Departamento de Física Teórica

Flavor violation of charged leptons in the Simplest Little Higgs model

Ph.D. Thesis

Andrea Lami

Advisors

Dr. Jorge Portolés Ibáñez, Dr. Pablo Roig Garcés.

Instituto de Física Corpuscular, IFIC/CSIC-UV

Valencia, February 2016.

JORGE PORTOLÉS IBÁÑEZ, Científico Titular del Consejo Superior de Investigaciones Científicas (CSIC), perteneciente al Instituto de Física Corpuscular (IFIC), centro mixto del CSIC y de la Universitat de València y PABLO ROIG GARCÉS, investigador titular 3-A del Departamento de Física del Centro de INVestigación y de ESTudios AVanzados (CINVESTAV) del Instituto Politécnico Nacional (IPN) de Ciudad de México,

CERTIFICAN:
Que la presente memoria " Flavor Violation of charged leptons in the Simplest Little Higgs model " ha sido realizada bajo su dirección en el IFIC y en el Departamento de Física Teórica de la Universitat de València, por ANDREA LAMI y constituye su Tesis para optar al grado de Doctor en Física.

Y para que así conste, en cumplimiento de la legislación vigente, presenta en el Departamento de Física Teórica de la Universitat de València la referida Tesis Doctoral, y firma el presente certificado.

Valencia, a 4 de Mayo de 2016.

Jorge Portolés Ibáñez　　Pablo Roig Garcés

Incontro

Resumen de la tesis

Esta tesis está compuesta de cuatro partes:

- En la primera parte me he dedicado a describir cómo funciona el modelo estándar (Capítulo 1), por qué esperamos violaciones de sabor para los procesos con leptones cargados (Capítulo 2) y un escenario posible fuera del modelo estándar (Capítulo 3) que justifique tales violaciones de sabor, la jerarquía de la masa del Higgs y que nos indique nueva física todavía no revelada.
- En la segunda parte he explicado en detalle los procedimientos para conseguir mis resultados en procesos específicos con violación de sabor leptónico de interés fenomenológico: desintegraciones hadrónicas del leptón tau (Capítulo 4) y desintegraciones leptónicas del Higgs (Capítulo 5), todo en el contexto del Simplest Little Higgs Model, explicado en la sección 3.4.
- En la tercera parte he vuelto a resumir todo lo que he hecho en este trabajo de tesis y he comentado los resultados finales (Capítulo 6). Además he escrito reglas, aproximaciones y trasformaciones necesarias para reproducir exactamente los cálculos que llevan a los resultados de los Capítulos 4 y 5.
- En la cuarta parte he incluido toda la literatura útil para comprender los conceptos que explico en la tesis.

Introduction

The *Standard Model* (SM) of particle physics is a theory that has passed many experimental tests. However, there are still fundamental questions and observations that cannot be answered or explained by it. One example is neutrino oscillations. The SM Lagrangian explicitly conserves lepton flavor in any given interaction. This feature does not arise from a gauge principle, it is an accidental symmetry of the SM which agreed with lepton flavor conservation as observed at that time. The observation of neutrino oscillations demonstrates that there is a *lepton flavor violation* (LFV) in the neutral sector.

Observations of the solar neutrino flux showed discrepancies from predictions made by the generally accepted model of the Sun. A significantly lower flux than anticipated [1] was measured. In later years, several other experiments confirmed this deficit [2], [3], [4]. One possible explanation of this result is neutrino oscillations: an electron neutrino changes its flavor during flight. These oscillations imply a mixing in the leptonic sector similar to that of quarks, and were proposed by S.M. Bilenky and B. Pontecorvo [5]. Such mixing requires massive neutrinos, however. The first direct evidence of neutrino oscillations was found by the Sudbury Neutrino Observatory in the year 2002 [6]. Since then, various experiments have confirmed the oscillations of neutrinos of different flavors, and although neutrino masses have not been measured so far, mass differences of the three mass eigenstates have been shown to be non vanishing [7].

If neutrino oscillations are taken into account, lepton number is no longer conserved for individual families and thus, many new decay modes become possible. Examples for decays accessible to low energy experiments are $\mu \to e\gamma$, $\mu \to 3e$ and $\tau \to \mu\gamma$.

Although the SM has been succesfully tested, it is commonly accepted that it constitutes only an effective theory which is valid up to an energy scale Λ (at least of several TeVs) where new physics enters and additional dynamic degrees of freedom become important. A renormalizable quantum field theory valid above this scale should satisfy the following requirements:

- Its gauge group must contain the SM gauge group.
- All SM degrees of freedom should be incorporated either as fundamental or as composite fields.
- At low energies it should reduce to the SM.

Table 1: Experimental upper limits on the branching ratios (B) of the radiative lepton decays.

Process	Experimental bound
$B\left(\tau \to \mu\gamma\right)$	4.4×10^{-8} [9], [10]
$B\left(\tau \to e\gamma\right)$	3.3×10^{-8} [9]
$B\left(\mu \to e\gamma\right)$	5.7×10^{-13} [11]

Table 2: Experimental upper limits on the branching ratios of the three body charged lepton decays.

Process	Experimental bound
$B\left(\tau^- \to \mu^-\mu^+\mu^-\right)$	2.1×10^{-8} [12]
$B\left(\tau^- \to e^-e^+e^-\right)$	2.7×10^{-8} [12]
$B\left(\tau^- \to e^-\mu^+\mu^-\right)$	2.7×10^{-8} [12]
$B\left(\tau^- \to \mu^-e^+\mu^-\right)$	1.7×10^{-8} [12]
$B\left(\mu^- \to e^-e^+e^-\right)$	1.0×10^{-12} [13]

Flavor observables, especially *flavor changing neutral current* (FCNC) processes are an excellent probe of new physics since they are suppressed in the SM and therefore sensitive even to small new physics contributions.

Especially the search for LFV is very promising since in the SM (minimally extended with massive neutrinos) all flavor violating effects in the charged lepton sector are proportional to the very small neutrino masses, the decay rates of heavy charged leptons into lighter ones are suppressed by the ratio $\frac{m_\nu^2}{M_W^2}$ and thus are by far too small to be measurable in any foreseeable experiment [8]. This in turn means that any observation of LFV would prove the existence of physics beyond the SM. In addition, LFV processes have the advantage of being "theoretically clean" because they can be computed precisely without problems with non perturbative QCD effects affecting similar observables in the quark sector. The current experimental situation and prospects for the search for charged lepton flavor violation are very promising.

In Tables 1 and 2 we list the experimental bounds on the radiative lepton decays $l_i \to l_f\gamma$ and on the three body lepton decays $l_i \to l_j l_k l_l$, respectively. Especially the limits on $\mu \to e$ transitions are very stringent due to constraints from the MEG and SINDRUM collaborations at the PSI and will be even further improved in the future, as next generation B factory experiments will aim to $\mathcal{O}(10^{-10})$ level for the τ LFV branching ratios.

We know that the lepton flavor symmetries are not exact in nature, since neutrino oscillations have been observed. The neutrino oscillations are induced due to the finite but tiny neutrino masses. This comes from the GIM mechanism in leptonic sector. In fact, $B\left(\mu \to e\gamma\right)$ is limited to be below 10^{-54} in the SM with the tiny neutrino masses. On the other hand, it is considered that the SM should be a low energy effective theory and new physics may appear at the TeV scale. Now we know that lepton flavor symmetries are not exact in nature, and we guess that the symmetries may be broken in the model. In that case, the charged LFV processes could be predicted with branching ratios accessible to experiments in near future.

In this thesis I tried to review briefly the SM (Part I, Chapter 1) and its frontiers with particular attention to the flavour violating processes involving charged leptons (Chapter 2). Then I summarized one of the options for the physics beyond the SM, the *Little Higgs* (LH) models in the Chapter 3.

After the dissertation of the Part I, I applied the *Simplest Little Higgs Model* (SLH), in the case of LFV τ decays into one and two pseudoscalars (Chapter 4).

In the Chapter 5 I discussed the contribution of the scalar Higgs in the LFV always in the framework of SLH.

I give the conclusion of my work in the Chapter 6.

Andrea Lami, Valencia, 4/5/2016

Publications

This thesis is based on the following publications:

Chapter 4
Lepton Flavor Violating Tau decays into one and two Pseudoscalars
By Andrea Lami, Jorge Portolés, Pablo Roig.
"Lepton Flavour Violation in Hadron Decays of the Tau Lepton in the Simplest Little Higgs Model".
27 January 2016 [e-Print Archive: 1601.07391]
Published in Physical Review D 93, 076008 (2016), DOI: 10.1103/PhysRevD.93.076008 .

Chapter 5
$H \to \ell\ell'$ in the Simplest Little Higgs Model
By Andrea Lami, Pablo Roig.
"$H \to \ell\ell'$ in the Simplest Little Higgs Model".
31 March 2016 [e-Print Archive: 1603.09663]
To be published.

Contents

Part I

The Standard Model and its extensions

Chapter 1

The Standard Model

1.1 Background

The main parts of the SM were introduced by Salam, Glashow and Weinberg independently around 1968. Although the correctness of the SM was not plainly apparent, the SM managed to predate the experimental evidence by years. The theory is considered ugly by most, but it is successful. The SM is our most confirmed model of elementary particle physics, yet its failures suggest that the story is not yet over.

To place the SM in context let us return to 1965. Tomonaga, Feynmann, and Schwinger had just won the Nobel prize for the renormalization of Quantum Electrodynamics. They calculated the magnetic moment of the electron and other observables using quantum field theory and regularization to separate out the infinite components of the theory from a finite contribution. Their calculations and the corresponding experiments verified the vital role of the subsequent renormalization in fundamental physics. The anomalous magnetic moment of the electron, which is calculated using the renormalization toolbox, today agrees with experiment to more than 13 significant digits. Unfortunately in 1965 the models explaining radioactive decay and the strong interaction were not renormalizable.

The leading theory was called the chiral V - A universal model of weak decays featuring four-fermion interactions in the combination of vector minus axial-vector currents. The V - A model could not be renormalized. Although gauge theory and renormalization explained the interaction of electrons with photons, gauge theory was not able to address the strong and weak forces. These forces were known to be short range forces. To make a force have a short range in *quantum field theory* (QFT), the mediating boson needed a mass. The Yukawa theory of scalar fields included such a term as an early model for the strong force with short range. The force law falls off as $\frac{e^{-rm}}{r^2}$ with both the classic inverse square law multiplied by an an exponential dampening with distance parameterized by the mass m. To give a gauge boson A_μ a short range, the Lagrangian would need a mass term like $m_A^2 A_\mu A^\mu$. This term violates gauge symmetry because when $A'_\mu = A_\mu + \partial_\mu \chi$ we see that $A_\mu A^\mu \neq A'_\mu A'^\mu$. Naively, one would think that gauge symmetry blocks all gauge bosons from having mass; and therefore, all gauge theories (Abelian and the non-Abelian ones) would obey force laws that scale as $\frac{1}{r^2}$. This would mean that all gauge theories would represent long-range forces

Table 1.1: General structure of the Standard Model fermions, divided in three families, and the Higgs.

	Leptons			Hadrons				Higgs
	(1,**2**,-1/2)	(1,1,1)		(**3**,**2**,1/6)	($\bar{\mathbf{3}}$,1,-2/3)	($\bar{\mathbf{3}}$,1,1/3)		(1,**2**,-1/2)
Family 1	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\bar{e}$		$\begin{pmatrix} u \\ d \end{pmatrix}$	$\bar{u}$	$\bar{d}$		
Family 2	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\bar{\mu}$		$\begin{pmatrix} c \\ s \end{pmatrix}$	$\bar{c}$	$\bar{s}$		
Family 3	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$\bar{\tau}$		$\begin{pmatrix} t \\ b \end{pmatrix}$	$\bar{t}$	$\bar{b}$		

similar to gravity and electromagnetism (each of which is mediated by a massless boson).

Apparently there are two solutions to this problem:

- The Higgs mechanism, which gives gauge bosons mass without violating gauge symmetry;
- A spontaneously created mass gap phenomena associated with non-Abelian gauge theories wich is yet not fully understood and seems related to the confinement of individual quarks within *Quantum Chromodynamics* (QCD).

The SM chooses the Higgs mechanism while the strong force seems to use the second option.

1.2 The Standard Model

The SM has a symmetry group of $SU(3)_C \times SU(2)_L \times U(1)_Y$ [14], [15] [16]. The C on $SU(3)_C$ stands for color. The L on $SU(2)_L$ means that it only acts on the left-handed states. The Y on $U(1)_Y$ stands for hypercharge and it is there to distinguish from $U(1)$ for electromagnetism. The gauge bosons content is therefore 8 "photon-like" fields from the adjoint of $SU(3)_C$, 3 from the adjoint $SU(2)_L$ and 1 from the Abelian $U(1)_Y$.

For each matter field we specify the representations and the charge under three symmetry groups as a triplet of numbers. A 1 for $SU(3)_C$ or $SU(2)_L$ is a singlet (it does not transform). The value in the third entry is the $U(1)_Y$ hypercharge. For example (1,**2**,-1/2) means that the field is a singlet under $SU(3)_C$, a doublet under $SU(2)_L$ and has a charge -1/2 under $U(1)_Y$, see Table 1.1.

1.2.1 The Gauge and Higgs Sector

In this section we will treat only the *Electroweak* (EW) part of the SM, the $SU(2)_L \times U(1)_Y$ gauge group. All begins with the Higgs complex scalar doublet field ϕ in the (**2**,-1/2) representation of $SU(2)_L \times U(1)_Y$. ϕ will be given a potential that spontaneously breaks the symmetry of the vacuum. The resulting vacuum expectation value will give mass to the gauge bosons with a special pattern, with the resulting broken theory having the symmetry $U(1)$ associated with QED. The pattern of masses of the massive gauge bosons will make the weak force a short range one.

The first step is to write down the covariant derivative:

$$(D_\mu\phi)_i = \partial_\mu\phi_i - i[g_2 W_{\mu a} T^a_{\mathbf{2}} + g_1 B_\mu Y]_{ij}\phi_j, \tag{1.1}$$

we denote the generators of the **2** representation of $SU(2)$:

$$T^a_{\mathbf{2}} = \frac{\sigma^a}{2}, \tag{1.2}$$

(for the Pauli matrices see Appendix B.1) and the gauge fields are W^a_μ. The generator of $U(1)_Y$ is the identity matrix, Y is the hypercharge (-1/2 in this case) and the $U(1)_Y$ gauge field is B_μ, g_1 and g_2 are coupling constants for the $U(1)_Y$ part and the $SU(2)_L$ part, respectively. Knowing that the generators of $SU(2)$ are the Pauli matrices, we can expand in matrix form, so the full covariant derivative is:

$$(D_\mu\phi)_i = \begin{pmatrix} \partial_\mu\phi_1 + \frac{i}{2}\left(g_2 W^3_\mu - g_1 B_\mu\right)\phi_1 + \frac{ig_2}{2}\left(W^1_\mu - iW^2_\mu\right)\phi_2 \\ \partial_\mu\phi_2 + \frac{ig_2}{2}\left(W^1_\mu + iW^2_\mu\right)\phi_1 - \frac{i}{2}\left(g_2 W^3_\mu + g_1 B_\mu\right)\phi_2 \end{pmatrix}. \tag{1.3}$$

We know that the Lagrangian will have the kinetic term and some potential:

$$\mathcal{L}_\phi = D_\mu\phi^\dagger D^\mu\phi - V\left(\phi^\dagger, \phi\right), \tag{1.4}$$

and the Higgs potential is:

$$V\left(\phi^\dagger, \phi\right) = \frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2. \tag{1.5}$$

With $\lambda > 0$ the minimum field configuration is not at $|\phi| = 0$, but at:

$$|\phi| = \frac{v}{\sqrt{2}}. \tag{1.6}$$

We can make a global $SU(2)$ transformation to put the entire *Vacuum Expectation Value* (VEV) on the first component of ϕ, and then make a global $U(1)$ transformation to make the field real. Therefore:

$$\langle\phi\rangle = \langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}, \tag{1.7}$$

and we expand ϕ around this new vacuum:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}. \tag{1.8}$$

In this case we have chosen the $SU(2)$ gauge to keep the second component null and the $U(1)$ phase to keep the first component real. Thus, $h(x)$ is a real scalar field. The other fluctuations turn out to be the longitudinal components of the massive gauge fields in the Higgs mechanism.

The gauge choice that keeps the Higgs in the first component is called the ***Unitary Gauge***. Although it is easiest to see the physical content, the ***Unitary Gauge*** is the most difficult to renormalize and perform advanced calculations with. When we plug the VEV into the kinetic term in the equation (1.4), we get:

$$\mathcal{L}_{\langle\phi\rangle} = -\frac{1}{8} \begin{pmatrix} v & 0 \end{pmatrix} \begin{pmatrix} g_2 W_\mu^3 - g_1 B_\mu & g_2 \left(W_\mu^1 - iW_\mu^2\right) \\ g_2 \left(W_\mu^1 + iW_\mu^2\right) & -g_2 W_\mu^3 - g_1 B_\mu \end{pmatrix}^2 \begin{pmatrix} v \\ 0 \end{pmatrix}. \tag{1.9}$$

To find the masses of the gauge bosons we can rewrite this matrix as:

$$\mathcal{L}_{\langle\phi\rangle} = -\frac{1}{8} v^2 V_\mu^T \begin{pmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & -g_1 g_2 \\ 0 & 0 & -g_1 g_2 & g_1^2 \end{pmatrix} V^\mu, \tag{1.10}$$

with $V_\mu^T = \left(W_\mu^1, W_\mu^2, W_\mu^3, B_\mu\right)$, so W_μ^1 and W_μ^2 have mass and are already diagonalized.

The submatrix formed by the other components has a determinant of 0 and therefore must have a 0 eigenvalue that keeps a massless gauge boson left, with a corresponding symmetry that keeps it massless under renormalization effects. The eigenvalues are 0, $-\frac{1}{8}v^2 g_2^2$, $-\frac{1}{8}v^2 g_2^2$ and $-\frac{1}{8}v^2\left(g_1^2 + g_2^2\right)$. The normalized eigenvector for the massless state A_μ is

$$V_A^T = (0, 0, g_1, g_2) \frac{1}{\sqrt{g_1^2 + g_2^2}}. \tag{1.11}$$

The eigenvector for the massive vector boson state Z_μ is

$$V_Z^T = (0, 0, g_2, -g_1) \frac{1}{\sqrt{g_1^2 + g_2^2}}, \tag{1.12}$$

where $Z_\mu = \frac{g_2 W_\mu^3 - g_1 B_\mu}{\sqrt{g_1^2 + g_2^2}}$ appeared in equation (1.9).

The format of the eigenvectors suggests a parametrization based on a right triangle with g_1 on one leg and g_2 on the other leg. Traditionally, this right triangle is used to describe the mixed states. The angle opposite to the leg with length g_1 is called the **Weak Mixing Angle**:

$$s_w = \sin\theta_w = \frac{g_1}{\sqrt{g_1^2+g_2^2}}, \qquad c_w = \cos\theta_w = \frac{g_2}{\sqrt{g_1^2+g_2^2}}. \tag{1.13}$$

Finally we can define the physical fields using a simple Euler rotations of the old fields:

$$W_\mu^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \tag{1.14}$$

$$W_\mu^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}, \tag{1.15}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{1.16}$$

where $W^\pm$ are chosen so that each term in the final Lagrangian will have an explicit $U(1)_{EM}$ gauge symmetry associated with charge conservation. We have to observe that the fields $W_\mu^\pm$ are linear combinatons of fields corresponding to non-Cartan generators of $SU(2)$, whereas Z_μ and A_μ are both linear combinations of fields corresponding to Cartan generators of $SU(2)$ and $U(1)$. For this reason the neutral fields will interact but do not change the charge, and the charged fields will interact and change the charge.
Through symmetry breaking, we have given mass to the $W_\mu^\pm$ and Z_μ while the A_μ remain massless:

$$M_W = \frac{g_2 v}{2}, \qquad M_Z = \frac{M_W}{c_w}. \tag{1.17}$$

These massive fields are the vector bosons, wich are the force carrying particles of the **Weak Force**. The A_μ remains massless because the $U(1)$ symmetry remains unbroken, these are the gauge group and field of **Electromagnetism**. The point of all this is that at very high energies (above the breaking of the $SU(2)\times U(1)$ symmetry), we have only a complex scalar Higgs field, along with four massless vector boson gauge fields $(W_\mu^1, W_\mu^2, W_\mu^3, B_\mu)$, each of which behaves massless like a photon.

At low energies, however, the $SU(2)\times U(1)$ symmetry is broken, and the low energy effective theory consists of a linear combination of the original four fields. The theory above the symmetry breaking scale is called the **Electroweak Theory** that below the breaking scale becomes two separate forces: the broken **Weak** and the unbroken **Electromagnetic**. This is the first and most basic example of unification within particle physics. At low energies, the electromagnetic and weak forces are separate, at high energies they unify into a single theory.
Before including leptons and quarks, we first write out the full Lagrangian for the effective field theory for $h(x)$ and the gauge fields. We start with the complete Lagrangian term for $h(x)$. We have written the original field ϕ as in equation (1.8). Our potential in equation (1.5) is now:

$$V\left(\phi^\dagger,\phi\right)=\frac{\lambda}{4}\left(\phi^\dagger\phi-\frac{v^2}{2}\right)^2=\frac{\lambda v^2h^2}{4}+\frac{\lambda vh^3}{4}+\frac{\lambda h^4}{16}. \tag{1.18}$$

The first term on the right hand side is a mass term giving the mass of the Higgs and the second two terms are self interaction vertices. The kinetic term for the Higgs will be the usual $-\frac{1}{2}\partial_\mu h\partial^\mu h$, knowing that in general the field strength for the gauge bosons is:

$$F_{\mu\nu}(x)=\frac{i}{g_2}\left[D_\mu,D_\nu\right]=\partial_\mu A_\nu-\partial_\nu A_\mu-ig_2\left[A_\mu,A_\nu\right], \tag{1.19}$$

and

$$F^{\mu\nu}=F_a^{\mu\nu}T^a\Rightarrow F_a^{\mu\nu}=2Tr\left(F^{\mu\nu}T^a\right). \tag{1.20}$$

The gauge field strength corresponding to the $U(1)$ symmetry will be

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu, \tag{1.21}$$

so we can now write the kinetic term:

$$\mathcal{L}_{Kin}=-\frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}. \tag{1.22}$$

At the end we can write the Lagrangian for the $SU(2)_L\times U(1)_Y$ gauge fields along with the Higgs field:

$$\begin{aligned}\mathcal{L}_{eff}=&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}-D^{\dagger\mu}W^{-\nu}D_\mu W^+_\nu+D^{\dagger\mu}W^{-\nu}D_\nu W^+_\mu\\&+ie\left(F^{\mu\nu}+\cot\theta_w Z^{\mu\nu}\right)W^+_\mu W^-_\nu\\&-\frac{1}{2}\left(\frac{e^2}{s_w^2}\right)\left(W^{+\mu}W^-_\mu W^{+\nu}W^-_\nu-W^{+\mu}W^+_\mu W^{-\nu}W^-_\nu\right)\\&-\left(M_W^2W^{+\mu}W^-_\mu+\frac{1}{2}M_Z^2Z^\mu Z_\mu\right)\left(1+\frac{h}{v}\right)^2\\&-\frac{1}{2}\partial^\mu h\partial_\mu h-\frac{1}{2}m_h^2h^2-\frac{1}{2}\frac{m_h^2}{v}h^3-\frac{1}{8}\frac{m_h^2}{v^2}h^4,\end{aligned} \tag{1.23}$$

where we have chosen the following definitions:

$$\begin{aligned}F_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu,\\Z_{\mu\nu}&=\partial_\mu Z_\nu-\partial_\nu Z_\mu,\\D_\mu&=\partial_\mu-ie\left(A_\mu+\cot\theta_w Z_\mu\right).\end{aligned} \tag{1.24}$$

The $\mathcal{L}_{eff}$ allows us to see what kinds of interactions will involve the Higgs and the vector bosons. The photon A_μ does not couple to h. However, the $W^\pm$ do. By expading the last but one line of (1.23), we see that the interaction is proportional to W^+W^-h. This interaction is responsible for charged weak gauge bosons fusing to form a Higgs.

1.2.2 The Lepton Sector and the origin of mass

There are six leptons arranged into three families or generations. Each family behaves nearly exactly the same way, so we will make three doublets, one for each generation and allow mixing between generations in the most general possible way.

The SM needs to simplify to the $V-A$ model at low energies. One characteristic of the $V-A$ model is that only left handed fields were included in the weak interactions. Because the neutrino only participates in the weak interactions (and gravity) there is only a left handed neutrino needed for a weak interactions theory. The electron exists in both a left handed and right handed state. The neutrino is added as part of a left handed $SU(2)_L$ doublet with the left handed electron,

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}. \tag{1.25}$$

This is why it is arranged as it is in Table 1.1 with the electron under the $(\mathbf{2}, -1/2)$ representation of $SU(2)_L \times U(1)_Y$. The right-handed electron is an $SU(2)_L$ singlet. The doublet L is a purely left-handed Weyl spinor [1]. The $SU(2)$ singlet field $\bar{e}$ is also a purely left-handed Weyl spinor and is in the $(1,1)$ representation. The neutrino ν_e is part of the L doublet and has no representation of its own. A priori there is no link between the e and $\bar{e}$.

Mimicking what we did in equation (1.1), we can write down the covariant derivative for each field,

$$(D_\mu L)_i = \partial_\mu L_i - ig_2 W_\mu^a (T^a)_{ij} L_j - ig_1 B_\mu Y_L L_i, \tag{1.26}$$
$$D_\mu \bar{e} = \partial_\mu \bar{e} - ig_1 B_\mu \bar{e}. \tag{1.27}$$

The field $\bar{e}$ has no $SU(2)$ term in its covariant derivative because the 1 representation of $SU(2)$ is the trivial representation, this means it doesn't carry $SU(2)$ charge. Also we know that:

$$Y_L = -\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{1.28}$$

Following the generic Lagrangian for the spin-1/2 fields ψ of mass m through which we can write out the Dirac equation (see Appendix D) [2],

[1]The Weyl Equation is a relativistic wave equation for describing massless spin $\frac{1}{2}$ particles.
[2]For a complete review of the Dirac Equation see [17].

$$\mathcal{L}_D = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi \Rightarrow \left(i\gamma^\mu \partial_\mu - m\right)\psi = 0, \tag{1.29}$$

we can find the kinetic term for both (massless) fields:

$$\mathcal{L}_{Kin} = iL^{\dagger j}\bar{\sigma}^\mu \left(D_\mu L\right)_j + i\bar{e}^\dagger \bar{\sigma}^\mu D_\mu \bar{e}. \tag{1.30}$$

If we try to add mass terms for L and $\bar{e}$ fields, the gauge symmetry is broken. For example Lorentz invariant combination of $\bar{e}_a \bar{e}_b \epsilon_{ab} = \bar{e}\bar{e}$ is $SU(2)_L$ invariant but violates $U(1)_Y$. The term has a net hypercharge of $+2$. Likewise $L^{ai}L^{bj}\epsilon_{ab}\epsilon_{ij}$ (where ϵ_{ab} is the totally antisymmetric Levi-Civita tensor, see Appendix B.3) is Lorentz invariant on the a,b indices and $SU(2)_L$ invariant on the i,j indices, but not under $U(1)_Y$ having a net hypercharge of -1. If we conjugate one of the terms, then we cannot form the Lorentz invariant combination and we lose Lorentz invariance.

Therefore we cannot add a mass term, but we know, experimentally that electrons and neutrinos have mass. We must incorporate mass into the theory and the Higgs mechanism comes another time to the rescue. So adding a direct mass term destroys gauge invariance, but we can add a Yukawa term:

$$\mathcal{L}_{Yuk} = -y\epsilon^{ij}\phi_j \left(L_j \bar{e}\right) + h.c., \tag{1.31}$$

where y is the Yukawa coupling constant, the $(L_j \bar{e})$ term indicates the Lorentz invariant combination of the spinor indices for L and $\bar{e}$ that are suppressed. We see that the term has a net hypercharge of 0, and the $SU(2)_L$ and Lorentz indices are all contracted to form singlets. At this point there is still no relationship between e and $\bar{e}$.

Now that we have added $\mathcal{L}_{Yuk}$ to the Lagrangian, we want to break the symmetry exactly as we did in the previous section. We continue to work in the ***Unitary Gauge*** where $\phi_2 = 0$, so:

$$\begin{aligned}\mathcal{L}_{Yuk} &= -\frac{1}{\sqrt{2}}y\left(v+h\right)\left(e\bar{e}\right) - \frac{1}{\sqrt{2}}y\left(v+h\right)\left(\bar{e}^\dagger e^\dagger\right)\\ &= -\frac{1}{\sqrt{2}}yv\bar{\Lambda}\Lambda - \frac{1}{\sqrt{2}}yvh\bar{\Lambda}\Lambda,\end{aligned} \tag{1.32}$$

where we are supposing that e and $\bar{e}$ are each one the hermitic of each other:

$$\Lambda = \begin{pmatrix} e \\ i\sigma^2 \bar{e}^\dagger \end{pmatrix} \tag{1.33}$$

is the Dirac field for the electron (e is the electron and $\bar{e}$ the positron).

Comparing (1.32) with (1.29) we see that it is a mass term for the electron and positron where $m_e = \frac{yv}{\sqrt{2}}$. Therefore when the theory undergoes spontaneous symmetry breaking, the fields e and $i\sigma^2\bar{e}^*$ which initially are completely unlinked join to form the left- and right-handed parts of the electron field Λ. In the presence of the $SU(2)_L$ symmetry, ν_e and e can be rotated into each other without affecting the theory. Now, we want a kinetic term for the neutrino. It is generally believed that neutrinos are described by Majorana fields so

$$\mathcal{N}' = \begin{pmatrix} \nu_e \\ i\sigma^2\nu_e^* \end{pmatrix}. \tag{1.34}$$

Now, we employ a trick because the kinetic term for Majorana fields has only one term, whereas the Dirac field sums over both Weyl spinors composing it. Hence we can work with the Dirac field

$$\mathcal{N} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}. \tag{1.35}$$

So, the Dirac kinetic term $i\bar{\mathcal{N}}\gamma^\mu\partial_\mu\mathcal{N}$ will result in the correct kinetic term from (1.30). Continuing with the symmetry breaking, we want to write the covariant derivative (1.26) and (1.27) in terms of our low energy gauge fields of the previous section (1.14-1.16), that correspond to Cartan generators and non-Cartan generators [18], [19] and act respectively like particles that change the charge of the particles they interact with, and particles that do not change that charge. Therefore to make the calculations simpler, we will break the covariante derivative up into the non-Cartan part:

$$g_2\left(W_\mu^1T^1 + W_\mu^2T^2\right) = \frac{g_2}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}, \tag{1.36}$$

and the Cartan part:

$$g_2W_\mu^3T^3 + g_1B_\mu Y = e\left(T^3 + Y\right)A_\mu + e\left(\cot\theta_w T^3 - \tan\theta_w Y\right)Z_\mu. \tag{1.37}$$

If A_μ is the electromagnetic field, and e is the electromagnetic charge, then $T^3 + Y$ must be the generator of electric charge.

Notice that the electromagnetic generator is a linear combination of the two Cartan generators of $SU(2)\times U(1)$. We know that

$$T^3 = \frac{\sigma^3}{2}, \tag{1.38}$$

and Y_L and $Y_{\bar{e}}$ are defined in (1.28), so we can find

$$T^3L = \frac{1}{2}\begin{pmatrix} \nu_e \\ -e \end{pmatrix}, \qquad Y_LL = -\frac{1}{2}\begin{pmatrix} \nu_e \\ -e \end{pmatrix}. \tag{1.39}$$

Further, $\bar{e}$ carries no T^3 charge, so its T^3 eigenvalues is 0, while $Y_{\bar{e}}$ is $+1$. Summarizing all this,

$$
\begin{aligned}
T^3\nu_e &= \frac{1}{2}\nu_e, \quad T^3 e = -\frac{1}{2}e, \qquad T^3\bar{e} = 0, \\
Y\nu_e &= -\frac{1}{2}\nu_e, \quad Ye = -\frac{1}{2}e, \qquad Y\bar{e} = \bar{e}.
\end{aligned}
\tag{1.40}
$$

Then defining the generator of electric charge to be $Q = T^3 + Y$, we have

$$
Q\nu_e = 0, \qquad Qe = -e, \qquad Q\bar{e} = e \tag{1.41}
$$

So the idea is that electrons/positrons and neutrinos interact with the gauge bosons: then the exchange of $W^{\pm}$ will reduce the process to a charged current weak interaction, the exchange of a Z will reduce to a neutral current weak interaction and the exchange of a photon will reduce to an electromagnetic interaction.

1.2.3 The Quark Sector

For simplicity we work with only one generation, extending to the others is then trivial. We define three fields: $\mathcal{Q}$, $\bar{u}$, $\bar{d}$, in the representation $(\mathbf{3}, \mathbf{2}, 1/6)$, $(\bar{\mathbf{3}}, 1, -2/3)$, and $(\bar{\mathbf{3}}, 1, 1/3)$ of $SU(3)_C \times SU(2)_L \times U(1)_Y$. The field $\mathcal{Q}$ will be the $SU(2)_L$ doublet

$$
\mathcal{Q} = \begin{pmatrix} u \\ d \end{pmatrix}. \tag{1.42}
$$

Following what we did with the leptons, we can write out the covariant derivative for all three fields:

$$
(D_\mu \mathcal{Q})_{\alpha i} = \partial_\mu \mathcal{Q}_{\alpha i} - i g_3 A^a_\mu \left(T^a_3\right)^\beta_\alpha \mathcal{Q}_{\beta i} - i g_2 W^a_\mu \left(T^a_2\right)^j_i q_{\beta j} - i g_1 \left(\frac{1}{6}\right) B_\mu \mathcal{Q}_{\alpha i}, \tag{1.43}
$$

$$
(D_\mu \bar{u})^\alpha = \partial_\mu \bar{u}^\alpha - i g_3 A^a_\mu \left(T^a_{\bar{3}}\right)^\alpha_\beta \bar{u}^\beta - i g_1 \left(-\frac{2}{3}\right) B_\mu \bar{u}^\alpha, \tag{1.44}
$$

$$
\left(D_\mu \bar{d}\right)^\alpha = \partial_\mu \bar{d}^\alpha - i g_3 A^a_\mu \left(T^a_{\bar{3}}\right)^\alpha_\beta \bar{d}^\beta - i g_1 \left(\frac{1}{3}\right) B_\mu \bar{d}^\alpha, \tag{1.45}
$$

where i is an $SU(2)_L$ index and α is an $SU(3)_C$ index. The $SU(3)$ index is lowered for the $\mathbf{3}$ representation and raised for the $\bar{\mathbf{3}}$ representation. The 8 generators of the $SU(3)_C$ acting on the $\mathbf{3}$ are given by T^a_3, and acting on the $\bar{\mathbf{3}}$ are given by $T^a_{\bar{3}} = -(T^a_3)^*$.

The vector field A^a_μ is the gluon field. Just as with leptons, we cannot write down a gauge invariant mass term for the quarks, but we can include a Yukawa term coupling these fields to the Higgs:

$$
\begin{aligned}
\mathcal{L} &= -y'\epsilon^{ij}\phi_j\mathcal{Q}_{\alpha j}\bar{d}^\alpha - y''\phi^{\dagger i}\mathcal{Q}_{\alpha i}\bar{u}^\alpha + h.c. \\
&= -\frac{y'}{\sqrt{2}}(v+h)\,\bar{\mathcal{D}}^\alpha\mathcal{D}_\alpha - \frac{y''}{\sqrt{2}}(v+h)\,\bar{\mathcal{U}}^\alpha\mathcal{U}_\alpha,
\end{aligned}
\tag{1.46}
$$

where,

$$
\mathcal{D}_\alpha = \begin{pmatrix} d_\alpha \\ i\sigma^2\bar{d}^\dagger_\alpha \end{pmatrix} \quad , \quad \mathcal{U}_\alpha = \begin{pmatrix} u_\alpha \\ i\sigma^2\bar{u}^\dagger_\alpha \end{pmatrix},
\tag{1.47}
$$

so also the quarks finally have acquired masses:

$$
m_d = \frac{y'v}{\sqrt{2}}, \qquad m_u = \frac{y''v}{\sqrt{2}}.
\tag{1.48}
$$

Again the spontaneous symmetry breaking of ϕ_i links a term in the doublet with the singlet $\bar{u}$ or $\bar{d}$ to provide a mass. It is again straightforward to find the electric charge eigenvalue for each field:

$$
Qu = \frac{2}{3}u, \qquad Qd = -\frac{1}{3}d, \qquad Q\bar{u} = -\frac{2}{3}\bar{u}, \qquad Q\bar{d} = \frac{1}{3}\bar{d}.
\tag{1.49}
$$

So we can collect all these terms and write out a complete Lagrangian. The primary idea to take away is that the $SU(2)_L$ doublet (1.42) behaves exactly as the lepton doublet (1.25) when interacting with the raising and lowering gauge particles $W^\pm$. This is why the u and d are arranged in the $SU(2)$ doublet $\mathcal{Q}$, and why this doublet carries the $SU(2)$ index i in the covariant derivative (1.43), whereas $\bar{u}$ and $\bar{d}$ carry only the $SU(3)$ index.

To represent the three generations we make three copies of the SM structure; one of each generation. However there is no symmetry preventing terms from different generations from coupling to each other, therefore the Yukawa couplings need to be generalized to allow a $\bar{c}$ to couple to $\mathcal{Q}$ from the first generation. Let us denote the generations with the indices A, B, C, the Yukawa couplings now becomes

$$
\mathcal{L}_{Yuk} = -Y^d_{AB}\phi\mathcal{Q}^A\bar{d}^B - Y^u_{AB}\phi^\dagger\mathcal{Q}^A\bar{u}^B - Y^e_{AB}\phi L^A\bar{e}^B + h.c.,
\tag{1.50}
$$

where we have suppressed the $SU(3)_C$ and the $SU(2)_L$ and the Lorentz indices.

Because neutrinos have mass, we should add several Yukawa terms to give the neutrino generations mass. For the moment, we stick to the original approximation where the neutrinos are massless. The kinetic terms for the fermion generations are invariant under global $SU(3)$ trasformations so we can redefine L and $\bar{e}$ to diagonalize Y^e. These entries will be proportional to the mass of the leptons. However if we diagonalize Y^u by redefining $\mathcal{Q}$ and $\bar{u}$ we are left without enough freedom to diagonalize Y^d. The remaining structure is called the *Cabibbo-Kobayashi-Maskawa* (CKM) matrix.

So now for the sake of convenience let us introduce the following notation for the quarks with charge $Q = \frac{2}{3}$,

$$u_i \; ; \; i = 1, 2, 3 \;\; (u, c, t) \, , \tag{1.51}$$

and similarly for the quarks with charge $Q = -\frac{1}{3}$,

$$b_i \; ; \; i = 1, 2, 3 \;\; (d, s, b) \, . \tag{1.52}$$

These are the quark states with the quantum numbers relevant to strong interactions, they are the constituents of the physical meson and baryon states.

The weak interactions do not conserve the strong isospin, they couple the u_i states to new states d_i, with electric charge $-\frac{1}{3}$, which are related to the states (1.52) by a unitarity transformation,

$$d_i = \sum_j V_{ij} b_j \qquad ; \qquad VV^\dagger = \mathbb{1}. \tag{1.53}$$

These new states are referred to as the *weak interaction eigenstates*. A way to visualize this relation is by assuming that the weak gauge bosons $W^\pm$ and Z couple to quark currents which contain the fields $u_i(x)$ and $d_i(x)$, but that the strong interaction Lagrangian contains quark mass terms which are not diagonal in the basis of the states d_i,

$$\mathcal{L}_{mass} = \sum_i m_i \bar{u}_i \,(x)\, u_i \,(x) + \sum_{ij} M_{ij} \bar{d}_i \,(x)\, d_j \,(x) \, . \tag{1.54}$$

The mass matrix M_{ij} can be diagonalized by means of the unitarity transformation V,

$$\sum_{ab} V^\dagger_{ia} M_{ab} V_{bj} = m_i \delta_{ij}. \tag{1.55}$$

These states b_j, (1.52), are then the *mass eigenstates* whilst the states d_j are the *weak eigenstates*.

In the case of two quark families,

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \tag{1.56}$$

where θ is called the Cabibbo angle.

