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COLOUR OCTET HIGGS MODEL

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Abstract

In this work we consider a model based on the Standard Model supplemented by a scalar having the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers $(\mathbf{8},\mathbf{2})_{1/2}$ which was proposed by A. V. Manohar and M. B. Wise [1]. As we show here, this model is consistent with the principle of minimal flavour violation and custodial $SU(2)_{L+R}$ symmetry can be implemented. We find its interactions with the SM particles and study the phenomenological implications of the existence of these new scalars. Constraints of the parameters are find studying the Higgs signal strengths, the oblique parameter S and the running of the strong coupling. Using the values of the Higgs signal strengths we perform a χ^2 test and also study specific limits that each Higgs signal strengths produces.

Contents

1 Introduction

The discovery of the Higgs boson on July of 2012 by the LHC has been one of the most important milestones of the decade in particle physics. This scalar particle was the last remnant of the Standard Model (SM) of particle physics and its discovery is the confirmation of the existence of a particle already predicted in the 60s [2–4]. The importance of this particle is that it solves the problem of generating the masses of the particles of the SM.

The "minimum" Higgs mechanism, which considers the existence of just one scalar, is not the only possibility for generating the masses of the SM. The discovery of the Higgs, therefore, can be understood not just like the confirmation of the mechanism needed to generate masses in the SM but like the discovery of the first element of a possible more extended scalar sector.

The SM, although has been extremely successful on many situations, has, as well, some lacks. This model is not able to explain the asymmetry between matter and antimatter, the origin of dark matter, how the masses of the neutrinos are generated or how the scales of the masses are produced. These lacks can be solved through extensions of the SM where more particles of higher energy are considered. For example, the problem of the neutrino masses could be solved considering the existence of new particles at higher scales which generate these masses through the so called seesaw mechanism [5].

In this work we focus on the extensions of the scalar sector of the SM. We consider the model of A. V. Manohar and M. B. Wise [1] which is based on the existence of a new scalar doublet, like the one of the SM, but with other quantum number, the colour, like the gluons responsible of the strong interaction. This model has additional imaginary parameters to the one of the SM. These are sources of CP violation which are needed for solving the problem of the asymmetry between matter and antimatter.

The aim of this work is to review this model developing its Lagrangian and calculating the interaction between the new scalars and the particles of the SM. Nowadays we have a huge among of data coming from the LHC which can be used to constrain the parameters of many models of new physics. In this work we also try to update the phenomenological analysis with the data coming from this powerful machine.

The work is organised as follows. In §2 we make a brief review of the SM developing its Lagrangian from gauge invariance and explaining how this symmetry is spontaneously broken by the vacuum. Furthermore, we explain how this spontaneous symmetry breaking can lead to the generation of the masses of the particles of the SM trough the Higgs mechanism. In §3 we explain the model of A. V. Manohar and M. B. Wise. We show the new particles that appear in this model, we calculate their masses and their interactions with the particles of the SM. Moreover, we show how these particles can be integrated out condensing their properties on operators of higher dimension. In §4 we analyse the phenomenological implications of this model. We present the constraints on the masses found by Ref. [6], we analyse the constraints that are generated by the Higgs signal strengths and by the oblique parameter S. We also study how the running of the strong coupling gets modified by the presence of these new scalars which can be used to find additional constraints on the masses. Finally in §5 we conclude the work. The Feynman rules of some interesting processes can be found in appendix A and some calculations at loop-level can be found developed step-by-step in appendix B.

2 The Standard Model

The Standard Model is a quantum field theory that explains the strong, weak and electromagnetic interactions combining special relativity and quantum mechanics. This theory is a gauge theory based on the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.[1] This means that imposing invariance to the Lagrangian under a local $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ transformation the interaction terms appear naturally in the theory, i.e., they are needed to preserve this symmetry. The first theory of this kind was Quantum Electrodynamics (QED) which was completed at the end of the 40s [7–10] by S. Tomonaga, J. Schwinger and R. Feynman, who were awarded with the Nobel prize in 1965 for this work. This gauge theory was based on the group $U(1)_Q$ and was able to explain the electromagnetic interactions with an accuracy never seen before. Actually, nowadays QED gives the most precise prediction of an observable, the anomalous magnetic moment of the electron [11], with an agreement between the theoretical prediction and the experiment measurement of 0.67 parts in a billion [12].

As a consequence of the success of the QED many physicists tried to develop gauge theories for the other interactions. The next gauge theory that was developed was the Yang-Mills theory in 1954 [13] based on the isospin $SU(2)$ symmetry, a non-abelian gauge theory. Although this theory did not explain correctly the strong interaction this work was the foundation for the following gauge theories. Its importance is such that both the strong and the weak interactions were explained with Yang-Mills theories.[2] The former interaction was explained by Quantum Chromodynamics (QCD) in 1973 [14], a Yang-Mills theory based on the symmetry group $SU(3)_C$ developed by Murray Gell-Mann, Harald Fritzsch and Heinrich Leutwyler. The latter by the electroweak theory that unifies the description of the weak and the electromagnetic interaction and is based on the $SU(2)_L \otimes U(1)_Y$ group. This theory was built in the 60s starting in 1961 with a paper of Glashow [15] where it was firstly proposed a $SU(2) \otimes U(1)$ model to describe the weak and electromagnetic interactions (also independently proposed by Salam and Ward [16] in 1964). The problem of these theories was that the symmetry breaking that gave mass to the gauge bosons responsible for the weak interaction was inserted by hand [17]. The solution to this problem was to generate the masses through a spontaneous symmetry breaking (SSB) of the gauge symmetry. However, the Goldstone theorem [18] apparently demonstrates that the SSB implies the generation of massless spin-zero bosons (the Nambu-Goldstone bosons) that had not been observed. This problem was solved when independently three different groups (Englert and Brout [2]; Higgs [3]; Guralnik, Hagen and Kibble [4]) showed that the Goldstone theorem does not apply on gauge theories. Finally in 1967 Weinberg [19] proposed what we know nowadays as electroweak theory.

2.1 Quantum Electrodynamics

To obtain the QED Lagrangian we start with the Lagrangian for a free Dirac fermion:

$$\mathcal{L}_0 = i\overline{\psi}(x)\gamma^\mu \partial_\mu \psi(x) - m\overline{\psi}(x)\psi(x). \tag{2.1}$$

This Lagrangian is clearly invariant under a global $U(1)_Q$ transformation

[1] Here subscripts C, L and Y refer respectively to colour, left part of the spinors and hypercharge ($Y = 2(Q + I_3)$, where Q is the electric charge and I_3 the third component of the isospin).

[2] As the theory of C. N. Yang and R. Mills was the first non-abelian gauge theory the term of Yang-Mills theory is used to describe such theories though they are based on a non-abelian group different than the one proposed by Yang and Mills.

$$\psi(x) \to \psi'(x) = \exp(iQ\theta)\psi(x), \tag{2.2}$$

where θ is a real constant and Q will be interpreted as the electric charge of the excitations of this field (in units of the elementary charge, e).

This Lagrangian is not invariant under a local gauge transformation though:

$$\psi(x) \to \psi'(x) = \exp(iQ\theta(x))(\partial_\mu + iQ\partial_\mu\theta(x))\psi(x). \tag{2.3}$$

This means that the phase can change without changing the physics but this change must be the same in the whole Universe, i.e., the parameter θ cannot depend on the space-time. This fact seems very unnatural and one would expect a total independence of the Lagrangian with the phase because this is not physical.

In order to obtain a local gauge invariant Lagrangian we have to add an extra piece which transforms properly to cancel the non-invariant terms of the Lagrangian, i.e., the term proportional to $\partial_\mu\theta(x)$ of Eq. (2.3). Since $\partial_\mu\theta(x)$ has a Lorentz index we need to introduce a new spin-1 field, $A_\mu(x)$ which transforms as:

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\theta. \tag{2.4}$$

So adding the term $-eQA_\mu(x)\overline{\psi}(x)\gamma^\mu\psi(x)$ to the Lagrangian of Eq. (2.1) we obtain a local gauge invariant Lagrangian. Due to the geometrical significance of the local gauge invariance it is convenient to interpret this term as part of a covariant derivative in such a way that our Lagrangian becomes:

$$\mathcal{L} = i\overline{\psi}(x)\gamma^\mu D_\mu\psi(x) - m\overline{\psi}(x)\psi(x), \tag{2.5}$$

where $D_\mu\psi(x) = [\partial_\mu + ieQA_\mu(x)]\psi(x)$. This covariant derivative transforms under local gauge transformation like the derivative does under global gauge transformations:

$$D_\mu\psi(x) \to \psi'(x) = \exp(iQ\theta(x))D_\mu\psi(x). \tag{2.6}$$

Finally, we would need to add the kinetic term of the gauge field if we want this to be a true propagating field:

$$\mathcal{L}_{\text{Kin}} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x), \tag{2.7}$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual electromagnetic strength, clearly invariant under local gauge transformations.

Therefore, interpreting the gauge field found here as the electromagnetic field we find the proper interaction between the fermion and the electromagnetic field. Note that this interaction arises naturally by imposing local gauge symmetry and it is not added by hand. This is the power of the gauge principle. Besides, gauge invariance forbids mass terms for the gauge field ($\frac{1}{2}m^2A^\mu A_\mu$), providing a natural explanation of the fact that the photon is massless.

2.2 Quantum Chromodynamics

In the 1960s the abundance of baryonic and mesonic states suggested the existence of a deeper level of elementary constituents of matter, the quarks. This new particles were proposed in 1964 [20,21] independently by Gell-Mann and Zweig in order to explain the Eightfold Way. Remember that the Eightfold Way was a classification of the baryons and mesons that Gell-Mann himself introduced in 1961 [22]. The Eightfold Way arranged the known mesons in octets and the known baryons in an octet of particles with spin 1/2 and a decuplet[3] of particles of spin 3/2.

The way in which the quark model explained the Eightfold Way is by considering that the baryons were composed of three fermions of spin 1/2, three quarks, and the mesons of a quark and an antiquark. Furthermore they consider that three different types (flavours) of quarks existed (u, d and s type).[4] With three different quarks they obtained nine instead of eight mesons, the missing meson was later discovered, and ten baryons for a given spin. Obtaining the baryon octet is a bit trickier and we refer the interested reader to Ref. [24].

The problem of this model was that it violated the Pauli exclusion principle, there were baryons constituted by three equal quarks. In order to avoid the violation of this principle O. W. Greenberg proposed that the quarks had another quantum number [25], the colour, such that each quark can have three different colours. Baryons and mesons are described by colour-singlet combinations:

$$B = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|q_\alpha q_\beta q_\gamma\rangle, \qquad M = \frac{1}{\sqrt{3}}\delta^{\alpha\beta}|q_\alpha \overline{q}_\beta\rangle. \tag{2.8}$$

As states with colour have not been observed it is assumed that the states of nature are colourless. This assumption is known as confinement hypothesis.

The existence of this new quantum number characteristic of the particles that feel the strong interactions suggested that colour plays the same role in the strong interactions as the electric charge in QED. Finally, in 1973, Murray Gell-Mann, Harald Fritzsch and Heinrich Leutwyler developed a gauge theory based on $SU(3)_{\text{Colour}}$ [14], the quantum chromodynamics theory (QCD), that explained the strong interactions.

In order to find the Lagrangian of QCD we start with the free Lagrangian of the fermions, as we did for QED:

$$\mathcal{L}_0 = \sum_f \overline{q}_f(i\gamma^\mu\partial_\mu - m_f)q_f, \tag{2.9}$$

where the subscript f refers to flavour and we have adopted a vector notation in colour space, $q_f^T = (q_f^1, q_f^2, q_f^3)$.

This Lagrangian is clearly invariant under global $SU(3)_C$ transformations:

$$q_f^\alpha \to (q_f^\alpha)' = U^\alpha_\beta q_f^\beta, \qquad UU^\dagger = U^\dagger U = 1 \qquad \det U = 1. \tag{2.10}$$

These $SU(3)$ matrices can be expressed in terms of the generators of the fundamental representation $\frac{1}{2}\lambda^a$ (a =1,2,...,8):

$$U = \exp\Big\{i\frac{\lambda^a}{2}\theta_a\Big\}. \tag{2.11}$$

[3] Actually one of the particles was missing so he proposed the existence of this new particle that was discovered in 1964 [23].

[4] Nowadays we know that there are actually six flavours (u, d, s, c, b and t type).

where θ_a are arbitrary constants and the $\frac{1}{2}\lambda^a$ matrices are traceless and form a Lie algebra:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = if^{abc}\frac{\lambda^c}{2}, \tag{2.12}$$

where f^{abc} are the $SU(3)_C$ structure constants, which are real and totally antisymmetric.

Like we did in the case of QED we impose local $SU(3)_C$ invariance in our Lagrangian. For doing so we add eight independent gauge parameters in our Lagrangian, the gluons, as we have eight independent generators. The covariant derivative will be in this case of the form:

$$D^\mu q_f = \left[\partial^\mu + ig_s\frac{\lambda^a}{2}G_a^\mu(x)\right]q_f = [\partial^\mu + ig_s G^\mu(x)]q_f. \tag{2.13}$$

This covariant derivative transforms under local $SU(3)_C$ transformations in the same way that a standard derivative does under global $SU(3)_C$ transformations, thanks to the specific transformation of the gauge fields:

$$D^\mu \to (D^\mu)' = UD^\mu U^\dagger, \qquad G^\mu = UG^\mu U^\dagger + \frac{i}{g_s}(\partial^\mu U)U^\dagger. \tag{2.14}$$

Finally, we have to add the kinetic term for the gluon field. The field strengths is the one corresponding to a non-Abelian field

$$G^{\mu\nu}(x) = -\frac{i}{g_s}[D^\mu, D^\nu] = \partial^\mu G^\nu - \partial^\nu G^\mu + ig_s[G^\mu, G^\nu] = \frac{\lambda^a}{2}G_a^{\mu\nu}(x), \tag{2.15}$$

which transforms under $SU(3)_C$ transformation like:

$$G^{\mu\nu} \to (G^{\mu\nu})' = UG^{\mu\nu}U^\dagger, \tag{2.16}$$

so the colour trace $\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) = \frac{1}{2}G_a^{\mu\nu}G^a_{\mu\nu}$ remains invariant. Therefore, the total Lagrangian will be:

$$\mathcal{L} = -\frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} + \sum_f \overline{q}_f(i\gamma^\mu D_\mu - m_f)q_f. \tag{2.17}$$

2.3 Electroweak Unification

As mentioned above, the weak interactions were explained by the Electroweak theory, a non-Abelian gauge theory based on the symmetry group $SU(2)_L \otimes U(1)_Y$. This theory unifies the description of the weak and electromagnetic interactions.

To understand properly this theory we have to take into account that the weak interactions do not conserve parity. This means that the spinors of our Lagrangian have to be split in their left and right part[5] because the left and right parts feel different interactions.

It is also important to identify the particle content of our theory. As mention in the previous section, there are six different types of quarks (six flavours) and each of which can be in one of the three different colour states. However, in this section, we can forget about colour because

[5]Note that the spinors are not an irreducible representation of the Poincaré group. This means that we can express the Lagrangian in terms of the decomposition of this reducible representation, the left and the right part of the spinors, which are the irreducible representation.

the electromagnetic and weak interactions are blind to it. With respect to the flavour, they are organised in three families in such a way that the left parts of two of the flavours form a doublet:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L, \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L, \tag{2.18}$$

so we have the family of the up and down quarks, the family of the charm and strange quarks and the family of the top and bottom quarks.

For the leptons we have as well three families where the left parts of them are also organised in doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L. \tag{2.19}$$

In order to simplify the calculation we will consider just one family,

$$\mathcal{L}_0 = i\overline{u}(x)\gamma^\mu\partial_\mu u(x) + i\overline{d}(x)\gamma^\mu\partial_\mu d(x) = \sum_{j=1}^{3} i\overline{\psi}_j(x)\gamma^\mu\partial_\mu\psi_j(x), \tag{2.20}$$

where we have introduced the notation

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad \psi_2(x) = u_R, \qquad \psi_3(x) = d_R. \tag{2.21}$$

This Lagrangian is invariant under global $SU(2)_L \otimes U(1)_Y$

$$\begin{aligned}
\psi_1(x) &\to \psi_1' = \exp\{iy_1\beta\}\exp\left\{i\frac{\sigma_j}{2}\alpha^j\right\}\psi_i(x), \\
\psi_2(x) &\to \psi_2' = \exp\{iy_2\beta\}\psi_2(x), \\
\psi_2(x) &\to \psi_3' = \exp\{iy_2\beta\}\psi_3(x),
\end{aligned} \tag{2.22}$$

where σ_j $(j = 1, 2, 3)$ are the the Pauli matrices, generators of the fundamental representation of the $SU(2)$ group. Clearly the right parts of the fields are singlets under $SU(2)_L$ transformations, so this transformation do not affect them. The parameters y_i are the hypercharges which will have to be set to a particular value to find the proper electromagnetic interaction.

As we did in QED and in QCD we require the Lagrangian to be invariant under local $SU(2)_L \otimes U(1)_Y$ transformations. We need therefore to introduce four new fields that change the standard derivatives by covariant derivatives:

$$\begin{aligned}
D_\mu\psi_1(x) &= \left[\partial_\mu + ig\frac{\sigma_j}{2}W_\mu^j(x) + ig'y_1B_\mu(x)\right]\psi_1(x), \\
D_\mu\psi_2(x) &= \left[\partial_\mu + ig'y_2B_\mu(x)\right]\psi_2(x), \\
D_\mu\psi_3(x) &= \left[\partial_\mu + ig'y_3B_\mu(x)\right]\psi_3(x).
\end{aligned} \tag{2.23}$$

This covariant derivatives have to transform under local $SU(2)_L \otimes U(1)_Y$ transformations in the same way that the usual derivative does under global $SU(2)_L \otimes U(1)_Y$ transfomations. This fixes the transformation of the gauge fields:

$$B_\mu(x) \to B'_\mu(x) = B_\mu(x) - \frac{1}{g'}\partial_\mu \beta(x), \tag{2.24}$$

$$\widetilde{W}_\mu(x) \to \widetilde{W}'_\mu(x) = U_L(x)\widetilde{W}_\mu U_L^\dagger(x) + \frac{i}{g}\partial_\mu U_L(x)U_L^\dagger, \tag{2.25}$$

where we have used the notation $\widetilde{W}_\mu(x) = \frac{\sigma_j}{2}W_\mu^j(x)$ and $U_L(x) = \exp\{i\frac{\sigma_j}{2}\alpha^j(x)\}$.

Besides of changing the usual derivative to this covariant derivative, we have to add the kinetic terms of the new fields if we want them to be true propagating fields. This kinetic terms are, like in QED and QCD, constructed from the corresponding field strengths:

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu, \tag{2.26}$$

$$\widetilde{W}^{\mu\nu}(x) = -\frac{i}{g}[(\partial^\mu + ig\widetilde{W}^\mu), (\partial^\nu + ig\widetilde{W}^\nu)] = \partial^\mu\widetilde{W}^\nu - \partial^\nu\widetilde{W}^\mu + ig_s[\widetilde{W}^\mu, \widetilde{W}^\nu] = \frac{\sigma^j}{2}\widetilde{W}_j^{\mu\nu}(x). \tag{2.27}$$

Once we have defined the field strengths we can construct the kinetic parts that will be like in QED for the B_μ field and like in QCD for the $\widetilde{W}_\mu$ field. The total Lagrangian becomes:

$$\mathcal{L} = i\overline{\psi}_j(x)\gamma^\mu D_\mu \psi^j(x) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W_i^{\mu\nu}, \tag{2.28}$$

where the sum over repeated indices is implicit.

Note that, as in the previous theories, gauge invariance forbids the existence of mass terms for the gauge bosons but in this case the mass terms for the fermions are also forbidden. Remember that such terms are of the form $m\overline{\psi}\psi = m(\overline{u}_R u_L + \overline{u}_L u_R)$ and these terms are not invariant under $SU(2)_L$ transformations. In the next section we will show how the Higgs mechanism solves this problem introducing the Spontaneous Symmetry Breaking but first, let us develop this Lagrangian to find the theory of QED.

