nu\rangle+\text{h.c.}$	$-8/3F^{-6}c_d^4\lambda_1^{\rm SP}-1/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}+2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}+1/6F^{-4}c_d^2\lambda_1^{\rm SP}-2/3F^{-4}c_d^2(\lambda_1^{\rm SP})^3-1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-13/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+3/4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-11/8F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3+1/12F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+1/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3-9/32F^{-2}\lambda_1^{\rm SP}+1/4F^{-2}(\lambda_1^{\rm SP})^3$
282	$\langle u_\mu\nabla^\mu Pu_\nu S\rangle\langle u^\nu\rangle+\text{h.c.}$	$-8/3F^{-6}c_d^4\lambda_1^{\rm SP}+2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+16/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}+5/3F^{-4}c_d^2\lambda_1^{\rm SP}-2F^{-4}c_d^2(\lambda_1^{\rm SP})^3-1/3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+5/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-4/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-17/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+4/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3+5/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3+13/48F^{-2}\lambda_1^{\rm SP}-1/3F^{-2}(\lambda_1^{\rm SP})^3$
283	$\langle u_\mu S\rangle\langle\nabla^\mu Pu\cdot u\rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}+F^{-4}c_d^2\lambda_1^{\rm SP}-F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+11/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-4/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+1/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+3/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP}-5/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+4/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3-11/48F^{-2}\lambda_1^{\rm SP}+1/6F^{-2}(\lambda_1^{\rm SP})^3$
284	$\langle u_\mu S\rangle\langle\nabla_\nu P\{u^\mu,u^\nu\}\rangle$	$-8F^{-6}c_d^4\lambda_1^{\rm SP}+2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}+12F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}+2F^{-4}c_d^2\lambda_1^{\rm SP}-4F^{-4}c_d^2(\lambda_1^{\rm SP})^3-F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-1/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-4/3F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-1/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3-10/3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+4/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3+19/48F^{-2}\lambda_1^{\rm SP}+1/6F^{-2}(\lambda_1^{\rm SP})^3$
285	$\langle S\rangle\langle\nabla_\mu Pu_\nu u^\mu u^\nu\rangle$	$2/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}-4/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP}-1/3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-1/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}+2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+1/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}+1/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-1/16F^{-2}\lambda_1^{\rm SP}$
286	$\langle S\rangle\langle u\cdot u\{u_\mu,\nabla^\mu P\}\rangle$	$-4F^{-6}c_d^4\lambda_1^{\rm SP}+2/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+8/3F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}+2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}-2F^{-4}c_d^2(\lambda_1^{\rm SP})^3-1/3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+5/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-4/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-11/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3-F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP}+F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+1/8F^{-2}\lambda_1^{\rm SP}-1/2F^{-2}(\lambda_1^{\rm SP})^3$

287	$\langle Su_\nu u^\mu u^\nu \rangle \langle \nabla^\mu P \rangle$	$-8/3F^{-6}c_d^4\lambda_1^{\rm SP} - 2/3F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 8/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 28/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/3F^{-4}c_d^2\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 14/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 13/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 13/12F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 7/2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 23/48F^{-2}\lambda_1^{\rm SP} - 1/6F^{-2}(\lambda_1^{\rm SP})^3$
288	$\langle Su_\nu \nabla^\mu P u^\nu \rangle \langle u_\mu \rangle$	$-8/3F^{-6}c_d^4\lambda_1^{\rm SP} - 8/3F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 28/3F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/3F^{-4}c_d^2\lambda_1^{\rm SP} - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 14/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/6F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 5/6F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 19/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 7/12F^{-2}\lambda_1^{\rm SP} - 1/3F^{-2}(\lambda_1^{\rm SP})^3$
289	$\langle S\{u_\mu, u_\nu\} \rangle \langle u^\nu \nabla^\mu P \rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 13/6F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 4F^{-2}(\lambda_2^{\rm SS})^2\lambda_1^{\rm SP} - 17/48F^{-2}\lambda_1^{\rm SP} - 1/6F^{-2}(\lambda_1^{\rm SP})^3$
290	$\langle S\{u_\mu, \nabla^\mu P\} \rangle \langle u \cdot u \rangle$	$-4F^{-6}c_d^4\lambda_1^{\rm SP} + F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 6F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 11/24F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/8F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} - 7/12F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 47/96F^{-2}\lambda_1^{\rm SP} - 2/3F^{-2}(\lambda_1^{\rm SP})^3$
291	$\langle S\{u_\mu, \nabla_\nu P\} \rangle \langle u^\mu u^\nu \rangle$	$2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3F^{-4}c_d^2\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 17/48F^{-2}\lambda_1^{\rm SP} + 1/6F^{-2}(\lambda_1^{\rm SP})^3$
292	$\langle Su \cdot u \rangle \langle u_\mu \nabla^\mu P \rangle$	$-8F^{-6}c_d^4\lambda_1^{\rm SP} + 2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 4F^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 3F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8/3F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 11/12F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 6F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 5/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 2F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP} + 11/3F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 10/3F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - 47/48F^{-2}\lambda_1^{\rm SP} + 4/3F^{-2}(\lambda_1^{\rm SP})^3$

293	$i\langle u_\mu f_+^{\mu\nu} P\nabla_\nu S\rangle + \text{h.c.}$	$1/6NF^{-4}c_d^2\lambda_1^{SP} + 1/6NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{SP} + 1/12NF^{-2}(\lambda_1^{SP})^3$
294	$i\langle u_\mu f_+^{\mu\nu}\nabla_\nu SP\rangle + \text{h.c.}$	$-1/3NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/8NF^{-2}\lambda_1^{SP} + 1/6NF^{-2}(\lambda_1^{SP})^3$
295	$i\langle Pu_\mu\nabla_\nu S f_+^{\mu\nu}\rangle + \text{h.c.}$	$-1/6NF^{-4}c_d^2\lambda_1^{SP} + 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/12NF^{-2}\lambda_1^{SP} - 1/12NF^{-2}(\lambda_1^{SP})^3$
296	$i\langle u_\mu f_+^{\mu\nu}\nabla_\nu PS\rangle + \text{h.c.}$	$1/3NF^{-4}c_d^2\lambda_1^{SP} + 1/6NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{SP}$
297	$i\langle u_\mu f_+^{\mu\nu} S\nabla_\nu P\rangle + \text{h.c.}$	$-1/6NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/8NF^{-2}\lambda_1^{SP} + 1/12NF^{-2}(\lambda_1^{SP})^3$
298	$i\langle Su_\mu\nabla_\nu P f_+^{\mu\nu}\rangle + \text{h.c.}$	$1/6NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/6NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/12NF^{-2}\lambda_1^{SP} + 1/12NF^{-2}(\lambda_1^{SP})^3$
299	$i\langle u_\mu\rangle\langle f_+^{\mu\nu}[\nabla_\nu S, P]\rangle$	$-1/3F^{-4}c_d^2\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/6F^{-2}(\lambda_1^{SP})^3$
300	$i\langle\nabla_\mu S\rangle\langle f_+^{\mu\nu}[P, u_\nu]\rangle$	$1/3F^{-4}c_d^2\lambda_1^{SP} + 1/12F^{-2}\lambda_1^{SP} - 1/6F^{-2}(\lambda_1^{SP})^3$
301	$i\langle P\rangle\langle f_+^{\mu\nu}[u_\mu, \nabla_\nu S]\rangle$	$1/3F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/12F^{-2}\lambda_1^{SP} - 1/3F^{-2}(\lambda_1^{SP})^3$
302	$i\langle u_\mu\rangle\langle f_+^{\mu\nu}[S, \nabla_\nu P]\rangle$	$-1/3F^{-4}c_d^2\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/6F^{-2}(\lambda_1^{SP})^3$
303	$i\langle\nabla_\mu P\rangle\langle f_+^{\mu\nu}[S, u_\nu]\rangle$	$-1/3F^{-4}c_d^2\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/12F^{-2}\lambda_1^{SP} - 1/6F^{-2}(\lambda_1^{SP})^3$
304	$i\langle S\rangle\langle f_+^{\mu\nu}[u_\mu, \nabla_\nu P]\rangle$	$1/12F^{-2}\lambda_1^{SP}$
305	$\langle u_\mu u_\nu PS f_-^{\mu\nu}\rangle + \text{h.c.}$	$-2/3NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 1/3NF^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 1/6NF^{-4}c_d^2\lambda_1^{SP} + 1/3NF^{-4}c_d^2(\lambda_1^{SP})^3 + 2/3NF^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/12NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 1/48NF^{-2}\lambda_1^{SP}$
306	$\langle u_\mu u_\nu SP f_-^{\mu\nu}\rangle + \text{h.c.}$	$-2/3NF^{-6}c_d^4\lambda_1^{SP} + 2/3NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 2/3NF^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 1/4NF^{-4}c_d^2\lambda_1^{SP} - 1/3NF^{-4}c_d^2(\lambda_1^{SP})^3 + 1/12NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - 2/3NF^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 1/16NF^{-2}\lambda_1^{SP} - 1/12NF^{-2}(\lambda_1^{SP})^3$
307	$\langle u_\mu f_-^{\mu\nu} u_\nu [S, P]\rangle$	$-2/3NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 1/3NF^{-4}c_d^2(\lambda_1^{SP})^3 + 2/3NF^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/3NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3$
308	$\langle u_\mu P u_\nu S f_-^{\mu\nu}\rangle + \text{h.c.}$	$-1/3NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 1/12NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 1/3NF^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/12NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/12NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/96NF^{-2}\lambda_1^{SP} + 1/24NF^{-2}(\lambda_1^{SP})^3$

309	$\langle u_\mu P f_-^{\mu\nu} u_\nu S \rangle + \text{h.c.}$	$-1/3NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 2/3NF^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 1/4NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 1/3NF^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/12NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/24NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/12NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/96NF^{-2}\lambda_1^{SP} + 1/24NF^{-2}(\lambda_1^{SP})^3$
310	$\langle S u_\mu u_\nu P f_-^{\mu\nu} \rangle + \text{h.c.}$	$-2/3NF^{-6}c_d^4\lambda_1^{SP} - 1/3NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4/3NF^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 1/3NF^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/24NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3NF^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/4NF^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/8NF^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/6NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/4NF^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/32NF^{-2}\lambda_1^{SP} + 1/8NF^{-2}(\lambda_1^{SP})^3$
311	$\langle u_\mu \rangle\langle u_\nu S P f_-^{\mu\nu} \rangle + \text{h.c.}$	$-2/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 2/3F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 1/3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 1/12F^{-4}c_d^2\lambda_1^{SP} + 1/3F^{-4}c_d^2(\lambda_1^{SP})^3 - 1/6F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 1/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/24F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/3F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2/3F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} + 1/12F^{-2}\lambda_2^{PP}\lambda_1^{SP} + 1/24F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/6F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 1/12F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/96F^{-2}\lambda_1^{SP} - 1/24F^{-2}(\lambda_1^{SP})^3$
312	$\langle u_\mu \rangle\langle S u_\nu P f_-^{\mu\nu} \rangle + \text{h.c.}$	$+2/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 2/3F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP}$
313	$\langle u_\mu \rangle\langle u_\nu P S f_-^{\mu\nu} \rangle + \text{h.c.}$	$1/3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 1/12F^{-4}c_d^2\lambda_1^{SP} - 1/3F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/6F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 1/3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/24F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/3F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} + 2/3F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/12F^{-2}\lambda_2^{PP}\lambda_1^{SP} - 1/24F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/6F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/12F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/96F^{-2}\lambda_1^{SP} + 1/24F^{-2}(\lambda_1^{SP})^3$
314	$\langle P \rangle\langle u_\mu u_\nu S f_-^{\mu\nu} \rangle + \text{h.c.}$	$1/3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 2/3F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 1/6F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/6F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/24F^{-2}\lambda_1^{SP} - 1/6F^{-2}(\lambda_1^{SP})^3$
315	$\langle S \rangle\langle u_\mu u_\nu P f_-^{\mu\nu} \rangle + \text{h.c.}$	$-4/3F^{-6}c_d^4\lambda_1^{SP} + F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 1/6F^{-4}c_d^2\lambda_1^{SP} + 1/24F^{-2}\lambda_1^{SP}$
316∗	$\langle f_-^{\mu\nu}\{\nabla_\mu P, \nabla_\nu S\} \rangle$	$1/3NF^{-4}c_d^2\lambda_1^{SP} - 1/6NF^{-2}(\lambda_1^{SP})^3$
317∗	$\langle \nabla_\mu P \rangle\langle f_-^{\mu\nu}\nabla_\nu S \rangle$	$2/3F^{-4}c_d^2\lambda_1^{SP} - 1/3F^{-2}(\lambda_1^{SP})^3$
318∗	$\langle \nabla_\mu S \rangle\langle f_-^{\mu\nu}\nabla_\nu P \rangle$	$-2/3F^{-4}c_d^2\lambda_1^{SP} + 1/3F^{-2}(\lambda_1^{SP})^3$
319	$i\langle S f_+^{\mu\nu} P f_{-\mu\nu} \rangle + \text{h.c.}$	$1/12NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - 1/12NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/24NF^{-2}\lambda_1^{SP} - 1/12NF^{-2}(\lambda_1^{SP})^3$