In the case of three quark families the most general V is a unitary matrix that depends on three real mixing angles θ_1, θ_2, θ_3, and one phase δ [20]:

$$V_{CKM} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}, \tag{1.57}$$

where $c_i = \cos\theta_i$ ans $s_i = \sin\theta_i$. Clearly, if $\theta_2 = \theta_3 = 0$ this matrix reduces to the Cabibbo case with only two families of quarks.

The parametrization (1.57) is only one of many possibilities. One often writes, more generally,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1.58}$$

Here subscript indices indicate the *charged current* vertex (CC-vertex) that appears multiplied with the corresponding matrix element. For example the quark vertex $\bar{u}dW$ responsible for the decay of the neutron $n \to p\ e\ \bar{\nu}_e$ is multiplied by V_{ud}.

Chapter 2

Flavor violation of charged leptons

2.1 Background

Although particle physics is presently dominated by the agreement of virtually all measurements with the predictions of the SM, there remain many unanswered questions. Almost seventy years after the discovery of the strange quark and after the first search for $\mu \to e\gamma$ [21], we still puzzle over the apparently redundant multiplicity of the generations of quarks and leptons.

In the quark sector, there are three generations of doublets wich are related by the CKM matrix. Unitarity of the CKM is necessary for the accomodation of a complex *charge parity* violation (CP-violation) phase parameter, possibly providing a clue to the generation puzzle. Violation of time reversal invariance may be related to the dominance of matter over anti-matter in the universe.

The situation in the lepton sector is simpler but no less intriguing. Three generations of leptons and their associated neutrinos appear to be isolated replications of each other, except for mass. The weak coupling strengths of the three generations to the gauge bosons are identical within considerable experimental precision [7]. Although quark flavor is not conserved, no confirmed mixing of electrically charged leptons has been observed at minute levels of experimental sensitivity. One can speculate that some global or accidental symmetry is responsible for the apparent conservation of lepton flavor or, as in many alternate theories, that flavor-violating interactions are suppressed by very high mass scales.

A neutrino is, by definition, a flavour eigenstate, in the sense that a neutrino is always produced with, or absorbed to give, a charged lepton (or antineutrino). However, as with the quarks and the CKM matrix, it is possible that the flavour eigenstates are not identical to the mass eigenstates.

A neutrino produced through the weak force in the decay of the muon, is described as the sum of (at least) two matter waves. As the neutrino travels through space (and depending on which masses are measured), these matter waves interfere with each other constructively or destructively. The interference causes first the disappearance and then the reappearance of the original type of neutrino. The interference can occur only if at least two matter waves have different masses. Thus, the mechanical oscillation starts from the assumption that the lepton weak and mass eigenstates are not the same and that one set is composed of mixtures of the other set. In other words, there must be mixing among the leptons as there is among the quarks.

The quarks within the composite particles (proton, neutron, Λ) start and end as pure mass eigenstates, and the fact that they are mixtures of weak states shows up through the action of the weak force. When a neutron decays through the weak force and the d quark transforms into a u, only a measurement of the decay rate reflects the degree to which a d quark is composed of the weak state d'.

In contrast, in neutrino oscillations experiments, the neutrinos always start and end as pure weak states. They are typically created through weak force processes of pion decay and muon decay, and they are typically detected through inverse beta decay and inverse muon decay, weak processes in which the neutrinos are transmuted back to their charged lepton partners. Between the point of creation and the point of detection, they propagate freely, and if they oscillate into a weak state from a different family, it is not through the action of the weak force, but rather through the pattern of interference that develops as the different mass eigenstates composing the original neutrino state evolve in time.

There are two types of neutrino oscillations experiments one could think of doing:

- The first is to start with a pure beam of known flavour ν_x, and look to see how many have disappeared. This is a "disappearance" experiment and measures the survival probability.

- The second type of experiment is an "appearance" experiment, in which one starts with a pure beam of known flavour ν_x and looks to see how many neutrinos of a different flavour ν_y are detected.

Most extensions of the SM tell us to expect mixing among leptons in analogy with mixing among quarks. But so far, those theories make no quantitative predictions on masses and mixing angles. Thus, neutrino oscillations experiments have a twofold purpose: first to confirm evidence for oscillations and then to make quantitative determinations of the neutrino masses and mixing angles.

Among the quarks, the amount of mixing is small and occurs primarily between the first two families. In all cases of interest, neutrino oscillations can be described accurately using the two mixing angle approximation. Applying the two family formalism to each experiment allows one to derive a range of possible values for the relevant mass difference of neutrinos and a range for the mixing angle. Input to the interpretation includes the neutrino energies in a particular experiment, the distance from source to detector, the expected neutrino flux, and the measured flux or probability.

The *Lepton Flavor Violation* (LFV) [1] reactions have continued to be pursued vigorously on many fronts. In most extensions of the SM, these reactions occur naturally. Actually, they are often difficult to eliminate without inventing seemingly artificial mechanisms. Progress on LFV experiments has spanned many orders of magnitude recently and there is

[1] For a complete review see [22].

no indication that the limits of achievable sensitivity have been exhausted. Since the LFV rates are supposedly suppressed by heavy force carriers, the virtual mass scales accessed by these experiments have been pushed to extremely high levels reaching the TeV.

LFV is absent in the SM with massless neutrinos. A minimal extension that includes right handed neutrinos allowing for small neutrino Dirac masses might allow LFV reactions such as $\mu \to e\gamma$ via diagrams like that shown in Figure 2.1. Even if the masses of the three known neutrinos were at their present experimental limits the branching ratio for $\mu \to e\gamma$ would be tiny [23], for this reason higher mass scales are being probed in some *grand unified theories* (GUTs).

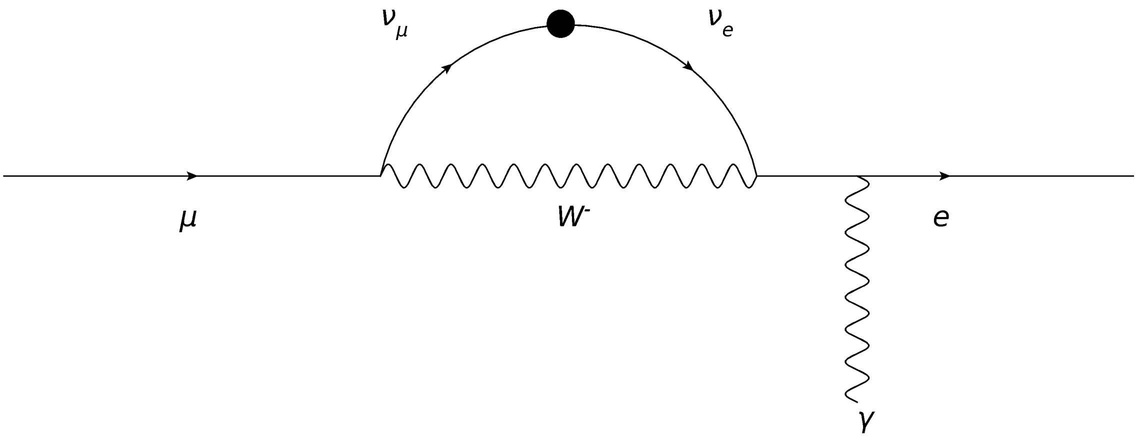


Figure 2.1: Diagram for $\mu \to e\gamma$ with massive neutrinos.

LFV reactions have continued to be pursued vigorously on many fronts including muon, tau [24] and kaon decays and in *ep* collisions at HERA [25]. Processes like the spontaneous conversion of muonium "atoms" to antimuonium ($\mu^+e^- \to \mu^-e^+$) appear in models containing Majorana neutrinos which are natural consequences of the "see-saw mechanism", a candidate approach to account for the tiny masses of neutrinos.

The search for manifestations of charged LFV constitutes the goal of several experiments, exclusively dedicated to look for signals of processes such as rare radiative as well as three-body decays and lepton conversion in muonic nuclei. In parallel to these low-energy searches, if the high-energy *Large Hadron Collider* (LHC) finds signatures of *Supersymmetry* (SUSY), it is then extremely appealing to consider SUSY models that can also accommodate neutrino oscillations (SUSY seesaw).

If the seesaw is indeed the source of both neutrino masses and leptonic mixings and accounts for low-energy LFV observables within future sensitivity reach, then these phenomena are expected to be observed at the LHC [26]. For this reason, we review how neutrino masses arise in seesaw models in the next subsection.

2.2 Neutrino masses and mixing in the Seesaw Model

In the Standard Model, particle masses are proportional to the strengths of the interactions between the particles and the Higgs bosons. Thus, the Dirac mass term like for example ν_e must be multiplied by some very small coupling strength such that his mass is at least 50,000 times smaller than the mass of the electron. But the electron and the electron neutrino are part of the same weak doublet, and there seems to be no reason why they should have such enormously different interaction strengths to the Higgs bosons.

In 1979, without introducing an arbitrarily small coupling strength to the Higgs bosons, Murray Gell-Mann, Pierre Ramond, and Richard Slansky invented a model that yields very small neutrino masses [27]. The two neutrino states ν_R and $\bar{\nu}_L$ that must be added to the theory to form the Dirac mass term could themselves be coupled to form a Majorana mass term. That term could also be added to the theory without violating any symmetry principle. Further, it could be assumed that the coefficient M of the Majorana mass term is very large. If the theory contains both Dirac mass term and this Majorana mass term, then the four components of the neutrino would no longer be states of definite mass m determined by the coefficient of the Dirac mass term.

Instead, the four components would split into two Majorana neutrinos, each made up of two components. One neutrino would have a very small mass, equal to $\frac{m^2}{M}$; the second neutrino would have a very large mass, approximately equal to M. The very light Majorana neutrino would mostly be the left handed neutrino that couples to the W, and the very heavy neutrino would mostly be a right handed neutrino that does not couple to the W. Similarly, the very light antineutrino would be mostly the original right handed antineutrino that couples to the W, and the very heavy antineutrino would be mostly a left handed antineutrino that does not couple to the W.

This so called seesaw mechanism in which the Dirac mass m is reduced by a factor of $\frac{m}{M}$ through the introduction of a large Majorana mass term has been used in many extensions of the SM to explain why neutrino masses are small. The large Majorana mass M is often associated with some new, weak gauge force that operates at a very high energy scale dictated by the mass of a new, very heavy gauge boson. The net result of this approach is that the neutrino seen at low energies is predicted to be mostly a Majorana particle.

We know that the conserved quantum numbers are always associated with exact symmetries, just as the conservation of electromagnetic charge is associated with $U(1)$ gauge invariance. On the other hand, there is no exact gauge symmetry associated with any of the lepton numbers.

Moreover, neutrinos have been seen to oscillate between their different flavours [28], [29], showing that the separate lepton flavours $L_{e,\mu,\tau}$ are indeed not conserved, though the conservation of total lepton number is still an open question. The observation of such oscillations strongly suggests that the neutrinos have different masses. Again, massless particles are generally associated with exact gauge symmetry, so in absence of any leptonic gauge symmetry, non zero lepton masses are to be expected. The conservation of lepton number is an accidental symmetry of the renormalizable terms in the SM lagrangian. However, one could easily add to the SM non-renormalizable terms that would generate neutrino masses, even without introducing a "right handed" neutrino [30].

The minimal renormalizable model of neutrino masses requires the introduction of weak singlet right handed neutrinos N [31], [32]. These will in general couple to the conventional weak doublet left handed neutrinos via Yukawa couplings Y_ν that yield Dirac masses $m_D \sim m_W$. In addition, these right handed neutrinos N can couple to themselves via Majorana masses $M \gg m_W$, since they do not require EW symmetry breaking. Combining the two types of mass terms, one obtains the seesaw mass matrix:

$$(\nu_L \quad N)\begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix}\begin{pmatrix} \nu_L \\ N \end{pmatrix}, \tag{2.1}$$

where each of the entries should be understood as a matrix in generation space.

In order to provide the two measured differences in squared neutrino masses, there must be at least two non zero masses, and hence at least two heavy singlet neutrinos N_i [31], [32]. Presumably, all three light neutrino masses are non zero, in which case there must be at least three N_i. This is indeed what happens in simple GUTs such as $SO(10)$, but some models [33] have more singlet neutrinos [34].

It is convenient to work in the field basis where the charged lepton masses $m_{l^\pm}$ and the heavy singlet neutrino mass matrix are real and diagonal. The seesaw neutrino mass matrix may then be diagonalized by a unitary transformation like that required for the quark mass matrices in the SM. In that case, it is well known that one can redefine the phases of the quark fields [35] so that the CKM matrix has just one CP violating phase [20]. We define V, the light neutrino mixing matrix first considered by *Maki, Nakagawa, Sakata* (MNS) [36],

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13}e^{-i\delta} \end{pmatrix}. \tag{2.2}$$

that represents the angles of mixing of the three lepton families. If Y_ν is the neutrino Dirac coupling matrix like in [37], we know that the leptogenesis [38] is proportional to the product $Y_\nu Y_\nu^\dagger$, which depends on just 1 CP violating phase, with two more phases appearing in higher orders, when one allows the heavy singlet neutrinos to be non degenerate [37].

In the minimal SUSY seesaw case, of the 18 parameters only 9 are accessible, a priori, in low energy neutrino physics experiments: m_{ν_1}, m_{ν_2}, m_{ν_3}, θ_{12}, θ_{23}, θ_{31}, δ, ϕ_1 and ϕ_2. The remaining 9 parameters may be characterized by an auxiliary Hermitean matrix of the following form [37], [39], [40]:

$$H = Y_\nu^\dagger D Y_\nu, \tag{2.3}$$

where D is an arbitrary real and diagonal matrix. In some definite models we can calculate H and checking with the experimental results, we can find the last 9 physical parameters.

A freely chosen model will in general violate the experimental upper limit on $\mu \to e\gamma$ but there are 2 parametrizations of H that suppress the leading contribution to this decay and correspond to two different suppressions for the tau decays. We can define two generic textures that correspond to:

$$H^1 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & d \\ 0 & d^\dagger & c \end{pmatrix} \Rightarrow \frac{B\left(\tau \to \mu\gamma\right)}{B\left(\tau \to e\gamma\right)} \ggg 1, \tag{2.4}$$

$$H^2 = \begin{pmatrix} a & 0 & d \\ 0 & b & 0 \\ d^\dagger & 0 & c \end{pmatrix} \Rightarrow \frac{B\left(\tau \to e\gamma\right)}{B\left(\tau \to \mu\gamma\right)} \ggg 1. \tag{2.5}$$

2.3 Flavor violation in charged leptonic processes

Charged Lepton Flavor Violation (CLFV) is a clear signal of new physics, it directly addresses the physics of flavor and of generations. The search for CLFV has continued from the early 1940's, when the muon was identified as a heavy partner of the electron, until today.

The most powerful searches have used the muon state or the tau state with additional contributions from the kaon system. The τ has an advantage since the GIM suppressions are smaller than in μ but given the high statistics available in muon beams, the muon searches have been the most powerful ones. The best limits have been set in the muon sector at the Paul Scherrer Institute (PSI) in Zurich, primarily $\mu \to e\gamma$ and $\mu N \to eN$ (muon to electron conversion, here N is the nucleon) along with a number of other muon processes. BABAR and BELLE have made significant measurements with tauons, and elegant kaon experiments at Brookhaven and Fermilab have produced important limits as well [41]. In the future, the flavor factories (and possibly an electron-ion collider) can be competitive. Both J-PARC and Fermilab are planning a new muon to electron conversion experiment, COMET and Mu2e respectively, to reach four orders of magnitude beyond current limits. PSI is discussing an innovative $\mu \to 3e$ search. It is possible to envisage another two orders of magnitude beyond Mu2e and COMET with upgrades to muon flux and new beams. J-PARC could build on COMET using innovative muon beam technology in PRISM/PRIME [41]. Fermilab's Project X has the potential to make intense muon and kaon beams that could push the limits of currently planned experiments another two orders of magnitude or study a signal by varying the Z of the target (where Z is the atomic number). High Z studies could illuminate the underlying physics of a signal (as explained in [42]), and must be pursued in the future despite the experimental difficulties.

2.3.1 $\mu \to e\gamma$ and $\mu \to 3e$

The first search for the process $\mu \to e\gamma$, was performed by Hincks and Pontecorvo [21], now if we use neutrino masses and mixings from the PDG [7], noting the recent observations of [43] we find, in the SM, $B\left(\mu \to e\gamma\right) = \mathcal{O}\left(10^{-54}\right)$, an effectively unmeasurable value. We can thus ignore any SM background. This is an important advantage of these searches since any signal is clear evidence for physics beyond the SM. There are two important backgrounds: the first is an intrinsic, "in-time" physics background from the inner bremsstrahlung radiative muon decay process $\mu^+ \to e^+\gamma\nu_e\bar{\nu_\mu}$, where the neutrinos carry off small momenta. The second set of backgrounds is "accidental". The search for $\mu \to e\gamma$ takes place in a sea of normal Michel decays: $\mu^+ \to e^+\nu_e\bar{\nu_\mu}$ [44].

The decay $\mu \to 3e$ is of great interest; it is sensitive to supersymmetry, LH scenarios, leptoquarks, and other physics models and is complementary to the other decay modes. The decay mode has signatures in a wide variety of beyond SM physics models [45]. This mode has been examined in Littlest Higgs scenarios [46] and in the *Simplest Little Higgs* (SLH) model [47], [48]. A new measurement of the branching ratio should strive to set a limit $< \mathcal{O}(10^{-16})$ to be competitive with existing limits and other planned measurements. Existing experiments have used stopped muons and muon decay at rest. In that case the outgoing electron and positrons can be tracked and the kinematic constraints $|\sum \overrightarrow{p}| = 0$ and $\sum E = m_e$, along with timing, can then be used to identify the rare decay. Unfortunately, this mode suffers from many of the same problems as $\mu \to e\gamma$. Because it is also a muon decay, $\mu \to 3e$ electrons are in the same momentum range as ordinary Michel decays. Therefore there are accidental backgrounds from Michel positrons that coincide with e^+e^- pairs from γ conversions or from other Michel positrons that undergo Bhabha scattering.

In the minimal SM with vanishing neutrino masses, lepton flavor is conserved separately for each generation. This is not necessarily true if new particles or new interactions beyond the SM are introduced. In SUSY models, the origin of LFV could be interactions at a very high energy scale, such as the GUT scale or the mass scale of a heavy right handed Majorana neutrino that appears in the see-saw mechanism. The effective Lagrangian for $\mu \to e\gamma$ process is given by [49]:

$$\mathcal{L}_{\mu\to e\gamma} = -\frac{4G_F}{\sqrt{2}}\left(m_\mu A_R \bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu} + m_\mu A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + h.c.\right), \tag{2.6}$$

where A_R and A_L are coupling constants that correspond to $\mu^+ \to e^+_R\gamma$ and $\mu^+ \to e^+_L\gamma$ processes, respectively. For $\mu^+ \to e^+e^+e^-$ decay and $\mu^- - e^-$ conversion, off shell photon emission also contributes. The general photonic $\mu - e$ transition amplitude is, then, written as

$$\mathcal{M}\left(\gamma\right)_\mu = \psi(p')\left[\gamma_\nu\left(F_L P_L + F_R P_R\right)\left(g^\nu_\mu - \frac{p_\mu p^\nu}{p^2}\right) + \frac{i\sigma_{\mu\nu}p^\nu}{m_\mu}\left(G_L P_L + G_R P_R\right)\right]\psi(q) \tag{2.7}$$

where q and p' are the μ^- and e^- four momenta, and $p = q - p'$ is the four momentum transfer. The form factors $F_{L;R}$, $G_{L;R}$ are functions of p^2.

The direct four fermion interactions could introduce $\mu^+ \to e^+e^+e^-$ decay and $\mu^- - e^-$ conversion. For the $\mu^+ \to e^+e^+e^-$ decay, the general four fermion couplings are given by

$$\mathcal{L}_{\mu\to 3e} = -\frac{G_F}{\sqrt{2}}\Big(g_1\bar{\mu}_R e_L \bar{e}_R e_L + g_2\bar{\mu}_L e_R \bar{e}_L e_R + g_3\bar{\mu}_R\gamma^\mu e_R \bar{e}_R\gamma_\mu e_R + g_4\bar{\mu}_L\gamma^\mu e_L \bar{e}_L\gamma_\mu e_L + \\ + g_5\bar{\mu}_R\gamma^\mu e_R \bar{e}_L\gamma_\mu e_L + +g_6\bar{\mu}_L\gamma^\mu e_L \bar{e}_R\gamma_\mu e_R + h.c.\Big). \tag{2.8}$$

The four fermion coupling constants g_n are determined by specific contributions of the models beyond the SM [50], [51].

2.3.2 Flavor violation in nuclei processes

The conversion of a muon captured by a nucleus into an electron has been one of the most powerful methods to search for CLFV. The core advantage of this mode is that the outgoing electron is monoenergetic at an energy far above the normal Michel endpoint:

$$E_{\mu e} = m_\mu - E_b - \frac{E_\mu^2}{2m_N}, \tag{2.9}$$

where $E_b \simeq \frac{\mathrm{Z}^2\alpha^2 m_\mu}{2}$ is the muonic binding energy, and the last term is from nuclear recoil energy and neglects variations of the weak interaction matrix element with energy. For Al (Z = 13), a currently favored candidate nucleus, the outgoing electron has energy $E_{\mu e} \simeq 104.96$ MeV. The measured quantity is:

$$B_{\mu e} = \frac{B\left(\mu^- N \to e^- N\right)}{B\left(\mu^- N \to \text{all captures}\right)}. \tag{2.10}$$

The normalization to all captures has calculational advantage since many details of the nuclear wavefunction cancel in the ratio. Detailed calculations have been performed by [52] and [42].

Normally one does not want to search for a single particle final state since it can be prone to accidental backgrounds but this conversion process is an exception. In this case, the electron stands out from the background: the Michel spectrum for free muon decay peaks and ends at 52.8 MeV. Typical experimental resolutions on the momentum of a 100 MeV electron are a few hundred keV or less, so there would effectively be no background if the muon were free. Hence muon-electron conversion does not suffer from accidental coincidences in the same manner as does $\mu \to e\gamma$ or $\mu \to 3e$ where one is searching for electrons near the peak of the Michel spectrum.

The $|\Delta L| = 2$ process

$$\mu^- + (\text{A,Z}) \to e^+(\text{A,Z-2}), \tag{2.11}$$

is of interest as well. As described in [53] this mode searches for $|\Delta L| = 2$ transitions with

$|\Delta L_e||\Delta L_\mu| = \pm 1$. The decay is intimately related to $K^+ \to \pi^- l^+ (l')^+$ transitions and neutrinoless double β decay [54].

The $|\Delta L| = 2$ process is in many ways similar to muon-electron conversion. A single positron is produced at

$$E_e = m_\mu - B_\mu - E_{recoil} - \Delta_{\text{Z-2}}, \tag{2.12}$$

where $\Delta_{\text{Z-2}}$ is the difference in nuclear binding energy between the final and initial nuclear states (the other terms are as in muon-electron conversion). However, this mode suffers from experimental difficulties not present in muon-electron conversion. First, since the initial and final nuclear states are different: it is not a coherent process; therefore it is not amplified by Z. Therefore the "intrinsic" rate is lower. Next, the enormous advantage of the monoenergetic electron of $\mu - e$ conversion does not apply. Since the initial and final states are different, the final nucleus can be in either the ground or excited states. If the excited state is a giant dipole resonance, the width of the final state is ≈ 20 MeV (together with a downward shift of about 20 MeV) and so the positron is far from monoenergetic.

Hydrogenic bound states of $\mu^+ e^-$ can convert to $\mu^- e^+$, violating individual electron and muon number by two units. This process is analogous to $K^0 \bar{K}^0$ mixing. One typically states the result of a search as an upper limit on an effective coupling analogous to G_F, where the exchange is mediated by such particles as a doubly charged Higgs, dileptonic gauge bosons, a heavy Majorana neutrino, or a supersymmetric R-parity violating τ-sneutrino [55] [2].

2.4 Flavor violation in the tau physics framework

The EW gauge structure has been successfully tested at the 0.1% to 1% level, confirming the SM framework [56]. The hadronic τ decays are important channels for studying strong interaction effects at low energies [57]. The τ is the only known lepton massive enough to decay into hadrons and with its semileptonic decays we can study the hadronic weak currents. Also the QCD coupling, $|V_{us}|$ and the strange quark mass have been studied with τ decay data. Actually the B Factories have explored LFV τ decays until $10^{-7} - 10^{-8}$ sensitivities, see Figure 2.2.

The τ is an interesting window in the "Three frontiers of Particle physics" [59] as shown in Figure 2.3. Physics with τ leptons in the final state probes decays of the Higgs, SUSY and new exotic particles. Physics with decays of τ leptons includes searches of LFV and CP-violation, tests of lepton universality and measurements of α_s, $g-2$ (for a complete review of tau physics see [60]).

The minimally extended SM, predicts charged LFV at a level that is suppressed by a factor of $\left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2$ at experimentally unobservable rates (here Δm^2 is the difference between the squared masses of neutrinos with different flavors, so $\left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \sim 10^{-48}$). Observation of LFV in the charged lepton sector would completely change our understanding of Nature, and

[2]For a more complete discussion of the muonic atoms see [54].

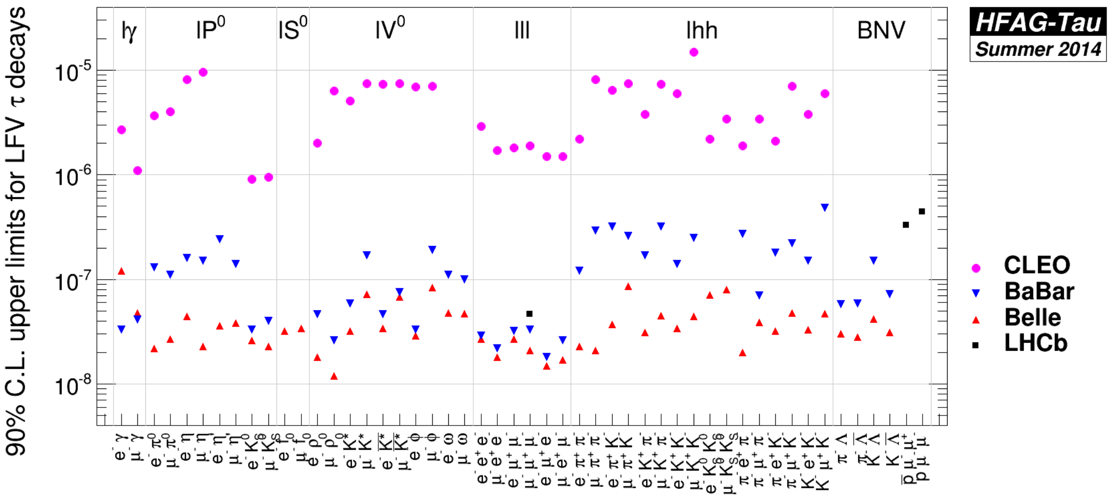


Figure 2.2: Tau LFV branching fraction upper limits summary plot [58].

herald a new era of discovery in particle physics. Such searches are recognized as discovery potential experiments in the near future. A complete review of the theoretical branchings of the tau decays channels can be found in [61]. The Belle II experiment at KEK accelerator is expected to answer fundamental questions relating to some of the unsolved mysteries of particle physics, such as the nature of the dark matter candidate, the origin of the CP violation and large baryon number asymmetry in the early universe, by searching for LFV as a signature of new physics, which can describe the Nature beyond the SM. Lepton flavour conservation is unique, because it is not associated with any conserved current. Most new physics models naturally include lepton flavour violating processes with predicted rates at the 10^{-9} level [62], which are slightly lower than the current experimental limits. The tau subgroup of Heavy Flavour Averaging Group has compiled upper limits up to the level of 10^{-8} for almost fifty possible LFV decays of the τ lepton [58]. Now is a very interesting era in the searches for LFV, as the current limits will improve by an order of magnitude in the next decade. With 50 ab^{-1} of data by 2020, the Belle II experiment is expected to be able to probe LFV in τ decays at the level of 10^{-9} [63] [64]. The observation of a new particle with a mass around 125 GeV consistent with the properties of a SM Higgs boson at the LHC experiments [65], [66] marks a beginning of the new era in particle physics. Higgs bosons decaying into τ leptons provide direct probe into the Yukawa coupling of the fermions, responsible for the origin of mass of all the quarks and leptons. In this mass region, decays to τ leptons have one of the largest branching ratios for both the neutral and charged Higgs bosons [67].

Using the data of pp collisions at 7 and 8 TeV, ATLAS [68], [69] and CMS [70], [71], experiments observe SM $H \to \tau^+\tau^-$ decays with significances of 1.1 σ and 1.5 σ, respectively. Searches for *minimal supersymmetric model* (MSSM) neutral Higgs and charged Higgs have also been performed by the ATLAS [72], [73], [74] and CMS [75], [76] experiments.

The τ leptons play a key role in understanding the true nature of the SM Higgs boson.

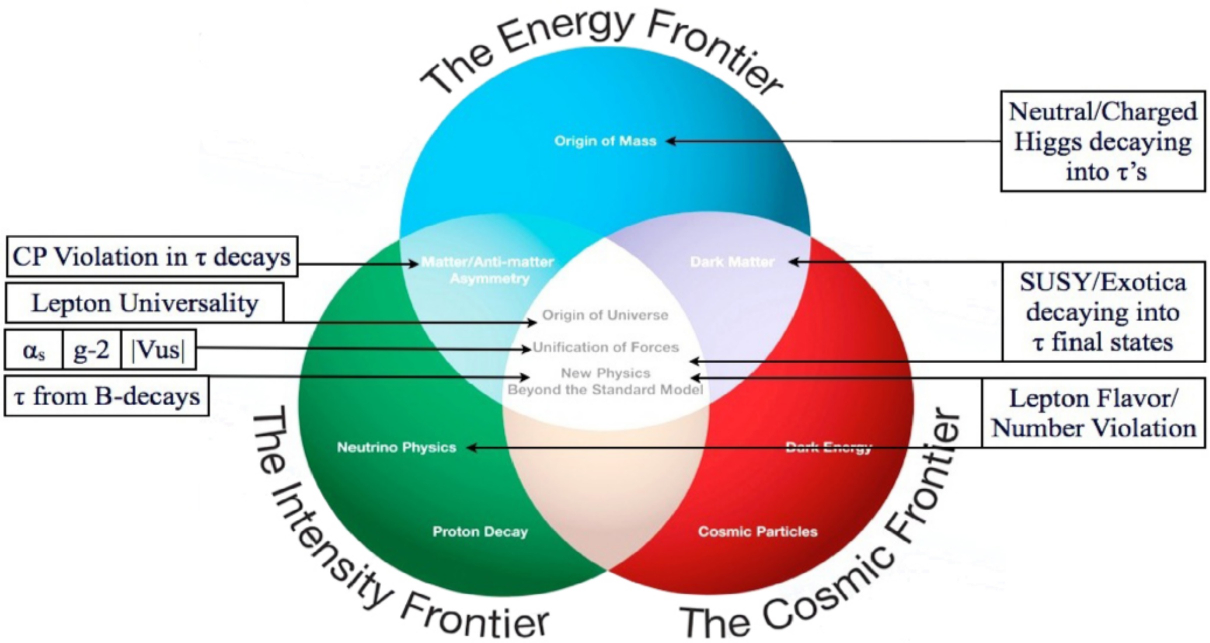


Figure 2.3: Impact of τ leptons on the frontiers of particle physics [59].

The di-boson coupling of a pure CP odd Higgs state being zero at tree level, the coupling of Higgs boson to fermions provides an unique probe into the CP nature of the Higgs, since the CP even and CP odd components in fermion pair decays can have the same magnitude [77]. Spin effects in the τ lepton pair production can help us to study the true nature of this new state at 125 GeV, which in the SM is expected to receive equal contributions from the left and right handed polarization states [78]. Measurements of the ratio

$$\frac{B\left(B \to D^{(*)}\tau\nu\right)}{B\left(B \to D^{(*)}l\nu\right)}, \tag{2.13}$$

where $l = e, \mu$, are sensitive to new physics contributions from charged Higgs boson in type-II *two Higgs doublet models* (2HDMs). In the SM, the τ polarization in $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ receives 70 (30)% contribution from left (right) handed states, while, in the 2HDMs, the contribution is purely from the right handed state. The excess over the SM prediction as observed in the BaBar data puts stringent constraints on such 2HDMs [79].

Chapter 3

Little Higgs models

3.1 Background

Many models exist where EW symmetry breaking is triggered by a light composite Higgs, which emerges from a strongly interacting sector as a pseudo-Goldstone boson. Two parameters broadly characterize these models:

- m_ρ, the mass scale of the new resonances,
- g_ρ, their coupling.

An effective low energy Lagrangian approach proves to be useful for LHC and ILC phenomenology below the scale m_ρ. One of the main goals of the LHC is to unveil the mechanism of EW symmetry breaking. A crucial issue that experiments should be able to settle is whether the dynamics responsible for symmetry breaking is weakly or strongly coupled. LEP1 has provided us with convincing indications in favor of weakly coupled dynamics. Indeed, the good agreement of EW precision measurements (or EW precision tests, EWPT) with the SM predictions showed that the new dynamics cannot significantly influence the properties of the Z boson, ruling out, for instance, the simplest forms of *Technicolor* (TC) models, which were viewed as the prototypes of a strongly interacting EW sector. Moreover, the best agreement between experiments and theory was obtained for a light Higgs, corresponding to a weakly coupled Higgs self interaction.

Finally, SUSY, which appeared to be the most favourable realization of a light Higgs with mass stabilized under quantum corrections, received a further boost by the LEP1 measurements of gauge coupling constants, found to be in accord with supersymmetric unification. The situation swayed back after the LEP2 results. The lack of discovery of a Higgs boson below 114 GeV or of any new states, has forced SUSY into fine tuning territory, partially undermining its original motivation. And now without any signal of SUSY from LHC, we need to consider new ways to explain the physical world. Moreover, new theoretical developments, mostly influenced by extra dimensions and by the connection between strongly interacting gauge theories and gravity on warped geometries, have led the way to the construction of new models of EW symmetry breaking [80], [81], [82], [83], [84] , [85].

Still, the complete replacement of the Higgs sector with strongly-interacting dynamics seemed hard to implement, mostly because of constraints from EW data. A more promising approach is to keep the Higgs boson as an effective field arising from new dynamics [86], [87] which becomes strong at a scale not much larger than the Fermi scale. There have been various attempts to realize such scenario, including the *Little Higgs* (LH) [81], Holographic Higgs as *Nambu-Goldstone bosons* (NGBs) [84] , [85] or not [88], and other variations.

3.2 Strongly Interacting Light Higgs couplings

The massive nature of the weak gauge bosons requires new degrees of freedom and/or new dynamics around the TeV scale to act as an ultraviolet moderator and ensure a proper decoupling at high energy of the longitudinal polarizations $W_L^{\pm}$, Z_L. It is remarkable that a simple elementary weak doublet not only provides the three NGBs that will become the spin-1 longitudinal degrees of freedom but also contains an extra physical scalar field, the Higgs boson, that screens the gauge boson non abelian self interaction contributions to scattering amplitudes and hence offers a consistent description of massive spin-1 gauge bosons. EWPT accumulated during the last 20 years, together with the absence of large FCNC, suggest that violent departures from this minimal Higgs mechanism are unlikely, and rather call for smooth deviations, at least at low energy. This provides a plausible motivation for considering a light Higgs boson emerging as a pseudo-NGB from a strongly coupled sector, the so called *Strongly Interacting Light Higgs* (SILH) scenario [89].

The effective SILH Lagrangian should be seen as an expansion in $\frac{v}{f}$ where $v = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx$ 246 GeV and f is the typical scale of the NGBs of the strong sector. Therefore, it can be used to describe composite Higgs models in the vicinity of the SM limit, $\frac{v}{f} \to 0$.

It should be stressed that the couplings of the Higgs boson in the SILH scenario are not the most general ones that would be allowed by the general principles of quantum field theory and the local and global symmetries of the models considered: for instance, the important anomalous couplings will have the same Lorentz structure as the SM ones. In principle, some couplings with a different Lorentz structure could also be expected, but these ones would be generated only via the exchange of heavy resonances of the strong sector and not directly by the strong dynamics of the NGBs, therefore they would be parametrically suppressed, at least by a factor $\left(\frac{f}{m_\rho}\right)^2$ ($m_\rho \gtrsim 2.5\ TeV$ is the typical mass scale of these resonances [89]).

The structure of the theories we want to consider is the following: in addition to the vector bosons and fermions of the SM, there exists a new sector responsible for EW symmetry breaking. Collectively indicating by g_{SM} the SM gauge and Yukawa couplings (basically the weak gauge coupling and the top quark Yukawa), we assume $g_{SM} \lesssim g_\rho \lesssim 4\pi$. The upper bound on g_ρ ensures that the loop expansion parameter $\sim \left(\frac{g_\rho}{4\pi}\right)^2$ is less than unity, while the limit $g_\rho \sim 4\pi$ corresponds to a maximally strongly coupled theory in the spirit of *naive* *dimensional analysis* (NDA) [90].

The Higgs multiplet is assumed to belong to the strong sector. The SM vector bosons and fermions are weakly coupled to the strong sector by means of the $SU(3) \times SU(2) \times U(1)$ gauge couplings and by means of proto-Yukawa interactions, namely interactions that in the low energy effective field theory will give rise to the SM Yukawas.

A second crucial assumption we are going to make is that in the limit $g_{SM} = 0$, $g_\rho \neq 0$ the Higgs doublet is an exact NGB. Two minimal possibilities in which the complex Higgs doublet spans the whole coset space are $SU(3) \to SU(2) \times U(1)$ and the custodially symmetric $SO(5) \to SO(4)$.

A mass term for the Higgs is generated at 1-loop and if the new dynamics is addressing the hierarchy problem, it should soften the sensitivity of the Higgs mass to short distances, that is to say below $\frac{1}{m_\rho}$. In interesting models, the Higgs mass parameter is thus expected to scale like $\frac{\alpha_{SM}}{4\pi} m_\rho$. Observation at the LHC of the new states with mass m_ρ will be the key signature of the various realizations of SILH.

The σ-model scale f is related to g_ρ and m_ρ by the equation

$$m_\rho = g_\rho f. \tag{3.1}$$

Fully strongly interacting theories, like QCD, correspond to $g_\rho \sim 4\pi$. In that case equation (3.1) expresses the usual NDA relation between the pion decay constant f and the mass scale of the QCD states. On the other hand, the theories we are considering represent a weakly coupled deviation of this QCD-like pattern. For $g_\rho < 4\pi$, the pure low energy effective σ-model description breaks down above a scale m_ρ, which is parametrically lower than the scale $\Lambda = 4\pi f$ where the σ-model would become strongly coupled. The coupling g_ρ precisely measures how strong the coupling of the σ-model can become before it is replaced by a more fundamental description.

A more interesting possibility arises when the strong sector is composite so that the corrections to the Higgs mass are screened above the “hadron” mass scale m_ρ. Moreover if the underlying theory is a large-N gauge theory, we also expect the hadrons to interact with a coupling

$$g_\rho = \frac{4\pi}{\sqrt{N}}, \tag{3.2}$$

which becomes weaker at large N. This is also basically the picture that holds in extradimensional constructions where the SM is represented by a weakly coupled boundary dynamics while the Higgs sector is part of a more strongly coupled bulk dynamics. Examples of this type are the Holographic Goldstones [85]. In these extradimensional realizations, the Kaluza-Klein mass and coupling play respectively the role of m_ρ and g_ρ, while the number of weakly coupled Kaluza-Klein modes below the cut-off can be basically interpreted as N.

Other models that basically fall into our class are LH models [80], where the scale m_ρ is represented by the masses of the partners of top quark, EW vector bosons and Higgs, the states that soften the quadratic correction to the Higgs mass. In LH models there is more parameter freedom, and the coupling g_ρ is more accurately described by a set of couplings that can range from weak ($\sim g_{SM}$) to strong ($\gg g_{SM}$). The effective Lagrangian describing a SILH involves higher dimensional operators.

There are two classes of higher dimensional operators:

- Those that are genuinely sensitive to the new strong force and will affect qualitatively the physics of the Higgs boson,
- Those that are sensitive to the spectrum of the resonances only and will simply act as form factors.