Let us look at the interaction of the W_μ bosons with the fermionic field, i.e., the term $-g\overline{\psi}_1\gamma^\mu\frac{\sigma_j}{2}W_\mu^j\psi_1$. Writing explicitly the Pauli matrices we obtain:

$$\frac{\sigma_j}{2}W_\mu^j = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^\dagger \\ \sqrt{2}W_\mu & -W_\mu^3 \end{pmatrix}, \tag{2.29}$$

where we have defined $W_\mu = (W_\mu^1 + iW_\mu^2)/\sqrt{2}$. We therefore construct a complex field (W_μ) with the two independent real gauge fields W_μ^1 and W_μ^2. The excitations of this field give rise to two charged bosons, W^+ and W^-, and its interaction with the fermions is given by:

$$\begin{aligned}\mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}}\Big\{W_\mu^\dagger[\overline{u_L}\gamma^\mu d_L + \overline{\nu_L}_e\gamma^\mu e_L] + \text{h. c.}\Big\}. \\ &= -\frac{g}{2\sqrt{2}}\Big\{W_\mu^\dagger[\overline{u}\gamma^\mu(1-\gamma_5)d + \overline{\nu}_e\gamma^\mu(1-\gamma_5)e] + \text{h. c.}\Big\}.\end{aligned} \tag{2.30}$$

In order to analyse the terms of the B_μ field, $-g'B_\mu\sum_{j=1}^3 y_j\overline{\psi}_j\gamma^\mu\psi_j$, we have to take into account that this gauge field cannot be the electromagnetic field, because the photons conserve parity. Therefore we have to find a linear combination of the B_μ and W_μ^3 fields, both neutral fields, in order to find the electromagnetic interaction:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{2.31}$$

The interaction of these new fields, Z_μ and A_μ, with the fermions is:

$$\mathcal{L}_{NC} = -\sum_{j=1}^{3} \overline{\psi}_j \gamma^\mu \Big\{ A_\mu \Big[g\frac{\sigma_3}{2}\sin\theta_W + g' y_j \cos\theta_W \Big] + Z_\mu \Big[g\frac{\sigma_3}{2}\cos\theta_W - g' y_j \sin\theta_W \Big] \Big\} \psi_j. \tag{2.32}$$

But in order to find the QED Lagrangian a precise relation have to be satisfied:

$$g\sin\theta_W = g'\cos\theta_W = e, \qquad\qquad Y = Q - T_3, \tag{2.33}$$

where[6] $T_3 = \sigma_3/2$ and Q is the electromagnetic charge operator:

$$Q_1 = \begin{pmatrix} Q_{u/\nu} & 0 \\ 0 & Q_{d/e} \end{pmatrix}, \qquad Q_2 = Q_{u/\nu}, \qquad Q_3 = Q_{d/e}. \tag{2.34}$$

The second equality of Eq. (2.33) provides the definition of the hypercharge in order to reproduce the QED theory, as we announced previously.

2.4 Spontaneous Symmetry Breaking

As we have seen before the $SU(2)_L \otimes U(1)_Y$ symmetry of the Electroweak theory forbids the mass terms for the gauge bosons and the fermions. To generate the masses for the fermions and the Z and $W^\pm$ bosons we need to break the gauge symmetry but we want to break it in a natural way without imposing it by hand. Moreover, it is crucial to have a gauge invariant Lagrangian to have a renormalisable theory. The most natural way for doing it is breaking the symmetry spontaneously by the vacuum. This is generated when we have an invariant Lagrangian under a group transformation, G, but the vacuum is degenerated, in such a way that the vacuum states are related through a transformation under the group G. All the vacuum states are equally provable but the Nature chooses one breaking the symmetry. This is what happens when we cool a ferromagnet under the Curie temperature, the magnetisation can be acquired in one direction or the reverse but one is chosen spontaneously.

One of the difficulties of applying spontaneous symmetry breaking on the electroweak theory was that it seemed to imply the appearance of massless spin-0 particles that have not been observed, this is the so called Goldstone theorem.

2.4.1 Goldstone Theorem

The Goldstone Theorem states that: "*In any field theory satisfying locality, Lorentz invariance and positive definite norm, if an exact continuous symmetry of the Lagrangian is not a symmetry of the physical vacuum, the theory must contain a massless spin zero particle(s) whose quantum numbers are those of the broken group generator(s).*"

Proof:

[6] Note that, in general, T_3 is the third generator of the fundamental representation of the $SU(2)$ group but, as we have chosen the Pauli matrices as our basis, $T_3 = \sigma_3/2$

Consider that we have a set of n real fields ϕ_i with a potential $V(\phi_i)$. Now consider that the Lagrangian is invariant under a group, G, that transforms the fields like:

$$\phi_i \to \phi_i' = \phi_i + \delta\phi_i = \phi_i + i\epsilon^a T^a_{ij}\phi_j, \tag{2.35}$$

where T^a are the generators of the group G.

The potential $V(\phi_i)$ is also invariant:

$$\delta V(\phi_i) = i\frac{\partial V}{\partial \phi_i}\epsilon^a T^a_{ij}\phi_j = 0 \quad \Rightarrow \quad \frac{\partial V}{\partial \phi_i} T^a_{ij}\phi_j = 0. \tag{2.36}$$

Let us consider that the potential is minimised by $\phi_i = v_i$

$$\left(\frac{\partial V}{\partial \phi_i}\right)_{\phi_i = v_i} = 0. \tag{2.37}$$

Deriving again on Eq. (2.36) we get

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_k} T^a_{ij}\phi_j + \frac{\partial V}{\partial \phi_i} T^a_{ik} = 0. \tag{2.38}$$

Evaluating Eq. (2.38) in the minimum and using Eq. (2.37) we obtain

$$\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_k}\right|_{\phi_j = v_j} T^a_{ij}\phi_j = 0 \quad \Rightarrow \quad M^2_{ij} T^a_{ij} v_j = 0. \tag{2.39}$$

Then, if the generator T^a is broken by the vacuum, $T^a_{ij}v_j \neq 0$, Eq. (2.39) implies the existence of a zero eigenvalue on the mass matrix, i.e., the existence of a particle of zero mass. If there is a subgroup $H \subset G$ that leaves the vacuum invariant the mass matrix has $(\dim G - \dim H)$ zero eigenvalues, one for each broken generator.

2.4.2 The Higgs Mechanism

Let us show how, in theories with a local gauge symmetry, the Goldstone theorem can be avoided, interpreting the Goldstone bosons merely as non-physical objects that can be eliminated of our Lagrangian thanks to the local gauge invariance. For doing so let us consider the Electroweak theory with an $SU(2)_L$ doublet of complex scalar fields,

$$\phi(x) = \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}. \tag{2.40}$$

The potential of this term would be:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger\phi)^2, \tag{2.41}$$

with $\lambda > 0$ to have a potential bounded from below and with $\mu^2 < 0$ to have a degenerated vacuum.

The minimum of this potential is localised at

$$\frac{\partial V(\phi)}{\partial |\phi|} = |\phi|(2\mu^2 + 4\lambda|\phi|^2) = 0 \Rightarrow |\phi| = \sqrt{\frac{-\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}, \tag{2.42}$$

where we have defined our $v = \sqrt{\frac{-\mu^2}{\lambda}}$.

The states whose vacuum expectation value (vev) satisfy Eq. (2.42) will be the states of minimum energy of the Lagrangian. As the electric charge must be conserved, the $\phi^{(+)}(x)$ field cannot acquire a vev, so the only possibility is that

$$|\langle 0|\phi^{(0)}|0\rangle| = \frac{v}{\sqrt{2}}. \tag{2.43}$$

When one of these vacuum states is chosen, $\langle 0|\phi^{(0)}|0\rangle = \frac{v}{\sqrt{2}}$ for instance, the $SU(2)_L \otimes U(1)_Y$ is broken to the electromagnetic subgroup $U(1)_Q$:

$$\begin{gathered}
T_1\langle 0|\phi^{(0)}|0\rangle = \frac{\sigma_1}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{v}{2\sqrt{2}} \\ 0 \end{pmatrix} \neq 0, \qquad T_2\langle 0|\phi^{(0)}|0\rangle = \frac{\sigma_2}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -i\frac{v}{2\sqrt{2}} \\ 0 \end{pmatrix} \neq 0, \\
T_3\langle 0|\phi^{(0)}|0\rangle = \frac{\sigma_3}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{v}{2\sqrt{2}} \end{pmatrix} \neq 0, \qquad Y\langle 0|\phi^{(0)}|0\rangle = \frac{I}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{2\sqrt{2}} \end{pmatrix} \neq 0, \\
Q\langle 0|\phi^{(0)}|0\rangle = (Y + T_3)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\end{gathered} \tag{2.44}$$

In Eq. (2.44) can be seen how the generators of the $SU(2)_L \otimes U(1)_Y$ are broken by the vacuum while the generator of the $U(1)_Q$ is not broken. The Goldstone Theorem would then imply the existence of three massless particles, three Goldstone bosons. Note, however, that if we parametrise the scalar doublet like:

$$\phi(x) = \exp\Big(i\frac{\sigma_i}{2}\theta^i(x)\Big)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{2.45}$$

the local gauge invariance allows us to rotate the field in such a way that the three Goldstone bosons, $\theta^i(x)$ disappear. This means that these particles are not physical in the context of local gauge theories, the Goldstone theorem does not apply exactly in the same way as before. The remaining three degrees of freedom become the longitudinal degree of freedom of the massive gauge bosons $W^{\pm}$ and Z, which acquire mass. This mass term appears in the kinetic term of the complex scalar field:

$$\begin{aligned}
\mathcal{L}_{\phi\mathrm{Kin}} = (D_\mu\phi)^\dagger D^\mu\phi &= ((\partial_\mu + ig\widetilde{W}_\mu + \frac{1}{2}ig'B_\mu)\phi)^\dagger(\partial^\mu + ig\widetilde{W}^\mu + \frac{1}{2}ig'B^\mu)\phi = \\
&= \frac{1}{2}\partial_\mu H\partial^\mu H + (v+H)^2\Big\{\frac{g^2}{4}W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu\Big\},
\end{aligned} \tag{2.46}$$

where we have already eliminated the Goldstone bosons through a gauge transformation. This gauge where we do not have Goldstone bosons is called unitary gauge.

Therefore, through this mechanism the gauge bosons $W^{\pm}$ and Z become massive with a mass of

$$M_Z\,\cos\theta_W = M_W = \frac{1}{2}vg. \tag{2.47}$$

As the degrees of freedom of the three Goldstone bosons have been eliminated and three gauge bosons have acquired a new degree of freedom, the longitudinal mode, it is often said that the

Goldstone bosons have been "eaten" by the gauge bosons. In this way we end with three massive gauge bosons and without any Goldstone boson so the Goldstone theorem is avoided.[7]

2.4.3 Fermion Masses

In the Electroweak theory the fermion mass terms were also forbidden by the $SU(2)_L$ symmetry because these terms mix the left and the right part of the fermions ($\mathcal{L} = -m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$). However, we can generate these masses through the SSB adding in our Lagrangian Yukawa terms that generate interactions between our fermions and the scalar doublet, terms that also preserve the gauge symmetry:

$$\mathcal{L}_Y = -\sum_{i,j=1}^{3}\left[\bar{l}_{L_i}Y^e_{ij}e_{R_j}\phi + \overline{Q}_{L_i}Y^d_{ij}d_{R_j}\phi + \overline{Q}_{L_i}Y^u_{ij}u_{R_j}\widetilde{\phi} + \text{h. c.}\right] \tag{2.48}$$

where the sum is over the three families and we have used the following notation:

$$l_{L_i} = \begin{pmatrix} \nu_i \\ e_i^- \end{pmatrix}_L, \qquad Q_{L_i} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \qquad \phi = \begin{pmatrix} \phi^{(+)} \\ \phi^0 \end{pmatrix}, \qquad \widetilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^{(-)} \end{pmatrix}, \tag{2.49}$$

where $\widetilde{\phi}$ is the charge conjugate of the scalar field in such a way that the term $\overline{Q}_{L_i}Y^u_{ij}u_{R_j}\widetilde{\phi}$ is invariant under the $U(1)_Y$ transformations (the sum of the hypercharge of $\overline{Q}_L$, u_R and $\widetilde{\phi}$ is 0).

Note that in this case we have considered the Lagrangian for the three families while in section 2.3, for simplicity, we only considered the Lagrangian for only one family. This is because the only terms that mix the families are the Yukawa terms so the generalisation of the Lagrangian of Eq. (2.28) is trivial.[8]

If we move to the unitary gauge, Eq. (2.48) becomes:

$$\mathcal{L}_Y = -\sum_{i,j=1}^{3}\left[\overline{e}_{L_i}M^e_{ij}e_{R_j} + \overline{d}_{L_i}M^d_{ij}d_{R_j} + \overline{u}_{L_i}M^u_{ij}u_{R_j}\right]\left(1 + \frac{H}{2}\right), \tag{2.50}$$

with

$$M^e_{ij} = \frac{v}{\sqrt{2}}Y^e_{ij}, \qquad M^d_{ij} = \frac{v}{\sqrt{2}}Y^d_{ij}, \qquad M^u_{ij} = \frac{v}{\sqrt{2}}Y^u_{ij}. \tag{2.51}$$

The mass matrices (M^e, M^d and M^u) are non-diagonal and complex. This means that, in principle, we have introduced 54 new parameters to our Lagrangian. However, we can use the symmetries of our Lagrangian to show that most of these parameters are redundant and that finally we end up with just 13 new parameters. In order to show this we will change from the interaction basis to the mass one, where the mass matrices become diagonal. This is also interesting because the particles that propagate are the particles with a definite mass, the eigenvectors of the mass matrices.

As mentioned before, the only term that mixes flavours is the one of Eq. (2.50), this means that the other parts of the Lagrangian are invariant under $U(3)_{Q_L} \otimes U(3)_{u_R} \otimes U(3)_{d_R} \otimes U(3)_{l_L} \otimes U(3)_{e_R}$

[7] Note that, as said before, this does not mean that the Goldstone theorem is wrong, just that it cannot be applied in such theories [4].

[8] It is enough considering that the u, d, ν and e fields are 3×3 diagonal matrices in the flavour space.

flavour transformations. This kind of transformations are often called Weak Basis Transformations (WBT). Performing this kind of transformations we can diagonalise M^e and M^u and make M^d Hermitian:

$$\begin{gathered}Q_L \to Q'_L = U_Q Q_L, \qquad u_R \to u'_R = U_u u_R, \qquad d_R \to d'_R = U_d d_R, \\ l_L \to l'_L = U_l l_L, \qquad e_R \to e'_R = U_e e_R \qquad \Rightarrow \\ \Rightarrow M^e \to U_l^\dagger M^e U_e = D^e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \qquad M^u \to U_Q^\dagger M^u U_u = D^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \\ M^d \to U_Q^\dagger M^d U_d = M'^d = M'^{d\dagger}\end{gathered} \tag{2.52}$$

Now we need to diagonalise the M'^d matrix. As this is a Hermitian matrix, we can diagonalise it through a unitary matrix, V. For doing it we introduce again a transformation but in this case a transformation that only affects the down part:

$$d_L \to V d_L, \quad d_R \to V d_R \quad \Rightarrow \quad M'^d \to V^\dagger M'^d V = D_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \tag{2.53}$$

With this final transformation we get a fully diagonal Yukawa part

$$\mathcal{L}_Y = -\Big[\overline{e}_L D^e e_R + \overline{d}_L D^d d_R + \overline{u}_L D^u u_R\Big]\Big(1 + \frac{H}{2}\Big), \tag{2.54}$$

but this transformation is not a WTB transformation, i.e., the other parts of our Lagrangian are not invariant under this transformation.

The Charged Currents sector of the Lagrangian, Eq. (2.30), mixes the up and the down chiralities of the fields, so this sector will be clearly modified by this transformation:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\Big\{W_\mu^\dagger[\overline{u_L}\gamma^\mu V d_L + \overline{\nu_{Le}}\gamma^\mu e_L] + \text{h. c.}\Big\}, \tag{2.55}$$

while the sector of the Neutral Currents, Eq. (2.32), does not mix the left and the right chiralites so this sector is invariant under this transformation. It is for this reason why we do not have Flavour Changing Neutral Currents (FCNC) at tree-level in the Standard Model.

Let us look now to the free parameters introduced by the V matrix, called Cabibbo-Kobayashi-Maskawa (CKM) matrix. Any unitary matrix can be expressed like $V = \exp(iH)$, where H is a Hermitian matrix. The Hermitian matrices have $N + \frac{N^2-N}{2}$ real parameters, where N is the dimension of the matrix ($N \times N$), and $\frac{N^2-N}{2}$ imaginary parameters. Therefore, the V matrix would have, in principle, $N + \frac{N^2-N}{2}$ imaginary parameters and $\frac{N^2-N}{2}$ real ones, because the imaginary become real and the real become imaginary due to the factor i. However, all the pieces of the Lagrangian but the one containing the CKM matrix are invariant under rephasing of the quark fields:

$$d \to K^d d = \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix} d, \qquad u \to K^u u = \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} u. \tag{2.56}$$

Therefore we can apply these matrices to the CKM matrix

$$V \to K^{u\dagger} V K^{d}, \tag{2.57}$$

which allows to eliminate $2N-1$ phases, and the total number of free imaginary parameters of the CKM matrix will be $N + \frac{N^2-N}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$. As we have three generations the CKM matrix will have one imaginary parameter and three real parameters. In addition we have the nine masses so in total 13 parameters, as mentioned before.

2.4.4 Gauge Fixing

In section 2.4.2 we saw how we could eliminate the Goldstone bosons using a particular gauge, the unitary gauge, but this is not the only possibility. In many situations, this particular gauge is not the most convenient in order to perform calculations, so it is interesting to introduce other gauges. Let us write the scalar doublet in the following way

$$\phi(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + iG^0(x)) \end{pmatrix}, \tag{2.58}$$

where the $G^{\pm}$ and G^0 represent the three Goldstone bosons that we eliminated in the unitary gauge.

If we now calculate the covariant derivative of the doublet we obtain

$$D_\mu \phi(x) = \begin{pmatrix} \partial_\mu G^+ + \frac{ig}{2} W_\mu^+ (v + H + iG^0) + ie(Z_\mu \cot 2\theta_W + A_\mu) G^+ \\ \frac{1}{\sqrt{2}} \partial_\mu (v + H + iG^0) + \frac{ig}{\sqrt{2}} W_\mu^- G^+ - \frac{ig}{2\sqrt{2}\cos\theta_W} Z_\mu (v + H + iG^0) \end{pmatrix}. \tag{2.59}$$

From this we can calculate the kinetic term with the Goldstone bosons

$$\begin{aligned}
(D^\mu \phi)^\dagger D_\mu \phi = {} & \frac{1}{2}(\partial_\mu H \partial^\mu H + \partial_\mu G^0 \partial^\mu G^0) + \partial_\mu G^- \partial^\mu G^+ + ie(Z_\mu \cot 2\theta_W + A_\mu) G^+ \overleftrightarrow{\partial^\mu} G^- \\
& + \left(\frac{g^2}{4} W_\mu^- W^{+\mu} + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu\right)\left((v+H)^2 + G^{0^2}\right) + \left(e^2 (A_\mu + Z_\mu \cot 2\theta_W)^2 + \frac{g^2}{2} W_\mu^- W^{+\mu}\right) G^+ G^- \\
& + \frac{ig}{2} W_\mu^- G^+ \overleftrightarrow{\partial^\mu} (v + H - iG^0) + \frac{ig}{2} W_\mu^+ (v + H + iG^0) \overleftrightarrow{\partial^\mu} G^- + \frac{g}{2\cos\theta_W} Z_\mu G^0 \overleftrightarrow{\partial^\mu} (v+H) + \\
& \frac{ge}{2}\Big(A_\mu - \tan\theta_W Z_\mu\Big)\Big(G^+ W^{-\mu}(v + H - iG^0) + G^- W^{+\mu}(v + H + iG^0)\Big),
\end{aligned} \tag{2.60}$$

where we have used the relation $A \overleftrightarrow{\partial^\mu} B = A\partial^\mu B - (\partial^\mu A) B$.

