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Lepton Flavor and Baryon Number Violation in charged lepton decays

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1 Introduction

The Standard Model (SM) of strong and electroweak interactions is a gauge theory based on the local gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ which correctly describes particle physics interactions with a great precision. However, there are some theoretical reasons to think that the SM cannot be a fundamental theory. In this sense, many Beyond Standard Model (BSM) theories have been developed in recent years.

Apart from the gauge local symmetry, the SM accidentally conserves other global symmetries such as the Lepton Family numbers, L_F, and the Lepton, L, and Baryon, B, numbers. These global symmetries are naturally violated in many BSM theories and consequently, the search of processes which violate them constitutes a good way to test new physics frameworks.

The first experimental evidence for violation of one of such symmetries came from neutrino mixing, giving rise to Lepton Flavor Violating (LFV) processes. In the charged sector, this mixing also induces LFV but with a probability far beyond the experimental reach. So far no LFV process has been measured in the charged sector. However, experimental precision has been increasing continuously in the recent years and there are some prospects for improvement in the recent future.

On the other hand, Baryon Number Violation (BNV) is believed to have occurred in the early universe, although the mechanism still remains unknown. Most of the extensions of the SM where BNV occurs require it to happen via B-L conserving processes. Among the BNV processes, we will focus on $\tau^+ \to p\mu^+\mu^-$ decay in this master thesis. This process is interesting as the first experimental limit on its branching ratio has been recently established by LHCb [1]. It is also interesting to investigate BNV involving quarks from the higher families. In this regard, we will complete our analysis of BNV decays by including $\tau^+ \to \Lambda\pi^+$ decay.

In the following, we will study LFV and BNV with $\Delta L = \Delta B$ decays in a model independent way based on an Effective Quantum Field Theory (EQFT) approach. In Section 2, we will present the SM Lagrangian in order to settle the notation. A general introduction to EQFT will be given in section 3. Finally, sections 4 and 5 are devoted to the study of LFV in tau and muon leptons decays and the BNV tau decays aforementioned, respectively.

2 The Standard Model Lagrangian

The SM matter content is summarized in Table 2.1 with weak-isospin, color and generation indices denoted by $j = 1, 2$, $\alpha = 1, 2, 3$ and $p = 1, 2, 3$, respectively. Chirality indices (L,R) of the fermion fields will be skipped in what follows. Complex conjugate of the Higgs field will always occur either as $\varphi^\dagger$ or $\tilde{\varphi}$, where $\tilde{\varphi}^j = \epsilon_{jk} \left(\varphi^k\right)^*$, being ϵ_{jk} the totally antisymmetric tensor with $\epsilon_{12} = 1$.

	Fermions					Scalars
Field	l^j_{Lp}	e_{Rp}	$q^{\alpha j}_{Lp}$	u^α_{Rp}	d^α_{Rp}	φ^j
Hypercharge	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 2.1: *The SM matter content.*

Along the master thesis, we will also use the notation where the left-handed doublets l and

q are decomposed as:

$$l_{Lp} = \begin{pmatrix} \nu_{\ell_p} \\ \ell_p \end{pmatrix}, \qquad q^\alpha_{Lp} = \begin{pmatrix} u^\alpha_{Lp} \\ d^\alpha_{Lp} \end{pmatrix}. \tag{2.1}$$

The SM Lagrangian, $\mathscr{L}^{(4)}_{SM}$, before Spontaneous Symmetry Breaking (SSB) is given by:

$$\begin{aligned}\mathscr{L}^{(4)}_{SM} =& -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda\left(\varphi^\dagger\varphi\right)^2 + \\ &+ i\left(\bar{l}\not{D}l + \bar{e}\not{D}e + \bar{q}\not{D}q + \bar{u}\not{D}u + \bar{d}\not{D}d\right) - \left(\bar{l}\Gamma_e e\varphi + \bar{q}\Gamma_u u\tilde{\varphi} + \bar{q}\Gamma_d d\varphi + h.c.\right),\end{aligned} \tag{2.2}$$

being the Yukawa couplings $\Gamma_{e,u,d}$ matrices in the generation space. Our sign convention for covariant derivatives is exemplified by:

$$(D_\mu q)^{\alpha j} = \left(\partial_\mu + ig_s T^A_{\alpha\beta}G^A_\mu + igS^I_{jk}W^I_\mu + ig'Y_q B_\mu\right)q^{\beta k}, \tag{2.3}$$

where $T^A = \frac{1}{2}\lambda^A$ and $S^I = \frac{1}{2}\tau^I$ are the SU(3) and SU(2) generators, while λ^A and τ^I are the Gell-Mann and Pauli matrices, respectively.

It is useful to define Hermitian derivative terms that contain $\varphi^\dagger \overleftarrow{D}_\mu\varphi \equiv (D_\mu\varphi)^\dagger\varphi$ as follows:

$$\begin{aligned}\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi &\equiv i\varphi^\dagger\left(D_\mu - \overleftarrow{D}_\mu\right)\varphi, \\ \varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi &\equiv i\varphi^\dagger\left(\tau^I D_\mu - \overleftarrow{D}_\mu\tau^I\right)\varphi.\end{aligned} \tag{2.4}$$

The gauge field strength tensors and their covariant derivatives take the following form:

$$\begin{aligned}G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f^{ABC}G^B_\mu G^C_\nu, & (D_\rho G_{\mu\nu})^A &= \partial_\rho G^A_{\mu\nu} - g_s f^{ABC}G^B_\rho G^C_{\mu\nu}, \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g f^{IJK}W^J_\mu W^K_\nu, & (D_\rho W_{\mu\nu})^I &= \partial_\rho W^I_{\mu\nu} - g f^{IJK}W^J_\rho W^K_{\mu\nu}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, & D_\rho B_{\mu\nu} &= \partial_\rho B_{\mu\nu}.\end{aligned} \tag{2.5}$$

Dual tensors are defined by $\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$ ($\epsilon_{0123} = 1$), where X stands for G^A, W^I or B.

3 Effective quantum field theories

Several reviews on EQFT can be found in the literature [2, 3, 4]. In the following, we will give a brief introduction to the topic. The starting point of effective theories consists in assuming that the dynamics of the systems at low energies (or large distances) cannot depend on the details of the systems at high energies (or small distances). Consequently, low energy physics can be described by an effective Lagrangian which contains only degrees of freedom which are relevant for the low energy scale, while additional degrees of freedom present at higher energies are excluded. Whenever we are talking about low or high energies, we are setting a scale and, at the same time, establishing a range of validity for the theory.

For example, when one studies the collision between two bodies, the description of the phenomenon, as well as the kinematic variables one uses, depend on the momentum transfer between the colliding bodies. Indeed, for small enough momentum transfers such that the atomic structure of the bodies cannot be seen, one can use Newtonian mechanics to describe the collision. Furthermore, characteristics of the bodies related to their atomic structure would be irrelevant in the description of the collision, two bodies with a very different inner structure but with the same mechanical characteristics such as mass or moments of inertia obey the same equations of motion. Nevertheless, if one increases the momentum transfer up to energies where the atomic structure plays a role, the results obtained with the classical model would not be valid anymore and quantum effects would start to be relevant.

The example we are considering is especial in the sense that we know the fundamental theory behind the effective one. It sometimes occurs that we do not know which is the fundamental theory. When this happens, effective field theories are useful not only to describe the low energy behavior but also to extract useful information about the fundamental theory by studying its deviations with respect to the dynamics in the boundaries of the region where it is valid.

In the realm of high energy physics, the same arguments remain valid and EQFT can be treated in a rigorous way. When describing physical phenomena in particle physics the standard procedure is first to determine the relevant particles, i.e the degrees of freedom of the theory, and the symmetries of the system. Once we know that, the most general local Lagrangian containing these fields and respecting these symmetries is constructed. The EQFT approach is particularly useful because it allows us disregard all the particles which are too heavy to be produced in the energy scale regime we are interested in. Eliminating such particles produces a great simplification.

This procedure can be realized by “integrating out” in the action all the fields which lie above some energy scale Λ. As a result, we obtain an effective action S^{Λ}_{eff} which will be a non-local function of the original fields. This can then be expanded in a series of local (effective) operators $\mathcal{O}_i$ containing only low-energy fields, with Λ-dependent coefficients:

$$S^{\Lambda}_{\text{eff}} = \int d^4x \mathcal{L}_{\text{eff}} = \int d^4x \sum_i \alpha_i \left(\Lambda\right) \mathcal{O}_i. \tag{3.1}$$

The effective Lagrangian, $\mathcal{L}_{\text{eff}}$, compiles all virtual heavy physics dynamics. However, when constructing the EQFT expansion, we have modified the high-energy behavior of the theory and hence it will only be valid at energy scales below Λ. Furthermore, the construction of the EQFT is subtle because the limit in which heavy energy scales are “removed” from the theory is non-trivial. In perturbation theory, the observables are expressed in terms of Feynman loop diagrams where integration is performed at every energy scale. As a result, the EQFT will be non-renormalizable. This problem is related to the presence of an infinite set of coupling which, in principle, should lead to a complete lack of predictability. This is not the case as one can define a hierarchy in the operators in the Lagrangian in a way that, to any order in this hierarchy, there is a finite set of couplings in the theory. At any given order in the expansion, one can therefore obtain a framework which will produce non-trivial predictions.

In particle physics there are numerous examples of EQFT. Among them, we will focus on Chiral Perturbation Theory (χPT) and the SM treated as an effective theory.

3.1 Chiral perturbation theory

In this section we will briefly present the general ideas underlying χPT. For a detailed review on χPT see [5, 6].

It is well established that Quantum Chromodynamics (QCD) is the theory of the strong interaction. Because QCD is an asymptotically free theory, it can be perturbatively expanded when high energy transfers are involved. QCD has been tested in this regime to a great precision. However, in low energy regimes QCD becomes strongly coupled and quarks and gluons confine to hadrons. Because of that, it is extremely difficult to use QCD to describe the dynamics of strongly interacting particles in terms of the fundamental degrees of freedom. We do not know how to derive the hadronic interactions directly from the fundamental QCD Lagrangian.

Nonetheless, at very low energies, $E \ll 1$ GeV, the strong interaction dynamics simplifies. In this regime, the relevant degrees of freedom are not quarks and gluons but the octet of pseudoscalar mesons. It is then when the use effective quantum field theories becomes convenient. χPT is an effective field theory where the transition between the fundamental and the effective level occurs via the spontaneous breakdown of the chiral symmetry generating light pseudo-Golstone bosons with $M \ll \Lambda_\chi \sim 1$ GeV [5].

In the absence of quark masses, the QCD Lagrangian,

$$\mathcal{L}^0_{QCD} = -\frac{1}{4} G^A_{\mu\nu} G^{\mu\nu}_A + i\overline{q}_L \gamma^\mu D_\mu q_L + i\overline{q}_R \gamma^\mu D_\mu q_R, \tag{3.2}$$

is invariant under global $G \equiv SU(N_f)_L \otimes SU(N_f)_R$ transformations[1].

As u, d and s quark masses are small, the chiral symmetry should be approximatively good in the light quark sector. However, this symmetry remains unseen in the hadronic spectrum. In fact, it is well known that hadrons can be classified in $SU(3)_V$ representations but degenerate multiplets with opposite parities do not appear. Moreover, the masses of the pseudoscalar meson octet are much lighter than the rest of hadrons. These experimental facts suggest that the chiral symmetry is spontaneously broken to $SU(3)_V$ and that the particles of the meson octet constitute the pseudo-Goldstone bosons which, according to Goldstone's theorem, should appear. The spontaneous chiral symmetry breaking follows the pattern:

$$G \equiv SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V, \tag{3.3}$$

and hence, eight generators are broken which correspond to the eight pseudoscalar mesons in the octet.

To realize a spontaneously broken symmetry in quantum field theory, we use the Callan, Coleman, Wess and Zumino formalism [7]. In the case of chiral symmetry, the Golstone fields are collected in a unitary matrix $u(\phi)$ which transforms as follows:

$$u(\phi) \xrightarrow{G} g_R u(\phi) h(g,\phi)^{-1} = h(g,\phi) u(\phi) g_L^{-1}, \tag{3.4}$$

with $(g_R, g_L) \in G$ and where we introduced the so called compensator field, $h(g,\phi)$, representing an element of the conserved subgroup $SU(N_f)_V$ which is needed to return to the given choice of coset representative. The unitary matrix is defined in the following way:

$$u(\phi) = e^{\frac{i}{\sqrt{2}F}\phi}, \qquad \phi = \frac{1}{\sqrt{2}} \sum_{i=1}^{8} \lambda_i \phi_i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & K^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}. \tag{3.5}$$

It is interesting to extend the model by including external classical fields which allow us to add electromagnetic and semileptonic weak interactions and the explicit breaking of chiral symmetry through the quark masses in the theory. In order to do that, we start from the QCD extended Lagrangian,

$$\mathcal{L}_{QCD} = \mathcal{L}^0_{QCD} + \overline{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \overline{q}(s - ip\gamma_5) q, \tag{3.6}$$

[1] Indeed, the classical Lagrangian is invariant under $U(N_f)_L \otimes U(N_f)_R = SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_V \otimes U(1)_A$. Nonetheless, the $U(1)_A$ symmetry is broken in the quantum level due to anomalies while the $U(1)_V$ symmetry is related to baryon number conservation and it is trivially realized in the meson sector.

where the external classical fields are defined as:

$$
\begin{aligned}
& l_\mu \equiv v_\mu - a_\mu = -eQA_\mu - \frac{e}{\sqrt{2}\sin\theta_W}\left(W_\mu^+ T_+ + h.c.\right)\ldots, \\
& r_\mu \equiv v_\mu + a_\mu = -eQA_\mu + \ldots, \\
& s = \mathcal{M} + \ldots,
\end{aligned} \tag{3.7}
$$

with $Q = \frac{1}{3}diag\,(2,-1,-1)$ the quark charge matrix, $\mathcal{M} = diag(m_u, m_d, \ldots)$ the quark mass matrix and

$$
T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.8}
$$

being V_{ij} the Kobayashi–Maskawa mixing matrix elements.

The inclusion of external fields promotes the global chiral symmetry G to a local one with transformations:

$$
\begin{aligned}
& q_L \stackrel{G}{\to} g_L q_L, \\
& q_R \stackrel{G}{\to} g_R q_R, \\
& r_\mu \stackrel{G}{\to} g_R r_\mu g_R^{-1} + i g_R \partial_\mu g_R^{-1}, \\
& l_\mu \stackrel{G}{\to} g_L l_\mu g_L^{-1} + i g_L \partial_\mu g_L^{-1}, \\
& s + ip \stackrel{G}{\to} g_R\,(s+ip)\,g_L^{-1}.
\end{aligned} \tag{3.9}
$$

The local symmetry can now be used to construct a generalized effective Lagrangian in the presence of external sources. To achieve that, we introduce the covariant derivatives

$$
D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu, \qquad\qquad D_\mu U \stackrel{G}{\to} g_R D_\mu U g_L^{-1}, \tag{3.10}
$$

where $U = u^2$ with u defined in Equation (3.5). The associated non-Abelian field strength tensors are given by:

$$
\begin{aligned}
& F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i\,[r^\mu, r^\nu]\,, \\
& F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i\,[l^\mu, l^\nu]\,.
\end{aligned} \tag{3.11}
$$

Once we have parametrized the Goldstone fields and introduced the classical external fields, to get a low-energy effective Lagrangian realization of QCD for the light-quark sector, one should write the most general Lagrangian involving the matrix $u\,(\phi)$ and the external fields consistent with the chiral symmetry [8]. In χPT, the high energy scale is $\Lambda_\chi \sim 1$ GeV, and hence in the low-energy domain the Lagrangian can be organized in terms of increasing powers of momenta based on the following chiral counting rules:

$$
\begin{aligned}
U \qquad & \mathcal{O}(1), \\
D_\mu U,\, v_\mu,\, a_\mu \qquad & \mathcal{O}(p), \\
F_{L,R}^{\mu\nu} \qquad & \mathcal{O}(p^2), \\
s, p \qquad & \mathcal{O}(p^2).
\end{aligned} \tag{3.12}
$$

Using these power counting rules, the most general low-energy effective chiral Lagrangian reads (Lorentz invariance requires an even power of momenta):

$$\mathcal{L}_{eff}(u,l,r,s,p)=\sum_{n\geq 1}\mathcal{L}_{2n}. \tag{3.13}$$

A similar formalism can be developed to study interactions involving baryons and vector resonances. The chiral Lagrangian involving pseudoscalar mesons, baryons and vector resonances will be presented in Subsection 5.1.