Simple rules control the size of these different operators and the effective Lagrangian generically takes the form [89]:

$$\begin{aligned}\mathcal{L}_{SILH} =& \frac{c_H}{2f^2}\left(\partial_\mu \mid H \mid^2\right)^2 + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 - \frac{c_6\lambda}{f^2} \mid H \mid^6 + \left(\frac{c_y y_f}{f^2} \mid H \mid^2 \bar{f}_L H f_R + h.c.\right) \\ &+ \frac{ic_W g}{2m_\rho^2}\left(H^\dagger \sigma^i \overleftrightarrow{D}^\mu H\right)\left(D^\nu W_{\mu\nu}\right)^i + \frac{ic_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\left(\partial^\nu B_{\mu\nu}\right) + \ldots\end{aligned} \tag{3.3}$$

where g, g' are the SM EW gauge couplings, λ is the SM Higgs quartic coupling and y_f is the SM Yukawa coupling to the fermions $f_{L,R}$. All the coefficients, c_H, c_T ..., appearing in (3.3) are expected to be of order one unless protected by some symmetry. For instance, in every model in which the strong sector preserves custodial symmetry, the coefficient c_T vanishes and only three coefficients, c_H, c_y and c_6, give sizable contributions to the Higgs (self-)couplings. The operator c_H gives a correction to the Higgs kinetic term which can be brought back to its canonical form at the price of a proper rescaling of the Higgs field, inducing a universal shift of the Higgs couplings by a factor $\frac{1-c_H x}{2}$. For the fermions, this universal shift adds up to the modification of the Yukawa interactions.

In this manner all the dominant corrections, the ones controlled by the strong operators, preserve the Lorentz structure of the SM interactions, while the form factor operators will also introduce couplings with a different Lorentz structure.

3.3 Little Higgs Models

We would like to believe that the SM, which does not include gravity, must break down at the Planck energy scale where the gravitational interactions become comparable in strength to other forces but there are serious theoretical reasons to believe that the SM breaks down much earlier, at the TeV scale. Several theoretical extensions of the SM, attempting to provide a more satisfactory picture of EW symmetry breaking and conjecture the structure of the theory at the TeV scale, have been proposed in the last three decades. Well known examples include SUSY models, such as the MSSM, and TCs which do not contain a Higgs boson, relying instead on strong dynamics to achieve EW symmetry breaking.

LH models incorporate a light composite Higgs boson and remain perturbative until a scale of order 10 TeV, as required by precision EW data. If a LH model is realized in nature, the predicted new particles should be observable at the LHC.

In this scenario, f is the energy scale where the composite nature of the Higgs becomes important.

Unfortunately, precision EW data rule out new strong interactions at scales below about 10 TeV [91]. To implement the composite Higgs without fine tuning, an additional mechanism is required to stabilize the "little hierarchy" between the Higgs mass and the strong interaction scale. In analogy with the pions of QCD, one can attempt to explain the lightness of the Higgs by interpreting it as a NGB corresponding to a spontaneously broken global symmetry of the new strongly interacting sector. However, gauge and Yukawa couplings of the Higgs, as well as its self couplings, must violate the global symmetry explicitly, since an exact NGB only has derivative interactions. Quantum effects involving the symmetry breaking interactions generate a potential, including a mass term, for the Higgs. The NGB nature of the Higgs is completely obliterated by quantum effects, and cannot be used to stabilize the little hierarchy.

A solution to this difficulty has been proposed by Arkani-Hamed, Cohen and Georgi [80]. They argued that the gauge and Yukawa interactions of the Higgs can be incorporated in such a way that a quadratically divergent one-loop contribution to the Higgs mass is not generated. The cancellation of this contribution occurs as a consequence of the special collective pattern in which the gauge and Yukawa couplings break the global symmetries. The remaining quantum loop contributions to m_h are much smaller, and no fine tuning is required to keep the Higgs sufficiently light if the strong coupling scale is of order 10 TeV: the little hierarchy is stabilized.

All the LH models contain new particles with expected masses around the 1 TeV scale. The interactions of these particles can be described within perturbation theory, and detailed predictions of their properties can be made. These states cancel the one loop quadratically divergent contributions to the Higgs mass from SM loops. They provide distinct signatures that can be searched for at future colliders, as well as induce calculable, and often sizable, corrections to precision EW observables [92].

At an energy scale of order 10 TeV, the LH description of physics becomes strongly coupled, and the LH model needs to be replaced by a more fundamental theory, its *ultraviolet* (UV) completion. The UV completion could be, for example, a QCD-like gauge theory with a "confinement" scale around 10 TeV. The idea of little Higgs models is to break the global symmetry in such a way that the mass of the Higgs is parametrically two loop factors smaller than $\Lambda \sim 10$ TeV instead of one loop. The idea is that any one global symmetry breaking coupling by itself leaves enough of the global symmetry intact so that the Higgs is still an exact NGB.

In order to work, the global symmetry group needs to be quite large, which implies the presence of extra particles typically at scale $f \sim 1$ TeV. Those particles are responsible for canceling the one loop quadratic divergences to the Higgs mass. Two ingredients are needed to build a LH model [93]:

1. *The spontaneous breaking* of a global symmetry. The mechanism by which the symmetry breaking happens is not specified. It could be strongly coupled physics at 10 TeV, or weakly coupled physics at 1 TeV. The breaking produces a set of NGBs, among which is the Higgs, and at low energies these link fields (like pions) are described by a non-linear σ-model field which is written as an exponential of the broken generators T_a of the global symmetry:
$$\Sigma = e^{i\pi^a T^a}. \tag{3.4}$$

2. *The collective symmetry breaking principle* imposes:

   - Extension of the gauge group. To achieve this, one needs to gauge a group larger than the SM gauge group, which breaks to the SM at the scale f. There will then be extra gauge bosons at the scale f that cancel the quadratically divergent contributions of the SM NGBs to the Higgs mass. Also this extension requires extra heavy fermions that cancel the quadratically divergent contribution of the SM top quark loop to the Higgs mass.
   - The Higgs doublet transforms under some extended global symmetry, which is not completely broken by any single interacting term.

3.3.1 Theory space models

Theory space models were the first little Higgs models and were inspired by the deconstruction of extra dimensional models where the Higgs is the fifth component of a gauge field [80], [81], [94], [95], [96], [97], [98], [99]. Theory spaces are sets of sites and links, also called moose diagrams. Sites represent gauge groups, and links are $N \times N$ non linear σ-model fields transforming as bifundamentals under the gauge groups associated with the sites they touch (see Figure 3.1).

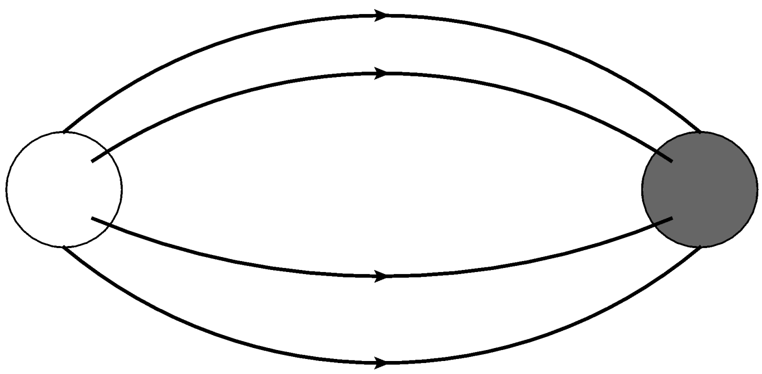


Figure 3.1: Moose diagram for the minimal moose. The open site corresponds to an $SU(3)$ gauge group, while the filled site corresponds to an $SU(2) \times U(1)$ gauge group.

Table 3.1: Little Higgs models classified by their type [93].

	Model	Global Group	Gauge Group	Features
Theory Space	Minimal moose	$SU(3)^8 \to SU(3)^4$	$SU(3) \times SU(2) \times U(1)$	Can contain extra light triplet and singlet scalars.
	Minimal moose with $SU(2)_C$	$S0(5)^8 \to S0(5)^4$	$SO(5) \times SU(2) \times U(1)$	Less constrained from EWPT.
	Moose with T-parity	$S0(5)^{10} \to S0(5)^5$	$(SU(2) \times U(1))^3$	Very few constraints from EWPT, large spectrum, complicated plaquettes.
Product gauge group models	Littlest Higgs	$SU(5) \to S0(5)$	$(SU(2) \times U(1))^2$	Minimal field content.
	$SU(6) \to Sp(6)$ model	$SU(6) \to Sp(6)$	$(SU(2) \times U(1))^2$	Small field content, contains a heavy vector-like quark doublet.
	Littlest Higgs with $SU(2)_C$	$S0(9) \to (SO(5) \times SO(4))$	$SU(2)^3 \times U(1)$	Less constraints from EWPT.
	Littlest Higgs with T-parity	$SU(5) \to SO(5)$	$(SU(2) \times U(1))^2$	Minimal field content, very few constraints from EWPT.
Simple gauge group models	$SU(3)$ simple group	$(SU(3) \times U(1))^2 \to (SU(2) \times U(1))^2$	$SU(3) \times U(1)$	Two Higgs multiplets. One physical Higgs doublet.
	$SU(4)$ simple group	$(SU(4) \times U(1))^4 \to (SU(3) \times U(1))^4$	$SU(4) \times U(1)$	Two Higgs doublets, large quartic.
	$SU(9) \to SU(8)$ simple group	$SU(9) \to SU(8)$	$SU(3) \times U(1)$	Two Higgs doublets, large quartic.

Each link breaks a global $SU(N)^2$ symmetry to the diagonal $SU(N)$. This results in the presence of NGBs. The gauge symmetry explicitly breaks the large global symmetry group. However, no single gauge coupling alone breaks enough symmetry to give the NGBs a mass.

In the Figure 3.1 we show the theory space of the “minimal moose”, the most simple little Higgs of this type. The kinetic term for the link fields is given by:

$$\sum_{j=1}^{4} \mid D_\mu \Sigma_j \mid^2, \tag{3.5}$$

with

$$D_\mu \Sigma_j = \partial_\mu \Sigma_j + i A_1 \Sigma_j - i \Sigma_j A_2, \tag{3.6}$$

where T^a are the generators of $SU(3)$. The global symmetry group is $SU(3)^4_L \times SU(3)^4_R$ broken down to the diagonal $SU(3)^4_D$, resulting in $4 \times 8 = 32$ NGBs. The spontaneous breaking of the global group also breaks the $SU(2)_L \times U(1)_L \times SU(3)_R$ gauge symmetry down to the diagonal $SU(2)_L \times U(1)_Y$ subgroup, which eats 8 NGBs leaving 24 pseudo-NGBs. The gauge group also explicitly breaks the global symmetry and the 24 pseudo-NGBs will get a potential generated by gauge interactions. Note that if only the $SU(3)$ gauge coupling is nonzero, there is an exact $SU(3)^4_L \times SU(3)_R$ global symmetry broken to the diagonal $SU(3)$. In this case there would be 32 NGBs, 8 of which would be eaten, leaving 24 exact NGBs. This tells us that we need the gauge couplings of both sites to generate any potential for the 24 pseudo-NGBs. The potential is in fact parametrically of the form:

$$\frac{g_1^2 g_2^2}{16\pi^2} \left(c_1 f^2 \Phi^2 + c_2 \Phi^4 + \dots \right), \tag{3.7}$$

where c_1 and c_2 are coefficients of order one and Φ represents the various pseudo-NGBs. We need to generate a large quartic coupling for ϕ and this can be achieved with “plaquette” interactions of the form

$$\lambda Tr \left[\Sigma_1 \Sigma_2^\dagger \Sigma_3 \Sigma_4^\dagger \right]. \tag{3.8}$$

These interactions do not respect the collective breaking principle for all the pseudo-NGBs, and they therefore give mass of order f to 8 of them. But still, each of these interactions respects enough global symmetries to protect the mass of the remaining 16 pseudo-NGBs, which consist of two Higgs doublets, two triplets and two singlets. The important feature of the plaquette terms is that, at tree level, they give no mass to the Higgs doublets, but they do give them an $\mathcal{O}(1)$ quartic coupling. The variety of little Higgs models that can be built from theory space is infinite.

3.3.2 Product gauge group models

The majority of LH models are product group models. The product group models have the following generic features:

- All the models contain a set of $SU(2)$ NGBs at the TeV scale, obtained from the diagonal breaking of two or more gauge groups down to $SU(2)_L$, and thus contain free parameters in the gauge sector from the independent gauge couplings.
- Since the collective symmetry breaking in the gauge sector is achieved by multiple gauged subgroups of the global symmetry, models can be built in which the SM Higgs doublet is embedded within a single non linear σ-model field; many product group models make this simple choice.
- The fermion sector of this class of models can usually be chosen to be very simple, involving only a single new vector-like quark.

The simplest incarnation of the product group class is the so called Littlest Higgs model [81]. In these models, the gauge groups are subgroups of a single global symmetry. The global group structure is $SU(5) \to SO(5)$. This generates $24 - 10 = 14$ NGBs that can be parametrized by the following nonlinear σ-model:

$$\Sigma = e^{i\Pi}\Sigma_0 e^{i\Pi^T} \qquad ; \qquad \Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbb{1} \\ 0 & 1 & 0 \\ \mathbb{1} & 0 & 0 \end{pmatrix}, \tag{3.9}$$

$$\Pi = \begin{pmatrix} 0 & \eta & \phi \\ \eta^\dagger & 0 & \eta^T \\ \phi^\dagger & \eta^* & 0 \end{pmatrix}, \tag{3.10}$$

where $\mathbb{1}$ is the 2×2 identity matrix, ϕ is an $SU(2)$ triplet and η is an $SU(2)$ doublet.

Two $SU(2) \times U(1)$ subgroups of $SU(5)$ are gauged with the following generators:

$$Q_{1a} = \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad ; \qquad Q_{1a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_a^* \end{pmatrix}, \tag{3.11}$$

$$Y_1 = \frac{1}{10}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \qquad ; \qquad Y_2 = \frac{1}{10}\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}. \tag{3.12}$$

The diagonal subgroup belongs to $SO(5)$ and is unbroken by the Σ vev. The gauging explicitly breaks the $SU(5)$ and generates a potential for the NGBs. Out of the 14 original NGBs, 4 are eaten by the gauge bosons that become massive. There are 10 left. If only one

$SU(2) \times U(1)$ gauge coupling constant is turned on, the global symmetry breaking pattern is $(SU(3) \times SU(2)) \rightarrow (SO(3) \times U(1))$. This leaves 7 exact NGBs, three of which are eaten, and four of which remain massless. These massless NGBs are the Higgs bosons, whose mass is protected by collective symmetry breaking.

An interesting feature of the littlest Higgs models (see Table 3.1) is that gauge boson loops generate the following operators:

$$f^4 \left(c_1 g_1^2 Tr \Sigma Q_{1a} \Sigma^* Q_{1a}^* + c_2 g_2^2 Tr \Sigma Q_{2a} \Sigma^* Q_{2a}^* \right). \tag{3.13}$$

These operators give a mass of order f for the EW triplet Φ, and generate a quartic coupling of order one for the Higgs. Therefore we do not need to add a plaquette term "by hand". There are many possible variations on the Littlest Higgs theme.

3.3.3 Simple gauge group models

The simple group models share two features that distinguish them from the product group models:

- All simple group models contain an $SU(N) \times U(1)$ gauge symmetry that is broken down to $SU(2)_L \times U(1)_Y$, yielding a set of TeV scale gauge bosons. The two gauge couplings of the $SU(N) \times U(1)$ are fixed in terms of the two SM $SU(2)_L \times U(1)_Y$ gauge couplings, leaving no free parameters in the gauge sector once the symmetry breaking scale is fixed.

- In order to implement the collective symmetry breaking, simple group models require at least two σ-model multiplets [93]. The SM Higgs doublet is embedded as a linear combination of the NGBs from these multiplets. This introduces at least one additional model parameter, which can be chosen as the ratio of the vevs of the σ-model multiplets. Moreover, due to the enlarged $SU(N)$ gauge symmetry, all SM fermion representations have to be extended to transform as fundamental (or antifundamental) representations of $SU(N)$, giving rise to additional heavy fermions in all three generations. The existence of multiple σ-model multiplets generically results in a more complicated structure for the fermion couplings to scalars. On the other hand, the existence of heavy fermion states in all three generations as required by the enlarged gauge symmetry provides extra experimental observables that in principle allow one to disentangle this more complicated structure. The "simplest" structure of the simple group class is the $SU(3)$ simple group model [100], that we briefly review now and we will develop in the Section (3.4).

In the previous models we could obtain the desired low energy gauge couplings in a way that respects the collective symmetry breaking by using a product of gauge groups. When one gauge coupling of the product was set to zero, the Higgs was exactly massless and that is how collective symmetry breaking was achieved. Each field alone "thinks" that it is the one breaking the symmetry and getting absorbed by the massive gauge bosons, and the couplings of both fields are needed to generate a potential for the uneaten pseudo-NGBs. The simplest model [100] of this type is an $SU(3) \times U(1)$ gauge theory broken down to $SU(2) \times U(1)$ by the vev of two different $SU(3)$ fundamentals:

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} \qquad ; \qquad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} . \tag{3.14}$$

The pseudo-NGBs can be parameterized by fluctuations around the vacuum:

$$\Phi_1(x) = e^{\frac{iT^a \pi_1^a}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} \qquad ; \qquad \Phi_2(x) = e^{\frac{iT^a \pi_2^a}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} . \tag{3.15}$$

The gauge couplings explicitly break the global symmetry, but couplings to both Φ_1 and Φ_2 are needed to generate a potential for the pseudo-NGBs. The Higgs mass is then suppressed relative to the f scale; however, the Higgs quartic coupling could be large.

An extra "plaquette" operator that breaks the $(SU(3) \times U(1))^2$ global symmetry must be added to give a large enough quartic coupling. Alternately, a large quartic can be produced if the theory is enlarged to an $SU(4)$ gauge theory with four fundamentals breaking it to $SU(2)$ [101].

Another model, consisting of a $SU(9) \to SU(8)$ global symmetry with $SU(3) \times U(1)$ gauged, contains two light Higgs doublets and also generates a large enough quartic [102].

Before we only needed an extra fermion in the top sector to cancel the one loop quadratic divergence of the SM top quark. Here, because of the extended gauge group, extra fermions for all generations of the SM are needed. Finally, because of the two vevs f_1, f_2, there is no simple relationship between the mass of the heavy vector bosons and the mass of the heavy top. As we will see next, this helps in avoiding constraints from EWPT.

3.3.4 Constraints

In the Littlest Higgs model, exchange of the new gauge bosons, as well as a vev for the heavy triplet, can all cause trouble. In product group models and theory space models, the couplings of the new gauge bosons can be written solely in terms of SM currents. In general, none of the dangerous couplings that give large contributions to these parameters are tied to the couplings that ensure the cancellation of the Higgs mass quadratic divergences, therefore it is in general possible to find regions of parameter space where the constraints are satisfied with reasonable fine tuning in the Higgs mass ($\sim 10\%$). However, the allowed region is in general quite small.

Since the top loop quadratic divergence is the largest, the heavy top quark partner cannot be too heavy without reintroducing fine-tuning, and this tends to push the models into a small corner of parameter space. In particular, in simple gauge group models the EWPT typically give strong constraints on $\sqrt{f_1^2 + f_2^2}$, while the heavy top quark partner mass is not directly tied to this combination, and can be made relatively light.

Several models have been built with the specific intention of satisfying more naturally the constraints of EWPT. T-parity tries to avoid tree-level exchange of the heavy states by making them odd under a new parity, while all the SM particles are even. This has the additional advantage of ensuring the presence of a stable heavy particle which could play the role of dark matter. The main drawback of this approach is that it requires the addition of one new TeV scale fermion for each of the fermions of the Standard Model. This in turns raises flavor questions similar to those in the MSSM. T-parity has been introduced in theory space models, where the parity has a nice geometric interpretation, and in product group models, but not in simple group models.

One can also analyze the scattering of all possible pairs of NGBs in LH models to find where unitarity is violated. The violation of unitarity at some scale indicates that the theory is not valid above that scale, or that perturbation theory has broken down. Due to the large number of NGBs in LH models, this unitarity analysis generically predicts an upper cutoff $\Lambda \simeq (3-4)\, f$ depending on the model, which is somewhat less than the $\Lambda \simeq (10-30)$ TeV usually quoted using NDA. There are also constraints on the scale f, for large $f \gg 1$ TeV, the Higgs mass is pulled up toward the scale f, destroying the desired hierarchy.

Therefore the desired hierarchy $v \ll f \ll \Lambda$ can be preserved, but the separation between each of these scales may only be a factor between 3 and 5 instead of 4π.

Because LH models have a cutoff at a relatively low scale $\Lambda \simeq 10$ TeV, the issues of dark matter, neutrino masses, and the baryon asymmetry of the universe can be deferred to energy scales above the cutoff. However, there have been some attempts to incorporate this physics within LH models themselves. Dark matter appears naturally as the lightest T-odd particle in LH models with T-parity [103]. Even without T-parity, theory space models often contain discrete symmetries, some part of which can remain unbroken even after EW symmetry breaking; the dark matter could then consist of a nonlinear σ-model field made stable by this accidental exact global symmetry [104].

There have been two main approaches to neutrino mass generation in LH models:

- Some models (such as the Littlest Higgs) contain a scalar triplet with a nonzero vev. This triplet can be used to generate neutrino Majorana masses through a lepton number violating coupling to two left-handed SM neutrinos [105].

- Simple group models naturally contain a pair of extra SM right gauge singlets N, N^C at the f scale due to the expansion of the lepton doublets into fundamentals of the enlarged gauge group. If lepton number is broken at a small scale $M \sim$ KeV, generating a small Majorana mass for N^C, then the SM neutrinos can get a radiatively generated Majorana mass [106] of the correct size through their mixing with N, without requiring extremely tiny Yukawa couplings.

3.4 Simplest Little Higgs model

The SM Higgs mass has quadratically quantum corrections which destabilize EW symmetry breaking, this means that we need a fine tuning to calculate theoretically the Higgs mass.

Because the W and Z masses are around 100 GeV, any natural extension of the SM must contain new physics at or below ~ 1 TeV in order to produce the physical standard masses [1].

Cheng and Low [108] pointed out that LH models can be constructed with a symmetry (T-parity) that forbids all tree level contributions from the new physics to EW observables while still allowing the loops necessary to calculate the Higgs mass. T parity is analogous to R parity in SUSY models.

The $SU(3)$ LH proposed by Kaplan and Schmaltz [101] allows a natural solution of the little hierarchy problem without T-parity. The model has regions of parameter space for which TeV scale particles only couple very weakly to SM fields in tree level interactions. This allows them to hide from EWPT while still conserve the Higgs mass. New fermion and gauge boson masses as low as 1 TeV could be consistent with the data.

In the original $SU(3)$ model [100] anomalies are not canceled in the low energy theory, thus requiring new structure at the cut-off. An anomaly free choice of fermion representations [109], which requires no spectators, provides a better fit to precision EWPT.

But first of all we have to explain the model, that consists in the extension of the SM $SU(2)_L \times U(1)_Y$ gauge group to $SU(3)_L \times U(1)_X$ in a minimal way. This entails enlarging $SU(2)$ doublets of the SM to $SU(3)$ triplets, adding $SU(3)$ gauge bosons, and writing $SU(3)$ invariant interactions which reproduce all the SM couplings when restricted to SM fields. Explicitly, a SM generation is embedded in $(SU(3)_C, SU(3)_L)_{U(1)_X}$ representations

$$\begin{aligned} \mathcal{Q} = (\mathit{3}, \mathit{3})_{\frac{1}{3}}, \qquad & L = (\mathit{1}, \mathit{3})_{-\frac{1}{3}}, \\ d_R = (\mathit{3}, \mathit{1})_{-\frac{1}{3}}, \qquad & l_R = (\mathit{1}, \mathit{1})_{-1}, \\ u_R = (\mathit{3}, \mathit{1})_{\frac{2}{3}}, \qquad U_R = (\mathit{3}, \mathit{1})_{\frac{2}{3}}, \qquad & N_R = (\mathit{1}, \mathit{1})_0, \end{aligned} \tag{3.16}$$

where there are two up quark singlet fields, one is the SM right handed up-type quark, the other obtains a large mass with the third components of the triplet $\mathcal{Q}$. Similarly, the singlet N_R is the Dirac partner of the third component of L.

The symmetry breaking, $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$, is achieved with aligned vevs for two complex triplet scalar fields

$$\Phi_1; \Phi_2 = (\mathit{1}, \mathit{3})_{-\frac{1}{3}} . \tag{3.17}$$

The gauge interactions of the model are uniquely determined by gauge invariance.

3.4.1 Scalar and Gauge sector

We will write the covariant derivative for this gauge group in the following form:

$$D_\mu = \partial_\mu - igA^a_\mu T_a + ig_x y_x B^x_\mu \qquad ; \qquad g_x = \frac{g t_w}{\sqrt{1 - \frac{t_w^2}{3}}}, \tag{3.18}$$

[1] The best EWPT coming from LHC data for the SLH is in [107].

with t_w the tangent of the weak angle and y_x the hypercharge. Writing the $SU(3)$ generators T_a in the fundamental representation (**3**), the $SU(3)$ part works out as follows:

$$A^a T_a = \frac{A_3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{A_8}{2\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0 & W^+ & Y^0\\ W^- & 0 & W'^-\\ Y^{0\dagger} & W'^+ & 0\end{pmatrix}. \tag{3.19}$$

Here we have introduced the definitions of the gauge bosons A_3, A_8, B^x, $W^\pm$, $W'^\pm$, Y^0 and $Y^{0\dagger}$. The value of the gauge coupling g_x is set by the requirement that the photon couples with the electric charge.

The scalar sector of the SLH is a non linear σ-model. The two scalar multiplets (3.17) transform as $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ under $SU(3)_1 \times SU(3)_2$, respectively that include the SM Higgs doublets as well as new NGBs. They can be expressed as follows:

$$\Phi_1 = exp\left(\frac{i\Theta'}{f}\right) exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\ 0\\ fc_\beta\end{pmatrix}, \tag{3.20}$$

$$\Phi_2 = exp\left(\frac{i\Theta'}{f}\right) exp\left(-\frac{i\Theta}{t_\beta f}\right)\begin{pmatrix}0\\ 0\\ fs_\beta\end{pmatrix}, \tag{3.21}$$

where

$$\Theta = \begin{pmatrix}0 & 0 & h^0\\ 0 & 0 & -\phi^-\\ h^{0\dagger} & -\phi^+ & 0\end{pmatrix} + \frac{\eta}{\sqrt{2}}\begin{pmatrix}\kappa & 0 & 0\\ 0 & \kappa & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{3.22}$$

$$\Theta' = \begin{pmatrix}0 & 0 & y^0\\ 0 & 0 & x^-\\ y^{0\dagger} & x^+ & 0\end{pmatrix} + \frac{z'}{\sqrt{2}}\begin{pmatrix}\kappa' & 0 & 0\\ 0 & \kappa' & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{3.23}$$

with $t_\beta = \tan\beta$ the ratio of the vevs of the two Higgs triplets. To introduce EW symmetry breaking we will substitute $h^0 = \frac{(v+H)}{\sqrt{2}} - i\chi$. The structure of the Θ and Θ' matrices is determined by the broken generators of the gauge symmetry [2] .

The two scalar triplets $\Phi_{1,2}$ which are responsible for $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$ breaking contain 10 real degrees of freedom [3] , five of them are eaten by the $SU(3)$ gauge

[2] One can set the first 2 elements of the diagonal matrices independently from the third element (κ, κ').

[3]

$$\begin{aligned}[SU(3)\times U(1)]^2 &\Rightarrow 2[(3^2-1)+1] = 18\\ [SU(2)\times U(1)]^2 &\Rightarrow 2[(2^2-1)+1] = 8\end{aligned} \tag{3.24}$$

so we have $18 - 8 = 10$ NGBs $\left\{y^0, y^{0\dagger}, x^+, x^-, \eta, z', \chi, \chi^\dagger, \phi^+, \phi^-\right\}$ that aren't physical particles.

bosons with TeV scale masses, four form the SM Higgs doublet $\binom{h^0}{-\phi^-}$ and one is a real scalar field η.

We want to work in the ***Unitary Gauge*** like in Section 1.2.1 so our multiplets will have only physical particles:

$$\varPhi_1 = exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix}, \tag{3.25}$$

$$\varPhi_2 = exp\left(-\frac{i\Theta}{t_\beta f}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}, \tag{3.26}$$

with

$$\Theta = \begin{pmatrix}0 & 0 & \frac{v+H}{\sqrt{2}}\\ 0 & 0 & 0\\ \frac{v+H}{\sqrt{2}} & 0 & 0\end{pmatrix}. \tag{3.27}$$

Since we did not include an operator which gives a quartic coupling for the Higgs, this must be generated dynamically. Explicitly, the radiative corrections should produce the standard model Higgs potential

$$V = m^2H^2 + \lambda H^4. \tag{3.28}$$

Above the scale f, the $SU(3)$ gauge symmetry is unbroken and the potential is best described in terms of the $SU(3)$ multiplets $\varPhi_i$, and it is easy to see that the most general potential is a function of the only gauge invariant term which depends on the Higgs, $\varPhi_1^\dagger\varPhi_2$. At the scale f, the $SU(3)$ partners of fermions and gauge bosons obtain masses, and the theory matches onto the SM. Below f, the Higgs potential receives the usual radiative corrections from top quark and gauge loops. We first discuss the potential generated above the scale f. The top Yukawa couplings and gauge couplings preserve a $U(1)$ symmetry under which $\varPhi_1$ and $\varPhi_2$ have opposite charges. Therefore the lowest dimensional operator which can be radiatively induced is $|\varPhi_1^\dagger\varPhi_2|^2$. This operator is already generated at one loop but only with a logarithmic divergence. The symmetry forbids any quadratically divergent contributions from gauge or Yukawa couplings. The radiatively generated potential alone generates a Higgs “soft mass squared” which is somewhat too large. Therefore we also include a tree level “μ” term (see (3.31)) which will partially cancel the Higgs mass. It explicitly breaks the spontaneously broken global $U(1)$ symmetry and gives a mass to the would be NGB η that must be near the weak scale M_W. The corrections to the potential (3.28) include a mass squared δm^2 and a quartic $\delta\lambda$ contributions [101]

$$\delta m^2 = -\frac{3}{8\pi^2}\left[\lambda_T^2 m_T^2 \log\left(\frac{\Lambda^2}{m_T^2}\right) - \frac{g^2}{4}M_{W'}^2 \log\left(\frac{\Lambda^2}{M_{W'}^2}\right) - \frac{g^2}{8}\left(1+t_w^2\right)M_{Z'}^2 \log\left(\frac{\Lambda^2}{M_{Z'}^2}\right)\right], \tag{3.29}$$

$$\delta\lambda = \frac{|\delta m^2|}{3f^2 c_\beta^2 s_\beta^2} + \frac{3}{16\pi^2}\left[\lambda_t^4 \log\left(\frac{m_T^2}{m_t^2}\right) - \frac{g^4}{8}\log\left(\frac{M_{W'}^2}{M_W^2}\right) - \frac{g^6}{16}\left(1+t_w^2\right)^2 \log\left(\frac{M_{Z'}^2}{M_Z^2}\right)\right]. \tag{3.30}$$

Assuming that there are no large direct contributions to the potential from physics at the cutoff we have

$$V_{total} = \left(\mu^2 \frac{1}{f^2 s_\beta^2 c_\beta^2} + \delta m^2\right) H^2 + \left(-\frac{\mu^2}{f^2 s_\beta^3 c_\beta^3} + \delta\lambda\right) H^4. \tag{3.31}$$

Note that the radiative contribution to the Higgs mass from the top loop is negative while the contribution to the quartic is positive. Thus we have radiative EW symmetry breaking and stability of the Higgs potential.

From the gauge invariant Lagrangian

$$\mathcal{L}_\Phi = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2. \tag{3.32}$$

we can readily obtain the charged gauge boson mass terms. As a first approximation we keep terms up to order $\frac{v^2}{f^2}$. To this order the charged boson sector is diagonal,

$$\mathcal{L}_\Phi \propto M_W^2 W_\mu^+ W^{-\mu} + M_{W'}^2 W'^+_\mu W'^{-\mu}. \tag{3.33}$$

This level of precision is sufficient everywhere except when obtaining the correct $\mathcal{O}\left(\frac{v^2}{f^2}\right)$ couplings for NGBs. We need to go up to order $\frac{v^4}{f^4}$ to obtain the corrections to the M_W and higher order corrections to the gauge boson eigenstates for these couplings. Taking our expansion up to order four we must rotate the original fields to obtain the physical states:

$$W^\pm \to W^\pm \pm \frac{iv^3}{3\sqrt{2}f^3}\left(\frac{c_\beta^3}{s_\beta} - \frac{s_\beta^3}{c_\beta}\right) W'^\pm, \tag{3.34}$$

$$W'^\pm \to W'^\pm \pm \frac{iv^3}{3\sqrt{2}f^3}\left(\frac{c_\beta^3}{s_\beta} - \frac{s_\beta^3}{c_\beta}\right) W^\pm. \tag{3.35}$$

Note that the physical states W and W' differ from the interaction states only by a term of order $\frac{v^3}{f^3}$. This difference is irrelevant almost everywhere in our calculation, but is important in determining the NGB states. The interaction fields W and W' will be considered equal to the physical fields elsewhere. The masses of the physical fields read:

$$M_W = \frac{gv}{2}\left[1 - \frac{v^2}{12f^2}\left(\frac{c_\beta^4}{s_\beta^2} + \frac{s_\beta^4}{c_\beta^2}\right)\right], \tag{3.36}$$

$$M'_W = \frac{gf}{\sqrt{2}}\left[1 - \frac{v^2}{4f^2} + \frac{v^4}{24f^4}\left(\frac{c_\beta^4}{s_\beta^2} + \frac{s_\beta^4}{c_\beta^2}\right)\right] \simeq \frac{gf}{\sqrt{2}}\left(1 - \frac{v^2}{4f^2}\right). \tag{3.37}$$

The neutral sector is already non-diagonal at order $\mathcal{O}\left(\frac{v^2}{f^2}\right)$ and requires some more work:

$$\mathcal{L}_\Phi \propto M_Y^2 Y^{0\mu} Y_\mu^{0\dagger} + \begin{pmatrix} A_3 & A_8 & B_x \end{pmatrix} \mathcal{M}_0 \begin{pmatrix} A_3 \\ A_8 \\ B_x \end{pmatrix}, \tag{3.38}$$

$$\mathcal{M}_0 = f^2 \begin{pmatrix} \frac{g^2v^2}{8f^2} & \frac{g^2v^2}{8\sqrt{3}f^2} & \frac{gg_xv^2}{12f^2} \\ \frac{g^2v^2}{8\sqrt{3}f^2} & \frac{g^2}{3} - \frac{g^2v^2}{8f^2} & -\frac{gg_x}{3\sqrt{3}} + \frac{gg_xv^2}{4\sqrt{3}f^2} \\ \frac{gg_xv^2}{12f^2} & -\frac{gg_x}{3\sqrt{3}} + \frac{gg_xv^2}{4\sqrt{3}f^2} & \frac{g_x^2}{9} \end{pmatrix}. \tag{3.39}$$

Diagonalizing this matrix, the masses at order $\mathcal{O}\left(\frac{v^2}{f^2}\right)$ are:

$$\mathcal{L}_\Phi \propto M_{Z'}^2 Z'_\mu Z'^\mu + M_Z^2 Z_\mu Z^\mu + M_Y^2 Y_\mu^0 Y^{0\mu}, \tag{3.40}$$

$$M_Z = \frac{gv}{2c_W}, \tag{3.41}$$

$$M_{Z'} = \frac{\sqrt{2}gf}{\sqrt{3 - t_W^2}}\left(1 - \frac{3 - t_W^2}{c_W^2}\frac{v^2}{16f^2}\right), \tag{3.42}$$

$$M_Y = \frac{gf}{\sqrt{2}}, \tag{3.43}$$

where the first order mixing matrix for gauge bosons is:

$$\begin{pmatrix} A_3 \\ A_8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ -\sqrt{\frac{3-t_W^2}{3}} & \frac{s_W^2}{\sqrt{3}c_W} & \frac{s_W}{\sqrt{3}} \\ \frac{t_W}{\sqrt{3}} & s_W\sqrt{\frac{3-t_W^2}{3}} & c_W\sqrt{\frac{3-t_W^2}{3}} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}. \tag{3.44}$$

Additionally, the physical Z and Z' states also require the replacements:

$$Z' \to Z' + \delta_Z Z \qquad ; \qquad Z \to Z - \delta_Z Z', \tag{3.45}$$

where

$$\delta_Z = \frac{(1 - t_W^2)\sqrt{3 - t_W^2}}{8c_W}\frac{v^2}{f^2}. \tag{3.46}$$

We don't need to find the Goldstone eigenvalues because in the ***Unitary Gauge*** we work only with physical particles. And finally we can develop the field strength tensors Lagrangian:

$$\mathcal{L} = -\frac{1}{2}Tr\left\{F_{\mu\nu}F^{\mu\nu}\right\} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \tag{3.47}$$

where

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] \qquad ; \qquad B_{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu, \tag{3.48}$$

from where we can find the couplings scalar-vector (that we don't have in the ***Unitary Gauge***) and purely vectorial (see Appendix F).

3.4.2 Fermion sector

To build the fermion sector of the model, the SM fermions have to be included in representations of the larger SLH gauge group. The simplest way to do this is to embed the SM fermions into $SU(3)$ triplets.

Then each lepton family consists of an $SU(3)$ left-handed triplet **3** and two right handed singlets **1**. There is no right-handed light neutrino:

$$L_i^T = \begin{pmatrix}\nu_L & l_L & iN_L\end{pmatrix}_i \qquad , \qquad l_{iR}, \qquad N_{iR}, \qquad i = \{1; 2; 3\}. \tag{3.49}$$

The structure of the quark fields depends on the embedding we select:

- Universal embedding:

  All three generations carry identical gauge quantum numbers and the $SU(3)_L \times U(1)_X$ gauge group is anomalous. The SM down type Yukawa matrix is equal to the matrix $\lambda_d \frac{f}{\Lambda}$. The up-type Yukawa matrices are more interesting as there are 6 quarks of charge $\frac{2}{3}$ which mix with each other. In general this leads to flavor changing neutral currents and is dangerous. Each quark family consists of an $SU(3)$ left-handed triplet **3** and three right handed singlets **1**:

$$\mathcal{Q}_i^T = \begin{pmatrix}u_L & d_L & U_L\end{pmatrix}_i \qquad ; \qquad u_{iR}, \qquad d_{iR}, \qquad U_{iR}. \tag{3.50}$$

- Anomaly free embedding:

  In the second configuration we cancel the $SU(3)_L$ anomaly by taking different charge assignments for the different generations of quark triplets (see Table 3.2).

$$\mathcal{Q}_1^T = \begin{pmatrix}d_L & -u_L & iD_L\end{pmatrix} \qquad ; \qquad d_R, \qquad u_R, \qquad D_R, \tag{3.51}$$

Table 3.2: Quantum numbers in the diferent embeddings.

Universal embedding							
Fermion	$\mathcal{Q}_{1,2}$	$\mathcal{Q}_3$	u_{iR}, U_{iR}	d_{iR}	L_i	N_{iR}	e_{iR}
Q charge	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	-1
$SU(3)$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
Anomaly free embedding							
Fermion	$\mathcal{Q}_{1,2}$	$\mathcal{Q}_3$	u_{iR}, T_{iR}	d_{iR}, D_{iR}, S_{iR}	L_i	N_{iR}	e_{iR}
Q charge	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	-1
$SU(3)$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$

$$\mathcal{Q}_2^T = \begin{pmatrix} s_L & -c_L & iS_L \end{pmatrix} \quad ; \quad s_R, \quad c_R, \quad S_R, \tag{3.52}$$

$$\mathcal{Q}_3^T = \begin{pmatrix} t_L & b_L & iT_L \end{pmatrix} \quad ; \quad t_R, \quad b_R, \quad T_R. \tag{3.53}$$

With this new charge assignment all anomalies cancel [109] which makes this model easier to complete in the ultraviolet regime [110].

In order to avoid FCNC the mass matrix of heavy partners needs to be sufficiently well aligned with the quark mass matrices. Note that in both embeddings the mixing of light fermions with their partners generates a coupling of the W and W' gauge bosons to a single SM fermion and his heavy partner. This opens the interesting possibility of single U, D production from fusion of weak gauge bosons with SM quarks ($d + W \to U$, $u + Z \to U$) and produces significant contributions in the box diagrams.