Note that we have quadratic interactions between the Goldstone bosons and the gauge fields,

$$iM_W W_\mu^+ \partial^\mu G^- - iM_W \partial^\mu G^+ W_\mu^- - M_Z \partial^\mu G^0 Z_\mu. \tag{2.61}$$

Since we need to fix the gauge in order to properly quantise the theory, we can remove these terms through an appropriate choice of the gauge-fixing term. For doing so we use the so called R_ξ gauges where the gauge-fixing term that we introduce is

$$\mathcal{L} = -\frac{1}{\xi}\left|\partial_\mu W_\mu^+ + i\xi M_W G^+\right|^2 - \frac{1}{2\xi}(\partial_\mu Z_\mu + \xi M_Z G^0)^2 - \frac{1}{2\xi}(\partial_\mu A_\mu)^2. \tag{2.62}$$

Note that this term can be added because our Lagrangian is gauge invariant. With this we eliminate the quadratic terms and, furthermore, we guarantee to have an invertible kinetic term, so we have a well defined propagator [26].

The unitary gauge is recovered in the limit $\xi \to \infty$ but we have other possibilities, like the Feynman-'t-Hooft gauge ($\xi = 1$) or the Landau gauge ($\xi \to 0$). As mentioned before the unitary gauge is not the most convenient in some cases so, as the result must be gauge invariant, sometimes it is convenient to use a different gauge.

2.4.5 Custodial Symmetry

The Custodial Symmetry is an approximate symmetry of the scalar Lagrangian that becomes exact on this part when $g' \to 0$ [27, 28]. Let us recover the scalar Lagrangian that was developed in section 2.4.3, but this time it is convenient to write it in terms of

$$\Sigma = \frac{1}{\sqrt{2}}(\tilde{\phi},\, \phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0*} & \phi^{(+)} \\ -\phi^{(-)} & \phi^{0} \end{pmatrix}, \tag{2.63}$$

$$\mathcal{L}_\phi = \mathrm{Tr}\big[(D^\mu\Sigma)^\dagger D_\mu\Sigma\big] - \mu^2\,\mathrm{Tr}\big[\Sigma^\dagger\Sigma\big] + \lambda\,\mathrm{Tr}\big[\Sigma^\dagger\Sigma\big], \tag{2.64}$$

where the covariant derivative is written as

$$D_\mu\Sigma = \partial_\mu\Sigma + \frac{i}{2}W^a_\mu\sigma^a\Sigma + \frac{i}{2}g'B_\mu\Sigma\sigma_3. \tag{2.65}$$

This Lagrangian, before the symmetry breaking, is invariant under $SU(2)_L$ transformations, $\Sigma \to L\Sigma$, as we have showed before. Furthermore, it is almost invariant under $SU(2)_R$ transformations where $\Sigma \to \Sigma R^\dagger$ thanks to the trace. The only term that is not invariant under this transformation is the one that goes with the hypercharge (and g'), because $\tilde{\phi}$ and ϕ have different hypercharges:

$$\mathrm{Tr}\big[\partial_\mu\Sigma^\dagger B^\mu\Sigma\sigma_3\big] \to \mathrm{Tr}\big[\partial_\mu\Sigma^\dagger B^\mu\Sigma R^\dagger\sigma_3 R\big] \quad \Rightarrow \quad R^\dagger\sigma_3 R \neq \sigma_3. \tag{2.66}$$

Therefore, the scalar part of our Lagrangian is invariant under $SU(2)_L \otimes SU(2)_R$ transformations in the limit in which $g' \to 0$ and before the SSB. Let us consider that we are on this limit and our Lagrangian gets spontaneously broken. In this situation the scalar field acquires a vev

$$\langle 0|\Sigma|0\rangle = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \tag{2.67}$$

so the scalar field is no longer invariant under $SU(2)_L \otimes SU(2)_R$:

$$L\langle 0|\Sigma|0\rangle R^\dagger \neq \langle 0|\Sigma|0\rangle. \tag{2.68}$$

However, the vev is still invariant under simultaneous $SU(2)_L$ and $SU(2)_R$ transformations with $L = R$, this is under the $SU(2)_{L+R}$ subgroup. This remaining symmetry is the so called custodial symmetry.

Therefore the vev breaks the $SU(2)_L \otimes SU(2)_R$ symmetry made of six generators into its $SU(2)_{L+R}$ subgroup made of three generators. Thus, in total we will have three broken generators which produce Goldstone boson that will be "eaten" by the three gauge bosons (W_1, W_2 and W_3) through the Higgs mechanism. This produces a degeneration on the masses of these bosons. This degeneration will also be satisfied at tree level even if the symmetry is broken by other terms of the Lagrangian, because it is this term the one that generates the masses at tree level. This result is a famous prediction of the SM and it is referred to it with the value of the ratio:

$$\rho_{\text{tree}} \equiv \frac{m_W^2}{m_{W_3}^2} \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1, \tag{2.69}$$

where m_Z is the mass of the Z boson and θ_W the mixing angle, $\cos^2\theta_W = \frac{g^2}{g^2+g'^2}$, which gives the separation from $g' = 0$.

Then, the fact of having a ρ parameter equal to one is a direct consequence of the custodial symmetry and this implies that the radiative corrections to this parameter will be proportional to the couplings of the terms that break this symmetry. Apart of the term proportional to g', the Yukawa Lagrangian also breaks this symmetry because the up-type quarks have a different mass than the down-type quarks. However, in the approximation of $m_u = m_d = m$ the Yukawa Lagrangian can be written like

$$\mathcal{L}_Y = -\frac{m}{v}(\bar{u}_L, \quad \bar{d}_L)\sqrt{2}\Sigma \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \text{h. c.}, \tag{2.70}$$

clearly invariant under custodial symmetry after the SSB. This implies that the radiative corrections to the ρ parameter due to massive fermions must disappear in this limit, and this is in fact the case [27]. In conclusion, this symmetry protects the ρ parameter to have a value different from 1 and hence its name.

Experimentally the value of this parameter has been precisely measured and has a value compatible with the prediction of the SM ($\rho_{exp} = 1.00037 \pm 0.00023$ [29]). It is interesting to analyse how this prediction changes in the framework of new physics. The introduction of one scalar doublet is not the only possibility in order to generate the masses of the particles and we can consider the situation in which other scalar particles are added. This parameter is the one that mostly constrains the possibility of having an exotic Higgs sector [30] and for this reason it is interesting to study the predictions of other models, i.e., the conditions that have to satisfy the scalar extensions in order to satisfy the Custodial Symmetry.

If we have a scalar sector containing N multiplets ϕ_i ($i = 1, ..., N$) belonging to different $SU(2)_L \otimes U(1)_Y$ representations (T_i, Y_i), each Higgs multiplet can be expressed as [31]

$$\Phi_i = \left[\Phi_i^{Q=Y_i+T_i}, \ldots, \Phi_i^+, \Phi_i^0, \Phi_i^-, \ldots \Phi_i^{Q=Y_i-T_i}\right]^T,$$
$$\text{with } \Phi_i^0 = \frac{1}{\sqrt{2c_i}}(H_i^0 + v_i + iz_i^0) \text{ for } Y_i \neq 0 \text{ fields}, \quad \Phi_i^0 = \frac{1}{\sqrt{2c_i}}(H_i^0 + v_i) \text{ for } Y_i = 0 \text{ fields}, \tag{2.71}$$

where v_i is the vev of the Φ_i multiplet and $c_i = 1(1/2)$ for a complex (real) scalar field.

In order to find an expression for the ρ_{tree} parameter we need to find the values of the W and Z vector boson masses. These masses are generated as a consequence of the vev, as we saw in section 2.4.2 . If we write the kinetic term of these scalars, like in Eq. (2.46), in terms of the generators:

$$\mathcal{L}_{\text{kin}} = \sum_i c_i |D_i^\mu \Phi_i|^2, \quad \text{with} \quad D_i^\mu = \partial^\mu - ig\sqrt{2}(T_i^+ W^{+\mu} + T_i^- W^{-\mu})$$
$$- i\frac{g}{\cos\theta_W}(T_i^3 - \sin^2\theta_W Q_i)Z_\mu - ieQ_iA^\mu, \tag{2.72}$$

with $T_i^{\pm}$ the $SU(2)_L$ ladder operators and T_i^3 the third component of the isospin operator, we trivially find that:

$$m_W^2 = \frac{g^2}{4}v^2, \quad m_Z^2 = \frac{g^2}{c_W^2}\sum_i v_i^2 Y_i^2, \text{ with } v^2 = 2\sum_i [T_i(T_i+1) - Y_i^2]v_i^2 = (246 \text{ GeV})^2. \tag{2.73}$$

Once we have the masses of these bosons we can get an expression for the electroweak ρ parameter:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos\theta_W} = \frac{\sum_i v_i^2 [T_i(T_i+1) - Y_i^2]}{2\sum_i v_i^2 Y_i^2}. \tag{2.74}$$

So from here, and taking into account that ρ has to be equal to one, we find that any scalar extension for the SM should satisfy:

$$T_i = \frac{1}{2}\left(-1 + \sqrt{1 + 12 Y_i^2}\right), \tag{2.75}$$

unless some fine-tuned cancellations were produced among the different multiplets, in which case this relation does not need to be satisfied.

This condition is trivially satisfied by singlets with zero hypercharge and doublets with $Y = \frac{1}{2}$, what makes these models the favoured candidates for an alternative to the minimal Higgs model.

3 Colour Octet Model

In this work we will focus on a model with an additional doublet of $SU(2)_L$ which will also be an octet of $SU(3)_C$ and will have hypercharge 1/2, i.e., a $(\mathbf{8},\mathbf{2})_{1/2}$ scalar. So our scalar sector will be

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^0 \end{pmatrix}, \qquad S^A = \begin{pmatrix} S^{+^A} \\ S^{0^A} \end{pmatrix}, \tag{3.1}$$

where the superscript $A = 1\dots8$ is the adjoint colour index.

Note that the scalar colour octet cannot acquire a vev because this would mean that the vacuum is not a singlet of colour, so colour would not be conserved. A similar thing happens in the SM with the charged scalars, they cannot have a vev because the electric charge has to be conserved. So the vev will be the same that in the SM

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v e^{i\theta} \end{pmatrix}, \qquad \langle 0|S^A|0\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3.2}$$

This vev will minimise the most general potential build with these two scalars which takes the form

$$\begin{aligned} V = {}& \frac{\lambda}{4}\left(\phi^{\dagger i}\phi_i - \frac{v^2}{2}\right)^2 + 2m_S^2 \operatorname{Tr} S^{\dagger i} S_i + \lambda_1 \phi^{\dagger i}\phi_i \operatorname{Tr} S^{\dagger j} S_j + \lambda_2 \phi^{\dagger i}\phi_j \operatorname{Tr} S^{\dagger j} S_i + [\lambda_3 \phi^{\dagger i}\phi^{\dagger j} \operatorname{Tr} S_i S_j \\ & + \lambda_4 \phi^{\dagger i} \operatorname{Tr} S^{\dagger j} S_j S_i + \lambda_5 \phi^{\dagger i} \operatorname{Tr} S^{\dagger j} S_i S_j + \text{h. c.}] + \lambda_6 \operatorname{Tr} S^{\dagger i} S_i S^{\dagger j} S_j + \lambda_7 \operatorname{Tr} S^{\dagger i} S_j S^{\dagger j} S_i \\ & + \lambda_8 \operatorname{Tr} S^{\dagger i} S_i \operatorname{Tr} S^{\dagger j} S_j + \lambda_9 \operatorname{Tr} S^{\dagger i} S_j \operatorname{Tr} S^{\dagger j} S_i + \lambda_{10} \operatorname{Tr} S_i S_j \operatorname{Tr} S^{\dagger i} S^{\dagger j} + \lambda_{11} \operatorname{Tr} S_i S_j \operatorname{Tr} S^{\dagger j} S^{\dagger i}, \end{aligned} \tag{3.3}$$

where i and j are $SU(2)$ indices, the traces are in colour space and we have used the notation $S = S^A T^A$, with T^A the generators of the $SU(3)$ group. All the parameters are real except λ_3, λ_4 and λ_5 but we can choose λ_3 real performing a phase rotation of the S fields. So we finally end with two complex parameters which will be a source of CP violation.

This potential is clearly minimised by the vev of Eq. (3.2) and does not have quadratic terms that mix the two scalars, because they are not singlets of $SU(3)_C$, so it is already written in the mass basis. We can now choose a particular vev and consider the excitations of the vacuum

$$\phi(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + iG^0(x)) \end{pmatrix}, \qquad S^A(x) = \begin{pmatrix} S^{+A}(x) \\ \frac{1}{\sqrt{2}}(S_R^{0A}(x) + iS_I^{0A}(x)) \end{pmatrix}, \tag{3.4}$$

where, like in the SM, the $G^\pm$ and G^0 are the Goldstone bosons that can be eliminated if we choose the unitary gauge and we have separated the neutral complex octets into a real CP-even scalar (S_R^{0A}) and a real CP-odd scalar (S_I^{0A}). In fact, the masses of these fields are split by the vev. Let us look at the first five terms of the potential which will be the responsible of producing the masses:

$$V \supset \frac{\lambda}{4}\left(\frac{(v+H)^2}{2} - \frac{v^2}{2}\right)^2 + T_F\left(2m_S^2 + \lambda_1\frac{(v+H)^2}{2}\right)S^{-A}S^{+A} + \frac{T_F}{2}\Bigg(2m_S^2 + (\lambda_1+\lambda_2+2\lambda_3)\frac{(v+H)^2}{2}\Bigg)S_R^{0A}S_R^{0A} + \frac{T_F}{2}\left(2m_S^2 + (\lambda_1+\lambda_2-2\lambda_3)\frac{(v+H)^2}{2}\right)S_I^{0A}S_I^{0A}, \tag{3.5}$$

where $T_F = 1/2$ is the index of the representation and we have used the unitary gauge. From this we trivially can get the masses

$$\begin{aligned} m_H^2 &= \frac{\lambda}{2}v^2H^2, & m_{S_R^{0A}}^2 &= m_S^2 + (\lambda_1+\lambda_2+2\lambda_3)\frac{v^2}{4}, \\ m_{S^\pm}^2 &= m_S^2 + \lambda_1\frac{v^2}{4}, & m_{S_I^{0A}}^2 &= m_S^2 + (\lambda_1+\lambda_2-2\lambda_3)\frac{v^2}{4}. \end{aligned} \tag{3.6}$$

3.1 Kinetic Term

The kinetic term of the ϕ scalar will be the same than in the SM (§2.4.2), as well as the gauge fixing term (§2.4.4) because our fields are totally decoupled. The kinetic term of our new scalars will be

$$\mathcal{L}_{S\text{Kin}} = 2\,\text{Tr}\big[(D_\mu S)^\dagger D^\mu S\big] \qquad \text{with} \qquad D_\mu S = \partial_\mu S + ig_s[G_\mu, S] + ig\widetilde{W}_\mu S + iy_S g' B_\mu S, \tag{3.7}$$

where the factor two in front generates the correct normalisation, the hypercharge will be $y_S = 1/2$, as mentioned before, and the trace is performed in the colour space.

Note that in this case in the covariant derivative we have a commutator between our scalar field and the gluon field. This term is needed in order to have a covariant derivative transforming like in Eq. (2.14) under $SU(3)_C$ transformations. This is trivial to see taking into account that our new scalar field belongs to the adjoint representation and therefore this will transform under the $SU(3)_C$ group as $S \to USU^\dagger$. The terms going with the $\widetilde{W}_\mu$ and the B_μ are the same that for the SM scalar field ϕ but, in this case, we will have scalar charged particles ($S^{\pm}$) and CP-odd neutral scalars ($S_I^{0^A}$) that couple to the photons and the $W^{\pm}$ and Z gauge bosons that cannot be eliminated in any gauge.

If we expand this kinetic term we find

$$\mathcal{L}_{S\text{Kin}} = 2T_F(\partial_\mu S^{A^\dagger} - g_s f^{ABC} G_\mu^B S^{C^\dagger} - ig\frac{\sigma_i}{2} W_\mu^i S^{A^\dagger} - iy_S g' B_\mu S^{A^\dagger})$$
$$\times (\partial^\mu S^A - g_s f^{ADE} G_\mu^D S^E + ig\frac{\sigma_i}{2} W_\mu^i S^A + iy_S g' B_\mu S^A) = \partial_\mu S^{A^\dagger} \partial^\mu S^A + \mathcal{L}_{SG} + \mathcal{L}_{SEW}, \tag{3.8}$$

where in the last step the Lagrangian $\mathcal{L}_{SG}$ refers to the coupling of our scalar particles with the gluons and the $\mathcal{L}_{SEW}$ to the coupling with the electroweak bosons. Now we can write $\mathcal{L}_{SG}$ in terms of the $S^{\pm^A}$ and $S_{R,I}^{0^A}$ fields

$$\mathcal{L}_{SG} = -g_s f^{ABC} G_\mu^B (S^{-^C} \overleftrightarrow{\partial^\mu} S^{+^A} + \frac{1}{2}(S_R^{0^C} \overleftrightarrow{\partial^\mu} S_R^{0^A} + S_I^{0^C} \overleftrightarrow{\partial^\mu} S_I^{0^A}))$$
$$+ g_s^2 f^{ABC} f^{ADE} G_\mu^B G_\mu^D (S^{-^C} S^{+^E} + \frac{1}{2}(S_R^{0^C} S_R^{0^E} + S_I^{0^C} S_I^{0^E})). \tag{3.9}$$

We can also expand the $\mathcal{L}_{SEW}$ term which will be, as mention before, like the one of the ϕ field without eliminating the Goldstone bosons, i.e., the one of Eq. (2.60),

$$\mathcal{L}_{SEW} = ie(Z_\mu \cot 2\theta_W + A_\mu) S^{+^A} \overleftrightarrow{\partial^\mu} S^{-^A} + \Big(\frac{g^2}{4} W_\mu^- W^{+\mu} + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu\Big)\Big((S_R^{0^A})^2 + (S_I^{0^A})^2\Big)$$
$$+ \Big(e^2(A_\mu + Z_\mu \cot 2\theta_W)^2 + \frac{g^2}{2} W_\mu^- W^{+\mu}\Big) S^{+^A} S^{-^A} + \frac{ig}{2} W_\mu^- S^{+^A} \overleftrightarrow{\partial^\mu} (S_R^{0^A} - iS_I^{0^A})$$
$$+ \frac{ig}{2} W_\mu^+ (S_R^{0^A} + iS_I^{0^A}) \overleftrightarrow{\partial^\mu} S^{-^A} + \frac{g}{2\cos\theta_W} Z_\mu S_I^{0^A} \overleftrightarrow{\partial^\mu} S_R^{0^A}$$
$$+ \frac{ge}{2}\Big(A_\mu - \tan\theta_W Z_\mu\Big)\Big(S^{+^A} W^{-\mu}(S_R^{0^A} - iS_I^{0^A}) + S^{-^A} W^{+\mu}(S_R^{0^A} + iS_I^{0^A})\Big). \tag{3.10}$$

3.2 Yukawa Sector

The interaction of this additional doublet with the quark sector cannot be arbitrary because this, in general, will generate FCNC at tree-level while experimentally the processes that involve these currents are extremely suppressed. In the SM, as we showed before, there are no FCNC at tree-level because the Yukawa couplings of the scalar sector and the mass matrices are proportional. So if the two Yukawa couplings matrices of the $(\mathbf{8},\mathbf{2})_{1/2}$ scalar are proportional to the ones of the SM there will not be FCNC. This condition is usually called Minimal Flavour Violation (MFV). Note that MFV is not a necessary condition to avoid the tree-level FCNC because we could have two non-proportional matrices that are diagonal in the same basis but it is a sufficient condition [1].