320	$i\langle f_+^{\mu\nu} f_{-\mu\nu} PS\rangle + \text{h.c.}$	$1/6NF^{-4}c_d^2\lambda_1^{SP} + 1/12NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - 1/16NF^{-2}\lambda_1^{SP} - 1/12NF^{-2}(\lambda_1^{SP})^3$
321	$i\langle f_+^{\mu\nu} f_{-\mu\nu} SP\rangle + \text{h.c.}$	$-1/6NF^{-4}c_d^2\lambda_1^{SP} - 1/12NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/6NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/48NF^{-2}\lambda_1^{SP}$
322	$i\langle P\rangle\langle f_+^{\mu\nu} f_{-\mu\nu} S\rangle + \text{h.c.}$	$1/6F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/6F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/3F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 1/24F^{-2}\lambda_1^{SP} - 1/6F^{-2}(\lambda_1^{SP})^3$
323	$i\langle S\rangle\langle P[f_+^{\mu\nu}, f_{-\mu\nu}]\rangle$	$1/24F^{-2}\lambda_1^{SP}$
324	$\langle u_\mu \chi_+ \nabla^\mu SP\rangle + \text{h.c.}$	$2NF^{-4}d_m\lambda_2^{PP}c_d + 1/2NF^{-4}c_d^2\lambda_1^{SP} + 1/4NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - 2NF^{-2}\lambda_2^{PP}\lambda_3^{PP}\lambda_1^{SP} + NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^3 + 1/16NF^{-2}\lambda_1^{SP}$
325	$\langle u_\mu \nabla^\mu S\chi_+ P\rangle + \text{h.c.}$	$3NF^{-6}d_mc_d^3 + 6NF^{-6}c_d^3c_m\lambda_1^{SP} - NF^{-4}d_m\lambda_1^{PP}c_d - 4NF^{-4}d_m\lambda_2^{PP}c_d - NF^{-4}d_mc_d\lambda_1^{SS} - 2NF^{-4}d_mc_d\lambda_2^{SS} + 8NF^{-4}d_mc_d(\lambda_1^{SP})^2 - 3/2NF^{-4}d_mc_d - 2NF^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP} - 4NF^{-4}\lambda_2^{PP}c_dc_m\lambda_1^{SP} + 3NF^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} - 2NF^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP} - 4NF^{-4}c_dc_m\lambda_2^{SS}\lambda_1^{SP} - 2NF^{-4}c_dc_m\lambda_1^{SP} + 12NF^{-4}c_dc_m(\lambda_1^{SP})^3 - 4NF^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 1/2NF^{-4}c_d^2\lambda_1^{SP} - NF^{-4}c_d^2\lambda_2^{SP} - NF^{-2}\lambda_1^{PP}\lambda_3^{PP}\lambda_1^{SP} - 1/2NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 1/2NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - NF^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2NF^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} - 1/2NF^{-2}\lambda_3^{PP}\lambda_1^{SP} + 3/2NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^3 + 2NF^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - NF^{-2}\lambda_2^{SS}\lambda_1^{SP} + 2NF^{-2}\lambda_2^{SS}\lambda_2^{SP} + 3/2NF^{-2}\lambda_3^{SS}\lambda_1^{SP} - 2NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - 1/2NF^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} + 1/4NF^{-2}(\lambda_1^{SP})^3$
326	$\langle u_\mu \nabla^\mu SP\chi_+\rangle + \text{h.c.}$	$3NF^{-6}d_mc_d^3 + 6NF^{-6}c_d^3c_m\lambda_1^{SP} - 2NF^{-4}d_m\lambda_1^{PP}c_d - 4NF^{-4}d_m\lambda_2^{PP}c_d - NF^{-4}d_mc_d\lambda_1^{SS} - 2NF^{-4}d_mc_d\lambda_2^{SS} + 9NF^{-4}d_mc_d(\lambda_1^{SP})^2 - 5/4NF^{-4}d_mc_d - 2NF^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP} - 4NF^{-4}\lambda_2^{PP}c_dc_m\lambda_1^{SP} + 3NF^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} - 2NF^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP} - 4NF^{-4}c_dc_m\lambda_2^{SS}\lambda_1^{SP} - 2NF^{-4}c_dc_m\lambda_1^{SP} + 12NF^{-4}c_dc_m(\lambda_1^{SP})^3 - 2NF^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 2NF^{-4}c_d^2\lambda_2^{SP} - 1/2NF^{-2}\lambda_1^{PP}\lambda_1^{SP} - NF^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2NF^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} - 3/4NF^{-2}\lambda_3^{PP}\lambda_1^{SP} + 3/2NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^3 - 1/2NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + NF^{-2}\lambda_1^{SS}\lambda_2^{SP} + 4NF^{-2}\lambda_2^{SS}\lambda_3^{SS}\lambda_1^{SP} - NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - 1/8NF^{-2}\lambda_1^{SP} - NF^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} + 1/2NF^{-2}(\lambda_1^{SP})^3 + 3/4NF^{-2}\lambda_2^{SP}$

327	$\langle u_\mu \chi_+ \nabla^\mu P S \rangle + \text{h.c.}$	$2NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/4NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2NF^{-2}\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/16NF^{-2}\lambda_1^{\mathrm{SP}}$
328	$\langle u_\mu \nabla^\mu P \chi_+ S \rangle + \text{h.c.}$	$-2NF^{-4}d_m\lambda_1^{\mathrm{PP}}c_d + 2NF^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2 - 3/2NF^{-4}d_mc_d + 1/4NF^{-4}c_dc_m\lambda_1^{\mathrm{SP}} - NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2NF^{-4}c_d^2\lambda_1^{\mathrm{SP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} - NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/2NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2NF^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + NF^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/2NF^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 1/2NF^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}}$
329	$\langle u_\mu \nabla^\mu P S \chi_+ \rangle + \text{h.c.}$	$-4NF^{-4}d_m\lambda_2^{\mathrm{PP}}c_d + NF^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2 + 1/4NF^{-4}c_dc_m\lambda_1^{\mathrm{SP}} - NF^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2NF^{-4}c_d^2\lambda_1^{\mathrm{SP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + NF^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} + 2NF^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/2NF^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/4NF^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/2NF^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - NF^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 3/4NF^{-2}\lambda_2^{\mathrm{SP}}$
330	$\langle u_\mu \rangle \langle \nabla^\mu S \{P, \chi_+\} \rangle$	$-2F^{-6}d_mc_d^3 - 4F^{-6}c_d^3c_m\lambda_1^{\mathrm{SP}} + F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d - 2F^{-4}d_mc_d\lambda_1^{\mathrm{SS}} - 3F^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2 + 7/4F^{-4}d_mc_d - 2F^{-4}\lambda_3^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 4F^{-4}c_dc_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3F^{-4}c_dc_m\lambda_1^{\mathrm{SP}} - 4F^{-4}c_dc_m(\lambda_1^{\mathrm{SP}})^3 + 2F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-4}c_d^2\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 5/4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} - 3/2F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 1/16F^{-2}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 1/4F^{-2}(\lambda_1^{\mathrm{SP}})^3 - 3/4F^{-2}\lambda_2^{\mathrm{SP}}$
331	$\langle \chi_+ \nabla^\mu S \rangle \langle u_\mu P \rangle$	$2F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d + 4F^{-4}d_m\lambda_2^{\mathrm{PP}}c_d - 2F^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2 - 1/2F^{-4}d_mc_d + 4F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-4}c_d^2\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} - 3F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 3/8F^{-2}\lambda_1^{\mathrm{SP}} + F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^3 - 3/2F^{-2}\lambda_2^{\mathrm{SP}}$
332	$\langle P \rangle \langle \chi_+ \{u_\mu, \nabla^\mu S\} \rangle$	$-8F^{-6}c_d^3c_m\lambda_1^{\mathrm{SP}} - 3F^{-4}d_m\lambda_1^{\mathrm{PP}}c_d + 3F^{-4}d_mc_d(\lambda_1^{\mathrm{SP}})^2 + 5/4F^{-4}d_mc_d - 4F^{-4}\lambda_1^{\mathrm{PP}}c_dc_m\lambda_1^{\mathrm{SP}} + F^{-4}c_dc_m\lambda_1^{\mathrm{SP}} + 4F^{-4}c_dc_m(\lambda_1^{\mathrm{SP}})^3 - 6F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-4}c_d^2\lambda_1^{\mathrm{SP}} - 3F^{-4}c_d^2\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 1/4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_3^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^3 + 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 4F^{-2}\lambda_2^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 3/2F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 3/16F^{-2}\lambda_1^{\mathrm{SP}} - 3/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 3/4F^{-2}(\lambda_1^{\mathrm{SP}})^3 + 3/4F^{-2}\lambda_2^{\mathrm{SP}}$

333	$\langle P\nabla^\mu S\rangle\langle u_\mu\chi_+\rangle$	$2F^{-4}d_m\lambda_1^{\rm PP}c_d - 2F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 + 1/2F^{-4}d_mc_d + 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 2F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^3 + 4F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 3F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/8F^{-2}\lambda_1^{\rm SP} + F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2F^{-2}(\lambda_1^{\rm SP})^3 - 3/2F^{-2}\lambda_2^{\rm SP}$
334	$\langle P\{u_\mu, \nabla^\mu S\}\rangle\langle\chi_+\rangle$	$-3F^{-4}d_m\lambda_1^{\rm PP}c_d - 6F^{-4}d_m\lambda_2^{\rm PP}c_d + 11F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 5/4F^{-4}d_mc_d - 4F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP} - 8F^{-4}\lambda_2^{\rm PP}c_dc_m\lambda_1^{\rm SP} - F^{-4}c_dc_m\lambda_1^{\rm SP} + 12F^{-4}c_dc_m(\lambda_1^{\rm SP})^3 - 6F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 3F^{-4}c_d^2\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^3 + 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} + 4F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 3/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 3F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - 1/16F^{-2}\lambda_1^{\rm SP} - 3/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/4F^{-2}(\lambda_1^{\rm SP})^3 + 3/4F^{-2}\lambda_2^{\rm SP}$
335	$\langle P\chi_+\rangle\langle u_\mu\nabla^\mu S\rangle$	$12F^{-6}d_mc_d^3 + 8F^{-6}c_d^3c_m\lambda_1^{\rm SP} - 6F^{-4}d_m\lambda_1^{\rm PP}c_d - 4F^{-4}d_mc_d\lambda_1^{\rm SS} - 8F^{-4}d_mc_d\lambda_2^{\rm SS} + 18F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 1/2F^{-4}d_mc_d - 8F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP} + 12F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP} - 8F^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 16F^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 4F^{-4}c_dc_m\lambda_1^{\rm SP} + 32F^{-4}c_dc_m(\lambda_1^{\rm SP})^3 - 12F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_1^{\rm SP} - 6F^{-4}c_d^2\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 4F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 8F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 7/2F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 8F^{-2}\lambda_3^{\rm PP}(\lambda_1^{\rm SP})^3 + 4F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} + 8F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 4F^{-2}\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 3F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 6F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - 3/8F^{-2}\lambda_1^{\rm SP} - 3F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/2F^{-2}(\lambda_1^{\rm SP})^3 + 3/2F^{-2}\lambda_2^{\rm SP}$