3.2 Effective operators in the SM Lagrangian

The SM has been tested to a great precision. So far, no significant discrepancy with experiment has been found [9]. Nonetheless, the SM presents some theoretical problems which should not appear in a fundamental theory. It is commonly accepted that the SM is an effective field theory up to some high energy scale, Λ, which might be the Planck scale, and thus remain unobservable, or an intermediate mass scale which is usually set around $1-10$ TeV. Consequently, it is expected that new particles and interactions appear as one probes higher energies.

The scientific community has put a great effort in constructing BSM theories which solve some of the unsatisfactory aspects that the SM presents. However, the abundance of BSM theories and the chance that none of them might be correct makes necessary the use of model independent techniques. This necessity leads us to EQFT approaches.

In this sense, if we assume that the SM correctly describes the dynamics of particles at low-energies, we can take it to be an effective low energy theory in which heavy fields (Φ) have been integrated out and where new physics is encoded in an expansion of higher-dimensional local operators which depend on SM fields (ψ) suppressed by the high energy scale,

$$\mathscr{L}_{NP}\left[\psi,\Phi\right]\Rightarrow\mathscr{L}_{EFT}=\mathscr{L}_{SM}+\sum_{d\geq 5}\frac{\alpha_n^d}{\Lambda^{d-4}}\mathcal{O}_n^d\left[\psi\right], \tag{3.14}$$

where $\mathscr{L}_{SM}$ is the usual SM Lagrangian, which only contains operators with dimension two and four, and where α_n^d are dimensionless and expected to be $\mathcal{O}(1)$.

In order to obtain the higher-dimensional operators, one uses a "theorem" which states that if one "writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles" [8].

Following this principle, a minimal basis of dimension six operators[2] assuming $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ symmetry has been enumerated in [10]. Afterwards, this set has been updated and reduced [11]. Moreover, B-violating operators with $\Delta B=\Delta L$ can be found in [12, 13, 14, 15]. We list the aforementioned dimension six operators in Tables 3.1 and 3.2.

[2]There is only one dimension five operator

$$\mathcal{O}_{\nu\nu}=\epsilon_{jk}\epsilon_{mn}\varphi^j\varphi^m\left(l_p^k\right)^T C l_r^n\equiv\left(\tilde{\varphi}^\dagger l_p\right)^T C\left(\tilde{\varphi}^\dagger l_r\right), \tag{3.15}$$

where $C=i\gamma^2\gamma^0$ is the charge conjugation matrix. $Q_{\nu\nu}$ violates lepton number and generates neutrino masses and mixings after electroweak symmetry breaking. As we are not interested in neutrinos in this master thesis, we will focus on dimension six operators.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_\varphi$	$(\varphi^\dagger\varphi)^3$	$\mathcal{O}_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$\mathcal{O}_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \tilde{\varphi})$
$\mathcal{O}_W$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^* (\varphi^\dagger D_\mu\varphi)$	$\mathcal{O}_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$\mathcal{O}_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$\mathcal{O}_{\varphi G}$	$\varphi^\dagger\varphi G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\varphi\tilde{G}}$	$\varphi^\dagger\varphi \tilde{G}^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$\mathcal{O}^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\varphi W}$	$\varphi^\dagger\varphi W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\tilde{\varphi} G^A_{\mu\nu}$	$\mathcal{O}_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\varphi\tilde{W}}$	$\varphi^\dagger\varphi \tilde{W}^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\tilde{\varphi} W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\varphi B}$	$\varphi^\dagger\varphi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{\varphi} B_{\mu\nu}$	$\mathcal{O}^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\varphi\tilde{B}}$	$\varphi^\dagger\varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi G^A_{\mu\nu}$	$\mathcal{O}_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi W^I_{\mu\nu}$	$\mathcal{O}_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\varphi\tilde{W}B}$	$\varphi^\dagger\tau^I\varphi \tilde{W}^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi B_{\mu\nu}$	$\mathcal{O}_{\varphi ud}$	$(\tilde{\varphi}^\dagger iD_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 3.1: *Dimension six operators other than four-fermion and B-violating ones. Indices p, r, s and t=1, 2, 3 indicate the generation and the dependence of the operators in these indices is not explicitly written. L and R indices have been dropped. X stands for G^A, W^I or B.*

4 Charged Lepton Flavor Violating decays

Lepton flavor conservation is an accidental symmetry of the SM which is not required by the gauge structure of the theory. The first evidence for LFV was neutrino oscillation. The introduction of a neutrino mass term in the SM also gives rise to LFV in the charged sector but with branching fractions smaller than $\sim 10^{-40}$. However, many BSM scenarios, such as Super Symmetry (SUSY) or Extra Dimensions, with new heavy particles entering into virtual loops, predict branching ratios in the range $10^{-12} - 10^{-14}$, which is close to the current experimental sensitivities. Therefore, observation of such processes would be an unambiguous sign of new physics beyond the SM, while improvements on the current limits would strictly constraint many new physics scenarios. Therefore, searching for LFV channels constitutes a good probe for new physics.

Moreover, on the experimental side, the sensitivity in the search of LFV has been increased by many orders of magnitude over the years mainly by Belle, Babar, MEG and SINDRUM collaborations [17, 18, 19, 20, 21, 22]. Figure 4.1 illustrates some of the experimental bounds on the branching fractions for LFV τ decays. Furthermore, new results have been recently published for LFV τ decays searches at LHCb [1] with a bound on $\tau^- \to \mu^-\mu^+\mu^-$ approaching the best experimental upper limit from Belle [21].

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\overline{e}_p\gamma_\mu e_r)(\overline{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\overline{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\overline{q}_p\gamma_\mu q_r)(\overline{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\overline{u}_p\gamma_\mu u_r)(\overline{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\overline{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\overline{q}_p\gamma_\mu\tau^I q_r)(\overline{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\overline{d}_p\gamma_\mu d_r)(\overline{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\overline{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\overline{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\overline{e}_p\gamma_\mu e_r)(\overline{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\overline{q}_p\gamma_\mu q_r)(\overline{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\overline{e}_p\gamma_\mu e_r)(\overline{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\overline{q}_p\gamma_\mu q_r)(\overline{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\overline{u}_p\gamma_\mu u_r)(\overline{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\overline{u}_p\gamma_\mu T^A u_r)(\overline{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\overline{q}_p\gamma_\mu q_r)(\overline{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\overline{q}_p\gamma_\mu T^A q_r)(\overline{d}_s\gamma^\mu T^A d_t)$

$(\overline{L}R)(\overline{R}L)$ and $(\overline{L}R)(\overline{L}R)$		B-violating $(\Delta B = \Delta L)$	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\overline{d}_s q_t^j)$	$\mathcal{O}_{RL}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\overline{(d_p^\alpha)^C}u_r^\beta\overline{(q_s^{\gamma i})^C}l_t^j$
$\mathcal{O}_{quqd}^{(1)}$	$(\overline{q}_p^j u_r)\epsilon_{jk}(\overline{q}_s^k d_t)$	$\mathcal{O}_{LR}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\overline{(q_p^{\alpha i})^C}q_r^{\beta j}\overline{(u_s^\gamma)^C}e_t$
$\mathcal{O}_{quqd}^{(8)}$	$(\overline{q}_p^j T^A u_r)\epsilon_{jk}(\overline{q}_s^k T^A d_t)$	$\mathcal{O}_{LL}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}\overline{(q_p^{\alpha i})^C}q_r^{\beta j}\overline{(q_s^{\gamma k})^C}l_t^l$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\overline{q}_s^k u_t)$	$\mathcal{O}_{RR}$	$\epsilon_{\alpha\beta\gamma}\overline{(d_p^\alpha)^C}u_r^\beta\overline{(u_s^\gamma)^C}e_t$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\overline{q}_s^k\sigma^{\mu\nu}u_t)$		

Table 3.2: *Four-fermion and B-violating operators. Indices p, r, s and t=1, 2, 3 indicate the generation and the dependence of the operators in these indices is not explicitly written. L and R indices have been dropped. In the case of B-violating operators, we only consider operators involving quarks u, d and s. Charge conjugation is defined such that* $\psi^C = C\overline{\psi}^T$ *with* $C = i\gamma^2\gamma^0$.

In this section we will use the EQFT extension of the SM introduced in Subsection 3.2 in order to enforce some constraints in $\ell_i \to \ell_j\gamma$ and $\ell_i \to \ell_j\ell_k\ell_l$ decays. Finally, an estimate of the new physics scale assuming that the operators have natural coefficients will be given.

4.1 $\ell_i \to \ell_j\gamma$ decays

By inspecting the set of operators given in Tables 3.1 and 3.2 it is clear those which contribute are:

$$\begin{aligned}
\mathcal{O}_{eW} &= \left(\bar{l}_p\sigma^{\mu\nu}e_r\right)\tau^I\varphi W_{\mu\nu}^I, \\
\mathcal{O}_{eB} &= \left(\bar{l}_p\sigma^{\mu\nu}e_r\right)\varphi B_{\mu\nu}, \\
\mathcal{O}_{e\varphi} &= \left(\varphi^\dagger\varphi\right)\left(\bar{l}_p e_r\varphi\right).
\end{aligned} \tag{4.1}$$

After SSB, the Higgs fields acquire a vev and the W^I and B fields mix in the mass diagonal

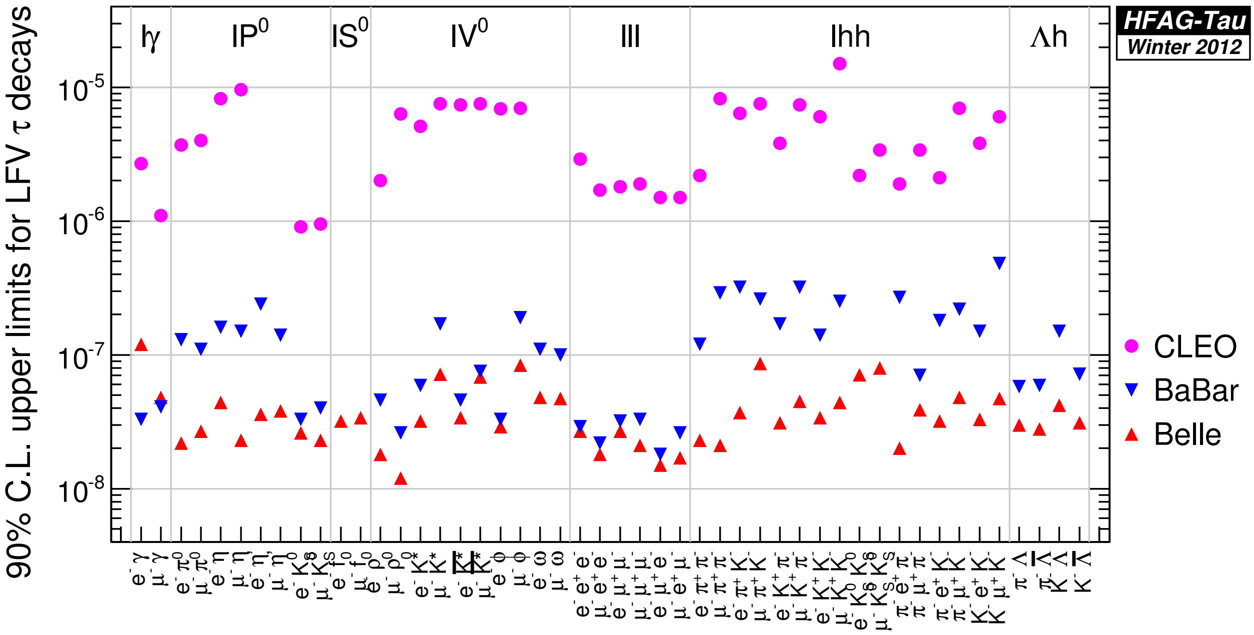

Figure 4.1: *Experimental bounds to branching ratios of hadronic and leptonic LFV τ decays. Figure from HFAG-Tau Report, Early 2012 [16].*

basis. The relevant operators for our process are then:

$$\mathcal{O}_{e\gamma} = \frac{v}{\sqrt{2}} \left(\bar{\ell}_p \sigma^{\mu\nu} e_r\right) F_{\mu\nu},$$
$$\mathcal{O}_{ev^3} = \frac{v^3}{2\sqrt{2}} \left(\bar{\ell}_p e_r\right), \tag{4.2}$$

where the convention for the Higgs vev is given by $\langle \varphi \rangle = v/\sqrt{2}$ with $v \simeq 246$ GeV and being $F_{\mu\nu}$ the photon field strength tensor. It can be shown, however, that the contribution of the last operator in (4.2) vanishes. Indeed, the matrix element for the diagrams where the operators $\mathcal{O}_{ev^3}$ appear (see Figure 4.2) is computed below. Also, notice that the operators in (4.2) with $p = i$ and $r = j$ are different than the ones with $p = j$ and $r = i$ and hence we have a different coupling for the different chiralities of the fields. For simplicity we will only consider left-handed coupling. Identical result can be obtained for the right-handed one. Then:

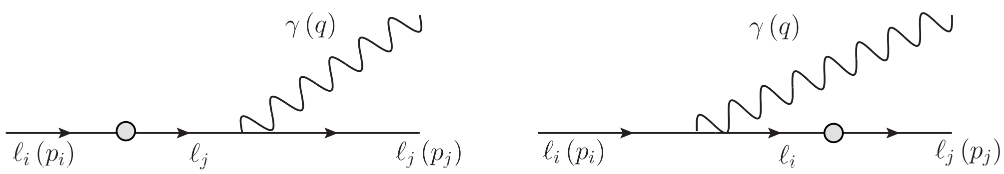

Figure 4.2: *Feynman diagrams for the decay $\ell_i \to \ell_j \gamma$ where the operators $\mathcal{O}_{ev^3}$ are involved. The circle stands for an $\mathcal{O}_{ev^3}$ vertex.*

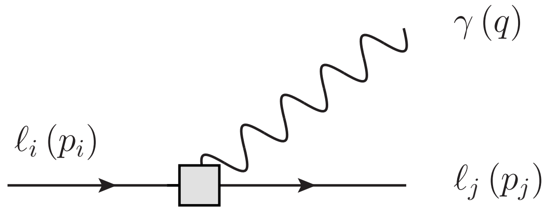

Figure 4.3: *Feynman diagram contributing to $\ell_i \to \ell_j\gamma$ decay. The square stands for $\mathcal{O}_{e\gamma}$ vertices.*

$$
\begin{aligned}
i\tilde{\mathscr{M}}_L &= \frac{i\beta_L v^3 e}{2\sqrt{2}\Lambda^2}\epsilon^\mu\left[\left(\overline{u}_j\gamma_\mu\frac{\not{p}_i + m_{\ell_j}}{m_{\ell_i}^2 - m_{\ell_j}^2}P_L u_i\right) + \left(\overline{u}_j P_L\frac{\not{p}_j + m_{\ell_i}}{m_{\ell_j}^2 - m_{\ell_i}^2}\gamma_\mu u_i\right)\right] \\
&= \frac{i\beta_L v^3 e}{2\sqrt{2}\Lambda^2}\epsilon^\mu\left[\left(\overline{u}_j\gamma_\mu\frac{m_{\ell_i}P_R + m_{\ell_j}P_L}{m_{\ell_i}^2 - m_{\ell_j}^2}u_i\right) + \left(\overline{u}_j\frac{m_{\ell_j}P_R + m_{\ell_i}P_L}{m_{\ell_j}^2 - m_{\ell_i}^2}\gamma_\mu u_i\right)\right] \\
&= \frac{i\beta_L v^3 e}{2\sqrt{2}\Lambda^2}\epsilon^\mu\left[\left(\overline{u}_j\frac{m_{\ell_i}P_L + m_{\ell_j}P_R}{m_{\ell_i}^2 - m_{\ell_j}^2}\gamma_\mu u_i\right) - \left(\overline{u}_j\frac{m_{\ell_j}P_R + m_{\ell_i}P_L}{m_{\ell_i}^2 - m_{\ell_j}^2}\gamma_\mu u_i\right)\right] \\
&= 0,
\end{aligned}
\tag{4.3}
$$

being β_L the coupling associated with $\mathcal{O}_{ev^3}$ and where we have used the Dirac equation in both terms and chirality projector properties. Therefore, as we see, the two diagrams cancel each other and, consequently, only the $\mathcal{O}_{e\gamma}$ operator contributes.