Imposing MFV, the Yukawa Lagrangian of the coloured scalars can be written, in the interaction basis, like

$$\mathcal{L}_{SY} = -\sum_{i,j=1}^{3} \Big[\eta_U Y_{ij}^d \overline{Q}_{L_i} S d_{R_j} + \eta_D Y_{ij}^u \overline{Q}_{L_i} \widetilde{S} u_{R_j} + \text{h. c.}\Big], \tag{3.11}$$

where η_U and η_D are complex constants.

In the mass basis this becomes

$$\mathcal{L}_{SY} = -\Big(\eta_U \frac{m_u^i}{v}\overline{u}_{Ri}T^A u_L^i(S_R^{0^A} + iS_I^{0^A}) + \text{h. c.}\Big) + \Big(\sqrt{2}\eta_U \frac{m_u^i}{v}\overline{u}_R^i V_{ij} T^A d_L^j S^{+^A} + \text{h. c.}\Big)$$
$$- \Big(\eta_D \frac{m_d^i}{v}\overline{d}_{Ri}T^A d_L^i(S_R^{0^A} - iS_I^{0^A}) + \text{h. c.}\Big) - \Big(\sqrt{2}\eta_D \frac{m_d^i}{v}\overline{d}_R^i V_{ij}^\dagger T^A u_L^j S^{-^A} + \text{h. c.}\Big). \tag{3.12}$$

From this Lagrangian we can see clearly that our new scalars can decay to quarks. Using the Feynman rules of appendix A we can calculate the decay width of these decays

$$\Gamma(S^+ \to t\bar{b}) = \frac{|\eta_U|^2}{16\pi m_S^3}\left(\frac{m_t}{v}\right)^2 |V_{tb}|^2 (m_S^2 - m_t^2)^2, \tag{3.13}$$

$$\Gamma(S_R^0 \to t\bar{t}) = \frac{m_S}{16\pi}\left(\frac{m_t}{v}\right)^2 \left[|\operatorname{Re}\eta_U|^2\left(1-\frac{4m_t^2}{m_S^2}\right)^{3/2} + |\operatorname{Im}\eta_U|^2\left(1-\frac{4m_t^2}{m_S^2}\right)^{1/2}\right], \tag{3.14}$$

$$\Gamma(S_I^0 \to t\bar{t}) = \frac{m_S}{16\pi}\left(\frac{m_t}{v}\right)^2 \left[|\operatorname{Re}\eta_U|^2\left(1-\frac{4m_t^2}{m_S^2}\right)^{1/2} + |\operatorname{Im}\eta_U|^2\left(1-\frac{4m_t^2}{m_S^2}\right)^{3/2}\right]. \tag{3.15}$$

3.3 Custodial Symmetry

In section 2.4.5 we showed how having a Lagrangian invariant under the custodial symmetry implies that the electroweak ρ parameter is equal to one. In this model the prediction at tree-level is still one but if we have terms in our Lagrangian that break this symmetry we can have radiative correction to this value. It is interesting to show under which conditions the potential of Eq. (3.3) remains invariant under the $SU(2)_{L+R}$ symmetry:

$$2\lambda_3 = \lambda_2, \qquad 2\lambda_6 = 2\lambda_7 = \lambda_{11}, \qquad \lambda_9 = \lambda_{10}, \qquad \lambda_4 = \lambda_5^*. \tag{3.16}$$

Under these conditions we easily can write the potential in terms of the bi-doublets,

$$\Sigma = \frac{1}{\sqrt{2}}(\tilde{\phi},\, \phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0*} & \phi^{(+)} \\ -\phi^{(-)} & \phi^0 \end{pmatrix} \qquad \mathcal{S}^A = \frac{1}{\sqrt{2}}(\widetilde{S}^A,\, S^A) = \frac{1}{\sqrt{2}}\begin{pmatrix} S^{0^{A*}} & S^{+^A} \\ -S^{-^A} & S^{0^A} \end{pmatrix}, \tag{3.17}$$

in order to make clear the invariance under custodial symmetry [32]:

$$V = \frac{\lambda}{4}\Big[\operatorname{Tr}\big(\Sigma^\dagger\Sigma\big) - v^2\Big]^2 + 2m_S^2 \operatorname{Tr}\Big(\mathcal{S}_A^\dagger \mathcal{S}_A\Big) + \lambda_1 \operatorname{Tr}\big(T^A T^B\big)\operatorname{Tr}\big(\Sigma^\dagger\Sigma\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\mathcal{S}_B\Big)$$
$$+ 4\lambda_3 \operatorname{Tr}\big(T^A T^B\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\Sigma\Big)\operatorname{Tr}\Big(\mathcal{S}_B^\dagger\Sigma\Big) + 4\operatorname{Tr}\big(T^A T^B T^C\big)\Big[\operatorname{Im}(\lambda_4) i \operatorname{Tr}\Big(\Sigma^\dagger \mathcal{S}_A \mathcal{S}_B^\dagger \mathcal{S}_C\Big) + \text{h. c.}\Big]$$
$$+ 4\operatorname{Re}(\lambda_4)\operatorname{Tr}\big(T^A T^B T^C\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\mathcal{S}_C\Big)\operatorname{Tr}\Big(\mathcal{S}_B^\dagger\Sigma\Big) + 4\lambda_6 \operatorname{Tr}\big(T^A T^B T^C T^D\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\mathcal{S}_B\Big)\operatorname{Tr}\Big(\mathcal{S}_C^\dagger\mathcal{S}_D\Big)$$
$$+ \lambda_8 \operatorname{Tr}\big(T^A T^B\big)\operatorname{Tr}\big(T^C T^D\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\mathcal{S}_B\Big)\operatorname{Tr}\Big(\mathcal{S}_C^\dagger\mathcal{S}_D\Big) + 2\lambda_9 \operatorname{Tr}\big(T^A T^B\big)\operatorname{Tr}\big(T^C T^D\big)\operatorname{Tr}\Big(\mathcal{S}_A^\dagger\mathcal{S}_C\Big)\operatorname{Tr}\Big(\mathcal{S}_B^\dagger\mathcal{S}_D\Big). \tag{3.18}$$

The contribution of the colour octet scalars to the ρ parameter was calculated in Ref. [1] for arbitrary values of the scalar-potential parameters (λ_i), obtaining

$$\Delta\rho = \frac{\alpha}{4\pi \sin^2\theta_W M_W^2}\Big[f(m_{S^\pm}, m_{S_R^0}) + f(m_{S^\pm}, m_{S_I^0}) - f(m_{S_R^0}, m_{S_I^0})\Big], \tag{3.19}$$

with

$$f(m_1, m_2) = m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2}\log\frac{m_1^2}{m_2^2}, \tag{3.20}$$

which is equal to 0 when the first condition of Eq. (3.16) is imposed, as expected.

3.4 Effective Field Theory

Taking into account the constraints on the masses of these particles that have been obtained [6], it seems reasonable to integrate out the $(\mathbf{8},\mathbf{2})_{1/2}$ scalars and work in the framework of an Effective Field Theory (EFT) [33].[9]

In this framework the physics of the new particles is captured in dimension six operators so we need to find the relation between the coefficients of the dimension six operators (the Wilson coefficients) and the parameters of our beyond SM theory. Once we have done this we can work with the Lagrangian of the dimensional six operators instead of working with the Lagrangian of our scalars which is useful because the calculations are easier. As phenomenologically the most interesting processes are $\sigma(gg \to h)$, $\Gamma(H \to \gamma\gamma)$ and $\Gamma(H \to Z\gamma)$ we will only consider the dimension six operators that contribute to these processes

$$\begin{aligned}\delta\mathcal{L} = &-\frac{c_G g_s^2}{2\Lambda^2} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu} - \frac{c_W}{2\Lambda^2} g^2 H^\dagger H W^a_{\mu\nu} W^{a\mu\nu} \\ &-\frac{c_B}{2\Lambda^2} g'^2 H^\dagger H B_{\mu\nu} B^{\mu\nu} - \frac{c_{WB}}{2\Lambda^2} g g' H^\dagger \sigma^a H B_{\mu\nu} W^{a\mu\nu}.\end{aligned} \tag{3.21}$$

In order to obtain the relation between the Wilson coefficients and the parameters of our theory we can consider a particular process in such a way that calculating it in both theories we find this relation.

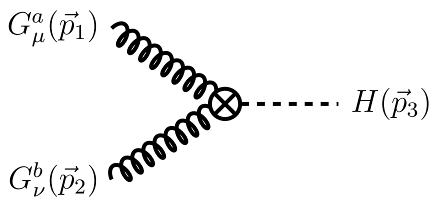
Let us first consider the process of gluon fusion, in which two gluons interact and produce a SM Higgs boson. In the SM this process is forbidden at tree-level and the first contribution is the one mediated by quarks. As it is proportional to the mass of the quarks the most important contribution will be the one that is mediated by the top quarks (Fig. 24). If we add the $(\mathbf{8},\mathbf{2})_{1/2}$ particles this process is still forbidden at tree-level but at one-loop-level we will have additional diagrams that are shown in Fig. 23. The amplitude of these diagrams in the limit of infinite mass has been calculated in the appendix B.1, Eq. (B.33). What we want, therefore, is to capture this contribution in the EFT in which this process in produced at tree-level. For doing so we just need to find the relation between the parameters of both theories. The only term that will contribute to this process in the EFT will be the one proportional to c_G:

$$\begin{aligned}\delta\mathcal{L} &\supset -\frac{c_G g_s^2}{2\Lambda^2}\frac{(v+H)^2}{2}(\partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f^{ABC} G^B_\mu G^C_\nu)(\partial^\mu G^{\nu A} - \partial^\nu G^{\mu A} - g_s f^{AB'C'} G^{\mu B'} G^{\nu C'}) \\ &\supset -\frac{c_G g_s^2}{2\Lambda^2} vH(\partial_\mu G^A_\nu - \partial_\nu G^A_\mu)(\partial^\mu G^{\nu A} - \partial^\nu G^{\mu A}),\end{aligned} \tag{3.22}$$

[9] A detailed analysis of how these constraints were obtained is made in section 4.1.

where we move to the unitary gauge. The Feynman rule of this last term is shown in Fig. 1. From this we can trivially find the relation between the Wilson coefficient c_G and the parameters of our theory

$$c_G = -\frac{2\lambda_1 + \lambda_2}{64\pi^2 m_S^2}. \tag{3.23}$$



$$i\frac{c_G g_s^2}{\Lambda^2} m_H^2 v \delta_{ab} \left(g^{\mu\nu} - \frac{2}{m_H^2} p_1^\nu p_2^\mu \right) \tag{3.24}$$

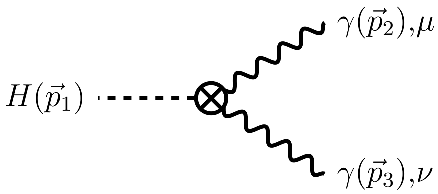
Figure 1: Gluon fusion generated by dimension-six operators.

Another interesting process is the decaying of the Higgs boson into two photons. The contribution of the colour octet scalars to this process, in the limit of infinity mass, have been calculated in appendix B.3, Eq. (B.40). In the EFT the contribution will come from

$$\begin{aligned}\delta\mathcal{L} &\supset -\frac{c_W}{2\Lambda^2} g^2 \frac{(v+H)^2}{2} W^3_{\mu\nu} W^{3\mu\nu} - \frac{c_B}{2\Lambda^2} g'^2 \frac{(v+H)^2}{2} B_{\mu\nu} B^{\mu\nu} + \frac{c_{WB}}{2\Lambda^2} g g' \frac{(v+H)^2}{2} B_{\mu\nu} W^{3\mu\nu} \\ &\supset -e\frac{c_W + c_B - c_{WB}}{2\Lambda^2} v H A_{\mu\nu} A^{\mu\nu} = -e\frac{c_W + c_B - c_{WB}}{2\Lambda^2} v H (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu).\end{aligned} \tag{3.25}$$

The Feynman rule is shown in Fig. 2 from which trivially we obtain:

$$\frac{c_{\gamma\gamma}}{\Lambda^2} = \frac{(c_W + c_B - c_{WB})}{\Lambda^2} = -\frac{\lambda_1}{24\pi^2 m_S^2}. \tag{3.26}$$



$$i\frac{(c_W + c_B - c_{WB})e^2}{\Lambda^2} m_H^2 v \left(g^{\mu\nu} - \frac{2}{m_H^2} p_2^\nu p_3^\mu \right) \tag{3.27}$$

Figure 2: SM Higgs going to two photons generated by dimension-six operators.

Similarly we can find the relation between the Wilson coefficients c_G, c_W and c_B:

$$\frac{c_G}{\Lambda^2} = \frac{3}{2}\frac{c_W}{\Lambda^2} = \frac{3}{2}\frac{c_B}{\Lambda^2} = -\frac{2\lambda_1 + \lambda_2}{64\pi^2 m_S^2}. \tag{3.28}$$

Finally combining Eq. (3.26) and Eq. (3.28) we find the expression for c_{WB}

$$\frac{c_{WB}}{\Lambda^2} = -\frac{\lambda_2}{48\pi^2 m_S^2}. \tag{3.29}$$

4 Phenomenological Analysis

In this section we will analyse the phenomenology of our model. The fact of having an extension of the scalar sector makes this model richer than the SM, in such a way that the new particles can interact with the SM particles and generate contributions to many processes, as we have seen in the previous section. Of course any extension of the SM has to reproduce its successful predictions and has to be compatible with the available data. Nowadays, the LHC is the most powerful machine available to measure observables and, therefore, in this section we will compare the predictions of our theory with the LHC data. Performing this comparison we will be able to find constraints on the parameters of our theory.

4.1 Colour Octet Production

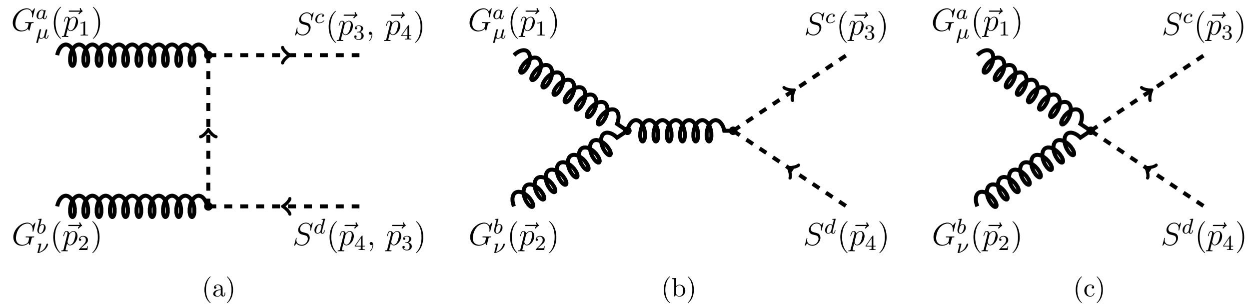


Figure 3: Pair production of colour octet scalars through gluon fusion.

Here we summarise the work made by Ref. [6] analysing the limits on the colour octet masses obtained from the analysis of the colour octet production. These scalars can be produced by pairs at tree-level through gluon fusion. The diagrams that contribute to this process are the ones showed on Fig. 3 and its cross section has been calculated in Ref. [1]:

$$\sigma(gg \to SS) = \frac{3\pi\alpha_s^2}{4s^2}\left\{\beta\left[3(4m_S^2+s)+\frac{1}{2}(10m_S^2-s)\right]-\frac{12m_S^2}{s}\left[m_S^2+(s-m_S^2)\log\left(\frac{1+\beta}{1-\beta}\right)\right]\right\}, \tag{4.1}$$

where

$$\beta = \sqrt{1-\frac{4m_S^2}{s}}. \tag{4.2}$$

This result is valid for charged scalars, for neutral scalars we have to add a factor 1/2 because the final state is formed of two identical particles.

The neutral scalars can also be produced in a single production mechanism through the diagrams of Fig. 4. For CP-even scalars the contribution of the diagrams 4b and 4c are proportional to $\mathrm{Re}(\lambda_4)+\mathrm{Re}(\lambda_5)$ and for CP-odd scalars they are proportional to $\mathrm{Im}(\lambda_4)+\mathrm{Im}(\lambda_5)$. Therefore, if we consider that the CP nonconservation is small, i.e. $\lambda_4 \approx \mathrm{Re}(\lambda_4)$ and $\lambda_5 \approx \mathrm{Re}(\lambda_5)$, the only diagram that contributes to the single S_I^0 production is the one of Fig. 4a. The decay width of the neutral colour scalars decaying to gluons was calculated in Ref. [34]

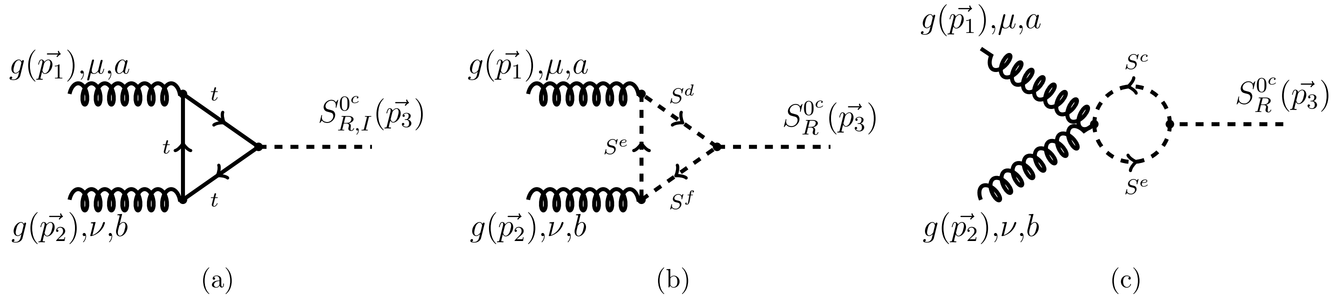


Figure 4: Single production of colour octet scalars through gluon fusion. In addition we will have the diagrams (a) and (b) with the internal propagators going anticlockwise.

$$\begin{aligned}\Gamma(S^0_R \to gg) =& \frac{m_S^3\alpha_s^2}{2^{11}\pi^3 v^2}\left(\frac{40}{3}\eta_U^2\left|I\left(\frac{m_t^2}{m_S^2}\right)\right|^2 - 60\eta_U(\lambda_4+\lambda_5)\frac{v^2}{m_S^2}\left(\frac{\pi^2}{9}-1\right)\mathrm{Re}\left(I\left(\frac{m_t^2}{m_S^2}\right)\right)\right.\\ &\left.+\frac{135}{2}(\lambda_4+\lambda_5)^2\frac{v^2}{m_S^2}\left(\frac{\pi^2}{9}-1\right)^2\right), \end{aligned} \tag{4.3}$$

$$\Gamma(S^0_I \to gg) = \frac{5}{3}\frac{\alpha_s^2 m_t^4}{2^{10}\pi^3 m_S v^2}\eta_U^2\left|I\left(\frac{m_t^2}{m_S^2}\right)\right|, \tag{4.4}$$

where

$$I(z) = 2z + z(4z-1)\mathcal{F}(z), \tag{4.5}$$

with $\mathcal{F}(z)$ defined on Eq. (B.29).

With this we can trivially obtain the cross section:

$$\sigma(gg\to S^0_R) = \frac{\pi^2}{8}\delta(s-m^2_{S^0_R})\Gamma(S^0_R\to gg), \qquad \sigma(gg\to S^0_I) = \frac{\pi^2}{8}\delta(s-m^2_{S^0_I})\Gamma(S^0_I\to gg). \tag{4.6}$$

These cross sections do not have physical meaning until we integrate them using the parton distribution functions (PDF) because we never have free gluons, they are confined in the protons. The PDF gives us the probability that a gluon of a proton carries a momentum xp if the proton carries a momentum p ($0 \le x \le 1$) and it is a function of the factorisation scale μ. In order to perform this integration it is interesting to relate the invariant mass of the two gluons (s) system and the one of the two proton system (S)

$$s = (p_{g_1}+p_{g_2})^2 = (x_1 p_{p_1} + x_2 p_{p_2})^2 = 2x_1x_2\, p_1\cdot p_2 = x_1x_2 S, \tag{4.7}$$

where we have considered that the mass of the protons is much smaller than the invariant mass of the two proton system.

