Minimal Lepton Flavor Violation and the Two-Higgs Doublet Model

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Contents

1 Introduction

The standard model of electroweak interactions due to Weinberg, Salam and Glashow [1–3] is a quantum field theory based on the principles of unitarity, causality, locality, Lorentz invariance and local gauge invariance under $SU(2)_L \otimes U(1)_Y$. Strong interactions are described by the gauge group $SU(3)_C$ and together with the electroweak sector form the standard model (SM) of elementary particle physics. We also require the theory to be renormalizable, though we now think of the SM as the low energy limit of a more fundamental theory, so that renormalizability can be dropped if we are looking for new physics effects.

To accommodate particle masses with the principle of gauge invariance, a complex scalar $SU(2)_L$-doublet, H, with a non-vanishing vacuum expectation value is introduced, so that the electroweak symmetry gets spontaneously broken: $SU(2)_L \otimes U(1)_Y \xrightarrow{SSB} U(1)_{EM}$. The matter content of the theory can be classified according to its transformation properties under the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ and the Lorentz group. The Higgs doublet $H(1, 2, +1/2)$ is a scalar under Lorentz transformations, while all the fermion fields are represented by Weyl fields of definite chirality,

$$
\begin{aligned}
L_L(1,2,-1/2) &= \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}, \qquad l_R(1,1,-1), \\
Q_L(3,2,+1/6) &= \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad u_R(3,1,+2/3), \quad d_R(3,1,-1/3)\,.
\end{aligned}
\tag{1}
$$

There are three families of fermions with the same quantum numbers and different masses, we say that each type of fermion comes in three flavors. Thus we should consider $\{\, Q_L, L_L, d_R, u_R, l_R \,\}$ as 3×1 vectors in flavor space. Gauge interactions corresponding to unbroken symmetries do not distinguish between the different families. All the flavor dynamics in the SM, i.e. interactions that distinguish between different flavors, is then generated by the electroweak sector. Neutrinos are massless in this model, so that it has to be extended if we want to account in a natural way for neutrino oscillations.

Even though the SM has been successful when compared to experiments, there are many questions that remain unanswered by the model, for example: why there seem to be three families of fermions and what is the origin of the flavor dynamics and the mass patterns observed. The large number of free parameters in the SM is also unsatisfactory. There are in total 18 free parameters: the strong gauge coupling, 2 electroweak gauge couplings, 2 in the scalar sector and 13 in the Yukawa sector (9 fermion masses, 3 mixing angles and 1 complex phase). It is not surprising that most of the parameters are needed to account for particle masses and the flavor dynamics. Any extension of the SM that accounts for neutrino masses and mixing introduces in general more parameters.

The electroweak sector of the SM suffers from the well known hierarchy problem [4–6]. Some of the most interesting extensions of the electroweak (EW) sector that remedy this problem, such as the minimal supersymmetric standard model (MSSM), necessarily introduce a second Higgs doublet. Extending the electroweak sector also seems necessary if we want to generate the baryon asymmetry of the universe at the EW scale, models with two Higgs doublets offer interesting possibilities in this respect [7, 8]. The general two-Higgs doublet model (2HDM) then provides a framework to study well motivated extensions of the SM and to discriminate between them by comparing to experimental data.

Different versions of the two-Higgs doublet model have been extensively studied, see for example Ref.[9] and references therein. Within the quark sector, the main constraint for such models comes from flavor changing neutral current processes which are severely suppressed in nature. The aligned two-Higgs doublet model (A2HDM) [10], based on the hypothesis of Yukawa alignment, eliminates non-diagonal couplings at tree level and allows for new flavor-blind sources of CP violation at the same time. Quantum corrections induce flavor changing neutral currents (FCNC's) at the loop level with a particular hierarchical structure, making it possible to accommodate masses for the scalar particles of the order of the EW scale with current low energy constraints. It should then be possible to test this scenario at both the intensity and precision frontier in experiments like the LHC and Super B factories. It has been shown that the Yukawa alignment condition can be derived from the hypothesis of minimal flavor violation (MFV) generalized to include flavor-blind phases [11].

In this thesis we propose different extensions of the MFV principle to the lepton sector of a 2HDM, considering different scenarios for neutrino masses. We define MFV for leptons inspired by Ref.[11] and the motivation to extend the A2HDM in order to account for neutrino masses. In Sec.(2) we develop the basic formalism of the 2HDM for the case of a CP conserving scalar potential. In Sec.(3) we discuss how the strong suppression of FCNC's observed in the quark sector imposes severe restrictions on the flavor sector of a general 2HDM. In Sec.(4), we consider different extensions of the MFV principle to the lepton sector of a 2HDM. In Sec.(5) we study the renormalization-group equations of the models developed previously with special attention on the induced FCNC's. Finally in Sec.(6) we study the constraints from the flavor violating decays $l_i \to l_j \gamma$ using the expressions obtained in the previous sections.

2 The Two-Higgs Doublet Model

In this section we describe the EW sector of the most general two-Higgs doublet model consistent with the $SU(2)_L \otimes U(1)_Y$ gauge symmetry principle. The presentation here will follow closely the one of Ref.[12], with some minor changes in the notation. We consider the same fermion content as in the SM and two Higgs doublets with hypercharge $Y = \frac{1}{2}$, denoted by ϕ_k, $k = \{1, 2\}$,

$$\phi_k = \begin{pmatrix} {\phi_k}^+ \\ {\phi_k}^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_k^1 + i\phi_k^2 \\ \phi_k^3 + i\phi_k^4 \end{pmatrix}. \tag{2}$$

The corresponding charge-conjugated fields are denoted by $\widetilde{\phi}_k$

$$\widetilde{\phi}_k = i\tau_2 {\phi_k}^* = \begin{pmatrix} {\phi_k}^{0\dagger} \\ -{\phi_k}^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_k^3 - i\phi_k^4 \\ -\phi_k^1 + i\phi_k^2 \end{pmatrix} . \tag{3}$$

The 2HDM is constructed as the most general renormalizable and gauge invariant Lagrangian and can be divided into three parts: the kinetic and gauge Lagrangian $\mathcal{L}_{\text{kinetic+gauge}}$, the Higgs sector $\mathcal{L}_{\text{Higgs}}$ and the Yukawa interactions contained in $\mathcal{L}_{\text{Y}}$,

$$\mathcal{L}_{\text{2HDM}} = \mathcal{L}_{\text{kinetic+gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Y}} . \tag{4}$$

Like in the SM, we have that $\mathcal{L}_{\text{Kinetic+gauge}} + \mathcal{L}_{\text{Higgs}}$ is invariant under the global flavor symmetry

$$SU(3)_q^3 = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} . \tag{5}$$

Flavor violation in the quark sector refers to interactions that break the flavor symmetry (5). In the 2HDM the source of all flavor violation are the Yukawa interactions $\mathcal{L}_{\text{Y}}$.

2.1 The Scalar Sector

The Higgs sector is responsible for the breaking of the EW symmetry and is described by

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + (D_\mu\phi_2)^\dagger(D^\mu\phi_2) - V(\phi_1,\phi_2) , \tag{6}$$

where $D_\mu = \partial_\mu - ig' Y B_\mu - ig\, T_i W_\mu^i$, is the covariant derivative associated with the gauge symmetry $SU(2)_L \otimes U(1)_Y$. The generators of $SU(2)$ are denoted by T_i and can be written in terms of the Pauli matrices τ_i as $T_i = \frac{1}{2}\tau_i$. The scalar potential $V(\phi_1,\phi_2)$ is given by

$$\begin{aligned} V = {} & m_{11}^2\ {\phi_1}^\dagger\phi_1 + m_{22}^2\ {\phi_2}^\dagger\phi_2 - \Big(m_{12}^2\ {\phi_1}^\dagger\phi_2 + h.c.\Big) + \frac{\lambda_1}{2}\ ({\phi_1}^\dagger\phi_1)^2 \\ & + \frac{\lambda_2}{2}\ ({\phi_2}^\dagger\phi_2)^2 + \lambda_3\ ({\phi_1}^\dagger\phi_1)({\phi_2}^\dagger\phi_2) + \lambda_4\ ({\phi_1}^\dagger\phi_2)({\phi_2}^\dagger\phi_1) \\ & + \Big\{\frac{\lambda_5}{2}({\phi_1}^\dagger\phi_2)^2 + \Big[\lambda_6({\phi_1}^\dagger\phi_1) + \lambda_7({\phi_2}^\dagger\phi_2)\Big]{\phi_1}^\dagger\phi_2 + h.c.\Big\}. \end{aligned} \tag{7}$$

All the parameters are real with the exception of m_{12}^2 and $\lambda_{5,6,7}$, h.c. stands for hermitian conjugate. We will assume that there is no explicit CP violation in the scalar potential, this implies that all the parameters in (7) are taken real. Assuming that the vacuum preserves the $U(1)_{\text{em}}$ symmetry we can expand the fields around their vacuum expectation values (VEVs)

$$\phi_a = e^{i\theta_a} \begin{pmatrix} \phi_a^+ \\ \frac{1}{\sqrt{2}}(v_a + \rho_a + i\eta_a) \end{pmatrix} . \tag{8}$$

Furthermore, using the invariance under $U(1)_{\rm em}$ we can take $\theta_1 = 0$, keeping the relative phase $\theta \equiv \theta_2 - \theta_1$. We assume that the scalar potential does not lead to spontaneous CP violation so that we take $\theta = 0$. We also define for future purposes $v = \sqrt{v_1^2 + v_2^2}$. The potential stationary conditions are obtained by requiring $\left.\frac{\partial V}{\partial \phi_i^\dagger}\right|_{\phi_j = v_j/\sqrt{2}} = 0$,

$$
\begin{aligned}
m_{11}^2 &= m_{12}^2 t_\beta - \frac{1}{2} v^2 \Big[\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2 + 3\lambda_6 s_\beta c_\beta + \lambda_7 s_\beta^2 t_\beta \Big] \,, \\
m_{22}^2 &= m_{12}^2 t_\beta^{-1} - \frac{1}{2} v^2 \Big[\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2 + \lambda_6 c_\beta^2 t_\beta^{-1} + 3\lambda_7 s_\beta c_\beta \Big] \,,
\end{aligned} \tag{9}
$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$, $c_\beta = \cos\beta = v_1/v$, $s_\beta = \sin\beta = v_2/v$ and $t_\beta = \tan\beta$. The terms quadratic in the fields that follow from (7) are

$$
V = m_{H^\pm}^2 \, H^+ H^- + \frac{1}{2} m_A^2 \, A^2 + \frac{1}{2} \begin{pmatrix} \rho_1 & \rho_2 \end{pmatrix} \mathcal{M} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} + \cdots \,. \tag{10}
$$

The physical scalar spectrum consists of a charged field H^+, a CP-odd scalar A and two CP-even scalars h and H. The CP-odd and charged scalar masses are given by

$$
\begin{aligned}
m_A^2 &= \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2} v^2 (2\lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) \,, \\
m_{H^\pm}^2 &= m_A^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4) \,.
\end{aligned} \tag{11}
$$

The neutral CP-even scalar states are found by diagonalizing the mass matrix $\mathcal{M}$ by means of an orthogonal transformation,

$$
\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} \,, \tag{12}
$$

with $c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$ and

$$
\mathcal{M}^2 \equiv m_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + \mathcal{B}^2 \,, \tag{13}
$$

where

$$
\mathcal{B}^2 = v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{pmatrix} \,. \tag{14}
$$

One then gets $m_H \geq m_h$,

$$
m_{H,h}^2 = \frac{1}{2} \Big[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2} \Big] \,. \tag{15}
$$

The angle α is fixed by

$$\begin{aligned}\sin(2\alpha) =& \frac{2\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{11}^2-\mathcal{M}_{22}^2)^2+4(\mathcal{M}_{12}^2)^2}}\,,\\ \cos(2\alpha) =& \frac{\mathcal{M}_{11}^2-\mathcal{M}_{22}^2}{\sqrt{(\mathcal{M}_{11}^2-\mathcal{M}_{22}^2)^2+4(\mathcal{M}_{12}^2)^2}}\,.\end{aligned} \tag{16}$$

We have been working so far in the basis of Higgs doublets defined by $\{\phi_1, \phi_2\}$, but we could have chosen any other basis connected to the latter by a unitary transformation, the kinetic term of the scalar sector (6) transforms covariantly under such transformation. Using this freedom we can look for a special basis where $\mathcal{L}_{\text{Kinetic+gauge}}$ and $\mathcal{L}_{\text{Y}}$ have a simpler form. Let us consider a general change of basis in the Higgs sector

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \rightarrow U \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}\,. \tag{17}$$

The rotation U is a general 2×2 unitary matrix . Particularly useful are those transformations that leave only one Higgs doublet with a VEV different from zero,

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \frac{1}{v}\begin{pmatrix}v_1 & v_2\\ -v_2 & v_1\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}\,. \tag{18}$$

We call such basis “the Higgs basis". Expanding Φ_1 and Φ_2 around their VEV we get

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+S_1+iG^0)\end{pmatrix}\,, \tag{19}$$

$$\Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2+iS_3)\end{pmatrix}\,. \tag{20}$$

The fields $\{G^0, S_1, S_2, S_3\}$ are hermitian Klein-Gordon fields, while G^+ and H^+ are complex Klein-Gordon fields. In the Higgs basis the Goldstone bosons and the physical charged scalar appear explicitly, the neutral mass eigenstates are obtained via an orthogonal transformation O,

$$\begin{pmatrix}H\\ h\\ A\end{pmatrix} = O\begin{pmatrix}S_1\\ S_2\\ S_3\end{pmatrix} = \begin{pmatrix}\cos(\alpha-\beta) & \sin(\alpha-\beta) & 0\\ -\sin(\alpha-\beta) & \cos(\alpha-\beta) & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}S_1\\ S_2\\ S_3\end{pmatrix}\,. \tag{21}$$

We can write the generic Higgs fields (8) in terms of the physical states and Goldstone bosons

$$\begin{aligned}\rho_1 + i\eta_1 &= c_\alpha H - s_\alpha h + ic_\beta G^0 - is_\beta A\,,\\ \rho_2 + i\eta_2 &= s_\alpha H + c_\alpha h + is_\beta G^0 + ic_\beta A\,,\\ {\phi_1}^{\pm} &= c_\beta G^{\pm} - s_\beta H^{\pm}\,,\\ {\phi_2}^{\pm} &= s_\beta G^{\pm} + c_\beta H^{\pm}\,.\end{aligned} \tag{22}$$

Working in the Higgs basis is particularly simple because the Goldstone fields G^0 and $G^\pm$ are contained in only one Higgs doublet Φ_1. It is important to note that the freedom under scalar basis transformations is not only useful for practical purposes but also has a fundamental significance since physical observables should also be independent of such basis changes. In particular the parameter $\tan\beta$ cannot be considered as a physical parameter in the general 2HDM since it is basis dependent [13].

As a final comment we remark that the scalar potential introduces many unknown parameters. This makes difficult to establish theoretical limits on the scalar boson masses. One can infer constraints on these parameters based on general assumptions like perturbativity, the unitarity of the S-matrix and the fact that the potential must be bounded from below (positivity), see Ref.[14] and references therein.

2.2 The Kinetic Sector

The fermion and gauge kinetic sectors of the 2HDM are the same as in the SM, we quote it here for completeness. The gauge fields have the following kinetic term

$$\mathcal{L}_{\text{gauge-kinetic}} = -\frac{1}{4}(F_1{}^{\mu\nu} F_{1\mu\nu} + F_2^{\mu\nu} F_{2\mu\nu} + F_3^{\mu\nu} F_{3\mu\nu} + F_Y^{\mu\nu} F_{Y\,\mu\nu})\,, \tag{23}$$

where the field strengths are given by

$$\begin{aligned} F_i^{\mu\nu} &= \partial^\mu W_i^\nu - \partial^\nu W_i^\mu + g\,\epsilon^{ijk}\, W_j^\mu\, W_k^\nu\,, \\ F_Y^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu\,. \end{aligned} \tag{24}$$

We have used the usual Levi-Civita tensor ϵ^{ijk}, with $\epsilon^{123} = 1$. One defines the physical gauge fields like in the SM by

$$\begin{pmatrix} B \\ W_3 \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}\,, \tag{25}$$

and $W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$. Then (23) is given by

$$\begin{aligned} \mathcal{L}_{\text{gauge-kinetic}} = &-(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu-}) + (\partial_\mu W_\nu^+)(\partial^\nu W^{\mu-}) \\ &- \frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu) + \frac{1}{2}(\partial_\mu A_\nu)(\partial^\nu A^\mu) - \frac{1}{2}(\partial_\mu Z_\nu)(\partial^\mu Z^\nu) + \frac{1}{2}(\partial_\mu Z_\nu)(\partial^\nu Z^\mu) \\ &+ \text{non} - \text{quadratic terms}\,. \end{aligned} \tag{26}$$

The covariant derivative, $D_\mu = \partial_\mu - ig'\,Y B_\mu - ig\,T_i W_\mu^i$, written in terms of the physical fields is given by

$$D_\mu = \partial_\mu + ieA_\mu Q - ig(W_\mu^+ T_+ + W^-{}_\mu T_-) - i\frac{g}{c_w} Z_\mu (T_3 - Q s_w^2)\,, \tag{27}$$

with $Q = Y + T_3$, $T_\pm = \frac{1}{\sqrt{2}}(T_1 \pm iT_2)$ and

$$g = \frac{e}{s_w} \quad , \quad g' = -\frac{e}{c_w}\,. \tag{28}$$

The gauge interactions of the different fermion multiplets f given in (1) are described by

$$\sum_{f'} i\bar{f}'\gamma^\mu D_\mu f' = \sum_{f'} i\bar{f}'\gamma^\mu \partial_\mu f' + \mathcal{L}_A + \mathcal{L}_Z + \mathcal{L}_W \,, \tag{29}$$

where

$$\begin{aligned}
\mathcal{L}_A =& -eA_\mu \left[\frac{2}{3}(\bar{u}'_L\,\gamma^\mu\,u'_L + \bar{u}'_R\,\gamma^\mu\,u'_R) - \frac{1}{3}(\bar{d}'_L\,\gamma^\mu\,d'_L + \bar{d}'_R\,\gamma^\mu\,d'_R) - (\bar{l}'_L\,\gamma^\mu\,l'_L + \bar{l}'_R\,\gamma^\mu\,l'_R)\right] \,, \\
\mathcal{L}_Z =& \frac{g}{2c_w} Z_\mu \Big[(1-\frac{4}{3}s_w^2)\,\bar{u}'_L\,\gamma^\mu\,u'_L - \frac{4}{3}s_w^2\,\bar{u}'_R\,\gamma^\mu\,u'_R + (-1+\frac{2}{3}s_w^2)\,\bar{d}'_L\,\gamma^\mu\,d'_L + \frac{2}{3}s_w^2\,\bar{d}'_R\,\gamma^\mu\,d'_R \\
&+\ (-1+2s_w^2)\,\bar{l}'_L\,\gamma^\mu\,l'_L + 2s_w^2\,\bar{l}'_R\,\gamma^\mu\,l'_R + \bar{\nu}'_L\,\gamma^\mu\,\nu'_L\Big] \,, \\
\mathcal{L}_W =& \frac{g}{\sqrt{2}}\left[W_\mu^+(\bar{u}'_L\,\gamma^\mu\,d'_L + \bar{\nu}'_L\,\gamma^\mu\,l'_L)\right] + \text{h.c.}\,.
\end{aligned} \tag{30}$$