Taking that into account, the matrix element for the $\ell_i \to \ell_j\gamma$ decay is given by (see Figure 4.3):

$$
i\mathscr{M} = -\frac{2v}{\sqrt{2}\Lambda^2}q_\mu\epsilon_\nu^*\left[\alpha_L\left(\overline{u}_j\sigma^{\mu\nu}P_L u_i\right) + \alpha_R\left(\overline{u}_j\sigma^{\mu\nu}P_R u_i\right)\right]. \tag{4.4}
$$

Again, notice that $\mathcal{O}_{e\gamma}$ contains two operators contributing to the process, one for each chirality of the field. Parameters α_L and α_R denote the coupling to the left and right chiralities of the ℓ_j field, respectively.

Taking the square and summing over photon polarization and final lepton spin and averaging over initial lepton spin we get:

$$
\begin{aligned}
\left|\overline{\mathscr{M}}\right|^2 = \frac{v^2}{\Lambda^4}\sum_{\lambda=1}^{2}\epsilon_\rho^*\epsilon_\sigma q_\mu q_\nu\Big[&\left|\alpha_L\right|^2\ \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_L\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_R\right\} \\
&+\left|\alpha_R\right|^2\ \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_R\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_L\right\} \\
&+\alpha_L\alpha_R^*\ \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_R\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_R\right\} \\
&+\alpha_L^*\alpha_R\ \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_L\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_L\right\}\Big].
\end{aligned}
\tag{4.5}
$$

In the system where the lepton ℓ_i is at rest, it is possible to choose the physical polarization vectors of the photon such that they simultaneously satisfy:

$$
\begin{aligned}
&\epsilon\left(\vec{q},\lambda\right)\cdot p_i = 0, \\
&\epsilon\left(\vec{q},\lambda\right)\cdot q = 0,
\end{aligned}
\tag{4.6}
$$

with $\lambda = 1, 2$. In this case,

$$\epsilon\left(\vec{q},\lambda\right) = \left(0, \vec{\epsilon}\left(\vec{q},\lambda\right)\right), \tag{4.7}$$

with $\vec{\epsilon}(\vec{q},\lambda)\cdot\vec{\epsilon}(\vec{q},\lambda) = 1$.

Using these properties, we can simplify the traces considerably as:

$$\begin{aligned} q_\mu \epsilon_\rho \sigma^{\mu\rho} &= \frac{i}{2}\left(\not{q}\not{\epsilon} - \not{\epsilon}\not{q}\right) \\ &= \frac{i}{2}\left(\not{q}\not{\epsilon} + \not{q}\not{\epsilon} - 2\cancel{\epsilon\cdot q}\right) \\ &= i\not{q}\not{\epsilon}, \end{aligned} \tag{4.8}$$

and therefore,

$$\begin{aligned} \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_R\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_L\right\} q_\mu q_\nu \epsilon_\rho \epsilon_\sigma &= -\ \mathrm{Tr}\left\{\not{p}_j\not{q}\not{\epsilon}\not{p}_i\not{q}\not{\epsilon}P_L\right\} \\ &= -\ \mathrm{Tr}\left\{\not{p}_j\not{q}\epsilon^2\not{p}_i\not{q}P_L\right\} \\ &= \ \mathrm{Tr}\left\{\not{q}\not{p}_j\right\}\left(p_i\cdot q\right) \\ &= 4\left(q\cdot p_j\right)\left(q\cdot p_i\right), \end{aligned} \tag{4.9}$$

where in the first line we have eliminated the terms with masses because of the chirality flip and where we have used the properties of the chirality projector as well as the properties of the polarization vector stated above. Finally, in the third line we have used that the photon is on-shell and thus $\not{q}\not{p}_i\not{q} = 2\left(q\cdot p_i\right)\not{q}$.

As the trace with a single γ_5 does not contribute, the result from the trace is the same for the term proportional to α_L and the term proportional to α_R. Finally, for the trace coming from the interference terms,

$$\begin{aligned} \mathrm{Tr}\left\{\left(\not{p}_j + m_{\ell_j}\right)\sigma^{\mu\rho}P_R\left(\not{p}_i + m_{\ell_i}\right)\sigma^{\nu\sigma}P_R\right\} q_\mu q_\nu \epsilon_\rho \epsilon_\sigma &= -m_{\ell_i}m_{\ell_j}\ \mathrm{Tr}\left\{\not{q}\not{\epsilon}\not{q}\not{\epsilon}P_R\right\} \\ &= m_{\ell_i}m_{\ell_j}\ \mathrm{Tr}\left\{\not{\epsilon}\not{q}\not{q}\not{\epsilon}P_R\right\} \\ &= 0, \end{aligned} \tag{4.10}$$

because the photon is on-shell, i.e. $q^2 = 0$.

Putting everything together (notice that there is an extra factor 2 coming from the sum over the photon physical polarizations) we get:

$$\left|\overline{\mathscr{M}}\right|^2 = \frac{8v^2}{\Lambda^4}\left(|\alpha_L| + |\alpha_R|^2\right)\left(q\cdot p_i\right)\left(q\cdot p_j\right). \tag{4.11}$$

Finally, as

$$p_i = q + p_j \Rightarrow \begin{cases} p_i^2 = m_{\ell_i}^2 = m_{\ell_j}^2 + 2\left(q\cdot p_j\right) \\ p_j^2 = m_{\ell_j}^2 = m_{\ell_i}^2 - 2\left(q\cdot p_i\right) \end{cases}, \tag{4.12}$$

we obtain:

$$\left|\overline{\mathscr{M}}\right|^2 = \frac{2v^2}{\Lambda^4}\left(|\alpha_L| + |\alpha_R|^2\right)\left(m_{\ell_i}^2 - m_{\ell_j}^2\right)^2. \tag{4.13}$$

The decay width is hence given by:

$$\Gamma\left(\ell_i \to \ell_j\gamma\right) = \frac{\sqrt{\lambda\left(m_{\ell_i}^2, m_{\ell_j}^2, 0\right)}}{16\pi m_{\ell_i}^3}\left|\overline{\mathscr{M}}\right|^2$$
$$= \frac{v^2}{8\pi m_{\ell_i}^3\Lambda^4}\left(m_{\ell_i}^2 - m_{\ell_j}^2\right)^3\left(|\alpha_L|^2 + |\alpha_R|^2\right), \tag{4.14}$$

where we have introduced the Källén function $\lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

4.2 $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ decays

The discussion about which operators contribute to these processes is more involved than the one from the previous section. By inspecting the set of operators given in Tables 3.1 and 3.2, we get the following operators which, in principle, contribute to the process:

$$\begin{aligned}
\mathcal{O}_{eW} &= \left(\bar{l}_p\sigma^{\mu\nu}e_r\right)\tau^I\varphi W_{\mu\nu}^I,\\
\mathcal{O}_{eB} &= \left(\bar{l}_p\sigma^{\mu\nu}e_r\right)\varphi B_{\mu\nu},\\
\mathcal{O}_{e\varphi} &= \left(\varphi^\dagger\varphi\right)\left(\bar{l}_p e_r\varphi\right),\\
\mathcal{O}_{\varphi l}^{(1)} &= \left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)\left(\bar{l}_p\gamma^\mu l_r\right),\\
\mathcal{O}_{\varphi l}^{(3)} &= \left(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi\right)\left(\bar{l}_p\tau^I\gamma^\mu l_r\right),\\
\mathcal{O}_{ll} &= \left(\bar{l}_p\gamma_\mu l_r\right)\left(\bar{l}_s\gamma^\mu l_t\right),\\
\mathcal{O}_{ee} &= \left(\bar{e}_p\gamma_\mu e_r\right)\left(\bar{e}_s\gamma^\mu e_t\right),\\
\mathcal{O}_{le} &= \left(\bar{l}_p\gamma_\mu l_r\right)\left(\bar{e}_s\gamma^\mu e_t\right).
\end{aligned}\tag{4.15}$$

After SSB, the relevant pieces from the previous operators are:

$$\begin{aligned}
\mathcal{O}_{e\gamma} &= \frac{v}{\sqrt{2}}\left(\bar{\ell}_p\sigma^{\mu\nu}e_r\right)F_{\mu\nu},\\
\mathcal{O}_{eZ} &= \frac{v}{\sqrt{2}}\left(\bar{\ell}_p\sigma^{\mu\nu}e_r\right)Z_{\mu\nu},\\
\mathcal{O}_{eH} &= \frac{v^2}{\sqrt{2}}H\left(\bar{\ell}_p e_r\right),\\
\mathcal{O}_{ev^3} &= \frac{v^3}{2\sqrt{2}}\left(\bar{\ell}_p e_r\right),\\
\mathcal{O}_{\ell H} &= i\frac{v}{\sqrt{2}}\partial_\mu H\left(\bar{\ell}_p\gamma^\mu\ell_r\right),\\
\mathcal{O}_{\ell\ell} &= \left(\bar{\ell}_p\gamma_\mu\ell_r\right)\left(\bar{\ell}_s\gamma^\mu\ell_t\right),\\
\mathcal{O}_{ee} &= \left(\bar{e}_p\gamma_\mu e_r\right)\left(\bar{e}_s\gamma^\mu e_t\right),\\
\mathcal{O}_{\ell e} &= \left(\bar{\ell}_p\gamma_\mu\ell_r\right)\left(\bar{e}_s\gamma^\mu e_t\right).
\end{aligned}\tag{4.16}$$

We can apply the same result that we got in the previous section to show that operators $\mathcal{O}_{ev^3}$ do not contribute so we will not consider them in the following. Still, there are far too many operators to obtain interesting constraints. However, not every operator contributes the same. By analyzing the structure of the diagrams involving the exchange of a virtual Higgs, which would be mediated by operators $\mathcal{O}_{eH}$ and $\mathcal{O}_{lH}$, it is straightforward to check that apart from the coefficients, which are expected to be $\mathcal{O}(1)$, their contribution is at most $\mathcal{O}\left(vm_\mu/m_H^2\right)$ and $\mathcal{O}\left(m_\mu m_\tau/m_H^2\right)$, respectively, while contribution due to the Z and photon exchange will be $\mathcal{O}\left(gvm_\tau/m_Z^2\right)$ and $\mathcal{O}\left(ev/m_\tau\right)$, respectively.

We see then that operators $\mathcal{O}_{e\gamma}$ are enhanced while the other operators are either suppressed (Z and H particles contribution) or $\mathcal{O}(1)$ (four fermion operators) and consequently it is reasonable to only consider $\mathcal{O}_{e\gamma}$ in the decay analyses.

However, if we assume that the high-energy theory is a general weakly coupled gauge theory consisting of scalars, fermions and vectors, operators $\mathcal{O}_{e\gamma}$ are loop generated [23] and therefore, naturalness implies that the coefficients should be $\mathcal{O}\left(1/16\pi^2\right)$ while four fermion operators might be tree-level generated so its coefficients are expected to be $\mathcal{O}(1)$. This suppression competes with vev enhancement. Nonetheless, if we impose that there are no LFV vertices between two SM fermions and a vector or a scalar, as it happens in some SUSY theories [24], all the operators are loop generated and the argument reinstates. Moreover, if the gauge theory is strongly coupled, the distinction between tree level and loop generated operators makes no sense [25] and hence $\mathcal{O}_{e\gamma}$ still dominates.

In the following, we will compute the decay $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ mediated by photon exchange contribution. Z exchange contribution, although expected to be suppressed, will also be included in the analysis because $\mathcal{O}_{e\gamma}$ and $\mathcal{O}_{eZ}$ have analogous structure and because it will allow us to test the arguments given above.

As it happened when studying the $\ell_i \to \ell_j\gamma$ decay, there are different contributions to the different chiralities from $\mathcal{O}_{e\gamma}$ and $\mathcal{O}_{eZ}$ operators which are denoted by the subindex L and R to determine left and right-handed chirality respectively. We will use $\alpha_{L,R}$ to denote the photon coupling and $\tilde{\alpha}_{L,R}$ for the Z coupling. At first order, these couplings are related to the W and the B couplings and the weak angle in the following way:

$$\begin{aligned}\alpha_{L,R} &= \alpha_{L,R}^W \sin\theta_W + \alpha_{L,R}^B \cos\theta_W,\\ \tilde{\alpha}_{L,R} &= \alpha_{L,R}^W \cos\theta_W - \alpha_{L,R}^B \sin\theta_W,\end{aligned} \tag{4.17}$$

where α^W and α^B denote the W and the B couplings respectively.

Finally, to perform the computation, we have to distinguish the cases when the leptons in the final state are the same and when they are different.

4.2.1 Different leptons in the final state

Applying Feynman rules to the diagram shown in Figure 4.4 we get the following matrix element:

$$\begin{aligned}i\mathscr{M} =& \frac{2\alpha_L v}{\sqrt{2}\Lambda^2} e\left\{\frac{q_\mu}{q^2}\left(\overline{u}_3\sigma^{\mu\nu}P_L u\right)\left(\overline{u}_2\gamma_\nu v_1\right)\right\} + \frac{2\tilde{\alpha}_L v}{\sqrt{2}\Lambda^2}\frac{g}{2c_W}\left\{\frac{q_\mu}{q^2-M_z^2}\left(g_{\nu\rho}-\frac{q_\nu q_\rho}{M_z^2}\right)\left(\overline{u}_3\sigma^{\mu\nu}P_L u\right)\right.\\ &\left.\times\left[\overline{u}_2\gamma^\rho\left(v_{\ell_k}+a_{\ell_k}\gamma_5\right)v_1\right]\right\} + (L\to R)\,.\end{aligned} \tag{4.18}$$

where $(L \to R)$ denote the interchange of the left-handed chirality to the right-handed one.

The terms proportional to $q_\mu q_\nu$ give no contribution due to the antisymmetry of $\sigma^{\mu\nu}$. More-

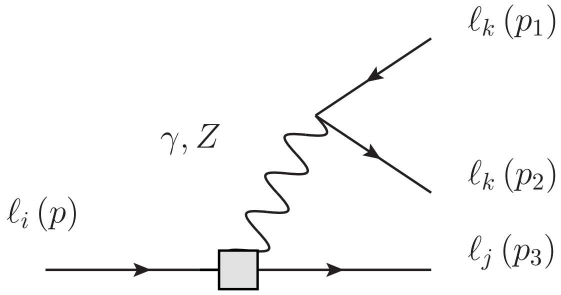

Figure 4.4: *Feynman diagram contributing to $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ decay. The square stands for $\mathcal{O}_{e\gamma}$ and $\mathcal{O}_{eZ}$ vertices.*

over, at $\mathcal{O}\left(1/\Lambda^2\right)$ we can substitute $g = e/s_W$,

$$\begin{aligned} i\mathscr{M} =& \frac{2ve}{\sqrt{2}\Lambda^2}\left\{\left[\frac{\alpha_L}{q^2} + \frac{\tilde{\alpha}_L v_{\ell_k}}{2 s_W c_W\left(q^2 - M_z^2\right)}\right]\left(\overline{u}_3 \sigma^{\mu\nu} P_L u\right) q_\mu \left(\overline{u}_2 \gamma_\nu v_1\right)\right. \\ & \left. + \frac{\tilde{\alpha}_L a_{\ell_k}}{2 s_W c_W\left(q^2 - M_z^2\right)}\left(\overline{u}_3 \sigma^{\mu\nu} P_L u\right) q_\mu \left(\overline{u}_2 \gamma_\nu \gamma_5 v_1\right) + (L \to R)\right\}. \end{aligned} \tag{4.19}$$

Before taking the square to get the amplitude, let us organize the calculation. First we define:

$$\begin{aligned} V_L &\equiv \frac{\alpha_L}{q^2} + \frac{v_{\ell_k}}{2 c_W s_W}\frac{\tilde{\alpha}_L}{\left(q^2 - M_z^2\right)}, \\ A_L &\equiv \frac{a_{\ell_k}}{2 c_W s_W}\frac{\tilde{\alpha}_L}{\left(q^2 - M_z^2\right)}, \end{aligned} \tag{4.20}$$

and analogously for right-handed coefficients.

Second, we can anticipate that, when squaring the amplitude, we will obtain the following traces:

$$\begin{aligned} I_1^{\mu\nu\rho\sigma} &\equiv \operatorname{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu} P_R \left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma} P_L\right\} = \operatorname{Tr}\left\{p_3 \sigma^{\mu\nu} \not{p} \sigma^{\rho\sigma} P_L\right\}, \\ I_2^{\mu\nu\rho\sigma} &\equiv \operatorname{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu} P_L \left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma} P_R\right\} = \operatorname{Tr}\left\{p_3 \sigma^{\mu\nu} \not{p} \sigma^{\rho\sigma} P_R\right\}, \\ I_3^{\mu\nu\rho\sigma} &\equiv \operatorname{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu} P_R \left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma} P_R\right\} = m_{\ell_i} m_{\ell_j} \operatorname{Tr}\left\{\sigma^{\mu\nu}\sigma^{\rho\sigma} P_R\right\}, \\ I_4^{\mu\nu\rho\sigma} &\equiv \operatorname{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu} P_L \left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma} P_L\right\} = m_{\ell_i} m_{\ell_j} \operatorname{Tr}\left\{\sigma^{\mu\nu}\sigma^{\rho\sigma} P_L\right\}, \\ G_{ij,\nu\sigma} &\equiv \operatorname{Tr}\left\{\left(\not{p}_2 + m_{\ell_k}\right)\gamma_\nu \left(V_i + A_i \gamma_5\right)\left(\not{p}_1 - m_{\ell_k}\right)\gamma_\sigma \left(V_j^* + A_j^* \gamma_5\right)\right\} \quad \text{for } i,j = L,R, \end{aligned} \tag{4.21}$$

where, in order to simplify, we have taken into account that:

1. γ_5 anticommutes with γ_μ and $\gamma_5^2 = I$.
2. The trace of an odd number of $\gamma' s$ is zero.