Furthermore, we can write the delta function of Eq. (4.6) in terms of x_1 and x_2

$$\delta(s-m^2_{S^0}) = \delta(x_1x_2S - m^2_{S^0}) = \frac{1}{x_1 S}\delta\left(x_2 - \frac{m_S^2}{x_1 S}\right). \tag{4.8}$$

The single colour octet scalar proton-proton cross section then becomes

$$\begin{aligned}\sigma(pp \to S^0_{R,I}) &= \frac{\pi^2}{8}\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 g(x_1,\mu) g(x_2,\mu)\delta(s - m^2_{S^0_{R,I}})\Gamma(S^0_{R,I} \to gg) \\ &= \frac{\pi^2}{8S}\int_{\frac{m^2_{S_{R,I}}}{S}}^1 \frac{\mathrm{d}x_1}{x_1} g(x_1,\mu) g\left(x_2, \frac{m_S^2}{x_1 S}\right)\Gamma(S^0_{R,I} \to gg), \end{aligned} \tag{4.9}$$

with $g(x,\mu)$ the PDF.

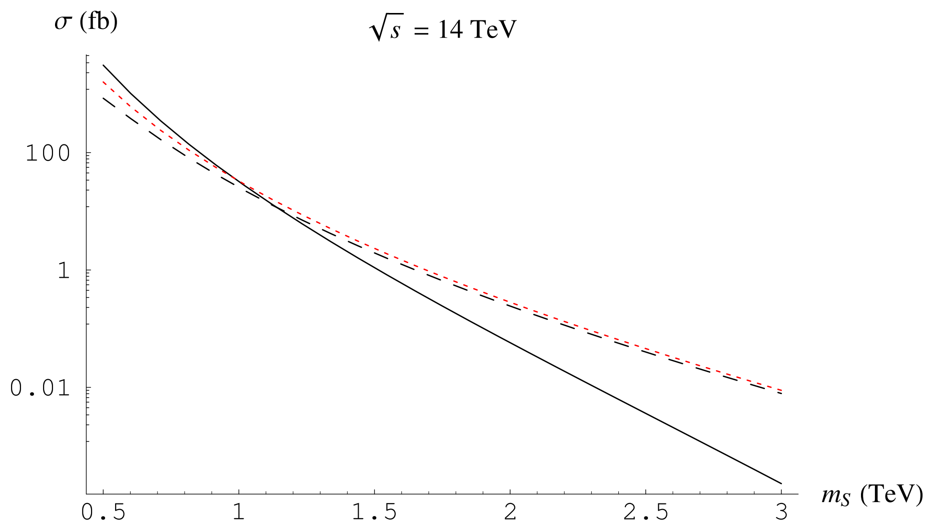


Figure 5: Figure taken from Ref. [34] where it is shown the dependence on the cross section for $pp \to S^0_{R,I} S^0_{R,I}$ (solid line), $pp \to S^0_R$ (dashed line) and $pp \to S^0_I$ (red dotted line). The values of the parameters where $\eta_U = 1$ and $\lambda_{4,5} \sim 1$, in such a way that the only important contribution to the single cross section is Fig. 4a.

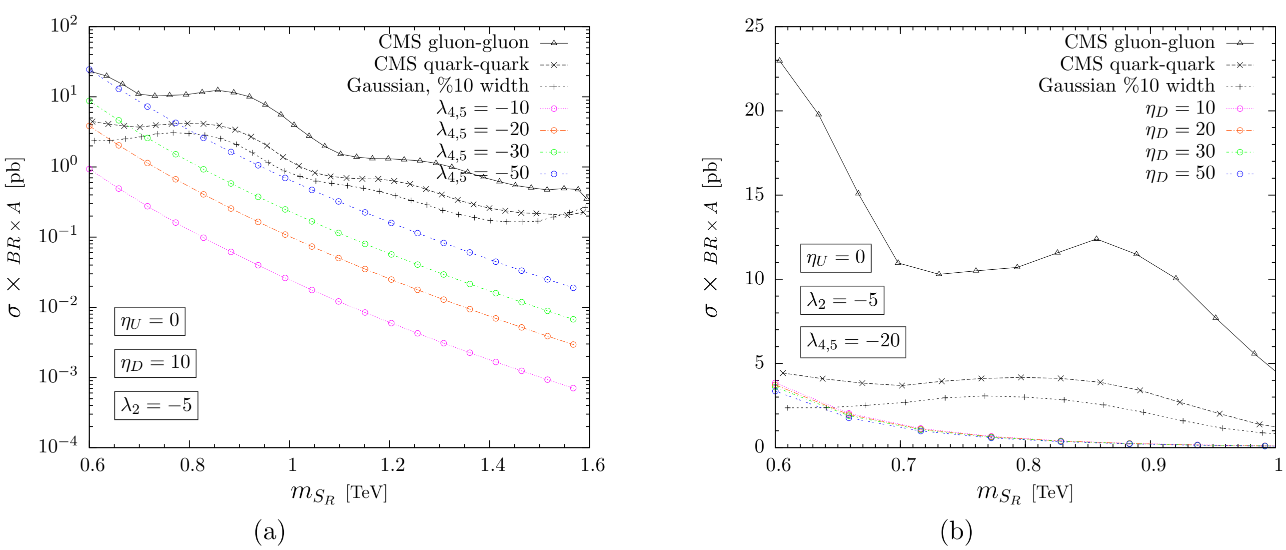
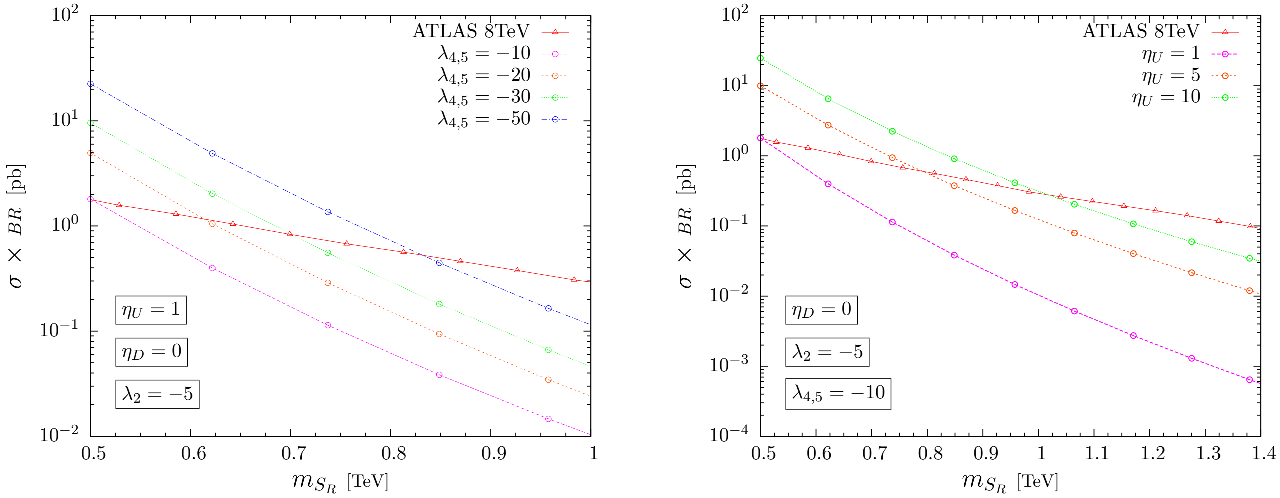
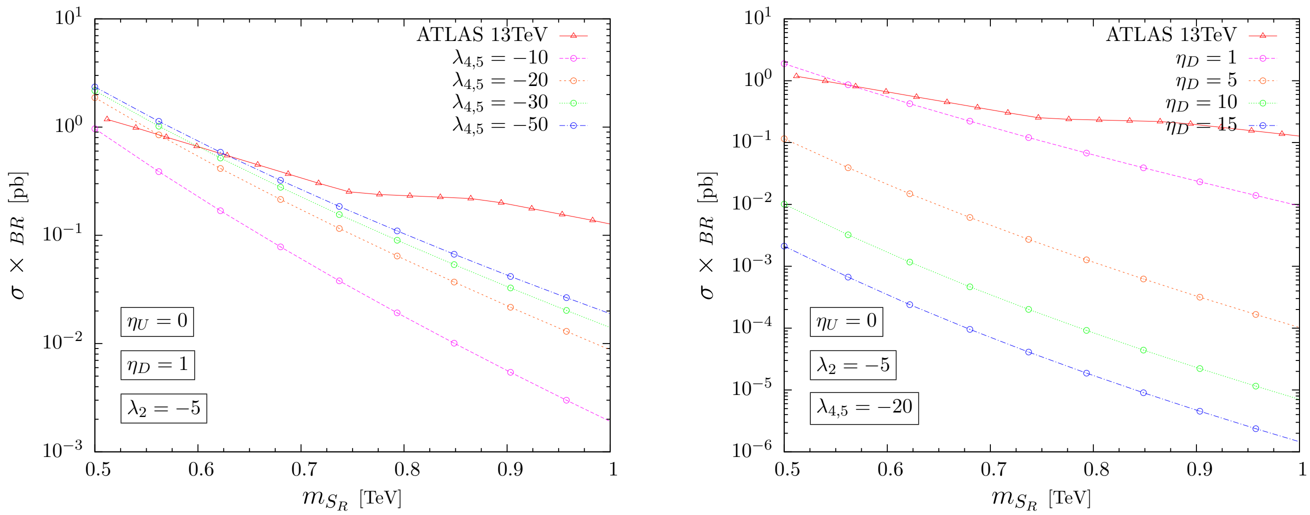


Figure 6: Production of single octet scalars (S^0_R) and decay to dijets compared with CMS data at 13 TeV taken from Ref. [6].



(a) (b)

Figure 7: Production of single octet scalars (S_R^0) and decay to $t\bar{t}-$pair compared with ATLAS data at 8 TeV taken from Ref. [6].



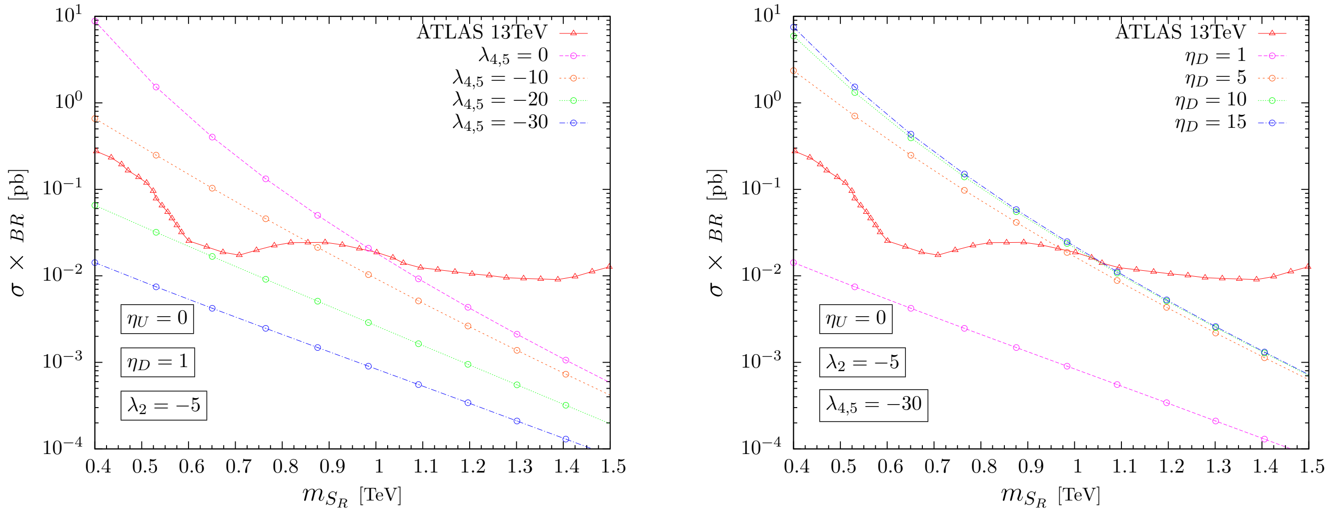
(a) (b)

Figure 8: Production of pair octet scalars ($S_R^0 S_R^0$) and decay to a pair of dijets compared with ATLAS data at 13 TeV taken from Ref. [6].

Similarly we obtain the proton-proton cross section for the pair production. It is interesting to notice that, although the single production is generated through loops, this becomes the dominant channel for masses of the scalars higher than 1 TeV, as can be seen in Fig. 5.

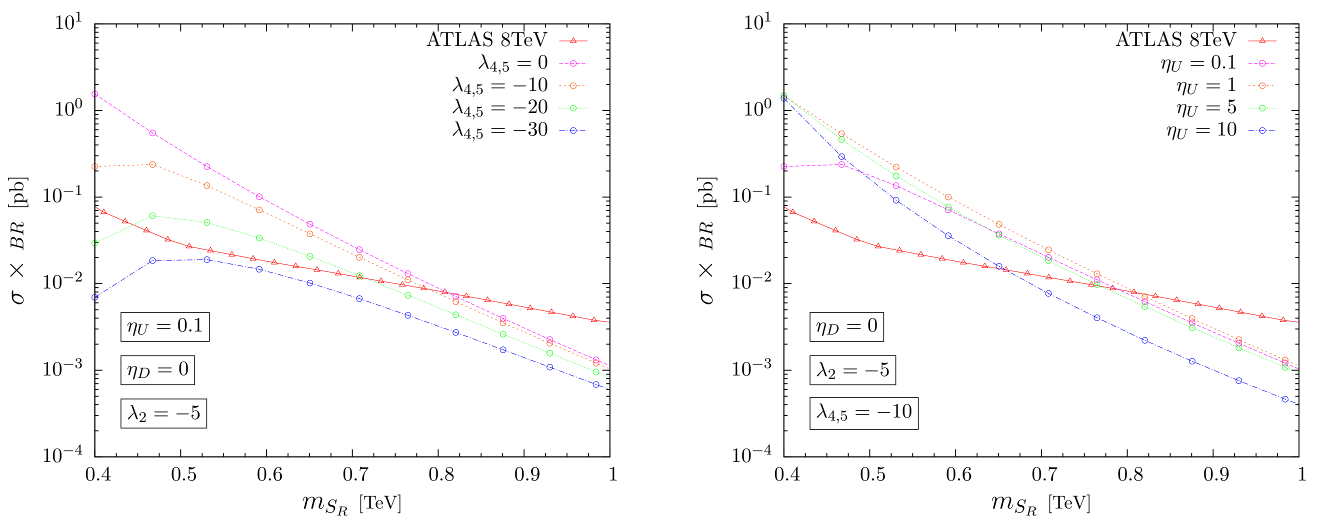
Once these particles are produced through gluon fusion they decay to other particles which are detected on the LHC. The decay channels of the single production analysed on Ref. [6] are to dijets and to a $t\bar{t}-$pair.

The decay to dijets will be the dominant channel when the decay to a $t\bar{t}-$pair and the process $S_{R,I}^0 \to S^{\pm}W^{\mp}j$ are suppressed. The former has zero contribution when $\eta_U = 0$ which is easy to see looking at Eqs. (3.14) and (3.15). The latter will be kinematically suppressed when the mass of the neutral scalar is smaller than the mass of the charged scalar plus the mass of the $W^{\pm}$ bosons.



(a) (b)

Figure 9: Production of pair octet scalars ($S_R^0 S_R^0$) and decay to four bottoms quarks compared with ATLAS data at 13 TeV taken from Ref. [6].



(a) (b)

Figure 10: Production of pair octet scalars ($S_R^0 S_R^0$) and decay to four tops quarks compared with ATLAS data at 8 TeV taken from Ref. [6].

Imposing custodial symmetry implies $m_{S_I^0} = m_{S^\pm}$ and choosing $\lambda_2 < 0$ we get $m_{S_R^0} < m_{S^\pm}$, in Ref. [6] was chosen $\lambda_2 = -5$.

The dominant channel that produces dijets is the decay to a $b\bar{b}-$pair for values of $\eta_D > 10$ and for smaller values of η_D the decay to gg starts to become relevant. On Fig. 6 we can see how the $m_{S_R^0}$ is constrained around 650 GeV for values of λ_4 around -20, for positive values of λ_4 almost the same result is obtained.

Analysing the $t\bar{t}-$pair production the limits that are obtained are $m_S > 500$ GeV as can be seen in Fig. 7.

For pair production the relevant decay channels will be the dijet pairs and the four top production with the same conditions than before. The constrains obtained from the pair dijet are shown on Fig. 8. In this figure we see that $m_S > 500\ GeV$ for $\eta_D \sim 1$ and $|\lambda_4| > 10$. For higher values of η_D there are not constraints because the relevant channel starts to be to four bottoms quarks. The constraints for the four bottoms are shown in Fig. 9 where masses smaller than 1 TeV are forbidden for values of $\eta_D > 5$ and $\lambda_4 \sim -30$.

Finally, the constraints obtained for four top quarks production are shown in Fig. 10. With this we also see that the masses are constrained to be higher than 700-800 GeV with values of $|\lambda_4| < 20$.

4.2 Higgs Signal Strengths

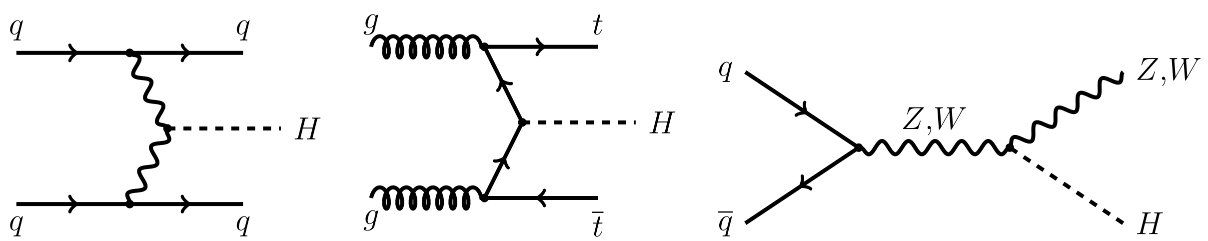


Figure 11: Production of SM Higgs through vector boson fusion (a), associated production with a $t\bar{t}$ (b) and associated production with a vector boson (c).

The LHC data for the Higgs physics is given in terms of the so called Higgs signal strengths. These are the measured cross section of the Higgs production times the branching ratio of the Higgs decay in units of the prediction of the SM. These strengths are measured for any production and decaying channel. At the LHC, the relevant production channels are gluon fusion (Fig. 24), vector boson fusion (Fig. 11a), associated production with a $t\bar{t}$ (Fig. 11b) and associated production with a vector boson (Fig. 11c). The relevant Higgs decay modes are the decay to two photons, the decay to two weak gauge bosons (one real and one virtual), the decay to two τ leptons and the decay to a $b\bar{b}$ pair. So, for instance, the Higgs signal strength for the Higgs production through gluon fusion and decaying to photons will be

$$\mu_F^{\gamma\gamma} = \frac{\sigma(pp \to H)\mathrm{Br}(H \to \gamma\gamma)}{\sigma(pp \to H)_{SM}\mathrm{Br}(H \to \gamma\gamma)_{SM}}. \tag{4.10}$$

Now we can think about the processes that are modified by the introduction of the colour octet scalars. In the previous section we saw that the new scalars modify the Higgs production through gluon fusion and the Higgs decay into two photons. The amplitude for the contributions of these particles to these processes has been calculated in the appendix B. Clearly the other production channels are not modified at tree-level by the scalars, neither the other decay modes. Furthermore, if we take into account that, analysing the available data, Ref. [6] found that the masses of these particles must be heavier than 500 GeV, we deduce that the Higgs cannot decay to these particles. Therefore, the total decay width of the Higgs boson remains almost invariant, we will neglect the contribution of the new scalars to the total decay width.