The fermion fields are primed to differentiate them from the mass eigenstates. The kinetic term of the scalar sector (6) is given in the Higgs basis by

$$\begin{aligned}
&(D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) = (\partial_\mu G^+)(\partial^\mu G^-) + \frac{1}{2}(\partial S_1)^2 + \frac{1}{2}(\partial G^0)^2 \\
&+ {M_W}^2 W_\mu{}^- W^{\mu+} + \frac{1}{2}{M_Z}^2 Z^2 + iM_W(W_\mu{}^-\partial^\mu G^+ - W_\mu{}^+\partial^\mu G^-) + M_Z Z_\mu \partial^\mu G^0 \\
&+ (\partial_\mu H^+)(\partial^\mu H^-) + \frac{1}{2}\Big((\partial S_2)^2 + (\partial S_3)^2\Big) + \mathcal{L}_{\phi^2 V} + \mathcal{L}_{\phi V^2} + \mathcal{L}_{\phi^2 V^2}\,.
\end{aligned} \tag{31}$$

Using the notation $A\overset{\leftrightarrow}{\partial}_\mu B \equiv A(\partial_\mu B) - (\partial_\mu A)B$, we have

$$\begin{aligned}
\mathcal{L}_{\phi^2 V} =& ie[A^\mu - \cot(2\theta_W)Z^\mu]\Big[(H^+\overset{\leftrightarrow}{\partial}_\mu H^-) + (G^+\overset{\leftrightarrow}{\partial}_\mu G^-)\Big] \\
&- \frac{e}{\sin(2\theta_{\rm W})} Z^\mu\Big[(G^0\overset{\leftrightarrow}{\partial}_\mu S_1) + (S_3\overset{\leftrightarrow}{\partial}_\mu S_2)\Big] \\
&- \frac{g}{2}W^{\mu+}\Big[(H^-\overset{\leftrightarrow}{\partial}_\mu S_3) - i(H^-\overset{\leftrightarrow}{\partial}_\mu S_2) + (G^-\overset{\leftrightarrow}{\partial}_\mu G^0) - i(G^-\overset{\leftrightarrow}{\partial}_\mu S_1)\Big] \\
&- \frac{g}{2}W^{\mu-}\Big[(H^+\overset{\leftrightarrow}{\partial}_\mu S_3) + i(H^+\overset{\leftrightarrow}{\partial}_\mu S_2) + (G^+\overset{\leftrightarrow}{\partial}_\mu G^0) + i(G^+\overset{\leftrightarrow}{\partial}_\mu S_1)\Big] \,,
\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\phi V^2} =& \frac{2}{v}S_1\Big[\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^- W^{\mu +}\Big] \\ & - (eM_W A^\mu + gM_Z \sin^2\theta_W Z^\mu)(G^+ W_\mu^- + G^- W_\mu^+)\,, \\ \mathcal{L}_{\phi^2 V^2} =& \frac{1}{v^2}\Big[\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^- W^{\mu +}\Big][H^2 + h^2 + A^2 + (G^0)^2] \\ & + \Big\{e^2[A^\mu - \cot(2\theta_W)Z^\mu]^2 + \frac{g^2}{2}W_\mu^- W^{\mu +}\Big\}(G^+G^- + H^+H^-) \\ & - \frac{eg}{2}S_1(A^\mu + \tan(\theta_W)Z^\mu)(G^+W_\mu^- + G^-W_\mu^+) \\ & - \frac{eg}{2}S_2(A^\mu + \tan(\theta_W)Z^\mu)(H^+W_\mu^- + H^-W_\mu^+) \\ & - \frac{eg}{2}iG^0(A^\mu + \tan(\theta_W)Z^\mu)(G^-W_\mu^+ - G^+W_\mu^-) \\ & - \frac{eg}{2}iS_3(A^\mu + \tan(\theta_W)Z^\mu)(H^-W_\mu^+ - H^+W_\mu^-)\,. \end{aligned} \tag{32}$$

The masses M_Z and M_W satisfy

$$v = \frac{2}{g}M_W = \frac{2c_w}{g}M_Z \approx 246 \text{ GeV}\,. \tag{33}$$

The gauge fixing Lagrangian $\mathcal{L}_{\rm GF}$ for a general R_ξ-gauge is given by

$$\begin{aligned}\mathcal{L}_{GF} =& -\frac{1}{2\xi_Z}\partial^\mu Z_\mu \partial^\nu Z_\nu + M_Z\, G^0 \partial^\mu Z_\mu - \frac{\xi_Z}{2}M_Z^2 (G^0)^2 \\ & - \frac{1}{\xi_W}\partial^\mu W_\mu^+ \partial^\nu W_\nu^- + iM_W(G^+\partial^\mu W_\mu^- - G^-\partial^\mu W_\mu^+) - \xi_W M_W^2 G^+G^- \\ & - \frac{1}{2\xi_A}\partial^\mu A_\mu \partial^\nu A_\nu\,, \end{aligned} \tag{34}$$

where the gauge fixing parameters ξ_Z, ξ_W and ξ_A are arbitrary non-negative real numbers.

Finally, to formulate the Fadeev-Popov ghost Lagrangian $\mathcal{L}_{\rm FP}$ one has to consider the variation of the gauge-fixing terms under an infinitesimal gauge transformation. An infinitesimal gauge transformation is parametrized by

$$U(x) = \exp(i\{g\sum_{k=1}^{3} T_k\, \omega_k(x) + g'\, Y\, \beta(x)\})\,, \tag{35}$$

where ω_k and β are infinitesimal parameters, T_k are the SU(2) generators in the fundamental representation and Y is the hypercharge. The scalar doublets in the Higgs basis transform under such transformation as

$$\begin{aligned}\delta\Phi_1 &= i\Big(g\sum_{k=1}^{3}\omega_k T_k + g'\frac{\beta}{2}\Big)\Phi_1\,, \\ \delta\tilde{\Phi}_1 &= i\Big(g\sum_{k=1}^{3}\omega_k T_k - g'\frac{\beta}{2}\Big)\Phi_1\,. \end{aligned} \tag{36}$$

From (36) one finds

$$\delta G^{\pm} = \pm i M_W \omega_{\pm} \pm i\Big[(-e\omega_A + g\frac{c_w^2 - s_w^2}{2c_w}\omega_Z)G^{\pm} + \frac{g}{2}(S_1 \pm iG^0)\omega_{\pm}\Big] ,$$

$$\delta G^0 = -M_Z\omega_Z - \frac{g}{2c_w}S_1\omega_Z + \frac{g}{2}(\omega_+ G^- + \omega_- G^+) , \tag{37}$$

where we have defined

$$\omega_{\pm} = \frac{1}{\sqrt{2}}(\omega_1 \mp i\omega_2) \quad , \quad \begin{pmatrix}\beta \\ \omega_3\end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w\end{pmatrix}\begin{pmatrix}\omega_A \\ \omega_Z\end{pmatrix} . \tag{38}$$

The gauge fields transform according to

$$\delta A_\mu = \partial_\mu \omega_A + ie(\omega_- W_\mu^+ - \omega_+ W_\mu^-) ,$$

$$\delta Z_\mu = \partial_\mu \omega_Z - igc_w(\omega_- W_\mu^+ - \omega_+ W_\mu^-) ,$$

$$\delta W_\mu^{\pm} = \partial_\mu \omega_{\pm} \pm igW_\mu^{\pm}(-s_w\omega_A + c_w\omega_Z) \mp ig(-s_w A_\mu + c_w Z_\mu)\omega_{\pm} . \tag{39}$$

The ghost Lagrangian for a general R_ξ-gauge is given by

$$\mathcal{L}_{\rm FP} = -\sum_i \Big[\bar{c}_Z \frac{\delta(\partial_\mu Z^\mu - \xi_Z M_Z G^0)}{\delta\omega_i} c_i + \bar{c}_A \frac{\delta(\partial_\mu A^\mu)}{\delta\omega_i} c_i$$

$$+ \bar{c}^{\,+} \frac{\delta(\partial^\mu W_\mu^+ - i\xi_W M_W G^+)}{\delta\omega_i} c_i + \bar{c}^{\,-} \frac{\delta(\partial^\mu W_\mu^- + i\xi_W M_W G^-)}{\delta\omega_i} c_i\Big] . \tag{40}$$

In (40) the sum the of ghost fields c_i runs over $\{c_Z, c_A, c^+, c^-\}$ and that of ω_i over $\{\omega_Z, \omega_A, \omega_+, \omega_-\}$. Taking into account (37) and (39) one gets

$$\mathcal{L}_{\rm FP} = -\,\bar{c}_Z\Big[\Big(\partial_\mu\partial^\mu + \xi_Z M_Z^2 + \frac{g}{2c_w}\xi_Z M_Z S_1\Big)c_Z - \frac{g}{2}\xi_Z M_Z G^+ c^- - \frac{g}{2}\xi_Z M_Z G^- c^+\Big]$$

$$- igc_w\partial^\mu\bar{c}_Z W_\mu^- c^+ + igc_w\partial^\mu\bar{c}_Z W_\mu^+ c^- - \bar{c}_A\partial_\mu\partial^\mu c_A - ie\partial^\mu\bar{c}_A W_\mu^- c^+ + ie\partial^\mu\bar{c}_A W_\mu^+ c^-$$

$$- \bar{c}^{\,+}\Big[\Big(\partial_\mu\partial^\mu + \xi_W M_W^2 + \frac{g}{2}\xi_W(S_1 + iG^0)\Big)c^+ + \xi_W g M_W \frac{c_w^2 - s_w^2}{2c_W}G^+ c_Z - e\xi_W M_W G^+ c_A\Big]$$

$$- ig\partial^\mu\bar{c}^{\,+}(-s_w A_\mu + c_w Z_\mu)c^+ + igc_w\partial^\mu\bar{c}^{\,+}W_\mu^+ c_Z - igs_w\partial^\mu\bar{c}^{\,+}W_\mu^+ c_A$$

$$- \bar{c}^{\,-}\Big[\Big(\partial_\mu\partial^\mu + g\xi_W M_W^2 + \frac{g}{2}\xi_W M_W(S_1 - iG^0)\Big)c^- + \xi_W g M_W \frac{c_w^2 - s_w^2}{2c_w}G^- c_Z - e\xi_W M_W G^-\Big)c_A\Big]$$

$$+ ig\partial^\mu\bar{c}^{\,-}(-s_w A_\mu + c_w Z_\mu)c^- - igc_w\partial^\mu\bar{c}^{\,-}W_\mu^- c_Z + igs_w\partial^\mu\bar{c}^{\,-}W_\mu^- c_A . \tag{41}$$

2.3 Yukawa Interactions

All the possible couplings of the fermion fields with the Higgs doublets consistent with gauge invariance and renormalizability are contained in the Yukawa Lagrangian $\mathcal{L}_Y$,

$$\mathcal{L}_Y \;=\; -\Big\{\bar{Q}_L'(\Gamma_1\phi_1 + \Gamma_2\phi_2)\,d_R' + \bar{Q}_L'(\Delta_1\widetilde{\phi}_1 + \Delta_2\widetilde{\phi}_2)\,u_R' + \bar{L}_L'(\Pi_1\phi_1 + \Pi_2\phi_2)\,l_R'\Big\} \;+\; \text{h.c.} , \tag{42}$$

where Γ_j, Δ_j and Π_j are arbitrary 3×3 complex matrices in flavor space. The primes over the fermion fields are meant to emphasize that these are not the mass eigenstates. The Lagrangian (42) cannot account for neutrino masses and mixing. In the following of this section we focus on the quark sector, the flavor lepton sector will be treated in detail in Sec.(4). Writing this Lagrangian in the Higgs basis we get

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\left\{\bar{Q}_L^{'}(M_d^{'}\Phi_1 + Y_d^{'}\Phi_2)\, d_R^{'} + \bar{Q}_L^{'}(M_u^{'}\tilde{\Phi}_1 + Y_u^{'}\tilde{\Phi}_2)\, u_R^{'}\right\} + \text{h.c.}\,, \tag{43}$$

with

$$\begin{aligned} M_d^{'} &= \frac{1}{\sqrt{2}}\,(v_1\Gamma_1 + v_2\Gamma_2\,)\,, \\ Y_d^{'} &= \frac{1}{\sqrt{2}}\,(v_1\Gamma_2 - v_2\Gamma_1)\,, \\ M_u^{'} &= \frac{1}{\sqrt{2}}\,(v_1\Delta_1 + v_2\Delta_2)\,, \\ Y_u^{'} &= \frac{1}{\sqrt{2}}\,(v_1\Delta_2 - v_2\Delta_1)\,. \end{aligned} \tag{44}$$

We can make bi-unitary transformations in flavor space to diagonalize the mass matrices $\{M_u^{'}, M_d^{'}\}$,

$$\begin{aligned} u_L^{'} &= U_L^u u_L \quad , \quad d_L^{'} = U_L^d d_L\,, \\ u_R^{'} &= U_R^u u_R \quad , \quad d_R^{'} = U_R^d d_R\,. \end{aligned} \tag{45}$$

We choose the unitary matrices $U_{L,R}^f$ $(f = u, d)$ so that

$$\begin{aligned} U_L^{u\dagger} M_u^{'} U_R^u &= M_u = diag(m_u, m_c, m_t)\,, \\ U_L^{d\dagger} M_d^{'} U_R^d &= M_d = diag(m_d, m_s, m_b)\,. \end{aligned} \tag{46}$$

We define the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V = U_L^{u\dagger} U_L^d$ and the Yukawa matrices by

$$\begin{aligned} Y_u &= U_L^{u\dagger}\, Y_u^{'}\, U_R^u\,, \\ Y_d &= U_L^{d\dagger}\, Y_d^{'}\, U_R^d\,. \end{aligned} \tag{47}$$

Note that Y_u and Y_d are arbitrary complex matrices. Expanding the Higgs fields around their VEV we can write the Yukawa interactions (omitting the Goldstone bosons) (42) as

$$\begin{aligned} \mathcal{L}_Y = &-(1 + \frac{S_1}{v})(\bar{d}\, M_d P_R\, d + \bar{u}\, M_u\, P_R\, u) - \frac{1}{v} S_2\Big[\bar{d}\, Y_d\, P_R\, d + \bar{u}\, Y_u\, P_R\, u\Big] \\ &- \frac{i}{v} S_3\left[\bar{d}\, Y_d\, P_R\, d - \bar{u}\, Y_u\, P_R\, u\right] \\ &- \frac{\sqrt{2}}{v} H^+ \bar{u}\, V\, Y_d\; P_R\, d + \frac{\sqrt{2}}{v} H^- \bar{d}\, V^\dagger\, Y_u\, P_R\, u + \text{h.c.}\,. \end{aligned} \tag{48}$$

In (48) $P_{L,R}$ are the usual chiral projectors, $P_L = \frac{1}{2}(1-\gamma_5)$ and $P_R = \frac{1}{2}(1+\gamma_5)$. Since Y_u and Y_d are in general non-diagonal they lead to flavor changing neutral couplings involving the fields S_2 and S_3. The Yukawa matrices Y_u and Y_d can also contain complex phases, giving rise to new sources of CP violation.

In terms of the fermion mass eigenstates we can write the charged-current interaction (30) as

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \left[W_\mu^+ \, \bar{u}_L \, V \, \gamma^\mu \, d_L \right] + \text{h.c.} \,. \tag{49}$$

The neutral interactions $\mathcal{L}_A$ and $\mathcal{L}_Z$ (30) are flavor diagonal in the mass eigenstate basis.

3 Natural Flavor Conservation and Minimal Flavor Violation in the Quark Sector

FCNC's arise in (48) because of the impossibility to simultaneously diagonalize two arbitrary complex matrices in general, in our case those of (44). Within the quark sector, flavor changing neutral current processes like $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing impose stringent bounds on the magnitude of these flavor non-diagonal couplings. One way to suppress the FCNC's is to assume that the particles that give rise to flavor changing neutral couplings are very heavy, in the TeV range [15].

More interesting scenarios, with the scalar particle masses close to the EW scale, can be obtained by making additional assumptions about the flavor sector of the model so as to protect it from large FCNC's. One way to eliminate non-diagonal terms in the Lagrangian is by imposing flavor-blind symmetries so that only one Higgs doublet couples to a given quark species, this is called natural flavor conservation [16, 17]. Another possibility to suppress FCNC's is provided by the hypothesis of minimal flavor violation (MFV) were all the flavor dynamics is related to the SM Yukawa couplings.

We can classify the different scenarios by considering the breaking of the largest group of unitary transformations that leave invariant the SM gauge and kinetic Lagrangians [11, 18]

$$G_g^q = SU(3)_q^3 \otimes U(1)_B \otimes U(1)_Y \otimes U(1)_{PQ} \,, \tag{50}$$

where $U(1)_B$ corresponds to baryon number, $U(1)_Y$ to hypercharge and $U(1)_{PQ}$ is the Peccei-Quin symmetry [19]. The transformation properties of the quark fields under $SU(3)_q^3$ in terms of the unitary matrices $\{V_L, V_U, V_D\}$ are

$$Q_L \to V_L \, Q_L, \qquad u_R \to V_U \, u_R, \qquad d_R \to V_D \, d_R \,. \tag{51}$$

Models with NFC can be obtained by imposing $U(1)_{PQ}$ or another flavor-blind discrete symmetry involving both right handed quarks and both Higgs fields. For different versions of the 2HDM with NFC see for example Ref.[9]. Minimal flavor violation as formulated in Ref.[18] is based on the assumption that $SU(3)_q^3$ is broken only by two independent 3×3 matrices $\lambda_{u,d}$, which transform as spurions under $SU(3)_q$ in the following way

$$\lambda_u \to V_L \, \lambda_u \, V_U^\dagger \quad , \quad \lambda_d \to V_L \, \lambda_d \, V_D^\dagger \,. \tag{52}$$

Then at first order in the breaking terms $\lambda_{u,d}$, the MFV hypothesis is equivalent to the alignment of the Yukawa matrices in Eq. (42) [10]

$$\begin{aligned} \Gamma_2 &= \xi_d^2 \lambda_d \quad , \quad \Gamma_1 = \xi_d^1 \lambda_d \, , \\ \Delta_2 &= \xi_u^{2*} \lambda_u \quad , \quad \Delta_1 = \xi_u^{1*} \lambda_u \, . \end{aligned} \tag{53}$$

Where $\vec{\xi_u} = \{\xi_u^1, \xi_u^2\}$ and $\vec{\xi_d} = \{\xi_d^1, \xi_d^2\}$ are arbitrary complex parameters since we assume that the breaking of CP and that of $SU(3)_q^3$ are decoupled. Note that we are using a similar notation for the gauge-fixing parameters $\{\xi_A, \xi_Z, \xi_W\}$ and in (53).