Now, it is straightforward to check that the amplitude takes the form:

$$\overline{|\mathscr{M}|}^2 = \frac{v^2 e^2}{\Lambda^4}\left(I_1^{\mu\nu\rho\sigma} G_{RR,\nu\sigma} + I_2^{\mu\nu\rho\sigma} G_{LL,\nu\sigma} + I_3^{\mu\nu\rho\sigma} G_{RL,\nu\sigma} + I_4^{\mu\nu\rho\sigma} G_{LR,\nu\sigma}\right) q_\mu q_\rho. \tag{4.22}$$

Finally, introducing the kinematic variables:

$$\begin{aligned} m_{12}^2 &= (p_1+p_2)^2 = (p-p_3)^2\,, \\ m_{13}^2 &= (p_1+p_3)^2 = (p-p_2)^2\,, \\ m_{23}^2 &= (p_2+p_3)^2 = (p-p_1)^2\,, \end{aligned} \tag{4.23}$$

with

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = \sum_i m_i^2, \tag{4.24}$$

and using FORM [26] for the traces computation and *Mathematica*© for the phase space integration, the decay widths are obtained. Results are shown in Table 4.1.

4.2.2 Identical leptons in the final state

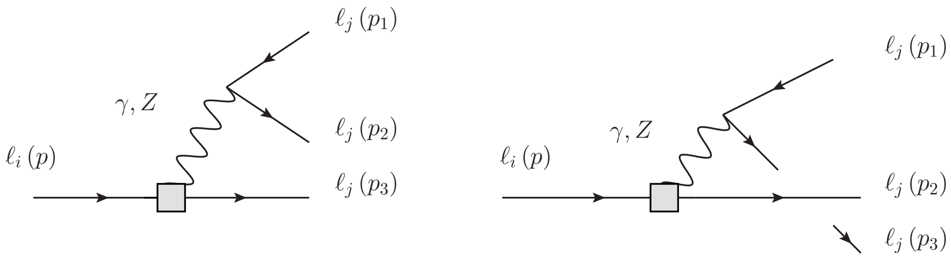

Figure 4.5: *Feynman diagrams contributing to* $\ell_i^\pm \to \ell_j^\pm \ell_j^+ \ell_j^-$ *decay. The square stands for* $\mathcal{O}_{e\gamma}$ *and* $\mathcal{O}_{eZ}$ *vertices.*

In this case, the decay amplitude reads:

$$\left|\overline{\mathscr{M}}\right|^2 = \left|\overline{\mathscr{M}}_{12}\right|^2 + \left|\overline{\mathscr{M}}_{13}\right|^2 + |\overline{\mathscr{M}}_{\rm Int}|^2, \tag{4.25}$$

where $\left|\overline{\mathscr{M}}_{12}\right|^2$ is the amplitude for the first diagram in Figure 4.5 which has been calculated previously and $\left|\overline{\mathscr{M}}_{13}\right|^2$ is the amplitude for the second diagram which is related to the previous one by crossing. Therefore, apart from the interference term between the diagrams, $\left|\overline{\mathscr{M}}_{\rm Int}\right|^2$, the computation of the process is immediate. Hence, we will focus exclusively in the interference term. Using the same notation, we have:

$$\begin{aligned} |\mathscr{M}_{\rm Int}|^2 = -\frac{2v^2e^2}{\Lambda^4} &\left\{\left[(\overline{u}_3\sigma^{\mu\nu}P_L u)\, q_\mu \left(\overline{u}_2\gamma_\nu \left(V_{(12),L} + A_{(12),L}\gamma_5\right) v_1\right) + (L\to R)\right]\right. \\ &\times \left.\left[(\overline{u}\sigma^{\rho\sigma}P_R u_2)\, q'_\rho \left(\overline{v}_1\gamma_\sigma \left(V^*_{(13),L} + A^*_{(13),L}\gamma_5\right) u_3\right) + (L\to R)\right] + h.c.\right\}, \end{aligned} \tag{4.26}$$

where now we have added an extra subindex in $A_{(ij)}$, $V_{(ij)}$ to indicate whether we have m_{12}^2 or m_{13}^2 in the propagator, i.e. whether the term comes from the first or the second diagram in

Figure 4.5. Expanding and taking traces, we will obtain the following terms:

$$
\begin{aligned}
J_1^{\mu\rho} \equiv \; & \mathrm{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu}P_R\left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma}P_L\left(\not{p}_2 + m_{\ell_j}\right)\gamma_\nu\left(V_{(12),R} + A_{(12),R}\gamma_5\right)\right. \\
& \left.\left(\not{p}_1 - m_{\ell_j}\right)\gamma_\sigma\left(V^*_{(13),R} + A^*_{(13),R}\gamma_5\right)\right\}, \\
J_2^{\mu\rho} \equiv \; & \mathrm{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu}P_L\left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma}P_R\left(\not{p}_2 + m_{\ell_j}\right)\gamma_\nu\left(V_{(12),L} + A_{(12),L}\gamma_5\right)\right. \\
& \left.\left(\not{p}_1 - m_{\mu}\right)\gamma_\sigma\left(V^*_{(13),L} + A^*_{(13),L}\gamma_5\right)\right\}, \\
J_3^{\mu\rho} \equiv \; & \mathrm{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu}P_R\left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma}P_R\left(\not{p}_2 + m_{\ell_j}\right)\gamma_\nu\left(V_{(12),R} + A_{(12),R}\gamma_5\right)\right. \\
& \left.\left(\not{p}_1 - m_{\ell_j}\right)\gamma_\sigma\left(V^*_{(13),L} + A^*_{(13),L}\gamma_5\right)\right\}, \\
J_4^{\mu\rho} \equiv \; & \mathrm{Tr}\left\{\left(\not{p}_3 + m_{\ell_j}\right)\sigma^{\mu\nu}P_L\left(\not{p} + m_{\ell_i}\right)\sigma^{\rho\sigma}P_L\left(\not{p}_2 + m_{\ell_j}\right)\gamma_\nu\left(V_{(12),L} + A_{(12),L}\gamma_5\right)\right. \\
& \left.\left(\not{p}_1 - m_{\ell_j}\right)\gamma_\sigma\left(V^*_{(13),R} + A^*_{(13),R}\gamma_5\right)\right\},
\end{aligned}
\tag{4.27}
$$

and, therefore,

$$
|\overline{\mathscr{M}}_{\mathrm{Int}}|^2 = -\frac{v^2 e^2}{\Lambda^4} q_\mu q'_\rho \left(J_1^{\mu\rho} + J_2^{\mu\rho} + J_3^{\mu\rho} + J_4^{\mu\rho} + h.c.\right). \tag{4.28}
$$

Once more, using FORM for the trace computation and *Mathematica*© for the phase space integration we have calculated the decay width. Results will be shown in Table 4.1. Notice that an extra factor 1/2 have to be included after integration in phase space because we have two identical particles in the final state.

4.3 Results, bounds to new physics and phenomenological constraints

From the analysis previously performed, we obtained the following structure for the decay widths:

$$
\begin{aligned}
\Gamma\left(\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-\right) = \frac{1}{\Lambda^4} & \left[A\left(|\alpha_L|^2 + |\alpha_R|^2\right) + B\ \mathrm{Re}\left\{\alpha_R\alpha_L^*\right\} + C_L|\tilde{\alpha}_L|^2\right. \\
& + C_R|\tilde{\alpha}_R|^2 + D_L\ \mathrm{Re}\left\{\alpha_L\tilde{\alpha}_L^*\right\} + D_R\ \mathrm{Re}\left\{\alpha_R\tilde{\alpha}_R^*\right\} \\
& \left. + E\left(\mathrm{Re}\left\{\alpha_R\tilde{\alpha}_L^*\right\} + \mathrm{Re}\left\{\tilde{\alpha}_R\alpha_L^*\right\}\right) + F\ \mathrm{Re}\left\{\tilde{\alpha}_R\tilde{\alpha}_L^*\right\}\right],
\end{aligned}
\tag{4.29}
$$

where the values of the coefficients are shown in Table 4.1. Remember that $\alpha_{L,R}$ is associated with the photon coupling while $\tilde{\alpha}_{L,R}$ is associated with the Z coupling.

As can be seen in Table 4.1, the Z boson exchange contribution is heavily suppressed with respect the photon exchange contribution. Indeed, every coefficient associated to terms with the couplings $\tilde{\alpha}_L$, $\tilde{\alpha}_R$ or both are several orders of magnitude smaller than the ones which only involve $\alpha_{L,R}$. This results can be perfectly understood when considering the discussion we made at the beginning of the section when we evaluated which operators dominate in the decay width computation. Also, notice that the interference term between the two photonic diagrams encoded in B is several orders of magnitude smaller than the leading term. The reason for this to happen is that, as we saw in $\ell_i \to \ell_j\gamma$ analysis, the interference term vanishes for a real photon.

The results we got from the previous analyses can be used to get a first idea on where the scale of new physics, Λ, is located. In order to do that, we assume that coefficients have

Coefficient	$\tau^\pm \to \mu^\pm\mu^+\mu^-$	$\tau^\pm \to \mu^\pm e^+e^-$	$\tau^\pm \to e^\pm\mu^+\mu^-$	$\tau^\pm \to e^\pm e^+e^-$	$\mu^\pm \to e^\pm e^+e^-$
A	30.64	137.84	28.05	142.08	1.74×10^{-2}
B	-2.51	-1.79	-8.67×10^{-3}	-1.13×10^{-2}	-3.99×10^{-5}
C_L	5.19×10^{-8}	5.10×10^{-8}	4.96×10^{-8}	5.94×10^{-8}	1.56×10^{-16}
C_R	5.37×10^{-8}	5.10×10^{-8}	4.96×10^{-8}	6.14×10^{-8}	1.62×10^{-16}
D_L	-7.46×10^{-5}	1.30×10^{-4}	1.27×10^{-4}	-1.39×10^{-4}	-1.03×10^{-10}
D_R	3.72×10^{-4}	1.30×10^{-4}	1.27×10^{-4}	4.52×10^{-4}	3.35×10^{-10}
E	-1.88×10^{-5}	-9.67×10^{-6}	-5.12×10^{-8}	-1.03×10^{-7}	-1.28×10^{-12}
F	-1.31×10^{-8}	-1.37×10^{-8}	-7.40×10^{-11}	-7.73×10^{-11}	-3.41×10^{-18}

Table 4.1: *Coefficients for the decay widths $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ and $\ell_i^\pm \to \ell_j^\pm \ell_j^+ \ell_j^-$ in units of GeV^5.*

natural values. One has to remember from the discussion performed in Subsection 4.2 that for a general weakly coupled gauge theory in the high energy sector, naturalness implies that the coefficients should be $\mathcal{O}(1/16\pi^2)$, as the dominant operators contributing to the process are loop generated. On the other hand, if the BSM theory is strongly coupled, naturalness suggests $\mathcal{O}(1)$ coefficients. Both scenarios are considered in Table 4.2 where bounds to the new physics scale, Λ, from LFV leptonic decays are obtained.

Process	Experimental branching ratio	Bound to new physics scale (TeV) Weakly coupled	Strongly coupled
$\tau^\pm \to \mu^\pm\gamma$	4.4×10^{-8} (BaBar)	$\Lambda \geq 57$	$\Lambda \geq 722$
	4.5×10^{-8} (Belle)		
$\tau^\pm \to e^\pm\gamma$	3.3×10^{-8} (BaBar)	$\Lambda \geq 62$	$\Lambda \geq 775$
	1.2×10^{-7} (Belle)		
$\mu^\pm \to e^\pm\gamma$	5.7×10^{-13} (MEG)	$\Lambda \geq 6039$	$\Lambda \geq 75892$
$\tau^\pm \to \mu^\pm\mu^+\mu^-$	2.1×10^{-8} (Belle)	$\Lambda \geq 15$	$\Lambda \geq 190$
	3.3×10^{-8} (BaBar)		
$\tau^\pm \to \mu^\pm e^+e^-$	1.8×10^{-8} (Belle)	$\Lambda \geq 23$	$\Lambda \geq 289$
	2.2×10^{-8} (BaBar)		
$\tau^\pm \to e^\pm\mu^+\mu^-$	2.7×10^{-8} (Belle)	$\Lambda \geq 14$	$\Lambda \geq 174$
	3.2×10^{-8} (BaBar)		
$\tau^\pm \to e^\pm e^+e^-$	2.7×10^{-8} (Belle)	$\Lambda \geq 21$	$\Lambda \geq 261$
	2.9×10^{-8} (BaBar)		
$\mu^\pm \to e^\pm e^+e^-$	1.0×10^{-12} (SINDRUM)	$\Lambda \geq 1469$	$\Lambda \geq 18461$

Table 4.2: *Bounds to new physics scale in an EQFT approach from experimental bounds to the branching ratios of LFV leptonic decays from Babar [19, 22], Belle [20, 21], MEG [17] and SINDRUM [18] collaborations.*

From the results we show in Table 4.2, it is clear that strongly coupled scenarios disfavor the possibility of a high energy LFV theory at the reach of future experiments. Moreover, LFV muon decays set a strong bound in the new physics scale in both scenarios while the bounds we obtained from tau LFV decays are not that severe.

As we do not know anything about the high energy theory, it could be possible that lepton universality is violated in LFV leptonic decays in such a way that LFV muon decays are suppressed while tau ones are not. The analysis of such scenarios is not the object of this master thesis. However, it is interesting to mention that there are some BSM theories which suggest such possibility. For example, a detailed examination of the $\mu - \tau$ LFV in the Unconstrained Minimal Supersymmetric framework shows that it is possible to accommodate LFV tau branching fractions as high as 10^{-7}, even with the strong experimental bounds on the LFV muon [27]. It seems thus reasonable to keep analyses of both decays separately as pure EQFT suggests.

On the other hand, as a consequence of the dominance of photon exchange contribution, some interesting phenomenological implications can be found for the LFV decays we studied. Indeed, as the interference terms for $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ and $\ell_i^\pm \to \ell_j^\pm \ell_j^+ \ell_j^-$ decays are small, we have the following fractions between branching ratios:

$$\frac{\mathcal{B}\left(\tau^\pm \to \mu^\pm e^+ e^-\right)}{\mathcal{B}\left(\tau^\pm \to \mu^\pm \gamma\right)} \simeq \frac{1}{82}, \qquad \frac{\mathcal{B}\left(\tau^\pm \to \mu^\pm \mu^+ \mu^-\right)}{\mathcal{B}\left(\tau^\pm \to \mu^\pm \gamma\right)} \simeq \frac{1}{370},$$
$$\frac{\mathcal{B}\left(\tau^\pm \to e^\pm \mu^+ \mu^-\right)}{\mathcal{B}\left(\tau^\pm \to e^\pm \gamma\right)} \simeq \frac{1}{481}, \qquad \frac{\mathcal{B}\left(\tau^\pm \to e^\pm e^+ e^-\right)}{\mathcal{B}\left(\tau^\pm \to \mu^\pm \gamma\right)} \simeq \frac{1}{95}. \tag{4.30}$$

These ratios are interesting as they allow us to impose stronger constraints on $\ell_i^\pm \to \ell_j^\pm \ell_k^+ \ell_k^-$ and $\ell_i^\pm \to \ell_j^\pm \ell_j^+ \ell_j^-$ decays from $\ell_i \to \ell_j \gamma$ ones.

Finally, we should mention that processes with LFV in more than one unit such as $\tau^- \to e^- e^- \mu^+$ were not included in the analysis because they can only be generated with four-fermion operators. Consequently, following the considerations from Subsection 4.2, their branching ratios are expected to be $\mathcal{O}\left(m_\tau / v^2 e\right)$ smaller than the ones from the decays we have studied.