336	$\langle u_\mu \{P,\chi_+\}\rangle\langle \nabla^\mu S\rangle$	$-2F^{-6}d_m c_d^3 - 4F^{-6}c_d^3 c_m\lambda_1^{\rm SP} + F^{-4}d_m\lambda_1^{\rm PP}c_d + 2F^{-4}d_m\lambda_2^{\rm PP}c_d - 2F^{-4}d_m c_d\lambda_1^{\rm SS} - 3F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 5/4F^{-4}d_m c_d - 2F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4F^{-4}c_d c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3F^{-4}c_d c_m\lambda_1^{\rm SP} - 4F^{-4}c_d c_m(\lambda_1^{\rm SP})^3 + 2F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-4}c_d^2\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - 2F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 7/4F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 3/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 + 3/16F^{-2}\lambda_1^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/4F^{-2}(\lambda_1^{\rm SP})^3 - 3/4F^{-2}\lambda_2^{\rm SP}$
337	$\langle u_\mu\rangle\langle \nabla^\mu P\{S,\chi_+\}\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d - F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 3/2F^{-4}d_m c_d - 1/2F^{-4}c_d c_m\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-4}c_d^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 7/4F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - 3/16F^{-2}\lambda_1^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3/4F^{-2}\lambda_2^{\rm SP}$
338	$\langle u_\mu \{S,\chi_+\}\rangle\langle \nabla^\mu P\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d - F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 3/2F^{-4}d_m c_d - 1/2F^{-4}c_d c_m\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-4}c_d^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/4F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 + 3/16F^{-2}\lambda_1^{\rm SP} + 1/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3/4F^{-2}\lambda_2^{\rm SP}$
339	$\langle u_\mu\chi_+\rangle\langle \nabla^\mu PS\rangle$	$-4F^{-4}d_m\lambda_1^{\rm PP}c_d - 2F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 3F^{-4}d_m c_d - F^{-4}c_d c_m\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 3/8F^{-2}\lambda_1^{\rm SP} + F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3/2F^{-2}\lambda_2^{\rm SP}$
340	$\langle \chi_+\nabla^\mu P\rangle\langle u_\mu S\rangle$	$-4F^{-4}d_m\lambda_1^{\rm PP}c_d - 2F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 3F^{-4}d_m c_d - F^{-4}c_d c_m\lambda_1^{\rm SP} + 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/8F^{-2}\lambda_1^{\rm SP} + F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3/2F^{-2}\lambda_2^{\rm SP}$
341	$\langle S\rangle\langle \chi_+\{u_\mu,\nabla^\mu P\}\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d - 4F^{-4}d_m\lambda_2^{\rm PP}c_d + 3F^{-4}d_m c_d(\lambda_1^{\rm SP})^2 - 3/2F^{-4}d_m c_d + 1/2F^{-4}c_d c_m\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-4}c_d^2\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 3/16F^{-2}\lambda_1^{\rm SP} - 3/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm SP}$

342	$\langle\, S\,\{u_\mu, \nabla^\mu P\}\,\rangle\langle\, \chi_+\,\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d - 4F^{-4}d_m\lambda_2^{\rm PP}c_d + 3F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 3/2F^{-4}d_mc_d + 1/2F^{-4}c_dc_m\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-4}c_d^2\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/16F^{-2}\lambda_1^{\rm SP} - 3/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm SP}$
343	$\langle\, S\chi_+\,\rangle\langle\, u_\mu\nabla^\mu P\,\rangle$	$-4F^{-4}d_m\lambda_1^{\rm PP}c_d - 8F^{-4}d_m\lambda_2^{\rm PP}c_d + 6F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 3F^{-4}d_mc_d + F^{-4}c_dc_m\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_1^{\rm SP} + 4F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} + 8F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 4F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 6F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - 3/8F^{-2}\lambda_1^{\rm SP} - 3F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/2F^{-2}\lambda_2^{\rm SP}$
344	$i\,\langle\, \{S, P\}\, u_\mu\chi_-u^\mu\,\rangle$	$2NF^{-6}c_d^3c_m\lambda_1^{\rm SP} - NF^{-6}c_d^4\lambda_1^{\rm SP} - NF^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 4NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_2^{\rm SP} - NF^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/4NF^{-4}c_dc_m\lambda_1^{\rm SP} + 1/2NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 5/8NF^{-4}c_d^2\lambda_1^{\rm SP} - 2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 4NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 - NF^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/4NF^{-2}(\lambda_1^{\rm SP})^3$
345	$i\,\langle\, Su\cdot u\chi_-P\,\rangle + \mathrm{h.c.}$	$-1/2NF^{-6}d_mc_d^3 - 2NF^{-6}c_d^3c_m\lambda_1^{\rm SP} + 2NF^{-6}c_d^4\lambda_2^{\rm SP} + 4NF^{-4}d_mc_d\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 - 1/2NF^{-4}d_mc_d\lambda_1^{\rm SS} + 1/2NF^{-4}d_mc_d\lambda_2^{\rm SS} - 1/8NF^{-4}d_mc_d + NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 3NF^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2NF^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/2NF^{-4}c_dc_m\lambda_1^{\rm SP} - 2NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 2NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 3NF^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 3/4NF^{-4}c_d^2\lambda_2^{\rm SP} + 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/8NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 5/2NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 - 1/2NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 3NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/8NF^{-2}\lambda_2^{\rm SP}$

346	$i\langle Su\cdot uP\chi_-\rangle+\text{h.c.}$	$-12NF^{-6}d_mc_d^3(\lambda_1^{SP})^2 - 1/2NF^{-6}d_mc_d^3 - 19NF^{-6}c_d^3c_m\lambda_1^{SP} + 11NF^{-6}c_d^4\lambda_1^{SP} + 2NF^{-6}c_d^4\lambda_2^{SP} + 6NF^{-4}d_mc_d\lambda_1^{SS}(\lambda_1^{SP})^2 - 1/2NF^{-4}d_mc_d\lambda_1^{SS} + 4NF^{-4}d_mc_d\lambda_2^{SS}(\lambda_1^{SP})^2 + 1/2NF^{-4}d_mc_d\lambda_2^{SS} + 1/2NF^{-4}d_mc_d(\lambda_1^{SP})^2 - 1/8NF^{-4}d_mc_d - 3NF^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP} + 5/2NF^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - NF^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} + 2NF^{-4}\lambda_2^{PP}c_dc_m\lambda_1^{SP} - 2NF^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - NF^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} + 7NF^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP} + 6NF^{-4}c_dc_m\lambda_2^{SS}\lambda_1^{SP} + 5/4NF^{-4}c_dc_m\lambda_1^{SP} - 4NF^{-4}c_dc_m(\lambda_1^{SP})^3 - 13/2NF^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + NF^{-4}c_d^2\lambda_1^{SS}\lambda_2^{SP} - 5NF^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 2NF^{-4}c_d^2\lambda_2^{SS}\lambda_2^{SP} + 3NF^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 11/8NF^{-4}c_d^2\lambda_1^{SP} - NF^{-4}c_d^2(\lambda_1^{SP})^2\lambda_2^{SP} + 11/2NF^{-4}c_d^2(\lambda_1^{SP})^3 + 1/2NF^{-4}c_d^2\lambda_2^{SP} + 1/4NF^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - NF^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_2^{SP} - 3/2NF^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + NF^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} - 3/16NF^{-2}\lambda_1^{PP}\lambda_1^{SP} + 2NF^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} - 1/4NF^{-2}\lambda_1^{PP}\lambda_2^{SP} + 3/2NF^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - NF^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/8NF^{-2}\lambda_2^{PP}\lambda_1^{SP} - NF^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} - NF^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} + NF^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} - 2NF^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} + 9/16NF^{-2}\lambda_1^{SS}\lambda_1^{SP} + 6NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 11/4NF^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/4NF^{-2}\lambda_1^{SS}\lambda_2^{SP} + NF^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} - 1/8NF^{-2}\lambda_2^{SS}\lambda_1^{SP} - NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} + 1/2NF^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 + 1/2NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 + 1/64NF^{-2}\lambda_1^{SP} - 1/16NF^{-2}(\lambda_1^{SP})^3 + 1/2NF^{-2}(\lambda_1^{SP})^4\lambda_2^{SP} - 1/16NF^{-2}\lambda_2^{SP}$

347	$i\,\langle\, S\chi_- u\cdot uP\,\rangle + \text{h.c.}$	$-12NF^{-6}d_m c_d^3(\lambda_1^{\rm SP})^2 - 19NF^{-6}c_d^3 c_m\lambda_1^{\rm SP} + 12NF^{-6}c_d^4\lambda_1^{\rm SP} + 2NF^{-4}d_m c_d\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 4NF^{-4}d_m c_d\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 3/2NF^{-4}d_m c_d(\lambda_1^{\rm SP})^2 - 3NF^{-4}\lambda_1^{\rm PP}c_d c_m\lambda_1^{\rm SP} + 2NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_2^{\rm SP} + 4NF^{-4}\lambda_2^{\rm PP}c_d c_m\lambda_1^{\rm SP} - 3NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} + 8NF^{-4}c_d c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4NF^{-4}c_d c_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 9/4NF^{-4}c_d c_m\lambda_1^{\rm SP} - 5NF^{-4}c_d c_m(\lambda_1^{\rm SP})^3 - 13/2NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 5NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 3NF^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 13/8NF^{-4}c_d^2\lambda_1^{\rm SP} - 2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 13/2NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 1/4NF^{-4}c_d^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/8NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} + 2NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} + NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 2NF^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/16NF^{-2}\lambda_1^{\rm SP} + NF^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/4NF^{-2}(\lambda_1^{\rm SP})^3 + 1/2NF^{-2}(\lambda_1^{\rm SP})^4\lambda_2^{\rm SP} + 1/8NF^{-2}\lambda_2^{\rm SP}$
348	$i\,\langle\, u_\nu S u^\nu\,\{P,\chi_-\}\,\rangle$	$-2NF^{-6}c_d^3 c_m\lambda_1^{\rm SP} + NF^{-6}c_d^4\lambda_1^{\rm SP} + 1/2NF^{-4}d_m c_d\lambda_1^{\rm SS} + 8NF^{-4}d_m c_d\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 - NF^{-4}d_m c_d\lambda_2^{\rm SS} - NF^{-4}d_m c_d(\lambda_1^{\rm SP})^2 + 1/8NF^{-4}d_m c_d - 1/2NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} + 2NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_2^{\rm SP} + 2NF^{-4}c_d c_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8NF^{-4}c_d c_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/4NF^{-4}c_d c_m\lambda_1^{\rm SP} - 2NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 6NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 4NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 3/4NF^{-4}c_d^2\lambda_1^{\rm SP} - NF^{-4}c_d^2\lambda_2^{\rm SP} - 1/4NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 3/2NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_2^{\rm SP} - 3/16NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - 1/2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/8NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 1/2NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 3/8NF^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} - 1/16NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 3/4NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/8NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 16NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 5NF^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 1/2NF^{-2}\lambda_2^{\rm SS}\lambda_2^{\rm SP} - 3/64NF^{-2}\lambda_1^{\rm SP} - 5/2NF^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 13/16NF^{-2}(\lambda_1^{\rm SP})^3$

349	$i\langle u_\nu P u^\nu \{S,\chi_-\}\rangle$	$4NF^{-6}c_d^3c_m\lambda_1^{\rm SP} - 3NF^{-6}c_d^4\lambda_1^{\rm SP} + NF^{-4}d_mc_d(\lambda_1^{\rm SP})^2 + 2NF^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP} - 3/2NF^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 6NF^{-4}\lambda_2^{\rm PP}c_dc_m\lambda_1^{\rm SP} + 4NF^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP} - NF^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 3NF^{-4}c_dc_m\lambda_1^{\rm SP} - NF^{-4}c_dc_m(\lambda_1^{\rm SP})^3 + NF^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 2NF^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - NF^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 7/4NF^{-4}c_d^2\lambda_1^{\rm SP} - 2NF^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + NF^{-4}c_d^2(\lambda_1^{\rm SP})^3 + 1/4NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 1/2NF^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 1/16NF^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - NF^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 2NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2NF^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 3/8NF^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} + 4NF^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2NF^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP} + 3/16NF^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 1/4NF^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3 + 3/8NF^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/8NF^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/64NF^{-2}\lambda_1^{\rm SP} - 3/16NF^{-2}(\lambda_1^{\rm SP})^3$
350	$i\langle u_\mu\rangle\langle u^\mu S\chi_- P\rangle + \text{h.c.}$	$-F^{-6}d_mc_d^3 - 4F^{-6}c_d^3c_m\lambda_1^{\rm SP} + 4F^{-6}c_d^4\lambda_2^{\rm SP} + 1/2F^{-4}d_mc_d\lambda_1^{\rm SS} + 1/2F^{-4}d_mc_d\lambda_2^{\rm SS} + 1/4F^{-4}d_mc_d + 2F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP} - F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP} - 2F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP} + 3F^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 6F^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5/4F^{-4}c_dc_m\lambda_1^{\rm SP} + F^{-4}c_dc_m(\lambda_1^{\rm SP})^3 - 4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 5F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_2^{\rm SP} + F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SP} + 4F^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 2F^{-4}c_d^2(\lambda_1^{\rm SP})^3 - 3/2F^{-4}c_d^2\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP} - F^{-2}\lambda_2^{\rm PP}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/8F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 3/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} - 4F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 3/4F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP} - 5F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3 + 3/8F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - 1/32F^{-2}\lambda_1^{\rm SP} - F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/8F^{-2}(\lambda_1^{\rm SP})^3 + F^{-2}(\lambda_1^{\rm SP})^4\lambda_2^{\rm SP} + 1/8F^{-2}\lambda_2^{\rm SP}$