The Yukawa alignment condition is the basis of the aligned two-Higgs doublet model (A2HDM) as defined in Ref.[10]. it was later shown that the A2HDM is equivalent to the MFV hypothesis (53) at fist order in the symmetry breaking terms $\lambda_{u,d}$ [11]. Writing the alignment condition (53) in terms of the mass and Yukawa matrices that appear in (43) we have

$$Y'_d = \varsigma_d M'_d \, , \qquad Y'_u = \varsigma_u^* M'_u \, , \qquad \varsigma_f = \frac{\xi_f - \tan\beta}{1 + \xi_f \tan\beta} \, , \qquad \tan\beta = v_2/v_1 \, . \tag{54}$$

In (54) we have used the definition $\xi_f = \xi_f^2/\xi_f^1$. The parameters ς_f are scalar-basis invariant. The condition of Yukawa alignment (53) guarantees that we can diagonalize simultaneously the mass and Yukawa matrices of (44), so that there are no FCNC's at tree level. The Yukawa interactions (48) become in this case

$$\begin{aligned} \mathcal{L}_Y = &- (1 + \frac{S_1}{v})(\bar{d}\, M_d P_R\, d + \bar{u}\, M_u\, P_R\, u) \\ &- \frac{1}{v} S_2 \Big[\varsigma_d\, \bar{d}\, M_d\, P_R\, d \, + \varsigma_u^*\, \bar{u}\, M_u\, P_R\, u\Big] \\ &- \frac{i}{v} S_3 \Big[\varsigma_d\, \bar{d}\, M_d\, P_R\, d - \varsigma_u^*\, \bar{u}\, M_u\, P_R\, u\Big] \\ &- \frac{\sqrt{2}}{v} \varsigma_d\, H^+ \bar{u}\, V\, M_d\, \, P_R\, d + \frac{\sqrt{2}}{v} \varsigma_u^*\, H^- \bar{d}\, V^\dagger\, M_u\, P_R\, u + \text{h.c.} \, . \end{aligned} \tag{55}$$

Considering the 2HDM as an effective theory below the scale Λ, we encounter higher order operators $O_i^{(k)}$ constructed from the 2HDM fields

$$\mathcal{L}_{\text{eff}}(E < \Lambda) = \mathcal{L}_{\text{2HDM}} + \sum_{i,k} \frac{1}{\Lambda^{k-4}}\, c_{i;k}\, O_i^{(k)} \, . \tag{56}$$

In the MFV framework $\mathcal{L}_{\text{eff}}$ must be invariant under $SU(3)_q^3$ once the transformation properties of the spurions are taken into account. The flavor structure of the higher order operators is then determined by the spurions $\lambda_{u,d}$. As is well known, the MFV principle allows us to avoid the new physics flavor problem and to accommodate new physics at the TeV scale [18]. The extension of MFV to the lepton sector (MLFV) can be done in many different ways and is in general less predictive than for the quark sector [20, 21].

4 Minimal Flavor Violation in the Lepton Sector

To understand the pattern of masses and mixing of the fundamental fermions is among the main challenges of particle physics. Comparing the flavor structure of the lepton and quark sectors may lead to important clues in the search for the underlying theory that generates such patterns. However, finding physically meaningful relations between the quark and lepton-flavor sectors is far from trivial. An example of this kind of efforts are the so-called Quark-Lepton Complementarity (QLC) relations, which relate angles of the quark mixing matrix to that of the lepton mixing matrix. However, as remarked by C. Jarlskog such relations are ill-defined since their definition is convention dependent [22].

Even though the hypothesis of minimal flavor violation was motivated by the strong suppression of FCNC's in the quark sector, it is interesting to look for extensions of this principle to the lepton sector. The interest is two-fold: supposing that the MFV structure of the quark flavor sector is generated by some underlying dynamics or symmetry principle, one can then expect that the lepton sector should also possess a similar structure. It is also interesting because extending the SM in order to account for neutrino masses can be done in many different ways and the principle of MFV can then be used as a guide in the model building or to make the model more predictive.

The principle of MFV has been extended to the lepton sector for different scenarios of neutrino masses for the case of one Higgs doublet, see for example Refs.[20, 21, 23, 24]. The case of two-Higgs doublets has not been studied in great detail yet, among some recent works we find Ref.[25] and Ref.[26]. In this section we consider minimal extensions of the SM with two-Higgs doublets following Refs.[20, 21]. We are interested in predictive scenarios where the flavor structure is determined in terms of the lepton mixing matrix and the light neutrino masses. We consider different ways to implement the MFV hypothesis with particular attention on the role of Yukawa alignment. The cases studied are:

Extended field content with lepton number conservation. There are 3 right-handed neutrinos and lepton number is conserved at the classical level, neutrinos are Dirac particles. The maximal flavor group is $SU(3)_l^3 \equiv SU(3)_{L_L} \otimes SU(3)_{\nu_R} \otimes SU(3)_{l_R}$.

Minimal field content. The same fermion content as in the SM, the neutrinos get their mass from the dimension five Weinberg operator [27]. The lepton flavor symmetry group is $SU(3)_{L_L} \otimes SU(3)_{l_R}$.

Extended field content without lepton number conservation. There are 3 right-handed neutrinos and lepton number is violated by the Majorana mass term for the right-handed neutrinos. We assume that the flavor symmetry group $SU(3)_l^3$ is broken down to the subgroup $SU(3)_{L_L} \otimes O(3)_{\nu_R} \otimes SU(3)_{l_R}$ by the Majorana mass matrix for the right-handed neutrinos.

It is important to notice that the physics responsible for the neutrino masses is expected to be at very high energy scales. Indeed see-saw models with natural Yukawa couplings require the heavy Majorana masses to be around $10^{10} - 10^{16}$ GeV. This is fundamentally different from the quark sector where the MFV principle is thought to constrain the flavor structure of TeV-scale new physics. One has to keep in mind that imposing a symmetry principle over such different scales is a very strong assumption.

4.1 Extended Field Content with Lepton Number Conservation.

One way to accommodate neutrino masses and mixing is to add to the fermion content of the SM three right-handed neutrinos, which we denote by a 3×1 vector in flavor space ν_R. The right-handed neutrinos are singlets under the gauge group $\nu_R(1,1,0)$. The lepton kinetic Lagrangian in this case is invariant under the group of transformations

$$G_g^l \equiv SU(3)_l^3 \otimes U(1)_L \otimes U(1)_Y \otimes U(1)_{l_R}\,, \tag{57}$$

where $U(1)_L$ corresponds to lepton number, $U(Y)$ to hypercharge. The transformation properties of the lepton fields under $SU(3)_l^3$ are

$$L_L \to V_L\, L_L, \qquad l_R \to V_l\, l_R, \qquad \nu_R \to V_\nu\, \nu_R\,, \tag{58}$$

where $\{V_L, V_l, V_\nu\}$ are generic unitary matrices.

The lepton Yukawa Lagrangian in a generic scalar basis $\{\phi_1, \phi_2\}$ is given by

$$\mathcal{L}_Y \;=\; -\left\{\bar{L}_L'(\Pi_1\phi_1 + \Pi_2\phi_2)\, l_R' + \bar{L}_L'(\Sigma_1\widetilde{\phi}_1 + \Sigma_2\widetilde{\phi}_2)\, \nu_R'\right\} \;+\; \text{h.c.}\;. \tag{59}$$

In the Higgs basis we have

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\left\{\bar{L}_L'(M_l'\Phi_1 + Y_l'\Phi_2)\, l_R' + \bar{L}_L'(M_\nu'\widetilde{\Phi}_1 + Y_\nu'\widetilde{\Phi}_2)\nu_R'\right\} + \text{h.c.}\,, \tag{60}$$

where

$$\begin{aligned}
M_l' &= \frac{1}{\sqrt{2}}\,(v_1\Pi_1 + v_2\Pi_2)\;, \\
Y_l' &= \frac{1}{\sqrt{2}}\,(v_1\Pi_2 - v_2\Pi_1)\;, \\
M_\nu' &= \frac{1}{\sqrt{2}}\,(v_1\Sigma_1 + v_2\Sigma_2)\;, \\
Y_\nu' &= \frac{1}{\sqrt{2}}\,(v_1\Sigma_2 - v_2\Sigma_1)\;.
\end{aligned} \tag{61}$$

In the case where neutrinos are Dirac particles and lepton number is conserved, one can implement the MFV principle in an analogous way to the quark sector. We assume that the flavor symmetry $SU(3)_l^3$ is broken only by two independent 3×3 matrices $\lambda_{l,\nu}$. These matrices transform as spurions under $SU(3)_l^3$ in the following way

$$\lambda_\nu \to V_L\, \lambda_\nu\, V_\nu^\dagger \qquad , \qquad \lambda_l \to V_L\, \lambda_l\, V_l^\dagger\,. \tag{62}$$

At first order in the symmetry breaking terms $\lambda_{l,\nu}$ the MFV hypothesis is then equivalent to the Yukawa alignment condition, analogous to (53),

$$\Pi_2 = \xi_l^2 \lambda_l \quad , \quad \Pi_1 = \xi_l^1 \lambda_l \, ,$$
$$\Sigma_2 = \xi_\nu^{2*} \lambda_\nu \quad , \quad \Sigma_1 = \xi_\nu^{1*} \lambda_\nu \, , \tag{63}$$

and defining $\xi_f = \xi_f^2/\xi_f^1$ we can write

$$Y_l' = \varsigma_l M_l' \, , \qquad Y_\nu' = \varsigma_\nu^* M_\nu' \, , \qquad \varsigma_f = \frac{\xi_f - \tan\beta}{1 + \xi_f \tan\beta} \, . \tag{64}$$

We consider the breaking of CP and of $SU(3)_l^3$ to be independent, so that $\vec{\xi_\nu} \equiv= \{\xi_\nu^1, \xi_\nu^2\}$ and $\vec{\xi_l} \equiv \{\xi_l^1, \xi_l^2\}$ are arbitrary complex parameters. A general bi-unitary transformation in the lepton sector is given by

$$\nu_R' = U_R^\nu \nu_R \;\; , \;\; \nu_L' = U_L^\nu \nu_L \, ,$$
$$l_R' = U_R^l l_R \;\; , \;\; l_L' = U_L^l l_L \, . \tag{65}$$

We can choose a specific basis where the mass matrices are diagonal, real and with positive eigenvalues,

$$U_L^{\nu\,\dagger} M_\nu' U_R^\nu = M_\nu = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \, ,$$
$$U_L^{l\,\dagger} M_l' U_R^l = M_l = diag(m_e, m_\mu, m_\tau) \, . \tag{66}$$

Defining the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) $U_{\rm PMNS} = U_L^{\nu\dagger} U_L^l$, the charged-current interaction $\mathcal{L}_W$ (30) takes the form

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \left[W_\mu^+ (\bar\nu_L \, U_{\rm PMNS} \, \gamma^\mu \, l_L) + \text{h.c.} \right] \, , \tag{67}$$

while the neutral interactions $\mathcal{L}_A$ and $\mathcal{L}_Z$ do not change since the right-handed neutrinos are singlets under the gauge group. The matrix $U_{\rm PMNS}$ can be parametrized in general as

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \, , \tag{68}$$

with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, the angles $\theta_{ij} \in [0, \pi/2]$ and the Dirac phase δ. The lepton Yukawa Lagrangian is given by

$$\begin{aligned} \mathcal{L}_Y = &- (1 + \frac{S_1}{v})(\bar l \, M_l P_R \, l + \bar\nu \, M_\nu \, P_R \, \nu) - \frac{1}{v} S_2 \Big[\varsigma_l \, \bar l \, M_l \, P_R \, l \, + \varsigma_\nu^* \, \bar\nu \, M_\nu \, P_R \, \nu \Big] \\ &- \frac{i}{v} S_3 \left[\varsigma_l \, \bar l \, M_l \, P_R \, l - \varsigma_\nu^* \, \bar\nu \, M_\nu \, P_R \, \nu \right] \\ &- \frac{\sqrt{2}}{v} \varsigma_l \, H^+ \bar\nu \, U_{\rm PMNS} \, M_l \;\, P_R \, l + \frac{\sqrt{2}}{v} \varsigma_\nu^* \, H^- \bar l \, U_{\rm PMNS}^\dagger \, Y_\nu \, P_R \, \nu + \text{h.c.} \, . \end{aligned} \tag{69}$$

4.2 Minimal Field Content

We now explore the scenario were the left-handed neutrinos get Majorana masses through an effective operator of dimension 5 known as the Weinberg operator [27], see Fig.(1). The kinetic part is invariant under the group transformations

$$SU(3)_{L_L} \otimes SU(3)_{l_R} \otimes U(1)_L \otimes U(1)_Y \,, \tag{70}$$

where $U(1)_L$ and $U(1)_Y$ correspond to total lepton number and hypercharge respectively. The lepton fields transform under (70) as

$$L_L \to V_L\, L_L, \qquad l_R \to V_l\, l_R \,. \tag{71}$$

The Lagrangian responsible for the neutrino Majorana masses is given by

$$\mathcal{L}_{eff} = -\frac{1}{\Lambda_{\rm LN}}\mathcal{O}_{eff} + \text{h.c.}\,, \tag{72}$$

where $\Lambda_{\rm LN}$ is the scale of the physics responsible for the breaking of $U(1)_L$. We assume that the breaking of $U(1)_L$ is independent from the breaking of the flavor symmetry $SU(3)_{L_L} \otimes SU(3)_{l_R}$. In full generality we can write $\mathcal{O}_{eff}$ in a general scalar basis as

$$\mathcal{O}_{eff} = \sum_{i,j=1}^{2} \sum_{a,b,c,d=1}^{2} \Big({L'_{La}}^T \kappa^{ij}\, C^{-1}\, L'_{Lc} \Big) \epsilon^{ab}\epsilon^{cd} \phi_{ib}\phi_{jd} \,, \tag{73}$$

where C is the charge conjugation operator[1] and κ^{ij} is a 3×3 matrix in flavor space for $i,j = 1,2$ that satisfies $\kappa^{ij} = (\kappa^{ji})^T$. The Weinberg operator $\mathcal{O}_{eff}$ introduces through κ^{ij} a new source of violation of the global flavor symmetry (70), specifically of the $SU(3)_{L_L}$ part, and it also violates lepton number by two units $\Delta L = 2$. In the Higgs basis $\mathcal{O}_{eff}$ is given by

$$\mathcal{O}_{eff} = \sum_{i,j=1}^{2} \sum_{a,b,c,d=1}^{2} \Big({L'_{La}}^T \kappa'^{ij}\, C^{-1}\, L'_{Lc} \Big) \epsilon^{ab}\epsilon^{cd} \Phi_{ib}\Phi_{jd} \,, \tag{74}$$

where

$$\begin{aligned}
\kappa'^{11} &= c_\beta^2 \kappa^{11} + c_\beta s_\beta \kappa^{12} + c_\beta s_\beta \kappa^{21} + s_\beta^2 \kappa^{22}\,, \\
\kappa'^{12} &= -c_\beta s_\beta \kappa^{11} + c_\beta^2 \kappa^{12} - s_\beta^2 \kappa^{21} + c_\beta s_\beta \kappa^{22}\,, \\
\kappa'^{21} &= -c_\beta s_\beta \kappa^{11} - s_\beta^2 \kappa^{12} + c_\beta^2 \kappa^{21} + c_\beta s_\beta \kappa^{22}\,, \\
\kappa'^{22} &= s_\beta^2 \kappa^{11} - c_\beta s_\beta \kappa^{12} - c_\beta s_\beta \kappa^{21} + c_\beta^2 \kappa^{22}\,.
\end{aligned} \tag{75}$$

[1]The charge conjugation operator C is given in the chiral representation by $C = i\gamma_2\gamma_0$.

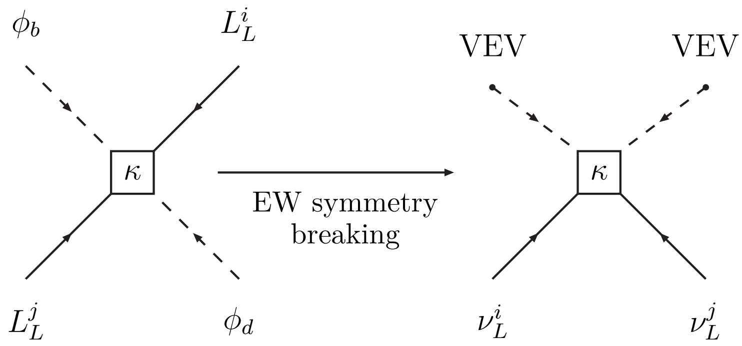


Figure 1: Effective Majorana mass for the neutrinos.

The lepton Yukawa sector (42) in the Higgs basis takes the form

$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v}\left\{\bar{L}_L'(M_l'\Phi_1 + Y_l'\Phi_2)\, l_R'\right\} + \text{h.c.}\,. \tag{76}$$

We assume that there are only two irreducible sources of lepton-flavor symmetry breaking: λ_l and g_ν, that transform under $SU(3)_{L_L} \otimes SU(3)_{l_R}$ as spurions in the following way

$$\lambda_l \to V_L\,\lambda_l\,V_l^\dagger \quad , \quad g_\nu \to V_L^*\,g_\nu\,V_L^\dagger\,. \tag{77}$$

At first order in λ_l we then get the alignment of the Yukawa matrices (63) and (64). At first order in g_ν we have

$$\kappa^{ij} = c_{ij}g_\nu\,, \qquad \kappa'^{ij} = c_{ij}'g_\nu\,, \tag{78}$$

with c_{ij} (c_{ij}') being complex parameters in a generic scalar basis (in the Higgs basis). Since $\kappa'^{ij} = \kappa'^{jiT}$ we have that $g_\nu = g_\nu^T$ and $c_{12}' = c_{21}'$. We can normalize g_ν so that $c_{11}' = 1$. After the electroweak symmetry breaking, the Weinberg operator gives rise to a Majorana mass term for the left-handed neutrinos

$$\mathcal{L}_{eff} \xrightarrow{SSB} \mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\nu_L'^T C^{-1} M_\nu' \nu_L' + \text{h.c.} + \cdots\,, \tag{79}$$

with

$$M_\nu' = \frac{v^2}{\Lambda_{\text{LN}}}\kappa'^{11} = \frac{v^2}{\Lambda_{\text{LN}}}g_\nu\,. \tag{80}$$

A general bi-unitary transformation of the lepton fields is given by

$$\begin{aligned} \nu_L' &= U_L^\nu \nu_L \\ l_R' &= U_R^l l_R\ , \qquad l_L' = U_L^l l_L\,. \end{aligned} \tag{81}$$

We can choose a particular basis where the charged lepton and neutrino mass matrices are diagonal and positive,

$$\begin{aligned} M_l &= U_L^{l\,\dagger} M_l' U_R^l = \text{diag}(m_e, m_\mu, m_\tau)\,, \\ M_\nu &= U_L^{\nu T} M_\nu' U_L^\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,. \end{aligned} \tag{82}$$

In this basis we have

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\left[W_\mu^+(\bar{\nu}_L\, U\, \gamma^\mu\, l_L + \text{h.c.}\right] , \tag{83}$$

where $U = U_L^\nu U_L^{l\,\dagger}$ is the mixing matrix and can be parametrized in general by,

$$U = U_{\text{PMNS}} \times \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) , \tag{84}$$

with U_{PMNS} as defined in (68). The extra phases $\{\alpha_{21}, \alpha_{31}\}$ appear because the neutrinos are Majorana particles in this case. The part of the effective Lagrangian linear in the scalar fields reads

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{v}{\Lambda_{\text{LN}}}\Big\{\nu_L'^{\,T}\kappa'^{11}C^{-1}\nu_L'(S_1+iG_0) - \nu_L'^{\,T}\kappa'^{11}C^{-1}l_L'\frac{1}{\sqrt{2}}G^+ - l_L'^{\,T}\kappa'^{11}C^{-1}\nu_L'\frac{1}{\sqrt{2}}G^+ \\ &+\nu_L'^{\,T}\kappa'^{12}C^{-1}\nu_L'\frac{1}{2}(S_2+iS_3) - \nu_L'^{\,T}\kappa'^{12}C^{-1}l_L'\frac{1}{\sqrt{2}}H^+ + \nu_L'^{\,T}\kappa'^{21}C^{-1}\nu_L'\frac{1}{2}(S_2+iS_3)\Big\} + h.c.\,.\end{aligned} \tag{85}$$

In the mass eigenstates basis we then have

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{1}{v}\Big\{\nu_L^T M_\nu C^{-1}\nu_L(S_1+iG_0) - \nu_L^T M_\nu U C^{-1} l_L\frac{1}{\sqrt{2}}G^+ - l_L^T M_\nu U^T C^{-1}\nu_L\frac{1}{\sqrt{2}}G^+ \\ &+\frac{c_{12}}{2}\nu_L^T M_\nu C^{-1}\nu_L(S_2+iS_3) - \frac{c_{12}}{\sqrt{2}}\nu_L^T M_\nu U C^{-1} l_L H^+ + \frac{c_{12}}{2}\nu_L^T M_\nu C^{-1}\nu_L(S_2+iS_3)\Big\} + h.c.\,.\end{aligned} \tag{86}$$

In (86) there are no FCNC's due to the MFV condition imposed on κ^{ij} (78).