5 ΔB = ΔL $\neq$ 0 decays

In order to explain the dominance of matter over antimatter in the universe BNV is required and so it is believed to have occurred in the early universe. In the SM, BNV can happen through nonperturbative effects which may give rise to processes which violate the combination B+L, but not the orthogonal combination B-L [28]. Nonetheless, the probability for this process to occur is extremely low and hence any measurement of BNV would have the track of new physics. Some BSM theories such as Grand Unified theories and models based on strings typically put quarks and leptons in the same multiplets which in general leads to B-violating processes which much larger probability. Moreover, in many BNV theories, B-L is conserved and no asymmetry in B-L can be generated.

In this master thesis, we will focus on the study of $\tau^+ \to p \mu^+ \mu^-$ and $\tau^+ \to \Lambda \pi^+$ decays in an EQFT framework. An experimental bound to $\tau^+ \to p \mu^+ \mu^-$ branching ratio has been established for the first time in LHCb [1] so its theoretical study is interesting. On the other side, BNV processes involving higher generations could play an important role in new physics scenarios. In this sense, a competitive bound to $\tau^+ \to \Lambda \pi^+$ branching ratio was set by the Belle Collaboration [29].

Because we will deal with hadrons in the final state, it is necessary to use χPT techniques in order to describe the hadronization process. Furthermore, vector resonances will play an important role in the calculation of $\tau^+ \to p \mu^+ \mu^-$ decay so we will have to use Resonance Chiral Theory (RχT) to compute the decay width. These tools will be introduced in the next subsection. In subsection 5.2, we will express the B-violating operators we have shown in Table 3.2 in terms of hadronic states. Finally, $\tau^+ \to p \mu^+ \mu^-$ and $\tau^+ \to \Lambda \pi^+$ decays will be computed in Subsections 5.3 and 5.4, respectively.

5.1 Chiral Lagrangian for mesons, baryons and vector resonances

As we are dealing with strongly interacting particles in a low energy regime, the use of χPT becomes mandatory. In Subsection 3.1, the general ideas of χPT have been introduced. In this section, we will follow a more practical approach where the Lagrangian which describes the dynamics of pseudoscalar mesons, baryons and vector resonances will be presented.

The chiral Lagrangian invariant under the chiral symmetry, $G = SU(3)_L \otimes SU(3)_R$, which describes mesons, baryons and vector resonances as well as electromagnetic and semileptonic weak interactions reads as follows:

$$\mathscr{L}(\phi, B, V) = \mathscr{L}_\chi^{(2)} + \mathscr{L}_B + \mathscr{L}_V + \mathscr{L}_{BV}, \tag{5.1}$$

being $\mathscr{L}_\chi^{(2)}$ the χPT Lagrangian for mesonic interactions up to $\mathcal{O}(p^2)$, which is given by [5, 6]:

$$\mathscr{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle, \tag{5.2}$$

with $\langle . \rangle$ denoting the trace in flavor space and being:

$$\begin{aligned} u_\mu &= i\left[u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right], \\ \chi_+ &= u^\dagger\chi u^\dagger + u\chi^\dagger u, \\ \chi &= 2B_0\left(s + ip\right). \end{aligned} \tag{5.3}$$

B_0 and F are constants whose numerical values are not fixed by symmetry requirements. F is related to the to the pion decay constant, $F \simeq F_\pi \simeq 92.4$ MeV, while B_0 is related to the quark condensate, $B_0 = -\langle 0|\overline{u}u|0\rangle/F^2$. The notation for the meson octet has been introduced in Subsection 3.1.

On the other hand, $\mathscr{L}_B$ describes baryon fields and their interaction with pseudoscalar mesons [30],

$$\mathscr{L}_B = \langle\overline{B}\left(i\nabla\!\!\!/ - M_B\right)B\rangle - \frac{D}{2}\langle\overline{B}\gamma^\mu\gamma_5\left\{u_\mu, B\right\}\rangle - \frac{F}{2}\langle\overline{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\rangle + \ldots, \tag{5.4}$$

where the dots stand for higher order terms and the $SU(3)$ octets of baryons B and $\overline{B}$ are defined as follows:

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \qquad \overline{B} = \begin{pmatrix} \frac{\overline{\Sigma}^0}{\sqrt{2}} + \frac{\overline{\Lambda}}{\sqrt{6}} & \overline{\Sigma}^- & -\overline{\Xi}^- \\ \overline{\Sigma}^+ & -\frac{\overline{\Sigma}^0}{\sqrt{2}} + \frac{\overline{\Lambda}}{\sqrt{6}} & \overline{\Xi}^0 \\ \overline{p} & \overline{n} & -\frac{2\overline{\Lambda}}{\sqrt{6}} \end{pmatrix}. \tag{5.5}$$

B transforms under $SU(3)_L \otimes SU(3)_R$ rotations as:

$$B \xrightarrow{G} h(g,\phi)\, B\, h(g,\phi)^{-1}, \tag{5.6}$$

being $h(g,\phi)$ the compensator field introduced in Equation (3.4).

The constants D and F may be determined by fitting the semi-leptonic decays $B \to B' e\overline{\nu}_e$ at tree level [31],

$$\begin{aligned} D &\simeq 0.80, \\ F &\simeq 0.50. \end{aligned} \tag{5.7}$$

The covariant derivative,

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B], \qquad \Gamma_\mu = \frac{1}{2}\left[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger\right], \tag{5.8}$$

is defined in such a way that $\nabla_\mu B$ also transforms as an octet under the action of the group.

Moreover, we want to characterize vector resonances and their interactions with the other fields. To the lowest order this is achieved by the following Lagrangian [32]:

$$\mathscr{L}_V = \mathscr{L}_K(V) + \mathscr{L}_2(V), \tag{5.9}$$

where $\mathscr{L}_K(V)$ describes the kinetic part:

$$\mathscr{L}_K(V) = -\frac{1}{2}\langle\nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2}V_{\mu\nu}V^{\mu\nu}\rangle, \tag{5.10}$$

being M_V the nonet mass in the limit where the chiral symmetry is exact and where $V_{\mu\nu}$ is:

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} \end{pmatrix}_{\mu\nu}, \tag{5.11}$$

with the transformation property:

$$V_{\mu\nu} \xrightarrow{G} h(g,\phi)\, V_{\mu\nu} h(g,\phi)^{-1}. \tag{5.12}$$

On the other hand, the interactions are encoded in $\mathscr{L}_2(V)$,

$$\mathscr{L}_2(V) = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle, \tag{5.13}$$

where we have defined,

$$f_+^{\mu\nu} = uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u, \tag{5.14}$$

being $F_{R,L}$ the field strength tensors of the left, l_μ, and right, r_μ, external sources and where F_V and G_V are real couplings whose values cannot be fixed by the chiral symmetry. They take the following phenomenological values [32]:

$$\begin{aligned} |F_V| &\simeq 154 \text{ MeV}, \\ |G_V| &\simeq 69 \text{ MeV}. \end{aligned} \tag{5.15}$$

It is important to notice that the interaction between the resonances and the photon field is encoded in $f_+^{\mu\nu}$,

$$f_+^{\mu\nu} = 2eQF^{\mu\nu} + \ldots, \tag{5.16}$$

with $F^{\mu\nu}$ the electromagnetic strength tensor and being $Q = \frac{1}{3}diag(2,-1,-1)$ the quark charge matrix. Given the form of Q and $V_{\mu\nu}$, it is clear that the photon will only directly couple to ρ^0 and ω_8 resonances. Indeed,

$$\frac{eF_V}{\sqrt{2}}F^{\mu\nu}\langle V_{\mu\nu}Q\rangle = \frac{eF_V}{2}F^{\mu\nu}\left(\rho^0 + \frac{\omega_8}{\sqrt{3}}\right)_{\mu\nu}. \tag{5.17}$$

Moreover, ω_1 and ω_8 fields mix in such a way that the physical fields are a combination of both fields. The mixing is given by:

$$\begin{pmatrix} \omega_8 \\ \omega_1 \end{pmatrix}_{\mu\nu} = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \phi \\ \omega \end{pmatrix}_{\mu\nu} . \tag{5.18}$$

For the mixing angle, it is phenomenologically motivated to take the so called ideal mixing where $\theta_V \approx 35^0$. Therefore,

$$\begin{aligned} \cos\theta_V &= \sqrt{\frac{2}{3}}, \\ \sin\theta_V &= \sqrt{\frac{1}{3}}, \end{aligned} \tag{5.19}$$

so the fields in the Lagrangian can be expressed in term of the physical fields in the following way:

$$\begin{aligned} \omega_{8,\mu\nu} &= \frac{1}{\sqrt{3}}\omega_{\mu\nu} + \sqrt{\frac{2}{3}}\phi_{\mu\nu}, \\ \omega_{1,\mu\nu} &= \sqrt{\frac{2}{3}}\omega_{\mu\nu} - \frac{1}{\sqrt{3}}\phi_{\mu\nu}. \end{aligned} \tag{5.20}$$

Finally, the interaction between octet mesons and vector resonances is governed by:

$$\mathscr{L}_{BV} = R_F\langle \overline{B}\sigma^{\mu\nu}\left[V_{\mu\nu}, B\right]\rangle + R_D\langle \overline{B}\sigma^{\mu\nu}\left\{V_{\mu\nu}, B\right\}\rangle + R_S\langle \overline{B}\sigma^{\mu\nu}B\rangle\langle V_{\mu\nu}\rangle. \tag{5.21}$$

The notation for the real coefficients $R_{F/D/S}$ has been established in [33], where higher order terms can also be found. The numerical values of $R_{F/D}$ can be fixed by measuring proton and neutron anomalous magnetic moments (see Appendix C):

$$\begin{aligned} R_F &\simeq -0.61, \\ R_D &\simeq -2.08. \end{aligned} \tag{5.22}$$

In the study of $\tau^+ \to p\mu^+\mu^-$ decay , the knowledge of $p \to pV^* \to p\gamma$ interaction will be needed. This interaction can be obtained from $\mathscr{L}_{BV}$:

$$\begin{aligned} \langle \overline{B}\sigma^{\mu\nu}\left[V_{\mu\nu}, B\right]\rangle &\overset{p\to pV^*}{\longrightarrow} \frac{1}{\sqrt{2}}\left(\overline{p}\sigma^{\mu\nu}p\right)\left(\rho^0 + \sqrt{3}\omega_8\right)_{\mu\nu}, \\ \langle \overline{B}\sigma^{\mu\nu}\left\{V_{\mu\nu}, B\right\}\rangle &\overset{p\to pV^*}{\longrightarrow} \frac{1}{\sqrt{2}}\left(\overline{p}\sigma^{\mu\nu}p\right)\left(\rho^0 - \frac{\omega_8}{\sqrt{3}}\right)_{\mu\nu}. \end{aligned} \tag{5.23}$$

In order to ease future computations, the rules associated to those vertices will be collected in Appendix B.

5.2 Interactions with $\Delta L = \Delta B \neq 0$ involving mesons, baryons and vector resonances

For $\tau^+ \to p\mu^+\mu^-$ and $\tau^+ \to \Lambda\pi^+$ decays, we have to study the dimension-six operators with $\Delta B = \Delta L$ presented in Table 3.2 where only quarks u, d and s are involved [12, 13, 14, 15]:

$$\begin{aligned} \mathscr{L}_{BL} =& \frac{1}{\Lambda^2}\Big[C_{RL}\mathcal{O}_{RL} + C_{LR}\mathcal{O}_{LR} + C_{LL}\mathcal{O}_{LL} + C_{RR}\mathcal{O}_{RR} + \\ & \tilde{C}_{RL}\tilde{\mathcal{O}}_{RL} + \tilde{C}_{LR}\tilde{\mathcal{O}}_{LR} + \tilde{C}_{LL}\tilde{\mathcal{O}}_{LL} + \tilde{C}_{RR}\tilde{\mathcal{O}}_{RR}\Big] + h.c. \end{aligned} \tag{5.24}$$

The operators without tilde only involve quarks from the first family and thus they generate interactions with $\Delta S = 0$:

$$\begin{aligned}
\mathcal{O}_{RL} &= \epsilon_{\alpha\beta\gamma}\overline{(d_R^\alpha)^C}u_R^\beta\overline{(u_L^\gamma)^C}\tau_L, \\
\mathcal{O}_{LR} &= \epsilon_{\alpha\beta\gamma}\overline{(d_L^\alpha)^C}u_L^\beta\overline{(u_R^\gamma)^C}\tau_R, \\
\mathcal{O}_{LL} &= \epsilon_{\alpha\beta\gamma}\overline{(d_L^\alpha)^C}u_L^\beta\overline{(u_L^\gamma)^C}\tau_L, \\
\mathcal{O}_{RR} &= \epsilon_{\alpha\beta\gamma}\overline{(d_R^\alpha)^C}u_R^\beta\overline{(u_R^\gamma)^C}\tau_R,
\end{aligned} \tag{5.25}$$

and operators with tilde contain the s quark and produce interactions with $|\Delta S| = 1$:

$$\begin{aligned}
\tilde{\mathcal{O}}_{RL} &= \epsilon_{\alpha\beta\gamma}\overline{(s_R^\alpha)^C}u_R^\beta\overline{(u_L^\gamma)^C}\tau_L, \\
\tilde{\mathcal{O}}_{LR} &= \epsilon_{\alpha\beta\gamma}\overline{(s_L^\alpha)^C}u_L^\beta\overline{(u_R^\gamma)^C}\tau_R, \\
\tilde{\mathcal{O}}_{LL} &= \epsilon_{\alpha\beta\gamma}\overline{(s_L^\alpha)^C}u_L^\beta\overline{(u_L^\gamma)^C}\tau_L, \\
\tilde{\mathcal{O}}_{RR} &= \epsilon_{\alpha\beta\gamma}\overline{(s_R^\alpha)^C}u_R^\beta\overline{(u_R^\gamma)^C}\tau_R.
\end{aligned} \tag{5.26}$$

Regarding the previous set of operators, it is interesting to mention that only $\mathcal{O}_{RL}$, $\mathcal{O}_{LR}$, $\tilde{\mathcal{O}}_{RL}$ and $\tilde{\mathcal{O}}_{RL}$ may be Fierz transformed to a product of vector currents. Therefore, if we assume that baryon non-conservation is due to the exchange of any sort of vector boson in the theory above the SM regime of validity as would be reasonable in a gauge theory, then only those operators contribute [14]. This assumption will be used in the decay width computations below.

It is known that quarks hadronize. Therefore, we need to express these operators in terms of meson and baryon fields. In order to do so, we use the same technique as in [12] which consists in analyzing how the operators transform under the action of $G = SU(3)_L \otimes SU(3)_R$ and finding a new set of operators in terms of mesonic and baryonic fields with the same transformation rules and the same quark content as the previous ones.

Hence, the first step to hadronize is to determine the transformation properties of operators from Equations (5.25) and (5.26). Let us take first:

$$\epsilon_{\alpha\beta\gamma}\overline{(q_{iR}^\alpha)^C}q_{jR}^\beta. \tag{5.27}$$

From the group theory viewpoint, we have that the operator transform as:

$$3_R \otimes 3_R = 3_R^* \oplus 6_R, \tag{5.28}$$

where 3^* tensors are antisymmetric in flavor space while 6 tensors are symmetric. However, it can be proven that the symmetric operator which arises from this decomposition vanishes. Indeed, we have that:

$$\begin{aligned}
\overline{(q_i)^C}q_j &= (q_i)^T Cq_j \\
&= \left[(q_i)^T Cq_j\right]^T \\
&= -(q_j)^T C^T q_i \\
&= (q_j)^T Cq_i \\
&= \overline{(q_j)^C}q_i,
\end{aligned} \tag{5.29}$$

where we have used the definition of charge conjugation over a spinor, $\psi^C = C\overline{\psi}^T$, that $C^T = -C$, the fact that the transpose of a scalar is equal to itself and that fermionic fields anticommute.

Therefore,

$$\begin{aligned}\epsilon_{\alpha\beta\gamma}\overline{(q_i^\alpha)^C} q_j^\beta &= \epsilon_{\alpha\beta\gamma}\overline{\left(q_j^\beta\right)^C} q_i^\alpha \\ &= -\epsilon_{\alpha\beta\gamma}\overline{\left(q_j^\alpha\right)^C} q_i^\beta,\end{aligned} \tag{5.30}$$

and consequently only the antisymmetric representation contributes. Hence,

$$\epsilon_{\alpha\beta\gamma}\overline{(q_{iR}^\alpha)^C} q_{jR}^\beta, \tag{5.31}$$

transform as 3_R^* under the chiral group.