The last part that we need for the complete description of the lepton interactions is:

$$\mathcal{L}_F = i\bar{\psi}_m \not{D} \psi_m \quad ; \quad \psi_m = \begin{pmatrix} L_m \\ l_{Rm} \\ N_{Rm} \end{pmatrix}. \tag{3.54}$$

The full mixing structure of the quark sector is much more complex than that of the lepton sector and, in general, all light quarks mix with other heavy and light quarks from every family.

Lepton masses follow from the Yukawa Lagrangian:

$$\mathcal{L}_Y \propto i\lambda_N^m \bar{N}_{Rm} \Phi_2^\dagger L_m + \frac{i\lambda_l^{mn}}{\Lambda} \bar{l}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j L_n^k + h.c., \tag{3.55}$$

where the quartic term preserves the global symmetry (L_m transforms as $(\mathbf{1}, \mathbf{3})$ under $SU(3)_1 \times SU(3)_2$) and λ_N can be taken diagonal after a proper field redefinition.

Firstly we need to determine the actual physical states of the leptons. Keeping only the mass terms to order $\mathcal{O}\left(\frac{v^2}{f^2}\right)$ we have the following Lagrangian:

$$\mathcal{L}_Y \propto -f s_\beta \lambda_N^m \left[\left(1 - \frac{\delta_\nu^2}{2}\right) \bar{N}_{Rm} N_{Lm} - \delta_\nu \bar{N}_{Rm} \nu_{Lm} \right] + \zeta_\beta \frac{fv}{\sqrt{2}\Lambda} \lambda_l^{mn} \bar{l}_{Rm} l_{Ln} + h.c., \tag{3.56}$$

where

$$\delta_\nu = -\frac{v}{\sqrt{2} f t_\beta} \qquad ; \qquad \zeta_\beta = \left[1 - \frac{v^2}{4f^2} - \frac{v^2}{12f^2}\left(\frac{s_\beta^4}{c_\beta^2} + \frac{c_\beta^4}{s_\beta^2}\right)\right]. \tag{3.57}$$

The matrices λ_N and λ_l are not necessarily aligned. Thus, in the basis where the former is diagonal, the latter mixes different light lepton flavors. Denoting the eigenvalues of λ_l as y_l^i, the light lepton masses are given by

$$m_{l_i} = -\zeta_\beta \frac{fv}{\sqrt{2}\Lambda} y_l^i, \tag{3.58}$$

whereas the left-handed components of the light physical fields are obtained by the replacement:

$$l_{L_m} \to (V_l l_L)_m = V_l^{mi} l_{Li}. \tag{3.59}$$

Furthermore, according to (3.56) each heavy neutrino is mixed just with the light neutrino of the same family. To separate them, we rotate only the left handed sector. To order $\mathcal{O}\left(\frac{v^2}{f^2}\right)$, the physical states for the neutrinos are given by:

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix}_m \to \left[\begin{pmatrix} 1 - \frac{\delta_\nu^2}{2} & -\delta_\nu \\ \delta_\nu & 1 - \frac{\delta_\nu^2}{2} \end{pmatrix} \begin{pmatrix} V_l \nu_L \\ N_l \end{pmatrix}\right]_m . \tag{3.60}$$

Since one can safely consider the SM neutrinos as massless, we have chosen to rotate them in the same way as the light charged leptons. Finally, the heavy neutrino masses are:

$$m_{N_i} = f s_\beta \lambda_N^i. \tag{3.61}$$

Corrections of order $\frac{v^2}{f^2}$ to vertices are only needed for particles involved in triangle diagrams and, since quarks only appear in box diagrams, $\frac{v}{f}$ precision is sufficient.

For the anomaly free embedding, the basic Yukawa Lagrangian reads:

$$\begin{aligned} \mathcal{L}_Y \propto & \lambda_1^t \bar{u}_{R3}^1 \Phi_1^\dagger Q_3 + i\lambda_2^t \bar{u}_{R3}^2 \Phi_2^\dagger Q_3 + i\frac{\lambda_b^m}{\Lambda} \bar{d}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j Q_3^k \\ & + i\lambda_1^{dn} \bar{d}_{Rn}^1 Q_n^T \Phi_1 + i\lambda_2^{dn} \bar{d}_{Rn}^2 Q_n^T \Phi_2 + i\frac{\lambda_u^{mn}}{\Lambda} \bar{u}_{Rm} \epsilon_{ijk} \Phi_1^{*i} \Phi_2^{*j} Q_n^k, \end{aligned} \tag{3.62}$$

where $n = 1, 2$; $i, j, k = 1, 2, 3$ are $SU(3)$ indices; d_{Rm} runs over (d_R, s_R, b_R, D_R, S_R) and u_{Rm} runs over (u_R, c_R, t_R, T_R); u_{R3}^1 and u_{R3}^2 are linear combinations of t_R and T_R; d_{R1}^n and d_{R2}^m are linear combinations of d_R and D_R for $n = 1$ and of s_R and S_R for $n = 2$:

$$T_R = \frac{\lambda_1^t c_\beta u_{R3}^1 + \lambda_2^t s_\beta u_{R3}^2}{\sqrt{\left(\lambda_1^t\right)^2 c_\beta^2 + \left(\lambda_2^t\right)^2 s_\beta^2}}, \qquad t_R = \frac{-\lambda_2^t s_\beta u_{R3}^1 + \lambda_1^t c_\beta u_{R3}^2}{\sqrt{\left(\lambda_1^t\right)^2 c_\beta^2 + \left(\lambda_2^t\right)^2 s_\beta^2}}, \tag{3.63}$$

$$D_R = \frac{\lambda_1^{d1} c_\beta d_{R1}^1 + \lambda_2^{d1} s_\beta d_{R1}^2}{\sqrt{\left(\lambda_1^{d1}\right)^2 c_\beta^2 + \left(\lambda_2^{d1}\right)^2 s_\beta^2}}, \qquad d_R = \frac{-\lambda_2^{d1} s_\beta d_{R1}^1 + \lambda_1^{d1} c_\beta d_{R1}^2}{\sqrt{\left(\lambda_1^{d1}\right)^2 c_\beta^2 + \left(\lambda_2^{d1}\right)^2 s_\beta^2}}, \tag{3.64}$$

$$S_R = \frac{\lambda_1^{d2} c_\beta d_{R2}^1 + \lambda_2^{d2} s_\beta d_{R2}^2}{\sqrt{\left(\lambda_1^{d2}\right)^2 c_\beta^2 + \left(\lambda_2^{d2}\right)^2 s_\beta^2}}, \qquad s_R = \frac{-\lambda_2^{d2} s_\beta d_{R2}^1 + \lambda_1^{d2} c_\beta d_{R2}^2}{\sqrt{\left(\lambda_1^{d2}\right)^2 c_\beta^2 + \left(\lambda_2^{d2}\right)^2 s_\beta^2}}, \tag{3.65}$$

We require the collective structure with different right handed quarks entering the Φ_1 and Φ_2 quartic Yukawa couplings. By a proper field redefinition, λ_1^d can be taken diagonal in general and, for simplicity and to avoid large quark flavor changing effects, we also assume λ_2^d to be diagonal [92].

Since we are interested in LFV we will assume no flavor mixing in the quark sector, which might otherwise dilute some of the effects we wish to highlight. We essentially set all the λ_b^m and λ_u^{mn} that mix different families or heavy and light quarks to zero and all others are fixed by the light quark masses. Only the heavy-light mixing within each family remains. We neglect terms proportional to $\frac{v^2}{f^2}$ and rotate the left-handed fields to obtain the physical quark states (heavy quark masses get corrections at this order that will be neglected as well):

$$T_L \to T_L + \delta_t t_L, \tag{3.66}$$

$$t_L \to t_L - \delta_t T_L, \tag{3.67}$$

$$D_L \to D_L + \delta_d d_L, \tag{3.68}$$

$$d_L \to d_L - \delta_d D_L, \tag{3.69}$$

$$S_L \to S_L + \delta_s s_L, \tag{3.70}$$

$$s_L \to s_L - \delta_s S_L, \tag{3.71}$$

where

$$\delta_t = \frac{v}{\sqrt{2} f} \frac{s_\beta c_\beta \left[\left(\lambda_1^t\right)^2 - \left(\lambda_2^t\right)^2\right]}{\left(\lambda_1^t\right)^2 c_\beta^2 + \left(\lambda_2^t\right)^2 s_\beta^2}, \tag{3.72}$$

$$\delta_d = -\frac{v}{\sqrt{2} f} \frac{s_\beta c_\beta \left[\left(\lambda_1^{d1}\right)^2 - \left(\lambda_2^{d1}\right)^2\right]}{\left(\lambda_1^{d1}\right)^2 c_\beta^2 + \left(\lambda_2^{d1}\right)^2 s_\beta^2}, \tag{3.73}$$

$$\delta_s = -\frac{v}{\sqrt{2} f} \frac{s_\beta c_\beta \left[\left(\lambda_1^{d2}\right)^2 - \left(\lambda_2^{d2}\right)^2\right]}{\left(\lambda_1^{d2}\right)^2 c_\beta^2 + \left(\lambda_2^{d2}\right)^2 s_\beta^2}, \tag{3.74}$$

are complex in general.

Taking all this into account we get the SM quark masses:

$$\begin{aligned} m_u &= -\frac{v}{\sqrt{2}}\frac{f}{\Lambda}\lambda_u^{11}, \\ m_c &= -\frac{v}{\sqrt{2}}\frac{f}{\Lambda}\lambda_u^{22}, \\ m_b &= -\frac{v}{\sqrt{2}}\frac{f}{\Lambda}\lambda_b^{3}, \\ m_t &= \frac{v}{\sqrt{2}}\frac{\lambda_1^t\lambda_2^t}{\sqrt{(\lambda_1^t)^2 c_\beta^2 + (\lambda_2^t)^2 s_\beta^2}}, \\ m_d &= -\frac{v}{\sqrt{2}}\frac{\lambda_1^{d1}\lambda_2^{d1}}{\sqrt{\left(\lambda_1^{d1}\right)^2 c_\beta^2 + \left(\lambda_2^{d1}\right)^2 s_\beta^2}}, \\ m_s &= -\frac{v}{\sqrt{2}}\frac{\lambda_1^{d2}\lambda_2^{d2}}{\sqrt{\left(\lambda_1^{d2}\right)^2 c_\beta^2 + \left(\lambda_2^{d2}\right)^2 s_\beta^2}}. \end{aligned} \tag{3.75}$$

The $\delta_{d,s,t}$ can be expressed in terms of the quark masses:

$$\delta_q = \pm\frac{v}{2\sqrt{2}f^2}\frac{1}{c_\beta s_\beta}\left(s_\beta^2 - c_\beta^2 + \epsilon\sqrt{1 - \frac{8c_\beta^2 s_\beta^2 f^2 m_q^2}{v^2 m_Q^2}}\right), \tag{3.76}$$

where the $+(-)$ sign stands for $q = t(d,s)$ and $\epsilon = \pm 1$ depending on the corresponding values of λ_1 and λ_2.

Like for the lepton sector we need the quark-gauge lagrangian to complete the review of the quark couplings, in the anomaly free embedding we have

$$\mathcal{L}_F = \bar{\mathcal{Q}}_m \not{D}_m^L \mathcal{Q}_m + \bar{u}_{Rm} i\not{D}^u u_{Rm} + \bar{d}_{Rm} i\not{D}^d d_{Rm} + \bar{T}_R i\not{D}^u T_R + \bar{D}_R i\not{D}^d D_R + \bar{S}_R i\not{D}^d S_R. \tag{3.77}$$

Remembering that the first two families are in the anti-fundamental representation:

$$\begin{aligned} D_{\{1,2\}\mu}^L &= \partial_\mu + igA_\mu^a T_a^*, \\ D_{3\mu}^L &= \partial_\mu - igA_\mu^a T_a + ig_x\frac{B_\mu^x}{3}, \\ D_\mu^u &= \partial_\mu + ig_x\frac{2B_\mu^x}{3}, \\ D_\mu^d &= \partial_\mu - ig_x\frac{B_\mu^x}{3}. \end{aligned} \tag{3.78}$$

Part II

Scientific Research

Chapter 4

Lepton Flavor Violating Tau decays into one and two Pseudoscalars

4.1 Background

Precise measurements of the lepton properties provide stringent tests of the SM and accurate determinations of its parameters. Moreover the hadronic τ decay modes constitute an ideal tool for studying low energy effects of the strong interaction in very clean conditions. The large mass of the τ opens the possibility to study many kinematically allowed exclusive decay modes and extract relevant dynamical information. Violations of flavour and CP conservation laws can also be searched for with τ decays. Being one of the fermions most strongly coupled to the scalar sector, the τ lepton is playing now a very important role at the LHC as a tool to test the Higgs properties and search for new physics at higher scales, for this reason we study the possibility of LFV in the hadronic decays of the τ beyond the SM. We use the Composite Higgs models frame where the Higgs is a composite particle and a NGB of the spontaneous breaking of a higher symmetry.

In particular, we consider the case of the SLH, where the initial group of symmetry is $SU(3)_L \times U(1)_X$, and LFV only happens through perturbative processes at one-loop with the assumption of the existence of a multiplet of heavy neutrinos. We study the decays $\tau \longrightarrow \mu \ \{P, V, PP\}$, where P (V) stands for a pseudoscalar (vector) meson. We use the ***Unitary Gauge***.

In the case of SLH these processes take place through various penguin diagrams $\tau \to \mu\gamma$, $\tau \to \mu Z$, $\tau \to \mu Z'$ (not $\tau \to \mu Y^0$ because Y^0 do not couples with the light quarks currents $\bar{q}q$ but only with heavy quarks $\bar{Q}Q$). In [111] one sees that the divergence cancellation of FCNC, for physical processes in the ***Unitary Gauge***, happens between penguins (see Sections 4.2, 4.3) and box diagrams (see Section 4.4). Therefore, we will have to cancel the penguin divergences with those from box diagrams, an example of this cancellation is given in Appendix C. The final hadrons will be one $\{\pi^0, \eta, \eta'\}$, and two $\left\{\pi^+\pi^-, K^+K^-, K^0\bar{K}^0\right\}$ pseudoscalar mesons, and $\{\rho, \phi\}$ vector mesons.

Bounds on branching ratios from B-factories will provide information on the cut-off scale of the SLH model.

$$A_{LFV} \propto \sum_i V_l^{i\tau} V_l^{i\mu} F[Q^2; M_{N_i}^2; \frac{v}{f}]. \tag{4.1}$$

We consider the phenomenology up to the SLH cut-off in order to provide a reasonable framework for TeV discovery at LHC or precision physics at the B-factories.

The SLH model extends the SM group, $SU(2)_L \times U(1)_Y$, to a gauge group $SU(3)_L \times U(1)_X$, that requires to enlarge the $SU(2)$ doublets of the SM into $SU(3)$ triplets, adding also other $SU(3)$ gauge bosons. Then the $SU(3)_L \times U(1)_X$ gauge symmetry breaks down spontaneously to the SM EW group by two complex scalar fields $\Phi_{1,2}$ (see equations (3.20), (3.21)), triplets under $SU(3)$ (see Section 3.4).

LFV decays in the SLH model arise at one loop level and they are driven by the presence of the heavy neutrinos N_i in connivance with the rotation of light lepton fields V_ℓ^{ij}. There are two generic topologies participating in this amplitude:

- Penguin diagrams, namely $\tau \to \mu\,\{\gamma, Z, Z'\}$, followed by $\{\gamma, Z, Z'\} \to q\bar{q}$,
- Box diagrams.

In principle there should be also a penguin contribution with a Higgs boson $\tau \to \mu H$. However the coupling of the Higgs to the light quarks, $H \to q\bar{q}$, has an intrinsic suppression due to the mass of the quarks and, therefore, we do not take this into account. In fact we will assume that light quarks are massless along all our calculation, and we will also neglect the muon mass.

As there is no contribution from the SLH model to our processes at tree level, the calculation, at one loop, has to be finite. In the following:

- $\tau(q) \to \mu(p') + \ldots$
- $\langle \Phi_1 \rangle = f c_\beta$, $\langle \Phi_2 \rangle = f s_\beta$, $f \sim$ 1 TeV,
- We calculate our amplitudes as an expansion, until the second order, of the parameter $x = \frac{v}{f}$,
- $p'^2 = m_\mu^2 = 0$,
- $q^2 = m_\tau^2$,
- The energy of the hadrons p^2 finite (where $q = p + p'$).

In the ***Unitary Gauge*** only physical states appear in the calculation. Our reasoning to understand which are the physical states goes as follows:

- In the Section (3.4.1) we have put $\Theta' = 0$. Fields in Θ' are the NGBs that, through the spontaneous breaking of the $SU(3) \times U(1) \to SU(2) \times U(1)$ give masses to the Y^0, $Y^{0\dagger}$, $W'^{\pm}$ and Z' gauge bosons.

- At higher energies the scalar fields in the Θ matrix are physical, however after the spontaneous breaking of the EW symmetry $SU(2)_L \times U(1)_Y \to U(1)$, only the field H is physical. Charged Higgs $\phi^{\pm}$ and the pseudoscalar χ are the NGBs that give mass to the EW gauge bosons, therefore they are also not physical after the EW spontaneous symmetry breaking.

The price to pay is that now we have to use the unitary propagator for the gauge vector bosons (see Appendix F) and the calculation looks more divergent. Therefore the cancellation of divergences is more subtle.

The general decomposition of the matrix element for a vector current V_α, can be written as:

$$\langle \mu\left(p'\right)|V_\alpha|\tau\left(q\right)\rangle = \bar{u}_\mu\left(p'\right)\left[p^2\gamma_\alpha F_1\left(p^2\right) + F_2\left(p^2\right) i m_\tau \sigma_{\alpha\nu} p^\nu + F_3\left(p^2\right) p_\alpha\right] u_\tau\left(q\right), \tag{4.2}$$

or

$$\langle \mu\left(p'\right)|V_\alpha|\tau\left(q\right)\rangle = \bar{u}_\mu\left(p'\right)\left[m_\tau G_1\left(p^2\right)\gamma_\alpha + G_2\left(p^2\right) p'^\alpha + G_3\left(p^2\right) p_\alpha\right] u_\tau\left(q\right), \tag{4.3}$$

where these two Lorentz structures are related through Gordon Identities (see Appendix D.2). Al the F_i, G_i form factors decompose in left and right parts that multiplicate $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$ respectively [1].

4.2 $\tau \to \mu\ \gamma^*$

We study the penguins of the tau decay through off shell photons, but for now we can estimate only the first part of the process. The hadronization is treated in the Section 4.5. All channels require the heavy neutrinos to produce the Flavour Violation.

We consider $\tau(q) \to \mu(p')\ \gamma^*(p)$ where q is the momentum of the tau, p' of the muon and p is the momentum of photon, so we consider:

$$\begin{aligned} p'^2 &= 0, \\ q^2 &= m_\tau^2, \end{aligned} \tag{4.4}$$

We have approximated all the integrations until the order $\mathcal{O}(x^2)$ and then we neglect the ratios

$$\frac{m_\tau^2}{M_N^2} \simeq \frac{m_\tau^2}{M_W^2} \simeq \frac{m_\tau^2}{M_{W'}^2} \simeq 0. \tag{4.5}$$

[1]If we assume that the light quarks have negligible masses with respect to the energy and the masses of the bosons we can exclude the contribution of p_α, the form factors F_3, G_3 see Appendix B.4.

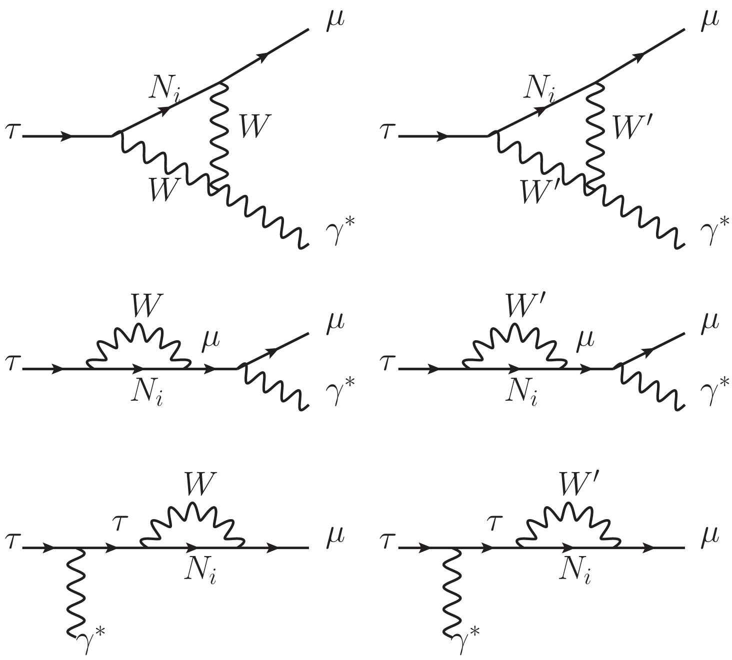


Figure 4.1: Penguin diagrams for $\tau \to \mu\gamma^*$ in the SLH model.

In the ***Unitary Gauge*** all divergent contributions, coming from the integrals A_m and B_0 (see Appendix E.1.1, E.1.2), nullify when we sum the penguin amplitudes of photon, Z and Z' with the boxes (see Appendix C). In the LFV case many contributions are multiplied by:

$$\sum_j V_{l_1 j} V^*_{l_2 j} = 0, \tag{4.6}$$

those who remain are those that multiply:

$$\sum_j V_{l_1 j} V^*_{l_2 j} F\left(M_{N_j}\right) \neq 0. \tag{4.7}$$

The diagrams contributing to the photon penguin are those in Figure 4.1 and the result is given by [2] :

$$\mathcal{M}\left(\gamma\right) = \frac{e^2}{Q^2}\frac{v^2}{f^2}\sum_j V_{\tau j} V^*_{\mu j} \psi(p')\left[p^2\gamma_\lambda\left(F^j_L P_L + F^j_R P_R\right) + i m_\tau \sigma_{\lambda\nu} p^\nu \left(G^j_L P_L + G^j_R P_R\right)\right]\psi(q, m_\tau)\times$$
$$\times\,\psi(p_2) Q_q \gamma^\lambda P_L \psi(p_1), \tag{4.8}$$

[2]Where we applied the trasformation of the Lorentz structure like in Appendix D.2.1.

where Q_q is the electric charge matrix of the quarks

$$Q_q = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{4.9}$$

and:

$$\begin{aligned} F_L^j &= \frac{\alpha_w}{4\pi}\frac{1}{16M_W^2}\left[\left(\frac{\chi_j^3\left(\chi_j^2-8\chi_j+13\right)}{\left(\chi_j-1\right)^4} - 4\delta_\nu^2\frac{M_{W'}^2}{M_W^2}\right)\ln\chi_j + \right. \\ &\qquad \left. + \frac{4\chi_j^5-19\chi_j^4+29\chi_j^3+5\chi_j^2-95\chi_j+40}{6\left(\chi_j-1\right)^3}\right], \\ G_R^j &= \frac{\alpha_w}{4\pi}\frac{1}{8M_W^2}\left[\left(\frac{\chi_j^3\left(2\chi_j+1\right)}{(\chi_j-1)^4} + 2\delta_\nu^2\frac{M_{W'}^2}{M_W^2}\right)\ln\chi_j + \right. \\ &\qquad \left. + \frac{6\chi_j^5-15\chi_j^4-35\chi_j^3+72\chi_j^2-66\chi_j+20}{6(\chi_j-1)^3}\right], \end{aligned} \tag{4.10}$$

where $\alpha_w = \frac{e^2}{4\pi s_w^2}$ and:

$$\chi_i = \frac{M_{N_i}^2}{M_{W'}^2} \propto x^0 \quad ; \quad \omega = \frac{M_W^2}{M_{W'}^2} \propto x^2. \tag{4.11}$$

4.3 $\tau \to \mu\ Z, Z'$

We consider $\tau(q) \to \mu(p')\ Z, Z'(p)$ so, like in the previous section, q is the momentum of the tau, p' of the muon and p is the momentum of the bosons Z and Z'; so we take:

$$\begin{aligned} p'^2 &= 0, \\ q^2 &= m_\tau^2. \end{aligned} \tag{4.12}$$

In this case we used the same integrations of the photons and the same approximations for the tau mass:

$$\frac{m_\tau^2}{M_N^2} \simeq \frac{m_\tau^2}{M_W^2} \simeq \frac{m_\tau^2}{{M_{W'}}^2} \simeq \frac{m_\tau^2}{M_Z^2} \simeq \frac{m_\tau^2}{{M_{Z'}}^2} \simeq 0, \tag{4.13}$$

also we ignore the SM neutrino masses:

$$m_{\nu_i} = 0, \tag{4.14}$$

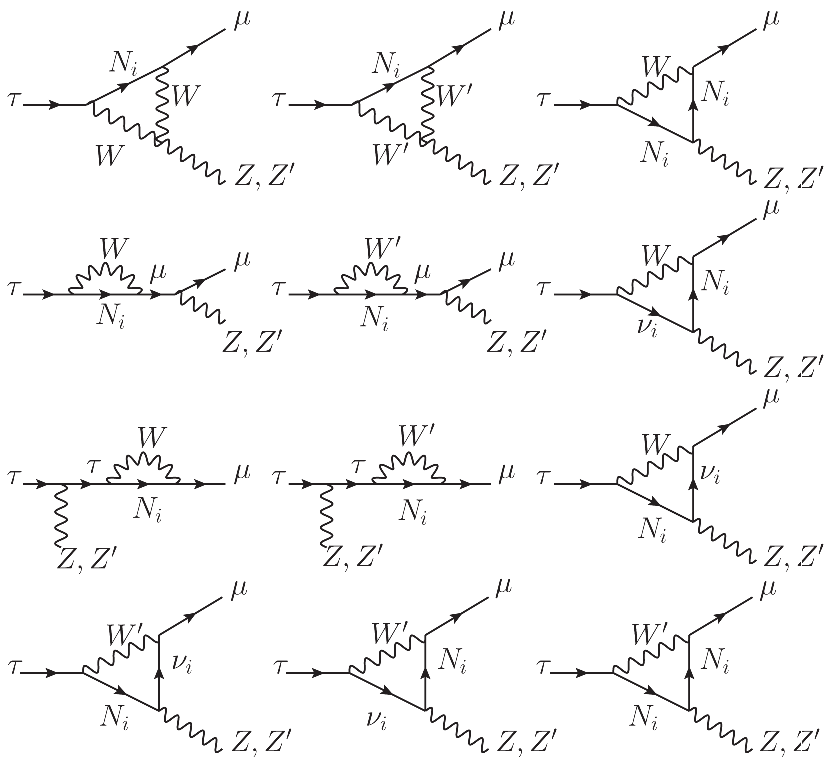


Figure 4.2: Penguin diagrams for $\tau \to \mu Z, Z'$ in the SLH model.

and finally we did additional approximations for p^2:

$$\frac{p^2}{M_N^2} \simeq \frac{p^2}{M_W^2} \simeq \frac{p^2}{{M_{W'}}^2} \simeq \frac{p^2}{M_Z^2} \simeq \frac{p^2}{{M_{Z'}}^2} \simeq 0. \tag{4.15}$$

All we said in the calculation of photon penguins remains valid but we have to distinguish the order until which we calculate the contributions to the amplitudes of

$$\begin{aligned} \mathcal{M}\,(Z) &\propto \mathcal{O}(x^2), \\ \mathcal{M}\,(Z') &\propto \mathcal{O}(x^0), \end{aligned} \tag{4.16}$$

because the Z boson has a standard propagador like the photon, of the order $\mathcal{O}(x^0)$ and the Z' haves a propagator of order $\mathcal{O}(x^2)$ that is an overall factor of the penguin amplitude. The penguin diagrams with Z and Z' are given in Figure 4.2.They give the following results:

$$
\begin{aligned}
\mathcal{M}(Z) &= \frac{g}{M_Z^2} \sum_j V_{\tau j} V_{\mu j}^* \psi(p') \left[\gamma_\lambda \left(H_L^j P_L + H_R^j P_R\right)\right] \psi(q, m_\tau) \times \\
&\qquad \times \psi(p_2) \left[\gamma^\lambda \left(Z_L P_L + Z_R P_R\right)\right] \psi(p_1), \\
\mathcal{M}(Z') &= \frac{g}{M_{Z'}^2} \sum_j V_{\tau j} V_{\mu j}^* \psi(p') \left[\gamma_\lambda \left(H_L^{j\prime} P_L + H_R^{j\prime} P_R\right)\right] \psi(q, m_\tau) \times \\
&\qquad \times \psi(p_2) \left[\gamma^\lambda \left(Z_L' P_L + Z_R' P_R\right)\right] \psi(p_1),
\end{aligned}
\tag{4.17}
$$

where now:

$$
\begin{aligned}
Z_L &= \frac{g}{c_w} \left(T_3^q - s_w^2 Q_q\right), \\
Z_R &= -\frac{g}{c_w} s_w^2 Q_q, \\
Z_L' &= \frac{g}{6} \sqrt{3 - t_w^2}\, \mathbb{1}, \\
Z_R' &= -\frac{g t_w^2}{\sqrt{3 - t_w^2}} Q_q,
\end{aligned}
\tag{4.18}
$$

being:

$$
T_3^q = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.
\tag{4.19}
$$

H_R^j and $H_R^{j\prime}$ in (4.17) are, again, $\mathcal{O}\left(\frac{m_\tau^2}{M_Z^2}\right)$ and we disregard them. For the left handed form factors we find:

$$
\begin{aligned}
H_L^j &= \frac{\alpha_w}{32\pi} \left\{ \frac{\delta_z}{c_w^2 \sqrt{3 - t_w^2}} \left[\left(3\chi_j(\chi_j - 2) - 2c_w^2(7\chi_j^2 - 14\chi_j + 4)\right) \frac{\chi_j \ln \chi_j}{(\chi_j - 1)^2} + \right.\right. \\
&\qquad \left. + \frac{-5\chi_j^2 + 5\chi_j + 6 + 6c_w^2\left(3\chi_j^2 - \chi_j - 4\right)}{2(\chi_j - 1)} \right] - \\
&\qquad \left. - \delta_\nu^2 \frac{2\chi_j^2 - 5\chi_j + 3}{c_w(\chi_j - 1)} \right\}, \\
H_L^{j\prime} &= \frac{\alpha_w}{32\pi} \frac{1}{c_w^2 \sqrt{3 - t_w^2}} \left[\left(3\chi_j(\chi_j - 2) - 2c_w^2(7\chi_j^2 - 14\chi_j + 4)\right) \frac{\chi_j \ln \chi_j}{(\chi_j - 1)^2} + \right. \\
&\qquad \left. + \frac{-5\chi_j^2 + 5\chi_j + 6 + 6c_w^2\left(3\chi_j^2 - \chi_j - 4\right)}{2(\chi_j - 1)} \right].
\end{aligned}
\tag{4.20}
$$

As we supposed in (4.16) the $H_L^{j\prime}$ corresponding to the Z' contributions, and that is very similar to the result for H_L^j, is $\mathcal{O}(x^2)$. This is due to the fact that the definitions of H_L^j and

$H_L^{j\,\prime}$ in (4.17) carry a factor of the inverse squared mass of the corresponding gauge boson in the penguin. Then the $\mathcal{M}\,(Z')$ amplitude conveys the leading suppression factor in this term.

4.4 Boxes

Finally we turn to evaluate the box diagrams in Figure 4.3. We proceed following the same approaches as in the case of the penguin diagrams. In addition we consider that the external momenta vanish.

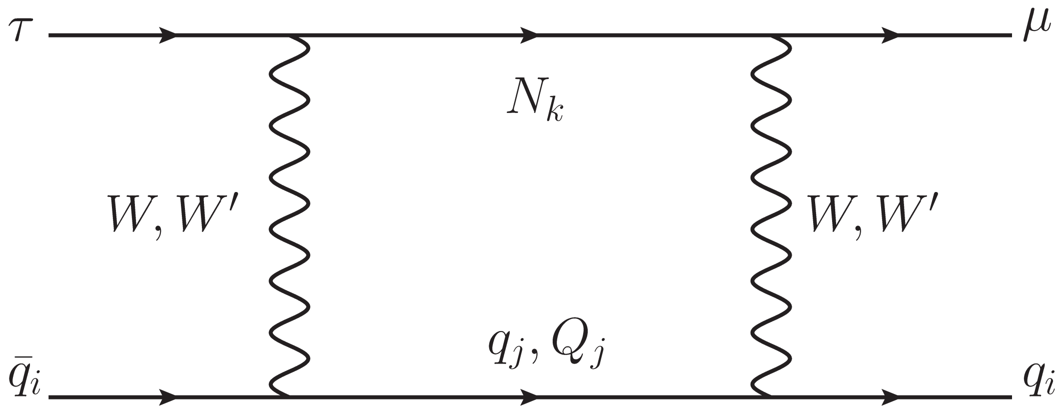


Figure 4.3: Box diagrams for $\tau \to \mu q\bar{q}$ in the SLH model. The internal quark states are $(u, \overline{u}) \to \{d, D\}$, $(d, \overline{d}) \to \{u\}$, $(s, \overline{s}) \to \{c\}$.

We did not consider the flavor mixing of quarks because we do not want to consider the CKM effect in these channels, even remembering that we want only final light quarks, the quark currents are:

$$\begin{aligned} u &\longrightarrow d \longrightarrow \bar{u} \\ u &\longrightarrow D \longrightarrow \bar{u} \\ d &\longrightarrow u \longrightarrow \bar{d} \\ s &\longrightarrow c \longrightarrow \bar{s}. \end{aligned} \tag{4.21}$$

In these diagrams we have approximated external null momenta like in Appendix E.4. The result is given by:

$$\mathcal{M}_{box} = g^2 \sum_q^{u,d,s} \sum_j V_{\tau j} V_{\mu j}^* B_q^j \psi(p')\gamma_\mu P_L \psi(q, m_\tau)\psi(p_2)\gamma^\mu P_L \psi(p_1), \tag{4.22}$$

where

$$B_q^j = \frac{\alpha_w}{64\pi}\left[\alpha_q^j \ln \chi_j + \beta_q^j \ln \delta + \gamma_q^j\right], \tag{4.23}$$

the remaining terms are given by:

$$
\begin{aligned}
\alpha_u^j =& \frac{1}{M_W^2(\chi_j-\delta)}\left\{\frac{3\chi_j\delta\delta_\nu(\delta_d^*+\delta_d)}{(\chi_j-1)} - \delta^2\delta_\nu(\delta_d^*+\delta_d) + \frac{\chi_j(6-13\chi_j)}{(\chi_j-1)^2}\frac{M_W^2}{M_{W'}^2} + \right.\\
&\left. + (\delta^2-6\delta)\frac{M_W^2}{M_{W'}^2} + \delta^2\delta_d^2\delta_\nu^2\frac{M_{W'}^2}{M_W^2}\right\},\\
\alpha_d^j =& \frac{3\delta_\nu}{M_W^2(\chi_j-1)}\left(\delta_d^*+\delta_d\right),\\
\alpha_s^j =& \frac{3\delta_\nu}{M_W^2(\chi_j-1)}\left(\delta_s^*+\delta_s\right),\\
\beta_u^j =& \frac{\delta^2}{M_W^2(\delta-\chi_j)}\left\{\delta_d^2\delta_\nu^2\frac{M_{W'}^2}{M_W^2} + \frac{\delta(\delta-8)}{(\delta-1)^2}\frac{M_W^2}{M_{W'}^2} - \delta_\nu(\delta_d^*+\delta_d)\frac{\delta^2-5\delta+4}{(\delta-1)^2}\right\},\\
\beta_d^j =& \beta_s^j = 0,\\
\gamma_u^j =& -\frac{1}{2M_W^2}\left\{3\delta_\nu^2\delta_d^2\chi_j\frac{M_{W'}^2}{M_W^2} + \frac{\delta(3\chi_j^2-16\chi_j+13)-3\chi_j^2+13\chi_j+4}{(\delta-1)(\chi_j-1)}\frac{M_W^2}{M_{W'}^2}\right\},\\
\gamma_d^j =& \frac{3}{2M_W^2}\delta_\nu(\delta_d^*+\delta_d)\chi_j,\\
\gamma_s^j =& \frac{3}{2M_W^2}\delta_\nu(\delta_s^*+\delta_s)\chi_j.
\end{aligned}
\tag{4.24}
$$

where:

$$
\delta = \frac{m_D^2}{M_{W'}^2} \propto x^0. \tag{4.25}
$$

The contribution of the box amplitudes is relevant in the ***Unitary Gauge*** not only for the handling of the divergences but also for their contribution to the convergent part of the processes. We will come back to this issue in Section 4.6.

4.5 Hadronization

In this work we study the LFV semileptonic τ decays:

- $\tau \to \mu PP$ with $PP = \pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0K^0$.
- $\tau \to \mu P$ with $P = \pi^0, \eta, \eta'$.
- $\tau \to \mu V$ with $V = \rho^0, \phi$.

We will analyze the importance of the various contributions, the γ, Z, Z', and W, W' in the boxes. The hadronisation of quark bilinears is performed within the chiral framework. Semileptonic decays of the τ lepton are a relatively clean scenario from the strong interaction point of view. Hadrons in the final state stem from the hadronization of quark bilinears, namely $\Psi\Gamma\Psi$, where Ψ is a vector in the $SU(3)$ flavour space and Γ is, in general, a matrix both in the spinor and the flavour space. An appropriate framework to handle the procedure of hadronisation is provided by the large-N_C expansion of $SU(N_C)_{QCD}$ [112], being N_C the number of colours. In short it states that in the $N_C \to \infty$ limit any Green function is given by meromorphic expressions provided by the tree level diagrams of a Lagrangian theory with an infinite spectrum of zero-width states.

A suitable tool to realise the $\frac{1}{N_C}$ expansion is provided by chiral Lagrangians. In those processes where hadron resonances do not play a dynamical role, χPT [113],[114] is the appropriate scheme to describe the strong interaction of NGBs (π, K and η). This is the case, for instance, of $\tau \to \mu P$ (being P short for a pseudoscalar meson). When resonances participate in the dynamics of the process, as in $\tau \to \mu PP$, it is necessary to include them as active degrees of freedom into the Lagrangian as it is properly done in the $R\chi T$ frame [115]. Hence we will make use of $R\chi T$, that naturally includes χPT, to hadronise the relevant currents that appear in the processes under study here. We consider bilinear light quark operators coupled to external sources and added to the massles QCD Lagrangian :

$$\mathcal{L}_{QCD} = \mathcal{L}^0_{QCD} + \bar{q}\left[\gamma_\mu\left(v^\mu + \gamma_5 a^\mu\right) - \left(s - ip\gamma_5\right)\right]q, \tag{4.26}$$

where vector v^μ, axial-vector a^μ, scalar s and pseudoscalar p fields are matrices in the flavour space, and $\mathcal{L}^0_{QCD}$ is the massless QCD Lagrangian. This Lagrangian density gives the QCD generating functional $\mathcal{Z}_{QCD}[v, a, s, p]$ as:

$$e^{i\mathcal{Z}_{QCD}[v,a,s,p]} = \int \left[DG_\mu\right]\left[Dq\right]\left[D\bar{q}\right] e^{i\int d^4x \mathcal{L}_{QCD}[q,\bar{q},G,v,a,s,p]} \tag{4.27}$$

In order to construct the corresponding Lagrangian theory in terms of the lightest hadron modes we need to specify them. The lightest $U(3)$ nonet of pseudoscalar mesons:

$$\phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}_0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix} \tag{4.28}$$

is realised nonlinearly into the unitary matrix in the flavour space:

$$u(\varphi) = e^{\frac{i\Phi}{\sqrt{2}F}}. \tag{4.29}$$

Hence the leading $\mathcal{O}(p^2)$ χPT $SU(3)_L \times SU(3)_R$ chiral Lagrangian is:

$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle, \tag{4.30}$$

where

$$\begin{aligned} u_\mu &= i\left[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger\right], \\ \chi_+ &= u^\dagger \chi u^\dagger + u\chi^\dagger u, \\ \chi &= 2B_0(s+ip), \end{aligned} \tag{4.31}$$

and $\langle \dots \rangle$ is short for a trace in the flavour space. Interactions with EW NGBs can be accommodated through the vector $v_\mu = \frac{r_\mu + l_\mu}{2}$ and axial vector $a_\mu = \frac{r_\mu - l_\mu}{2}$ external fields. The scalar field s incorporates explicit chiral symmetry breaking through the quark masses $s = M + \dots$ and, finally, $F_\pi \simeq 92.4\ MeV$ is the pion decay constant and $B_0 F_\pi^2 = -\langle 0|\bar{\psi}\psi|0\rangle_0$ in the chiral limit. The chiral tensor χ provides masses to the NGBs through the external scalar field, as can be seen in (4.31). Indeed in the isospin limit we have:

$$\chi = 2B_0\mathcal{M} + \dots = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix} + \cdots \tag{4.32}$$

Hence we identify:

$$\begin{aligned} B_0 m_u &= B_0 m_d = \frac{m_\pi^2}{2}, \\ B_0 m_s &= m_K^2 - \frac{m_\pi^2}{2}. \end{aligned} \tag{4.33}$$

In the simplest approximation, mass eigenstates η and η' are defined from the octet η_8 and singlet η_0 states through the rotation:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}, \tag{4.34}$$

where $\theta \simeq -18°$.