351	$i\langle u_\mu \rangle \langle u^\mu \chi_- PS \rangle + \text{h.c.}$	$-F^{-6} d_m c_d^3 - 4F^{-6} c_d^3 c_m \lambda_1^{\text{SP}} + 4F^{-6} c_d^4 \lambda_2^{\text{SP}} + 1/2F^{-4} d_m c_d \lambda_1^{\text{SS}} + 1/2F^{-4} d_m c_d \lambda_2^{\text{SS}} + 1/4F^{-4} d_m c_d - 1/2F^{-4} \lambda_1^{\text{PP}} c_d^2 \lambda_1^{\text{SP}} + 2F^{-4} \lambda_1^{\text{PP}} c_d^2 \lambda_2^{\text{SP}} + 2F^{-4} \lambda_2^{\text{PP}} c_d^2 \lambda_1^{\text{SP}} + 3F^{-4} c_d c_m \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + 2F^{-4} c_d c_m \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} + F^{-4} c_d c_m \lambda_1^{\text{SP}} + 1/2F^{-4} c_d^2 \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} - 4F^{-4} c_d^2 \lambda_1^{\text{SS}} \lambda_2^{\text{SP}} - 2F^{-4} c_d^2 \lambda_2^{\text{SS}} \lambda_2^{\text{SP}} - 1/8F^{-4} c_d^2 \lambda_1^{\text{SP}} + 4F^{-4} c_d^2 (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - 3/2F^{-4} c_d^2 (\lambda_1^{\text{SP}})^3 - 1/4F^{-2} \lambda_1^{\text{PP}} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + 1/2F^{-2} \lambda_1^{\text{PP}} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} - 2F^{-2} \lambda_1^{\text{PP}} \lambda_2^{\text{SS}} \lambda_2^{\text{SP}} - 3/16F^{-2} \lambda_1^{\text{PP}} \lambda_1^{\text{SP}} - 1/2F^{-2} \lambda_2^{\text{PP}} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} - F^{-2} \lambda_2^{\text{PP}} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} + 3/8F^{-2} \lambda_2^{\text{PP}} \lambda_1^{\text{SP}} + 1/2F^{-2} \lambda_3^{\text{PP}} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + 3/8F^{-2} \lambda_3^{\text{PP}} \lambda_1^{\text{SP}} - 9/16F^{-2} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} - 2F^{-2} \lambda_1^{\text{SS}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} + 3/4F^{-2} \lambda_1^{\text{SS}} (\lambda_1^{\text{SP}})^3 + F^{-2} \lambda_1^{\text{SS}} \lambda_2^{\text{SP}} + 1/8F^{-2} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} - F^{-2} \lambda_2^{\text{SS}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} + 1/2F^{-2} \lambda_2^{\text{SS}} (\lambda_1^{\text{SP}})^3 - 1/2F^{-2} \lambda_2^{\text{SS}} \lambda_2^{\text{SP}} - 3/64F^{-2} \lambda_1^{\text{SP}} + 3/4F^{-2} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - 3/16F^{-2} (\lambda_1^{\text{SP}})^3 - 1/4F^{-2} \lambda_2^{\text{SP}}$
352	$i\langle u_\mu \rangle \langle u^\mu \chi_- SP \rangle + \text{h.c.}$	$2F^{-4} \lambda_1^{\text{PP}} c_d c_m \lambda_1^{\text{SP}} - 3/2F^{-4} \lambda_1^{\text{PP}} c_d^2 \lambda_1^{\text{SP}} - 2F^{-4} \lambda_1^{\text{PP}} c_d^2 \lambda_2^{\text{SP}} + 2F^{-4} \lambda_2^{\text{PP}} c_d c_m \lambda_1^{\text{SP}} - 2F^{-4} \lambda_2^{\text{PP}} c_d^2 \lambda_1^{\text{SP}} + 4F^{-4} \lambda_2^{\text{PP}} c_d^2 \lambda_2^{\text{SP}} - 2F^{-4} c_d c_m \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + 12F^{-4} c_d c_m \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} - 9/4F^{-4} c_d c_m \lambda_1^{\text{SP}} - F^{-4} c_d c_m (\lambda_1^{\text{SP}})^3 + 1/2F^{-4} c_d^2 \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} - 9F^{-4} c_d^2 \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} + F^{-4} c_d^2 \lambda_3^{\text{SS}} \lambda_1^{\text{SP}} + 15/8F^{-4} c_d^2 \lambda_1^{\text{SP}} + 1/2F^{-4} c_d^2 (\lambda_1^{\text{SP}})^3 + 1/2F^{-4} c_d^2 \lambda_2^{\text{SP}} + 1/4F^{-2} \lambda_1^{\text{PP}} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + 1/2F^{-2} \lambda_1^{\text{PP}} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} - 1/2F^{-2} \lambda_1^{\text{PP}} \lambda_3^{\text{SS}} \lambda_1^{\text{SP}} + 1/16F^{-2} \lambda_1^{\text{PP}} \lambda_1^{\text{SP}} - F^{-2} \lambda_1^{\text{PP}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} + F^{-2} \lambda_1^{\text{PP}} \lambda_2^{\text{SP}} + 1/2F^{-2} \lambda_2^{\text{PP}} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} - 2F^{-2} \lambda_2^{\text{PP}} \lambda_1^{\text{SS}} \lambda_2^{\text{SP}} + F^{-2} \lambda_2^{\text{PP}} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} + 1/8F^{-2} \lambda_2^{\text{PP}} \lambda_1^{\text{SP}} - F^{-2} \lambda_2^{\text{PP}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - 1/2F^{-2} \lambda_2^{\text{PP}} \lambda_2^{\text{SP}} + 3/16F^{-2} \lambda_1^{\text{SS}} \lambda_1^{\text{SP}} + F^{-2} \lambda_1^{\text{SS}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - 1/4F^{-2} \lambda_1^{\text{SS}} (\lambda_1^{\text{SP}})^3 - 3/8F^{-2} \lambda_2^{\text{SS}} \lambda_1^{\text{SP}} - 4F^{-2} \lambda_2^{\text{SS}} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - F^{-2} \lambda_2^{\text{SS}} (\lambda_1^{\text{SP}})^3 + 4F^{-2} (\lambda_2^{\text{SS}})^2 \lambda_1^{\text{SP}} - 3/8F^{-2} \lambda_3^{\text{SS}} \lambda_1^{\text{SP}} - 5/64F^{-2} \lambda_1^{\text{SP}} + 1/4F^{-2} (\lambda_1^{\text{SP}})^2 \lambda_2^{\text{SP}} - 1/16F^{-2} (\lambda_1^{\text{SP}})^3 + F^{-2} (\lambda_1^{\text{SP}})^4 \lambda_2^{\text{SP}} - 1/4F^{-2} \lambda_2^{\text{SP}}$

353	$i\langle u_\mu \chi_- \rangle \langle u^\mu \{S,P\}\rangle$	$-2F^{-6}d_m c_d^3 - 8F^{-6}c_d^3 c_m \lambda_1^{SP} + 4F^{-6}c_d^4\lambda_1^{SP} + F^{-4}d_m c_d \lambda_1^{SS} + F^{-4}d_m c_d \lambda_2^{SS} + 1/2F^{-4}d_m c_d + 2F^{-4}\lambda_1^{PP}c_d c_m \lambda_1^{SP} - F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} + 2F^{-4}\lambda_2^{PP}c_d c_m\lambda_1^{SP} - 2F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_2^{PP}c_d^2\lambda_2^{SP} - 2F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}c_d c_m \lambda_1^{SS}\lambda_1^{SP} + 12F^{-4}c_d c_m \lambda_2^{SS}\lambda_1^{SP} - 2F^{-4}c_d c_m(\lambda_1^{SP})^3 - 3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 6F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-4}c_d^2\lambda_1^{SP} + F^{-4}c_d^2(\lambda_1^{SP})^3 + F^{-4}c_d^2\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_2^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_2^{SP} - F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} - 1/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} + F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_2^{SP} - F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_2^{PP}\lambda_2^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} + F^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_1^{SP} + 1/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 4F^{-2}\lambda_2^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_2^{SS}\lambda_2^{SP} + F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 + 3/32F^{-2}\lambda_1^{SP} + 1/8F^{-2}(\lambda_1^{SP})^3 - 3/8F^{-2}\lambda_2^{SP}$
354	$i\langle u_\mu P\rangle\langle u^\mu\{S,\chi_-\}\rangle$	$-2F^{-6}d_m c_d^3 - 12F^{-6}c_d^3 c_m\lambda_1^{SP} + 8F^{-6}c_d^4\lambda_1^{SP} + F^{-4}d_m c_d\lambda_1^{SS} + F^{-4}d_m c_d\lambda_2^{SS} + 1/2F^{-4}d_m c_d + 4F^{-4}\lambda_1^{PP}c_d c_m\lambda_1^{SP} - 3F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} + 6F^{-4}\lambda_2^{PP}c_d c_m\lambda_1^{SP} - 6F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} - 2F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} + 20F^{-4}c_d c_m\lambda_2^{SS}\lambda_1^{SP} + 1/2F^{-4}c_d c_m\lambda_1^{SP} - 4F^{-4}c_d c_m(\lambda_1^{SP})^3 - 3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 16F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 3/4F^{-4}c_d^2\lambda_1^{SP} + 4F^{-4}c_d^2(\lambda_1^{SP})^2\lambda_2^{SP} + 3F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_2^{SP} + F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_2^{SP} - F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} + F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_2^{SP} + 2F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_2^{SP} - F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} + 5/2F^{-2}\lambda_2^{PP}\lambda_2^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} + F^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_1^{SP} + 1/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 4F^{-2}\lambda_2^{SS}\lambda_3^{SS}\lambda_1^{SP} + 1/2F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 8F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 + 1/2F^{-2}\lambda_2^{SS}\lambda_2^{SP} + 4F^{-2}(\lambda_2^{SS})^2\lambda_1^{SP} + F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - 3/32F^{-2}\lambda_1^{SP} - 1/8F^{-2}(\lambda_1^{SP})^3 + 2F^{-2}(\lambda_1^{SP})^4\lambda_2^{SP} - 1/8F^{-2}\lambda_2^{SP}$