Using the previous result, it clear that the piece:

$$\epsilon_{\alpha\beta\gamma}\overline{(q_{iR}^\alpha)^C} q_{jR}^\beta \overline{(q_{kR}^\gamma)^C} e_{lR}, \tag{5.32}$$

transforms as $3_R^* \otimes 3_R = 8_R \oplus 1_R$ while it is a singlet under $SU(3)_L$. Nonetheless, the tensor[3] $(1,1)$ is proportional to:

$$\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\overline{(q_{iR}^\alpha)^C} q_{jR}^\beta \overline{(q_{kR}^\gamma)^C} e_{lR}, \tag{5.33}$$

and, since we are interested in operators with two equal quarks, the pure singlet contribution does not appear. Therefore, we finally obtain the result that the operator $\mathcal{O}_{RR}$ transforms as $(1,8)$.

A similar analysis can be performed with the other operators. It is immediate to get that the B-L conserving operators previously presented transform under $G = SU(3)_L \otimes SU(3)_R$ as shown in Table 5.1.

Dimension-six operator	Chirality	Transformation
$\mathcal{O}_{RL}$, $\tilde{\mathcal{O}}_{RL}$	RRLL	$(3,3^*)$
$\mathcal{O}_{LR}$, $\tilde{\mathcal{O}}_{LR}$	LLRR	$(3^*,3)$
$\mathcal{O}_{LL}$, $\tilde{\mathcal{O}}_{LL}$	LLLL	$(8,1)$
$\mathcal{O}_{RR}$, $\tilde{\mathcal{O}}_{RR}$	RRRR	$(1,8)$

Table 5.1: *Transformation properties of the BNV dimension six operators under the chiral group.*

Now that we know how the operators in terms of quarks transform, we need to find a combination of hadronic fields with the same transformation properties. Taking the transformation rules of the fields shown in (3.4) and (5.6), one can prove that the following hadronic operators transform as in Table 5.2.

[3] In the following, we use the notation $(x\,,x)$ to indicate the transformation properties of the tensors under the chiral group $G = (SU(3)_L, SU(3)_R)$

For instance, let us check the transformation properties of:

$$\begin{aligned}
u^\dagger B u^\dagger \xrightarrow{G} & \left[h\left(g,\phi\right)\, u\, g_L^{-1}\right]^\dagger h\left(g,\phi\right)\, B\, h\left(g,\phi\right)^{-1} \left[g_R\, u\, h\left(g,\phi\right)^{-1}\right]^\dagger \\
& = g_L\, u^\dagger\, h^\dagger h\, B\, h^\dagger\, h\, u^\dagger\, g_R^\dagger \\
& = g_L\, u^\dagger\, B\, u^\dagger\, g_R^{-1},
\end{aligned} \tag{5.34}$$

where we have used that $h^\dagger = h^{-1}$, $g_L^\dagger = g_L^{-1}$ and $g_R^\dagger = g_R^{-1}$. Finally, we have to show that this transformation corresponds to a $(3, 3^*)$ under the chiral group. Denoting by X a tensor with the same transformation properties as $u^\dagger B u^\dagger$ we have:

$$X_{ab} \xrightarrow{G} \left(g_L\right)_{ac}\, X_{cd}\, \left(g_R^\dagger\right)_{db} = \left(g_L\right)_{ac} \left(g_R^*\right)_{bd}\, X_{cd} \tag{5.35}$$

where g_R^* is the tensor associated to transformations in the adjoint representation. A similar analysis can be performed with the other terms in Table 5.2.

Operator	Transformation
$u^\dagger B u^\dagger$	$(3, 3^*)$
uBu	$(3^*, 3)$
$u^\dagger B u$	$(8, 1)$
$uBu^\dagger$	$(1, 8)$

Table 5.2: *Combination of baryon and meson fields with the same transformation properties under the chiral group as the BNV dimension six operators.*

Finally, in order to correctly obtain the interaction, we have to select the hadrons with the same valence quarks as the quark fields in the operators. To achieve that, we use the following projectors:

$$P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \tilde{P} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{5.36}$$

where the P selects the uud quark content and is associated with operators without tilde while $\tilde{P}$ selects uus valence quarks and is related to operators with tilde.

As a corollary of the analysis we have done, we get that the relevant interactions for $\tau^+ \to p\mu^+\mu^-$ and $\tau^+ \to \Lambda\pi^+$ decays in terms of baryons and vector resonances are encoded in the following Lagrangian:

$$\begin{aligned}
\mathscr{L}_{BL} = & \frac{1}{\Lambda^2} \left[C_{RL}\mathcal{O}^h_{RL} + C_{LR}\mathcal{O}^h_{LR} + C_{LL}\mathcal{O}^h_{LL} + C_{RR}\mathcal{O}^h_{RR} + \right. \\
& \left. \tilde{C}_{RL}\tilde{\mathcal{O}}^h_{RL} + \tilde{C}_{LR}\tilde{\mathcal{O}}^h_{LR} + \tilde{C}_{LL}\tilde{\mathcal{O}}^h_{LL} + \tilde{C}_{RR}\tilde{\mathcal{O}}^h_{RR}\right] + h.c.,
\end{aligned} \tag{5.37}$$

with:

$$\begin{aligned}
\mathcal{O}^h_{RL} &= \alpha\overline{(\tau_L)^C}\langle P u^\dagger B_L u^\dagger\rangle,\\
\mathcal{O}^h_{LR} &= -\alpha\overline{(\tau_R)^C}\langle P u B_R u\rangle,\\
\mathcal{O}^h_{LL} &= \beta\overline{(\tau_L)^C}\langle P u^\dagger B_L u\rangle,\\
\mathcal{O}^h_{RR} &= -\beta\overline{(\tau_R)^C}\langle P u B_R u^\dagger\rangle,\\
\tilde{\mathcal{O}}^h_{RL} &= \gamma\overline{(\tau_L)^C}\langle \tilde{P} u^\dagger B_L u^\dagger\rangle,\\
\tilde{\mathcal{O}}^h_{LR} &= -\gamma\overline{(\tau_R)^C}\langle \tilde{P} u B_R u\rangle,\\
\tilde{\mathcal{O}}^h_{LL} &= \delta\overline{(\tau_L)^C}\langle \tilde{P} u^\dagger B_L u\rangle,\\
\tilde{\mathcal{O}}^h_{RR} &= -\delta\overline{(\tau_R)^C}\langle \tilde{P} u B_R u^\dagger\rangle.
\end{aligned} \tag{5.38}$$

The strong coefficients α, β, γ and δ are defined in terms of the matrix elements:

$$\begin{aligned}
\langle 0 \mid \mathcal{O}_{RL} \mid p\rangle &= \alpha P_L u_p,\\
\langle 0 \mid \mathcal{O}_{LL} \mid p\rangle &= \beta P_L u_p,\\
\langle 0 \mid \mathcal{O}_{LR} \mid p\rangle &= -\alpha P_R u_p,\\
\langle 0 \mid \mathcal{O}_{RR} \mid p\rangle &= -\beta P_R u_p,\\
\langle 0 \mid \tilde{\mathcal{O}}_{RL} \mid \Sigma^+\rangle &= \gamma P_L u_{\Sigma^+},\\
\langle 0 \mid \tilde{\mathcal{O}}_{LL} \mid \Sigma^+\rangle &= \delta P_L u_{\Sigma^+},\\
\langle 0 \mid \tilde{\mathcal{O}}_{LR} \mid \Sigma^+\rangle &= -\gamma P_L u_{\Sigma^+},\\
\langle 0 \mid \tilde{\mathcal{O}}_{RR} \mid \Sigma^+\rangle &= -\delta P_L u_{\Sigma^+}.
\end{aligned} \tag{5.39}$$

Due to parity conservation, the strong coefficients for $\mathcal{O}^h_{LR}$ and $\mathcal{O}^h_{RL}$ are equal up to a relative minus sign. The same happens for $\mathcal{O}^h_{LL}$ and $\mathcal{O}^h_{RR}$ and with the corresponding operators with tilde. Moreover, using $SU(3)$ symmetry, the strong parameters are related so that $\gamma = \alpha$ and $\delta = \beta$. This is explicitly tested in [34] where the parameters where calculated under some simplifications. Parameters α and β are known to satisfy the constraint $|\alpha| = |\beta|$ [35]. Lattice QCD calculation of the proton decay matrix element by CP-PACS and JLQCD Collaborations give the value [36]:

$$\begin{aligned}
|\alpha| &= 0.0090(09)\left({}^{+5}_{-19}\right)\ \text{GeV}^3,\\
|\beta| &= 0.0096(09)\left({}^{+6}_{-20}\right)\ \text{GeV}^3,
\end{aligned} \tag{5.40}$$

with α and β in a relatively opposite sign and evaluated at the scale $Q = 2$ GeV.

A more recent lattice computation from RBC-UKQCD collaboration gives a slightly different value for the parameters evaluated at the same scale, $Q = 2$ GeV [37]:

$$\begin{aligned}
\alpha &= -0.0112(25)\ \text{GeV}^3,\\
\beta &= 0.0120(26)\ \text{GeV}^3,
\end{aligned} \tag{5.41}$$

where the phase convention has been chosen in such a way that the parameters α and β are real.

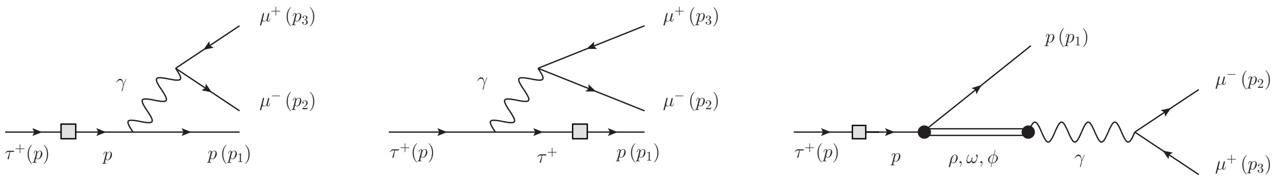

Figure 5.1: *Feynman diagrams contributing to $\tau^+ \to p\mu^+\mu^-$ decay width at tree-level order. The squares denote vertices which violate lepton and baryon number coming from dimension six operators while the circles indicate $R\chi T$ interactions.*

5.3 $\tau^+ \to p\mu^+\mu^-$ decay

From the Lagrangian derived in Sections 5.1 and 5.2 we can obtain the Feynman rules necessary for computing $\tau^+ \to p\mu^+\mu^-$ decay width. These rules are given in Appendix B.

One should notice that the first and second diagrams in Figure 5.1 cancel each other in a similar way to what it happened with the LFV processes. The cancellation of the two diagrams at the matrix element level is shown below. Once more, we have a different coupling for the different chiralities of the fields. In what follows, only the left-handed coupling is considered:

$$\begin{aligned}
i\tilde{\mathscr{M}}_L &= \frac{ie^2 A_L}{q^2\Lambda^2}\left(\overline{u}_2\gamma^\mu v_3\right)\left[\left(\overline{u}_1\gamma_\mu\frac{\not p + m_\tau}{m_P^2 - m_\tau^2}P_L C\overline{u}^T\right) + \left(\overline{u}_1 P_L C\frac{\not p_1^T + m_P}{m_\tau^2 - m_P^2}\gamma_\mu^T\overline{u}^T\right)\right] \\
&= \frac{ie^2 A_L}{q^2\Lambda^2}\left(\overline{u}_2\gamma^\mu v_3\right)\left[\left(\overline{u}_1\gamma_\mu\frac{\not p + m_\tau}{m_P^2 - m_\tau^2}P_L C\overline{u}^T\right) + \left(\overline{u}_1 P_L\frac{\not p_1 + m_P}{m_\tau^2 - m_P^2}\gamma_\mu C\overline{u}^T\right)\right] \\
&= \frac{ie^2 A_L}{q^2\Lambda^2}\left(\overline{u}_2\gamma^\mu v_3\right)\left[\left(\overline{u}_1\gamma_\mu\frac{m_P P_R + m_\tau P_L}{m_P^2 - m_\tau^2}C\overline{u}^T\right) + \left(\overline{u}_1\frac{m_\tau P_R + m_P P_L}{m_\tau^2 - m_P^2}\gamma_\mu C\overline{u}^T\right)\right] \\
&= \frac{ie^2 A_L}{q^2\Lambda^2}\left(\overline{u}_2\gamma^\mu v_3\right)\left[\left(\overline{u}_1\frac{m_P P_L + m_\tau P_R}{m_P^2 - m_\tau^2}\gamma_\mu C\overline{u}^T\right) - \left(\overline{u}_1\frac{m_\tau P_R + m_P P_L}{m_P^2 - m_\tau^2}\gamma_\mu C\overline{u}^T\right)\right] \\
&= 0,
\end{aligned} \tag{5.42}$$

with the definitions:

$$\begin{aligned}
A_L &\equiv \alpha C_{RL} + \beta C_{LL}, \\
A_R &\equiv -\alpha C_{LR} - \beta C_{RR}.
\end{aligned} \tag{5.43}$$

and where the Dirac equation and chirality projector and charge conjugation properties has been used. From the previous result it is straightforward to check the cancellation for the right-handed coupling too.

Hence, the matrix element for the decay is given only by the third diagram in Figure 5.1. This matrix element reads:

$$\begin{aligned}
i\mathscr{M}_{L,R} &= -\frac{e^2 F_V A_{L,R}}{\sqrt{2}q^2\Lambda^2\left(m_\tau^2 - m_P^2\right)}\left[\frac{R_F + R_D}{M_\rho^2 - q^2} + \frac{R_F - \frac{R_D}{3}}{\sqrt{3}}\left(\frac{1}{M_\omega^2 - q^2} + \frac{\sqrt{2}}{M_\phi^2 - q^2}\right)\right] \\
&\quad \times \left(q_\mu g_{\nu\rho} - q_\nu g_{\mu\rho}\right)\left(\overline{u}_2\gamma^\rho v_3\right)\left[\overline{u}_1\sigma^{\mu\nu}\left(\not p + m_P\right)P_{L,R}C\overline{u}^T\right] \\
&= -\frac{\sqrt{2}e^2 F_V A_{L,R}}{q^2\Lambda^2\left(m_\tau^2 - m_P^2\right)}\left[\frac{R_F + R_D}{M_\rho^2 - q^2} + \frac{R_F - \frac{R_D}{3}}{\sqrt{3}}\left(\frac{1}{M_\omega^2 - q^2} + \frac{\sqrt{2}}{M_\phi^2 - q^2}\right)\right] \\
&\quad \times q_\mu\left(\overline{u}_2\gamma_\nu v_3\right)\left[\overline{u}_1\sigma^{\mu\nu}\left(\not p + m_P\right)P_{L,R}C\overline{u}^T\right],
\end{aligned} \tag{5.44}$$

Therefore,

$$i\mathscr{M} = i\mathscr{M}_L + i\mathscr{M}_R, \tag{5.45}$$

and consequently

$$|\mathscr{M}|^2 = |\mathscr{M}_L|^2 + |\mathscr{M}_R|^2 + 2\,\mathrm{Re}\left\{\mathscr{M}_L\left(\mathscr{M}_R\right)^*\right\}. \tag{5.46}$$

Let us compute the piece with the charge conjugation matrix first,

$$\begin{aligned} X_{L,R} &= \overline{u}_1\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}C\overline{u}^T, \\ \left(X_{L,R}\right)^* &= \left[\overline{u}_1\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}C\overline{u}^T\right]^* \\ &= \left(\overline{u}^T\right)^\dagger C^\dagger P_{L,R}\left(\gamma^0\not{p}\gamma^0+m_P\right)\gamma^0\sigma^{\mu\nu}\gamma^0\overline{u}_1^\dagger \\ &= -u^TC^\dagger P_{R,L}\left(\not{p}+m_P\right)\sigma^{\mu\nu}u_1, \end{aligned} \tag{5.47}$$

hence,

$$\begin{aligned} \sum_{pol} X_{L,R}\left(X_{L',R'}\right)^* &= -\sum_{pol} Tr\left\{u^TC^\dagger P_{R',L'}\left(\not{p}+m_P\right)\sigma^{\rho\sigma}u_1\overline{u}_1\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}C\overline{u}^T\right\} \\ &= -\sum_{pol} Tr\left\{C^\dagger\left(u\overline{u}\right)^TCP_{R',L'}\left(\not{p}+m_P\right)\sigma^{\rho\sigma}u_1\overline{u}_1\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}\right\} \\ &= -Tr\left\{C^\dagger\left(\not{p}^T+m_\tau\right)CP_{R',L'}\left(\not{p}+m_P\right)\sigma^{\rho\sigma}\left(\not{p}_1+m_P\right)\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}\right\} \\ &= Tr\left\{\left(\not{p}-m_\tau\right)P_{R',L'}\left(\not{p}+m_P\right)\sigma^{\rho\sigma}\left(\not{p}_1+m_P\right)\sigma^{\mu\nu}\left(\not{p}+m_P\right)P_{L,R}\right\}. \end{aligned} \tag{5.48}$$

When calculating the decay width, one has to be careful because we go over the resonance pole in the 3-body phase space integration. Indeed, during the phase space integration:

$$4m_\mu^2 < q^2 < \left(m_\tau - m_P\right)^2 \tag{5.49}$$

and so, we go over the ρ and ω resonance poles but not over the ϕ pole. For this reason, it is necessary to add their decay width into the propagator. This is performed in the narrow-width approximation by modifying the propagator in the following way:

$$\frac{1}{q^2-M^2} \to \frac{1}{q^2-M^2+iM\Gamma}, \tag{5.50}$$

being M the mass of the resonance and Γ its decay width.