Now let us calculate the Higgs signal strengths of the Higgs being produced by any process but gluon fusion and decaying to two photons. For these cases the cross section of the Higgs production in our model is the same than in the SM and the only modification will come from the decay

$$\mu^{\gamma\gamma} = \frac{\sigma(pp \to H)\mathrm{Br}(H \to \gamma\gamma)}{\sigma(pp \to H)_{SM}\mathrm{Br}(H \to \gamma\gamma)_{SM}} = \frac{\mathrm{Br}(H \to \gamma\gamma)}{\mathrm{Br}(H \to \gamma\gamma)_{SM}} = \frac{\Gamma(H \to \gamma\gamma)}{\Gamma(H \to \gamma\gamma)_{SM}} = \frac{|\mathcal{M}^{\gamma\gamma}|^2}{|\mathcal{M}^{\gamma\gamma}|^2_{SM}}. \quad (4.11)$$

The value for the SM amplitude was taken form Ref. [35] and adding our contribution we find

$$|\mathcal{M}^{\gamma\gamma}|^2 = \frac{\alpha^2}{256\pi^3}\frac{m_H^3}{v^2}\Bigg|\left(2 + 12\frac{m_W^2}{m_H^2} - 6\frac{m_W^2}{m_H^2}\left(2 - 4\frac{m_W^2}{m_H^2}\mathcal{F}\left(\frac{m_W^2}{m_H^2}\right)\right)\right)$$
$$- \frac{32}{3}\frac{m_t^2}{m_H^2}\left(1 - \frac{1}{2}\left(1 - \frac{m_t^2}{m_H^2}\mathcal{F}\left(\frac{m_t^2}{m_H^2}\right)\right)\right) + \frac{16\lambda_1 v^2 T_F}{m_H^2}\left(1 + 2\frac{m_S^2}{m_H^2}\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)\right)\Bigg|^2, \quad (4.12)$$

the SM prediction is the same doing $\lambda_1 = 0$.

As the mass of our scalars is much bigger than the Higgs mass we can proceed like in section 3.4 and expand the function $\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)$ around infinity

$$\frac{16\lambda_1 v^2 T_F}{m_H^2}\left(1 + 2\frac{m_S^2}{m_H^2}\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)\right) = -\frac{4v^2 T_F}{3}\frac{\lambda_1}{m_S^2}, \quad (4.13)$$

in such a way that this function just depends on λ_1/m_S^2.

Now we can calculate the strength of the Higgs being produced by gluon fusion and decaying to anything but two photons

$$\mu_F = \frac{\sigma(pp \to H)}{\sigma(pp \to H)_{SM}} = \frac{|\mathcal{M}_{ggF}|^2}{|\mathcal{M}_{ggF}|^2_{SM}}. \quad (4.14)$$

Combining the results of appendix B we obtain

$$|\mathcal{M}_{ggF}|^2 = \left(\frac{\alpha_s}{\pi}\right)^2\left|\frac{2m_t^2}{v^2}\left(-2 + \left(1 - 4\frac{m_t^2}{m_H^2}\right)\mathcal{F}\left(\frac{m_t^2}{m_H^2}\right)\right) + 3(2\lambda_1 + \lambda_2)^2 v^2\left(1 + 2\frac{m_S^2}{m_H^2}\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)\right)\right|^2, \quad (4.15)$$

where again expand $\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)$ around infinity

$$3(2\lambda_1 + \lambda_2)^2 v^2\left(1 + 2\frac{m_S^2}{m_H^2}\mathcal{F}\left(\frac{m_S^2}{m_H^2}\right)\right) = -\frac{v^2}{4}\frac{(2\lambda_1 + \lambda_2)}{m_S^2}, \quad (4.16)$$

in such a way that we have just dependence on two variables λ_1/m_S^2 and λ_2/m_S^2.

Finally the strength of the Higgs produced by gluon fusion and decaying to two photons will be simply the product of these two

$$\mu_F^{\gamma\gamma} = \mu_F \mu^{\gamma\gamma}. \quad (4.17)$$

Once we have calculated the theoretical predictions on our model we can compare them with the experimental data, the data that we used came from Ref. [36] and it is shown in Tab. 1.

The first thing we can do is to find constraints on λ_1/m_S^2. For doing so we calculate the range in which λ_1/m_S^2 can vary in order to reproduce the experimental value of the $\mu_{VBF}^{\gamma\gamma}$ at a 95% confidence level. We took this strength because it is the best measured of the ones that just depend on λ_1/m_S^2. The result was that λ_1/m_S^2 must be constrained between -102 TeV^{-2} and 120 TeV^{-2}.

Once we have calculated the allowed region for λ_1/m_S^2 we can use the result for constraining the λ_2/m_S^2 parameter using other strengths. As the other strengths depend on both parameters we cannot find a general region in which λ_2/m_S^2 can vary, but the range of λ_2/m_S^2 will depend on the value that λ_1/m_S^2 takes. In Fig. 12 can be seen the allowed region for both parameters using the data for μ_F^{ZZ}, μ_F^{WW} and $\mu_F^{\gamma\gamma}$, which are the three best measured. Furthermore, this figure also shows the superposition of the allowed regions for μ_F^{WW} and $\mu_F^{\gamma\gamma}$.[10] Looking at Fig. 12d we can see that the lower allowed value for λ_1/m_S^2 is -60 TeV^{-2} which is a harder constraint than the obtained using the $\mu_{VBF}^{\gamma\gamma}$ Higgs signal strength. The upper allowed value using $\mu_{VBF}^{\gamma\gamma}$ is still better so we can conclude that $\lambda_1/m_S^2 \in [-60, 120]$ TeV^{-2} at a 95% CL.

Production/Decay	$H \to \gamma\gamma$	$H \to ZZ$	$H \to WW$	$H \to \tau\tau$	$H \to bb$
ggF	$1.1^{+0.23}_{-0.22}$	$1.13^{+0.34}_{-0.31}$	$0.84^{+0.17}_{-0.17}$	$1.0^{+0.6}_{-0.6}$	–
VBF	$1.3^{+0.5}_{-0.5}$	$0.1^{+1.1}_{-0.6}$	$1.2^{+0.4}_{-0.4}$	$1.3^{+0.4}_{-0.4}$	–
WH	$0.5^{+1.3}_{-1.2}$	–	$1.6^{+1.2}_{-1.0}$	$-1.4^{+1.4}_{-1.4}$	$1.0^{+0.5}_{-0.5}$
ZH	$0.5^{+3.0}_{-2.5}$	–	$5.9^{+2.6}_{-2.2}$	$2.2^{+2.2}_{-1.8}$	$0.4^{+0.4}_{-0.4}$
$t\bar{t}$	$2.2^{+1.6}_{-1.3}$	–	$5^{+1.8}_{-1.7}$	$-1.9^{+3.7}_{-3.3}$	$1.1^{+1.0}_{-1.0}$

Table 1: Experimental values for the Higgs signal strengths obtained from Ref. [36].

The next step is to use all the data of Tab. 1 to find the values of our parameters that best fit these experimental data. In order to do this we will do a similar analysis than the one that Ref. [37] made for the A2HDM model. First of all we need to minimise the χ^2 function:

$$\chi^2 = \sum_k \frac{(\mu_k - \hat{\mu}_k)^2}{\sigma_k^2}, \tag{4.18}$$

where σ_k is the standard deviation of the experimental measurement $\hat{\mu}_k$ and μ_k is the theoretical prediction. If the errors are not symmetric, as they are similar, we symmetrise them

$$\delta\hat{\mu}_k = \sqrt{\frac{(\delta\hat{\mu}_+)^2 + (\delta\hat{\mu}_-)^2}{2}}. \tag{4.19}$$

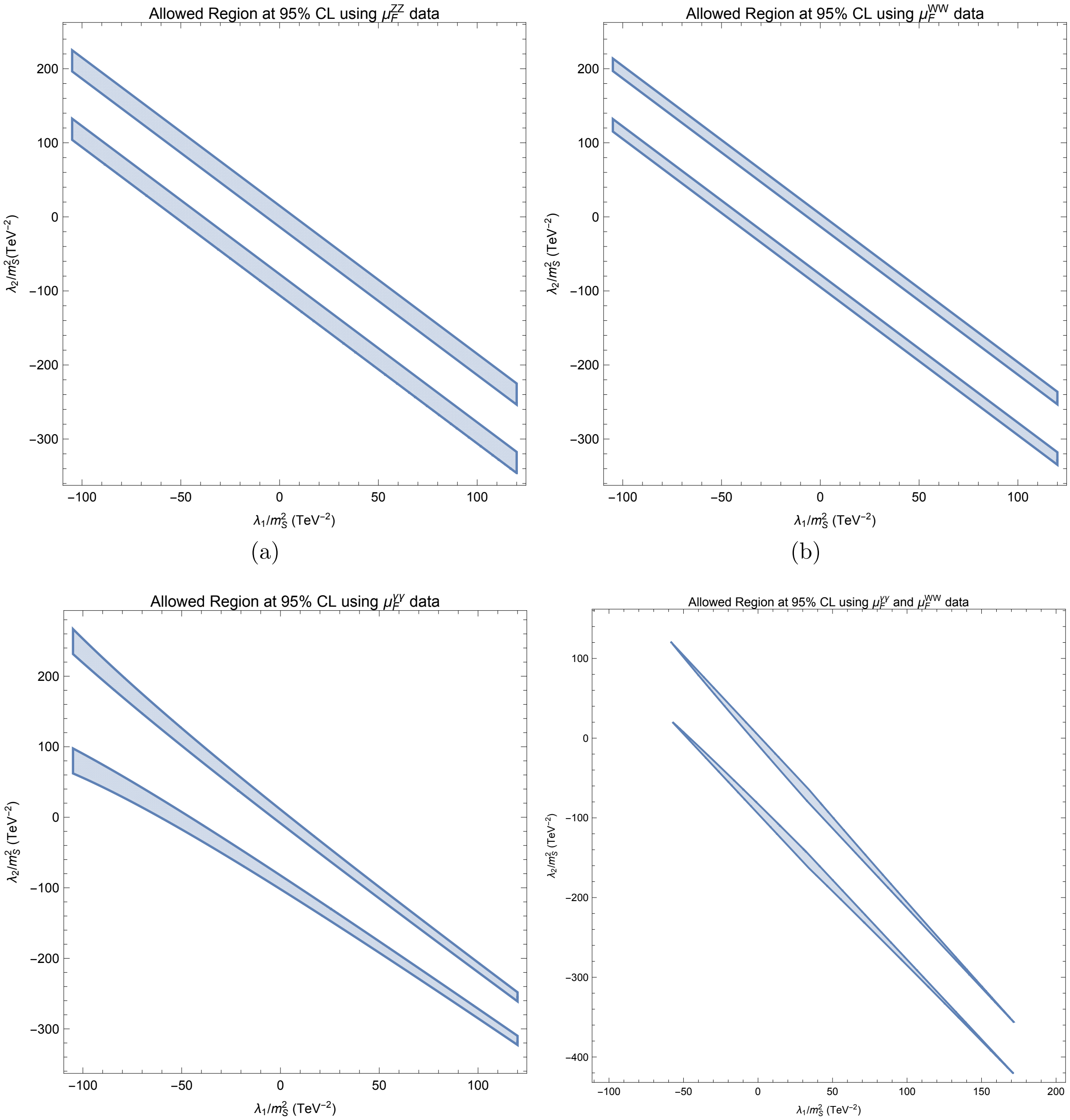
Performing this minimisation we obtain that the minimum is $\chi^2_{\min} = 19.27$ which normalised by the degrees of freedom

$$\text{d.o.f.} = \#\mu + \#\text{parameters} - 1 = 17, \tag{4.20}$$

gives $\chi^2/\text{d.o.f.} = 1.13$ which is an acceptable value.

For calculating the allowed region of the parameters we used the $\Delta\chi^2$ values shown in Tab. 2, extracted from Ref. [38]. In this table we present how much can the χ^2 function vary in terms of

[10] Note that the contribution of our particles to μ_F^{ZZ} and μ_F^{WW} is exactly the same because they do not contribute to the decay. However, the contribution to $\mu_F^{\gamma\gamma}$ is not exactly the same because they do contribute to the decay to two photons, this is why this combined plot is interesting. The fact of using μ_F^{WW} for this combination is because it has less error.



(a) (b) (c) (d)

Figure 12: Constraints of the λ_1/m_S^2 and λ_2/m_S^2 parameters using the data from the Higgs signal strengths. Figures (a), (b) and (c) shows the constraints of these parameters using the μ_F^{ZZ}, μ_F^{WW} and $\mu_F^{\gamma\gamma}$ Higgs signal strengths, respectively, at a 95% CL. The last one (d) shows the superposition of the constraints obtained by (b) and (c).

the degrees of freedom and the confidence level. As we have two degrees of freedom we should use the second column of this table and the result is showed in Fig. 13 for 68.3% and 90% of CL. The SM is reproduced when these parameters are 0, the red cross in this figure represents these values. Clearly we can see that the SM is compatible with the data at one sigma. However, the values of these parameters that minimise the χ^2 are not 0 but the ones of the green cross in the figure. If we consider that one of the two parameters that we have is set at the value of the minimum and we consider then that we have just one degree of freedom we can calculate the error of the other.

CL/d. o. f.	1	2	3	4	5	6
68.3%	1.00	2.30	3.53	4.72	5.89	7.04
90%	2.71	4.61	6.25	7.78	9.24	10.6
99%	6.63	9.21	11.3	13.3	15.1	16.8
99.73%	9.00	11.8	14.2	16.3	18.2	20.1
99.99%	15.1	18.4	21.1	23.5	25.7	27.8

Table 2: $\Delta\chi^2$ as a function of confidence level and degrees of freedom.

Doing so we can give the values for the best fit with an uncertainty

$$\frac{\lambda_1}{m_S^2} = 22.8^{+2.7}_{-1.4} \ \ TeV^{-2} \qquad\qquad \frac{\lambda_2}{m_S^2} = -50.5^{+2.7}_{-2.8} \ \ TeV^{-2}. \tag{4.21}$$

It is important to emphasise that in Eq. 4.21 the uncertainties have been calculated fixing one of the parameters to the best fit value and considering just one degree of freedom but in reality we have two degrees of freedom, so the real allowed values are the ones showed in Fig. 13, this equation is just added for completeness. The fact that in this equation the values of these parameters that reproduce the SM are discarded does not mean that they are not allowed because they are allowed, as can be see in the figure.

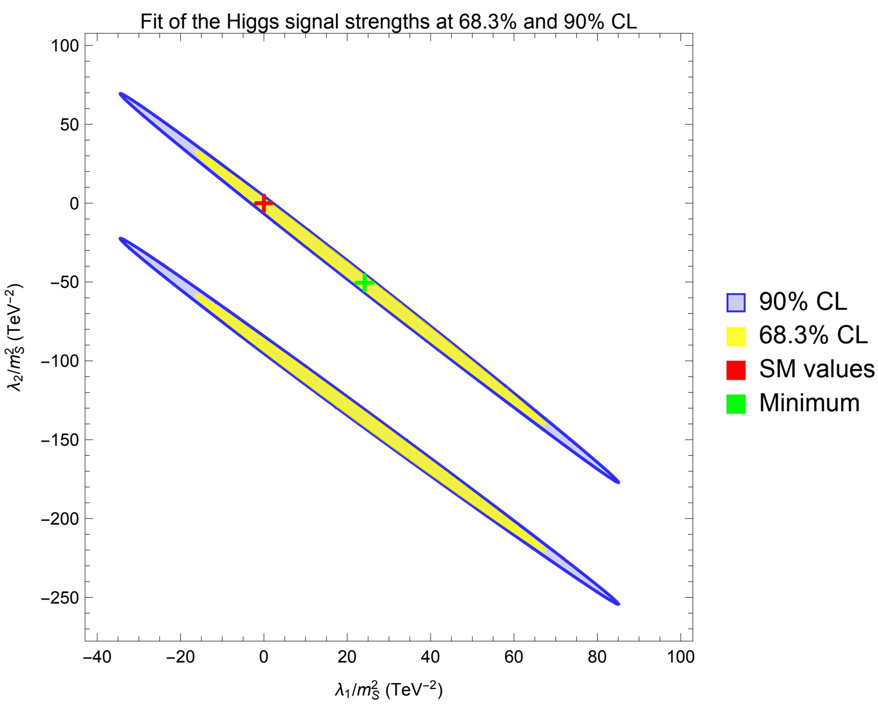


Figure 13: Best fit for λ_1/m_S^2 and λ_2/m_S^2 using Higgs signal strengths. The yellow region corresponds to the values obtained at a 68.3% of CL and the blue region at a 90% of CL. The red cross is the value of the parameters that represent the SM, both at 0, and the green cross the point with the lowest value of χ^2.

4.3 Wilson Coefficients

The Wilson coefficient c_{WB} is related to the oblique parameter S [1]

$$\frac{c_{WB}}{\Lambda^2} = -\frac{1}{8\pi v^2} S. \tag{4.22}$$

This oblique parameter is associated with the contribution of new physics to the difference between the Z self-energy at the scale of M_Z^2 and 0:

$$\frac{\alpha(M_Z)}{4\sin\theta_W\cos\theta_W} S = \frac{\Pi_{ZZ}^{\text{new}}(M_Z^2) - \Pi_{ZZ}^{\text{new}}(0)}{M_Z^2}, \tag{4.23}$$

where the fine structure constant and the electroweak mixing angle are expressed in the $\overline{MS}$ scheme.

Using Eq. (4.22) and the experimental value of this parameter coming from Ref. [29] we could find constraints on the value of λ_2/m_S^2. The experimental value is $S = 0.05 \pm 0.10$ and this constrains $\lambda_2/m_S^2 \in [-45, 77]\ \ TeV^{-2}$ at a 95% of confidence level.

The other Wilson coefficients are not constrained by precision electroweak physics so we cannot use them in order to constrain the parameters of our theory.

4.4 Running Gauge Coupling

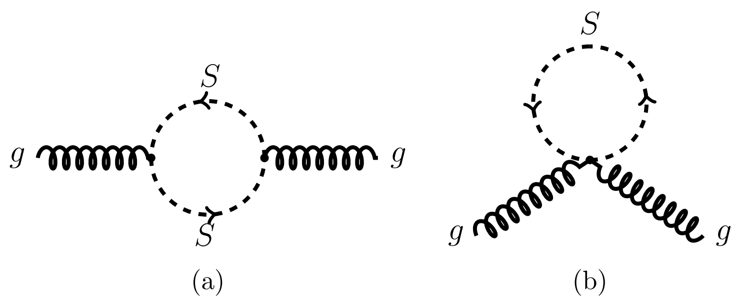


Figure 14: Contributions of our scalars to the gluon self-energy.

The fact of having new colour octet scalars affects to the gluon self energy which changes the running of the strong coupling. The new diagrams are shown in Fig. 14 and their divergent part in the $\overline{MS}$ scheme is

$$\Delta\Pi_{ab}^{\mu\nu} = \Delta\Pi_S \delta_{ab}(q^\mu q^\nu - q^2 g^{\mu\nu}) = -\frac{4}{3} g_s^2 \frac{\mu^{2\epsilon}}{(4\pi)^2} \frac{1}{\hat{\epsilon}} \delta_{ab}(q^\mu q^\nu - q^2 g^{\mu\nu}), \tag{4.24}$$

where q is the momentum of the gluon and $1/\hat{\epsilon} = 1/\epsilon + \gamma_E - \log 4\pi$.