The hadronisation of a final state of two pseudoscalars is driven by vector and scalar resonances though the latter, because their higher masses, play a lesser role. We will introduce the vector resonances in the antisymmetric formalism; hence the nonet of resonance fields $V_{\mu\nu}$ (see [115]) is defined by analogy with (4.28) with the same flavour structure. By demanding the chiral symmetry invariance the resonance Lagrangian reads:

$$\mathcal{L}_V = \mathcal{L}_{kin}^V + \mathcal{L}_{(2)}^V, \tag{4.35}$$

where

$$\begin{aligned}\mathcal{L}_{kin}^{V} &= -\frac{1}{2}\langle \nabla^{\lambda} V_{\lambda\mu} \nabla_{\nu} V^{\nu\mu} \rangle + \frac{M_V^2}{4}\langle V_{\mu\nu} V^{\mu\nu} \rangle, \\ \mathcal{L}_{(2)}^{V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu} \rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu} u^{\mu} u^{\nu} \rangle,\end{aligned} \tag{4.36}$$

and in the latter the subscript (2) indicates the chiral order of the tensor accompanying $V_{\mu\nu}$. In (4.36) we have used the definitions:

$$\begin{aligned}\nabla_{\mu} X &\equiv \partial_{\mu} X + [\Gamma_{\mu}, X]\,, \\ \Gamma_{\mu} &= \frac{1}{2}\left[u^{\dagger}\left(\partial_{\mu} - i r_{\mu}\right) u + u\left(\partial_{\mu} - i l_{\mu}\right) u^{\dagger}\right], \\ f_+^{\mu\nu} &= u F_L^{\mu\nu} u^{\dagger} + u^{\dagger} F_R^{\mu\nu} u,\end{aligned} \tag{4.37}$$

being $F_{L,R}^{\mu\nu}$ the field strength tensors associated with the external right and left fields. The couplings F_V and G_V are real. Accordingly our $R\chi T$ framework is provided by:

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{\chi}^{(2)} + \mathcal{L}_V, \tag{4.38}$$

and the contribution of the low modes to the QCD functional is formally given by:

$$e^{i\mathcal{Z}_{QCD}[v,a,s,p]}|_{low\ modes} = \int [Du]\,[DV]\, e^{i\int d^4x \mathcal{L}_{R\chi T}[u,V,v,a,s,p]}. \tag{4.39}$$

With this identification we can already carry out the hadronization of the bilinear quark currents included in (4.26) by taking the appropriate partial derivatives, with respect to the external auxiliary fields, of the functional action,

$$\begin{aligned}V_{\mu}^{i} &= \bar{q}\gamma_{\mu}\frac{\lambda^i}{2}q = \frac{\partial \mathcal{L}_{R\chi T}}{\partial v_i^{\mu}}|_{j=0}, \\ A_{\mu}^{i} &= \bar{q}\gamma_{\mu}\gamma_5\frac{\lambda^i}{2}q = \frac{\partial \mathcal{L}_{R\chi T}}{\partial a_i^{\mu}}|_{j=0}, \\ S^{i} &= -\bar{q}\lambda^i q = \frac{\partial \mathcal{L}_{R\chi T}}{\partial s_i}|_{j=0}, \\ P^{i} &= \bar{q}i\gamma_5\lambda^i q = \frac{\partial \mathcal{L}_{R\chi T}}{\partial p_i}|_{j=0},\end{aligned} \tag{4.40}$$

where $j = 0$ indicates that all external currents are set to zero. This gives (see [116]):

$$\begin{aligned}
V_\mu^i &= \frac{F_\pi^2}{4}\langle\lambda^i\left(uu_\mu u^\dagger - u^\dagger u_\mu u\right)\rangle - \frac{F_V}{2\sqrt{2}}\langle\lambda^i\partial^\nu\left(u^\dagger V_{\mu\nu}u + uV_{\mu\nu}u^\dagger\right)\rangle,\\
A_\mu^i &= \frac{F_\pi^2}{4}\langle\lambda^i\left(uu_\mu u^\dagger + u^\dagger u_\mu u\right)\rangle,\\
S^i &= \frac{B_0F_\pi^2}{2}\langle\lambda^i\left(u^\dagger u^\dagger + uu\right)\rangle,\\
P^i &= \frac{iB_0F_\pi^2}{2}\langle\lambda^i\left(u^\dagger u^\dagger - uu\right)\rangle.
\end{aligned} \tag{4.41}$$

With these expressions we are able to hadronise the final states in $\tau\to\mu PP$, $\tau\to\mu P$ and $\tau\to\mu V$ processes.

4.5.1 $\tau(q)\to\mu(p')\ P(p)$

Only the axial-vector current contributes and that means that $\mathcal{M}_\gamma$ does not participate. The axial-vector current is determined from the leading $\mathcal{O}(p^2)$ chiral Lagrangian and we get, for $P=\{\pi^0,\eta,\eta'\}$:

$$\begin{aligned}
\mathcal{T}_Z(P) =& -i\frac{g^2}{2c_w}\frac{F_\pi}{M_Z^2}Z(P)\sum_j V^{j\mu *}V^{j\tau}\psi(p')\left[\not{p}\left(H_L^jP_L + H_R^jP_R\right)\right]\psi(p),\\
\mathcal{T}_{Z'}(P) =& i\frac{g^2}{4\sqrt{9-3t_w^2}}\frac{F_\pi}{M_{Z'}^2}Z'(P)\sum_j V^{j\mu *}V^{j\tau}\psi(p')\left[\not{p}\left({H_L^j}'P_L + {H_R^j}'P_R\right)\right]\psi(p).
\end{aligned} \tag{4.42}$$

Here F_π is the decay constant of the pion and the $Z(P)$, $Z'(P)$ factors are given in Table 4.1. Finally:

$$\mathcal{T}_{box}(P) = -ig^2F_\pi\sum_j V^{j\mu *}V^{j\tau}B^j(P)\psi(p')\left[\not{p}P_L\right]\psi(p). \tag{4.43}$$

where $B^j(P)$ factors are given in Table 4.1.

The width of these processes, with $\mathcal{T}(P)=\mathcal{T}_Z(P)+\mathcal{T}_{Z'}(P)+\mathcal{T}_{box}(P)$, is given by [116]:

$$B\left(\tau\to\mu P\right) = \frac{1}{4\pi}\frac{\sqrt{\lambda(m_\tau^2,m_\mu^2,m_P^2)}}{m_\tau^2\Gamma_\tau}\frac{1}{2}\sum_{i,f}|\mathcal{T}(P)|^2, \tag{4.44}$$

where

$$\lambda(x,y,z) = (x+y-z)^2 - 4xy, \tag{4.45}$$

and

	$P=\pi^0$	$P=\eta$	$P=\eta'$
$Z(P)$	1	$\frac{1}{\sqrt{6}}\left(\sin\theta_\eta+\sqrt{2}\cos\theta_\eta\right)$	$\frac{1}{\sqrt{6}}\left(\sqrt{2}\sin\theta_\eta-\cos\theta_\eta\right)$
$Z'(P)$	$\sqrt{3}t_w^2$	$\cos\theta_\eta t_w^2-\sqrt{2}\sin\theta_\eta\,(3-t_w^2)$	$\sin\theta_\eta t_w^2+\sqrt{2}\cos\theta_\eta\,(3-t_w^2)$
$B_j(P)$	$\frac{1}{2}\left(B_d^j-B_u^j\right)$	$\frac{1}{2\sqrt{3}}\left[\left(\sqrt{2}\sin\theta_\eta-\cos\theta_\eta\right)B_u^j\right.$ $\left.+\left(2\sqrt{2}\sin\theta_\eta+\cos\theta_\eta\right)B_d^j\right]$	$\frac{1}{2\sqrt{3}}\left[\left(\sin\theta_\eta-2\sqrt{2}\cos\theta_\eta\right)B_d^j\right.$ $\left.-\left(\sin\theta_\eta+\sqrt{2}\cos\theta_\eta\right)B_u^j\right]$

Table 4.1: Factors appearing in (4.42), (4.43). The mixing between the octet (η_8) and the singlet (η_0) components of the nonet of pseudoscalar mesons is parameterized by the angle $\theta_\eta \simeq -18°$. The functions B_q^j are given in (4.23).

$$\sum_{i,f}|\mathcal{T}(P)|^2 = \frac{1}{2m_\tau}\sum_{k,l}\left[\left(m_\tau^2+m_\mu^2-m_P^2\right)\left(a_P^k a_P^{l*}+b_P^k b_P^{l*}\right)+2m_\mu m_\tau\left(a_P^k a_P^{l*}-b_P^k b_P^{l*}\right)\right], \tag{4.46}$$

with $k,l=Z,Z',B$. Defining $\Delta_{\tau\mu}=m_\tau-m_\mu$, $\Sigma_{\tau\mu}=m_\tau+m_\mu$ we have :

$$\begin{aligned}
a_P^Z &= -\frac{g^2F_\pi}{4c_W M_Z^2}\Delta_{\tau\mu}Z(P)\sum_j V^{j\mu*}V^{j\tau}\left(H_R^j+H_L^j\right),\\
a_P^{Z'} &= \frac{g^2F_\pi}{8\sqrt{9-3t_W^2}M_{Z'}^2}\Delta_{\tau\mu}Z'(P)\sum_j V^{j\mu*}V^{j\tau}\left({H_R^j}'+{H_L^j}'\right),\\
a_P^B &= -\frac{g^2F_\pi}{2}\Delta_{\tau\mu}\sum_j V^{j\mu*}V^{j\tau}B_j(P),\\
b_P^Z &= \frac{g^2F_\pi}{4c_W M_Z^2}\Sigma_{\tau\mu}Z(P)\sum_j V^{j\mu*}V^{j\tau}\left(H_R^j-H_L^j\right),\\
b_P^{Z'} &= -\frac{g^2F_\pi}{8\sqrt{9-3t_W^2}M_{Z'}^2}\Sigma_{\tau\mu}Z'(P)\sum_j V^{j\mu*}V^{j\tau}\left({H_R^j}'-{H_L^j}'\right),\\
b_P^B &= -\frac{g^2F_\pi}{2}\Sigma_{\tau\mu}\sum_j V^{j\mu*}V^{j\tau}B_j(P).
\end{aligned} \tag{4.47}$$

4.5.2 $\tau(q) \to \mu(p')\ \{P(p_+)P(p_-); V(p)\}$

The semileptonic $\tau \to \mu\{PP;V\}$ channels can be mediated by a photon, Z and Z'. The $\{Z, Z'\}$-mediated contributions are expected to be much smaller than the γ-mediated contribution due to the suppression of the boson masses in the propagators. This has been shown to happen in the leptonic channels like $\tau \to 3\mu$, where the Z-mediated contribution to its branching ratio has been estimated to be a factor $10^{-3} - 10^{-5}$ smaller than the γ-mediated contribution [117].

The photon that decays into two pseudoscalars mesons is driven by the electromagnetic current:

$$V_\mu^{em} = \sum_q^{u,d,s} Q_q \bar{q}\gamma_\mu q = V_\mu^3 + \frac{V_\mu^8}{\sqrt{3}}, \tag{4.48}$$

the electromagnetic form factor is then defined as:

$$\langle P^+(p_+)P^-(p_-)|V_\mu^{em}|0\rangle = (p_+ - p_-)_\mu F_V^{PP}(p^2) \tag{4.49}$$

where $k = p_+ + p_-$ with $p_\pm$ is the momentum of the $P^\pm$ meson and $F_V^{PP}(s)$ is steered by both $I = 1$ and $I = 0$ vector resonances, in particular the $\rho(770)$ that is the lightest of hadron resonance. Due to the $p^2 = 0$ pole of the photon propagator this is, by far, the dominant contribution to this hadronic final state. Hence the result is more sensitive to the construction of this form factor. The authors of [116] have elaborated a more complete expression than the one provided by the vector current in (4.41) though it reduces to this one in the $N_C \to \infty$ limit, including only one multiplet of resonances and at $p^2 \ll M_\rho$. A proper construction of $F_V^{PP}(p^2)$ is given in Appendix B of [116],

After the hadronization we find:

$$\begin{aligned}
\mathcal{T}_\gamma(P) =& \frac{e^2}{p^2}\frac{v^2}{f^2} F_V^{PP}(p^2) \sum_j V^{j\mu *}V^{j\tau} \bar{\psi}(p') \Big[p^2 (\not{p}_+ - \not{p}_-) \left(F_L^j P_L + F_R^j P_R\right) + \\
&+ 2 i m_\tau p_+^\lambda \sigma_{\lambda\nu} p_-^\nu \left(G_L^j P_L + G_R^j P_R\right) \Big] \psi(p), \\
\mathcal{T}_Z(P) =& g^2 \frac{2s_w^2 - 1}{2c_w M_Z^2} F_V^{PP}(p^2) \sum_j V^{j\mu *}V^{j\tau} \bar{\psi}(p')(\not{p}_+ - \not{p}_-) \left(H_L^j P_L + H_R^j P_R\right) \psi(p), \\
\mathcal{T}_{Z'}(P) =& -g^2 \frac{t_w^2}{4M_{Z'}^2\sqrt{3 - t_w^2}} F_V^{PP}(p^2) \sum_j V^{j\mu *}V^{j\tau} \bar{\psi}(p')(\not{p}_+ - \not{p}_-) \left(H_L^{j\prime} P_L + H_R^{j\prime} P_R\right) \psi(p), \\
\mathcal{T}_{box}(P) =& \frac{g^2}{2} F_V^{PP}(p^2) \sum_j V^{j\mu *}V^{j\tau} \left(B_u^j - B_d^j\right) \bar{\psi}(p')(\not{p}_+ - \not{p}_-) P_L \psi(p).
\end{aligned} \tag{4.50}$$

The branching ratio for the process is:

$$B\left(\tau \to \mu PP\right) = \frac{k_{PP}}{64\pi^3 m_\tau^2 \Gamma_\tau} \int_{p_{min}^2}^{p_{max}^2} ds \int_{t_{min}}^{t_{max}} dt \frac{1}{2} \sum_{i,f} |\mathcal{T}(P)|^2, \tag{4.51}$$

where k_{PP} is 1 for $PP = \pi^+\pi^-, K^+K^-, K^0\bar{K}^0$ and $\frac{1}{2}$ for $PP = \pi^0\pi^0$. In addition:

$$\begin{aligned} t_{min}^{max} &= \frac{(m_\tau^2 - m_\mu^2)^2 - \left(\sqrt{\lambda(p^2, m_P^2, m_P^2)} \mp \sqrt{\lambda(m_\tau^2, p^2, m_\mu^2)}\right)^2}{4p^2}, \\ p_{min}^2 &= 4m_P^2, \\ p_{max}^2 &= (m_\tau - m_\mu)^2. \end{aligned} \tag{4.52}$$

With the (4.51) we could plot the results in the Figures 4.4, 4.6, 4.5, 4.7, 4.8, 4.9, 4.10, 4.11.

We would like to consider also the decays into a vector resonance, namely $V = \rho, \phi$. From a quantum field theory point of view, a resonance is not an asymptotic state and, indeed, a vector decays strongly into a pair of pseudoscalar mesons. When an experiment *measures* a final state with a vector resonance, in fact what is measuring is a pair of pseudoscalar mesons with a squared total mass approaching m_V^2. Hence the definition of a resonance from an experimental point of view is uncertain. Actually the chiral nature of the lightest pseudoscalar mesons relies on this occurrence and two pions into a $J = I = 1$ state are indistinguishable from a $\rho(770)$ meson, for instance. As a consequence the channels $\tau \to \mu V$ are related with $\tau \to \mu PP$ that we discussed above. We follow the proposal of reference [116].

The outcome of this circumstance is that the branching ratio of $\tau \to \mu V$ is obtained from that of the $\tau \to \mu PP$ by trying to implement the experimental procedure, that is, focusing in two pseudoscalar mesons on the mass (and width) of the resonance. That is:

$$\begin{aligned} B(\tau \to \mu\rho) =& B(\tau \to \mu\pi^+\pi^-)\Big|_\rho, \\ B(\tau \to \mu\phi) =& B(\tau \to \mu K^+K^-)\Big|_\phi + B(\tau \to \mu K^0\overline{K^0})\Big|_\phi, \end{aligned} \tag{4.53}$$

where the two pseudoscalars branching ratio is the one given by (4.51) but where the p_{min}^{max} limits of integration are now specified by:

$$p_{min}^{max} = M_\rho^2 \pm \frac{1}{2} M_\rho \Gamma_\rho(M_\rho^2), \tag{4.54}$$

and

$$p_{min}^{max} = M_\phi^2 \pm \frac{1}{2} M_\phi \Gamma_\phi(M_\phi^2), \tag{4.55}$$

respectively. Here the full widths of ρ and ϕ are taken from [118]. We think that this definition of the branching ratios into vector mesons approaches the experimental interpretation and provides a reasonable estimate of them.

4.6 Numerical Results

The provision of numerical estimates for our LFV branching ratios, from our results in the previous section, requires an all inclusive discussion of the parameters of the SLH model that we have employed:

- Scale of compositeness f. As commented in the Introduction almost everyone expects some new physics around 1 TeV, and going up. We could fix the scale of compositeness in the SLH model as that $f \sim 1$ TeV. However analyses of the model from Higgs data and EW Observables [107], [119] seem to indicate that, at 95 % C.L., values of $f \lesssim 3.5$ TeV should be excluded for our model. That, of course, also delays the appearance of a strongly coupled region. For definiteness we choose a range $2 < f < 10$ TeV in order to furnish our results.

- Heavy neutrinos. "Little" neutrinos drive the dynamics of LFV lepton decays. Inherited from the SM setting we have three different heavy neutrinos that appear in the amplitudes that we can write, generically, as:

$$\mathcal{M} = \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} A\left(\chi_j\right), \tag{4.56}$$

  with j adding over the three families and $A(\chi_j)$ a generic function of $\chi_j = M_{N_j}^2/M_{W'}^2$. We do not have any information on the mixing matrix elements V_ℓ^{ik} and we have to keep at least two non degenerated families in order to have a non-vanishing result; as a consequence we will give our numerical results assuming only two families and, accordingly, one mixing angle. Hence we will have:

$$\mathcal{M} = \sin\theta \cos\theta \left[A(\chi_1) - A(\chi_2)\right]. \tag{4.57}$$

  In [48] it can be observed that, from LFV tau decays into leptons within the SLH model and for $f \simeq 1$ TeV, experimental bounds require $\sin 2\theta = 2 \sin\theta \cos\theta < 0.05$ and even smaller from muon-electron conversion in nuclei. As we propose higher values for the scale of compositeness we will take, for the numerical determinations $\sin 2\theta \simeq 0.25$, though we will also study the variation of the branching ratios in function of this parameter.

  "Little" neutrino masses are also unknown. Experimental bounds on these masses are rather loose and very much model dependent. However, we will take into account the results in [48] pointing to $\chi_1\chi_2 \lesssim 0.01$ and $\sqrt{\frac{\chi_1}{\chi_2}} - \sqrt{\frac{\chi_2}{\chi_1}} \lesssim 0.05$. Given our larger values for f we will use ($\chi_2 > \chi_1$ is assumed, our spectrum cannot be degenerated) $0 \leq \chi_1 \leq 0.25$ and $1.1\chi_1 \leq \chi_2 \leq 10\chi_1$, where the latter limits of χ_2 correspond to the nearly degenerate and large mass splitting cases, respectively.

- $\tan\beta = \frac{f_1}{f_2}$. The ratio of the two vevs from the spontaneous breaking of the upper symmetry is also an unknown parameter in our model. The mixing between a "little" and a light neutrino, parameterized by δ_ν, can give us a hint. Phenomenological analyses indicate that $\delta_\nu < 0.05$ [48], [120], [121]. Therefore from (3.57) we obtain that $|f t_\beta| \gtrsim 3.5$ TeV. We will take, as a value of reference, $t_\beta = 5$ and will explore the range $1 < t_\beta < 10$.

- Quark parameters. As commented above we do not consider flavour-mixing in the quark sector. The redefinition of fields that diagonalizes the mixing between "little" and light quarks is parameterized by the δ_p parameters that appear in the box amplitude, (4.24), for $p = d, s$. We follow the proposal of [92] and assume that the mixing effects in the down quark sector are suppressed in the $t_\beta > 1$ regime. This is analogous to what happens in the neutrino case. It implies:

$$\delta_d \simeq \delta_s \simeq -\delta_\nu. \tag{4.58}$$

  Finally, in the box diagrams also appears the ratio $\delta = \frac{m_D^2}{M_{W'}^2}$ that involves the mass of the "little" down quark D. In all the numerical evaluations we take $\delta = 1$.

The input of SM parameters and masses is taken from the PDG [118]. In particular we will take $\sin^2\theta_w = 0.23$, $F_\pi = 0.0922$ GeV and $\theta_\eta = -18^\circ$. Although we will present our results for LFV tau decays into a muon and hadrons, the results on the decay to an electron should be essentially the same because we have expanded the mass of the outgoing charged lepton over heavy masses in our calculation. Hence the only difference between both channels is, essentially, one of phase space that would turn out to be tiny in any case due to the relative high mass of the tau lepton, and the lepton flavour mixing matrix elements. Provided that the latter are of the same order of magnitude, we consider our results to be valid for both decays: $\tau \to \ell$hadrons for $\ell = e, \mu$.

The present upper bounds on the LFV hadron tau decays branching ratios are collected in Table 4.2. All these bounds originate in the excellent work carried out by both BaBar and Belle experiments in the last ten years. It can be seen that present limits stand at the 10^{-8} level. Super B Factories, like the SuperKEKB/Belle II project [122] will give the next step. Hadron decays of the tau lepton are almost background free, although efficiencies are different from channel to channel. All in all, expected sensitivities are in the range of $B(\tau \to \ell\,\text{hadrons}) \sim (2-6)\times 10^{-10}$.
In Figure 4.4 we show the dependence on the scale of compositeness f of the branching ratios (normalized to the upper bounds in Table 4.2) in the LFV hadron decays under study. We use $\chi_1 = 0.25$, $\chi_2 = 10\,\chi_1$, $t_\beta = 5$ and $\sin 2\theta = 0.25$. The plotted range for the scale of

Process	$B \times 10^8$ (90 % C.L.) [118]	
	$\ell = \mu$	$\ell = e$
$\tau \to \ell\,\gamma$	< 4.4	< 3.3
$\tau \to \ell\,\pi^0$	< 11.0	< 8.0
$\tau \to \ell\,\eta$	< 6.5	< 9.2
$\tau \to \ell\,\eta'$	< 13.0	< 16.0
$\tau \to \ell\,\pi^+\pi^-$	< 2.1	< 2.3
$\tau \to \ell\,K^+K^-$	< 4.4	< 3.4
$\tau \to \ell\,K_S\overline{K}_S$	< 8.0	< 7.1
$\tau \to \ell\,\rho^0$	< 1.2	< 1.8
$\tau \to \ell\,\phi$	< 8.4	< 3.1

Table 4.2: Experimental upper bounds, at 90 % C.L., on the branching ratios of the LFV decays $\tau \to \ell(P, V, PP)$ for $\ell = \mu, e$, studied in this article. We quote them from the PDG [118].

compositeness seems the most natural in these models, however a higher value of f might also make sense. In any case Figure 4.4 indicates clearly the trend of the prediction. It can be seen that, in the most optimistic case, for low values of f, our results imply branching ratios at least four orders of magnitude smaller than present limits.

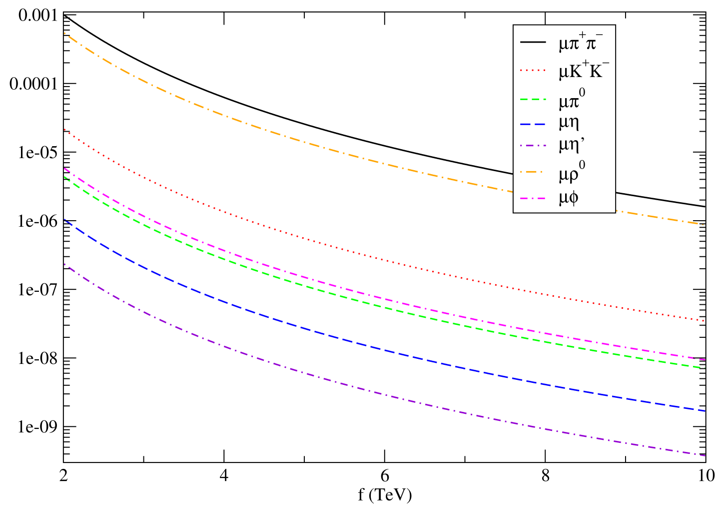


Figure 4.4: Dependence of the scale of compositeness f for the branching ratios of LFV tau decays into hadrons in the SLH model. They are normalized to the present upper bounds in Table 4.2, i.e. a value of 1 in the y-axis indicates the present upper limit. The input parameters are: $t_\beta = 5$; $\chi_1 = \frac{M_{N_1}}{M_{W'}} = 0.25$; $\chi_2 = 5\chi_1$; $\sin(2\theta) = 0.25$.

The dependence on the “little” neutrino masses is collected in Figure 4.6 and Figure 4.5. In Figure 4.5 we assume the cases of a small splitting: $\chi_2 = 1.1\,\chi_1$ and a large one $\chi_2 = 10\,\chi_1$. In the Figure 4.6 we can see the effect produced by the large splitting in heavy neutrino masses when the second neutrino reaches and goes over the mass of the heavy gauge boson W'. Naturally a small splitting produces branching ratios much smaller due to the unitarity of the lepton mixing matrix (see (4.57)).

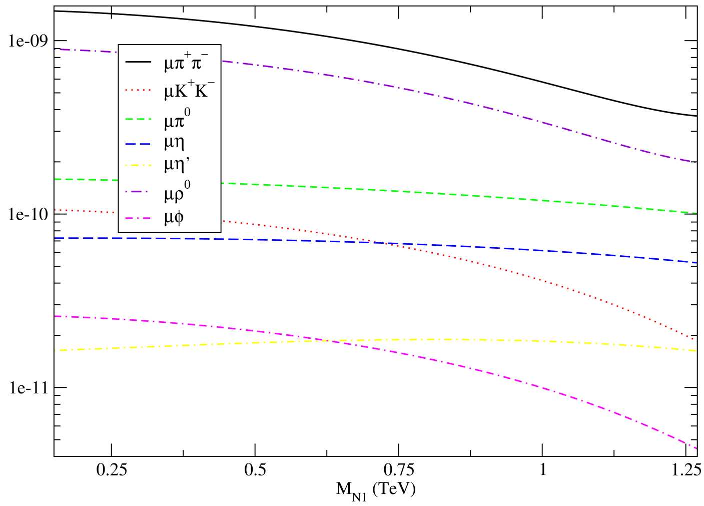


Figure 4.5: Dependence of the LFV branching ratios on M_{N_1}. We assume a small splitting of the heavy neutrino spectrum: $\chi_2 = 1.1\,\chi_1$. We input $t_\beta = 5$, $\sin 2\theta = 0.25$ and $f = 6$ TeV. Normalization as in Figure 4.4.

In the Figures 4.7, 4.8 we show the dependence of the branching ratios respectively on the parameters $\tan\beta$ and $\sin 2\theta$. It can be seen that the dependence on t_β is rather mild for $t_\beta \gtrsim 3$.

Correlations between different branching ratios are shown in the Figures 4.9, 4.10, 4.11. The branching ratio of $\tau \to \mu\gamma$ has been obtained from the SLH prediction for $\mu \to e\gamma$ in Ref. [48]. In these figures we have not normalized the branching ratios to the upper bounds as we did in previous figures. In the Figures 4.9 and 4.10 we show $B(\tau \to \mu\pi^+\pi^-)$ and $B(\tau \to \mu\pi^0)$ versus $B(\tau \to \mu\gamma)$ and the vertical red line indicates the measured present upper bound for the later decay. That would leave hadron branching ratios, at the most, of $\mathcal{O}(10^{-12}) - \mathcal{O}(10^{-14})$ as we already commented in the previous discussion. In the Figure 4.11 we plot $B(\tau \to \mu\pi^0)$ versus $B(\tau \to \mu\pi^+\pi^-)$ and it is shown that both are highly correlated (we turn to this point later).

We would like to turn now to comment a property of our calculation in the SLH model. This is related with the relative weight of the different contributions. Supersymmetric scenarios seem to indicate that, in the ’t Hooft-Feynman gauge, box diagrams provide

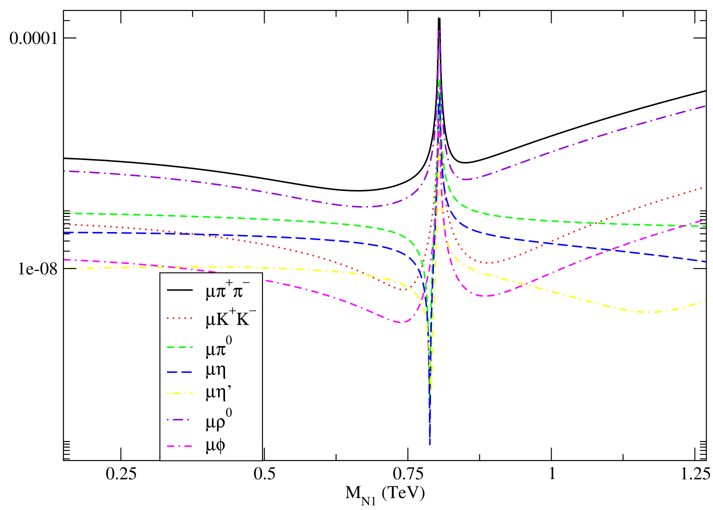


Figure 4.6: Dependence of the LFV branching ratios on M_{N_1}. We assume a large splitting: $\chi_1 = 10\,\chi_2$. We input $t_\beta = 5$, $\sin 2\theta = 0.25$ and $f = 6$ TeV. Normalization as in Figure 4.4.

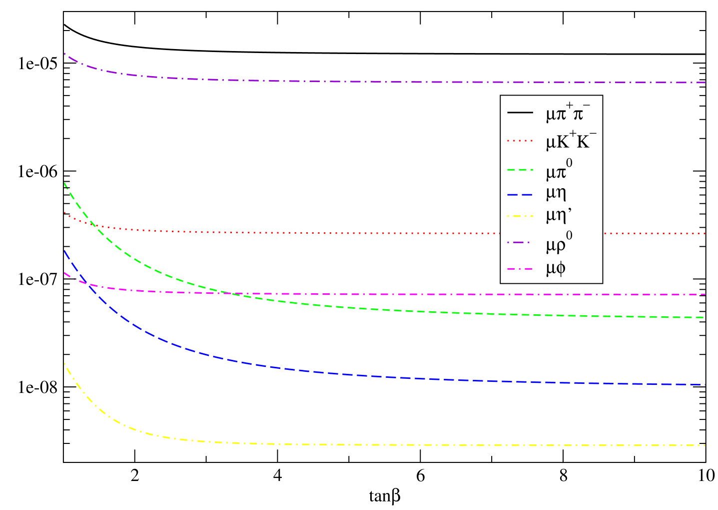


Figure 4.7: Dependence of the LFV branching ratios on $\tan\beta$. We input $(f, \sin 2\theta) = (6\,\mathrm{TeV}, 0.25)$, $\chi_1 = 0.25$, $\chi_2 = 10\,\chi_1$. Normalization as in Figure 4.4.

negligible contributions in comparison with photon-penguin diagrams in leptonic processes [123], [117]. However it has been pointed out that this might not be the case in other

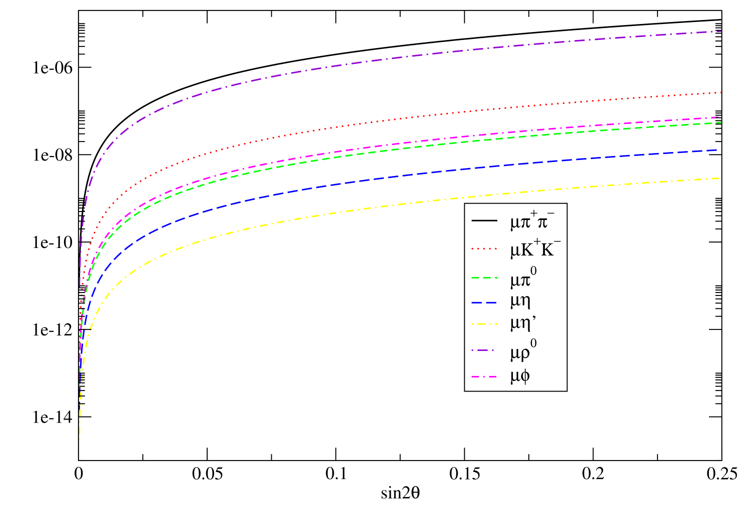


Figure 4.8: Dependence of the LFV branching ratios on $\sin 2\theta$. We input $(f, t_\beta) = (6\,\text{TeV}, 5)$, $\chi_1 = 0.25$, $\chi_2 = 10\,\chi_1$. Normalization as in Figure 4.4.

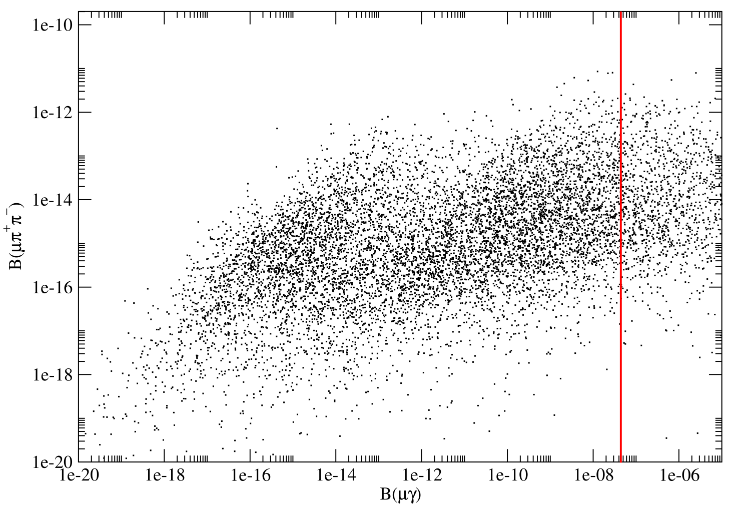


Figure 4.9: Scattered plot that show correlation between different branching ratios: $B(\tau \to \mu\pi^+\pi^-)$ versus $B(\tau \to \mu\gamma)$. We vary $2 < f < 10$ TeV; $0 < \sin(2\theta) < 0.25$; $1 < t_\beta < 10$; $0 < \chi_1 < 0.25$ and $\chi_2 = a\chi_1$ for $1.1 < a < 10$. Red line indicate the present upper bound for $B(\tau \to \mu\gamma)$ at 90% C.L..

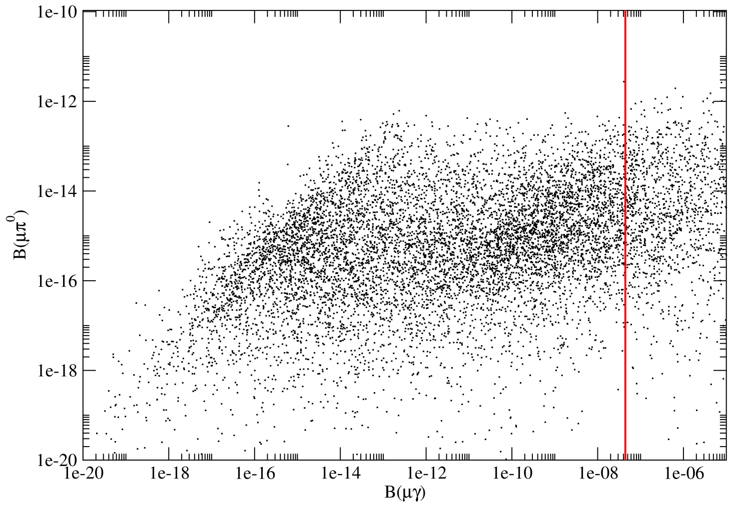


Figure 4.10: Scattered plot that show correlation between different branching ratios: $B\left(\tau \rightarrow \mu\pi^0\right)$ versus $B\left(\tau \rightarrow \mu\gamma\right)$. We vary $2 < f < 10$ TeV; $0 < \sin\left(2\theta\right) < 0.25$; $1 < t_\beta < 10$; $0 < \chi_1 < 0.25$ and $\chi_2 = a\chi_1$ for $1.1 < a < 10$. Red line indicate the present upper bound for $B\left(\tau \rightarrow \mu\gamma\right)$ at 90% C.L..

models. For instance, in the Littlest Higgs model with T-parity both contributions are of the same order in $\mu \rightarrow ee\bar{e}$ [47] and the same happens in the same purely leptonic processes within the SLH model [48]. Notwithstanding in LFV hadron decays of the tau lepton within this later model, that we have studied in this article, we do not reach the same conclusion, at least in the *Unitary Gauge*. In Figure 4.12 we show the dependence on f for the different contributions for $\tau \rightarrow \mu\pi^+\pi^-$. It can be seen that in hadron decays photon-penguin diagrams dominate over the rest of contributions. Box diagrams give a small input although one can see that they interfere destructively with the photon ones. Meanwhile the Z- and Z'-penguin diagrams are negligible. In Figure 4.13 we plot the analogous comparison for $\tau \rightarrow \mu\pi^0$ where, obviously, there are not photon-penguin diagrams contributing. Then box diagrams give the bulk of the branching ratio though with a non negligible positive interference of the Z penguin contribution. In Figure 4.11 we noticed the high correlation between both $\tau \rightarrow \mu\pi^+\pi^-$ and $\tau \rightarrow \mu\pi^0$ decays. This seems eye catching because, as we have seen, both processes are dominated by different contributions: the first by the photon-penguin diagrams and the second by the boxes.

As commented at the beginning of this chapter we did not include Higgs penguin contribution on the basis that their couplings to light quarks are suppressed by their masses. In [124], [125] it was pointed out that a Higgs could couple, through a one loop of heavy quarks to two gluons able to hadronize into one or two pseudoscalars and, at least in the latter case, give a relevant contribution comparable with the one of the photon penguin amplitude.

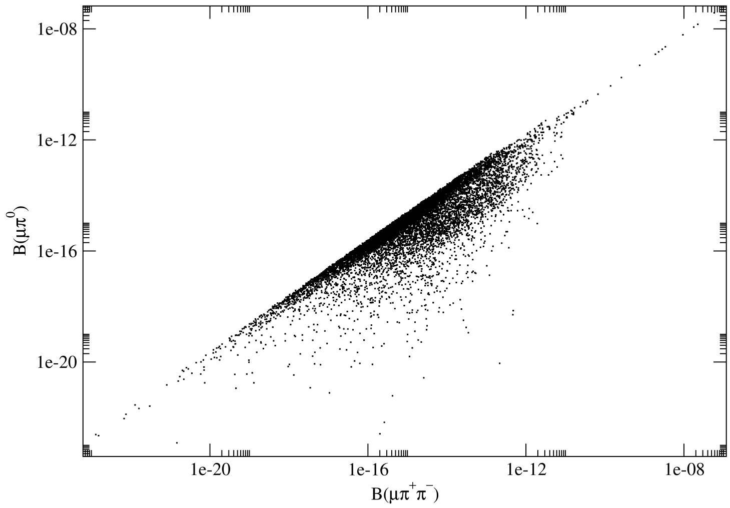


Figure 4.11: Scattered plot that show correlation between different branching ratios: $B\left(\tau \to \mu\pi^0\right)$ versus $B\left(\tau \to \mu\pi^+\pi^-\right)$. We vary $2 < f < 10$ TeV; $0 < \sin(2\theta) < 0.25$; $1 < t_\beta < 10$; $0 < \chi_1 < 0.25$ and $\chi_2 = a\chi_1$ for $1.1 < a < 10$.