The ρ resonance decay width is given by [38]:

$$\Gamma_\rho\left(q^2\right) = \frac{M_\rho q^2}{96\pi F^2}\left[\sigma_\pi^3\theta\left(q^2-4m_\pi^2\right)+\frac{1}{2}\sigma_K^3\theta\left(q^2-4m_K^2\right)\right], \tag{5.51}$$

with:

$$\sigma_P = \sqrt{1-4m_P^2/q^2}. \tag{5.52}$$

For ω and ϕ resonances decay widths, as they are narrow, we take their constant values from Particle Date Group (PDG) [9].

Now it is immediate to compute the amplitude:

$$\Gamma_{\tau^+\to p\mu^+\mu^-} = \frac{1}{\Lambda^4}\left[1.78\times 10^{-5}\ \mathrm{GeV}^5\left(|A_L|^2+|A_R|^2\right) - 2.12\times 10^{-5}\ \mathrm{GeV}^5\ \ \mathrm{Re}\left\{A_L A_R^*\right\}\right]. \tag{5.53}$$

If we assume that BNV takes places through a heavy vector boson exchange in the high energy theory we have learned from the previous Subsection that we can neglect $\tilde{\mathcal{O}}^h_{LL}$ and $\tilde{\mathcal{O}}^h_{RR}$ contribution. As a result, the decay width is given by:

$$\Gamma_{\tau^+\to p\mu^+\mu^-} = \frac{1}{\Lambda^4}\left[2.23\times 10^{-9}\ \mathrm{GeV}^5\left(|C_{RL}|^2+|C_{LR}|^2\right) + 2.66\times 10^{-9}\ \mathrm{GeV}^5\ \ \mathrm{Re}\left\{C_{RL} C_{LR}^*\right\}\right]. \tag{5.54}$$

Finally, from the experimental bound to the process we can set a bound to the new physics scale under naturalness condition, i.e taking the undetermined coefficients to be $\mathcal{O}(1)$. Using $\mathcal{B}\left(\tau^+\to p\mu^+\mu^-\right) < 3.3\times 10^{-7}$ from LHCb [1] we obtain:

$$\begin{aligned}\Lambda_+ &\geq 0.3\ \mathrm{TeV},\\ \Lambda_- &\geq 0.2\ \mathrm{TeV},\end{aligned} \tag{5.55}$$

where the subindex indicate whether the interference term adds in a constructive or destructive way, respectively.

5.4 $\tau^+\to\Lambda\pi^+$ decay

Given the results from Subsection 5.2, we need to extract the relevant contributions to $\tau^\pm\to\Lambda\pi^\pm$ and $\tau^\pm\to\overline{\Lambda}\pi^\pm$ decays. The first thing one should notice is that the dimension six BNV operators conserve B-L and hence processes $\tau^-\to\Lambda\pi^-$ and $\tau^+\to\overline{\Lambda}\pi^+$ cannot be generated. We will focus then in the computation of $\tau^+\to\Lambda\pi^+$ decay width.

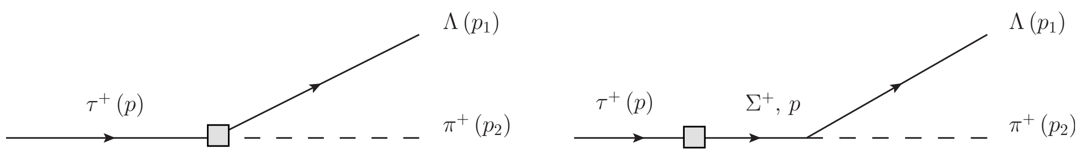

Figure 5.2: *Tree level Feynman diagrams contributing to $\tau^+\to\Lambda\pi^+$ decay. The square stand for BNV interactions from the dimension six operators introduced in Subsection 5.2*

There are two graphs contributing to the process (see Figure 5.2): direct BNV tau decay into Λ and π and indirect decay mediated by a Σ baryon or a proton. However, the vertex $p\to\Lambda\pi^+$ violates strangeness so it can only occur through weak interactions. Therefore, we expect it to be suppressed by at least a factor of the Fermi constant, G_F, and thus we can neglect it in our analysis.

Consequently, we need to obtain $\tau^+\to\Sigma^+$ and $\tau^+\to\Lambda\pi^+$ vertices. These interactions can be readily determined from the operator analysis performed in Subsection 5.2. Indeed, as these processes require the presence of a strange quark, only operators in Equation (5.26) will contribute. Expanding hadronized operators from Equation (5.38) up to second order in the meson fields and keeping the baryons we are interested in only, we get:

$$
\begin{aligned}
\tilde{\mathcal{O}}^h_{RL} &= \gamma\overline{(\tau_L)^C}\left[\langle PB_L\rangle - \frac{i}{\sqrt{2}F}\left(\langle PB_L\phi\rangle + \langle P\phi B_L\rangle\right) + \dots\right] \\
&= \gamma\overline{\tau_L^C}\Sigma_L^+ - \frac{i\gamma}{\sqrt{3}F}\overline{(\tau_L)^C}\Lambda_L\pi^+ + \dots, \\
\tilde{\mathcal{O}}^h_{LR} &= -\gamma\overline{(\tau_R)^C}\left[\langle PB_R\rangle + \frac{i}{\sqrt{2}F}\left(\langle PB_R\phi\rangle + \langle P\phi B_R\rangle\right) + \dots\right] \\
&= -\gamma\overline{\tau_R^C}\Sigma_R^+ - \frac{i\gamma}{\sqrt{3}F}\overline{(\tau_R)^C}\Lambda_R\pi^+ + \dots, \\
\tilde{\mathcal{O}}^h_{LL} &= \delta\overline{(\tau_L)^C}\left[\langle PB_L\rangle + \frac{i}{\sqrt{2}F}\left(\langle PB_L\phi\rangle - \langle P\phi B_L\rangle\right) + \dots\right] \\
&= \delta\overline{\tau_L^C}\Sigma_L^+ + \dots, \\
\tilde{\mathcal{O}}^h_{RR} &= -\delta\overline{(\tau_R)^C}\left[\langle PB_R\rangle - \frac{i}{\sqrt{2}F}\left(\langle PB_R\phi\rangle - \langle P\phi B_R\rangle\right) + \dots\right] \\
&= -\delta\overline{\tau_R^C}\Sigma_L^+ + \dots,
\end{aligned} \tag{5.56}
$$

where the dots stand for terms involving baryons and mesons we are not interested in. Remember that we have $\gamma = \alpha$ and $\delta = \beta$ in the $SU(3)$ limit. We will use these identities in what follows.

On the other hand, the interaction $\Sigma^- \to \Lambda\pi^-$ can be extracted from the baryon Lagrangian in Equation (5.4). Expanding the traces and keeping only the relevant pieces we obtain:

$$
\mathscr{L}_B = \frac{D}{\sqrt{3}F}\left(\overline{\Sigma}^-\gamma^\mu\gamma_5\Lambda\partial_\mu\pi^- + \overline{\Lambda}\gamma^\mu\gamma_5\Sigma^+\partial_\mu\pi^-\right) + h.c. + \dots, \tag{5.57}
$$

with the dots denoting interactions which are irrelevant to $\tau^+ \to \Lambda\pi^+$ decay.

Now that we have all the ingredients, it is straightforward to perform the computation. Applying the Feynman rules compiled in Appendix B we obtain the following matrix element for the decay:

$$
\begin{aligned}
i\mathscr{M} = &\frac{\alpha}{\sqrt{3}F\Lambda^2}\overline{u}_1\left(\tilde{C}_{RL}P_L + \tilde{C}_{RL}P_R\right)C\overline{u}^T + \frac{D}{\sqrt{3}F\Lambda^2}p_{2,\mu}\overline{u}_1\gamma^\mu\gamma_5\frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \\
&\left[\left(\alpha\tilde{C}_{RL} + \beta\tilde{C}_{LL}\right)P_L - \left(\alpha\tilde{C}_{LR} + \beta\tilde{C}_{RR}\right)P_R\right]C\overline{u}^T.
\end{aligned} \tag{5.58}
$$

Squaring, summing over Λ spin and averaging over τ spin we get the decay amplitude:

$$
|\overline{\mathscr{M}}|^2 = \frac{1}{12F^2\Lambda^4}\left[I_1 + I_2 + 2\ \mathrm{Re}\left\{I_3\right\}\right], \tag{5.59}
$$

where we have defined:

$$
\begin{aligned}
I_1 &\equiv \alpha^2\sum_{pol}Tr\left\{u_1\overline{u}_1\left(\tilde{C}_{RL}P_L + \tilde{C}_{LR}P_R\right)C\overline{u}^T u^T C\left(\tilde{C}^*_{RL}P_R + \tilde{C}^*_{LR}P_L\right)\right\} \\
&= \alpha^2 Tr\left\{\left(\not{p}_1 + m_\Lambda\right)\left(\tilde{C}_{RL}P_L + \tilde{C}_{LR}P_R\right)\left(\not{p} - m_\tau\right)\left(\tilde{C}^*_{RL}P_R + \tilde{C}^*_{LR}P_L\right)\right\},
\end{aligned}
$$

$$I_2 \equiv D^2 p_{2,\mu} p_{2,\nu} \sum_{pol} Tr \left\{ u_1 \overline{u}_1 \gamma^\mu \gamma_5 \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right) P_L - \left(\alpha \tilde{C}_{LR} + \beta \tilde{C}_{RR} \right) P_R \right] \right.$$

$$\left. C \overline{u}^T u^T C \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right)^* P_R - \left(\alpha \tilde{C}_{LR} - \beta \tilde{C}_{RR} \right)^* P_L \right] \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \gamma^\nu \gamma_5 \right\}$$

$$= D^2 p_{2,\mu} p_{2,\nu} Tr \left\{ \left(\not{p}_1 + m_\Lambda \right) \gamma^\mu \gamma_5 \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right) P_L - \left(\alpha \tilde{C}_{LR} + \beta \tilde{C}_{RR} \right) P_R \right] \right.$$

$$\left. \left(\not{p} - m_\tau \right) \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right)^* P_R - \left(\alpha \tilde{C}_{LR} + \beta \tilde{C}_{RR} \right)^* P_L \right] \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \gamma^\nu \gamma_5 \right\}, \quad (5.60)$$

$$I_3 \equiv \alpha D \sum_{pol} p_{2,\mu} Tr \left\{ u_1 \overline{u}_1 \left(\tilde{C}_{LL} P_L + \tilde{C}_{RR} P_R \right) C \overline{u}^T u^T C \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right)^* P_R \right. \right.$$

$$\left. \left. - \left(\alpha \tilde{C}_{LR} + \beta \tilde{C}_{RR} \right)^* P_L \right] \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \gamma^\mu \gamma_5 \right\}$$

$$= \alpha D p_{2,\mu} Tr \left\{ \left(\not{p}_1 + m_\Lambda \right) \left(\tilde{C}_{RL} P_L + \tilde{C}_{LR} P_R \right) \left(\not{p} - m_\tau \right) \left[\left(\alpha \tilde{C}_{RL} + \beta \tilde{C}_{LL} \right)^* P_R \right. \right.$$

$$\left. \left. - \left(\alpha \tilde{C}_{LR} + \beta \tilde{C}_{RR} \right)^* P_L \right] \frac{\not{p} + m_\Sigma}{m_\tau^2 - m_\Sigma^2} \gamma^\mu \gamma_5 \right\}.$$

Coefficient	a_1	a_2	a_3	a_4	a_5	a_6
$10^3 \times$Value (GeV^5)	4.41	−0.57	1.57	2.89	−2.77	−4.29

Table 5.3: *Coefficients for the decay width $\tau^+ \to \Lambda\pi^+$ in units of GeV^5.*

Calculating the traces we get the following expression for the decay width:

$$\Gamma\left(\tau^+ \to \Lambda\pi^+\right) = \frac{1}{\Lambda^4} \left[a_1 \left(\left|\tilde{C}_{LR}\right|^2 + \left|\tilde{C}_{RL}\right|^2 \right) + a_2 \ \mathrm{Re}\left\{ \tilde{C}_{LR} \tilde{C}_{RL}^* \right\} + a_3 \left(\left|\tilde{C}_{LL}\right|^2 + \left|\tilde{C}_{RR}\right|^2 \right) \right.$$

$$+ a_4 \ \mathrm{Re}\left\{ \tilde{C}_{LL} \tilde{C}_{RR}^* \right\} + a_5 \left(\mathrm{Re}\left\{ \tilde{C}_{LR} \tilde{C}_{LL}^* \right\} \ \mathrm{Re}\left\{ \tilde{C}_{RL} \tilde{C}_{RR}^* \right\} \right) +$$

$$\left. + a_6 \left(\mathrm{Re}\left\{ \tilde{C}_{RL} \tilde{C}_{LL}^* \right\} + \ \mathrm{Re}\left\{ \tilde{C}_{LR} \tilde{C}_{RR}^* \right\} \right) \right]. \quad (5.61)$$

Numerical analysis shows that both diagrams in Figure 5.2 (with only the Σ baryon in the right diagram) give contributions of the same order.

On the other hand, under the assumption that in the BSM theory BNV takes place through the exchange of a heavy vector boson, we invoke the argument given at the beginning of Subsection 5.2 and the decay width simplifies:

$$\Gamma\left(\tau^+ \to \Lambda\pi^+\right) = \frac{1}{\Lambda^4} \left[a_1 \left(\left|\tilde{C}_{LR}\right|^2 + \left|\tilde{C}_{RL}\right|^2 \right) + a_2 \ \mathrm{Re}\left\{ \tilde{C}_{LR} \tilde{C}_{RL}^* \right\} \right]. \quad (5.62)$$

Once more, we can impose naturalness condition over the undetermined coefficients to set a bound to the new physics scale using the experimental bound to the branching fraction for the decay. Using the best experimental upper limit, $\mathcal{B}\left(\tau^+ \to \Lambda\pi^+\right) < 1.4 \times 10^{-7}$, from Belle [29] we get:

$$\Lambda \geq 13 \text{ TeV}. \quad (5.63)$$

5.5 Results and conclusions

It is important to remember that the only dimension six operators which violate baryon number are B-L conserving [14, 15]. As a consequence, every other B-violating operator will be suppressed by at least one power of the higher energy scale. Therefore, one naively expect the B-L conserving BNV processes to be dominant. Hence, the analysis of B-L conserving BNV processes is specially interesting. The study of $\tau^+ \to p\mu^+\mu^-$ and $\tau^+ \to \Lambda\pi^+$ decays we have realized in the previous Subsections will be helpful to constrain such BNV processes.

Nevertheless, we should point out that the operators we have used have been also applied to study proton decay [12] but with electrons as leptonic fields instead of taus. Based on the universality principle, one could therefore argue that the operators contributing to $\tau^+ \to p\mu^+\mu^-$ decay width should be extremely suppressed because of proton decays analyses. However, as we have remarked in the LFV decays analysis, one should be careful when imposing such principles as we do not know anything about the high energy theory. It could happen that, for some reason, the BSM theory favors B-L tau decays over the ones involving electrons or muons. Nevertheless, it does not seem to be the case [39].

Even with BNV processes involving quarks from the first family heavily constrained, BNV processes where higher generations appear could be relevant. In this regard, the study of decays involving quarks from the second and third family in a model independent way could be interesting. In this sense, we have analyzed $\tau^+ \to \Lambda\pi^+$ decay in an EQFT framework. From the computation of this decay, it can be seen that this process is enhanced by six orders of magnitude over $\tau^+ \to p\mu^+\mu^-$ decay. For this reason, under naturalness hypothesis we expect $\tau^+ \to \Lambda\pi^+$ process to be more favorable to be observed. Indeed, before doing the computation one would expect this to happen because apart from the BNV vertex $\tau^+ \to p\mu^+\mu^-$ decay occurs through electromagnetic interactions while $\tau^+ \to \Lambda\pi^+$ decay is due to strong interactions only. In this sense, we would naively expect a difference $\mathcal{O}(\alpha^2) \simeq 10^{-6}$ which is the factor observed.