Using the value of the SM for the other processes that contribute to the gluon self energy we obtain that the total divergent part is

$$\Delta\Pi = g_s^2 \frac{\mu^{2\epsilon}}{(4\pi)^2} \left(\left(\frac{5}{6} - \frac{1}{2}\xi \right) C_A - \frac{4}{3} T_F n_F \right) \frac{1}{\hat{\epsilon}}. \tag{4.25}$$

Using this and the SM values for the divergent parts of the vertex corrections we obtain easily the $\beta-$function:

$$\frac{\mu}{\alpha_s(\mu)}\frac{\mathrm{d}}{\mathrm{d}\mu}\alpha_s(\mu) = \frac{\alpha_s}{\pi}\beta_1 + \dots = \frac{\alpha_s}{\pi}\left[\frac{2}{3}n_F T_F - \frac{7}{6}C_A\right] + \dots \tag{4.26}$$

Now we can integrate this equation in order to find the running of the strong coupling constant in this theory

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha_s(Q_0^2)}{2\pi}\log\left(\frac{Q^2}{Q_0^2}\right)}. \tag{4.27}$$

Using this equation and having an initial experimental input we can find the values for the strong coupling constant at any energy. The values of this constant have been measured at different scales, as can be seen in Fig. 15, and our theory must produce a running compatible with such measurements. The best measurement of this constant that has been performed is the one at the scale of the Z boson mass, $\alpha_s(M_Z) = 0.1181 \pm 0.0011$. We can take this value like the input and predict with it the values for the other energies. For doing so we have to take into account that the contribution of these particles will start at the scale of its mass because at a smaller scale they can be integrated out.[11] Therefore, at the M_Z scale we should use the β_1 function predicted by the SM,

$$\beta_1^{SM} = \frac{2}{3}T_F n_F - \frac{11}{6}C_A, \tag{4.28}$$

and with just 5 flavours until we get the scale of the top quark mass.

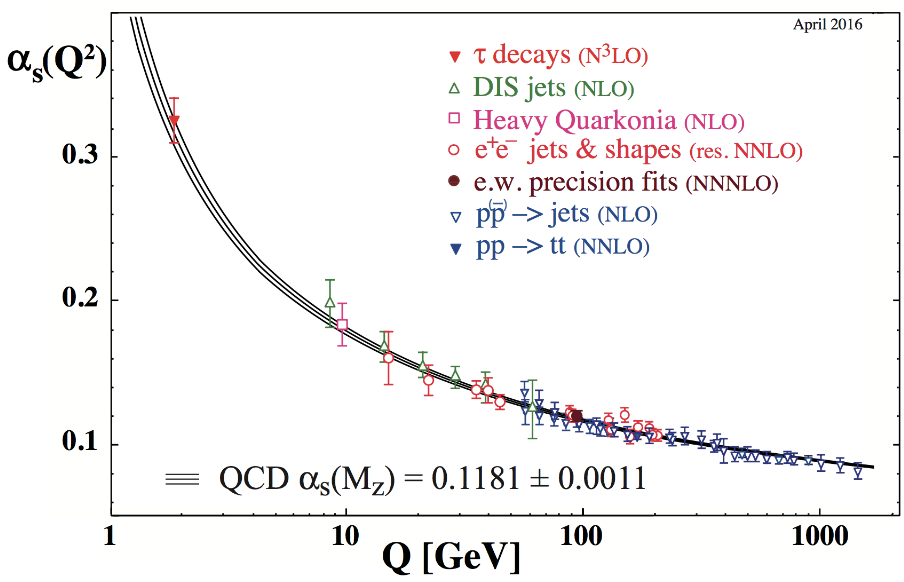


Figure 15: Summary of the measurements of α_s and the predictions of the SM taken from Ref. [29].

Taking this into account we can plot the running of α_s considering that our new scalars have a mass around 500 GeV and compare it with the experimental data taken form Ref. [39]. If we look at the last experimental value of Fig. 16 we see that scalars with a mass smaller than 500 GeV are

[11] Actually, if we could solve this theory exactly, without a perturbative development, the result at low scale should be the same for both theories.

forbidden up to 2σ. It is important to remark that this calculation have been performed at leading order (LO). We have just calculated the running of α_s at LO and for consistency we have used the matching condition at LO, $\alpha_s^{SM}(m_S) = \alpha_s^{\text{new}}(m_S)$. This has a strong dependence on the matching point so we should take this result carefully. In order to obtain a solid result we should go further to the next order, and calculate carefully the matching conditions, where the dependence on the matching point will not be so strong.

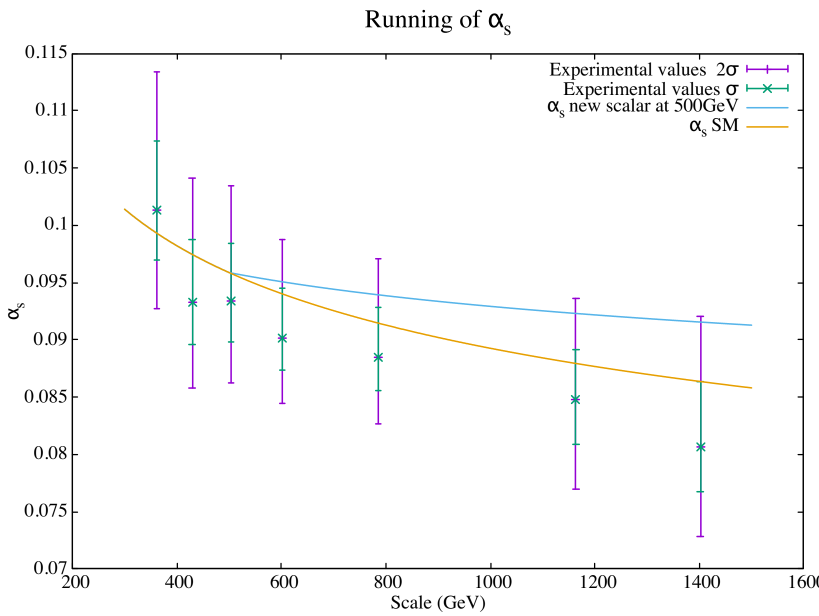


Figure 16: Experimental values of α_s and its running at LO for the SM and for the SM extended with colour octet scalars of 500 GeV.

5 Conclusions

In this work we have made a brief review of the SM, developing its Lagrangian from the principle of gauge invariance. We have showed that this symmetry brings to particles without mass and that this problem could be solved considering that this symmetry is spontaneously broken by the vacuum. We also explained that this SSB, in principle, leaded to the existence of massless particles, the Goldstone bosons, which had not been observed. This problem, as explained before, was definitely solved by Ref. [2–4] who showed that these particles are not physical in gauge theories.

Once the SM was understood properly we studied a escalar extension of this model, the model of A. V. Manohar and M. B. Wise [1]. In this model an additional $SU(2)_L$ doublet and $SU(3)_C$ octet is added to the SM. So, we have two new charged scalars $S^{\pm^a}$, one new CP-even neutral scalar $S_R^{0^a}$ and one new CP-odd neutral scalar $S_I^{0^a}$ which can appear with any of the eight possible adjoint colours.

The fact of having new scalars with colour produces a total decoupling between these new scalars and the Higgs doublet. These scalars have different quantum numbers than the Higgs doublet so they cannot mix with this doublet like in other extensions as the 2HDM or Higgs triplet models. Furthermore, as a consequence of the conservation of the colour the colour scalars cannot adquiere a vev and, therefore, the vev of the SM must be produced entirely by the SM Higgs doublet.

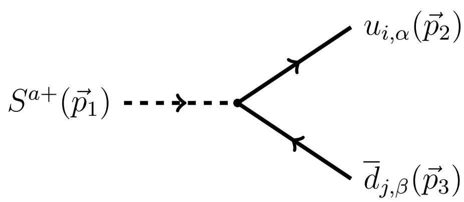
The colour makes also this model richer because new interactions appear. In this work we have shown that the covariant derivative leads, apart of coupling to the electroweak bosons like in the 2HDM, to couplings with the gluons. In the Yukawa sector we have considered MFV in such a way that the Yukawa matrices of the new scalars were equal to the ones of the SM except for some complex proportionality constants. With this we avoid the existence of FCNC at tree-level, extremely suppressed experimentally.

Taking into account the actual limits on the masses of these scalars, we found that these particles have a high mass compared to the electroweak scale. This fact allowed us to integrate out these particles, condensing their physics in operators of dimension six. In particular we only considered the operators that are related to Higgs physics, extremely important for the processes of the LHC.

In this work a phenomenological analysis have also been performed. We analysed the limits on the masses found by Ref. [6] and we studied the Higgs signal strengths. The allowed range for the parameters studied is still wide for values of the masses around 500 GeV. With masses of this order the constraints to the parameters are outside their perturbative region but around this region, although these are worse for higher masses. However, the Higgs signal strengths have considerable errors and when these reduce, thanks to the increase of luminosity of the LHC, better constraints will be obtained. The oblique parameter S produces interesting constraints when the mass of these scalars is around 500 GeV which are inside the perturbative region of the constrained parameters.

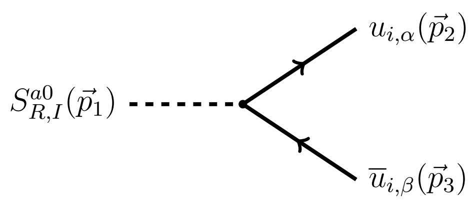
Finally, we studied the contributions of the scalars to the running of α_s. We made an initial study at LO where we found interesting bounds for the mass of these scalars although for conclusive results it is necessary to go beyond LO. At this order the dependence with the matching point is strong and it is, therefore, necessary to make the calculation at higher orders to reduce this dependence. We leave this for a future study. Anyway, if we look at Fig. 16 the trend seem to be that light masses are forbidden but we have to be careful with this statement.

A Feynman Rules



$$i\frac{\sqrt{2}}{2v}\eta_U m_{u_i} V_{ij}\lambda^a_{\alpha\beta}\mathcal{P}_L - i\frac{\sqrt{2}}{2v}\eta^*_D m_{d_j} V_{ji}\lambda^a_{\beta\alpha}\mathcal{P}_R \tag{A.1}$$

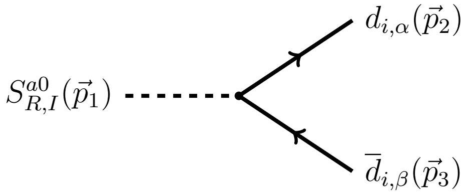
Figure 17: Charged colour octet decaying to quarks.



$$\text{Even Scalar:} \quad -i\frac{1}{2v}m_{u_i}\lambda^a_{\alpha\beta}(\eta_U\mathcal{P}_L + \eta^*_U\mathcal{P}_R) \tag{A.2}$$

$$\text{Odd Scalar:} \quad \frac{1}{2v}m_{u_i}\lambda^a_{\alpha\beta}(\eta_U\mathcal{P}_L - \eta^*_U\mathcal{P}_R) \tag{A.3}$$

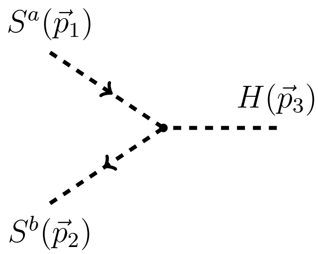
Figure 18: Neutral colour octet decaying to up-type quarks



$$\text{Even Scalar:} \quad -i\frac{1}{2v}m_{d_i}\lambda^a_{\alpha\beta}(\eta_D\mathcal{P}_L + \eta_D^*\mathcal{P}_R) \tag{A.4}$$

$$\text{Odd Scalar:} \quad \frac{1}{2v}m_{d_i}\lambda^a_{\alpha\beta}(\eta_D^*\mathcal{P}_R - \eta_D\mathcal{P}_L) \tag{A.5}$$

Figure 19: Neutral colour octet decaying to down-type quarks

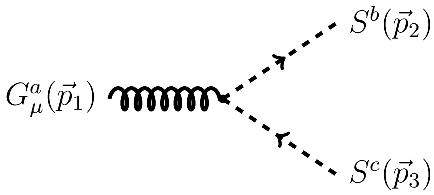


$$\text{Charged:} \quad -iT_Fv\lambda_1 \tag{A.6}$$

$$\text{Even Scalar:} \quad -\frac{i}{2}T_Fv(\lambda_1+\lambda_2+2\lambda_3) \tag{A.7}$$

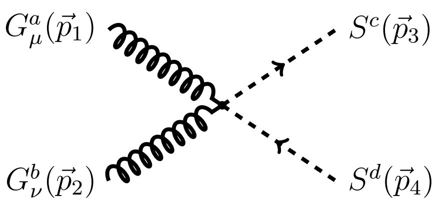
$$\text{Odd Scalar:} \quad -\frac{i}{2}T_Fv(\lambda_1+\lambda_2-2\lambda_3) \tag{A.8}$$

Figure 20: Colour octet particles to SM Higgs



$$-g_sf^{abc}(p_2^\mu - p_3^\mu) \tag{A.9}$$

Figure 21: Gluon decaying to any of the three colour octet scalars ($S^{+^b}S^{-^c}$, $S_R^{0^b}S_R^{0^c}$ and $S_I^{0^b}S_I^{0^c}$). The momenta are incoming.



$$2ig_s^2f^{ace}f^{bde}g_{\mu\nu} \tag{A.10}$$

Figure 22: Vertex of two gluons going to any of the three colour octet scalars ($S^{+^b}S^{-^c}$, $S_R^{0^b}S_R^{0^c}$ and $S_I^{0^b}S_I^{0^c}$).

B Explicit Calculations

B.1 Gluon Fusion Mediated By Colour Octet Scalars

Let us calculate the contribution of the colour octet scalars to the Higgs production through gluon fusion, Fig. 23. The calculation of this process is almost the same if the internal propagators are charged colour octets, even neutral colour octets or odd neutral colour octets, the only difference will be the coupling between the scalars and the Higgs, Fig. 20, and the mass. Therefore we will do the calculation for the case of charged colour octets and we will generalise this result afterwards for the other two cases.

Let us start with the amplitude for the first diagram, Fig. 23b:

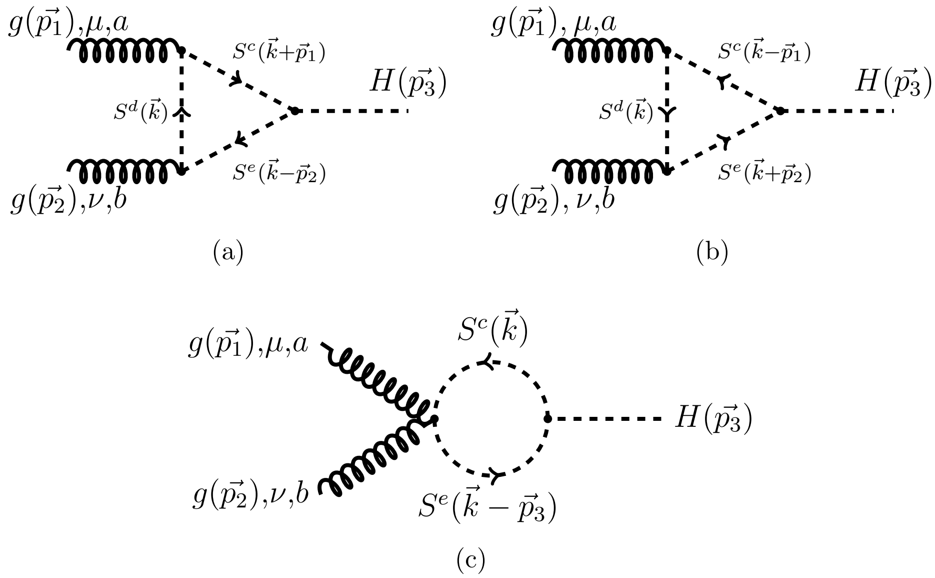


Figure 23: Production of SM Higgs through gluon fusion mediated by colour scalars.

$$i\mathcal{M}_a = iT_F\lambda_1 v g_s^2 f^{acd} f^{bcd} \epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)\int\frac{\mathrm{d}^4k}{(2\pi)^4}(2k^\mu + p_1^\mu)(2k^\nu - p_2^\nu)\frac{i}{(k+p_1)^2-m_S^2}\frac{i}{(k-p_2)^2-m_S^2}\frac{i}{k^2-m_S^2}. \tag{B.1}$$

As the gluons have transversal polarisation, $\epsilon_\mu^a(p_1)p_1^\mu = 0$, we can eliminate some terms:

$$i\mathcal{M}_a = 4T_F\lambda_1 v g_s^2 f^{acd} f^{bcd} \epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{((k+p_1)^2-m_S^2)((k-p_2)^2-m_S^2)(k^2-m_S^2)}. \tag{B.2}$$

Let us now focus on the integral, in order to calculate it we will use the Feynman parametrisation:

$$\frac{1}{ABC} = \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{\Gamma(3)}{[Ax+By+C(1-x-y)]^3}, \tag{B.3}$$

using this our integral becomes:

$$\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{((k+p_1)^2-m_S^2)((k-p_2)^2-m_S^2)(k^2-m_S^2)} = \int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\Gamma(3)k^\mu k^\nu}{[(k-p_2y+p_1x)^2-a^2]^3}, \tag{B.4}$$

where

$$a^2 = m_S^2 - 2p_1\cdot p_2 xy. \tag{B.5}$$

Now we use dimensional regularisation to perform the integral on k:

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{[(k-p_2y+p_1x)^2-a^2]^3} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{(k+p_2y-p_1x)^\mu (k+p_2y-p_1x)^\nu}{[k^2-a^2]^3} \Rightarrow$$
$$\Rightarrow \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu - p_2^\mu p_1^\nu xy}{[k^2-a^2]^3}, \tag{B.6}$$

where we have changed variables, from $(k-p_2y+p_1x)^\mu$ to k^μ. Moreover, in the last step we have used the transversality to remove terms that afterwards will not contribute and we have eliminated also the terms with an odd number of k^μ in the numerator, which are 0 in dimensional regularisation.

Now in order to solve this integral we will use the following identity of dimensional regularisation:

$$\mathcal{J}(D,\alpha,\beta,a^2) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{(k^2)^\alpha}{[k^2-a^2+i\epsilon]^\beta} = \frac{i}{(4\pi)^{D/2}} (a^2)^{D/2} (-a^2)^{\alpha-\beta} \frac{\Gamma(\beta-\alpha-D/2)\Gamma(\alpha+D/2)}{\Gamma(\beta)\Gamma(D/2)}, \tag{B.7}$$

and

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{(k^2)^\alpha}{[k^2-a^2+i\epsilon]^\beta} k^\mu k^\nu = \frac{g^{\mu\nu}}{D} \mathcal{J}(D,\alpha+1,\beta,a^2). \tag{B.8}$$

This last identity is easy to obtain if we notice that the integral is odd over a change of sign of the components of the momentum k^μ when $\mu \neq \nu$ because this implies that, in this case, the integral is equal to minus itself so it must be 0. However, if $\mu = \nu$ we change twice the sign so the integral becomes even over a change of sign in the components of the momentum. With this we find that $g^{\mu\nu}$ is the only dirac structure that can be constructed and the proportionality is trivially find taking the trace on both sides. Similarly we can also see why the integrals with an odd number of k^μ in the numerator are zero, these terms have been already removed, as mentioned before.

Therefore to solve the integral of Eq. (B.6) we can split it in two integrals, the one proportional to $k^\mu k^\nu$ and the other, let us perform the first one:

$$I^{\mu\nu} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{[k^2-a^2]^3} = \frac{g^{\mu\nu}}{D} \mathcal{J}(D,1,3,a^2) = \frac{g^{\mu\nu}}{4+2\epsilon} \frac{i}{(4\pi)^2} \mu^{2\epsilon} \left(\frac{a^2}{4\pi\mu^2}\right)^\epsilon \frac{\Gamma(-\epsilon)}{\Gamma(3)} \frac{\Gamma(3+\epsilon)}{\Gamma(2+\epsilon)}, \tag{B.9}$$

where we have used $D = 4+2\epsilon$. Now we expand for ϵ going to zero:

$$I^{\mu\nu} = \frac{g^{\mu\nu}}{2(2+\epsilon)} \frac{i}{2(4\pi)^2} \mu^{2\epsilon} \Big(1+\epsilon \log\left(\frac{a^2}{4\pi\mu^2}\right)\Big) \Big(-\frac{1}{\epsilon}-\gamma_E\Big)(2+\epsilon) = -g^{\mu\nu} \frac{i}{4(4\pi)^2} \mu^{2\epsilon} \left(\frac{1}{\hat{\epsilon}} + \log\left(\frac{a^2}{\mu^2}\right)\right), \tag{B.10}$$

with $1/\hat{\epsilon} = 1/\epsilon + \gamma_E - \log(4\pi)$ and where γ_E is the Euler constant.