4.3 Extended Field Content without Lepton Number Conservation.

Without imposing lepton number conservation, the gauge symmetry principle allows the right-handed neutrinos to have a Majorana mass term together with (59),

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2}\nu_R'^{\,T}\, C^{-1}\, M_R'\, \nu_R' + \text{h.c.}\,. \tag{87}$$

In Eq. (87), M_R' is a 3×3 symmetric complex matrix because fermionic fields anti commute. The lepton kinetic Lagrangian in this case is invariant under the flavor transformations $SU(3)_l^3$. The lepton fields transform under this symmetry as (51). The Majorana mass term violates $U(1)_R$ and introduce a new source of violation of the flavor symmetry $SU(3)_l^3$. We can recover the invariance under $SU(3)_l^3$ by considering that the Yukawa matrices transform as spurions according to (62) and

$$M_R' \to V_\nu^*\, M_R'\, V_\nu^\dagger\,. \tag{88}$$

There are many different ways to define MLFV according to how one defines the irreducible sources of lepton-flavor symmetry breaking. Following Ref.[20] we assume that the

Majorana mass matrix M_R breaks the $SU(3)_{\nu_R}$ symmetry to $O(3)_{\nu_R}$. This implies that M_R is proportional to the identity in flavor space, $M_R^{ij} = M_R^0\,\delta^{ij}$. We will assume also that there are only two irreducible sources $\lambda_{l,\nu}$ that break $SU(3)_{L_L} \otimes SU(3)_{l_R}$ in the Yukawa sector.

The Yukawa sector in this case is given by (59) in a general scalar basis and (60) in the Higgs basis. At first order in the symmetry breaking terms $\lambda_{l,\nu}$ the MFV hypothesis is then equivalent to the Yukawa alignment condition (63) and (64). At energies well below the right-handed Majorana mass scale M_R^0, one can consider an effective theory in which the right-handed neutrinos are integrated out, indeed for heavy right-handed neutrinos one can neglect the kinetic term in the equation of motion

$$0 \approx \frac{\partial(\mathcal{L}_Y + \mathcal{L}_{\text{Majorana}})}{\partial \nu_{Rj}} = M_R {\nu_R}^T C^{-1} - \bar{L}_L\Big(\Sigma_1\widetilde{\phi_1} + \Sigma_2\widetilde{\phi_2}\Big)\,. \tag{89}$$

One then obtains

$$\nu_R = -\frac{1}{M_R}\Big(\Sigma_1^T\widetilde{\phi_1}^T + \Sigma_2^T\widetilde{\phi_2}^T\Big)C\bar{L}_L^T\,. \tag{90}$$

Substituting (90) into $\mathcal{L}_Y + \mathcal{L}_{\text{Majorana}}$ and taking into account (63) gives the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{1}{2M_R}\bar{L_L}\bigg((\xi_\nu^{1*})^2\lambda_\nu\widetilde{\phi_1}\lambda_\nu^T\widetilde{\phi_1}^T + \xi_\nu^{1*}\xi_\nu^{2*}\lambda_\nu\widetilde{\phi_2}\lambda_\nu^T\widetilde{\phi_1}^T \\ &+ \xi_\nu^{2*}\xi_\nu^{1*}\lambda_\nu\widetilde{\phi_1}\lambda_\nu^T\widetilde{\phi_2} + (\xi_\nu^{2*})^2\lambda_\nu\widetilde{\phi_2}\lambda_\nu^T\widetilde{\phi_2}\bigg)C\bar{L}_L^T + \text{h.c.}\,.\end{aligned} \tag{91}$$

We can rewrite (91) in a similar form to (73)

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{1}{2M_R}\Big\{(\xi_\nu^{1*})^2(L_L^T\tau_2\phi_1)\lambda_\nu\lambda_\nu^TC^{-1}(\phi_1^T\tau_2L_L) + \xi_\nu^{1*}\xi_\nu^{2*}(L_L^T\tau_2\phi_2)\lambda_\nu\lambda_\nu^TC^{-1}(\phi_1^T\tau_2L_L) \\ &+ \xi_\nu^{2*}\xi_\nu^{1*}(L_L^T\tau_2\phi_1)\lambda_\nu\lambda_\nu^TC^{-1}(\phi_2^T\tau_2L_L) + (\xi_\nu^{2*})^2(L_L^T\tau_2\phi_2)\lambda_\nu\lambda_\nu^TC^{-1}(\phi_2^T\tau_2L_L)\Big\} + \text{h.c.}\,.\end{aligned} \tag{92}$$

Matching the Lagrangian (72) to (92) we obtain the low energy parameters of the minimal field content scenario in terms of high energy parameters related to the right-handed neutrinos. The Yukawa alignment condition naturally leads to the MFV hypothesis (78) with $g_\nu \propto \lambda_\nu\lambda_\nu^T$.

Returning to the complete Lagrangian $\mathcal{L}_Y + \mathcal{L}_{\text{Majorana}}$, a bi-unitary transformation in the lepton sector is given by

$$\begin{aligned} l_L^{'} &= U_L^l l_L \quad , \quad \nu_L^{'} = U_L^\nu \nu_L\,, \\ l_R^{'} &= U_R^l l_R \quad , \quad \nu_R^{'} = U_R^\nu \nu_R\,. \end{aligned} \tag{93}$$

We will work in a particular basis where M_l and M_R are diagonal and real while M_ν is an arbitrary complex matrix. The mass terms obtained are then

$$\mathcal{L}_{\text{mass}} = -\bar{l}_L\,M_l\,l_R + \frac{1}{2}\left({\nu_L'}^T\,,\,(\nu_R')^{cT}\right)C^{-1}\mathcal{M}^*\begin{pmatrix}\nu_L'\\(\nu_R')^c\end{pmatrix} + \text{h.c.}\,, \tag{94}$$

where $(\psi_L)^c \equiv C\gamma_0^T(\psi_L)^*$ and $\mathcal{M}$ is a 6×6 matrix given by

$$\mathcal{M} = \begin{pmatrix} 0 & M_\nu \\ {M_\nu}^T & M_R \end{pmatrix} . \tag{95}$$

To find the neutrino mass eigenstates we need to diagonalize $\mathcal{M}$,

$$V^T \mathcal{M}^* V = \mathcal{D} . \tag{96}$$

In (96), $\mathcal{D}$ is the mass matrix of the physical light and heavy neutrinos,

$$\mathcal{D} \equiv \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix} = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_1, M_2, M_3) , \tag{97}$$

and the matrix V can be written in general as

$$V = \begin{pmatrix} K & Q \\ S & T \end{pmatrix} . \tag{98}$$

Assuming that the mass scale of the right-handed neutrinos is much larger than the EW scale, one can diagonalize $\mathcal{M}$ by an expansion in terms of $\epsilon \sim v/M_R$. In the following we will neglect terms of $O(\epsilon^2)$. Eq.(96) then implies the following conditions

$$S^\dagger \approx -K^\dagger\, M_\nu\, {M_R}^{-1} , \tag{99}$$

$$d \approx -K^\dagger\, M_\nu\, {M_R}^{-1}\, {M_\nu}^T\, K^* , \tag{100}$$

$$T \approx 1 , \tag{101}$$

$$D \approx M_R , \tag{102}$$

$$Q \approx M_\nu\, D^{-1} . \tag{103}$$

The physical neutrino states $\{\nu_L, N_L\}$ are given by

$$\begin{pmatrix} {\nu_L}' \\ {\nu'_R}^c \end{pmatrix} = V \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} , \tag{104}$$

so that we can diagonalize the mass term (94) by the following set of transformations:

$$\nu'_L = K\,\nu_L + Q\,N_L \ , \qquad \nu'_R = S^*\,\nu_L^c + N_L^c . \tag{105}$$

We can write then

$$\mathcal{L}_{\mathrm{mass}} = -\bar{l}_L\, M_l\, l_R + \frac{1}{2}(\bar{\nu}_L^c\ \bar{N}_L^c)\,\mathcal{D} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} + \mathrm{h.c.} . \tag{106}$$

The leptonic charged and neutral current interactions in the mass basis take the form

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \Big[W_\mu^+ (\bar{l}_L\, \gamma_\mu\, K\, \nu_L + \bar{l}_L\, \gamma_\mu\, Q\, N_L)\Big] + \text{h.c.}\,,$$

$$\begin{aligned} \mathcal{L}_Z =& \frac{g}{2c_w} Z_\mu [(-1+2s_w^2)\, \bar{l}_L \gamma^\mu l_L + 2s_w^2\, \bar{l}_R \gamma^\mu l_R \\ &+ \bar{\nu}_L\, \gamma^\mu\, \nu_L + \bar{\nu}_L\, K^\dagger\, Q\, \gamma^\mu N_L + \bar{N}_L\, Q^\dagger\, K\, \gamma^\mu \nu_L + \bar{N}_L\, Q^\dagger\, Q\, \gamma^\mu N_L]\,, \end{aligned} \tag{107}$$

where K can be identified then with the lepton mixing matrix given in (84).

Expanding the Higgs fields around the VEV we get the Yukawa interactions in the Higgs basis

$$\begin{aligned} \mathcal{L}_Y + \mathcal{L}_{\text{Majorana}} = \mathcal{L}_{\text{mass}} + \Bigg\{ &-\frac{S_1}{v} \Big[\bar{l}_L\, M_l\, l_R + (\bar{\nu}_L\, K^\dagger + \bar{N}_L\, Q^\dagger)\, M_\nu\, (S^*\, \nu_L^c + \, N_L^c)\Big] \\ &- \frac{1}{v} S_2 \Big[\varsigma_l\, \bar{l}_L\, M_l\, l_R + \varsigma_\nu^* (\bar{\nu}_L\, K^\dagger + \bar{N}_L\, Q^\dagger) M_\nu (S^* \nu_L^c + N_L^c)\Big] \\ &- \frac{i}{v} S_3 \left[\varsigma_l\, l_L\, M_l\, l_R - \varsigma_\nu^*\, (\bar{\nu}_L\, K^\dagger + \bar{N}_L\, Q^\dagger)\, M_\nu\, (S^* \nu_L^c + N_L^c)\right] \\ &+ \frac{\sqrt{2}}{v} H^+ \left[\varsigma_\nu\, (\bar{\nu}_L^c\, S^T + \bar{N}_L^c)\, M_\nu{}^\dagger\, U_L^l\, l_L - \varsigma_l\, (\bar{\nu}_L\, K^\dagger + \, \bar{N}_L Q^\dagger)\, M_l'\, U_R^l\, l_R\right] \\ &+ \text{h.c.} \Bigg\}\,. \end{aligned} \tag{108}$$

Multiplying (100) from both sides by $\sqrt{d^{-1}}$ we get

$$\mathbf{1} = -\left(\sqrt{M_R^{-1}}\ M_\nu{}^T\, K^*\, \sqrt{d^{-1}}\right)^T \left(\sqrt{M_R^{-1}}\ M_\nu{}^T\, K^*\, \sqrt{d^{-1}}\right)\,. \tag{109}$$

A general solution to (109) is given by

$$\sqrt{M_R^{-1}}\, M_\nu{}^T = i\, R\, \sqrt{d}\, K^T\,, \tag{110}$$

where R is an orthogonal complex matrix, this is known as the Casas and Ibarra parameterization [28]. A complex orthogonal matrix R can be decomposed as

$$R = O\, H\,, \tag{111}$$

where O is a real orthogonal matrix and H is a complex orthogonal and hermitian matrix. By using the invariance under $O(3)_{\nu_R}$ we can choose a basis of right-handed fields such that $O = 1$. We can parametrize H in the following way

$$H = e^{i\Omega} = I - \frac{\cosh r - 1}{r^2} \Omega^2 + i \frac{\sinh r}{r} \Omega\,, \quad \Omega = \begin{pmatrix} 0 & \phi_1 & \phi_2 \\ -\phi_1 & 0 & \phi_3 \\ -\phi_2 & -\phi_3 & 0 \end{pmatrix}\,, \tag{112}$$

where $r = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2}$ and ϕ_j are real parameters. Using (110) we get

$$\begin{aligned}
B &\equiv K^\dagger\, M_\nu = i\sqrt{d}\, R\, \sqrt{M_R}\,, \\
F &\equiv Q^\dagger\, M_\nu = M_R^{-1}(i\sqrt{d}\, R\, \sqrt{M_R})^\dagger(i\sqrt{d}\, R\, \sqrt{M_R})\,, \\
E &\equiv Q^\dagger\, M_\nu\, S^* = -M_R^{-1}(i\sqrt{d}\, R\, \sqrt{M_R})^\dagger d\,.
\end{aligned} \tag{113}$$

From the previous identities we can also derive

$$S^T\, M_\nu\, U_L^l \approx d\, K\,, \qquad M_\nu\, U_L^l \approx F^\dagger\, Q^\dagger\,, \qquad K^\dagger\, M_\nu\, S^* \approx d\,. \tag{114}$$

Now we can express (108) in terms of R, Q, the lepton mixing matrix and the physical masses $\{M_l,\, d,\, M_R\}$

$$\begin{aligned}
\mathcal{L}_Y + \mathcal{L}_{\text{Majorana}} = \mathcal{L}_{\text{mass}} + \Bigg\{ & -\frac{S_1}{v}\Big[\bar{l}_L\, M_l\, l_R + \bar{\nu}_L\, d\, \nu_L^c + \bar{N}_L\, E\, \nu_L^c + \bar{\nu}_L\, B\, N_L^c + \bar{N}_L\, F\, N_L^c\Big] \\
& - \frac{1}{v} S_2 \Big[\varsigma_l\, \bar{l}_L\, M_l\, l_R + \varsigma_\nu^* (\bar{\nu}_L\, d\, \nu_L^c + \bar{\nu}_L\, B\, N_L^c + \bar{N}_L\, E\, \nu_L^c + \bar{N}_L\, F\, N_L^c)\Big] \\
& - \frac{i}{v} S_3 \Big[\varsigma_l\, l_L\, M_l\, l_R - \varsigma_\nu^* (\bar{\nu}_L\, d\, \nu_L^c + \bar{\nu}_L\, B\, N_L^c + \bar{N}_L\, E\, \nu_L^c + \bar{N}_L\, F\, N_L^c)\Big] \\
& + \frac{\sqrt{2}}{v} H^+ \Big[\varsigma_\nu\, (\bar{\nu}_L^c\, d\, K\, l_L + \bar{N}_L^c\, F^\dagger\, Q^\dagger\, l_L) - \varsigma_l\, (\bar{\nu}_L\, K^\dagger\, M_l\, l_R + \bar{N}_L\, Q^\dagger\, M_l\, l_R)\Big] \\
& + \text{h.c.} \Bigg\}\,.
\end{aligned} \tag{115}$$

At this point we can compare (115) with the MFV model proposed in Ref.[25]. In Ref.[25] a Z_4 symmetry is imposed on the Lagrangian to constrain the possible couplings of the Yukawa sector. As a consequence of the particular charge assignments under Z_4, there are no Higgs mediated FCNC's in the light neutrino sector. In the charged lepton sector there are FCNC's at tree level determined by the lepton mixing matrix. Flavor changing neutral couplings with heavy neutrinos are parametrized by the neutrino masses d, M_R and the matrix R, which in this case is block diagonal as a consequence of the Z_4 symmetry imposed.

On the other hand, in (115) the Yukawa alignment condition produces no Higgs mediated FCNC's for both the light neutrinos and the charged leptons. The flavor changing neutral couplings with the heavy neutrinos are also parametrized by d, M_R and the matrix R, a general orthogonal and hermitian complex matrix. It is also important to notice in (115) the presence of new flavor-blind CP violating phases ς_ν and ς_l, in close analogy with the quark sector of the A2HDM [10]. The Yukawa sector of the *extended field content* scenario studied in Refs.[20, 29] is recovered in the limit $\varsigma_{l,\nu} = 0$.

5 Radiative Corrections

Now we study the one-loop renormalization-group equations (RGE) for the parameters of the different scenarios considered in the previous section. We are interested in the effects of the RGE running from a high energy scale (at which the Yukawa alignment condition is supposed to hold) down to the electroweak scale $\sim M_Z$. For previous studies of the MLFV hypothesis under RGE evolution see for example Refs.[26, 30]. We first consider the case of Dirac neutrinos and lepton number conservation, see Sec.(4.1).

5.1 Extended Field Content with Lepton Number Conservation

It is well known that even though the Lagrangian (69) does not contain tree-level FCNC's, quantum corrections will in general induce non-diagonal neutral couplings. Only models with NFC remain stable under the renormalization group [31]. Since flavor changing neutral current processes are strongly suppressed, one has to examine whether this class of models are still viable.