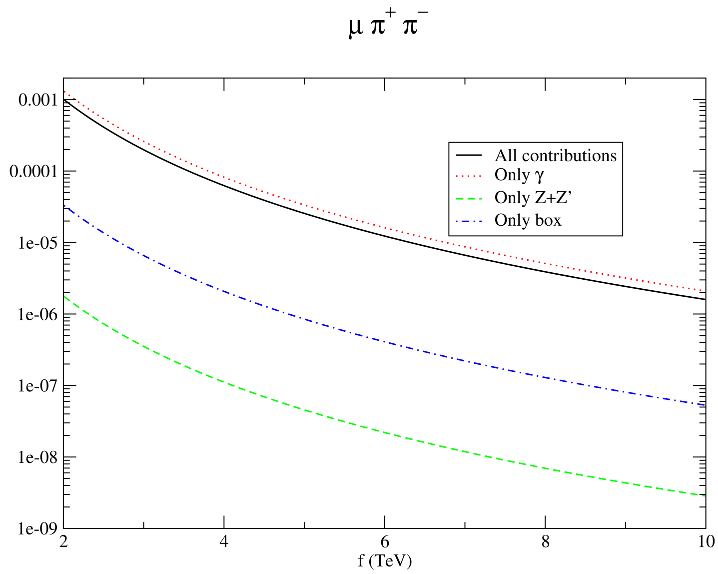


Figure 4.12: Contributions to $B\left(\tau \to \mu\pi^+\pi^-\right)$ of the different penguins with final bosons $\gamma; Z; Z'$, of the boxes and their sum. Normalization as in Figure 4.4.

This is indeed a two loop calculation in our framework and we have not considered to sum

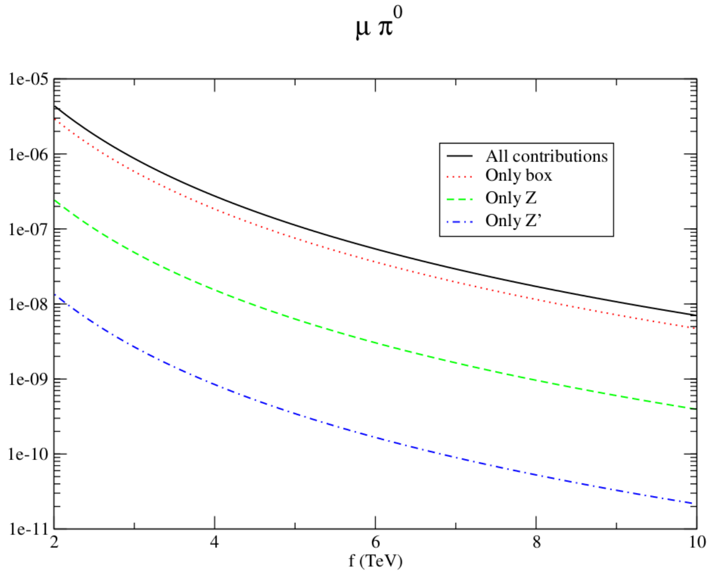


Figure 4.13: Contributions to $B\left(\tau \to \mu\pi^0\right)$ of the different penguins with final bosons $Z; Z'$, of the boxes and their sum. Normalization as in Figure 4.4.

this addition. In our opinion this could change our results by a factor not larger than $\mathcal{O}(1)$ and therefore it would not change our main conclusions.

In [48] it was indicated that, at least in LFV decays of the muon into leptons and muon conversion in nuclei, the behaviour of the SLH model is very similar to the Littlest Higgs with T-parity. If that assertion could be extended to the hadron decays of the tau lepton, as it seems rather sensible, we would definitely conclude that Little Higgs models predict a high suppression for these channels. It is now the turn of the flavour factories to clarify this issue.

Chapter 5

$H \to \ell\ell'$ in the Simplest Little Higgs Model

5.1 Background

Both ATLAS and CMS have announced the discovery of a Higgs-like resonance with a mass of 125 GeV [65], [126], further supported by combined Tevatron data [127]. An interesting question is whether the properties of this resonance are consistent with the SM Higgs boson. Deviations from the SM predictions could point to the existence of a secondary mechanism of EW symmetry breaking or to other types of new physics not too far above the EW scale. While there is a large ongoing experimental effort to measure precisely the decay rates into the channels that dominate for the SM Higgs, it is equally important to search for Higgs decays into channels that are subdominant or absent in the SM. For instance, since the couplings of the Higgs boson to quarks of the first two generations and to leptons are suppressed by small Yukawa couplings in the SM, new physics contributions can easily dominate over the SM predictions.

The indirect constraints on many flavor violating Higgs decays are rather weak. In particular, the branching ratios $B\left(H \to \tau\mu\right)$ and $B\left(H \to \tau e\right)$ can reach up to 10% [128], [129]. For the channels $H \to \tau\mu$ and $H \to \tau e$ the LHC is placing limits that are comparable to or even stronger than those from rare τ decays. CMS provides the best fit branching fraction of $H \to \tau\mu$ [130]. We emphasize that large deviations from the SM do not require very exotic flavor structures. A branching ratio $B\left(H \to \tau\mu\right)$ comparable to the one for $B\left(H \to \tau\tau\right)$, or a $B\left(H \to \mu\mu\right)$ a few times larger than in the SM can arise in many models of flavor [128], [129], [131], [132], [133], [134], [135], [136], [137], in this thesis we also computed LFV Higgs decays within the SLH model.

In fact, there may already be experimental hints that the Higgs couplings to fermions may not be SM like. For instance, the BaBar collaboration recently announced a 3.4 σ indication of flavor universality violation in $b \to c\tau\nu$ transitions [79], which can be explained for instance by an extended Higgs sector with nontrivial flavor structure [138].

The presence of LFV couplings would allow $\mu \to e$, $\tau \to \mu$ and $\tau \to e$ transitions to proceed via a virtual Higgs boson [139], [140]. The experimental limits on these have recently been translated into constraints on the branching fractions $B(H \to \mu e,\ \tau\mu,\ \tau e)$ [141], [128]. The $\mu \to e$ transition is strongly constrained by null search results for $\mu \to e\gamma$ [118], $B(H \to \mu e) < 10^{-8}$. However, the constraints on $\tau \to \mu$ and $\tau \to e$ are much less stringent. Especially the last reinterpretation of the ATLAS $H \to \tau\tau$ search results in terms of LFV decays by an independent group has been used to set limits at the 95% confidence level (CL) of $B(H \to \tau\mu) < 13\%$, $B(H \to \tau e) < 13\%$ [141].

5.2 $H \to \tau\ \mu$

We studied the diagrams reported in Figure 5.1, where q is the momentum of the Higgs, p' of the muon and p is the momentum of the tau. We consider:

$$\begin{aligned} p'^2 &= 0, \\ p^2 &= m_\tau^2, \\ q^2 &= M_H^2, \end{aligned} \tag{5.1}$$

In this case we used the same integrations of the Chapter 4, in the limit $\frac{m_\tau}{M} \to 0$, also we ignored the SM neutrino masses $m_{\nu_i} = 0$.

In these LFV Higgs decays there is at least a lepton which can be considered massless and, thus, with fixed helicity. Then, the matrix element for the $H \to \tau\ell$ decays can be written as:

$$\mathcal{M}_H = -\frac{ie^2 \sum_j V_{\tau j} V^*_{\mu j} m_\tau}{s_w^2 16\pi^2 M_W^2} \left(O(\chi_j) \log\left(\frac{\chi_j}{\omega}\right) + P(\chi_j) \right) \psi(p') P_R \psi(q, m_\tau), \tag{5.2}$$

where:

$$O(\chi_j) = \frac{\delta_\nu}{16 M_W^2 \sqrt{\chi_j}} \left[\frac{e^2 v \delta_\nu \sqrt{\chi_j}}{s_w} \left(M_H^2 - 13 M_W^2\right) + 2\sqrt{2} \lambda_{N_j} M_{W'} M_W^2 c_\beta \left(\delta_\nu^2 \chi_j - 3\omega\right) \right] \tag{5.3}$$

and

$$P(\chi_j) = \delta_\nu \left(P_1(\chi_j) - \frac{v e^2}{s_w} P_2(\chi_j) \right) \tag{5.4}$$

with:

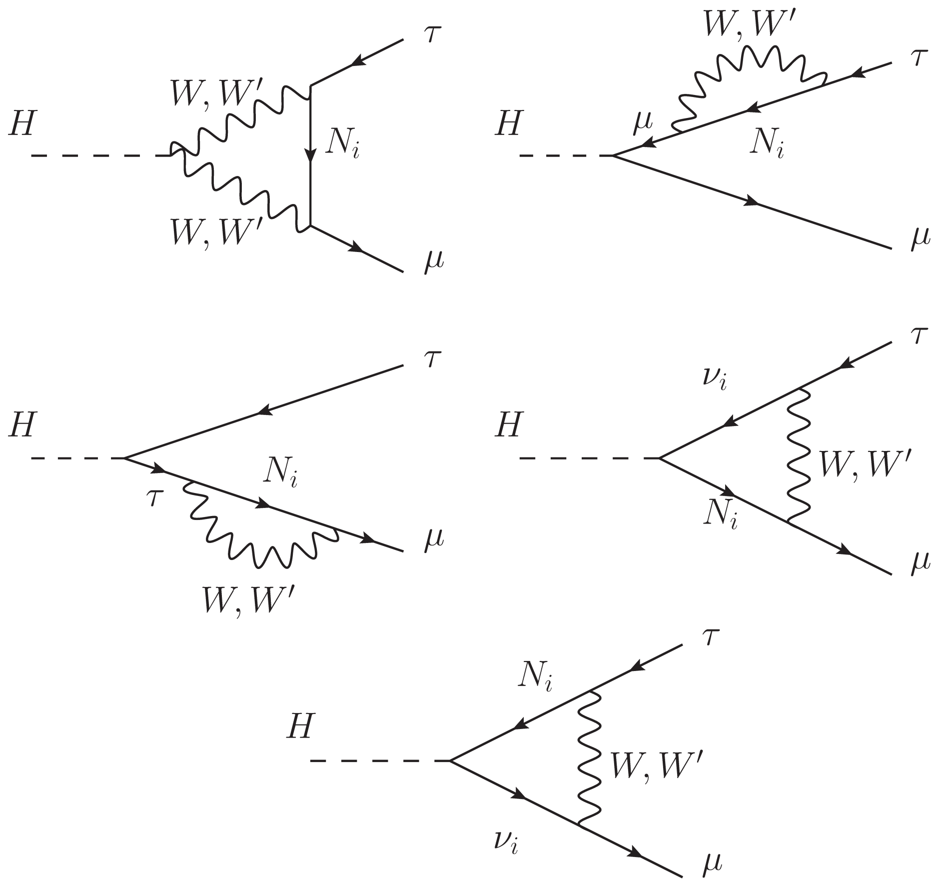


Figure 5.1: Feynman diagrams for $H \to \tau\mu$ decays in the SLH model.

$$
\begin{aligned}
P_1(\chi_j) =& \lambda_{N_j} c_\beta \frac{\delta_\nu^2 (\chi_j - 1) M_N^2 + 2\left(\chi_j^2 - \chi_j + 2\right) M_W^2 - 14 (\chi_j - 1) M_H^2}{8\sqrt{2} (\chi_j - 1) M_N} \\
P_2(\chi_j) =& \cot(2\beta) \frac{M_H^2 \left(3\chi_j^3 - 9\chi_j^2 + 8\chi_j - 2\right) + M_W^2 \chi_j \left(12\chi_j^2 - 23\chi_j + 10\right)}{8 M_{W'} M_W \chi_j (\chi_j - 1)} + \\
& + \frac{\delta_\nu}{192 M_W^6 \sin^2(2\beta)} \Big\{ 2M_H^6 + 14 M_H^4 M_W^2 + M_H^2 M_W^4 (29 - 40\chi_j) - \cos(4\beta) \Big[2M_H^6 + \\
& + 14 M_H^4 M_W^2 + M_H^2 M_W^4 (24\chi_j + 29) + 8M_W^6 (7 - 12\chi_j) \Big] + 8 M_W^6 (20\chi_j + 7) \Big\} + \\
& + \frac{\omega \left(2\chi_j^4 + 20\chi_j^3 - 74\chi_j^2 + 35\chi_j - 1\right)}{48 (\chi_j - 1)^2}
\end{aligned}
\tag{5.5}
$$

then we can calculate the branching ratio like in the Chapter 4:

$$B\left(H\to\tau\mu\right)=\frac{(M_H^2-m_\tau^2)^2\alpha^2 m_\tau^2}{256\pi^3 M_H^3\Gamma_H M_W^4 s_w^4}\left(\frac{\sin2\theta}{2}\right)^2\left[O(\chi_1)\log\left(\frac{\chi_1}{\omega}\right)-O(\chi_2)\log\left(\frac{\chi_2}{\omega}\right)+\right. \\ \left.+P(\chi_1)-P(\chi_2)\right]^2. \tag{5.6}$$

where $\alpha=\frac{e^2}{4\pi}$.

In our computation we kept terms of subleading order (v^3/f^3) and checked for accidental numerical enhancements of these before neglecting them. After checking its irrelevance, we omitted one such a term in $O(\chi_j)$ and another one in $P(\chi_j)$. With these simplifications, equation (5.2) reads:

$$\mathcal{M}_H=-\frac{iv\alpha^2\delta_\nu m_\tau\sum_j V_{\tau j}V_{\mu j}^*}{s_w^3 M_W^2}\left[O\log\chi_j-P(\chi_j)\right]\bar{\psi}(p')P_R\psi(q,m_\tau)\,, \tag{5.7}$$

where:

$$O=\frac{\delta_\nu}{16}\left(M_H^2-13M_W^2\right) \tag{5.8}$$

and

$$P(\chi_j)=\cot\left(2\beta\right)\frac{M_H^2\left(3\chi_j^3-9\chi_j^2+8\chi_j-2\right)+M_W^2\chi_j\left(12\chi_j^2-23\chi_j+10\right)}{8M_{W'}M_W\chi_j\left(\chi_j-1\right)}+ \\ +\frac{\omega\left(2\chi_j^4+20\chi_j^3-74\chi_j^2+35\chi_j-1\right)}{48\left(\chi_j-1\right)^2}+ \\ +\frac{\delta_\nu\chi_j}{24M_W^2\sin^2\left(2\beta\right)}\left[-5M_H^2+3\cos\left(4\beta\right)\left(4M_W^2-M_H^2\right)\right]. \tag{5.9}$$

The corresponding branching ratio is:

$$B\left(H\to\tau\mu\right)=\frac{(M_H^2-m_\tau^2)^2\alpha^4 v^2\delta_\nu^2 m_\tau^2}{16\pi M_H^3\Gamma_H M_W^4 s_w^6}\left(\frac{\sin2\theta}{2}\right)^2\left[O\log\left(\frac{\chi_1}{\chi_2}\right)+P(\chi_2)-\right. \\ \left.-P(\chi_1)\right]^2. \tag{5.10}$$

5.3 Numerical results

LFV Higgs decays arise at one-loop level in the SLH model and are possible because the heavy neutrinos N_k couple to either charged lepton, ℓ_i, irrespective of its flavor. In the considered $H \to \ell\ell'$ decays all the topologies sketched in Figure 5.1 contribute.

Since the Higgs boson couples not only to a pair of $W^{(\prime)}$ but also to WW', the first topology gives rise to four different diagrams [1]. Then, there are 12 different diagrams at this order working in the ***Unitary Gauge***, where only physical degrees of freedom appear and the number of diagrams is reduced.

The contributions of self-energy type (second and third diagrams) are proportional to m_ℓ and will thus be neglected for $\ell = e,\ \mu$. For definiteness we include our results for the $H \to \tau\ell$ decay. There is an overall dependence on the heaviest final-state lepton mass, which shows that the decay rate $H \to \ell\ell'$ vanishes in the limit of massless decay products. Therefore, and in absence of a mechanism of lepton universality violation in the SLH model, we will have $B(H \to \tau\mu) = B(H \to \tau e) = \frac{m_\tau^2}{m_\mu^2} B(H \to \mu e)$, which suppresses the latter decay rate by a factor $\simeq 283$. Given this trivial proportionality, we will only be plotting $B(H \to \tau\ell)$ in Figures 5.2, 5.3, 5.4 and 5.5.

Within this setting, there are two different mass scales in the problem: those of $\mathcal{O}(v)$ (M_W and M_H) and those of $\mathcal{O}(f)$ (M_N and $M_{W'}$). In Chapter 4 we used $\omega = \frac{M_W^2}{M_{W'}^2} \simeq \frac{v^2}{f^2} << 1$ to characterize the ratio between two separated scales and $\chi_j = \frac{M_{N_j}^2}{M_{W'}^2} \simeq \mathcal{O}(1)$ for that of two high scales. Since there were only three mass scales in the study of semileptonic LFV tau decays within this model, all mass ratios could be expressed in terms of ω and χ_j (unless there is a very strong hierarchy between the different heavy neutrino flavors). In the present study, there is M_H, as well. This entails the appearance of four small ratios between a light and a heavy particle mass: $\frac{M_H^2}{M_{N_j}^2} \simeq \frac{M_W^2}{M_{N_j}^2} \simeq \frac{M_H^2}{M_{W'}^2} \simeq \frac{M_W^2}{M_{W'}^2} = \omega \simeq \frac{v^2}{f^2} << 1$ (we recall that $\delta_\nu \simeq \frac{v}{f}$ as well) and two involving particles with similar masses: $\frac{M_W^2}{M_H^2} \simeq \frac{M_{N_j}^2}{M_{W'}^2} = \chi_j \simeq \mathcal{O}(1)$.

Our analytical expressions of the section 5.2 are simplified in the limit of only two heavy neutrinos that we followed in Chapter 4. In the numerical analysis we have to stick to the choices argued in our previous work in Chapter 4, that we recall in the following:

- We have varied the scale of compositeness between 2 and 10 TeVs. Lower values are in tension with electroweak precision observables and larger figures enter the region where a UV completion of the SLH model (that would become strongly coupled) starts to be expected.

- The LFV processes are possible in the SLH model because of the presence of the heavy neutrinos. The dependence of the amplitude on their contribution is

[1] We point out that exchanging $W \leftrightarrow W'$ in the diagrams built with the HWW' vertex yields two different results, as can easily be shown.

$$\mathcal{M} = \sum_j V_\ell^{j\mu *} V_\ell^{j\tau} A\left(\chi_j\right). \tag{5.11}$$

Assuming two families and one mixing angle, this can be written:

$$\mathcal{M} = \sin\theta\cos\theta\left[A(\chi_1) - A(\chi_2)\right]. \tag{5.12}$$

We used this simplification to write the equations (5.7), (5.8), (5.9), (5.10). Particularly, since the terms with $\log(\omega)$ are independent on χ_j, they do not contribute and are not quoted in our results. This, by the way, prevents the appearance of a moderately large $\log(\chi_j/\omega)$. In the numerics, we used the limits $0 \leq \chi_1 \leq 0.25$ and $1.1\chi_1 \leq \chi_2 \leq 10\chi_1$ and $\sin 2\theta \leq 0.25$, consistent with current data.

- Finally, the ratio of the two vevs in the model, $\tan\beta$, is also a free parameter of the SLH model. We took the range $1 < \tan\beta < 10$ for it.

We have performed a scan of the parameter space limited by the above restrictions and plotted $B(H \to \tau\ell)$ as a function of one parameter in turn (f, M_{N_1}, $\tan\beta$ and $\sin 2\theta$) in Figures 5.2, 5.3, 5.4 and 5.5.

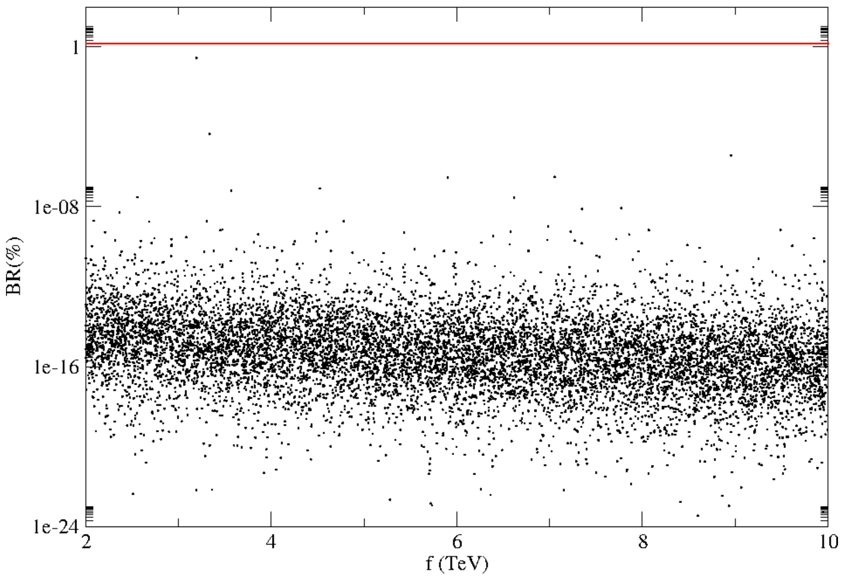


Figure 5.2: Dependence of the scale of compositeness, f, of the branching ratio of the $H \to \tau\ell$ decays in the SLH model. The red line shows the 95% CL upper bound by CMS.

The general trend is that the SLH model produces $H \to \tau\ell$ decay widths which are eight to ten orders of magnitude smaller than the $B \simeq \mathcal{O}(\%)$ hinted by CMS. As expected, if the SLH model is to satisfy the bounds on $H \to \mu e$ set by $\mu \to e\gamma$ ($B(H \to \mu e) \lesssim 10^{-8}$ [129]) as it does, it must fall way too short to explain the CMS signal, as a consequence

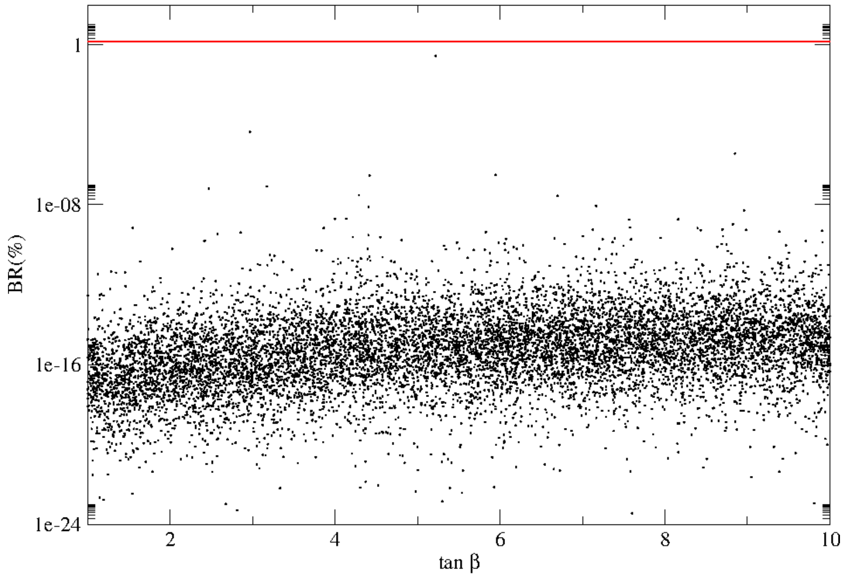


Figure 5.3: Dependence on the ratio of the two vevs, $\tan\beta$, of the branching ratio of the $H \to \tau\ell$ decays in the SLH model. The red line shows the 95% CL upper bound by CMS.

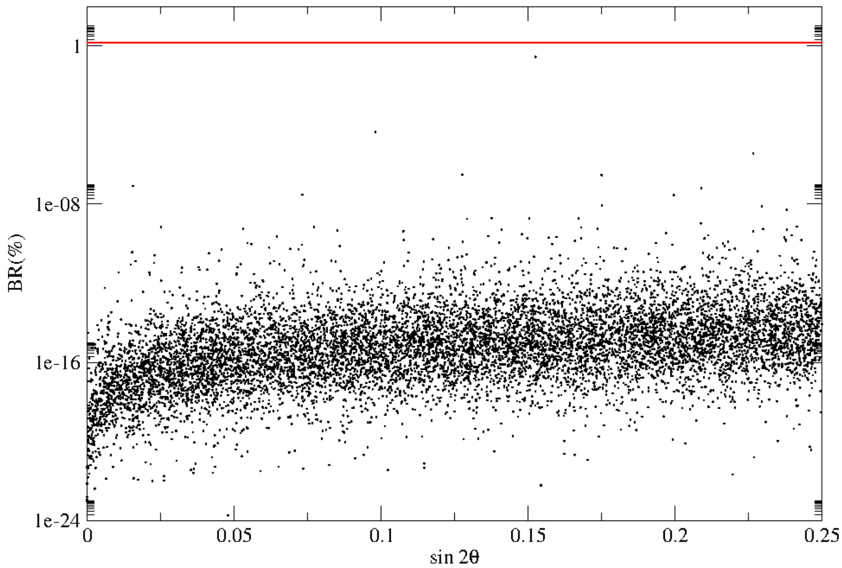


Figure 5.4: Dependence on the mixing angle between the two heavy leptons, $\sin 2\theta$, of the branching ratio of the $H \to \tau\ell$ decays in the SLH model. The red line shows the 95% CL upper bound by CMS.

of its lepton universality. Besides this, very mild variations are appreciated in Figures 5.2, 5.3, 5.4 and 5.5 with respect to the independent variables. Decay probabilities are slightly larger for smaller f and M_{N_1} and for larger $\tan\beta$ and $\sin 2\theta$.

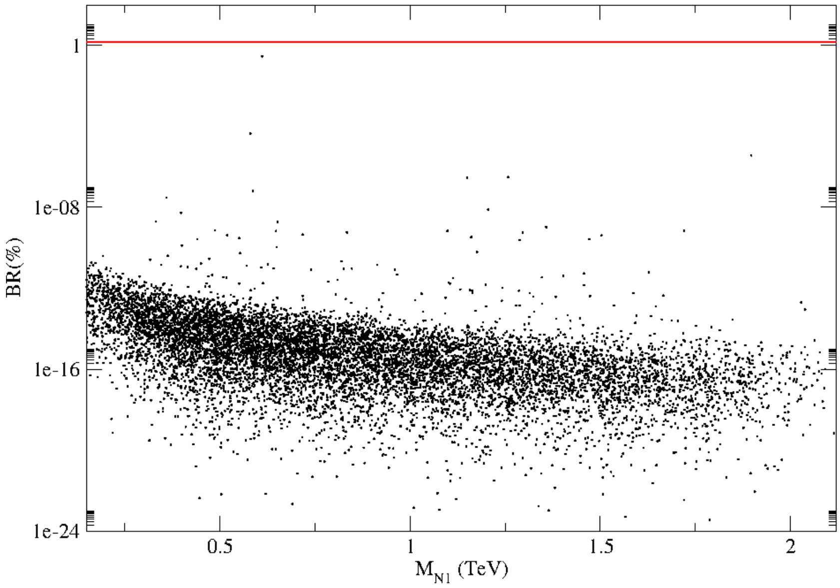


Figure 5.5: Dependence on the largest mass of the heavy neutrinos, M_{N_1}, of the branching ratio of the $H \to \tau\ell$ decays in the SLH model. The red line shows the 95% CL upper bound by CMS.

Part III

Conclusions and Appendices

Chapter 6

Conclusions

In this thesis I tried to review briefly the SM (Part I, Chapter 1) and his frontiers with particular attention to processes involving flavour violation of charged lepton (Chapter 2) then I summarized one of the options for physics beyond the SM, the LH models in the Chapter 3.

Lepton Flavour Violating decays are, due to their high suppression in the SM, an excellent benchmark where to look for new physics. Though present upper bounds are very tight both in $\mu \to e\gamma$ and other muonic decays into leptons where one could expect that LFV, if any, will be first observed, tau physics provide the unique property of being the only lepton decaying into hadrons and, consequently, offer a new scenario that, moreover, has been thoroughly explored in B-factories like Babar and Belle. Present upper limits on branching ratios of the studied processes are of $\mathcal{O}(10^{-8})$ and future flavour factories, like Belle II, could lower those up to two orders of magnitude. Therefore the study of LFV hadron decays of the tau lepton is all important in order to face the near future experimental status.

In Chapter 4 I have analysed LFV decays of the tau lepton into one pseudoscalar, one vector or two pseudoscalar mesons in the SLH, characterized as a composite Higgs model with a simple group $SU(3)_L \times U(1)_X$ and with a scale of compositeness $f \sim 1$ TeV were a feature of collective symmetry breaking [142] occurs providing a light Higgs boson. This model has interesting features like a reduced extension of the spectrum of gauge bosons and fermions over the SM ones and a small number of unknown parameters. In contrast the model has no custodial symmetry [143], [144], though its lack does not bring large unwanted corrections. For the inclusion of the quark sector I use the anomaly free embedding that does not need the role of an ultraviolet completion in order to cancel a gauge anomaly in the extended sector. The model has already been confronted with LHC data [119], [107] and keeps its strength waiting for more precise determinations.

I have considered the study of several hadron decays of the tau lepton, i.e. $\tau \to \mu(P, V, PP)$ decays where P is short for a pseudoscalar meson and V for a vector one. The leading amplitude for these decays, in the SLH model, is given by a one loop contribution dynamically driven by the mixing of the light charged leptons, τ and μ with the heavy "little" neutrinos of the model. I have carried out the calculation at leading order in the $\frac{v}{f}$ expansion and our results are $\mathcal{O}\left(\frac{v^2}{f^2}\right)$. For the numerical determination of the branching ratios I have considered previous constraints on the input constants of the model, although I have allowed their variation rather prodigally in order to convey the generic pattern of the predictions. Hence I have studied the dependence of the branching ratios on the relevant parameters of the model.

I conclude that the predictions of the SLH model for theses processes are, typically, between 5 and 8 orders of magnitude smaller than present upper bounds and, therefore, out of reach for the foreseen next flavour factories. An observation of any of these decays by Belle II not only would signal new physics but also would falsify the SLH model.

In Chapter 5 I calculated $H \to \ell\ell'$ always in the framework of SLH. LFV although extremely suppressed in the SM extended with right handed neutrinos, it may appear at measurable rates in several well-motivated new physics models. On the other hand, the discovery of the Higgs boson at the LHC has brought a new scenario to search for LFV in the decays of this scalar. An elegant solution to the hierarchy problem on the Higgs mass is provided by the LH models. In Chapter 5 we have considered the SLH model (one of the simplest realizations of these ideas) against the ATLAS and CMS limits on $B(H \to \tau\mu)$, of order percent. Given the couplings of the Higgs to the leptons in the SLH, is not surprising that the SLH cannot simultaneously account for the tiny rate at which the $H \to \mu e$ decays must proceed and also for a measurable signal at LHC. We have found that a $B(H \to \mu e)$ as low as 10^{-12} is obtained naturally, with $B(H \to \tau\ell)$ only enhanced by a factor of order 300. Thus, the confirmation of the CMS hint would rule out the SLH model, as it will do a measurement at Belle-II of semileptonic LFV tau decays [145].

Appendix A

Dirac Matrices

A.1 Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu},$$

$$\gamma^0 = \beta \quad ; \quad \vec{\gamma} = \beta\vec{\alpha},$$

$$\begin{gathered}
\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \\
= -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^3\gamma^2\gamma^1\gamma^0 = {\gamma_5}^+, \\
{\gamma_5}^2 = I, \\
\{\gamma^5, \gamma^\mu\} = 0, \\
\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \Rightarrow \gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}, \\
[\gamma^5, \sigma^{\mu\nu}] = 0 \Rightarrow \gamma^5\sigma^{\mu\nu} = \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}, \\
\gamma_5\gamma^0\vec{\gamma} = \vec{\Sigma} \equiv \Sigma^i \equiv \frac{1}{2}\varepsilon_{ijk}\sigma^{jk}.
\end{gathered}$$

A.2 Hermitian conjugated

$$\begin{gathered}
\gamma^0\gamma^\mu\gamma^0 = {\gamma^\mu}^+, \\
\gamma^0\gamma_5\gamma^0 = -{\gamma_5}^+ = -\gamma_5, \\
\gamma^0(\gamma_5\gamma^\mu)\gamma^0 = (\gamma_5\gamma^\mu)^+, \\
\gamma^0\sigma^{\mu\nu}\gamma^0 = (\sigma^{\mu\nu})^+,
\end{gathered}$$

If ψ_i is a spinor and Γ is a 4×4 matrix:

$$(\bar{\psi}_1\Gamma\psi_2)^\dagger = \bar{\psi}_2(\gamma_0\Gamma^\dagger\gamma_0)\psi_1.$$

A.3 Charge conjugation

$$C\gamma_\mu C^{-1} = -{\gamma_\mu}^T,$$
$$C\gamma_5 C^{-1} = {\gamma_5}^T$$
$$C\sigma_{\mu\nu} C^{-1} = -{\sigma_{\mu\nu}}^T,$$
$$C(\gamma_5\gamma_\mu)C^{-1} = (\gamma_5\gamma_\mu)^T.$$

A.4 Representations

Dirac representation:

$$\gamma^0 = \beta = \sigma^3 \otimes I = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

$$\vec{\alpha} = \sigma^1 \otimes \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix},$$

$$\vec{\gamma} = \beta\vec{\alpha} = i\sigma^2 \otimes \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix},$$

$$\gamma_5 = \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = \sigma^1 \otimes I,$$

$$\gamma^5\gamma^0 = -i\sigma^2 \otimes I = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix},$$

$$\gamma^5\vec{\gamma} = -\sigma^3 \otimes \vec{\sigma} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

,

$$\gamma^5\gamma^0\vec{\gamma} = \vec{\Sigma} = I \otimes \vec{\sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix},$$

$$\sigma^{0i} = i\sigma^1 \otimes \sigma^i = i\alpha^i = i\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix},$$

$$\sigma^{ij} = \varepsilon_{ijk} I \otimes \sigma^k = \varepsilon_{ijk}\Sigma^k = \varepsilon_{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix},$$

$$C = i\gamma^2\gamma^0 = -i\sigma^1 \otimes \sigma^2 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}.$$

Majorana representation:

$$\gamma^0 = \beta = \sigma^1 \otimes \sigma^2 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\alpha^1 = -\sigma^1 \otimes \sigma^1 = \begin{pmatrix} 0 & -\sigma^1 \\ -\sigma^1 & 0 \end{pmatrix},$$

$$\alpha^2 = \sigma^3 \otimes I = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

$$\alpha^3 = -\sigma^1 \otimes \sigma^3 = \begin{pmatrix} 0 & -\sigma^3 \\ -\sigma^3 & 0 \end{pmatrix},$$

$$\gamma^1 = iI \otimes \sigma^3 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix},$$

$$\gamma^2 = -i\sigma^2 \otimes \sigma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = -iI \otimes \sigma^1 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix},$$

$$\gamma_5 = \gamma^5 = \sigma^3 \otimes \sigma^2 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix},$$

$$C = -i\sigma^1 \otimes \sigma^2 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}.$$

Correlation between the representations of Dirac and Majorana:

$$\gamma^\mu_{Majorana} = U\gamma^\mu_{Dirac}U^\dagger \quad / \quad U = U^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} I & \sigma^2 \\ \sigma^2 & -I \end{pmatrix}.$$

Chiral representation:

$$\gamma^0 = \beta = -\sigma^1 \otimes I = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix},$$

$$\vec{\alpha} = \sigma^3 \otimes \vec{\sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix},$$

$$\vec{\gamma} = i\sigma^2 \otimes \vec{\sigma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix},$$

$$\gamma_5 = \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

$$C = -i\sigma^3 \otimes \sigma^2 = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix},$$

$$\sigma^{0i} = i\begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix},$$

$$\sigma^{ij} = \varepsilon_{ijk}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}.$$

Correlation between the chiral representation and Dirac representation:

$$\gamma^\mu_{chirale} = U\gamma^\mu_{Dirac}U^\dagger \quad / \quad U = \frac{1}{\sqrt{2}}(1+\gamma^5\gamma^0) = \frac{1}{\sqrt{2}}\begin{pmatrix} I & -I \\ I & I \end{pmatrix},$$

All the representations satisfy:

$$C^T = C^\dagger = -C, \qquad CC^\dagger = C^\dagger C = I, \qquad C^2 = -I.$$

A.5 Indices contractions

D dimensions:

$$\begin{aligned}
\not{a}\not{b} &= a\cdot b - i\sigma_{\mu\nu}a^\mu b^\nu, \\
\gamma^\lambda\gamma_\lambda &= 4, \\
\gamma^\lambda\gamma^\mu\gamma_\lambda &= (2-D)\,\gamma^\mu, \\
\gamma^\lambda\gamma^\mu\gamma^\nu\gamma_\lambda &= 4g^{\mu\nu} + (D-4)\,\gamma^\mu\gamma^\nu, \\
\gamma^\lambda\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\lambda &= -2\gamma^\rho\gamma^\nu\gamma^\mu + (4-D)\,\gamma^\mu\gamma^\nu\gamma^\rho, \\
\gamma^\lambda\sigma^{\mu\nu}\gamma_\lambda &= 0,
\end{aligned}$$

4 dimensions:

$$\begin{aligned}
tr(\gamma^5\gamma^\mu) &= 0, \\
tr(\gamma^\mu\gamma^\nu) &= 4g^{\mu\nu}, \\
tr(\sigma^{\mu\nu}) &= 0, \\
tr(\gamma^\mu\gamma^\nu\gamma^5) &= 0, \\
tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}), \\
tr(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= -4i\varepsilon^{\mu\nu\rho\sigma} = 4i\varepsilon_{\mu\nu\rho\sigma}, \\
tr(\not{a}_1\not{a}_2\cdots\not{a}_{2n}) &= tr(\not{a}_{2n}\cdots\not{a}_2\not{a}_1),
\end{aligned}$$

Appendix B

Others Matrices and Algebraic relations

B.1 Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

B.2 Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

B.3 Levi-Civita tensor

$$\epsilon^{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{if } \{\mu,\nu,\rho,\sigma\} \text{ is a even permutation of } \{0,1,2,3\} \\ -1 & \text{if is a odd permutation} \\ 0 & \text{all the other permutations} \end{cases},$$

$$\begin{aligned}
&\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}, \\
&\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu`\nu`\rho`\sigma`} = -\det(g^{\alpha\alpha'}) \quad / \quad \alpha = \mu,\nu,\rho,\sigma, \alpha' = \mu',\nu',\rho',\sigma', \\
&\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu}{}^{\nu`\rho`\sigma`} = -\det(g^{\alpha\alpha'}) \quad / \quad \alpha = \nu,\rho,\sigma, \alpha' = \nu',\rho',\sigma', \\
&\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu}{}^{\rho`\sigma`} = -2(g^{\rho\rho'}g^{\sigma\sigma'} - g^{\rho\sigma'}g^{\rho'\sigma}), \\
&\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho}{}^{\sigma`} = -6g^{\sigma\sigma'}, \\
&\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -24,
\end{aligned},$$

in 3 dimensions:

$$\epsilon^{ijk} = \epsilon_{ijk} = 1 \qquad \text{if i,j,k is even permutation of 1,2,3.}$$

B.4 Contribution of p_α

When we assume that the light quarks have negligible masses, respect the energy and the masses of the bosons, we can exclude the contribution of p_α, the form factors F_3,G_3, because:

$$\begin{aligned} \mathcal{M}_{\tau\to\mu+V} &\propto \left[a_3\left(p^2\right)P_L + b_3\left(p^2\right)P_R\right]p^\alpha \\ \mathcal{M}_{V\to\bar{q}q} &= \bar{u}_q\left[a\left(p^2\right)P_L + b\left(p^2\right)P_R\right]\gamma^\beta u_q, \end{aligned} \tag{B.1}$$

so if we add the vector propagator (the photon case is simpler but the final result is the same) and remembering that we are interested only to the contribution of "p", schematically:

$$\begin{aligned} \mathcal{M}_{\tau\to\mu+V}\left(-i\frac{g_{\alpha\beta}-\frac{p_\alpha p_\beta}{M_V^2}}{p^2-M_V^2}\right)\mathcal{M}_{V\to\bar{q}q} \to \mathcal{M}_{\tau\to\mu+V+\bar{q}q} \propto& \\ &\propto \bar{u}_q\left(c_1+c_2\gamma_5\right)\not{p}u_q = \\ &= c_1\bar{u}_q\left(m_q-m_q\right)u_q + c_2\bar{u}_q\gamma_5\left(\not{p}_{\bar{q}}+\not{p}_q\right)u_q = \\ &= c_2\bar{u}_q\left(-m_q\gamma_5-\gamma_5 m_q\right)u_q = \\ &= -2m_q c_2\bar{u}_q\gamma_5 u_q \simeq \\ &\simeq 0, \end{aligned} \tag{B.2}$$

because we supposed $m_q = \{m_u, m_d, m_s\}$ negligible.