The second integral is performed in a similar way but is simpler because there are no singularities:

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{p_2^\mu p_1^\nu xy}{[k^2-a^2]^3} = p_2^\mu p_1^\nu xy \mathcal{J}(4+2\epsilon,0,3,a^2) = -\frac{i}{2(4\pi)^2 a^2} p_2^\mu p_1^\nu xy. \tag{B.11}$$

So the amplitude becomes:

$$\mathcal{M}_a = \mathcal{C}_{\mu\nu}\left\{\frac{1}{2}g^{\mu\nu}\mu^{2\epsilon}\Big[\frac{1}{\hat{\epsilon}}+2\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\log\left(\frac{a^2}{\mu^2}\right)\Big] - 2p_2^{\mu}p_1^{\nu}\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\ \frac{xy}{a^2}\right\} \tag{B.12}$$

with

$$\mathcal{C}_{\mu\nu} = -\frac{2T_F}{(4\pi)^2}\lambda_1 v g_s^2 f^{acd} f^{bcd}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2) \tag{B.13}$$

The amplitude of the second diagram, Fig. 23a, is clearly the same than the one of the first so we will have twice this contribution. Now we can calculate the third one, Fig. 23c:

$$i\mathcal{M}_c = -iT_F\lambda_1 v 2ig_s^2 f^{acd}f^{bcd}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)g^{\mu\nu}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m_S^2}\frac{i}{(k-p_1-p_2)^2-m_S^2}. \tag{B.14}$$

The integral can be performed in a similar way than the other one using the following Feynman parametrisation:

$$\frac{1}{AB} = \int_0^1 \mathrm{d}x\frac{1}{[Ax+B(1-x)]^2}. \tag{B.15}$$

Then the integral becomes:

$$\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2-m_S^2}\frac{1}{(k-p_1-p_2)^2-m_S^2} = \int_0^1\mathrm{d}x\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{[(k-x(p_1+p_2))^2-b^2]^2} = \int_0^1\mathrm{d}x\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{[k^2-b^2]}, \tag{B.16}$$

where we have changed of variables form k^μ to $k^\mu + x(p_1^\mu + p_2^\mu)$ and $b^2 = 2p_1\cdot p_2(x-1)x + m_S^2$.

Now we perform the integral on k using dimensional regularisation:

$$\int\frac{\mathrm{d}^Dk}{(2\pi)^D}\frac{1}{[k^2-b^2]} = \mathcal{J}(D,0,2,b^2) = \frac{-i}{(4\pi)^2}\mu^{2\epsilon}\left(\frac{1}{\hat{\epsilon}}+\log\left(\frac{b^2}{\mu^2}\right)\right), \tag{B.17}$$

where, like before, we set $D = 4 + 2\epsilon$.

The amplitude becomes, therefore,

$$\mathcal{M}_c = -\mathcal{C}_{\mu\nu}g^{\mu\nu}\mu^{2\epsilon}\left(\frac{1}{\hat{\epsilon}}+\int_0^1\mathrm{d}x\log\left(\frac{b^2}{\mu^2}\right)\right). \tag{B.18}$$

Now we can calculate the total amplitude:

$$2\mathcal{M}_a+\mathcal{M}_c = \mathcal{C}_{\mu\nu}\Big\{g^{\mu\nu}\mu^{2\epsilon}\Big[2\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\log\left(\frac{a^2}{\mu^2}\right) - \int_0^1\mathrm{d}x\log\left(\frac{b^2}{\mu^2}\right)\Big] - 4p_2^\mu p_1^\nu\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\ \frac{xy}{a^2}\Big\}. \tag{B.19}$$

Note that, as expected, the divergences cancel. This is necessary in order to have a renormalisable theory because we do not have any other structure in our Lagrangian that could cancel this divergence. Furthermore, thanks to the Ward identities we know that this amplitude must be proportional to $(p_1\cdot p_2 g^{\mu\nu} - p_1^\mu p_2^\nu)$. Let us simplify this equation in order to show it:

$$
\begin{aligned}
&2\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\log\left(\frac{a^2}{\mu^2}\right)-\int_0^1\mathrm{d}x\log\left(\frac{b^2}{\mu^2}\right)=2\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\Big[\cancel{\log\left(\frac{2p_1\cdot p_2}{\mu^2}\right)}+\log(n-xy)\Big]+\\
&-\int_0^1\mathrm{d}x\Big[\cancel{\log\left(\frac{2p_1\cdot p_2}{\mu^2}\right)}+\log(n-x(1-x))\Big]=2\int_0^1\mathrm{d}x\int_0^{x(1-x)}\mathrm{d}u\frac{\log(n-u)}{x}-\int_0^1\mathrm{d}x\log(n-x(1-x))=\\
&=2\int_0^1\mathrm{d}x\Big[-\frac{n}{x}\log\left(1-\frac{x(1-x)}{n}\right)+(x-1)+\cancel{(1-x)\log(n-x(1-x))}\Big]-\cancel{\int_0^1\mathrm{d}x\log(n-x(1-x))},
\end{aligned}
\tag{B.20}
$$

where $n=\frac{m_S}{2p_1\cdot p_2}$. The other integral becomes:

$$
-4\int_0^1\mathrm{d}x\int_0^{1-x}\mathrm{d}y\;\frac{xy}{a^2}=\frac{-4}{2p_1\cdot p_2}\int_0^1\mathrm{d}x\int_0^{x(1-x)}\mathrm{d}u\;\frac{1}{x}\frac{u}{n-u}=\frac{4}{2p_1\cdot p_2}\int_0^1\mathrm{d}x\Big[(1-x)+\frac{n}{x}\log\left(1-\frac{x(1-x)}{n}\right)\Big],
\tag{B.21}
$$

in such a way that the total amplitude is, indeed, proportional to $(p_1\cdot p_2 g^{\mu\nu}-p_1^\mu p_2^\nu)$.

In order to calculate the last integral,

$$
I=\int_0^1\mathrm{d}x\frac{1}{x}\log\left(1-\frac{x(1-x)}{n}\right),
\tag{B.22}
$$

is convenient to define $\xi=1/n$. This integral will have poles when $\xi>4$ so first we will calculate it for $\xi<4$. This integral can be performed derivating and integrating with respect to ξ

$$
\frac{\mathrm{d}I}{\mathrm{d}\xi}=\int_0^1\mathrm{d}x\frac{x-1}{1-\xi x(1-x)}=\frac{-2}{\sqrt{\xi(4-\xi)}}\arcsin\left(\frac{\sqrt{\xi}}{2}\right)\Rightarrow I=-2\arcsin^2(\sqrt{\xi}/2)
\tag{B.23}
$$

Now let us perform the integral for $\xi>4$. As this has poles we should recover the factor $i\epsilon$ of the propagator, basically where we have a mass square we have to add a term $-i\epsilon$. We will do the integration by parts first of all to obtain:

$$
\begin{aligned}
I&=\xi\int_0^1\mathrm{d}x\frac{1-2x}{1-\xi x(1-x)-i\epsilon}\log x=\int_0^1\mathrm{d}x\frac{1-2x}{(x-x_-+i\epsilon)(x-x_+-i\epsilon)}\log x=\\
&=\int_0^1\mathrm{d}x\frac{1-2x}{\beta}\log x\left(\frac{1}{x-x_+-i\epsilon}-\frac{1}{x-x_-+i\epsilon}\right)=\mathcal{P}\left(\int_0^1\mathrm{d}x\frac{1-2x}{\beta}\log x\Big(\frac{1}{x-x_+}-\frac{1}{x-x_-}\Big)\right)\\
&+\frac{i\pi}{\beta}\int_0^1\mathrm{d}x(\delta(x-x_+)+\delta(x-x_-))(1-2x)\log x=\mathcal{P}\big(I\big)-\frac{i\pi}{\beta}\log\left(\frac{1+\beta}{1-\beta}\right),
\end{aligned}
\tag{B.24}
$$

where we have used the Sokhotski-Plemelj theorem,

$$
\lim_{\epsilon\to0^+}\int\frac{f(x)}{x\pm i\epsilon}\mathrm{d}x=\mathcal{P}\left(\int\frac{f(x)}{x}\mathrm{d}x\right)\mp i\pi\int f(x)\delta(x)\mathrm{d}x,
\tag{B.25}
$$

and

$$
x_\pm=\frac{1}{2}(1\pm\sqrt{1-4/\xi})=\frac{1}{2}(1\pm\beta).
\tag{B.26}
$$

Finally we solve the last remaining integral in a similar way than for the case without poles:

$$\mathcal{P}\left(\frac{\mathrm{d}I}{\mathrm{d}\xi}\right) = \mathcal{P}\left(\int_0^1 \mathrm{d}x \frac{x-1}{1-\xi x(1-x)}\right) = \mathcal{P}\left(\frac{-1}{2\beta\xi}\int_0^1 \mathrm{d}x \left(\frac{1}{x-x_+} - \frac{1}{x-x_-}\right)\right) = \frac{1}{\beta\xi}\log\left(\frac{1+\beta}{1-\beta}\right) \Rightarrow$$
$$\Rightarrow I = \int \mathrm{d}\beta \frac{\xi^2\beta}{2}\frac{1}{\beta\xi}\log\left(\frac{1+\beta}{1-\beta}\right) = \frac{1}{2}\log^2\left(\frac{1+\beta}{1-\beta}\right) - \frac{1}{2}\pi^2, \tag{B.27}$$

where we have set the integration constant to $-\frac{1}{2}\pi^2$ in order to have a continuous function.

With this the total amplitude becomes:

$$2\mathcal{M}_a + \mathcal{M}_c = \mathcal{C}_{\mu\nu}(1 + 2n\mathcal{F}(n))\left(-g^{\mu\nu} + \frac{1}{p_1\cdot p_2}p_2^\mu p_1^\nu\right), \tag{B.28}$$

with

$$\mathcal{F}(n) = \begin{cases} \frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-4n}}{1-\sqrt{1-4n}}\right) - i\pi\right]^2 & n < 1/4 \\ -2\arcsin^2(1/\sqrt{4n}) & n > 1/4 \end{cases}. \tag{B.29}$$

Remember that we have to add the contributions for the neutral scalar colour octets. If we consider that the scalar particles have the same mass we obtain

$$2\mathcal{M}_a + \mathcal{M}_c = -\frac{2T_F}{(4\pi)^2}(2\lambda_1+\lambda_2)vg_s^2 C_A\delta_{ab}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)(1+2n\mathcal{F}(n))\left(-g^{\mu\nu} + \frac{1}{p_1\cdot p_2}p_2^\mu p_1^\nu\right). \tag{B.30}$$

The cross section will be:

$$\sigma = \int \mathrm{d}Q_1 \frac{\sum|\overline{\mathcal{M}}|^2}{2\sqrt{\lambda(s,0,0)}} = \frac{\pi\sum|\overline{\mathcal{M}}|^2}{s}\delta(s-m_h^2) = \frac{\pi\sum|\mathcal{M}|^2}{256s}\delta(s-m_h^2) \tag{B.31}$$

with

$$|2\mathcal{M}_a + \mathcal{M}_c|^2 = \frac{T_F^2}{2}\left(\frac{\alpha_s}{\pi}\right)^2(2\lambda_1+\lambda_2)^2 v^2 C_A^2(N^2-1)|1+2n\mathcal{F}(n)|^2. \tag{B.32}$$

If we make the approximation for scalars with a very high mass with respect to the Higgs mass, i. e. we expand the function $\mathcal{F}(n)$ in $n\to\infty$, and we move to the centre of mass frame we get:

$$2\mathcal{M}_a + \mathcal{M}_c \approx -\frac{2T_F}{(4\pi)^2}(2\lambda_1+\lambda_2)vg_s^2 C_A\delta_{ab}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)\frac{m_H^2}{12m_S^2}\left(g^{\mu\nu} - \frac{2}{m_H^2}p_2^\mu p_1^\nu\right). \tag{B.33}$$

B.2 Gluon Fusion Mediated By Top-Quark

The amplitude of the first diagram will be

$$\mathcal{M}_a = -ig_s^2\frac{m_t}{v}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)T_{\alpha\beta}^a T_{\beta\alpha}^b\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left[\gamma^\mu(\not{k}+\not{p}_1+m_t)(\not{k}-\not{p}_2+m_t)\gamma^\nu(\not{k}+m_t)\right]}{[k^2-m_t^2][(k^2+p_1)^2-m_t^2][(k-p_2)^2-m_t^2]}, \tag{B.34}$$

where we can expand the trace in order to obtain

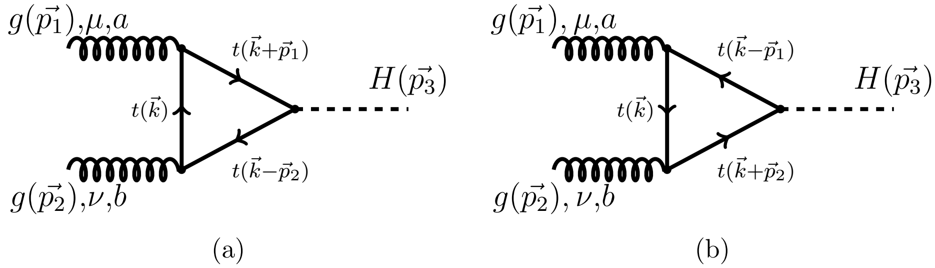


Figure 24: Production of SM Higgs through gluon fusion mediated by top quarks.

$$\begin{aligned}
&\mathrm{Tr}\Big[\gamma^\mu(\not{k}+\not{p}_1+m_t)(\not{k}-\not{p}_2+m_t)\gamma^\nu(\not{k}+m_t)\Big] = \\
&= 4m_t(p_2^\mu p_1^\nu + 4k^\mu k^\nu - 2k^\mu p_2^\nu + 2p_1^\mu k^\nu - p_1^\mu p_2^\nu + g^{\mu\nu}(m_t^2 - p_1\cdot p_2) - g^{\mu\nu}k^2) \Rightarrow \\
&\Rightarrow 4m_t(p_2^\mu p_1^\nu + 4k^\mu k^\nu + g^{\mu\nu}(m_t^2 - p_1\cdot p_2) - g^{\mu\nu}k^2), \qquad \text{(B.35)}
\end{aligned}$$

where in the last step we have eliminated the terms that will vanish due to the transversality of the gluons $\epsilon_\mu^a(p_1)p_1^\mu = 0$. With this the amplitude becomes

$$\mathcal{M}_a = -ig_s^2\frac{m_t}{v}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)T_F\delta_{ab}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{4m_t(p_2^\mu p_1^\nu + 4k^\mu k^\nu + g^{\mu\nu}(m_t^2 - p_1\cdot p_2) - g^{\mu\nu}k^2)}{[k^2-m_t^2][(k^2+p_1)^2-m_t^2][(k-p_2)^2-m_t^2]}. \qquad \text{(B.36)}$$

In order to perform these integrals we proceeded like in the previous section: first we use the Feynman parametrisation and after that we use dimensional regularisation. Using this and taking into account that the second diagram is equal to the first one we obtain

$$\mathcal{M} = \frac{g_s^2}{4v\pi^2}\epsilon_\mu^a\epsilon_\nu^b\delta_{ab}n_t(-2+(1-4n_t)\mathcal{F}(n_t))(p_2^\mu p_1^\nu - g^{\mu\nu}p_1\cdot p_2) \qquad \text{(B.37)}$$

with $n_t = \frac{m_t^2}{2p_1\cdot p_2}$, in the CM:

$$|\mathcal{M}|^2 = \frac{4m_t^4}{v^2}\left(\frac{\alpha_s}{\pi}\right)^2|-2+(1-4n_t)\mathcal{F}(n_t)|^2 \qquad \text{(B.38)}$$

B.3 Higgs Decaying to Two Photons

Now let us calculate the contributions of our particles to the process of the Higgs boson decaying to two photons. As we can see in Fig. 25 the diagrams are extremely similar to the ones of the gluon fusion mediated by the colour octet scalars, although this time only the charged scalar contributes to the process. The result will be, therefore, very similar, the only difference is that this time we do not have the $SU(3)$ structure constants (f^{abc}), we just have a delta in colour space, and instead of having the strong coupling (g_s) we have the electromagnetic coupling (e). Therefore regarding the couplings the only difference is a factor $(N^2-1)/C_A$ because we change $\delta_{ab}C_A$ by $\delta_{aa} = N^2-1 = 8$:

$$\mathcal{M} = -\frac{16T_F}{(4\pi)^2}\lambda_1 v e^2\epsilon_\mu(p_2)\epsilon_\nu(p_3)\Big(1+2n\mathcal{F}(n)\Big)\left(-g^{\mu\nu}+\frac{1}{p_2\cdot p_3}p_3^\mu p_2^\nu\right), \qquad \text{(B.39)}$$

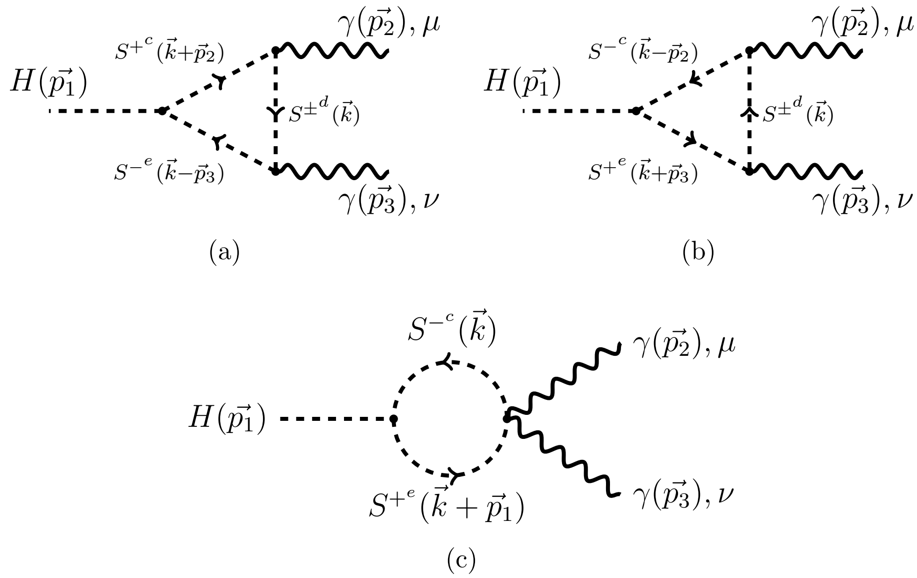


Figure 25: Higgs decaying to two photons.

where again $n = \frac{m_S}{2p_1 \cdot p_2}$.

Like in section B.1 we can make the approximation for scalars heavy compared to the Higgs boson and we get

$$\mathcal{M} \approx -\frac{16T_F}{(4\pi)^2}\lambda_1 v e^2 \epsilon_\mu(p_1)\epsilon_\nu(p_2)\frac{m_H^2}{12m_S^2}\left(g^{\mu\nu} - \frac{2}{m_H^2}p_2^\mu p_1^\nu\right). \tag{B.40}$$

And the decay width will be

$$\Gamma = \frac{1}{2}\frac{\sqrt{\lambda(m_H^2,0,0)}}{64\pi^2 m_H^3}4\pi|\mathcal{M}|^2 = \frac{T_F^2\lambda_1^2 v^2\alpha^2}{\pi^3 m_H}|1+2n\mathcal{F}(n)|^2. \tag{B.41}$$

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