The one-loop RGE for the Yukawa couplings are [25]

$$
\begin{aligned}
16\pi^2\mu\frac{d}{d\mu}\Gamma_k \equiv \beta_{\Gamma_k} = {} & a_\Gamma \Gamma_k + \\
& + \sum_{l=1}^{2}\Big[3\mathrm{Tr}\Big(\Gamma_k\Gamma_l^\dagger + \Delta_k^\dagger\Delta_l\Big) + \mathrm{Tr}\Big(\Pi_k\Pi_l^\dagger + \Sigma_k^\dagger\Sigma_l\Big)\Big]\Gamma_l \\
& + \sum_{l=1}^{2}\Big(-2\Delta_l\Delta_k^\dagger\Gamma_l + \Gamma_k\Gamma_l^\dagger\Gamma_l + \frac{1}{2}\Delta_l\Delta_l^\dagger\Gamma_k + \frac{1}{2}\Gamma_l\Gamma_l^\dagger\Gamma_k\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\mu\frac{d}{d\mu}\Delta_k \equiv \beta_{\Delta_k} = {} & a_\Delta \Delta_k + \\
& + \sum_{l=1}^{2}\Big[3\mathrm{Tr}\Big(\Delta_k\Delta_l^\dagger + \Gamma_k^\dagger\Gamma_l\Big) + \mathrm{Tr}\Big(\Sigma_k\Sigma_l^\dagger + \Pi_k^\dagger\Pi_l\Big)\Big]\Delta_l \\
& + \sum_{l=1}^{2}\Big(-2\Gamma_l\Gamma_k^\dagger\Delta_l + \Delta_k\Delta_l^\dagger\Delta_l + \frac{1}{2}\Gamma_l\Gamma_l^\dagger\Delta_k + \frac{1}{2}\Delta_l\Delta_l^\dagger\Delta_k\Big)\,,
\end{aligned}
$$

$$16\pi^2\mu\frac{d}{d\mu}\Pi_k \equiv \beta_{\Pi_k} = a_\Pi \Pi_k +$$
$$+\sum_{l=1}^{2}\Big[3\mathrm{Tr}\Big(\Gamma_k\Gamma_l^\dagger + \Delta_k^\dagger\Delta_l\Big) + \mathrm{Tr}\Big(\Pi_k\Pi_l^\dagger + \Sigma_k^\dagger\Sigma_l\Big)\Big]\Pi_l$$
$$+\sum_{l=1}^{2}\Big(-2\Sigma_l\Sigma_k^\dagger\Pi_l + \Pi_k\Pi_l^\dagger\Pi_l + \frac{1}{2}\Sigma_l\Sigma_l^\dagger\Pi_k + \frac{1}{2}\Pi_l\Pi_l^\dagger\Pi_k\Big)\,,$$

$$16\pi^2\mu\frac{d}{d\mu}\Sigma_k \equiv \beta_{\Sigma_k} = a_\Sigma \Sigma_k +$$
$$+\sum_{l=1}^{2}\Big[3\mathrm{Tr}\Big(\Delta_k\Delta_l^\dagger + \Gamma_k^\dagger\Gamma_l\Big) + \mathrm{Tr}\Big(\Sigma_k\Sigma_l^\dagger + \Pi_k^\dagger\Pi_l\Big)\Big]\Sigma_l$$
$$+\sum_{l=1}^{2}\Big(-2\Pi_l\Pi_k^\dagger\Sigma_l + \Sigma_k\Sigma_l^\dagger\Sigma_l + \frac{1}{2}\Pi_l\Pi_l^\dagger\Sigma_k + \frac{1}{2}\Sigma_l\Sigma_l^\dagger\Sigma_k\Big)\,, \tag{116}$$

where μ is the renormalization scale. The coefficients $\{a_\Gamma, a_\Pi, a_\Delta, a_\Sigma\}$ are given by

$$a_\Gamma = -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2\,, \qquad a_\Delta = -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\,,$$
$$a_\Pi = -\frac{9}{4}g^2 - \frac{15}{4}g'^2\,, \qquad a_\Sigma = -\frac{9}{4}g^2 - \frac{3}{4}g'^2\,, \tag{117}$$

where g_s, g and g' are the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ gauge coupling constants respectively. If we neglect the Yukawa matrices Σ_k that couple with ν_R we recover the expressions given in Refs.[34, 41].

The running of the coupling constants is the same as in the SM and is given at one-loop by

$$16\pi^2\mu\frac{d}{d\mu}g_j = -C_j g_j^3\,. \tag{118}$$

Denoting by n_q the number of effective quark flavors we have

$$C_1 = -\frac{1}{3} - \frac{10}{9}n_q \quad , \quad C_2 = 7 - \frac{2}{3}n_q \quad , \quad C_3 = \frac{1}{3}(11N_c - 2n_q)\,. \tag{119}$$

Solving the RGE for the coupling constants we get

$$g_j(M_Z)^2 = \frac{g_j(\Lambda)^2}{1 + \frac{C_j}{8\pi^2}g_j(\Lambda)^2\log\Big(\frac{M_Z}{\Lambda}\Big)} = g_j(\Lambda)^2\sum_{n=0}\Big[-\frac{C_j}{8\pi^2}g_j(\Lambda)^2\log\Big(\frac{M_Z}{\Lambda}\Big)\Big]^n\,. \tag{120}$$

We can study for which values of the parameters $\{\xi_u, \xi_d, \xi_l, \xi_\nu\}$ the alignment condition is stable under the renormalization group. This kind of analysis was made for the case of a trivial leptonic sector (42) in Ref.[31]. Imposing the stability conditions

$$\frac{d}{d\mu}\Gamma_2 = \xi_d\frac{d}{d\mu}\Gamma_1 \quad , \quad \frac{d}{d\mu}\Delta_2 = \xi_u^*\frac{d}{d\mu}\Delta_1\,,$$
$$\frac{d}{d\mu}\Pi_2 = \xi_l\frac{d}{d\mu}\Pi_1 \quad , \quad \frac{d}{d\mu}\Sigma_2 = \xi_\nu^*\frac{d}{d\mu}\Sigma_1\,, \tag{121}$$

we get the following set of equations

$$
\begin{aligned}
&(\xi_u-\xi_d)(1+\xi_u^*\xi_d)=0\,, \qquad &(\xi_d^*-\xi_u^*)(1+\xi_d\xi_u^*)=0\,,\\
&(\xi_l-\xi_d)(1+\xi_l^*\xi_d)=0\,, \qquad &(\xi_l^*-\xi_u^*)(1+\xi_l\xi_u^*)=0\,,\\
&(\xi_\nu-\xi_d)(1+\xi_\nu^*\xi_d)=0\,, \qquad &(\xi_\nu^*-\xi_u^*)(1+\xi_\nu\xi_u^*)=0\,,\\
\\
&(\xi_d-\xi_l)(1+\xi_d^*\xi_l)=0\,, \qquad &(\xi_u^*-\xi_\nu^*)(1+\xi_\nu^*\xi_u)=0\,,\\
&(\xi_u-\xi_l)(1+\xi_u^*\xi_l)=0\,, \qquad &(\xi_d^*-\xi_\nu^*)(1+\xi_\nu^*\xi_d)=0\,,\\
&(\xi_\nu-\xi_l)(1+\xi_l\xi_\nu^*)=0\,, \qquad &(\xi_l^*-\xi_\nu^*)(1+\xi_\nu^*\xi_l)=0\,.
\end{aligned} \tag{122}
$$

Note that in (121) we have neglected the running of the parameters ξ_f. Solving (122) one finds 8 different solutions

$$
\begin{aligned}
&\xi_d=\xi_l=\xi_u=\xi_\nu\,, \qquad &\xi_d^*=-\frac{1}{\xi_l}=\xi_u^*=\xi_\nu^*\,,\\
&\xi_d=\xi_l=-\frac{1}{\xi_u^*}=-\frac{1}{\xi_\nu^*}\,, \qquad &\xi_d^*=\xi_u^*=-\frac{1}{\xi_l}=-\frac{1}{\xi_\nu}\,,\\
&\xi_d=\xi_l=\xi_u=-\frac{1}{\xi_\nu^*}\,, \qquad &\xi_u^*=\xi_l^*=-\frac{1}{\xi_d}=\xi_\nu^*\,,\\
&\xi_d=\xi_l=-\frac{1}{\xi_u^*}=\xi_\nu\,, \qquad &\xi_u^*=\xi_l^*=-\frac{1}{\xi_d}=-\frac{1}{\xi_\nu}\,.
\end{aligned} \tag{123}
$$

The solutions (123) correspond to the possible implementations of a $\mathcal{Z}_2$ symmetry in which each fermion type couples to only one Higgs doublet, generalizing the results of Ref.[31].

We now consider the *leading log* (LL) approximation to solve the RGE (116)

$$
\frac{dY_k}{d\mu}=\frac{1}{16\pi^2\mu}\beta_{Y_k}(\Lambda)\Rightarrow Y_k(M_Z)\approx Y_k(\Lambda)+\frac{1}{16\pi^2}\beta_{Y_k}(\Lambda)\log\left(\frac{M_Z}{\Lambda}\right). \tag{124}
$$

In this approximation the running of the gauge couplings is not taken into account. Using (124) one gets the following scalar mediated FCNC's from (60),

$$
\begin{aligned}
\mathcal{L}_{\rm FCNC}=-\Big\{&\bar{u}_L\Delta_u\Big[\cos(\alpha-\beta)h+\sin(\alpha-\beta)H-iA\Big]u_R\\
&+\bar{d}_L\Delta_d\Big[\cos(\alpha-\beta)h+\sin(\alpha-\beta)H+iA\Big]d_R\\
&+\bar{\nu}_L\Delta_\nu\Big[\cos(\alpha-\beta)h+\sin(\alpha-\beta)H-iA\Big]\nu_R\\
&+\bar{l}_L\Delta_l\Big[\cos(\alpha-\beta)h+\sin(\alpha-\beta)H+iA\Big]l_R\Big\}+\text{h.c.}\,.
\end{aligned} \tag{125}
$$

From (47) and (48) we have

$$
\Delta_u=\frac{1}{v}Y_u=\frac{1}{v}U_L^{u\dagger}Y_u'U_R^u\quad,\qquad \Delta_d=\frac{1}{v}Y_d=\frac{1}{v}U_L^{d\dagger}Y_d'U_R^d\,, \tag{126}
$$

with analogous expressions for the lepton sector. The result we obtain for the non-diagonal matrices Δ_f agrees with the expression found in Ref.[32]

$$
\begin{aligned}
\Delta_u^{\text{off-diag}} &= -\frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_u^*\varsigma_d)(\varsigma_d^*-\varsigma_u^*)\Big(V(M_d^{\text{diag}})^2 V^\dagger M_u^{\text{diag}}\Big)^{\text{off-diag}}\,,\\
\Delta_d^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_u^*\varsigma_d)(\varsigma_d-\varsigma_u)\Big(V^\dagger(M_u^{\text{diag}})^2 V M_d^{\text{diag}}\Big)^{\text{off-diag}}\,,
\end{aligned}
\tag{127}
$$

$$
\begin{aligned}
\Delta_\nu^{\text{off-diag}} &= -\frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_\nu^*\varsigma_l)(\varsigma_l^*-\varsigma_\nu^*)\Big(U_{\text{PMNS}}(M_l^{\text{diag}})^2 U_{\text{PMNS}}^\dagger M_\nu^{\text{diag}}\Big)^{\text{off-diag}}\,,\\
\Delta_l^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_\nu^*\varsigma_l)(\varsigma_l-\varsigma_\nu)\Big(U_{\text{PMNS}}^\dagger(M_\nu^{\text{diag}})^2 U_{\text{PMNS}} M_l^{\text{diag}}\Big)^{\text{off-diag}}\,.
\end{aligned}
\tag{128}
$$

Note that charged lepton flavor violating couplings are suppressed compared to the neutrino ones due to the smallness of the neutrino masses. From (127) and (128) it is also clear that there is a GIM cancellation mechanism suppressing the induced FCNCs. The GIM cancellation is exact for degenerate fermions and is expected to be very effective for the charged leptons since the neutrino mass differences are very small compared to $v \sim 246$ GeV. Numerical simulations of the RGE for the quark Yukawa couplings performed in Ref.[33] showed that the error associated to the LL approximation in expressions (127) can be as large as a factor of 2.5, so that it is not negligible.

As remarked in Ref.[10], the quantum corrections (127) and (128) are invariant under the following flavor-dependent phase transformations of the fermion mass eigenstates

$$
\begin{aligned}
f_X^i &\to e^{i\alpha_i^{f,X}} f_X^i \quad , \quad M_{f,ij} \to e^{i\alpha_i^{f,L}} M_{f,ij} e^{-i\alpha_j^{f,R}}\,,\\
V_{ij} &\to e^{i\alpha_i^{u,L}} V_{ij} e^{-i\alpha_j^{d,L}} \quad , \quad (U_{\text{PMNS}})_{ij} \to e^{i\alpha_i^{\nu,L}} (U_{\text{PMNS}})_{ij} e^{-i\alpha_j^{l,L}}\,,
\end{aligned}
\tag{129}
$$

where $f = d, u, l, \nu$ and $X = L, R$. Because of this symmetry, quantum corrections to the A2HDM have a specific flavor structure. The only allowed local FCNC structures in the quark sector are of the form $\bar{u}_L\, V(M_d M_d^\dagger)^n V^\dagger (M_u M_u^\dagger)^m M_u\, u_R$, $\bar{d}_L\, V^\dagger (M_u M_u^\dagger)^n V (M_d M_d^\dagger)^m M_d\, d_R$ or similar terms with extra factors of V and mass matrices [10]. This argument can be extended to the lepton sector for the case of Dirac neutrinos in a straightforward way.

5.2 Minimal Field Content

In the case of Sec.(4.2), we found that due to the condition (78), which is supposed to hold at an energy scale Λ, there are no FCNC's in (86). One expects that quantum corrections will in general generate some flavor non-diagonal couplings. To write the renormalization group equations for κ^{ij} (73), it is useful to use the following parametrization of the scalar potential (7)

$$
V = \text{quadratic terms} + \sum_{i,j,k,l=1}^{2} \lambda_{ijkl}(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_l)\,,
\tag{130}
$$

so that

$$\frac{\lambda_1}{2} = \lambda_{1111} \quad , \quad \frac{\lambda_2}{2} = \lambda_{2222} \quad , \quad \lambda_3 = \lambda_{2211} + \lambda_{1122}\,,$$
$$\lambda_4 = \lambda_{1221} + \lambda_{2112} \quad , \quad \lambda_5 = \lambda_{1212} + \lambda_{2121}\,,$$
$$\lambda_6 = \lambda_{1112} + \lambda_{1211} = \lambda_{2111} + \lambda_{1121} \quad , \quad \lambda_7 = \lambda_{2212} + \lambda_{1222} = \lambda_{2122} + \lambda_{2221}\,. \tag{131}$$

Using this notation we have [34],

$$16\pi^2\mu\frac{d}{d\mu}\kappa^{ij} = \beta_{\kappa^{ij}} = -3g^2\kappa^{ij} + 4\sum_{k,l=1}^{2}\lambda_{kilj}\kappa^{kl} + \sum_{k=1}^{2}\Big(T_{ki}\kappa^{kj} + T_{kj}\kappa^{ik}\Big) + \kappa^{ij}P + P^T\kappa^{ij}$$
$$+ 2\sum_{k=1}^{2}\Big\{\kappa^{kj}\Pi_i^\dagger\Pi_k - (\kappa^{ik}+\kappa^{ki})\Pi_j^\dagger\Pi_k + \Pi_k^T\Pi_j^*\kappa^{ik} - \Pi_k^T\Pi_i^*(\kappa^{kj}+\kappa^{jk})\Big\}\,, \tag{132}$$

where

$$T_{ij} = \mathrm{tr}\Big[\Pi_i\Pi_j^\dagger\Big] + 3\,\mathrm{tr}\Big[\Gamma_i\Gamma_j^\dagger + \Delta_i^\dagger\Delta_j\Big] \quad ; \quad P = \frac{1}{2}\sum_{k=1}^{2}\Pi_k^\dagger\Pi_k\,. \tag{133}$$

The Yukawa couplings follow the RGE given in (116) setting $\Sigma_1 = \Sigma_2 = 0$.

We can solve (132) by using the LL approximation

$$\kappa^{ij}(M_Z) = \kappa^{ij}(\Lambda) + \frac{1}{16\pi^2}\beta_{\kappa^{ij}}(\Lambda)\log\Big(\frac{M_Z}{\Lambda}\Big)\,. \tag{134}$$

We define the quantity $\hat{v} = (c_\beta, s_\beta)$ and use the notation $\hat{v}\cdot\vec{\xi}_f \equiv c_\beta\xi_f^1 + s_\beta\xi_f^2$ together with $|\vec{\xi}_f|^2 = |\xi_f^1|^2 + |\xi_f^2|^2$. Solving (135) by imposing the MFV condition (78) at the energy scale Λ, we obtain that quantum corrections induce FCNC's at the scale $\sim M_Z$

$$\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{11}(M_Z)U_L^\nu)^{\text{off-diag}} = \Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[|\vec{\xi}_l|^2 - 4|\xi_l^1|^2 - 4c_{12}\xi_l^{1*}\xi_l^2\Big]$$
$$+ (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[|\vec{\xi}_l|^2 + 8|\xi_l^1|^2 + 8c_{12}\xi_l^2\xi_l^{1*}\Big]\Big\}$$
$$\times\frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\hat{v}\cdot\vec{\xi}_l|^2}\,,$$

$$\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{12}(M_Z)U_L^\nu)^{\text{off-diag}} = \Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{12}|\vec{\xi}_l|^2 + 4c_{12}|\xi_l^1|^2$$
$$- 8\xi_l^1\xi_l^{2*} + 4c_{22}\xi_l^2\xi_l^{1*} - 8c_{12}|\xi_l^2|^2\Big]$$
$$+ (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{12}|\vec{\xi}_l|^2 + 8c_{12}\xi_l^1\xi_l^{2*} + 8c_{22}|\xi_l^2|^2\Big]\Big\}$$
$$\times\frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\hat{v}\cdot\vec{\xi}_l|^2}\,,$$

$$
\begin{aligned}
\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{21}(M_Z)U_L^{\nu})^{\text{off-diag}} = \Big\{&(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{12}|\vec{\xi}_l|^2 \\
&+ 4\xi_l^1\xi_l^{2*} - 8c_{22}\xi_l^{1*}\xi_l^2 - 4c_{12}|\xi_l^1|^2 + 4c_{12}|\xi_l^2|^2\Big] \\
&+ (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{12}|\vec{\xi}_l|^2 + 8c_{12}\xi_l^{1*}\xi_l^2 + 8c_{22}|\xi_l^1|^2\Big]\Big\} \\
&\times \frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\hat{v}\cdot\vec{\xi}_l|^2}\,,
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{22}(M_Z)U_L^{\nu})^{\text{off-diag}} = \Big\{&(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{22}|\vec{\xi}_l|^2 + 4c_{12}\xi_l^{2*}\xi_l^1 - 8c_{12}\xi_l^1\xi_l^{2*} \\
&+ (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{22}|\vec{\xi}_l|^2 + 8c_{12}\xi_l^{2*}\xi_l^1\Big]\Big\} \\
&\times \frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\hat{v}\cdot\vec{\xi}_l|^2}\,. \qquad (135)
\end{aligned}
$$

Note that in this case the induced FCNCs are also GIM suppressed. Due to the unitarity of the lepton mixing matrix U, the expressions obtained in (135) vanish in the limit of degenerate charged leptons.