355	$i\langle u_\mu S\rangle\langle u^\mu\{P,\chi_-\}\rangle$	$-2F^{-6}d_mc_d^3-8F^{-6}c_d^3c_m\lambda_1^{\rm SP}+8F^{-6}c_d^4\lambda_2^{\rm SP}+F^{-4}d_mc_d\lambda_1^{\rm SS}+F^{-4}d_mc_d\lambda_2^{\rm SS}+1/2F^{-4}d_mc_d+2F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP}-F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_2^{\rm SP}+2F^{-4}\lambda_2^{\rm PP}c_dc_m\lambda_1^{\rm SP}+4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_2^{\rm SP}-2F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP}+12F^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP}-2F^{-4}c_dc_m(\lambda_1^{\rm SP})^3-F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}-4F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_2^{\rm SP}-2F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}-8F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_2^{\rm SP}+2F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP}+5/4F^{-4}c_d^2\lambda_1^{\rm SP}+4F^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}-F^{-4}c_d^2(\lambda_1^{\rm SP})^3-2F^{-4}c_d^2\lambda_2^{\rm SP}+1/2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_2^{\rm SP}-2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_2^{\rm SP}-F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP}-1/8F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}-2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_2^{\rm SP}-4F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_2^{\rm SP}-F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP}-1/4F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}+1/2F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP}+F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-1/8F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}+1/2F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3-4F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP}-7/4F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}+F^{-2}\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^3+5/2F^{-2}\lambda_2^{\rm SS}\lambda_2^{\rm SP}+F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3+5/32F^{-2}\lambda_1^{\rm SP}+1/8F^{-2}(\lambda_1^{\rm SP})^3-1/8F^{-2}\lambda_2^{\rm SP}$
356	$i\langle\chi_-\rangle\langle u\cdot u\{S,P\}\rangle$	$-24F^{-6}d_mc_d^3(\lambda_1^{\rm SP})^2-F^{-6}d_mc_d^3-28F^{-6}c_d^3c_m\lambda_1^{\rm SP}+14F^{-6}c_d^4\lambda_1^{\rm SP}+12F^{-4}d_mc_d\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2-F^{-4}d_mc_d\lambda_1^{\rm SS}+8F^{-4}d_mc_d\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2+F^{-4}d_mc_d\lambda_2^{\rm SS}+F^{-4}d_mc_d(\lambda_1^{\rm SP})^2-1/4F^{-4}d_mc_d-4F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP}+5/2F^{-4}\lambda_1^{\rm PP}c_d^2\lambda_1^{\rm SP}+2F^{-4}\lambda_2^{\rm PP}c_dc_m\lambda_1^{\rm SP}-2F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_1^{\rm SP}+4F^{-4}\lambda_2^{\rm PP}c_d^2\lambda_2^{\rm SP}-F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP}+13F^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP}+10F^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP}+3/4F^{-4}c_dc_m\lambda_1^{\rm SP}-F^{-4}c_dc_m(\lambda_1^{\rm SP})^3-19/2F^{-4}c_d^2\lambda_1^{\rm SS}\lambda_1^{\rm SP}-5F^{-4}c_d^2\lambda_2^{\rm SS}\lambda_1^{\rm SP}+6F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP}+1/8F^{-4}c_d^2\lambda_1^{\rm SP}-16F^{-4}c_d^2(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}+9/2F^{-4}c_d^2(\lambda_1^{\rm SP})^3+1/2F^{-4}c_d^2\lambda_2^{\rm SP}+1/4F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}-F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SS}\lambda_2^{\rm SP}-1/2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+2F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP}+3/16F^{-2}\lambda_1^{\rm PP}\lambda_1^{\rm SP}+1/4F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP}+1/2F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-4F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SS}\lambda_2^{\rm SP}-F^{-2}\lambda_2^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP}-1/8F^{-2}\lambda_2^{\rm PP}\lambda_1^{\rm SP}-2F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SS}\lambda_1^{\rm SP}-2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP}-9/16F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP}+12F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}-13/4F^{-2}\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^3+1/4F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP}+F^{-2}(\lambda_1^{\rm SS})^2\lambda_1^{\rm SP}+1/8F^{-2}\lambda_2^{\rm SS}\lambda_1^{\rm SP}+1/2F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3-3/64F^{-2}\lambda_1^{\rm SP}+1/16F^{-2}(\lambda_1^{\rm SP})^3+3/16F^{-2}\lambda_2^{\rm SP}$

357	$i\langle \chi_- \{S,P\}\rangle\langle u\cdot u\rangle$	$-F^{-6}d_m c_d^3 - 4F^{-6}c_d^3 c_m\lambda_1^{\mathrm{SP}} + 4F^{-6}c_d^4\lambda_2^{\mathrm{SP}} + 1/2F^{-4}d_m c_d\lambda_1^{\mathrm{SS}} - 3/8F^{-4}d_m c_d + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} - 7/2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 2F^{-4}\lambda_2^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} - F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - F^{-4}\lambda_3^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 10F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-4}c_d c_m\lambda_1^{\mathrm{SP}} - 6F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^3 - 11/2F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} - F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 7/8F^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 9/2F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 + 5/2F^{-4}c_d^2\lambda_2^{\mathrm{SP}} + 1/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 5/16F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 5/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} - 1/2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/8F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 9/16F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 5F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 1/4F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 5/4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + F^{-2}(\lambda_1^{\mathrm{SS}})^2\lambda_1^{\mathrm{SP}} - 3/8F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 3/8F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 5/64F^{-2}\lambda_1^{\mathrm{SP}} + 5/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 11/16F^{-2}(\lambda_1^{\mathrm{SP}})^3 + F^{-2}(\lambda_1^{\mathrm{SP}})^4\lambda_2^{\mathrm{SP}} + 3/16F^{-2}\lambda_2^{\mathrm{SP}}$
358	$i\langle S\rangle\langle P\{\chi_-, u\cdot u\}\rangle$	$-8F^{-6}d_m c_d^3(\lambda_1^{\mathrm{SP}})^2 - F^{-6}d_m c_d^3 - 16F^{-6}c_d^3 c_m\lambda_1^{\mathrm{SP}} + 6F^{-6}c_d^4\lambda_1^{\mathrm{SP}} + 4F^{-6}c_d^4\lambda_2^{\mathrm{SP}} + 4F^{-4}d_m c_d\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2 + 1/2F^{-4}d_m c_d\lambda_1^{\mathrm{SS}} - F^{-4}d_m c_d(\lambda_1^{\mathrm{SP}})^2 - 3/8F^{-4}d_m c_d - 4F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} + 2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 2F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SP}} + 2F^{-4}\lambda_2^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} - F^{-4}\lambda_3^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 12F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 5/4F^{-4}c_d c_m\lambda_1^{\mathrm{SP}} - F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^3 - 8F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 6F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/4F^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 + F^{-4}c_d^2\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 1/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/8F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/8F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 1/2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 1/4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + F^{-2}(\lambda_1^{\mathrm{SS}})^2\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 1/32F^{-2}\lambda_1^{\mathrm{SP}} + 3/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^3 + 1/16F^{-2}\lambda_2^{\mathrm{SP}}$

359	$i\langle S\{\chi_-, u\cdot u\}\rangle\langle P\rangle$	$-F^{-6}d_m c_d^3 - 2F^{-6}c_d^3 c_m\lambda_1^{\mathrm{SP}} - F^{-4}d_m c_d\lambda_1^{\mathrm{SS}} + F^{-4}d_m c_d\lambda_2^{\mathrm{SS}} - 1/4F^{-4}d_m c_d + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} - 3F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SP}} - F^{-4}\lambda_3^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 5F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-4}c_d c_m\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/2F^{-4}c_d c_m\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^3 - 5F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 3F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 + 3/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} - 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 1/4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_3^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/8F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 3/2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 1/4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + F^{-2}(\lambda_1^{\mathrm{SS}})^2\lambda_1^{\mathrm{SP}} - 1/4F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 3/8F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 + 1/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + F^{-2}(\lambda_1^{\mathrm{SP}})^4\lambda_2^{\mathrm{SP}} + 1/16F^{-2}\lambda_2^{\mathrm{SP}}$
360	$i\langle Su\cdot u\rangle\langle P\chi_-\rangle$	$-2F^{-6}d_m c_d^3 - 8F^{-6}c_d^3 c_m\lambda_1^{\mathrm{SP}} + 8F^{-6}c_d^4\lambda_2^{\mathrm{SP}} - F^{-4}d_m c_d\lambda_1^{\mathrm{SS}} - 1/4F^{-4}d_m c_d + 2F^{-4}\lambda_1^{\mathrm{PP}}c_d c_m\lambda_1^{\mathrm{SP}} - F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}\lambda_1^{\mathrm{PP}}c_d^2\lambda_2^{\mathrm{SP}} + 2F^{-4}\lambda_2^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} - 2F^{-4}\lambda_3^{\mathrm{PP}}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d c_m\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/2F^{-4}c_d c_m\lambda_1^{\mathrm{SP}} - 2F^{-4}c_d c_m(\lambda_1^{\mathrm{SP}})^3 - 5F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d^2\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 2F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4F^{-4}c_d^2\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 2F^{-4}c_d^2\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 1/4F^{-4}c_d^2\lambda_1^{\mathrm{SP}} + 4F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - F^{-4}c_d^2(\lambda_1^{\mathrm{SP}})^3 + 2F^{-4}c_d^2\lambda_2^{\mathrm{SP}} + 5/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 4F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} - F^{-2}\lambda_1^{\mathrm{PP}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 5/8F^{-2}\lambda_1^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} + 1/2F^{-2}\lambda_1^{\mathrm{PP}}\lambda_2^{\mathrm{SP}} + 3F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 2F^{-2}\lambda_2^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/4F^{-2}\lambda_2^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 3F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 2F^{-2}\lambda_3^{\mathrm{PP}}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 3/4F^{-2}\lambda_3^{\mathrm{PP}}\lambda_1^{\mathrm{SP}} - 4F^{-2}\lambda_1^{\mathrm{SS}}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} - 3/8F^{-2}\lambda_1^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 14F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 11/2F^{-2}\lambda_1^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 1/2F^{-2}\lambda_1^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 2F^{-2}(\lambda_1^{\mathrm{SS}})^2\lambda_1^{\mathrm{SP}} + 3/2F^{-2}\lambda_2^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + 12F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} - 4F^{-2}\lambda_2^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 3F^{-2}\lambda_2^{\mathrm{SS}}\lambda_2^{\mathrm{SP}} + 3/4F^{-2}\lambda_3^{\mathrm{SS}}\lambda_1^{\mathrm{SP}} + F^{-2}\lambda_3^{\mathrm{SS}}(\lambda_1^{\mathrm{SP}})^3 - 5/32F^{-2}\lambda_1^{\mathrm{SP}} - 7/2F^{-2}(\lambda_1^{\mathrm{SP}})^2\lambda_2^{\mathrm{SP}} + 11/8F^{-2}(\lambda_1^{\mathrm{SP}})^3 + 1/8F^{-2}\lambda_2^{\mathrm{SP}}$

361	$i\langle S\chi_-\rangle\langle Pu\cdot u\rangle$	$-16F^{-6}d_m c_d^3(\lambda_1^{SP})^2 - 2F^{-6}d_m c_d^3 - 36F^{-6}c_d^3 c_m \lambda_1^{SP} + 24F^{-6}c_d^4\lambda_1^{SP} + 8F^{-4}d_m c_d \lambda_1^{SS}(\lambda_1^{SP})^2 + F^{-4}d_m c_d\lambda_1^{SS} - 2F^{-4}d_m c_d(\lambda_1^{SP})^2 - 3/4F^{-4}d_m c_d - 8F^{-4}\lambda_1^{PP}c_d c_m\lambda_1^{SP} + 5F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} - 4F^{-4}\lambda_2^{PP}c_d c_m\lambda_1^{SP} + 4F^{-4}\lambda_2^{PP}c_d^2\lambda_2^{SP} - 2F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} + 28F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} - F^{-4}c_d c_m\lambda_1^{SP} - 16F^{-4}c_d c_m(\lambda_1^{SP})^3 - 23F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 10F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} + 10F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 3/4F^{-4}c_d^2\lambda_1^{SP} + 20F^{-4}c_d^2(\lambda_1^{SP})^2\lambda_2^{SP} + 15F^{-4}c_d^2(\lambda_1^{SP})^3 - F^{-4}c_d^2\lambda_2^{SP} - 3/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_2^{SP} - 3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} + 3F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} + 6F^{-2}\lambda_1^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{PP}\lambda_2^{SP} - 4F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_2^{SP} - 2F^{-2}\lambda_2^{PP}\lambda_2^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} + 4F^{-2}\lambda_2^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} - 3F^{-2}\lambda_2^{PP}\lambda_2^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 3/4F^{-2}\lambda_3^{PP}\lambda_1^{SP} + 4F^{-2}\lambda_1^{SS}\lambda_2^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} + 7/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 8F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 + 1/2F^{-2}\lambda_1^{SS}\lambda_2^{SP} + 2F^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} + 3/4F^{-2}\lambda_2^{SS}\lambda_1^{SP} - F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - 3/4F^{-2}\lambda_3^{SS}\lambda_1^{SP} + F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 + 3/32F^{-2}\lambda_1^{SP} + 1/2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} - 5/8F^{-2}(\lambda_1^{SP})^3 + 2F^{-2}(\lambda_1^{SP})^4\lambda_2^{SP} + 1/8F^{-2}\lambda_2^{SP}$
362	$i\langle SP\rangle\langle\chi_-u\cdot u\rangle$	$-2F^{-6}d_m c_d^3 - 8F^{-6}c_d^3c_m\lambda_1^{SP} + 4F^{-6}c_d^4\lambda_1^{SP} + F^{-4}d_m c_d\lambda_1^{SS} - 3/4F^{-4}d_m c_d + 2F^{-4}\lambda_1^{PP}c_d c_m\lambda_1^{SP} - F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} - 2F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} + 12F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} - 5/2F^{-4}c_d c_m\lambda_1^{SP} - 2F^{-4}c_d c_m(\lambda_1^{SP})^3 - 7F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} + 3/4F^{-4}c_d^2\lambda_1^{SP} + F^{-4}c_d^2(\lambda_1^{SP})^3 + 2F^{-4}c_d^2\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_2^{SP} - F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} + 3/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{PP}\lambda_2^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 3/4F^{-2}\lambda_3^{PP}\lambda_1^{SP} - 4F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - 5/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/2F^{-2}\lambda_1^{SS}\lambda_2^{SP} + 2F^{-2}(\lambda_1^{SS})^2\lambda_1^{SP} + 3/4F^{-2}\lambda_3^{SS}\lambda_1^{SP} + F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - 3/32F^{-2}\lambda_1^{SP} + 2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} - 3/8F^{-2}(\lambda_1^{SP})^3 + 3/8F^{-2}\lambda_2^{SP}$