Appendix C

Cancellation of divergences in the Unitary Gauge

We study the channel:

$$\tau^- (q) \to \mu (p') \, e^+ (p_2) \, e^- (p_1) \, ,$$

in the *Unitary Gauge*, with the convention:

$$p = q - p' = p_1 + p_2,$$

so for simplicity we define:

$$\begin{aligned} \bar{u} (p_1) \, \gamma_\mu v (p_2) &= [L (e) + R (e)]_\mu \, , \\ \bar{u} (p_1) \, \gamma_\mu P_L (p_2) &= L (e)_\mu \, , \\ \bar{u} (p_1) \, \gamma_\mu P_R (p_2) &= R (e)_\mu \, , \\ \bar{u} (p') \, \gamma_\mu P_L (q) &= L (2)_\mu \, . \end{aligned} \tag{C.1}$$

For the LFV we have the global factor:

$$F = \sum_i V_{\tau i} V^*_{i\mu} M^2_{n_i} \delta^2_\nu e^4,$$

if we calculate the divergent contributions, multiplying λ_∞ (E.1) of the penguins with "x" virtual boson D_x, we find:

$$
\begin{aligned}
D_\gamma &= \frac{F}{8s_w^2 M_W^4}\left(L\left(e\right)+R\left(e\right)\right)_\mu \cdot L^\mu\left(2\right), \\
D_Z &= \frac{F}{16s_w^2 M_W^4}\left[L\left(e\right)_\mu \cdot L^\mu\left(2\right) - 2\left(L\left(e\right)+R\left(e\right)\right)_\mu \cdot L^\mu\left(2\right)\right], \\
D_{Z'} &= \frac{F}{p^2-M_{Z'}^2}\frac{\kappa}{2c_w^2\left(3-t_w^2\right)}\left(2s_w^2 R\left(e\right)_\mu \cdot L\left(2\right)^\mu - \cos\left(2\theta_w\right)L\left(e\right)_\mu \cdot L\left(2\right)^\mu\right),
\end{aligned}
\tag{C.2}
$$

where:

$$
\kappa = \frac{1}{s_w^4 M_{Z'}^2}\left\{\frac{M_{Z'}^2-p^2}{8M_W^2}\left[2t_\beta^2-\left(1-t_w^2\right)\right]+\frac{3}{8}\frac{\cos\left(2\theta_w\right)}{c_w^4}-\frac{1}{3-t_w^2}-ct_\beta^2\right\}. \tag{C.3}
$$

Now we write the results for the divergent parts of the boxes formed with A and B bosons and C and D fermions, $D\left(ABCD\right)$:

$$
\begin{aligned}
D\left(WWN\nu\right) &= -\frac{F}{16s_w^4 M_W^4}L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(W'W'N\nu\right) &= -\frac{F}{16s_w^4 M_{W'}^4}L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(WWNN\right) &= -\frac{F}{8s_w^4 M_W^4}\delta_\nu^2 L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(W'W'NN\right) &= -\frac{F}{8s_w^4 M_{W'}^4}L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(WW'N\nu\right) &= -\frac{F}{16s_w^4 M_W^2 M_{W'}^2}L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(W'WN\nu\right) &= D\left(WW'N\nu\right), \\
D\left(WW'NN\right) &= -\frac{F}{8s_w^4 M_W^2 M_{W'}^2}L\left(e\right)_\mu \cdot L^\mu\left(2\right), \\
D\left(W'WNN\right) &= D\left(WW'NN\right).
\end{aligned}
\tag{C.4}
$$

We can see that keeping the leading order in the $\frac{v}{f}$ expansion, only the divergence $D\left(W'WN\nu\right)$ remains, so:

$$
D_\gamma + D_Z + D\left(WWN\nu\right) = 0 + \mathcal{O}\left(x^3\right) \tag{C.5}
$$

Appendix D

Dirac Equation

The Dirac equation is [146]:

$$\left(i\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - m\right)\psi(x) = 0$$

where:

$$\begin{aligned} \textit{particle wave:} \quad \psi^{(+)}_{p,s} &= \frac{u(p,s)}{\sqrt{2p^0V}} exp(-ip_{\mu}x^{\mu}) \\ \textit{antiparticle wave:} \quad \psi^{(-)}_{p,s} &= \frac{v(p,s)}{\sqrt{2p^0V}} exp(ip_{\mu}x^{\mu}) \end{aligned} \tag{D.1}$$

$\left(\text{ here } s \text{ is the polarization of the particle, } s = \pm\frac{1}{2}\right)$

then the Dirac equation in the center of mass become:

$$\begin{aligned} (\not p - m)u(p,s) &= 0 \\ (\not p + m)v(p,s) &= 0 \end{aligned} \quad / \quad \gamma^{\mu}p_{\mu} = \gamma^0 p^0 - \vec{\gamma}\cdot\vec{p} = \not p$$

and finally we have the solutions of Dirac equation $\left(p^0 = E_p \equiv \sqrt{m^2 + \vec{p}^2}\right)$:

$$\begin{aligned} u(p,s) &= \sqrt{p^0+m}\begin{pmatrix} \chi_s \\ \frac{\sigma^i\cdot\vec{p}}{p^0+m}\cdot\chi_s \end{pmatrix} \\ v(p,s) &= \sqrt{p^0+m}\begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{p^0+m}\cdot\chi_s \\ \chi_s \end{pmatrix} \end{aligned} \quad / \quad \chi_{+\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_{-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{D.2}$$

the normalization of (D.1) allow to stay only one particle in the volume V:

$$\int_V d^3x \psi^{(\varepsilon')\dagger}_{\vec{p}'s'}(x)\psi^{(\varepsilon)}_{\vec{p}s}(x) = \begin{cases} 1 & \Leftrightarrow \{\vec{p}' = \vec{p}, s' = s, \varepsilon' = \varepsilon\} \\ 0 \end{cases}$$

$$u^{\dagger}(p,s)u(p,s) = v^{\dagger}(p,s)v(p,s) = 2p^0$$

The adjoint spinors:

$$\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$$
$$\bar{u}(p,s) = u^\dagger(p,s)\gamma^0$$
$$\bar{u}(p,s)(\not{p} - m) = 0$$
$$\bar{v}(p,s) = v(p,s)^\dagger\gamma^0$$
$$\bar{v}(p,s)(\not{p} + m) = 0$$

Normalizations of the polarizations:

$$\bar{u}_\alpha(p,s)u_\beta(p,s) = \bar{u}^{(\alpha)}(p)u^{(\beta)}(p) = \delta^{\alpha,\beta}$$
$$\bar{v}_\alpha(p,s)v_\beta(p,s) = \bar{v}^{(\alpha)}(p)v^{(\beta)}(p) = -\delta^{\alpha,\beta}$$
$$\bar{v}_\alpha(p,s)u_\beta(p,s) = \bar{v}^{(\alpha)}(p)u^{(\beta)}(p) = \bar{u}^{(\alpha)}(p)v^{(\beta)}(p) = 0$$

The density:

$$\bar{u}^{(\alpha)}(p)\gamma^0 u^{(\beta)}(p) = u^{\dagger(\alpha)}(p)u^{(\beta)}(p) = \bar{u}^{(\alpha)}(\tilde{p})u^{(\beta)}(p) = \frac{E_p}{m}\sigma^{\alpha\beta}$$
$$\bar{v}^{(\alpha)}(p)\gamma^0 v^{(\beta)}(p) = v^{\dagger(\alpha)}(p)v^{(\beta)}(p) = -\bar{v}^{(\alpha)}(\tilde{p})v^{(\beta)}(p) = \frac{E_p}{m}\sigma^{\alpha\beta}$$
$$\tilde{p} = (p^0, -\vec{p})$$

D.1 Projectors on the energy states

$$\begin{aligned}\Lambda_+(p) &= \sum_s u_\alpha(p,s)\bar{u}_\beta(p,s) = (\not{p} + m)_{\alpha\beta}\\ \Lambda_-(p) &= \sum_s v_\alpha(p,s)\bar{v}_\beta(p,s) = (\not{p} - m)_{\alpha\beta}\end{aligned} \tag{D.3}$$

$$(u\bar{u} = u \otimes \bar{u} \qquad \bar{u}u = \bar{u}\cdot u)$$

$$u_\alpha(p,s)\bar{u}_\beta(p,s) = \frac{1}{2}[(\not{p} + m)(1 + \gamma^5\not{s})]_{\alpha\beta}$$
$$v_\alpha(p,s)\bar{v}_\beta(p,s) = \frac{1}{2}[(\not{p} - m)(1 + \gamma^5\not{s})]_{\alpha\beta}$$

D.2 Gordon identities

$$\bar{u}^{(\alpha)}(p)\gamma^\mu u^{(\beta)}(q) = \frac{1}{2m}\bar{u}^{(\alpha)}(p)[(p+q)^\mu + i\sigma^{\mu\nu}(p-q)_\nu]u^{(\beta)}(q)$$
$$\bar{u}^{(\alpha)}(p)\gamma^\mu\gamma^5 u^{(\beta)}(q) = \frac{1}{2m}\bar{u}^{(\alpha)}(p)[(p-q)^\mu\gamma^5 + i\sigma^{\mu\nu}(p+q)_\nu\gamma^5]u^{(\beta)}(q)$$

where:

$$\bar{u}^{(\alpha)}(p)\not{q}u^{(\beta)}(q) = \delta^{\alpha\beta}\frac{pq}{m}u^{(\alpha)\dagger}(p) = \vec{\alpha}u^{(\beta)}(p) = \delta^{\alpha\beta}\frac{\vec{p}}{m} \tag{D.4}$$

D.2.1 Change of Lorentz structure

$$\begin{aligned}
\bar{u}(p')\sigma_{\mu\nu}p^{\nu}u(q) &= -i\bar{u}(p')\left(g_{\mu\nu} - \gamma_\mu\gamma_\nu\right)p^\nu u(q) \\
&= -i\bar{u}(p')\left[(q-p')_\mu - \gamma_\mu\left(\not{q} - \not{p}'\right)\right]u(q) \\
&= -i\bar{u}(p')\left[(q-p')_\mu - \gamma_\mu m_\tau + 2p'_\mu - \not{p}'\gamma_\mu\right]u(q) \\
&= -i\bar{u}(p')\left[(q+p')_\mu - \gamma_\mu\left(m_\tau + m_\mu\right)\right]u(q)
\end{aligned} \tag{D.5}$$

$$\begin{aligned}
\bar{u}(p')\sigma_{\mu\nu}p^{\nu}\gamma_5 u(q) &= -i\bar{u}(p')\left[(q-p')_\mu - \gamma_\mu\left(-m_\tau - \not{p}'\right)\right]\gamma_5 u(q) \\
&= -i\bar{u}(p')\left[(q+p')_\mu - \gamma_\mu\left(-m_\tau + m_\mu\right)\right]\gamma_5 u(q)
\end{aligned} \tag{D.6}$$

if we define

$$P_L = \frac{1-\gamma_5}{2} \qquad ; \qquad P_R = \frac{1+\gamma_5}{2},$$

rewrite

$$p' = \frac{q+p'}{2} - \frac{q-p'}{2} = \frac{q+p'}{2} - \frac{p}{2}$$

and using (D.5), (D.6):

$$\begin{aligned}
\bar{u}(p')p'_\mu P_L u(q) &= \frac{1}{2}\bar{u}(p')\left[\left(\frac{q+p'}{2} - \frac{p}{2}\right)(1-\gamma_5)\right]u(q) \\
&= \frac{1}{4}\bar{u}(p')\{[i\sigma_{\mu\nu}p^\nu + \gamma_\mu(m_\tau + m_\mu)] - [i\sigma_{\mu\nu}p^\nu + \gamma_\mu(m_\mu - m_\tau)]\gamma_5 - \\
&\qquad - p_\mu(1-\gamma_5)\}u(q) \\
&= \frac{1}{2}\bar{u}(p')\left\{i\sigma_{\mu\nu}p^\nu P_L + \gamma_\mu m_\tau P_R + \gamma_\mu m_\mu P_L - p_\mu P_L\right\}u(q)
\end{aligned} \tag{D.7}$$

$$\begin{aligned}
\bar{u}(p')p'_\mu P_R u(q) &= \frac{1}{2}\bar{u}(p')\left[\left(\frac{q+p'}{2} - \frac{p}{2}\right)(1+\gamma_5)\right]u(q) \\
&= \frac{1}{2}\bar{u}(p')\left\{i\sigma_{\mu\nu}p^\nu P_R + \gamma_\mu m_\tau P_L + \gamma_\mu m_\mu P_R - p_\mu P_R\right\}u(q)
\end{aligned} \tag{D.8}$$

D.3 Fierz Identities

If we take the spinors $\bar{a}$ e b of 2 different particles, we can construct 16 bilinear terms [147], these terms can be combined in 5 different Lorentz covariant modes:

Covariant quantities		Components
$\bar{a}b$	scalar (S)	1
$\bar{a}\gamma^\alpha b$	vector (V)	4
$\sqrt{\frac{1}{2}}\bar{a}\sigma^{\alpha\beta}b$	tensor (T)	6
$\bar{a}\gamma^5\gamma^\alpha b$	axial vector (A)	4
$\bar{a}\gamma^5 b$	pseudoscalar (P)	1

Using 4 bispinors $\bar{a}$, b, $\bar{c}$ and d we have 5 Lorentz scalars:

$$\begin{array}{cc} (\bar{a}b)(\bar{c}d) & S-variant \\ (\bar{a}\gamma_\alpha b)(\bar{c}\gamma^\alpha d) & V-variant \\ \frac{1}{2}(\bar{a}\sigma_{\alpha\beta}b)(\bar{c}\sigma^{\alpha\beta}d) & T-variant \\ (\bar{a}\gamma_\alpha\gamma_5 b)(\bar{c}\gamma^\alpha\gamma^5 d) & A-variant \\ (\bar{a}\gamma_5 b)(\bar{c}\gamma^5 d) & P-variant \end{array}$$

Each one of these variants ($O_k = 1, \gamma_\alpha, \frac{\sigma_{\alpha\beta}}{\sqrt{2}}, \gamma_5\gamma_\alpha, \gamma_5$) can be expressed like

$$(\bar{a}O_i b)(\bar{c}O^i d) = \sum_k c_{ik}(\bar{a}O_k d)(\bar{c}O^k b)$$

where c_{ik} are:

	S	V	T	A	P
S	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$
V	1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
T	$-\frac{3}{2}$	0	$-\frac{1}{2}$	0	$-\frac{3}{2}$
A	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	1
P	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

The 5 amplitudes aren't orthogonal [1]. More in general the Fierz Matrix is the same for pseudoscalars, this is very important for the weak processes because we can use the identities:

$$\begin{aligned} \bar{a}\gamma_\alpha(1-\gamma_5)b\cdot\bar{c}\gamma^\alpha(1-\gamma_5)d &= -\bar{a}\gamma_\alpha(1-\gamma_5)d\cdot\bar{c}\gamma^\alpha(1-\gamma_5)b \\ \bar{a}\gamma_\alpha(1-\gamma_5)b\cdot\bar{c}\gamma^\alpha(1+\gamma_5)d &= 2\bar{a}(1+\gamma_5)d\cdot\bar{c}(1-\gamma_5)b \end{aligned} \tag{D.9}$$

[1]The permutation $b \leftrightarrow d$ have 2 symmetric combinations: 3(S+P)-T ; 2(S-P)+V+A, and 3 antisymmetric combinations: V-A ; S+P+T ; 2(S-P)-(V+A).

Appendix E

Passarino-Veltman Integrals

The divergent part of the integrals is (in our case we used $D = 4 - 2\varepsilon$):

$$\lambda_\infty = \frac{1}{\varepsilon} + \gamma + \ln 4\pi - 1 + \ln \nu^2. \tag{E.1}$$

To calculate our integrals we expanded on the squared transfer momenta over the squared heavy masses:

$$\begin{gathered}
(k-p)^2 - M^2 = (k^2 - M^2)\left[1 + \frac{p^2 - 2k\cdot p}{k^2 - M^2}\right] \Rightarrow \\
\Rightarrow \frac{1}{(k-p)^2 - M^2} \approxeq \frac{1}{k^2 - M^2}\left[1 - \frac{p^2 - 2k\cdot p}{k^2 - M^2} + \frac{(p^2 - 2k\cdot p)^2}{(k^2 - M^2)^2}\right],
\end{gathered} \tag{E.2}$$

$$\begin{gathered}
(k-p')^2 - M^2 = (k^2 - M^2)\left[1 + \frac{p'^2 - 2k\cdot p'}{k^2 - M^2}\right] \approxeq (k^2 - M^2)\left[1 + \frac{-2k\cdot p'}{k^2 - M^2}\right] \Rightarrow \\
\Rightarrow \frac{1}{(k-p')^2 - M^2} \approxeq \frac{1}{k^2 - M^2}\left[1 + \frac{2k\cdot p'}{k^2 - M^2} + 4\frac{(2k\cdot p')^2}{(k^2 - M^2)^2}\right] \approxeq \\
\approxeq \frac{1}{k^2 - M^2}\left[1 + \frac{2k\cdot p'}{k^2 - M^2}\right],
\end{gathered} \tag{E.3}$$

In the sections E.1.1, E.1.2, E.1.3, E.2 and E.4 we assume $q = p + p'$ and $p'^2 = m_\mu^2 \approx 0$.

E.1 Scalar Integrals

E.1.1 One point function integrals

$$A_n[m^2] = \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{1}{(k^2 - m^2)^n} \tag{E.4}$$

$$\begin{aligned}
A_0[m^2] &= 0,\\
A_1[m^2] &= \frac{-i}{16\pi^2} m^2 \left[\lambda_\infty + \ln\frac{m^2}{\nu^2}\right],\\
A_2[m^2] &= \frac{-i}{16\pi^2} \left[\lambda_\infty + 1 + \ln\frac{m^2}{\nu^2}\right],\\
A_3[m^2] &= \frac{-i}{16\pi^2}\frac{1}{2m^2},\\
A_4[m^2] &= \frac{i}{16\pi^2}\frac{1}{6m^4},\\
A_5[m^2] &= \frac{-i}{16\pi^2}\frac{1}{12m^6},\\
A_n[m^2] &= \frac{1}{n-1}\frac{\mathrm{d}A^{n-1}}{\mathrm{d}m^2} \qquad ; \qquad n \geq 2.
\end{aligned} \tag{E.5}$$

E.1.2 Two point function integrals

Using (E.2) approximation:

$$\begin{aligned}
B_0[p^2, m_0^2, m_1^2] = &\int \frac{\mathrm{d}^\mathrm{D} k}{(2\pi)^D} \frac{1}{[(k+p)^2 - m_0^2]\,(k^2 - m_1^2)} \approx\\
\approx &\int \frac{\mathrm{d}^\mathrm{D} k}{(2\pi)^D} \frac{1}{(k^2 - m_0^2)(k^2 - m_1^2)} \left[1 - \frac{p^2 - 2k\cdot p}{k^2 - M^2} + \frac{(p^2 - 2k\cdot p)^2}{(k^2 - M^2)^2}\right] =\\
= &\frac{16\pi^4}{(m_0^2 - m_1^2)}\Bigg[-p^2(2m_1^2 + p^2)A_3[m_1^2] +\\
&+ \frac{m_0^2 B_0[0, m_0^2, m_0^2]\left((m_0^2 - m_1^2)^2 + m_1^2 p^2 + p^4\right)}{(m_0^2 - m_1^2)^2} -\\
&- \frac{B_0[0, m_1^2, m_1^2]\left((m_1^3 - m_0^2 m_1)^2 + m_0^2 m_1^2 p^2 + m_0^2 p^4\right)}{(m_0^2 - m_1^2)^2} +\\
&+ \frac{-p^2(m_0^2 - 3m_1^2) + 2(m_0^2 - m_1^2)^2 + 2p^4}{2(m_0^2 - m_1^2)}\Bigg],
\end{aligned} \tag{E.6}$$

and:

$$\begin{aligned}
B_0[p^2, m^2, m^2] = &\int \frac{\mathrm{d}^\mathrm{D} k}{(2\pi)^D} \frac{1}{(k^2 - m^2)^2} - \frac{p^2}{(k^2 - m^2)^3} + \frac{p^4}{(k^2 - m^2)^4} + 4\frac{(p\cdot k)^2}{(k^2 - m^2)^4} =\\
= &A_2[m^2] + p^2(m^2 + p^2)A_4[m^2],
\end{aligned} \tag{E.7}$$

where:

$$B_0[0, m_0^2, m_1^2] = \frac{-i}{16\pi^2}\left[\lambda_\infty + \frac{m_0^2 \ln\frac{m_0^2}{\nu^2} - m_1^2 \ln\frac{m_1^2}{\nu^2}}{m_0^2 - m_1^2}\right], \tag{E.8}$$

$$B_0[0, m^2, m^2] = A_2[m^2]. \tag{E.9}$$

E.1.3 Three point function integrals

Using (E.2), (E.3) approximations:

$$\begin{aligned} C_0[p'^2, q^2, p^2, m_a^2, m_b^2, m_a^2] = &\int \frac{\mathrm{d^D}k}{(2\pi)^D} \frac{1}{(k^2 - m_b^2)\left[(k-p')^2 - m_a^2\right]\left[(k-q)^2 - m_a^2\right]} \approx \\ \approx &\frac{16\pi^4}{12 m_a^4 \left(\frac{m_b^2}{m_a^2} - 1\right)^4} \left\{ \left(\frac{m_b^2}{m_a^2} - 1\right)\left[(12 m_a^2 \left(\frac{m_b^2}{m_a^2} - 1\right)^2 + \right.\right. \\ &\left. + 3q^2\left(1 + 5\frac{m_b^2}{m_a^2}\right) + p^2\left(1 - 5\frac{m_b^2}{m_a^2} - 2\frac{m_b^4}{m_a^4}\right)\right] - \\ &\left. - 6\frac{m_b^2}{m_a^2}\ln\frac{m_b^2}{m_a^2}\left[2m_a^2\left(\frac{m_b^2}{m_a^2} - 1\right)^2 - p^2\frac{m_b^2}{m_a^2} + q^2\left(2 + \frac{m_b^2}{m_a^2}\right)\right]\right\} \end{aligned}, \tag{E.10}$$

$$\begin{aligned} C_0[p'^2, q^2, p^2, 0, m_b^2, m_c^2] = &\int \frac{\mathrm{d^D}k}{(2\pi)^D} \frac{1}{(k^2 - m_b^2)(k-p')^2\left[(k-q)^2 - m_c^2\right]} = \\ = &- \frac{16\pi^4}{2\frac{m_c^4}{m_b^4}\left(\frac{m_c^2}{m_b^2} - 1\right)^4 m_b^6} \left\{ \left(\frac{m_c^2}{m_b^2} - 1\right)\left\{p^4 + \frac{m_c^2}{m_b^2}p^2(m_b^2 - 5p^2 + \right.\right. \\ &+ q^2) + \frac{m_c^6}{m_b^6}m_b^2\left(p^2 + 2q^2\right) - \frac{m_c^4}{m_b^4}\left[2p^4 - 5p^2q^2 + 2m_b^2(p^2 + \right. \\ &\left.\left. + q^2)\right]\right\} - \frac{m_c^4}{m_b^4}\ln\frac{m_c^2}{m_b^2}\left\{2\left(\frac{m_c^2}{m_b^2} - 1\right)^3 m_b^4 + \right. \\ &+ \left(\frac{m_c^2}{m_b^2} - 1\right)m_b^2\left[\left(\frac{m_c^2}{m_b^2} - 1\right)p^2 + \left(1 + \frac{m_c^2}{m_b^2}\right)q^2\right] + \\ &\left.\left. + 2p^2\left[-3p^2 + \left(2 + \frac{m_c^2}{m_b^2}\right)q^2\right]\right\}\right\} \end{aligned}, \tag{E.11}$$

$$
\begin{aligned}
&C_0[p'^2, q^2, p^2, m_a^2, m_b^2, 0] = \\
&= C_0[p'^2, q^2, p^2, m_a^2, 0, m_b^2] = \int \frac{\mathrm{d}^{\mathrm{D}} k}{(2\pi)^D} \frac{1}{(k^2 - m_b^2)\,[(k-p')^2 - m_a^2]\,(k-q)^2} = \\
&= - \frac{16\pi^4}{2\frac{m_b^4}{m_a^4}\left(\frac{m_b^2}{m_a^2} - 1\right)^5 m_a^{10}} \left\{ \left(\frac{m_b^2}{m_a^2} - 1\right) \left\{ \frac{m_b^{10}}{m_a^{10}} m_a^4 p^2 \left(m_a^2 + p^2\right) + \right.\right. \\
&+ \left(m_a^4 + m_a^2 p^2 + p^4\right) q^4 - \frac{m_b^8}{m_a^8} \Big[6 m_a^4 p^4 - m_a^2 p^4 q^2 + \\
&+ m_a^6 \left(3p^2 + q^2\right) \Big] - \frac{m_b^2}{m_a^2} q^2 \Big[-m_a^6 + 6 m_a^4 q^2 + 7 p^4 q^2 + m_a^2 \big(p^4 + \\
&+ 9 p^2 q^2\big) \Big] + \frac{m_b^6}{m_a^6} \Big[p^4 q^4 + 3 m_a^6 \left(p^2 + q^2\right) - m_a^2 p^2 q^2 \left(9p^2 + q^2\right) + \\
&+ m_a^4 \left(3p^4 + + 2q^4\right) \Big] - \frac{m_b^4}{m_a^4} \Big[7 p^4 q^4 + 27 m_a^2 p^2 q^2 \left(p^2 + q^2\right) + \\
&+ m_a^6 \left(p^2 + 3q^2\right) - m_a^4 \left(2p^4 + 3q^4\right) \Big] \Big\} + \\
&+ \frac{m_b^4}{m_a^4} \ln \frac{m_b^2}{m_a^2} \left\{ 2 \left(\frac{m_b^2}{m_a^2} - 1\right)^4 m_a^8 + 12 p^4 q^4 - \right. \\
&- \left(\frac{m_b^2}{m_a^2} - 1\right)^3 m_a^6 \left(p^2 - q^2\right) + 12 m_a^2 p^2 q^2 \left[\left(1 + 2\frac{m_b^2}{m_a^2}\right) p^2 + \right. \\
&\left.\left.\left. + \left(2 + \frac{m_b^2}{m_a^2}\right) q^2 \right] + 6 m_a^4 \left[\frac{m_b^4}{m_a^4} p^4 + q^4 - \frac{m_b^2}{m_a^2}\left(p^4 + q^4\right) \right] \right\} \right\}
\end{aligned}
\tag{E.12}
$$

E.2 Vector Integrals

Using (E.2) approximation:

$$
\begin{aligned}
B_1[p^2, m_0^2, m_1^2] &= \int \frac{\mathrm{d}^{\mathrm{D}} k}{(2\pi)^D} \frac{k^\mu}{(k^2 - m_0^2)\,[(k-p)^2 - m_1^2]} = \\
&= i\pi^2 p^\mu \left[\frac{B_0[p^2, m_0^2, m_1^2]}{2} + \right. \\
&\left. + \frac{m_0^2 - m_1^2}{2p^2} \Big(B_0[p^2, m_0^2, m_1^2] - B_0[0, m_0^2, m_1^2] \Big) \right],
\end{aligned}
\tag{E.13}
$$

$$
\begin{aligned}
I_2 = &\int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\mu}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=&Aa_\mu + Bq_\mu \Rightarrow \begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{a^2q^2-(a\cdot q)^2} \begin{pmatrix} q^2 & -a\cdot q \\ -a\cdot q & a^2 \end{pmatrix} \begin{pmatrix} a_\mu I_2 \\ q_\mu I_2 \end{pmatrix},
\end{aligned}
\tag{E.14}
$$

where:

$$
\begin{aligned}
a_\mu I_2 = &\int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k\cdot a}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
= &\pm\frac{1}{2}B_0[q^2,m_1^2,m_3^2] - B_0[b^2,m_2^2,m_3^2] + (m_2^2-a^2-m_1^2)C_0[a^2,q^2,b^2,m_2^2,m_1^2,m_3^2], \\
q_\mu I_2 = &\int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k\cdot q}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
= &\pm\frac{1}{2}B_0[a^2,m_1^2,m_2^2] - B_0[b^2,m_2^2,m_3^2] + (m_3^2-q^2-m_1^2)C_0[a^2,q^2,b^2,m_2^2,m_1^2,m_3^2].
\end{aligned}
\tag{E.15}
$$

E.3 Tensor Integrals

If $q = a + b$:

$$
\begin{aligned}
I_T = &\int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k_\mu k_\nu}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=&A'g_{\mu\nu} + B'a_\mu a_\nu + C'q_\mu q_\nu + D'(a_\mu q_\nu + a_\nu q_\mu) \Rightarrow \\
\Rightarrow &\begin{cases}
\boxed{1} = g_{\mu\nu}I_T = DA' + a^2B' + q^2C' + 2a\cdot qD' \\
\boxed{2} = a_\mu a_\nu I_T = a^2A' + a^4B' + (a\cdot q)^2C' + 2a^2a\cdot qD' \\
\boxed{3} = q_\mu q_\nu I_T = q^2A' + (a\cdot q)^2B' + q^4C' + 2q^2a\cdot qD' \\
\boxed{4} = a_\mu q_\nu I_T = a\cdot qA' + a^2a\cdot qB' + q^2a\cdot qC' + [a^2q^2+(a\cdot q)^2]D'
\end{cases}
\end{aligned}
\tag{E.16}
$$

where:

$$
\begin{aligned}
\boxed{1} =& \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^2}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=& B_0[b^2,m_2^2,m_3^2] + m_1^2 C_0[a^2,q^2,b^2,m_2^2,m_1^2,m_3^2], \\
\boxed{2} =& \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k\cdot a}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=& \frac{1}{2}\left[-a\cdot q B_1[q^2,m_1^2,m_3^2] + a\cdot b B_1[b^2,m_2^2,m_3^2] + a^2 B_0[b^2,m_2^2,m_3^2] + (m_2^2-a^2-m_1^2)a_\mu I_2\right], \\
\boxed{3} =& \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k\cdot q}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=& \frac{1}{2}\left[-a\cdot q B_1[a^2,m_1^2,m_2^2] + q\cdot b B_1[b^2,m_2^2,m_3^2] + a\cdot q B_0[b^2,m_2^2,m_3^2] + (m_3^2-q^2-m_1^2)q_\mu I_2\right], \\
\boxed{4} =& \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{(k\cdot a)(k\cdot q)}{(k^2-m_1^2)\,[(k\pm a)^2-m_2^2]\,[(k\pm q)^2-m_3^2]} = \\
=& \frac{1}{2}\left[-a^2 B_1[a^2,m_1^2,m_2^2] + a\cdot b B_1[b^2,m_2^2,m_3^2] + a^2 B_0[b^2,m_2^2,m_3^2] + (m_3^2-q^2-m_1^2)a_\mu I_2\right],
\end{aligned}
\tag{E.17}
$$

where:

$$
\begin{aligned}
A' =& \frac{-\boxed{2}q^2 + a^2(-\boxed{3} + q^2\boxed{1}) + 2\boxed{4}a\cdot q - \boxed{1}(a\cdot q)^2}{(D-2)(a^2q^2-(a\cdot q)^2)} \\
B' =& -\frac{(D-1)\boxed{2}q^4 + a^2q^2(\boxed{3} - q^2\boxed{1}) - 2(D-1)q^2\boxed{4}a\cdot q + ((D-2)\boxed{3} + q^2\boxed{1})(a\cdot q)^2}{(D-2)(a^2q^2-(a\cdot q)^2)^2} \\
C' =& -\frac{a^2\boxed{2}q^2 + a^4((D-1)\boxed{3} - q^2\boxed{1}) - 2a^2(D-1)\boxed{4}a\cdot q + ((D-2)\boxed{2} + a^2\boxed{1})(a\cdot q)^2}{(D-2)(a^2q^2-(a\cdot q)^2)^2} \\
D' =& -\frac{a^2(D-2)q^2\boxed{4} + ((1-D)\boxed{2}q^2 + a^2((1-D)\boxed{3} + q^2\boxed{1}))a\cdot q + D\boxed{4}(a\cdot q)^2 - \boxed{1}(a\cdot q)^3}{(D-2)(a^2q^2-(a\cdot q)^2)^2}
\end{aligned}
\tag{E.18}
$$

E.4 Integrals for boxes

When the integral converges we can approximate:

$$
\begin{aligned}
p_1^2 &= m_q^2 \simeq 0 \\
p_2^2 &= m_q^2 \simeq 0 \\
p'^2 &= m_\mu^2 \simeq 0, p_1^\alpha \quad \approx 0 \\
p_2^\alpha &\approx 0 \\
p'^\alpha &\approx 0
\end{aligned}
\tag{E.19}
$$

so:

$$
\begin{aligned}
I_{box1} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^2}{(k^2-M_N^2)\,[(k-q)^2-M_{V1}^2]\,[(k-p')^2-M_{V2}^2]\,\left[(k-p'-p_2)^2-m_q^2\right]} \simeq \\
&\simeq \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^2}{(k^2-M_N^2)\,[k^2-M_{V1}^2]\,[k^2-M_{V2}^2]\,\left[k^2-m_q^2\right]} = \\
&= C_0[0,0,0,M_{V1}^2,M_{V2}^2,m_q^2] + M_N^2 D_0[0,0,0,M_N^2,M_{V1}^2,M_{V2}^2,m_q^2]
\end{aligned} \tag{E.20}
$$

$$
\begin{aligned}
I_{box2} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta}{(k^2-M_N^2)\,[(k-q)^2-M_{V1}^2]\,[(k-p')^2-M_{V2}^2]\,\left[(k-p'-p_2)^2-m_q^2\right]} \simeq \\
&\simeq \frac{g^{\alpha\beta}}{D} I_{box1}
\end{aligned} \tag{E.21}
$$

$$
\begin{aligned}
I_{box3} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta}{\left(k^2-m_q^2\right)(k^2-M_{V1}^2)\,(k^2-M_{V2}^2)} = \\
&= \frac{g_{\alpha\beta}}{D}\Big(B_0[0,m_q^2,M_{V2}^2] + M_{V1}^2 C_0[0,0,0,m_q^2,M_{V1}^2,M_{V2}^2]\Big) \Rightarrow \\
\Rightarrow I_{box3}^{div} &= \frac{g_{\alpha\beta}}{D} B_0[0,m_q^2,M_{V2}^2]
\end{aligned} \tag{E.22}
$$

$$
\begin{aligned}
I_{box4} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta k^2}{(k^2-M_N^2)\,[(k-q)^2-M_{V1}^2]\,[(k-p')^2-M_{V2}^2]\,\left[(k-p'-p_2)^2-m_q^2\right]} \simeq \\
&\simeq \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta}{[(k-q)^2-M_{V1}^2]\,[(k-p')^2-M_{V2}^2]\,\left[(k-p'-p_2)^2-m_q^2\right]} + \\
&\quad + M_N^2 \frac{k^\alpha k^\beta}{(k^2-M_N^2)\,[(k-q)^2-M_{V1}^2]\,[(k-p')^2-M_{V2}^2]\,\left[(k-p'-p_2)^2-m_q^2\right]} = \\
&= \int \frac{\mathrm{d}k'}{2\pi^D} \frac{(k'^\alpha + p'^\alpha + p_2^\alpha)(k'^\beta + p'^\beta + p_2^\beta)}{[(k'-p_1)^2-M_{V1}^2]\,[(k'+p_2)^2-M_{V2}^2]\,\left(k'^2-m_q^2\right)} + M_N^2 I_{box2} \simeq \\
&\simeq I_{box3} + M_N^2 I_{box2}
\end{aligned} \tag{E.23}
$$

and if I_{box4} multiplied by $p_i = \{p_1, p_2, p'\}$:

$$
I_{box4} \cdot p_i^\alpha = I_{box3}^{div} \cdot p_i^\alpha \tag{E.24}
$$

$$
\begin{aligned}
I_{box5} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta k^\eta k^\rho}{\left(k^2 - m_q^2\right)(k^2 - M_{V1}^2)(k^2 - M_{V2}^2)} = \\
&= \frac{g_{\eta\rho} g_{\alpha\beta} + g_{\eta\alpha} g_{\rho\beta} + g_{\eta\beta} g_{\alpha\rho}}{D(D+2)} \Big(B_0[0, m_q^2, M_{V2}^2] + 2M_{V2}^2 C_0[0,0,0,m_q^2, M_{V1}^2, M_{V2}^2] + \\
&\quad + M_{V2}^4 D_0[0,0,0,m_q^2, M_{V1}^2, M_{V2}^2] \Big)
\end{aligned} \tag{E.25}
$$

$$
\begin{aligned}
I_{box6} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta k^\eta k^2}{(k^2 - M_N^2)\left[(k-q)^2 - M_{V1}^2\right]\left[(k-p')^2 - M_{V2}^2\right]\left[(k-p'-p_2)^2 - m_q^2\right]} \simeq \\
&\simeq 2\, I_{box5}\big|_{M_{V1} \leftrightarrow M_{V2}} + I_{box3}^{div} \cdot (p_2^\eta g_{\alpha\beta} + p'^\eta g_{\alpha\beta} + p_2^\alpha + p'^\alpha + g_{\alpha\beta} p_2 \cdot p'),
\end{aligned} \tag{E.26}
$$

also in this case, if I_{box6} multiplied by $p_i = \{p_1, p_2, p'\}$

$$
I_{box6} \cdot p_i^\alpha = 2\, I_{box5}^{div}\big|_{M_{V1} \leftrightarrow M_{V2}} p_i^\alpha + I_{box3}^{div} \cdot (p_2^\eta p_i^\beta + p'^\eta p_i^\beta + p_2 \cdot p_i + p' \cdot p_i + p_2 \cdot p' p_i^\beta), \tag{E.27}
$$

$$
I_{box7} = \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta}{(k^2 - m_q^2)(k^2 - M_{V2}^2)} = \frac{g_{\alpha\beta}}{D} \left(A_0[m_q^2] + M_{V2}^2 B_0[0, m_q^2, M_{V2}^2] \right) \tag{E.28}
$$

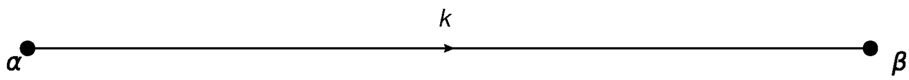
$$
\begin{aligned}
I_{box8} &= \int \frac{\mathrm{d}^{\mathrm{D}}k}{(2\pi)^D} \frac{k^\alpha k^\beta k^4}{(k^2 - M_N^2)\left[(k-q)^2 - M_{V1}^2\right]\left[(k-p')^2 - M_{V2}^2\right]\left[(k-p'-p_2)^2 - m_q^2\right]} \simeq \\
&\simeq \Bigg\{ \left[M_{V1}^2 + \frac{4}{D}(p_1 \cdot p') + \left(2 - \frac{4}{D}\right)(p_2 \cdot p') \right] + \left(2 + \frac{2}{D}\right)(p_2 + p') \cdot \\
&\quad \cdot (p_1 + p') \Bigg\} I_{box3} + I_{box7} + (p_2^\alpha + p'^\alpha)(p_2^\beta + p'^\beta) B_0[0, m_q^2, M_{V2}^2] + M_N^2 I_{box4},
\end{aligned} \tag{E.29}
$$

also in this case, if I_{box8} multiplied by $p_i = \{p_1, p_2, p'\}$, then $\{I_{box4}, I_{box3}\} \to I_{box3}^{div}$.