5.3 Extended Field Content without Lepton Number Conservation

Renormalization group effects also play an important role in the case of right-handed Majorana neutrinos. In Sec.(4.3) we assumed that the right-handed neutrino Majorana mass is flavor universal. This condition is supposed to hold at a very high energy scale Λ and is not generally true at other scales due to quantum corrections. The renormalization group evolution of the Majorana mass M_R (87) is given by

$$
16\pi^2\mu\frac{dM_R}{d\mu} =(\Sigma_1\Sigma_1^\dagger + \Sigma_2\Sigma_2^\dagger)M_R + M_R^T(\Sigma_1\Sigma_1^\dagger + \Sigma_2\Sigma_2^\dagger)^T\,, \qquad (136)
$$

where Σ_j are the Yukawa matrices of (59). We are interested in the renormalization group evolution of M_R from Λ to the scale M_R^0. Solving (136) in the LL approximation we get

$$
\begin{aligned}
M_R(M_R^0) \approx{}& M_R(\Lambda) \\
&+ \frac{1}{16\pi^2}\Big[(\Sigma_1\Sigma_1^\dagger + \Sigma_2\Sigma_2^\dagger)M_R^0 + M_R^0(\Sigma_1\Sigma_1^\dagger + \Sigma_2\Sigma_2^\dagger)^T\Big](\Lambda)\log\Big(\frac{M_R^0}{\Lambda}\Big)\,. \qquad (137)
\end{aligned}
$$

Using the condition (63) we find the radiative corrections for the majorana neutrino mass,

$$
\begin{aligned}
M_R(M_R^0) \approx{}& M_R(\Lambda) \\
&+ \frac{1}{16\pi^2}\Big[(\lambda_\nu\lambda_\nu^\dagger + \lambda_\nu\lambda_\nu^\dagger)M_R^0 + M_R^0(\lambda_\nu\lambda_\nu^\dagger + \lambda_\nu\lambda_\nu^\dagger)^T\Big](\Lambda)\log\Big(\frac{M_R^0}{\Lambda}\Big)|\vec{\xi}_\nu|^2\,. \qquad (138)
\end{aligned}
$$

6 Phenomenology: $l_i \to l_j\gamma$ Transitions

Flavor changing processes in the charged lepton sector have not been observed so far. The simplest flavor changing transitions among charged leptons $l_i \to l_j\gamma$ are strongly constrained experimentally, recent limits obtained for the decay of a muon to an electron and a photon, set the upper bound $\mathrm{B}(\mu \to e\gamma) < 2.4 \times 10^{-12}$ at 90% C.L. [35]. For the other decays we have the 90% C.L bounds [36],

$$\begin{aligned} \mathrm{B}(\tau \to e\gamma) &< 3.3 \times 10^{-8}\,, \\ \mathrm{B}(\tau \to \mu\gamma) &< 4.4 \times 10^{-8}\,. \end{aligned} \tag{139}$$

In this section we will perform a phenomenological analysis of $l_i \to l_j\gamma$ transitions for the case of Dirac neutrinos and minimal field content. In Sec.(5.1) we derived analytical expressions in the LL approximation for the radiative induced flavor changing neutral couplings (128). In Sec.(6.1) we use this result to estimate the contribution of the flavor changing neutral Higgs bosons to the decay $\mu \to e\gamma$. Then in Sec.(6.2) we constrain the effective flavor violating operators by studying the transitions $l_i \to l_j\gamma$. We study for which values of Λ_{LN} and the scale of flavor symmetry breaking Λ it is possible to observe these decays experimentally and we also analyze the correlations between the different decay modes. We will analyze branching ratios normalized as

$$\mathrm{B}(l_i \to l_j\gamma) = \frac{\Gamma(l_i \to l_j\gamma)}{\Gamma(l_i \to l_j\nu_i\bar{\nu}_j)}\,. \tag{140}$$

6.1 Flavor Changing Neutral Higgs Boson Contributions to $\mu \to e\gamma$

It is known that within the SM modified by the presence of neutrino mass terms (Dirac, Majorana or type I see-saw), the leading contribution to the decay $\mu \to e\gamma$ is too small to be observed. For example in the case of Dirac neutrino mass terms one obtains $\mathrm{B}(\mu \to e\gamma) < 10^{-40}$. The observation of such decay would then be a clear signal of physics beyond the SM, furthermore many extensions of the SM predict a sizable enhancement of the $\mu \to e\gamma$ decay rate to levels close to the present experimental limits.

In Sec.(5) we derived the FCNCs induced by radiative corrections from a high energy scale Λ to the EW scale $\sim M_Z$. In the presence of flavor changing neutral couplings certain two-loop graphs may dominate over the one-loop contribution as realized by Bjorken and Weinberg [37]. This can be understood because the one-loop contribution involving virtual scalars have three chirality flips while certain two-loop graphs have only one (see Figs.(2) and (3)). This decay has been studied previously for the case of multi-Higgs doublets and general flavor changing couplings at two-loops [38, 39]. We will use the expressions derived in Ref.[38] to constrain the parameters of the model developed in Sec.(4.1). For completeness we quote the relevant results derived in Ref.[38] with the original notation. We will then establish the relation with our particular model.

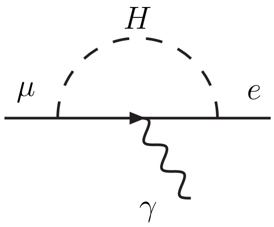


Figure 2: A one-loop Feynman diagram for $\mu \to e\gamma$ with flavor changing neutral Higgs couplings. There are two chirality flips in the Yukawa couplings and one in the fermion propagator.

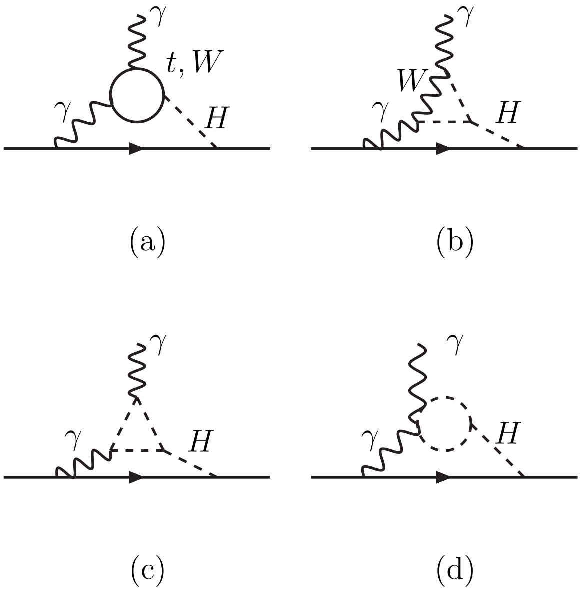


Figure 3: Dominant two-loop Feynman diagrams for $\mu \to e\gamma$. In diagram (a) there is also a contribution from ghost fields and Goldstone bosons in the internal loop. The dashed lines in the internal loop of (b), (c) and (d) correspond to Goldstone bosons and ghost fields.

The flavor changing neutral couplings of the physical scalars $H_a = \{H, h, A\}$ are parametrized by

$$\begin{aligned}\mathcal{L} = &-\frac{m_t}{v}\bar{t}(\Delta^a_{tt}P_L + \Delta^{a\,*}_{tt}P_R)tH_a \\ &- \frac{\sqrt{m_\mu m_e}}{v}\bar{e}(\Delta^{aL}_{e\mu}P_L + \Delta^{aR}_{e\mu}P_R)\mu H_a + gM_W\cos\phi_a W^+W^-H_a + \cdots . \end{aligned} \tag{141}$$

In our notation we have the Higgs mixing angles: $\phi_H = \alpha - \beta$ corresponding to H and $\phi_h = \alpha - \beta + \frac{\pi}{2}$ for h, see (21) and (32). The dominant two-loop contribution arises from W boson and top exchange [38]. We neglect diagrams with internal Z bosons since their contribution is suppressed by a factor of $\frac{(1-4\sin^2\theta_W)}{4\sin^2\theta_W} \sim 0.087$ with respect to the diagrams where the Z is replaced by a photon. The dominant two-loop amplitudes are then

$$A^{L,R\ H\gamma\gamma}_{W\ \text{loop}}(\mu \to e\gamma) = -\sum_a \cos\phi_a \Delta^{aL,R}_{e\mu}\Big[3f(z_a) + 5g(z_a) + \frac{3}{4}g(z_a) + \frac{3}{4}h(z_a)\Big],$$
$$A^{L,R\ H\gamma\gamma}_{t\ \text{loop}}(\mu \to e\gamma) = 3Q_t^2 \sum_a \Delta^{a\ L,R}_{e\mu} 2\Big[\text{Re}\Delta^a_{tt} f(z_{ta}) - i\lambda_5^{L,R}\text{Im}\Delta^a_{tt} g(z_{ta})\Big]. \tag{142}$$

where $z_a = \frac{M_W^2}{M_{H_a}^2}$, $z_{ta} = \frac{m_t^2}{M_{H_a}^2}$; $\lambda_5^L = -1$, $\lambda_5^R = 1$. The functions $f(z)$, $g(z)$ and $h(z)$ are defined as

$$f(z) = \frac{1}{2}z\int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-z}\log\Big(\frac{x(1-x)}{z}\Big),$$
$$g(z) = \frac{1}{2}z\int_0^1 \frac{1}{x(1-x)-z}\log\Big(\frac{x(1-x)}{z}\Big),$$
$$h(z) = \frac{z}{2}\int_0^1 \frac{dx}{z-x(1-x)}\Big[1 + \frac{z}{z-x(1-x)}\log\Big(\frac{x(1-x)}{z}\Big)\Big]. \tag{143}$$

These functions are of order $O(1)$ for the range of scalar masses we are interested (see Fig.(4)).

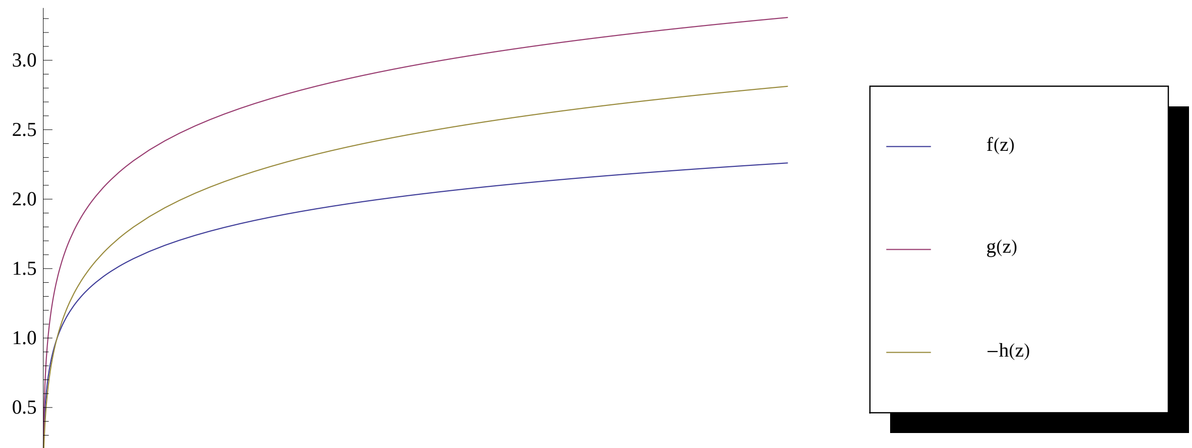


Figure 4: Numerical values for the functions $f(z)$, $g(z)$ and $-h(z)$.

The branching ratio is given by

$$\text{B}(\mu \to e\gamma) = \frac{3}{4}\Big(\frac{\alpha}{\pi}\Big)^3 \frac{m_e}{m_\mu}\Big(\frac{1}{2}|A_L|^2 + \frac{1}{2}|A_R|^2\Big), \tag{144}$$

where $\alpha = e^2/(4\pi)$ is the fine structure constant and $A_{L,R} = A_{t\ \text{loop}}^{L,R\ H\gamma\gamma} + A_{W\ \text{loop}}^{L,R\ H\gamma\gamma}$, see (142). For the case of Dirac neutrinos (see Sec.(5.1)) we have

$$
\begin{aligned}
\Delta_{e\mu}^{h\ R} &= \frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)_{12}\cos(\alpha-\beta)\,,\\
\Delta_{e\mu}^{H\ R} &= \frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)_{12}\sin(\alpha-\beta)\,,\\
\Delta_{e\mu}^{A\ R} &= i\frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)_{12}\,,\\
\Delta_{e\mu}^{h\ L} &= \frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)^*_{21}\cos(\alpha-\beta)\,,\\
\Delta_{e\mu}^{H\ L} &= \frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)^*_{21}\sin(\alpha-\beta)\,,\\
\Delta_{e\mu}^{A\ L} &= -i\frac{v}{\sqrt{m_\mu m_e}}(\Delta_l)^*_{21}\,.
\end{aligned}
\tag{145}
$$

The matrix Δ_l is defined in (128), in particular

$$
(\Delta_l)_{12} = \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_\nu^*\varsigma_l)(\varsigma_l-\varsigma_\nu)m_\mu\sum_i (U_{\text{PMNS}})^*_{i1}(U_{\text{PMNS}})_{i2}m^2_{\nu i}\,. \tag{146}
$$

Note that there is a GIM cancellation mechanism at play, for degenerate neutrinos $(\Delta_l)_{12} = 0$ because of the unitarity of the PMNS matrix U_{PMNS}. This can be seen in a more general way if we consider that

$$
\Delta_l^{\text{off}-\text{diag}} \propto (U^\dagger_{\text{PMNS}}(M_\nu^{\text{diag}})^2 U_{\text{PMNS}} M_l^{\text{diag}})^{\text{off}-\text{diag}}\,. \tag{147}
$$

The GIM cancellation is very effective since the neutrino mass differences are very small compared to the electroweak scale $v = 246$ GeV, for example taking $m_{\nu_1} = 1$ eV and normal hierarchy we find

$$
(\Delta_l)_{12} \approx 10^{-31}\times\log\Big(\frac{M_Z}{\Lambda}\Big)(1+\varsigma_\nu^*\varsigma_l)(\varsigma_l-\varsigma_\nu)\,. \tag{148}
$$

One obtains that the two-loop contributions (142) to $\text{B}(\mu\to e\gamma)$ are negligible even for $\Lambda = 10^{19}$ GeV due to the strong suppression of the flavor changing couplings (146). This conclusion also holds for the decays $\tau\to\mu\gamma$ and $\tau\to e\gamma$. The GIM cancellation mechanism makes the one- and two-loop contributions from flavor changing neutral scalars negligible.

6.2 Constraining LFV Effective Operators from $l_i \to l_j\gamma$

Within the MFV framework there are effective operators suppressed by the scale Λ at which the flavor symmetry is broken,

$$
\mathcal{L} = \frac{1}{\Lambda^2}\Big(\sum_{a=1}^{2}\sum_{j=1}^{2} c^{(a)}_{j,RL}O^{(j)}_{a,RL} + \text{h.c.}\Big)\,. \tag{149}
$$

In the following part we neglect renormalization group effects on these effective operators. We are interested in effective operators that conserve lepton number since otherwise they would be suppressed by the scale $\Lambda_{\rm LN}$, supposed to be much higher than Λ. This implies that the lowest order operators that contribute are of dimension 6. Since $\mu \to e\gamma$ is a magnetic transition the relevant operators have the form $\bar{l}^i_R \Gamma L^j_L$, where Γ must transform like $(\bar{3},1,3)$ under $SU(3)^3_l$ in the spurion sense. In a general scalar basis one has

$$\begin{aligned} O^{(1)}_{a,RL} &= g' \phi^\dagger_a \bar{l}_R \sigma_{\mu\nu}\, \Gamma\, L_L F^{\mu\nu}_Y \,, \\ O^{(2)}_{a,RL} &= g \phi^\dagger_a \bar{l}_R \sigma_{\mu\nu} \tau_i \Gamma L_L F^{\mu\nu}_i \,. \end{aligned} \tag{150}$$

We can also write the effective Lagrangian (149) in the Higgs basis

$$\mathcal{L} = \frac{1}{\Lambda^2}\Big(\sum_{a=1}^{2}\sum_{j=1}^{2} c'^{(a)}_{j,RL} O^{(j)}_{a,RL} + \text{h.c.}\Big) \,. \tag{151}$$

According to (18) we have

$$\begin{aligned} c'^{(a)}_{1,RL} &= c^{(a)}_{1,RL} c_\beta + c^{(a)}_{2,RL} s_\beta \,, \\ c'^{(a)}_{2,RL} &= -c^{(a)}_{1,RL} s_\beta + c^{(a)}_{2,RL} c_\beta \,, \end{aligned} \tag{152}$$

where $a = \{1,2\}$. In the framework of Dirac neutrinos there is only one spurion with the required transformation property $\Gamma = \lambda_l$ (62), we neglect terms of second order in λ_l. From (61) we then have that

$$\text{diag}(m_e, m_\mu, m_\tau) = \frac{v}{\sqrt{2}} (c_\beta \xi^1_l + s_\beta\, \xi^2_l) \lambda_l \,. \tag{153}$$

Since λ_l is flavor diagonal the operators in (150) do not contribute to the decay $\mu \to e\gamma$. Note that the quantity $\hat{v} \cdot \vec{\xi_l} \equiv (c_\beta\, \xi^1_l + s_\beta\, \xi^2_l)$ is invariant under scalar basis transformations (17).

In the case of the minimal field content (see Sec.(4.2)) we have another irreducible source of flavor symmetry breaking g_ν (77). Since we are interested in flavor transitions among charged leptons we choose to work in the basis where λ_l is diagonal. Thus we have

$$\begin{aligned} g_\nu &= \frac{\Lambda_{\rm LN}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger \,, \\ \lambda_l &= \frac{\sqrt{2}}{v(\hat{v}\cdot \vec{\xi_l})} \text{diag}(m_e, m_\mu, m_\tau) \,. \end{aligned} \tag{154}$$

As in Ref.[20], we now have a spurion combination transforming like $(\bar{3},1,3)$ given by $\lambda_l (g^\dagger_\nu g_\nu)$. Hence for the case of minimal field content we have

$$\begin{aligned} O^{(1)}_{a,RL} &= g' \Phi^\dagger_a \bar{l}_R \sigma_{\mu\nu} \lambda_l (g^\dagger_\nu g_\nu) L_L F^{\mu\nu}_Y \,, \\ O^{(2)}_{a,RL} &= g \Phi^\dagger_a \bar{l}_R \sigma_{\mu\nu} \tau_i \lambda_l (g^\dagger_\nu g_\nu) L_L F^{\mu\nu}_i \,. \end{aligned} \tag{155}$$

The branching ratio in the limit $m_e \ll m_\mu$ is given by

$$\mathrm{B}(\mu \to e\gamma) = 96\pi^2 e^2 \frac{v^4}{|\hat{v}\cdot\vec{\xi}_l|^2\Lambda^4}|(g_\nu^\dagger g_\nu)_{\mu e}|^2 |c'^{(2)}_{1,RL} - c'^{(1)}_{1,RL}|^2 \,. \tag{156}$$

The branching ratio (156) we obtain for the minimal field content scenario is multiplied with respect to the equivalent expression in Ref.[20] by the flavor universal factor $|\hat{v}\cdot\vec{\xi}_l|^{-2}$. From (154) we have

$$(g_\nu^\dagger g_\nu)_{ij} = \frac{\Lambda_{\mathrm{LN}}^2}{v^4}(U\mathrm{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2)U^\dagger)_{ij} = \frac{\Lambda_{\mathrm{LN}}^2}{v^4}(m_{\nu 1}^2\delta_{ij} + U_{i\mu}U_{j\mu}^*\Delta m_{\mathrm{sol}}^2 \pm U_{i\tau}U_{j\tau}^*\Delta m_{\mathrm{atm}}^2) \,. \tag{157}$$