363	$i\langle u_\nu S u^\nu \chi_-\rangle\langle P\rangle$	$F^{-4}d_m c_d\lambda_1^{SS} - 2F^{-4}d_m c_d\lambda_2^{SS} + 1/4F^{-4}d_m c_d - F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} + 2F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} - 4F^{-4}c_d c_m\lambda_2^{SS}\lambda_1^{SP} + F^{-4}c_d c_m\lambda_1^{SP} - F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} + 2F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 3/4F^{-4}c_d^2\lambda_1^{SP} - 4F^{-4}c_d^2(\lambda_1^{SP})^2\lambda_2^{SP} + F^{-4}c_d^2(\lambda_1^{SP})^3 + F^{-4}c_d^2\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} + 3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_2^{SP} - 3/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} + 3/4F^{-2}\lambda_3^{PP}\lambda_1^{SP} + 1/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} + 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 1/4F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 12F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 3F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 - F^{-2}\lambda_2^{SS}\lambda_2^{SP} + 3/32F^{-2}\lambda_1^{SP} - 3/2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} + 3/8F^{-2}(\lambda_1^{SP})^3$
364	$i\langle u_\nu S u^\nu P\rangle\langle \chi_-\rangle$	$F^{-4}d_m c_d\lambda_1^{SS} + 16F^{-4}d_m c_d\lambda_2^{SS}(\lambda_1^{SP})^2 - 2F^{-4}d_m c_d\lambda_2^{SS} + 1/4F^{-4}d_m c_d + 2F^{-4}\lambda_1^{PP}c_d c_m\lambda_1^{SP} - 2F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_d^2\lambda_2^{SP} - 8F^{-4}\lambda_2^{PP}c_d c_m\lambda_1^{SP} + 4F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 2F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} + 12F^{-4}c_d c_m\lambda_2^{SS}\lambda_1^{SP} + 3F^{-4}c_d c_m\lambda_1^{SP} - 6F^{-4}c_d^2\lambda_2^{SS}\lambda_1^{SP} - 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 3/2F^{-4}c_d^2\lambda_1^{SP} - F^{-4}c_d^2\lambda_2^{SP} + 3F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_1^{PP}\lambda_2^{SS}\lambda_2^{SP} - F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SP} - F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_2^{SP} + 4F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/4F^{-2}\lambda_2^{PP}\lambda_1^{SP} + F^{-2}\lambda_2^{PP}\lambda_2^{SP} + F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} + 3/4F^{-2}\lambda_3^{PP}\lambda_1^{SP} + 1/2F^{-2}\lambda_1^{SS}\lambda_1^{SP} - 3/4F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 24F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 6F^{-2}\lambda_2^{SS}(\lambda_1^{SP})^3 + F^{-2}\lambda_2^{SS}\lambda_2^{SP} - 3/4F^{-2}\lambda_3^{SS}\lambda_1^{SP} - 2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} + 1/2F^{-2}(\lambda_1^{SP})^3$
365	$i\langle u_\nu P u^\nu \chi_-\rangle\langle S\rangle$	$8F^{-6}c_d^3 c_m\lambda_1^{SP} - 4F^{-6}c_d^4\lambda_1^{SP} + 2F^{-4}\lambda_1^{PP}c_d c_m\lambda_1^{SP} - F^{-4}\lambda_1^{PP}c_d^2\lambda_1^{SP} - 8F^{-4}\lambda_2^{PP}c_d c_m\lambda_1^{SP} + 2F^{-4}\lambda_2^{PP}c_d^2\lambda_1^{SP} + 8F^{-4}\lambda_2^{PP}c_d^2\lambda_2^{SP} - 4F^{-4}c_d c_m\lambda_1^{SS}\lambda_1^{SP} + F^{-4}c_d c_m\lambda_1^{SP} + 3F^{-4}c_d^2\lambda_1^{SS}\lambda_1^{SP} - 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} - 3/4F^{-4}c_d^2\lambda_1^{SP} - 4F^{-4}c_d^2(\lambda_1^{SP})^2\lambda_2^{SP} + F^{-4}c_d^2(\lambda_1^{SP})^3 + 1/2F^{-2}\lambda_1^{PP}\lambda_1^{SS}\lambda_1^{SP} - F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} - 1/8F^{-2}\lambda_1^{PP}\lambda_1^{SP} - F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_2^{PP}\lambda_1^{SS}\lambda_2^{SP} + 4F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} + 3/4F^{-2}\lambda_2^{PP}\lambda_1^{SP} - F^{-2}\lambda_2^{PP}\lambda_2^{SP} + 3/8F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 1/2F^{-2}\lambda_1^{SS}(\lambda_1^{SP})^3 - 3/4F^{-2}\lambda_3^{SS}\lambda_1^{SP} - 3/32F^{-2}\lambda_1^{SP} + 1/2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP} - 1/8F^{-2}(\lambda_1^{SP})^3$

366	$i\langle \chi_-\{\nabla^\mu P,\nabla_\mu S\}\rangle$	$4NF^{-4}d_mc_d(\lambda_1^{SP})^2+3NF^{-4}c_dc_m\lambda_1^{SP}-3/2NF^{-4}c_d^2\lambda_1^{SP}+3NF^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}-3/4NF^{-2}(\lambda_1^{SP})^3$
367	$i\langle \chi_-\rangle\langle \nabla_\mu P\nabla^\mu S\rangle$	$8F^{-4}d_mc_d(\lambda_1^{SP})^2+6F^{-4}c_dc_m\lambda_1^{SP}-3F^{-4}c_d^2\lambda_1^{SP}+6F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}-3/2F^{-2}(\lambda_1^{SP})^3$
368	$i\langle \chi_-\nabla_\mu P\rangle\langle \nabla^\mu S\rangle$	$-2F^{-4}c_dc_m\lambda_1^{SP}+F^{-4}c_d^2\lambda_1^{SP}-2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}+1/2F^{-2}(\lambda_1^{SP})^3$
369	$i\langle \chi_-\nabla_\mu S\rangle\langle \nabla^\mu P\rangle$	$-2F^{-4}c_dc_m\lambda_1^{SP}+F^{-4}c_d^2\lambda_1^{SP}-2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}+1/2F^{-2}(\lambda_1^{SP})^3$
370	$i\langle S\chi_+\chi_-P\rangle+\mathrm{h.c.}$	$-2NF^{-4}d_m\lambda_2^{PP}c_d+4NF^{-4}d_mc_d\lambda_3^{SS}(\lambda_1^{SP})^2-1/2NF^{-4}d_mc_d\lambda_3^{SS}-4NF^{-4}d_mc_d\lambda_1^{SP}\lambda_2^{SP}+2NF^{-4}d_mc_d(\lambda_1^{SP})^2-1/4NF^{-4}d_mc_d+1/8NF^{-4}d_mc_m+2NF^{-4}c_dc_m\lambda_3^{SS}\lambda_1^{SP}-NF^{-4}c_dc_m\lambda_1^{SP}+3/2NF^{-4}c_dc_m\lambda_2^{SP}-2NF^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP}+2NF^{-4}c_d^2\lambda_3^{SS}\lambda_2^{SP}+1/2NF^{-4}c_m^2\lambda_1^{SP}+1/2NF^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP}+1/2NF^{-2}\lambda_1^{PP}\lambda_2^{SP}+NF^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP}-NF^{-2}\lambda_3^{PP}\lambda_3^{SS}\lambda_1^{SP}-NF^{-2}\lambda_3^{PP}\lambda_2^{SP}+1/4NF^{-2}\lambda_1^{SS}\lambda_1^{SP}-1/2NF^{-2}\lambda_1^{SS}\lambda_2^{SP}+1/8NF^{-2}\lambda_3^{SS}\lambda_1^{SP}+8NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2\lambda_2^{SP}-5/2NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3+3NF^{-2}\lambda_1^{SP}(\lambda_2^{SP})^2-NF^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}+1/4NF^{-2}\lambda_2^{SP}$
371	$i\langle S\chi_+P\chi_-\rangle+\mathrm{h.c.}$	$-12NF^{-6}d_mc_d^2c_m(\lambda_1^{SP})^2-8NF^{-6}d_mc_d^2c_m+5NF^{-6}d_mc_d^3-6NF^{-6}d_m^2c_d^2\lambda_1^{SP}-16NF^{-6}c_d^2c_m^2\lambda_1^{SP}+10NF^{-6}c_d^3c_m\lambda_1^{SP}-2NF^{-4}d_m\lambda_1^{PP}c_d+NF^{-4}d_m\lambda_1^{PP}c_m-2NF^{-4}d_m\lambda_2^{PP}c_d+2NF^{-4}d_m\lambda_2^{PP}c_m+2NF^{-4}d_m\lambda_3^{PP}c_d-NF^{-4}d_mc_d\lambda_1^{SS}-2NF^{-4}d_mc_d\lambda_2^{SS}+4NF^{-4}d_mc_d\lambda_3^{SS}(\lambda_1^{SP})^2+1/2NF^{-4}d_mc_d\lambda_3^{SS}-4NF^{-4}d_mc_d\lambda_1^{SP}\lambda_2^{SP}+6NF^{-4}d_mc_d(\lambda_1^{SP})^2-NF^{-4}d_mc_d+2NF^{-4}d_mc_m\lambda_1^{SS}(\lambda_1^{SP})^2+NF^{-4}d_mc_m\lambda_1^{SS}+4NF^{-4}d_mc_m\lambda_2^{SS}(\lambda_1^{SP})^2+2NF^{-4}d_mc_m\lambda_2^{SS}-7/2NF^{-4}d_mc_m(\lambda_1^{SP})^2+5/8NF^{-4}d_mc_m+NF^{-4}d_m^2\lambda_1^{SS}\lambda_1^{SP}+2NF^{-4}d_m^2\lambda_2^{SS}\lambda_1^{SP}+1/4NF^{-4}d_m^2\lambda_1^{SP}-NF^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP}+NF^{-4}\lambda_1^{PP}c_m^2\lambda_1^{SP}-2NF^{-4}\lambda_2^{PP}c_dc_m\lambda_1^{SP}+2NF^{-4}\lambda_2^{PP}c_m^2\lambda_1^{SP}-4NF^{-4}\lambda_3^{PP}c_dc_m\lambda_1^{SP}+3NF^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP}-2NF^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP}-4NF^{-4}c_dc_m\lambda_2^{SS}\lambda_1^{SP}+6NF^{-4}c_dc_m\lambda_3^{SS}\lambda_1^{SP}-7/4NF^{-4}c_dc_m\lambda_1^{SP}-4NF^{-4}c_dc_m(\lambda_1^{SP})^2\lambda_2^{SP}+7NF^{-4}c_dc_m(\lambda_1^{SP})^3+5/2NF^{-4}c_dc_m\lambda_2^{SP}-4NF^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP}+2NF^{-4}c_d^2\lambda_3^{SS}\lambda_2^{SP}-NF^{-4}c_d^2\lambda_2^{SP}+2NF^{-4}c_m^2\lambda_1^{SS}\lambda_1^{SP}+4NF^{-4}c_m^2\lambda_2^{SS}\lambda_1^{SP}+5/4NF^{-4}c_m^2\lambda_1^{SP}-5NF^{-4}c_m^2(\lambda_1^{SP})^3+1/2NF^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP}-1/2NF^{-2}\lambda_1^{PP}\lambda_2^{SP}+NF^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP}+NF^{-2}\lambda_2^{PP}\lambda_2^{SP}-1/2NF^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP}-NF^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP}-1/8NF^{-2}\lambda_3^{PP}\lambda_1^{SP}+2NF^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2\lambda_2^{SP}+NF^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP}+1/4NF^{-2}\lambda_1^{SS}\lambda_1^{SP}-1/2NF^{-2}\lambda_1^{SS}\lambda_2^{SP}-1/2NF^{-2}\lambda_2^{SS}\lambda_1^{SP}+NF^{-2}\lambda_2^{SS}\lambda_2^{SP}+3/8NF^{-2}\lambda_3^{SS}\lambda_1^{SP}+6NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2\lambda_2^{SP}-5/2NF^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3-2NF^{-2}(\lambda_3^{SS})^2\lambda_1^{SP}+2NF^{-2}\lambda_1^{SP}(\lambda_2^{SP})^2-1/2NF^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}$