Finally, we have calculated a lower limit to the new physics scale from the BNV we have studied. The analysis shows that while the experimental bounds are far from being constraining in $\tau^+ \to p\mu^+\mu^-$ decay, the bounds to $\tau^+ \to \Lambda\pi^+$ process are around 10 TeV so if we assume that new physics including BNV interactions take place around $10 - 100$ TeV we are approaching the region where this decay should be observed.

6 Summary and final conclusions

In this master thesis we have studied LFV and BNV tau and muon lepton decays in an EQFT framework. As a first approach, a systematic study of LFV leptonic decays has been done using the EQFT extension of the SM. We have carefully analyzed the structure of the different effective operators contributing to the processes and we have determined the dominance of one subset of them under some assumptions over the new physics theory. Using this analysis, we have obtained several constraints in the new physics scale and some interesting phenomenological fractions between branching ratios. As a result, we have concluded that scenarios where the high-energy theory is a heavily coupled LFV gauge theory are severely constrained. Moreover, we have resolved that if LFV occurs at a scale near the SSB scale, it has to be in such a way that it takes place in the tau sector while being suppressed in the muon one.

After studying LFV leptonic decays, we have examined BNV $\tau^+ \to p\mu^+\mu^-$ decay. In other to do that, we had to present the RχT formalism and a Lagrangian for baryons and their interactions. After that, we have hadronized the BNV violating dimension six operators and we have finished with $\tau^+ \to p\mu^+\mu^-$ decay computation. From the effective operator examination we have concluded that, as there is no dimension six BNV operator which violates B-L conservation, it seems natural that B-L conserving BNV decays are enhanced over every other BNV decay. Finally, in order to complete the analysis of BNV tau decays, we have studied $\tau^+ \to \Lambda\pi^+$ decay which involves quarks from the second family. This decay analysis has shown that under naturalness condition this process is more likely to occur than $\tau^+ \to p\mu^+\mu^-$ decay by six orders of magnitude.

It is worth to mention that the analysis we present in this master thesis is not the end of the story. Specially in the hadronic sector, there is a lot of work to do. Indeed, some new processes can be studied with minor modifications to the presented formalism. As an example, the study of $\tau \to \mu\pi\pi$ LFV or $\tau^+ \to p\pi^0\pi^0$ BNV decays, with similar experimental branching ratios as the decays considered here, can be readily computed. The analysis of such processes could serve as an starting point for Ph.D. project.

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A Antisymmetric formalism for spin-1 particles

The antisymmetric formalism is more convenient than the Proca formalism when dealing with resonances in RχT. Among the main advantages of using the antisymmetric instead of the Proca formalism, we can stress the fact that axial and pseudoscalar resonances do not mix at first order in the chiral expansion and that the effective action to lowest chiral order simplifies because it is not necessary to consider terms of order $\mathcal{O}(p^4)$ to guarantee a good behavior at high energies, as it happens with Proca formalism.

As antisymmetric formalism has been less popular in the literature than the Proca formalism, we will present it below. We will follow [32]. Let us consider a general Lagrangian which is quadratic in the antisymmetric tensor $W_{\mu\nu} = -W_{\nu\mu}$,

$$\mathscr{L} = a\partial^\mu W_{\mu\nu}\partial_\rho W^{\rho\nu} + b\partial^\rho W_{\mu\nu}\partial_\rho W^{\mu\nu} + cW_{\mu\nu}W^{\mu\nu}, \tag{A.1}$$

with a, b and c arbitrary constants. The field $W^{\mu\nu}$ has six degrees of freedom while we only need three in order to correctly describe a spin-1 massive field. In order to reduce the number of degrees of freedom, we need to choose the constants conveniently.

The equations of motion of this Lagrangian are:

$$a\left(\partial^\mu\partial_\sigma W^{\sigma\nu} - \partial^\nu\partial_\sigma W^{\sigma\mu}\right) + 2b\partial^\sigma\partial_\sigma W^{\mu\nu} - 2cW^{\mu\nu} = 0. \tag{A.2}$$

In components,

$$\begin{aligned}&(a+2b)\,\ddot{W}^{0i} + a\partial_l\dot{W}^{li} - a\partial^i\partial_l W^{l0} - 2\left(b\Delta + c\right)W^{0i} = 0,\\ &2b\ddot{W}^{ik} + a\left[\partial^i\left(\dot{W}^{0k} + \partial_l\dot{W}^{lk}\right) - \partial^k\left(\dot{W}^{0i} + \partial_l W^{li}\right)\right] - 2\left(b\Delta + c\right)W^{ik} = 0,\end{aligned} \tag{A.3}$$

where the dots denote time derivatives. Now, if we take $a + 2b = 0$, the fields W^{0i} do not propagate, while the other three fields W^{ik} are frozen when $b = 0$. Moreover, the propagator of $W^{\mu\nu}$ contains poles in $k^2 = -c/b$ y $k^2 = -2c/(a+2b)$ which disappear for $b = 0$ and $a + b = 0$ respectively. If we choose $a = -1/2$, $b = 0$ and $c = M^2/4$, we have,

$$\mathscr{L} = -\frac{1}{2}\partial^\mu W_{\mu\nu}\partial_\rho W^{\rho\nu} + \frac{1}{4}M^2 W_{\mu\nu}W^{\mu\nu}, \tag{A.4}$$

where we get the following equations of motion:

$$\partial^\mu\partial_\rho W^{\sigma\nu} - \partial^\nu\partial_\sigma W^{\sigma\mu} + M^2 W^{\mu\nu} = 0. \tag{A.5}$$

Now, if we define:

$$W_\mu = M^{-1}\partial^\nu W_{\mu\nu}, \tag{A.6}$$

the equation of motion takes the form:

$$\partial_\rho\left(\partial^\rho W^\mu - \partial^\mu W^\rho\right) + M^2 W^\mu = 0, \tag{A.7}$$

which is the Proca equation.

From the Lagrangian one derives the free propagator:

$$\begin{aligned}\langle 0 \mid T\left\{W_{\mu\nu}(x), W_{\rho\sigma}(y)\right\} \mid 0\rangle = iM^{-2}\int \frac{d^4k e^{-ik(x-y)}}{(2\pi)^4\left(M^2 - k^2 - i\epsilon\right)} \\ \times\left[g_{\mu\rho}g_{\nu\sigma}\left(M^2 - k^2\right) + g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - (\mu \leftrightarrow \nu)\right],\end{aligned} \tag{A.8}$$

which correspond to the normalization:

$$\langle 0 \mid W_{\mu\nu} \mid W, p\rangle = iM^{-1}\left[p_\mu\epsilon_\nu(p) - p_\nu\epsilon_\mu(p)\right], \tag{A.9}$$

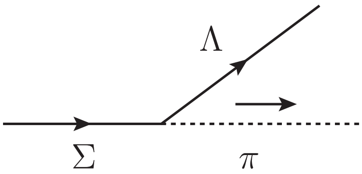
or

$$\langle 0 \mid W_\mu \mid W, p\rangle = \epsilon_\nu(p). \tag{A.10}$$

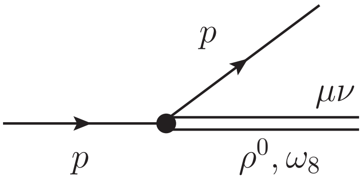
B Feynman rules

In order to ease the understanding of the decay width calculations we made along this master thesis, the Feynman rules which have been used are listed below.

B.1 χPT, RχT and Standard Model vertices

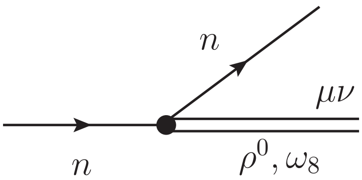


$\frac{D}{\sqrt{3}F} p_\pi^\mu \gamma_\mu \gamma_5$



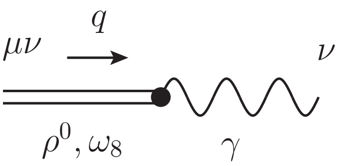
$\rho^0 : \frac{i}{\sqrt{2}} \left(R_F + R_D\right) \sigma_{\mu\nu}$

$\omega_8 : i\sqrt{\frac{3}{2}} \left(R_F - \frac{R_D}{3}\right) \sigma_{\mu\nu}$



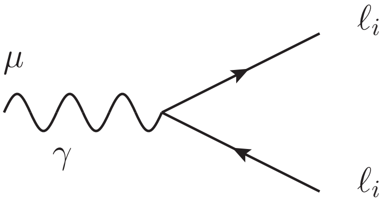
$\rho^0 : -\frac{i}{\sqrt{2}} \left(R_F + R_D\right) \sigma_{\mu\nu}$

$\omega_8 : i\sqrt{\frac{3}{2}} \left(R_F - \frac{R_D}{3}\right) \sigma_{\mu\nu}$

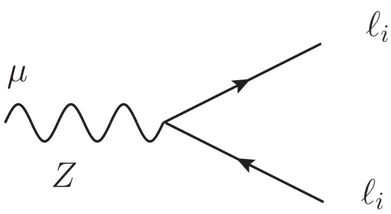


$\rho^0 : -eF_V q_\mu$

$\omega_8 : -\frac{eF_V}{\sqrt{3}} q_\mu$

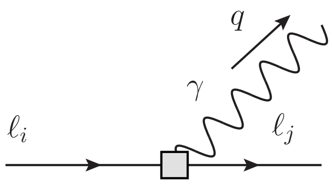


$ieQ\gamma_\mu$

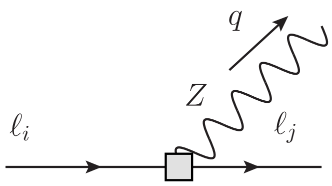


$-\frac{ig}{c_W} \gamma_\mu \left(v_{\ell_i} + a_{\ell_i} \gamma_5\right)$

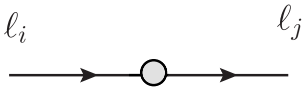
B.2 Lepton Flavor and Baryon Number Violating vertices



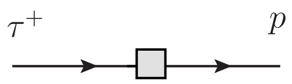
$$-2q_\nu\sigma^{\mu\nu}\left(P_L\alpha_L + P_R\alpha_R\right)$$



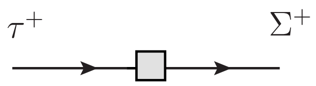
$$-2q_\nu\sigma^{\mu\nu}\left(P_L\tilde{\alpha}_L + P_R\tilde{\alpha}_R\right)$$



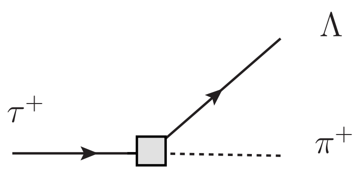
$$\frac{iv^3}{2\sqrt{2}}\left(\beta_L P_L + \beta_R P_R\right)$$



$$i\left(\alpha C_{RL} + \beta C_{LL}\right)CP_L - i\left(\alpha C_{LR} + \beta C_{RR}\right)CP_R$$

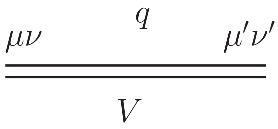


$$i\left(\alpha\tilde{C}_{RL} + \beta\tilde{C}_{LL}\right)CP_L - i\left(\alpha\tilde{C}_{LR} + \beta\tilde{C}_{RR}\right)CP_R$$

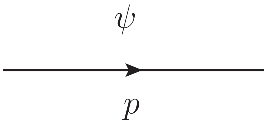


$$\frac{\alpha}{\sqrt{3}F}\left(\tilde{C}_{RL}P_L + \tilde{C}_{RL}P_R\right)$$

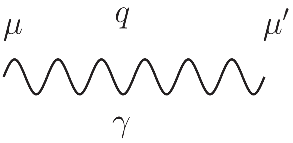
B.3 Propagators



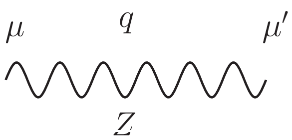
$$\frac{i}{M_V^2(M_V^2-q^2)}\left[g_{\mu\mu'}g_{\nu\nu'}\left(M_V^2-q^2\right)+g_{\mu\mu'}q_\nu q_{\nu'}-g_{\mu\nu'}q_\nu q_{\mu'}-(\mu\leftrightarrow\nu)\right]$$



$$\frac{i}{\not{p}-m}$$



$$-\frac{ig_{\mu\mu'}}{q^2}$$



$$-\frac{i}{q^2-M_Z^2}\left[g_{\mu\mu'}-\frac{q_\mu q_{\mu'}}{M_Z^2}\right]$$

C Proton and neutron anomalous magnetic moments

The structure of the nucleon electromagnetic form factor is given by:

$$\langle N(p')\mid\mathcal{J}_\mu\mid N(p)\rangle=e\overline{u}(p')\left\{\gamma_\mu F_1^N(t)+\frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^N(t)\right\}u(p),\qquad N=p,n,\tag{C.1}$$

with $t=q^2=(p'-p)^2$ the invariant momentum transfer squared, $\mathcal{J}_\mu=\overline{q}Q\gamma_\mu q$ the isovector quark current, and m_N the nucleon mass. In this section, we are interested in the calculation of the anomalous magnetic moments which is encoded in F_2^N. At first order in the chiral expansion, these quantities are completely determined by vector resonance exchange (see Figure C.1) and therefore their computation is useful to determine the phenomenological value of R_D and R_F constants in the chiral Lagrangian from Equation (5.21).

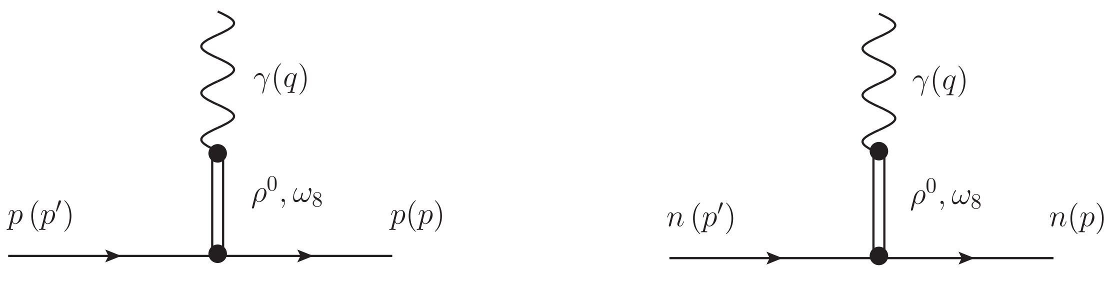


Figure C.1: *Feynman diagrams contributing to nucleon anomalous magnetic moments at tree level order. The dots stand for $R\chi T$ interactions.*

Using Feynman rules in Appendix B, it is straightforward to get the matrix element for diagrams in Figure C.1. In the chiral limit,

$$i\mathcal{M}_p = -\frac{2\sqrt{2}eF_V}{t-M_V^2}\left(R_F+\frac{R_D}{3}\right)\overline{u}_p\left(p'\right)\sigma^{\mu\nu}u_p\left(p\right)q_\nu\epsilon_\mu,$$
$$i\mathcal{M}_n = \frac{4\sqrt{2}eF_V}{3\left(t-M_V^2\right)}R_D\overline{u}_n\left(p'\right)\sigma^{\mu\nu}u_n\left(p\right)q_\nu\epsilon_\mu, \tag{C.2}$$

where $\mathcal{M}_p$ and $\mathcal{M}_n$ are the proton and neutron matrix element respectively. These matrix elements can be directly related to F_2^N so that,

$$F_2^p(t) = \frac{4\sqrt{2}F_V m_p}{t-M_V^2}\left(R_F+\frac{R_D}{3}\right),$$
$$F_2^n(t) = -\frac{8\sqrt{2}F_V m_n}{3\left(t-M_V^2\right)}R_D. \tag{C.3}$$

Now, taking the experimental values for the nucleon anomalous magnetic moment available at PDG [9]:

$$F_2^p(0) = 1.793,$$
$$F_2^n(0) = -1.913, \tag{C.4}$$

we get the following phenomenological values for R_F and R_D:

$$R_F \simeq -0.61,$$
$$R_D \simeq -2.08. \tag{C.5}$$

A similar analysis using a Lagrangian where the resonances have been integrated out and where the anomalous magnetic moment of more baryons has been taken into account in order to fit R_F and R_D is done in [40]. The result we have obtained is compatible with the one in that paper at tree level order. Values for the constants at second order in the chiral expansion can also be found in [40].

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