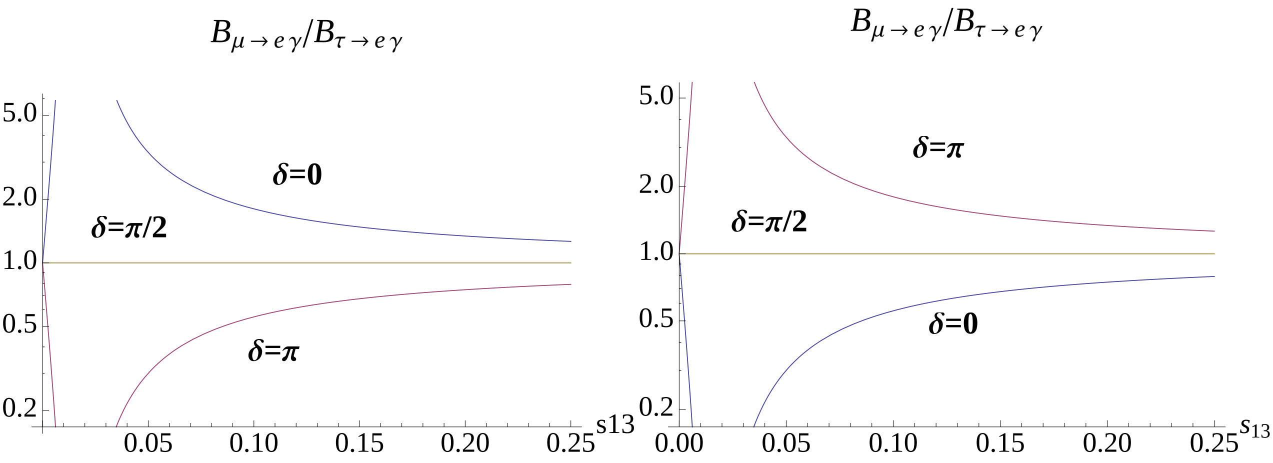
By using the parametrization of the lepton mixing matrix given in (84) we find

$$\begin{aligned}
(g_\nu^\dagger g_\nu)_{\mu e} &\approx \frac{\Lambda_{\mathrm{LN}}^2}{v^4}\frac{1}{\sqrt{2}}(s_{12}c_{12}\Delta m_{\mathrm{sol}}^2 \pm s_{13}e^{i\delta}\Delta m_{\mathrm{atm}}^2) \,, \\
(g_\nu^\dagger g_\nu)_{\tau e} &\approx \frac{\Lambda_{\mathrm{LN}}^2}{v^4}\frac{1}{\sqrt{2}}(-s_{12}c_{12}\Delta m_{\mathrm{sol}}^2 \pm s_{13}e^{i\delta}\Delta m_{\mathrm{atm}}^2) \,, \\
(g_\nu^\dagger g_\nu)_{\tau \mu} &\approx \frac{\Lambda_{\mathrm{LN}}^2}{v^4}\frac{1}{2}(-c_{12}^2\Delta m_{\mathrm{sol}}^2 \pm \Delta m_{\mathrm{atm}}^2) \,.
\end{aligned} \tag{158}$$

For the numerical analysis we use the values $\Delta m_{\mathrm{sol}}^2 = 8\times 10^{-5}$ eV^2, $\Delta m_{\mathrm{atm}}^2 = 2.5\times 10^{-3}$ eV^2 and $\theta_{12} = 33\,^\circ$. The angle θ_{13} is experimentally bounded $\theta_{13} < \pi/13$ [36]. The plus sign in (158) is for normal hierarchy $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$ and the minus sign is for inverted hierarchy $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$. By looking at (156) we note that even though $\mathrm{B}(l_i \to l_j\gamma)$ depends strongly on the scales Λ and Λ_{LN}, the ratio of different LFV rates only depends on low energy parameters. In Fig.(5) and Fig.(6) we compare the different LFV transition rates for normal and inverted hierarchy of neutrino masses. Since the flavor universal factor $|\hat{v}\cdot\vec{\xi}_l|$ cancels in the ratio of different LFV transition rates, we find as in Ref.[20] that $\mathrm{B}(\tau \to \mu\gamma) \gg \mathrm{B}(\tau \to e\gamma) \sim \mathrm{B}(\mu \to e\gamma)$.

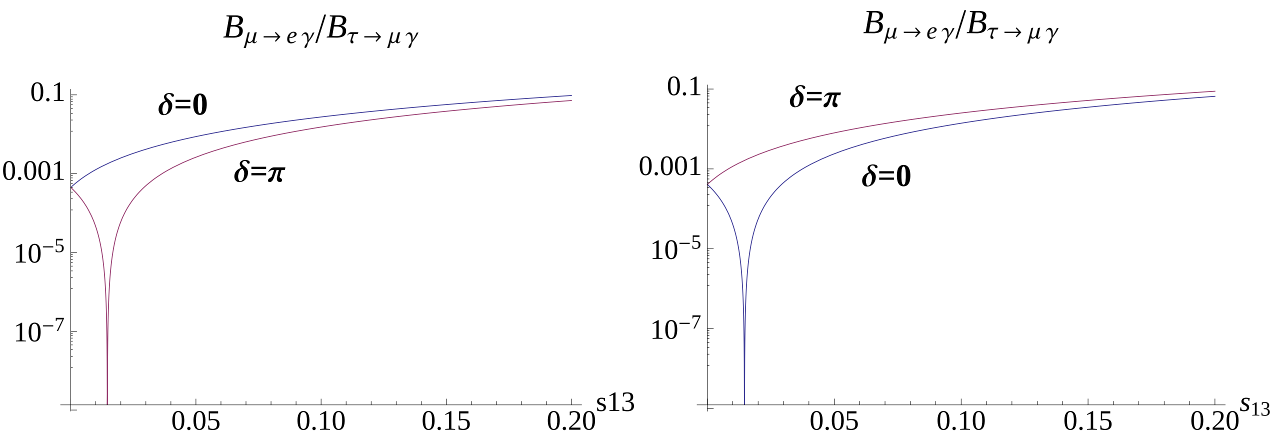
The decays $l_i \to l_j\gamma$ would be experimentally observable only if there is a large hierarchy between the scales of lepton-number and lepton-flavor violation. Observable rates for $\mu \to e\gamma$ requires approximately $\Lambda \gtrsim 10^9\Lambda_{\mathrm{LN}}$, in Fig.(7) we plot $\mathrm{B}(\mu \to e\gamma)$ and $\mathrm{B}(\tau \to \mu\gamma)$ as a function of s_{13} taking $((c'^{(2)}_{1,RL} - c'^{(1)}_{1,RL})^{1/2}\Lambda_{\mathrm{LN}}/\Lambda)^4 = 10^{10}$. According to (156), the factor $|\hat{v}\cdot\vec{\xi}_l|^{-2}$ can lower the required hierarchy between these scales if $\hat{v}\cdot\vec{\xi}_l$ is very small.

As mentioned in Ref.[20] the requirement of perturbativity of the spurion g_ν and the limits on neutrino masses, $m_\nu < 2$ eV, imply the upper bound $\Lambda_{\mathrm{LN}} \lesssim 10^{13}$ GeV. Interestingly one then finds that $\mu \to e\gamma$ is in the observable range for the expected values $\Lambda \sim 1 - 10$ TeV. The dependence of $\mathrm{B}(\mu \to e\gamma)$ on the CP phase δ is shown in Fig.(8) assuming normal hierarchy, $(c'^{(2)}_{1,RL} - c'^{(1)}_{1,RL})^{1/2}\Lambda_{\mathrm{LN}}/\Lambda)^4 = 10^{10}$, $\hat{v}\cdot\vec{\xi}_l = 1$ and taking s_{13} in the interval $[0.01, 0.1]$. It is important to take into account that the decay $\mu \to e\gamma$ can be accidentally suppressed with respect to $\tau \to \mu\gamma$ for a particular combination of s_{13} and δ, see Fig.(8).



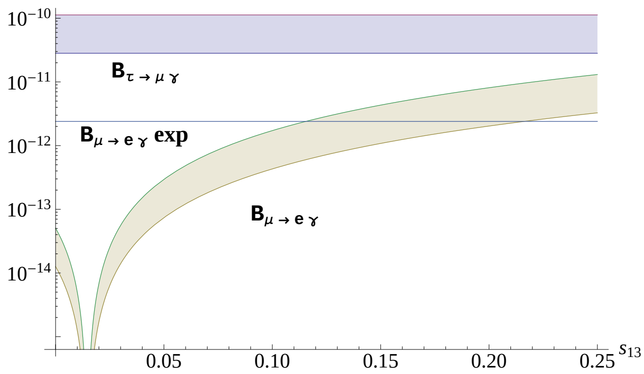
(a) normal hierarchy (b) inverted hierarchy

Figure 5: Ratio $\mathrm{B}(\mu \to e\gamma)/\mathrm{B}(\tau \to e\gamma)$ as a function of s_{13} for normal (a) and inverted (b) hierarchies.

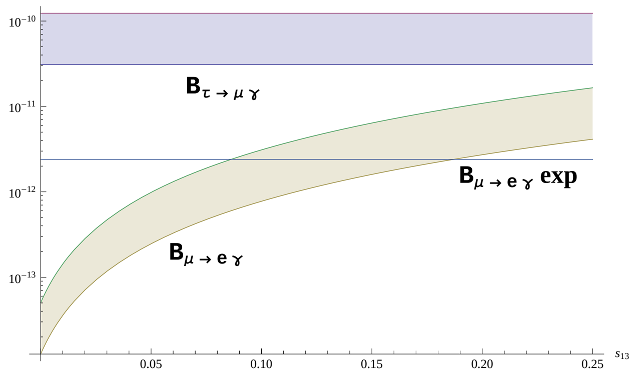


(a) normal hierarchy (b) inverted hierarchy

Figure 6: Ratio $\mathrm{B}(\mu \to e\gamma)/\mathrm{B}(\tau \to \mu\gamma)$ as a function of s_{13} for normal (a) and inverted (b) hierarchies.



(a) normal hierarchy



(b) inverted hierarchy

Figure 7: Branching ratio $\mathrm{B}(\mu\to e\gamma)$ and $\mathrm{B}(\tau\to\mu\gamma)$ as a function of s_{13} for $((c'^{(2)}_{1,RL} - c'^{(1)}_{1,RL})^{1/2}\Lambda_{\mathrm{LN}}/\Lambda)^4 = 10^{10}$. The CP phase is fixed $\delta=\pi$ and the shading is generated by varying $|\hat{v}\cdot\vec{\xi}_l|$ in the interval $[0.5, 1]$. The current experimental upper bound on $\mathrm{B}(\mu\to e\gamma)$ is also shown.

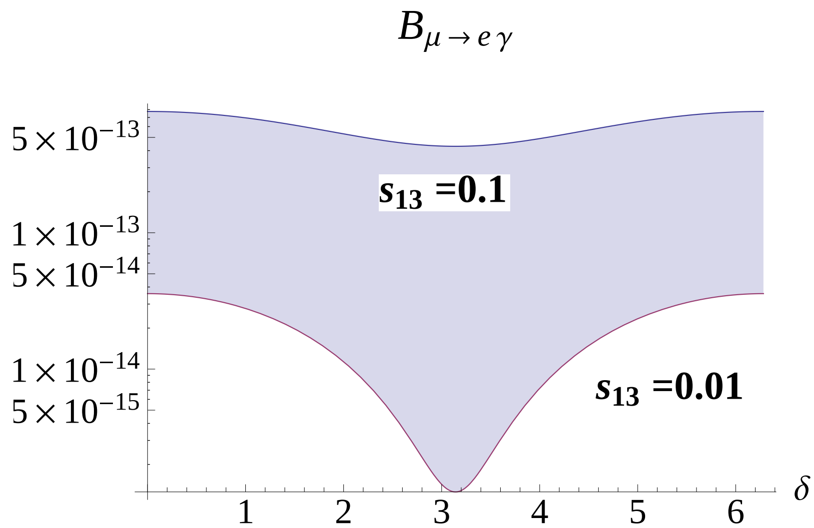


Figure 8: Branching ratio $\mathrm{B}(\mu\to e\gamma)$ as a function of δ for $(\Lambda_{\mathrm{LN}}((c'^{(2)}_{1,RL} - c'^{(1)}_{1,RL})^{1/2}/\Lambda)^4 = 10^{10}$ and $\hat{v}\cdot\vec{\xi}_l = 1$. The shading corresponds to a variation of s_{13} in the interval $[0.01, 0.1]$ and normal hierarchy is assumed.

7 Summary and Conclusions

In this thesis we provide possible extensions of the A2HDM [10] to the lepton sector. We consider Yukawa alignment as arising from an MFV principle following Ref.[11]. We define the MFV principle for three different scenarios of neutrino masses

- Extended field content and lepton number conservation. Dirac neutrinos.
- Minimal field content.
- Extended field content without lepton number conservation.

In the case of Dirac neutrinos we then obtain expressions for the leptonic sector completely analogous to those found for quarks in Refs.[10, 33]. We studied the effect of radiative corrections and obtained analytical expressions for the flavor changing neutral couplings in the LL approximation for the case of Dirac neutrinos and minimal field content. We also consider the one-loop RGE for the right-handed Majorana mass matrix. This is relevant for leptogenesis since radiative corrections break the degeneracy imposed at a high energy scale [29, 30].

By using general two-loop expressions for $\text{B}(\mu \to e\gamma)$ [38], we find that the contribution of the radiatively generated FCNCs to the decay $\mu \to e\gamma$ is negligible even for $\Lambda \sim 10^{19}$ GeV due to the strong GIM suppression. The GIM cancellation mechanism is very effective because the neutrino mass differences are very small compared to the EW scale. We argue that these contributions are also negligible for the other lepton flavor violating decays $\tau \to \mu\gamma$ and $\tau \to e\gamma$.

In Sec.(6.2) we study the contribution of dimension 6 effective operators to the decays $l_i \to l_j\gamma$. As found in Ref.[20], observable rates for $l_i \to l_j\gamma$ transitions are obtained if there is a large hierarchy between the scale of lepton number violation Λ_{LN} and the flavor symmetry breaking scale Λ. For example the observation of $\mu \to e\gamma$ requires approximately $\Lambda \gtrsim 10^9\Lambda_{\text{LN}}$. We find that $\text{B}(l_i \to l_j\gamma)$ is multiplied with respect to the expression found in Ref.[20] by a flavor universal factor $|\hat{v} \cdot \vec{\xi}_l|^{-2}$. For very small values of $\hat{v} \cdot \vec{\xi}_l$ the required hierarchy between Λ and Λ_{LN} would be relaxed. The same particular pattern found in Ref.[20] is also found in this case among the different decay channels, $\text{B}(\tau \to \mu\gamma) \gg \text{B}(\tau \to e\gamma) \sim \text{B}(\mu \to e\gamma)$. This pattern is a clear prediction of the MFLV hypothesis and is expressed exclusively in terms of neutrino masses and mixing parameters. The branching ratios $\text{B}(l_i \to l_j\gamma)$ increase with s_{13}, with the exception of $\text{B}(\tau \to \mu\gamma)$ which is independent of this parameter.

The recent evidence for a non-zero θ_{13} at 90% C.L [40], together with the expected sensitivity improvements $\text{B}(\mu \to e\gamma) = O(10^{-13})$ at MEG and $\text{B}(\tau \to \mu\gamma) = O(10^{-19})$ in Super B factories, open the possibility to test the MFV hypothesis in the near future. The non-observation of none of these decays on the other hand would only set stringent bounds on the ratio $(\Lambda_{\text{LN}}/\Lambda)^4|\hat{v} \cdot \xi_l|^{-2}$ and based only on this analysis, will not disprove the MFV hypothesis.

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A Feynman Rules

In this section we provide Feynman rules associated to the scalar fields. We provide the rules in a general R_ξ-gauge. The Feynman rules for the three and four point Higgs vertices in the most general CP-conserving 2HDM, see (7), are listed in Ref.[14]. We quote the Feynman rules in terms of the states $\{S_1, S_2, S_3\}$ for simplicity, these are given in terms of the physical particles $\{H, h, A\}$ by (21). We use the notation $\phi_i^0 = \{h, H, A\}$ for the physical neutral scalars. The scalar propagators are given by

$$\overset{k}{\underset{\phi_i^0}{\dashrightarrow}} = \frac{i}{k^2 - m^2_{\phi_i^0}} \qquad \overset{k}{\underset{H^\pm}{\dashrightarrow}} = \frac{i}{k^2 - m^2_{H^\pm}}$$

$$\overset{k}{\underset{G^\pm}{\dashrightarrow}} = \frac{i}{k^2 - \xi_W M_W^2} \qquad \overset{k}{\underset{G^0}{\dashrightarrow}} = \frac{i}{k^2 - \xi_Z M_Z^2}$$

Figure 9: Propagators of the scalar fields.

In Fig.(11) we quote the gauge interactions of the scalar and in Fig.(10) the scalar-ghost interactions. The Feynman rules given in Fig.(12) arise from the Lagrangians (55) and (69) corresponding to the case of Dirac neutrinos and lepton number conservation.

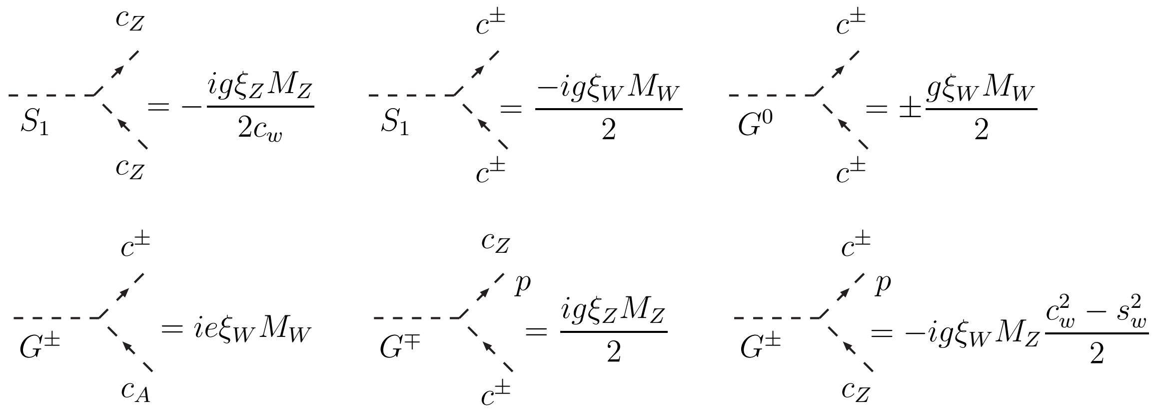


Figure 10: Ghost vertices from (41).