372	$i\langle S\chi_-\chi_+P\rangle + \text{h.c.}$	$-12NF^{-6}d_mc_d^2c_m(\lambda_1^{\text{SP}})^2 - 8NF^{-6}d_mc_d^2c_m + 5NF^{-6}d_mc_d^3 - 6NF^{-6}d_m^2c_d^2\lambda_1^{\text{SP}} - 16NF^{-6}c_d^2c_m^2\lambda_1^{\text{SP}} + 10NF^{-6}c_d^3c_m\lambda_1^{\text{SP}} - NF^{-4}d_m\lambda_1^{\text{PP}}c_d + NF^{-4}d_m\lambda_1^{\text{PP}}c_m - 2NF^{-4}d_m\lambda_2^{\text{PP}}c_d + 2NF^{-4}d_m\lambda_2^{\text{PP}}c_m - NF^{-4}d_mc_d\lambda_1^{\text{SS}} - 2NF^{-4}d_mc_d\lambda_2^{\text{SS}} + NF^{-4}d_mc_d\lambda_3^{\text{SS}} + 5NF^{-4}d_mc_d(\lambda_1^{\text{SP}})^2 - 1/2NF^{-4}d_mc_d + 2NF^{-4}d_mc_m\lambda_1^{\text{SS}}(\lambda_1^{\text{SP}})^2 + NF^{-4}d_mc_m\lambda_1^{\text{SS}} + 4NF^{-4}d_mc_m\lambda_2^{\text{SS}}(\lambda_1^{\text{SP}})^2 + 2NF^{-4}d_mc_m\lambda_2^{\text{SS}} - 7/2NF^{-4}d_mc_m(\lambda_1^{\text{SP}})^2 + 1/2NF^{-4}d_mc_m + NF^{-4}d_m^2\lambda_1^{\text{SS}}\lambda_1^{\text{SP}} + 2NF^{-4}d_m^2\lambda_2^{\text{SS}}\lambda_1^{\text{SP}} + 1/4NF^{-4}d_m^2\lambda_1^{\text{SP}} - NF^{-4}\lambda_1^{\text{PP}}c_dc_m\lambda_1^{\text{SP}} + NF^{-4}\lambda_1^{\text{PP}}c_m^2\lambda_1^{\text{SP}} - 2NF^{-4}\lambda_2^{\text{PP}}c_dc_m\lambda_1^{\text{SP}} + 2NF^{-4}\lambda_2^{\text{PP}}c_m^2\lambda_1^{\text{SP}} - 4NF^{-4}\lambda_3^{\text{PP}}c_dc_m\lambda_1^{\text{SP}} + 3NF^{-4}\lambda_3^{\text{PP}}c_d^2\lambda_1^{\text{SP}} - 2NF^{-4}c_dc_m\lambda_1^{\text{SS}}\lambda_1^{\text{SP}} - 4NF^{-4}c_dc_m\lambda_2^{\text{SS}}\lambda_1^{\text{SP}} + 8NF^{-4}c_dc_m\lambda_3^{\text{SS}}\lambda_1^{\text{SP}} - 3/4NF^{-4}c_dc_m\lambda_1^{\text{SP}} - 4NF^{-4}c_dc_m(\lambda_1^{\text{SP}})^2\lambda_2^{\text{SP}} + 7NF^{-4}c_dc_m(\lambda_1^{\text{SP}})^3 + 3NF^{-4}c_dc_m\lambda_2^{\text{SP}} - 4NF^{-4}c_d^2\lambda_3^{\text{SS}}\lambda_1^{\text{SP}} - 2NF^{-4}c_d^2\lambda_2^{\text{SP}} + 2NF^{-4}c_m^2\lambda_1^{\text{SS}}\lambda_1^{\text{SP}} + 4NF^{-4}c_m^2\lambda_2^{\text{SS}}\lambda_1^{\text{SP}} + 3/4NF^{-4}c_m^2\lambda_1^{\text{SP}} - 5NF^{-4}c_m^2(\lambda_1^{\text{SP}})^3 - 1/2NF^{-2}\lambda_1^{\text{PP}}\lambda_2^{\text{SP}} - 1/2NF^{-2}\lambda_3^{\text{PP}}\lambda_1^{\text{SS}}\lambda_1^{\text{SP}} - NF^{-2}\lambda_3^{\text{PP}}\lambda_2^{\text{SS}}\lambda_1^{\text{SP}} + NF^{-2}\lambda_3^{\text{PP}}\lambda_3^{\text{SS}}\lambda_1^{\text{SP}} - 1/8NF^{-2}\lambda_3^{\text{PP}}\lambda_1^{\text{SP}} + 2NF^{-2}\lambda_3^{\text{PP}}(\lambda_1^{\text{SP}})^2\lambda_2^{\text{SP}} - 1/4NF^{-2}\lambda_1^{\text{SS}}\lambda_1^{\text{SP}} + 1/2NF^{-2}\lambda_1^{\text{SS}}\lambda_2^{\text{SP}} + 2NF^{-2}\lambda_2^{\text{SS}}\lambda_3^{\text{SS}}\lambda_1^{\text{SP}} + 1/2NF^{-2}\lambda_3^{\text{SS}}\lambda_1^{\text{SP}} - 2NF^{-2}\lambda_3^{\text{SS}}(\lambda_1^{\text{SP}})^2\lambda_2^{\text{SP}} - NF^{-2}\lambda_3^{\text{SS}}\lambda_2^{\text{SP}} + NF^{-2}\lambda_1^{\text{SP}}(\lambda_2^{\text{SP}})^2 + 1/4NF^{-2}\lambda_2^{\text{SP}}$

373	$i\langle S\rangle\langle P\{\chi_-,\chi_+\}\rangle$	$-8F^{-6}d_mc_d^2c_m(\lambda_1^{SP})^2 - 4F^{-6}d_mc_d^2c_m + 2F^{-6}d_mc_d^3 - 4F^{-6}d_m^2c_d^2\lambda_1^{SP} - 8F^{-6}c_d^2c_m^2\lambda_1^{SP} + 4F^{-6}c_d^3c_m\lambda_1^{SP} - F^{-4}d_m\lambda_1^{PP}c_d - 2F^{-4}d_m\lambda_2^{PP}c_d + 2F^{-4}d_m\lambda_3^{PP}c_d - 2F^{-4}d_mc_d\lambda_1^{SS} + 2F^{-4}d_mc_d\lambda_3^{SS} - 8F^{-4}d_mc_d\lambda_1^{SP}\lambda_2^{SP} + 3F^{-4}d_mc_d(\lambda_1^{SP})^2 - 1/4F^{-4}d_mc_d + 4F^{-4}d_mc_m\lambda_1^{SS}(\lambda_1^{SP})^2 + 2F^{-4}d_mc_m\lambda_1^{SS} - F^{-4}d_mc_m(\lambda_1^{SP})^2 - 1/4F^{-4}d_mc_m + 2F^{-4}d_m^2\lambda_1^{SS}\lambda_1^{SP} - 1/2F^{-4}d_m^2\lambda_1^{SP} - 4F^{-4}\lambda_3^{PP}c_dc_m\lambda_1^{SP} + 2F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} - 4F^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP} + 8F^{-4}c_dc_m\lambda_3^{SS}\lambda_1^{SP} - F^{-4}c_dc_m\lambda_1^{SP} + F^{-4}c_dc_m\lambda_2^{SP} - 2F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} + F^{-4}c_d^2\lambda_2^{SP} + 4F^{-4}c_m^2\lambda_1^{SS}\lambda_1^{SP} + 1/2F^{-2}\lambda_1^{PP}\lambda_2^{SP} + F^{-2}\lambda_2^{PP}\lambda_2^{SP} - F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_3^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/4F^{-2}\lambda_3^{PP}\lambda_1^{SP} - F^{-2}\lambda_3^{PP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/2F^{-2}\lambda_1^{SS}\lambda_2^{SP} + 1/2F^{-2}\lambda_2^{SS}\lambda_1^{SP} - F^{-2}\lambda_2^{SS}\lambda_2^{SP} + 1/4F^{-2}\lambda_3^{SS}\lambda_1^{SP} - F^{-2}\lambda_3^{SS}\lambda_2^{SP} - 2F^{-2}(\lambda_3^{SS})^2\lambda_1^{SP} + 4F^{-2}\lambda_1^{SP}(\lambda_2^{SP})^2 - 3/2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}$
374	$i\langle S\{\chi_-,\chi_+\}\rangle\langle P\rangle$	$4F^{-6}c_d^2c_m^2\lambda_1^{SP} - 4F^{-6}c_d^3c_m\lambda_1^{SP} - 3F^{-4}d_m\lambda_1^{PP}c_d + 2F^{-4}d_m\lambda_1^{PP}c_m + 2F^{-4}d_m\lambda_3^{PP}c_d - F^{-4}d_mc_d\lambda_3^{SS} - 4F^{-4}d_mc_d\lambda_1^{SP}\lambda_2^{SP} + 3F^{-4}d_mc_d(\lambda_1^{SP})^2 + 1/4F^{-4}d_mc_d - 2F^{-4}d_mc_m(\lambda_1^{SP})^2 - 1/4F^{-4}d_mc_m - 2F^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP} + 2F^{-4}\lambda_1^{PP}c_m^2\lambda_1^{SP} + 4F^{-4}c_dc_m\lambda_3^{SS}\lambda_1^{SP} - 3F^{-4}c_dc_m\lambda_1^{SP} + 2F^{-4}c_dc_m(\lambda_1^{SP})^3 + 4F^{-4}c_dc_m\lambda_2^{SP} - 4F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} + 2F^{-4}c_d^2\lambda_1^{SP} - 3F^{-4}c_d^2\lambda_2^{SP} + 1/2F^{-4}c_m^2\lambda_1^{SP} - 2F^{-4}c_m^2(\lambda_1^{SP})^3 + F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} + 1/2F^{-2}\lambda_1^{PP}\lambda_2^{SP} - F^{-2}\lambda_2^{PP}\lambda_2^{SP} - 2F^{-2}\lambda_3^{PP}\lambda_3^{SS}\lambda_1^{SP} - F^{-2}\lambda_3^{PP}\lambda_2^{SP} + F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/4F^{-2}\lambda_1^{SS}\lambda_1^{SP} + 1/2F^{-2}\lambda_1^{SS}\lambda_2^{SP} + 2F^{-2}\lambda_2^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_2^{SS}\lambda_1^{SP} + F^{-2}\lambda_2^{SS}\lambda_2^{SP} + 1/2F^{-2}\lambda_3^{SS}\lambda_1^{SP} - F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - F^{-2}\lambda_3^{SS}\lambda_2^{SP} - 2F^{-2}(\lambda_3^{SS})^2\lambda_1^{SP} + 1/2F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}$