Appendix F

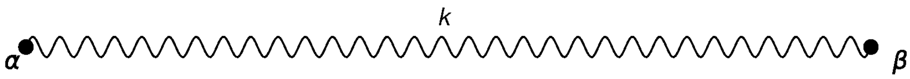
Feynman rules of the SLH model in the Unitary Gauge

Fermion propagator:



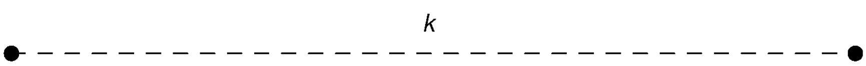
$$\left(\frac{i}{\not{k} - m}\right)_{\beta\alpha} = \left(i\frac{\not{k} + m}{k^2 - m^2}\right)_{\beta\alpha} \tag{F.1}$$

Vector propagator:



$$-\,i\frac{g_{\alpha\beta} - \frac{k_\alpha k_\beta}{m^2}}{k^2 - m^2} \tag{F.2}$$

Scalar propagator:



$$\frac{i}{k^2 - m^2} \tag{F.3}$$

F.1 Vertices

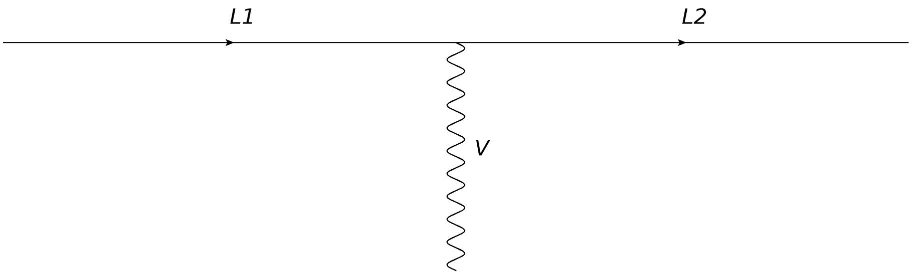


Table F.1: Couplings fermion-fermion-vector in the FCC case.

$L1L2$	$V = W$	$V = W'$
$l_i^- \nu_i$	$i\frac{e}{\sqrt{2}s_w}\left(1-\frac{\delta_\nu^2}{2}\right)\bar{\nu}_i\gamma_\mu P_L l_i$	$\frac{e}{\sqrt{2}s_w}\delta_\nu\bar{\nu}_i\gamma_\mu P_L l_i$
$l_i^- N_j$	$-i\frac{e}{\sqrt{2}s_w}\delta_\nu V_{ij}\bar{N}_j\gamma_\mu P_L l_i$	$\frac{e}{\sqrt{2}s_w}\left(1-\frac{\delta_\nu^2}{2}\right)V_{ij}\bar{N}_j\gamma_\mu P_L l_i$
du	$i\frac{e}{\sqrt{2}s_w}\bar{u}\gamma_\mu P_L d$	$-\delta_d\frac{e}{\sqrt{2}s_w}\bar{u}\gamma_\mu P_L d$
ud	$i\frac{e}{\sqrt{2}s_w}\bar{d}\gamma_\mu P_L u$	$\delta_d\frac{e}{\sqrt{2}s_w}\bar{d}\gamma_\mu P_L u$
Du	$-i\delta_d\frac{e}{\sqrt{2}s_w}\bar{u}\gamma_\mu P_L D$	$-\frac{e}{\sqrt{2}s_w}\bar{u}\gamma_\mu P_L D$
uD	$-i\delta_d\frac{e}{\sqrt{2}s_w}\bar{D}\gamma_\mu P_L u$	$\frac{e}{\sqrt{2}s_w}\bar{D}\gamma_\mu P_L u$
cs	$i\frac{e}{\sqrt{2}s_w}\bar{s}\gamma_\mu P_L c$	$\delta_s\frac{e}{\sqrt{2}s_w}\bar{s}\gamma_\mu P_L c$
sc	$i\frac{e}{\sqrt{2}s_w}\bar{c}\gamma_\mu P_L s$	$-\delta_s\frac{e}{\sqrt{2}s_w}\bar{c}\gamma_\mu P_L s$

Table F.2: Couplings fermion-fermion-γ_μ in the FNC case.

$L1L2$	$V = \gamma_\mu$
$\nu_i N_i$	0
$N_i N_j$	0
$l_i l_i$	$ie\bar{l}_i\gamma_\mu P_L l_i$
uu	$-i\frac{2e}{3}\bar{u}\gamma_\mu P_L u$
dd	$i\frac{e}{3}\bar{d}\gamma_\mu P_L d$
ss	$i\frac{e}{3}\bar{s}\gamma_\mu P_L s$

Table F.3: Couplings fermion-fermion-Z_μ in the FNC case.

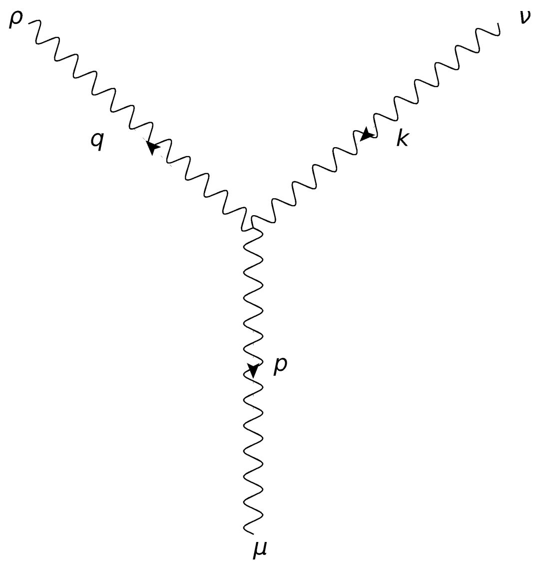
$L1L2$	$V = Z_\mu$
$\nu_i N_i$	$-i\frac{e}{2s_w c_w}\delta_\nu V_{ij}\bar{N}_i\gamma_\mu P_L \nu_i$
$N_i N_j$	$i\frac{e}{s_w}\left(\frac{\delta_\nu^2}{2c_w} + \frac{\delta_Z}{\sqrt{3-t_w^2}}\right)\bar{N}_j\gamma_\mu P_L N_i$
$l_i l_i$	$ie\left(A_L \bar{l}_i\gamma_\mu P_L l_i + A_R \bar{l}_i\gamma_\mu P_R l_i\right)$
uu	$ie\left(\frac{3-t_w^2}{6t_w}\bar{u}\gamma_\mu P_L u - \frac{2t_w}{3}\bar{u}\gamma_\mu P_R u\right)$
dd	$ie\left(-\frac{3+t_w^2}{6t_w}\bar{d}\gamma_\mu P_L d + \frac{t_w}{3}\bar{d}\gamma_\mu P_R d\right)$
ss	$-ie\frac{3+t_w^2}{6t_w}\bar{s}\gamma_\mu P_L s$

Table F.4: Couplings fermion-fermion-Z'_μ in the FNC case.

$L1L2$	$V = Z'_\mu$
$\nu_i N_i$	$i\frac{e}{2s_w}\sqrt{3-t_w^2}\delta_\nu V_{ij}\bar{N}_i\gamma_\mu P_L \nu_i$
$N_i N_j$	$i\frac{e}{2s_w}\frac{2-\delta_\nu^2\left(3-t_w^2\right)}{\sqrt{3-t_w^2}}\bar{N}_j\gamma_\mu P_L N_i$
$l_i l_i$	$ie\left(B_L \bar{l}_i\gamma_\mu P_L l_i + B_R \bar{l}_i\gamma_\mu P_R l_i\right)$
uu	$ie\left(\frac{\sqrt{3-t_w^2}}{6s_w}\bar{u}\gamma_\mu P_L u - \frac{2t_w}{3c_w\sqrt{3-t_w^2}}\bar{u}\gamma_\mu P_R u\right)$
dd	$ie\left(\frac{\sqrt{3-t_w^2}}{6s_w}\bar{d}\gamma_\mu P_L d + \frac{t_w}{3c_w\sqrt{3-t_w^2}}\bar{d}\gamma_\mu P_R d\right)$
ss	$ie\frac{\sqrt{3-t_w^2}}{6s_w}\bar{s}\gamma_\mu P_L s$

where:

$$\begin{aligned}
A_L &= \frac{2s_w^2-1}{2s_w c_w} - \frac{\delta_Z}{2s_w c_w^2}\frac{1-2s_w^2}{\sqrt{3-t_w^2}}, \\
A_R &= t_w + \frac{\delta_Z s_w}{c_w^2\sqrt{3-t_w^2}}, \\
B_L &= \frac{2s_w^2-1}{2s_w c_w^2\sqrt{3-t_w^2}} + \frac{\delta_Z\left(1-2s_w^2\right)}{2s_w c_w}, \\
B_R &= \frac{s_w}{c_w^2\sqrt{3-t_w^2}} - \delta_Z t_w.
\end{aligned} \tag{F.4}$$



$$V_{\mu\nu\rho} = ieC \left[(p+k)_\rho \, g_{\mu\nu} + (q-p)_\nu \, g_{\mu\rho} - (k+q)_\mu \, g_{\nu\rho} \right] \tag{F.5}$$

Table F.5: Couplings vector-vector-vector.

$V_{\mu\nu\rho}$	C
AW^+W^-	-1
ZW^+W^-	$\frac{c_w}{s_w}$
$Z'W^+W^-$	$-\delta_Z \frac{c_w}{s_w}$
$AW'^+W'^-$	-1
$ZW'^+W'^-$	$\frac{c_w^2 - s_w^2 + c_w \delta_Z \sqrt{3-t_w^2}}{2c_w s_w}$
$Z'W'^+W'^-$	$\frac{c_w \sqrt{3-t_w^2} - \delta_Z \left(c_w^2 - s_w^2\right)}{2c_w s_w}$

where we are using the definitions (3.46), (3.57):

$$\delta_Z = \frac{(1-t_W^2)\sqrt{3-t_W^2}}{8c_W} x^2 \qquad ; \qquad \delta_\nu = -\frac{x}{\sqrt{2}t_\beta} \qquad ; \qquad x \equiv \frac{v}{f} \tag{F.6}$$

Part IV

Bibliography

Bibliography

[1] B. T. Cleveland, Timothy Daily, Raymond Davis, Jr., James R. Distel, Kenneth Lande, C. K. Lee, Paul S. Wildenhain, and Jack Ullman. Measurement of the solar electron neutrino flux with the Homestake chlorine detector. *Astrophys. J.*, 496:505–526, 1998. doi: 10.1086/305343.

[2] P. Anselmann et al. Status report on the GALLEX experiment. *Nuovo Cim.*, C15: 917–929, 1992. doi: 10.1007/BF02506685.

[3] J. N. Abdurashitov et al. Solar neutrino flux measurements by the Soviet-American Gallium Experiment (SAGE) for half the 22 year solar cycle. *J. Exp. Theor. Phys.*, 95:181–193, 2002. doi: 10.1134/1.1506424. [Zh. Eksp. Teor. Fiz.122,211(2002)].

[4] M. Altmann et al. Complete results for five years of GNO solar neutrino observations. *Phys. Lett.*, B616:174–190, 2005. doi: 10.1016/j.physletb.2005.04.068.

[5] Samoil M. Bilenky and B. Pontecorvo. Lepton Mixing and Neutrino Oscillations. *Phys. Rept.*, 41:225–261, 1978. doi: 10.1016/0370-1573(78)90095-9.

[6] Q. R. Ahmad et al. Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory. *Phys. Rev. Lett.*, 89:011301, 2002. doi: 10.1103/PhysRevLett.89.011301.

[7] J. Beringer et al. Review of Particle Physics (RPP). *Phys. Rev.*, D86:010001, 2012. doi: 10.1103/PhysRevD.86.010001.

[8] E. O. Iltan and I. Turan. Lepton flavor violating $Z \to l^+l^-$ decay in the general Higgs doublet model. *Phys. Rev.*, D65:013001, 2002. doi: 10.1103/PhysRevD.65.013001.

[9] Bernard Aubert et al. Searches for Lepton Flavor Violation in the Decays $\tau^\pm \to e^\pm\gamma$ and $\tau^\pm \to \mu^\pm\gamma$. *Phys. Rev. Lett.*, 104:021802, 2010. doi: 10.1103/PhysRevLett.104.021802.

[10] K. Hayasaka et al. New search for $\tau \to \mu\gamma$ and $\tau \to e\gamma$ decays at Belle. *Phys. Lett.*, B666:16–22, 2008. doi: 10.1016/j.physletb.2008.06.056.

[11] et al. Adam. New constraint on the existence of the $\mu^+ \to e^+\gamma$ decay. *Phys. Rev. Lett.*, 110:201801, May 2013. doi: 10.1103/PhysRevLett.110.201801. URL http://link.aps.org/doi/10.1103/PhysRevLett.110.201801.

[12] K. Hayasaka et al. Search for Lepton Flavor Violating Tau Decays into Three Leptons with 719 Million Produced Tau+Tau- Pairs. *Phys. Lett.*, B687:139–143, 2010. doi: 10.1016/j.physletb.2010.03.037.

[13] U. Bellgardt et al. Search for the Decay $\mu^+ \to e^+e^+e^-$. *Nucl. Phys.*, B299:1, 1988. doi: 10.1016/0550-3213(88)90462-2.

[14] Mark Srednicki. *Quantum Field Theory*. Cambridge Univ. Press, Cambridge, 2007. URL https://cds.cern.ch/record/1019751.

[15] W.N. Cottingham and D.A. Greenwood. *Introduction to the Standard Model*. Cambridge Univ. Press, Cambridge, 2007.

[16] M.E. Peskin and Schroeder. *An Introduction to Quantum Field Theory*. Westview Press, 1995.

[17] G. Naber. *Topology, Geometry, and Gauge fields interactions*. Springer, 2000.

[18] R. Gilmore. *Lie Groups, Lie Algebras and Some of Their Applications*. Dover, 2006.

[19] R. Gilmore. *Lie Groups, Physics, and Geometry*. Cambridge Univ. Press, Cambridge, 2008.

[20] Makoto Kobayashi and Toshihide Maskawa. CP Violation in the Renormalizable Theory of Weak Interaction. *Prog. Theor. Phys.*, 49:652–657, 1973. doi: 10.1143/PTP.49.652.

[21] E. P. Hincks and B. Pontecorvo. The absorption of charged particles from the 2.2-mu-sec meson decay. *Phys. Rev.*, 74:697–698, 1948. doi: 10.1103/PhysRev.74.697.

[22] Frank F. Deppisch. Lepton Flavour Violation and Flavour Symmetries. *Fortsch. Phys.*, 61:622–644, 2013. doi: 10.1002/prop.201200126.

[23] A. A. Gvozdev, A. V. Kuznetsov, N. V. Mikheev, and L. A. Vasilevskaya. Three types of fermion mixing and possible manifestations of a Pati-Salam leptoquark in the low-energy processes. In *'95 electroweak interactions, unified theories. Proceedings, Leptonic Session of the 30th Rencontre de Moriond, Moriond Particle Physics Meetings, Les Arcs, France, March 11-18, 1995*, pages 321–326, 1995. URL https://inspirehep.net/record/394779/files/C95-03-11-1_321-326.pdf.

[24] et al. Aktas. Tau lepton production in ep collisions at hera. 48(3):699–714, 2006. ISSN 1434-6044. URL http://dx.doi.org/10.1140/epjc/s10052-006-0028-2.

[25] B. Foster. Lectures on HERA physics. In *Corfu Summer Institute on Elementary Particle Physics (Corfu 2001) Corfu, Greece, August 31-September 20, 2001*, 2002. URL http://alice.cern.ch/format/showfull?sysnb=2315307.

[26] Ana M. Teixeira, Asmaa Abada, Antonio J. R. Figueiredo, and Jorge C. Romao. Phenomenology of LFV at low-energies and at the LHC: strategies to probe the SUSY seesaw. *Nucl. Phys. Proc. Suppl.*, 218:50–55, 2011. doi: 10.1016/j.nuclphysbps.2011.06.010.

[27] Murray Gell-Mann, Pierre Ramond, and Richard Slansky. Complex Spinors and Unified Theories. *Conf. Proc.*, C790927:315–321, 1979.

[28] Y. Fukuda et al. Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567, 1998. doi: 10.1103/PhysRevLett.81.1562.

[29] Q. R. Ahmad et al. Measurement of day and night neutrino energy spectra at SNO and constraints on neutrino mixing parameters. *Phys. Rev. Lett.*, 89:011302, 2002. doi: 10.1103/PhysRevLett.89.011302.

[30] Riccardo Barbieri, John R. Ellis, and Mary K. Gaillard. Neutrino Masses and Oscillations in SU(5). *Phys. Lett.*, B90:249, 1980. doi: 10.1016/0370-2693(80)90734-0.

[31] P. H. Frampton, S. L. Glashow, and T. Yanagida. Cosmological sign of neutrino CP violation. *Phys. Lett.*, B548:119–121, 2002. doi: 10.1016/S0370-2693(02)02853-8.

[32] T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi, and M. Tanimoto. CP violation in neutrino oscillation and leptogenesis. *Phys. Rev. Lett.*, 89:231601, 2002. doi: 10.1103/PhysRevLett.89.231601.

[33] John Ellis, S. Kelley, and D.V. Nanopoulos. Precision lep data, supersymmetric guts and string unification. *Physics Letters B*, 249(3):441 – 448, 1990. ISSN 0370-2693. doi: http://dx.doi.org/10.1016/0370-2693(90)91013-2. URL http://www.sciencedirect.com/science/article/pii/0370269390910132.

[34] John R. Ellis, M. E. Gomez, G. K. Leontaris, S. Lola, and Dimitri V. Nanopoulos. Charged lepton flavor violation in the light of the Super-Kamiokande data. *Eur. Phys. J.*, C14:319–334, 2000. doi: 10.1007/s100520000357.

[35] Jonathan Richard Ellis, Mary Katherin Gaillard, and Dimitri V Nanopoulos. Left-handed currents and CP violation. *Nucl. Phys. B.*

[36] Ziro Maki, Masami Nakagawa, and Shoichi Sakata. Remarks on the unified model of elementary particles. *Prog. Theor. Phys.*, 28:870–880, 1962. doi: 10.1143/PTP.28.870.

[37] John R. Ellis, Junji Hisano, Martti Raidal, and Yasuhiro Shimizu. Lepton electric dipole moments in nondegenerate supersymmetric seesaw models. *Phys. Lett.*, B528: 86–96, 2002. doi: 10.1016/S0370-2693(02)01197-8.

[38] M. Fukugita and T. Yanagida. Baryogenesis Without Grand Unification. *Phys. Lett.*, B174:45, 1986. doi: 10.1016/0370-2693(86)91126-3.

[39] Sacha Davidson and Alejandro Ibarra. Determining seesaw parameters from weak scale measurements? *JHEP*, 09:013, 2001. doi: 10.1088/1126-6708/2001/09/013.

[40] John R. Ellis, Junji Hisano, Martti Raidal, and Yasuhiro Shimizu. A New parametrization of the seesaw mechanism and applications in supersymmetric models. *Phys. Rev.*, D66:115013, 2002. doi: 10.1103/PhysRevD.66.115013.

[41] Andre de Gouvea and Petr Vogel. Lepton Flavor and Number Conservation, and Physics Beyond the Standard Model. *Prog. Part. Nucl. Phys.*, 71:75–92, 2013. doi: 10.1016/j.ppnp.2013.03.006.

[42] Vincenzo Cirigliano, Ryuichiro Kitano, Yasuhiro Okada, and Paula Tuzon. On the model discriminating power of $\mu \to e$ conversion in nuclei. *Phys. Rev.*, D80:013002, 2009. doi: 10.1103/PhysRevD.80.013002.

[43] F. P. An et al. Observation of electron-antineutrino disappearance at Daya Bay. *Phys. Rev. Lett.*, 108:171803, 2012. doi: 10.1103/PhysRevLett.108.171803.

[44] L Michel. Interaction between four half-spin particles and the decay of the Î¼-meson. *Proceedings of the Physical Society. Section A*, 63(12):1371, 1950. URL http://stacks.iop.org/0370-1298/63/i=12/a=512.

[45] A Blondel, A Bravar, M Pohl, S Bachmann, N Berger, A Schöning, D Wiedner, P Fischer, I Perić, M Hildebrandt, et al. Letter of intent for an experiment to search for the decay μâ eee, 2012.

[46] Monika Blanke, Andrzej J. Buras, Bjoern Duling, Anton Poschenrieder, and Cecilia Tarantino. Charged Lepton Flavour Violation and $(g-2)(\mu)$ in the Littlest Higgs Model with T-Parity: A Clear Distinction from Supersymmetry. *JHEP*, 05:013, 2007. doi: 10.1088/1126-6708/2007/05/013.

[47] F. del Aguila, J. de Blas, and M. Perez-Victoria. Effects of new leptons in Electroweak Precision Data. *Phys. Rev.*, D78:013010, 2008. doi: 10.1103/PhysRevD.78.013010.

[48] Francisco del Aguila, Jose I. Illana, and Mark D. Jenkins. Lepton flavor violation in the Simplest Little Higgs model. *JHEP*, 03:080, 2011. doi: 10.1007/JHEP03(2011)080.

[49] Yoshitaka Kuno and Yasuhiro Okada. Muon decay and physics beyond the standard model. *Rev. Mod. Phys.*, 73:151–202, 2001. doi: 10.1103/RevModPhys.73.151.

[50] Nima Arkani-Hamed, Christopher D. Carone, Lawrence J. Hall, and Hitoshi Murayama. Supersymmetric framework for a dynamical fermion mass hierarchy. *Phys. Rev.*, D54:7032–7050, 1996. doi: 10.1103/PhysRevD.54.7032.

[51] S Dimopoulos and Gian Francesco Giudice. Naturalness constraints in supersymmetric theories with non-universal soft terms. *Physics Letters B*, 357(4):573–578, 1995.

[52] Ryuichiro Kitano, Masafumi Koike, and Yasuhiro Okada. Erratum: Detailed calculation of lepton flavor violating muon-electron conversion rate for various nuclei [phys. rev. d **66** , 096002 (2002)]. *Phys. Rev. D*, 76:059902, Sep 2007. doi: 10.1103/PhysRevD.76.059902. URL http://link.aps.org/doi/10.1103/PhysRevD.76.059902.

[53] Laurence S. Littenberg and Robert Shrock. Implications of improved upper bounds on $|\Delta L| = 2$ processes. *Phys. Lett.*, B491:285–290, 2000. doi: 10.1016/S0370-2693(00)01041-8.

[54] Yoshitaka Kuno. Future lfv experiments. In *Proceedings of Flavor Physics and CP Violation 2010. May 25-29, 2010. Turin, Italy. Published online at http://pos. sissa. it/cgi-bin/reader/conf. cgi? confid= 116, id. 49*, volume 1, page 49, 2010.

[55] Boyang Liu. Muonium-antimuonium oscillations in an extended minimal supersymmetric standard model with right-handed neutrinos. *Physical Review D*, 79(1):015001, 2009.

[56] Antonio Pich. The Standard model of electroweak interactions. In *High-energy physics. Proceedings, European School, Aronsborg, Sweden, June 18-July 1, 2006*, 2007. URL http://doc.cern.ch/yellowrep/2007/2007-005/cern-2007-005.pdf.

[57] Antonio Pich. Selected topics on tau physics. *Acta Phys. Polon.*, B38:3449–3458, 2007.

[58] Y. Amhis et al. Averages of b-hadron, c-hadron, and τ-lepton properties as of summer 2014. 2014.

[59] *Fundamental Physics at the Intensity Frontier*, 2012. doi: 10.2172/1042577. URL http://inspirehep.net/record/1114323/files/arXiv:1205.2671.pdf.

[60] Antonio Pich. Precision tau physics. *Progress in Particle and Nuclear Physics*, 75: 41–85, 2014.

[61] Andrea Brignole and Anna Rossi. Anatomy and phenomenology of mu-tau lepton flavor violation in the MSSM. *Nucl. Phys.*, B701:3–53, 2004. doi: 10.1016/j.nuclphysb.2004.08.037.

[62] A Vicente. Lepton flavor violation beyond the mssm. *Advances in High Energy Physics*, 2015:686572, 2015.

[63] Daiji Kimura, Kang Young Lee, and Takuya Morozumi. The Form factors of $\tau \to K\pi(\eta)\nu$ and the predictions for CP violation beyond the standard model. *PTEP*, 2013:053B03, 2013. doi: 10.1093/ptep/ptu107,10.1093/ptep/ptt013. [Erratum: PTEP2014,no.8,089202(2014)].

[64] Ken Kiers, Kevin Little, Alakabha Datta, David London, Makiko Nagashima, and Alejandro Szynkman. CP violation in $\tau \to K\pi\pi\nu(\tau)$. *Phys. Rev.*, D78:113008, 2008. doi: 10.1103/PhysRevD.78.113008.

[65] Georges Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Phys. Lett.*, B716:1–29, 2012. doi: 10.1016/j.physletb.2012.08.020.

[66] Serguei Chatrchyan et al. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Phys. Lett.*, B716:30–61, 2012. doi: 10.1016/j.physletb.2012.08.021.

[67] S. Dittmaier et al. Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables. 2011. doi: 10.5170/CERN-2011-002.

[68] Georges Aad et al. Search for the Standard Model Higgs boson in the H to $\tau^+\tau^-$ decay mode in $\sqrt{s} = 7$ TeV pp collisions with ATLAS. *JHEP*, 09:070, 2012. doi: 10.1007/JHEP09(2012)070.

[69] Search for the Standard Model Higgs boson in H-> tau tau decays in proton-proton collisions with the ATLAS detector. 2012.

[70] Serguei Chatrchyan et al. Search for neutral Higgs bosons decaying to tau pairs in pp collisions at $\sqrt{s} = 7$ TeV. *Phys. Lett.*, B713:68–90, 2012. doi: 10.1016/j.physletb.2012.05.028.

[71] Search for the standard model Higgs boson decaying to tau pairs produced in association with a W or Z boson with the CMS experiment in pp collisions at p s = 7 and 8 TeV. 2013.

[72] Georges Aad et al. Search for neutral Higgs bosons of the minimal supersymmetric standard model in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector. *JHEP*, 11:056, 2014. doi: 10.1007/JHEP11(2014)056.

[73] Georges Aad et al. Search for charged Higgs bosons decaying via $H^+ \to \tau\nu$ in top quark pair events using pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector. *JHEP*, 06:039, 2012. doi: 10.1007/JHEP06(2012)039.

[74] Georges Aad et al. Search for charged Higgs bosons through the violation of lepton universality in $t\bar{t}$ events using pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS experiment. *JHEP*, 03:076, 2013. doi: 10.1007/JHEP03(2013)076.

[75] Vardan Khachatryan et al. Search for neutral MSSM Higgs bosons decaying to a pair of tau leptons in pp collisions. *JHEP*, 10:160, 2014. doi: 10.1007/JHEP10(2014)160.

[76] Serguei Chatrchyan et al. Search for a light charged Higgs boson in top quark decays in pp collisions at $\sqrt{s} = 7$ TeV. *JHEP*, 07:143, 2012. doi: 10.1007/JHEP07(2012)143.

[77] P. S. Bhupal Dev, A. Djouadi, R. M. Godbole, M. M. Muhlleitner, and S. D. Rindani. Determining the CP properties of the Higgs boson. *Phys. Rev. Lett.*, 100:051801, 2008. doi: 10.1103/PhysRevLett.100.051801.

[78] T. Pierzchala, E. Richter-Was, Z. Was, and M. Worek. Spin effects in tau lepton pair production at LHC. *Acta Phys. Polon.*, B32:1277–1296, 2001.

[79] J. P. Lees et al. Evidence for an excess of $\bar{B} \to D^{(*)}\tau^- \bar{\nu}_\tau$ decays. *Phys. Rev. Lett.*, 109:101802, 2012. doi: 10.1103/PhysRevLett.109.101802.

[80] Nima Arkani-Hamed, Andrew G. Cohen, and Howard Georgi. Electroweak symmetry breaking from dimensional deconstruction. *Phys. Lett.*, B513:232–240, 2001. doi: 10.1016/S0370-2693(01)00741-9.

[81] N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson. The Littlest Higgs. *JHEP*, 07:034, 2002. doi: 10.1088/1126-6708/2002/07/034.

[82] Claudio A. Scrucca, Marco Serone, and Luca Silvestrini. Electroweak symmetry breaking and fermion masses from extra dimensions. *Nucl. Phys.*, B669:128–158, 2003. doi: 10.1016/j.nuclphysb.2003.07.013.

[83] Csaba Csaki, Christophe Grojean, Luigi Pilo, and John Terning. Towards a realistic model of Higgsless electroweak symmetry breaking. *Phys. Rev. Lett.*, 92:101802, 2004. doi: 10.1103/PhysRevLett.92.101802.

[84] Roberto Contino, Yasunori Nomura, and Alex Pomarol. Higgs as a holographic pseudoGoldstone boson. *Nucl. Phys.*, B671:148–174, 2003. doi: 10.1016/j.nuclphysb.2003.08.027.

[85] Kaustubh Agashe, Roberto Contino, and Alex Pomarol. The Minimal composite Higgs model. *Nucl. Phys.*, B719:165–187, 2005. doi: 10.1016/j.nuclphysb.2005.04.035.

[86] David B Kaplan and Howard Georgi. $su(2) \times u(1)$ breaking by vacuum misalignment. *Physics Letters B*, 136(3):183–186, 1984.

[87] Savas Dimopoulos and John Preskill. Massless composites with massive constituents. *Nuclear Physics B*, 199(2):206–222, 1982.

[88] Kaustubh Agashe, Antonio Delgado, Michael J. May, and Raman Sundrum. RS1, custodial isospin and precision tests. *JHEP*, 08:050, 2003. doi: 10.1088/1126-6708/2003/08/050.

[89] G. F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi. The Strongly-Interacting Light Higgs. *JHEP*, 06:045, 2007. doi: 10.1088/1126-6708/2007/06/045.

[90] Aneesh Manohar and Howard Georgi. Chiral quarks and the non-relativistic quark model. *Nuclear Physics B*, 234(1):189–212, 1984.

[91] Maxim Perelstein. Little Higgs models and T parity. *Pramana*, 67:813–820, 2006. doi: 10.1007/s12043-006-0094-x.

[92] Tao Han, Heather E. Logan, and Lian-Tao Wang. Smoking-gun signatures of little Higgs models. *JHEP*, 01:099, 2006. doi: 10.1088/1126-6708/2006/01/099.

[93] *Workshop on CP Studies and Non-Standard Higgs Physics*, 2006. URL http://weblib.cern.ch/abstract?CERN-2006-009.

[94] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire, and Jay G. Wacker. The Minimal moose for a little Higgs. *JHEP*, 08:021, 2002. doi: 10.1088/1126-6708/2002/08/021.

[95] Hsin-Chia Cheng and Ian Low. TeV symmetry and the little hierarchy problem. *JHEP*, 09:051, 2003. doi: 10.1088/1126-6708/2003/09/051.

[96] Nima Arkani-Hamed, Andrew G. Cohen, Thomas Gregoire, and Jay G. Wacker. Phenomenology of electroweak symmetry breaking from theory space. *JHEP*, 08:020, 2002.

[97] Nima Arkani-Hamed, Andrew G. Cohen, and Howard Georgi. (De)constructing dimensions. *Phys. Rev. Lett.*, 86:4757–4761, 2001. doi: 10.1103/PhysRevLett.86.4757.

[98] Christopher T. Hill, Stefan Pokorski, and Jing Wang. Gauge invariant effective Lagrangian for Kaluza-Klein modes. *Phys. Rev.*, D64:105005, 2001. doi: 10.1103/PhysRevD.64.105005.

[99] Hsin-Chia Cheng, Christopher T. Hill, Stefan Pokorski, and Jing Wang. The Standard model in the latticized bulk. *Phys. Rev.*, D64:065007, 2001. doi: 10.1103/PhysRevD.64.065007.

[100] Martin Schmaltz. The Simplest little Higgs. *JHEP*, 08:056, 2004. doi: 10.1088/1126-6708/2004/08/056.

[101] David E. Kaplan and Martin Schmaltz. The Little Higgs from a simple group. *JHEP*, 10:039, 2003. doi: 10.1088/1126-6708/2003/10/039.

[102] Witold Skiba and John Terning. A Simple model of two little Higgses. *Phys. Rev.*, D68:075001, 2003. doi: 10.1103/PhysRevD.68.075001.

[103] Jay Hubisz and Patrick Meade. Phenomenology of the littlest Higgs with T-parity. *Phys. Rev.*, D71:035016, 2005. doi: 10.1103/PhysRevD.71.035016.

[104] Andreas Birkedal-Hansen and Jay G. Wacker. Scalar dark matter from theory space. *Phys. Rev.*, D69:065022, 2004. doi: 10.1103/PhysRevD.69.065022.

[105] W. Kilian and J. Reuter. The Low-energy structure of little Higgs models. *Phys. Rev.*, D70:015004, 2004. doi: 10.1103/PhysRevD.70.015004.

[106] F. del Aguila, M. Masip, and J. L. Padilla. A Little Higgs model of neutrino masses. *Phys. Lett.*, B627:131–136, 2005. doi: 10.1016/j.physletb.2005.08.115.

[107] JÃ¼rgen Reuter, Marco Tonini, and Maikel de Vries. Little Higgs Model Limits from LHC - Input for Snowmass 2013. In *Snowmass 2013: Workshop on Energy Frontier Seattle, USA, June 30-July 3, 2013*, 2013. URL http://inspirehep.net/record/1243423/files/arXiv:1307.5010.pdf.

[108] Hsin-Chia Cheng and Ian Low. Little hierarchy, little Higgses, and a little symmetry. *JHEP*, 08:061, 2004. doi: 10.1088/1126-6708/2004/08/061.

[109] Otto C. W. Kong. A Completed chiral fermionic sector model with little Higgs. 2003.

[110] David E. Kaplan, Martin Schmaltz, and Witold Skiba. Little Higgses and turtles. *Phys. Rev.*, D70:075009, 2004. doi: 10.1103/PhysRevD.70.075009.

[111] Xiao-Gang He, Jusak Tandean, and G. Valencia. Penguin and Box Diagrams in Unitary Gauge. *Eur. Phys. J.*, C64:681–687, 2009. doi: 10.1140/epjc/s10052-009-1162-4.

[112] Edward Witten. Current algebra theorems for the $u(1)$ "goldstone boson". *Nuclear Physics B*, 156(2):269–283, 1979.

[113] Juerg Gasser and Heinrich Leutwyler. Chiral perturbation theory to one loop. *Annals of Physics*, 158(1):142–210, 1984.

[114] J. Gasser and H. Leutwyler. Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark. *Nucl. Phys.*, B250:465, 1985. doi: 10.1016/0550-3213(85)90492-4.

[115] John F Donoghue, Carlos Ramirez, and German Valencia. Spectrum of qcd and chiral lagrangians of the strong and weak interactions. *Physical Review D*, 39(7):1947, 1989.

[116] E. Arganda, M. J. Herrero, and J. Portoles. Lepton flavour violating semileptonic tau decays in constrained MSSM-seesaw scenarios. *JHEP*, 06:079, 2008. doi: 10.1088/1126-6708/2008/06/079.

[117] Ernesto Arganda and Maria J. Herrero. Testing supersymmetry with lepton flavor violating tau and mu decays. *Phys. Rev.*, D73:055003, 2006. doi: 10.1103/PhysRevD.73.055003.

[118] K. A. Olive et al. Review of Particle Physics. *Chin. Phys.*, C38:090001, 2014. doi: 10.1088/1674-1137/38/9/090001.

[119] J. Reuter and M. Tonini. Can the 125 GeV Higgs be the Little Higgs? *JHEP*, 02:077, 2013. doi: 10.1007/JHEP02(2013)077.

[120] J. de Blas. Electroweak limits on physics beyond the Standard Model. *EPJ Web Conf.*, 60:19008, 2013. doi: 10.1051/epjconf/20136019008.

[121] Frank F. Deppisch, P. S. Bhupal Dev, and Apostolos Pilaftsis. Neutrinos and Collider Physics. *New J. Phys.*, 17(7):075019, 2015. doi: 10.1088/1367-2630/17/7/075019.

[122] B. A. Shwartz. The Belle II Experiment. *Nucl. Part. Phys. Proc.*, 260:233–237, 2015. doi: 10.1016/j.nuclphysbps.2015.02.049.

[123] J. Hisano, T. Moroi, K. Tobe, and Masahiro Yamaguchi. Lepton flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model. *Phys. Rev.*, D53:2442–2459, 1996. doi: 10.1103/PhysRevD.53.2442.

[124] Alejandro Celis, Vincenzo Cirigliano, and Emilie Passemar. Lepton flavor violation in the Higgs sector and the role of hadronic τ-lepton decays. *Phys. Rev.*, D89:013008, 2014. doi: 10.1103/PhysRevD.89.013008.

[125] Alejandro Celis, Vincenzo Cirigliano, and Emilie Passemar. Model-discriminating power of lepton flavor violating τ decays. *Phys. Rev.*, D89(9):095014, 2014. doi: 10.1103/PhysRevD.89.095014.

[126] Serguei Chatrchyan et al. Observation of a new boson with mass near 125 GeV in pp collisions at $\sqrt{s} = 7$ and 8 TeV. *JHEP*, 06:081, 2013. doi: 10.1007/JHEP06(2013)081.

[127] T. Aaltonen et al. Search for the standard model Higgs boson decaying to a bb pair in events with two oppositely-charged leptons using the full CDF data set. *Phys. Rev. Lett.*, 109:111803, 2012. doi: 10.1103/PhysRevLett.109.111803.

[128] Gianluca Blankenburg, John Ellis, and Gino Isidori. Flavour-Changing Decays of a 125 GeV Higgs-like Particle. *Phys. Lett.*, B712:386–390, 2012. doi: 10.1016/j.physletb.2012.05.007.

[129] Alejandro Celis, Vincenzo Cirigliano, and Emilie Passemar. Lepton flavor violation in the higgs sector and the role of hadronic τ-lepton decays. *Physical Review D*, 89(1):013008, 2014.

[130] Vardan Khachatryan et al. Search for Lepton-Flavour-Violating Decays of the Higgs Boson. *Phys. Lett.*, B749:337–362, 2015. doi: 10.1016/j.physletb.2015.07.053.

[131] J. G. Körner, A. Pilaftsis, and K. Schilcher. Leptonic CP asymmetries in flavor-changing H^0 decays. *Phys. Rev. D*, 47:1080–1086, Feb 1993. doi: 10.1103/PhysRevD.47.1080. URL http://link.aps.org/doi/10.1103/PhysRevD.47.1080.

[132] J Lorenzo Diaz-Cruz and JJ Toscano. Lepton flavor violating decays of higgs bosons beyond the standard model. *Physical Review D*, 62(11):116005, 2000.

[133] Tao Han and Danny Marfatia. $h \to \mu\tau$ at hadron colliders. *Physical review letters*, 86(8):1442, 2001.

[134] J Lorenzo Diaz-Cruz. A more flavored higgs boson in supersymmetric models. *Journal of High Energy Physics*, 2003(05):036, 2003.

[135] Andrea Brignole and Anna Rossi. Lepton flavour violating decays of supersymmetric higgs bosons. *Physics Letters B*, 566(3):217–225, 2003.

[136] Ernesto Arganda, Ana M Curiel, María J Herrero, and David Temes. Lepton flavor violating higgs boson decays from massive seesaw neutrinos. *Physical Review D*, 71 (3):035011, 2005.

[137] James D Bjorken and Steven Weinberg. Mechanism for nonconservation of muon number. *Physical Review Letters*, 38(12):622, 1977.

[138] Svjetlana Fajfer, Jernej F. Kamenik, Ivan Nisandzic, and Jure Zupan. Implications of Lepton Flavor Universality Violations in B Decays. *Phys. Rev. Lett.*, 109:161801, 2012. doi: 10.1103/PhysRevLett.109.161801.

[139] L F Li. Properties of Higgs particles. 1980. URL https://cds.cern.ch/record/123371.

[140] O Shanker. Flavour violation, scalar particles and leptoquarks. *Nuclear Physics B*, 206(2):253–272, 1982.

[141] Roni Harnik, Joachim Kopp, and Jure Zupan. Flavor Violating Higgs Decays. *JHEP*, 03:026, 2013. doi: 10.1007/JHEP03(2013)026.

[142] Brando Bellazzini, Csaba Csáki, and Javi Serra. Composite Higgses. *Eur. Phys. J.*, C74(5):2766, 2014. doi: 10.1140/epjc/s10052-014-2766-x.

[143] Spencer Chang and Jay G Wacker. Little higgs models and custodial su (2). *Physical Review D*, 69(3):035002, 2004.

[144] Spencer Chang. A 'Littlest Higgs' model with custodial SU(2) symmetry. *JHEP*, 12: 057, 2003. doi: 10.1088/1126-6708/2003/12/057.

[145] A. Lami, J. Portoles, and P. Roig. Lepton flavor violation in hadronic decays of the tau lepton in the simplest little Higgs model. *Phys. Rev.*, D93(7):076008, 2016. doi: 10.1103/PhysRevD.93.076008.

[146] C Itzykson and JB Zuber. Quantum field theory, mcgraw-hillbook company, 1980.

[147] LB Okun. Leptons and quarks north, 1982.