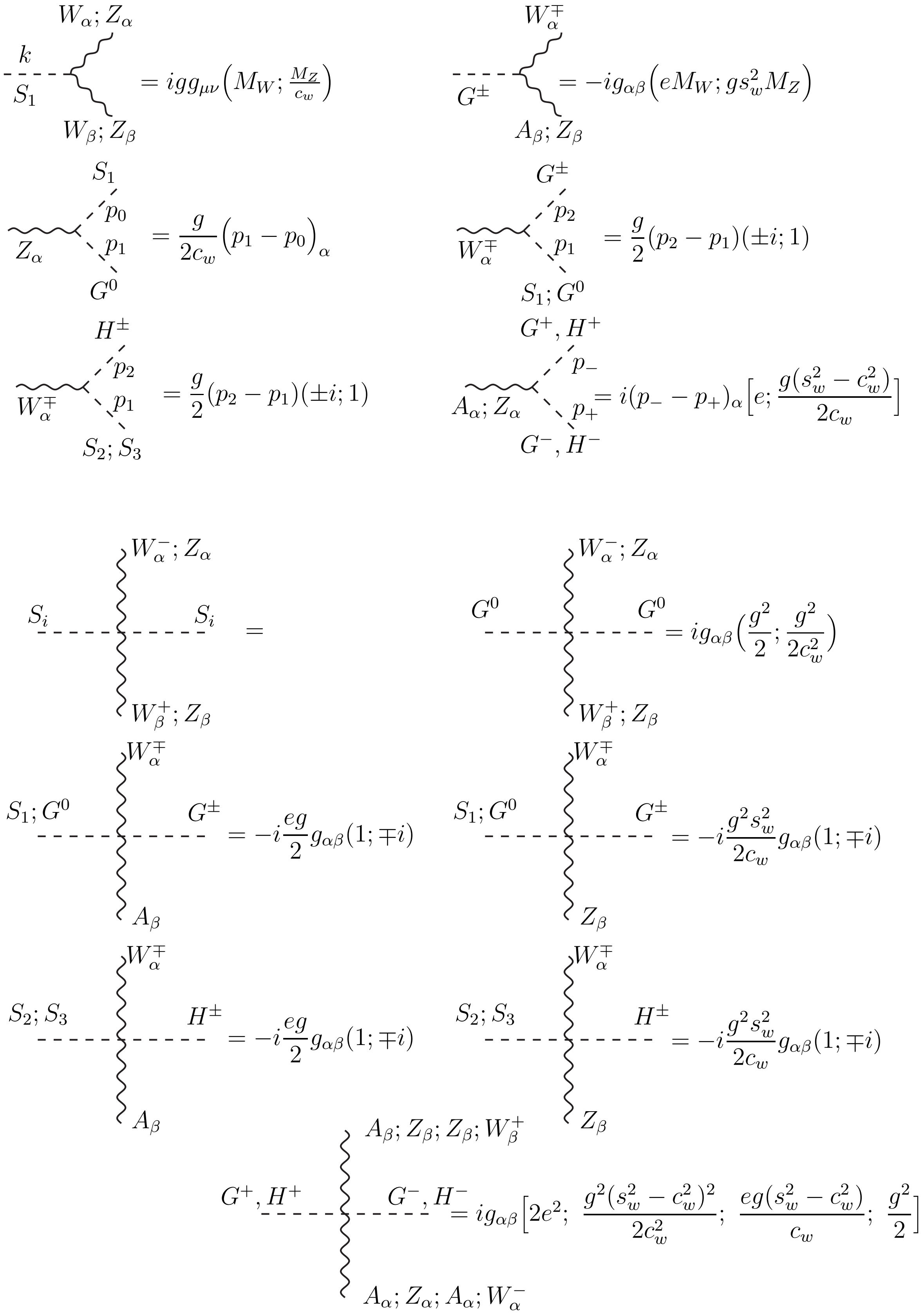
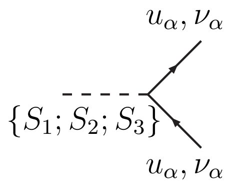
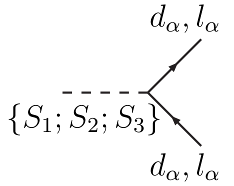


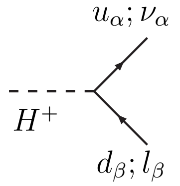
Figure 11: Gauge interactions of the scalars from (32), all momenta are taken as incoming into the vertex.



$$= \frac{g m_\alpha}{2M_W}\Big\{-i; -i(\varsigma^*_{u,\nu}P_R + \varsigma_{u,\nu}P_L); (\varsigma^*_{u,\nu}P_R - \varsigma_{u,\nu}P_L)\Big\}$$



$$= \frac{g m_\alpha}{2M_W}\Big\{-i; -i(\varsigma^*_{u,\nu}P_R + \varsigma_{u,\nu}P_L); (\varsigma^*_{u,\nu}P_R - \varsigma_{u,\nu}P_L)\Big\}$$



$$= \frac{ig}{\sqrt{2}M_W}\Big\{\varsigma_u m_{u_\alpha} V_{\alpha\beta}P_L - \varsigma_d V_{\alpha\beta} m_{d_\beta} P_R; \varsigma_\nu m_{\nu_\alpha} U_{\alpha\beta} P_L - \varsigma_l U_{\alpha\beta} m_{l_\beta} P_R\Big\}$$

Figure 12: Yukawa interactions of the physical scalars for the case of Dirac neutrinos and lepton number conservation, see (55) and (69).

B Renormalization Group Equations

In this appendix we provide a derivation of the RGE for the Yukawa couplings, only taking into account the quark sector. We concentrate on the flavor changing part of these equations, so that we are not going to discuss the gauge contributions since these are flavor diagonal before EW symmetry breaking. We use the finite cutoff interpretation of the RGE following Ref.[41] with an UV cutoff E and in the following we only quote the cutoff-dependent parts $\propto \ln(E^2)$ of the Green functions. Within this scheme, the loop integrals in momentum space are given by

$$\int \frac{d^4q}{(2\pi)^4} = \frac{1}{2}\frac{1}{(2\pi)^4}\int d\Omega_4 \int_{m^2}^{E^2} dq^2 q^2\,, \tag{B.1}$$

where m is an arbitrary fixed parameter. The non vanishing contributions to the one-loop two-point Green function $-iG^{(ij)}_{k,l}(p^2,E^2)$ for $k,l=1,2$ and $i,j=1,2,3,4$; represented diagrammatically in Fig.(13) are

$$\begin{aligned}
-iG^{(jj)}_{k,l}(p^2,E^2) =& i\frac{N_c}{32\pi^2}p^2\ln(\frac{E^2}{m^2})\mathrm{tr}\Big[\Delta_k\Delta_l^\dagger+\Delta_l\Delta_k^\dagger+\Gamma_k\Gamma_l^\dagger+\Gamma_l\Gamma_k^\dagger\Big]\,,\\
-iG^{(j,j')}_{k,l}(p^2,E^2) =& (-1)^j\frac{N_c}{32\pi^2}p^2\ln(\frac{E^2}{m^2})\mathrm{tr}\Big[\Delta_k\Delta_l^\dagger-\Delta_l\Delta_k^\dagger-\Gamma_k\Gamma_l^\dagger+\Gamma_l\Gamma_k^\dagger\Big]\,,
\end{aligned} \tag{B.2}$$

where $(jj')=(12)=(21)=(34)=(43)$.

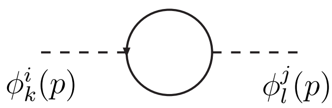


Figure 13: Diagrams that contribute to the Green function $-i\Sigma^{(ij)}_{k,l}(p^2,E^2)$.

The two-point Green function for incoming u^i and outgoing u^j, denoted by $-iG(p;E;u^i,u^j)$ is represented in Fig.(14). One obtains that

$$-iG(p;E;u^i,u^j) = \frac{i}{64\pi^2}\ln(\frac{E^2}{m})\not{p}\Big\{2(1+\gamma_5)\sum_{l=1}^{2}\Big[\Delta_l^\dagger\Delta_l\Big]_{ji}$$
$$+(1-\gamma_5)\sum_{l=1}^{2}\Big[\Delta_l\Delta_l^\dagger+\Gamma_l\Gamma_l^\dagger\Big]_{ji}\Big\},$$

$$-iG(p;E;d^i,d^j) = \frac{i}{64\pi^2}\ln(\frac{E^2}{m})\not{p}\Big\{2(1+\gamma_5)\sum_{l=1}^{2}\Big[\Gamma_l^\dagger\Gamma_l\Big]_{ji}$$
$$+(1-\gamma_5)\sum_{l=1}^{2}\Big[\Gamma_l\Gamma_l^\dagger+\Delta_l\Delta_l^\dagger\Big]_{ji}\Big\}. \quad \text{(B.3)}$$

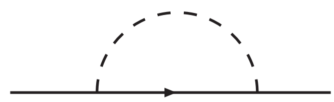


Figure 14: Diagrams that contribute to the Green function $-i\Sigma(p;E;u^i,u^j)$.

The vertex correction, showed in Fig.(15) gives rise to the three-point Green functions

$$G^{(3)}(k,p;E;u^i;u^j;\phi_l^3) = -\frac{i}{32\pi^2\sqrt{2}}\ln(\frac{E^2}{m^2})\sum_{r=1}^{2}\Big\{(1+\gamma_5)\Big[\Gamma_r\Gamma_l^\dagger\Delta_r\Big]_{ji}$$
$$+(1-\gamma_5)\Big[\Delta_r^\dagger\Gamma_l\Gamma_r^\dagger\Big]_{ji}\Big\},$$

$$G^{(3)}(k,p;E;d^i;d^j;\phi_l^3) = -\frac{i}{32\pi^2\sqrt{2}}\ln(\frac{E^2}{m^2})\sum_{r=1}^{2}\Big\{(1+\gamma_5)\Big[\Delta_r\Delta_l^\dagger\Gamma_r\Big]_{ji}$$
$$+(1-\gamma_5)\Big[\Gamma_r^\dagger\Delta_l\Delta_r^\dagger\Big]_{ji}\Big\}. \quad \text{(B.4)}$$

The RGE are obtained by requiring that Green functions in the theory with finite cut-of E are equal to those with cut-off $E+dE$, this condition determines how the fields and couplings must change with the cut-off scale. Imposing that $G(p^2,E^2) = G(p^2,E+dE)$ one obtains

$$\frac{16\pi^2}{N_c}d\phi_k(E) = -d\log E\sum_{l=1}^{2}\mathrm{Tr}\Big[\Delta_k\Delta_l^\dagger+\Gamma_l\Gamma_k^\dagger\Big]\phi_k(E)\,. \quad \text{(B.5)}$$

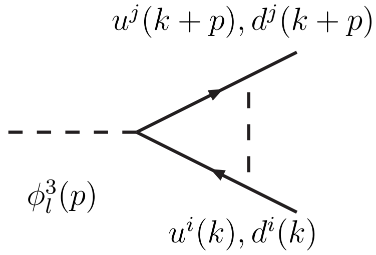


Figure 15: Diagram contributing to the Green function $G^{(3)}(k,p;E;u^i;u^j;\phi^3_l)$.

Similarly imposing $G(p;E;u^i,u^j) = G(p;E+dE;u^i,u^j)$ we get the following cut-off dependence of the quark fields

$$dq^k(E)_{L,R} = df_q(E)_{kl}{}^{(L,R)} q^l(E)_{L,R} \,. \tag{B.6}$$

with

$$\begin{aligned}
df_u(E)^{(L)}_{ij} = df_d(E)^{(L)}_{ij} &= -\frac{(d\log E^2)}{64\pi^2}\sum_{k=1}^{2}\Big[\Delta_k\Delta_k^\dagger + \Gamma_k\Gamma_k^\dagger\Big]_{ij}(E)\,,\\
df_u(E)^{(R)}_{ij} &= -\frac{2(d\log E^2)}{64\pi^2}\sum_{k=1}^{2}\Big[\Delta_k^\dagger\Delta_k\Big]_{ij}(E)\,,\\
df_d(E)^{(R)}_{ij} &= -\frac{2(d\log E^2)}{64\pi^2}\sum_{k=1}^{2}\Big[\Gamma_k^\dagger\Gamma_k\Big]_{ij}(E)\,.
\end{aligned} \tag{B.7}$$

Finally, from the condition $G^{(3)}(k,p;E;u^i,u^j;\phi^3_l) = G^{(3)}(k,p;E+dE;u^i,u^j;\phi^3_l)$ we get the RGE for the Yukawa couplings

$$\begin{aligned}
\frac{d\Delta_k}{d\log E} =& -\frac{1}{16\pi^2}\Big\{-N_c\sum_{l=1}^{2}\mathrm{Tr}\Big[\Delta_k\Delta_l^\dagger + \Gamma_l\Gamma_k^\dagger\Big]\Delta_l\\
&-\frac{1}{2}\sum_{l=1}^{2}\Big[(\Delta_l\Delta_l^\dagger + \Gamma_l\Gamma_l^\dagger)\Delta_k + 2\Delta_k\Delta_l^\dagger\Delta_l\Big] + 2\sum_{l=1}^{2}\Big[\Gamma_l\Gamma_k^\dagger\Delta_l\Big]\Big\}\,,\\
\frac{d\Gamma_k}{d\log E} =& -\frac{1}{16\pi^2}\Big\{-N_c\sum_{l=1}^{2}\mathrm{Tr}\Big[\Gamma_k\Gamma_l^\dagger + \Delta_l\Delta_k^\dagger\Big]\Gamma_l\\
&-\frac{1}{2}\sum_{l=1}^{2}\Big[(\Gamma_l\Gamma_l^\dagger + \Delta_l\Delta_l^\dagger)\Gamma_k + 2\Gamma_k\Gamma_l^\dagger\Gamma_l\Big] + 2\sum_{l=1}^{2}\Big[\Delta_l\Delta_k^\dagger\Gamma_l\Big]\Big\}\,.
\end{aligned} \tag{B.8}$$

C Parametrizing Yukawa Alignment

Yukawa alignment was originally parametrized in Ref.[10] as (53). Recently a different parametrization of Yukawa alignment was proposed in Ref.[33]

$$\begin{aligned}
\Gamma_1 &= \cos(\psi_d)\lambda_d \quad , \quad \Gamma_2 = \sin(\psi_d)\lambda_d\,,\\
\Delta_1 &= \cos(\psi_u)\lambda_u \quad , \quad \Delta_2 = \sin(\psi_u)\lambda_u\,.
\end{aligned} \tag{C.1}$$

where $\{\psi_u, \psi_d\}$ are arbitrary complex parameters. This parametrization is equivalent to the one proposed in Ref.[10] in the following way

$$\xi_d = \tan\psi_d \qquad , \qquad \xi_u^* = \tan\psi_u\,, \tag{C.2}$$

and

$$\varsigma_d = \tan(\psi_d - \beta) \qquad , \varsigma_u^* = \tan(\psi_u - \beta)\,. \tag{C.3}$$

Using the parametrization of the Yukawa alignment condition of [33], one obtains for (127) and (128) the following expressions

$$\begin{aligned}
\Delta_u^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)\frac{\cos(\psi_d-\psi_u)\sin(\psi_u-\psi_d^*)}{|\cos(\psi_d-\beta)|^2\cos^2(\psi_u-\beta)}\Big(V(M_d^{\text{diag}})^2 V^\dagger M_u^{\text{diag}}\Big)^{\text{off-diag}}\,,\\
\Delta_d^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)\frac{\cos(\psi_u-\psi_d)\sin(\psi_d-\psi_u^*)}{|\cos(\psi_u-\beta)|^2\cos^2(\psi_d-\beta)}\Big(V^\dagger(M_u^{\text{diag}})^2 V M_d^{\text{diag}}\Big)^{\text{off-diag}}\,,
\end{aligned} \tag{C.4}$$

$$\begin{aligned}
\Delta_\nu^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)\frac{\cos(\psi_l-\psi_\nu)\sin(\psi_\nu-\psi_l^*)}{|\cos(\psi_l-\beta)|^2\cos^2(\psi_\nu-\beta)}\Big(U_{\text{PMNS}}(M_l^{\text{diag}})^2 U_{\text{PMNS}}^\dagger M_\nu^{\text{diag}}\Big)^{\text{off-diag}}\,,\\
\Delta_l^{\text{off-diag}} &= \frac{1}{4\pi^2 v^3}\log\Big(\frac{M_Z}{\Lambda}\Big)\frac{\cos(\psi_\nu-\psi_l)\sin(\psi_l-\psi_\nu^*)}{|\cos(\psi_\nu-\beta)|^2\cos^2(\psi_l-\beta)}\Big(U_{\text{PMNS}}^\dagger(M_\nu^{\text{diag}})^2 U_{\text{PMNS}} M_l^{\text{diag}}\Big)^{\text{off-diag}}\,.
\end{aligned} \tag{C.5}$$

The expressions (135) obtained in Sec.(5.2) can also be written in the parametrization of [33],

$$\begin{aligned}
\frac{1}{\Lambda_{\text{LN}}}(U_L^{\nu T}\kappa^{11}(M_Z)U_L^\nu)^{\text{off-diag}} &= \Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[\cosh(2\text{Im}\psi_l) - 4|\cos(\psi_l)|^2 - 4c_{12}\sin(\psi_l)\cos(\psi_l^*)\Big]\\
&\quad + (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[\cosh(2\text{Im}\psi_l)\\
&\quad + 8|\cos(\psi_l)|^2 + 8c_{12}\cos(\psi_l^*)\sin(\psi_l)\Big]\Big\}\frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\cos(\beta-\psi_l)|^2}\,,
\end{aligned} \tag{C.6}$$

$$\begin{aligned}
\frac{1}{\Lambda_{\text{LN}}}(U_L^{\nu T}\kappa^{12}(M_Z)U_L^\nu)^{\text{off-diag}} &= \Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{12}\cosh(2\text{Im}\psi_l) + 4c_{12}|\cos(\psi_l)|^2\\
&\quad - 8\sin(\psi_l^*)\cos(\psi_l) + 4c_{22}\cos(\psi_l^*)\sin(\psi_l) - 8c_{12}|\sin(\psi_l)|^2\Big]\\
&\quad + (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{12}\cosh(2\text{Im}\psi_l)\\
&\quad + 8c_{12}\cos(\psi_l)\sin^*(\psi_l) + 8c_{22}|\sin(\psi_l)|^2\Big]\Big\}\frac{\log\Big(\frac{M_Z}{\Lambda}\Big)}{16\pi^2 v^4|\cos(\beta-\psi_l)|^2}\,,
\end{aligned} \tag{C.7}$$

$$
\begin{aligned}
\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{21}(M_Z)U_L^{\nu})^{\text{off-diag}} = &\Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{12}\cosh(2\text{Im}\psi_l) \\
&+ 4\sin(\psi_l^*)\cos(\psi_l) - 8c_{22}\cos(\psi_l^*)\sin(\psi_l) - 4c_{12}|\cos(\psi_l)|^2 \\
&+ 4c_{12}|\sin(\psi_l)|^2\Big] + (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{12}\cosh(2\text{Im}\psi_l) \\
&+ 8c_{12}\cos(\psi_l^*)\sin(\psi_l) + 8c_{22}|\cos(\psi_l)|^2\Big]\Big\}\frac{\log\left(\frac{M_Z}{\Lambda}\right)}{16\pi^2 v^4|\cos(\beta-\psi_l)|^2}\,,
\end{aligned}
\tag{C.8}
$$

$$
\begin{aligned}
\frac{1}{\Lambda_{\rm LN}}(U_L^{\nu T}\kappa^{22}(M_Z)U_L^{\nu})^{\text{off-diag}} = &\Big\{(M_\nu U^\dagger M_l^2 U)^{\text{off-diag}}\Big[c_{22}\cosh(2\text{Im}\psi_l) + 4c_{12}\sin(\psi_l^*)\cos(\psi_l) \\
&- 8c_{12}\cos(\psi_l)\sin(\psi_l^*) \\
&+ (U^T M_l^2 U^* M_\nu)^{\text{off-diag}}\Big[c_{22}\cosh(2\text{Im}\psi_l) \\
&+ 8c_{12}\cos(\psi_l)\sin(\psi_l^*)\Big]\Big\}\frac{\log\left(\frac{M_Z}{\Lambda}\right)}{16\pi^2 v^4|\cos(\beta-\psi_l)|^2}\,.
\end{aligned}
\tag{C.9}
$$

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