375	$i\langle S\chi_+\rangle\langle P\chi_-\rangle$	$-2F^{-4}d_m\lambda_1^{PP}c_d - 4F^{-4}d_m\lambda_2^{PP}c_d + 4F^{-4}d_m\lambda_3^{PP}c_d - 2F^{-4}d_mc_d\lambda_3^{SS} - 16F^{-4}d_mc_d\lambda_1^{SP}\lambda_2^{SP} + 6F^{-4}d_mc_d(\lambda_1^{SP})^2 - 3/2F^{-4}d_mc_d + 1/2F^{-4}d_mc_m - 4F^{-4}c_dc_m\lambda_1^{SP} + 2F^{-4}c_dc_m\lambda_2^{SP} - 4F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} + 8F^{-4}c_d^2\lambda_3^{SS}\lambda_2^{SP} + 2F^{-4}c_d^2\lambda_2^{SP} + 2F^{-4}c_m^2\lambda_1^{SP} + 2F^{-2}\lambda_1^{PP}\lambda_3^{SS}\lambda_1^{SP} + F^{-2}\lambda_1^{PP}\lambda_2^{SP} + 4F^{-2}\lambda_2^{PP}\lambda_3^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_2^{PP}\lambda_2^{SP} - 4F^{-2}\lambda_3^{PP}\lambda_3^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_3^{PP}\lambda_2^{SP} + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{SS}\lambda_1^{SP} + F^{-2}\lambda_1^{SS}\lambda_2^{SP} + F^{-2}\lambda_2^{SS}\lambda_1^{SP} - 2F^{-2}\lambda_2^{SS}\lambda_2^{SP} + F^{-2}\lambda_3^{SS}\lambda_1^{SP} + 16F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 6F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^3 - 2F^{-2}\lambda_3^{SS}\lambda_2^{SP} - 4F^{-2}(\lambda_3^{SS})^2\lambda_1^{SP} + 8F^{-2}\lambda_1^{SP}(\lambda_2^{SP})^2 - 3F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}$
376	$i\langle S\chi_-\rangle\langle P\chi_+\rangle$	$-16F^{-6}d_mc_d^2c_m(\lambda_1^{SP})^2 - 16F^{-6}d_mc_d^2c_m + 12F^{-6}d_mc_d^3 - 8F^{-6}d_m^2c_d^2\lambda_1^{SP} - 24F^{-6}c_d^2c_m^2\lambda_1^{SP} + 16F^{-6}c_d^3c_m\lambda_1^{SP} - 6F^{-4}d_m\lambda_1^{PP}c_d + 4F^{-4}d_m\lambda_1^{PP}c_m + 4F^{-4}d_m\lambda_3^{PP}c_d - 4F^{-4}d_mc_d\lambda_1^{SS} - 4F^{-4}d_mc_d\lambda_2^{SS} + 4F^{-4}d_mc_d\lambda_3^{SS} + 10F^{-4}d_mc_d(\lambda_1^{SP})^2 + 1/2F^{-4}d_mc_d + 8F^{-4}d_mc_m\lambda_1^{SS}(\lambda_1^{SP})^2 + 4F^{-4}d_mc_m\lambda_1^{SS} - 10F^{-4}d_mc_m(\lambda_1^{SP})^2 - 3/2F^{-4}d_mc_m + 4F^{-4}d_m^2\lambda_1^{SS}\lambda_1^{SP} - F^{-4}d_m^2\lambda_1^{SP} - 4F^{-4}\lambda_1^{PP}c_dc_m\lambda_1^{SP} + 4F^{-4}\lambda_1^{PP}c_m^2\lambda_1^{SP} - 16F^{-4}\lambda_3^{PP}c_dc_m\lambda_1^{SP} + 12F^{-4}\lambda_3^{PP}c_d^2\lambda_1^{SP} - 8F^{-4}c_dc_m\lambda_1^{SS}\lambda_1^{SP} - 8F^{-4}c_dc_m\lambda_2^{SS}\lambda_1^{SP} + 24F^{-4}c_dc_m\lambda_3^{SS}\lambda_1^{SP} - 6F^{-4}c_dc_m\lambda_1^{SP} + 16F^{-4}c_dc_m(\lambda_1^{SP})^2\lambda_2^{SP} + 12F^{-4}c_dc_m(\lambda_1^{SP})^3 + 8F^{-4}c_dc_m\lambda_2^{SP} - 12F^{-4}c_d^2\lambda_3^{SS}\lambda_1^{SP} + 4F^{-4}c_d^2\lambda_1^{SP} - 6F^{-4}c_d^2\lambda_2^{SP} + 8F^{-4}c_m^2\lambda_1^{SS}\lambda_1^{SP} - F^{-4}c_m^2\lambda_1^{SP} - 12F^{-4}c_m^2(\lambda_1^{SP})^3 + F^{-2}\lambda_1^{PP}\lambda_2^{SP} - 2F^{-2}\lambda_2^{PP}\lambda_2^{SP} - 2F^{-2}\lambda_3^{PP}\lambda_1^{SS}\lambda_1^{SP} - 4F^{-2}\lambda_3^{PP}\lambda_2^{SS}\lambda_1^{SP} + 4F^{-2}\lambda_3^{PP}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_3^{PP}\lambda_1^{SP} + 8F^{-2}\lambda_3^{PP}(\lambda_1^{SP})^2\lambda_2^{SP} - 2F^{-2}\lambda_3^{PP}\lambda_2^{SP} + 2F^{-2}\lambda_1^{SS}\lambda_3^{SS}\lambda_1^{SP} - 1/2F^{-2}\lambda_1^{SS}\lambda_1^{SP} + F^{-2}\lambda_1^{SS}\lambda_2^{SP} + 4F^{-2}\lambda_2^{SS}\lambda_3^{SS}\lambda_1^{SP} - F^{-2}\lambda_2^{SS}\lambda_1^{SP} + 2F^{-2}\lambda_2^{SS}\lambda_2^{SP} + 3/2F^{-2}\lambda_3^{SS}\lambda_1^{SP} - 8F^{-2}\lambda_3^{SS}(\lambda_1^{SP})^2\lambda_2^{SP} - 2F^{-2}\lambda_3^{SS}\lambda_2^{SP} - 4F^{-2}(\lambda_3^{SS})^2\lambda_1^{SP} + F^{-2}(\lambda_1^{SP})^2\lambda_2^{SP}$

377	$i\langle SP\rangle\langle\chi_-\chi_+\rangle$	$-2F^{-4}d_m\lambda_1^{\rm PP}c_d + 4F^{-4}d_m\lambda_3^{\rm PP}c_d - 8F^{-4}d_mc_d\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 2F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 1/2F^{-4}d_mc_d + 1/2F^{-4}d_mc_m + 8F^{-4}c_dc_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 5F^{-4}c_dc_m\lambda_1^{\rm SP} + 4F^{-4}c_dc_m\lambda_2^{\rm SP} - 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_d^2\lambda_1^{\rm SP} - 2F^{-4}c_d^2\lambda_2^{\rm SP} + 2F^{-4}c_m^2\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - 2F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SP} + 2F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 2F^{-2}\lambda_3^{\rm SS}\lambda_2^{\rm SP} - 4F^{-2}(\lambda_3^{\rm SS})^2\lambda_1^{\rm SP} + 4F^{-2}\lambda_1^{\rm SP}(\lambda_2^{\rm SP})^2 - F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}$
378	$i\langle\chi_-\rangle\langle\chi_+\{S,P\}\rangle$	$-24F^{-6}d_mc_d^2c_m(\lambda_1^{\rm SP})^2 - 12F^{-6}d_mc_d^2c_m + 6F^{-6}d_mc_d^3 - 12F^{-6}d_m^2c_d^2\lambda_1^{\rm SP} - 24F^{-6}c_d^2c_m^2\lambda_1^{\rm SP} + 12F^{-6}c_d^3c_m\lambda_1^{\rm SP} - F^{-4}d_m\lambda_1^{\rm PP}c_d + 2F^{-4}d_m\lambda_3^{\rm PP}c_d - 2F^{-4}d_mc_d\lambda_1^{\rm SS} - 2F^{-4}d_mc_d\lambda_2^{\rm SS} + 8F^{-4}d_mc_d\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2 + F^{-4}d_mc_d\lambda_3^{\rm SS} - 8F^{-4}d_mc_d\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 3F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 1/4F^{-4}d_mc_d + 4F^{-4}d_mc_m\lambda_1^{\rm SS}(\lambda_1^{\rm SP})^2 + 2F^{-4}d_mc_m\lambda_1^{\rm SS} + 8F^{-4}d_mc_m\lambda_2^{\rm SS}(\lambda_1^{\rm SP})^2 + 4F^{-4}d_mc_m\lambda_2^{\rm SS} + F^{-4}d_mc_m(\lambda_1^{\rm SP})^2 + 3/4F^{-4}d_mc_m + 2F^{-4}d_m^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 4F^{-4}d_m^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-4}d_m^2\lambda_1^{\rm SP} - 4F^{-4}\lambda_3^{\rm PP}c_dc_m\lambda_1^{\rm SP} + 2F^{-4}\lambda_3^{\rm PP}c_d^2\lambda_1^{\rm SP} - 4F^{-4}c_dc_m\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 4F^{-4}c_dc_m\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 12F^{-4}c_dc_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 5/2F^{-4}c_dc_m\lambda_1^{\rm SP} - 16F^{-4}c_dc_m(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 4F^{-4}c_dc_m(\lambda_1^{\rm SP})^3 + 6F^{-4}c_dc_m\lambda_2^{\rm SP} - 4F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} + F^{-4}c_d^2\lambda_1^{\rm SP} - 3F^{-4}c_d^2\lambda_2^{\rm SP} + 4F^{-4}c_m^2\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 8F^{-4}c_m^2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + 2F^{-4}c_m^2\lambda_1^{\rm SP} + F^{-2}\lambda_1^{\rm PP}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_3^{\rm PP}\lambda_1^{\rm SP} - F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} + 1/2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} + 12F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - 3F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^3 - F^{-2}\lambda_3^{\rm SS}\lambda_2^{\rm SP} - 2F^{-2}(\lambda_3^{\rm SS})^2\lambda_1^{\rm SP} + 6F^{-2}\lambda_1^{\rm SP}(\lambda_2^{\rm SP})^2 - 3/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP}$

379	$i\langle\chi_-\{S,P\}\rangle\langle\chi_+\rangle$	$-3F^{-4}d_m\lambda_1^{\rm PP}c_d + 2F^{-4}d_m\lambda_1^{\rm PP}c_m - 6F^{-4}d_m\lambda_2^{\rm PP}c_d + 4F^{-4}d_m\lambda_2^{\rm PP}c_m + 2F^{-4}d_m\lambda_3^{\rm PP}c_d - 8F^{-4}d_mc_d\lambda_1^{\rm SP}\lambda_2^{\rm SP} + 9F^{-4}d_mc_d(\lambda_1^{\rm SP})^2 - 5/4F^{-4}d_mc_d - 6F^{-4}d_mc_m(\lambda_1^{\rm SP})^2 + 3/4F^{-4}d_mc_m - 2F^{-4}\lambda_1^{\rm PP}c_dc_m\lambda_1^{\rm SP} + 2F^{-4}\lambda_1^{\rm PP}c_m^2\lambda_1^{\rm SP} - 4F^{-4}\lambda_2^{\rm PP}c_dc_m\lambda_1^{\rm SP} + 4F^{-4}\lambda_2^{\rm PP}c_m^2\lambda_1^{\rm SP} + 8F^{-4}c_dc_m\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 5/2F^{-4}c_dc_m\lambda_1^{\rm SP} + 6F^{-4}c_dc_m(\lambda_1^{\rm SP})^3 + 3F^{-4}c_dc_m\lambda_2^{\rm SP} - 6F^{-4}c_d^2\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-4}c_d^2\lambda_2^{\rm SP} + 3/2F^{-4}c_m^2\lambda_1^{\rm SP} - 6F^{-4}c_m^2(\lambda_1^{\rm SP})^3 - 1/2F^{-2}\lambda_1^{\rm PP}\lambda_2^{\rm SP} + F^{-2}\lambda_2^{\rm PP}\lambda_2^{\rm SP} - F^{-2}\lambda_3^{\rm PP}\lambda_2^{\rm SP} + F^{-2}\lambda_1^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} + 1/4F^{-2}\lambda_1^{\rm SS}\lambda_1^{\rm SP} - 1/2F^{-2}\lambda_1^{\rm SS}\lambda_2^{\rm SP} + 2F^{-2}\lambda_2^{\rm SS}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - F^{-2}1/2\lambda_2^{\rm SS}\lambda_1^{\rm SP} + F^{-2}\lambda_2^{\rm SS}\lambda_2^{\rm SP} + 3/4F^{-2}\lambda_3^{\rm SS}\lambda_1^{\rm SP} - 4F^{-2}\lambda_3^{\rm SS}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} - F^{-2}\lambda_3^{\rm SS}\lambda_2^{\rm SP} - 2F^{-2}(\lambda_3^{\rm SS})^2\lambda_1^{\rm SP} + 2F^{-2}\lambda_1^{\rm SP}(\lambda_2^{\rm SP})^2 - 1/2F^{-2}(\lambda_1^{\rm SP})^2\lambda_2^{\rm SP} + 1/2F^{-2}\lambda_2^{\rm SP}$

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