K. R. Schubert, TU Dresden

IMFP04 Alicante, 1-5 March 2004

Four Lectures on „Quark Mixing and CP Violation“

Standard Model for the Charged Weak Interaction

Measurements of CKM Matrix Elements

Four Different Types of CP Violation

Results on CP Violation in K- and B-Meson Decays

Tests of CPT Invariance

The Standard Charged Weak Interaction

Standard Model of Elementary Particle Physics

= Extension of Dirac's equation
by the gauge principle
and by „zoology".

Dirac 1928:

$$L = \bar{e}\,\gamma^{\mu} i\partial_{\mu} e - m\,\bar{e}\,e \quad ;$$

$$e = e(x) = (e_L^-, e_R^-, e_L^+, e_R^+)$$

Weyl 1929:

$$e(x) \rightarrow e(x)\cdot e^{i\alpha(x)}$$

$$\Rightarrow \quad i\partial_{\mu} \rightarrow i\partial_{\mu} - q\,A_{\mu}$$

Paul Adrien Maurice Dirac
(1902 – 1984)

Glashow 1961:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \text{ and } e_R \text{ instead of } e, \quad \begin{pmatrix} \nu_L(x) \\ e_L(x) \end{pmatrix} \to U(x) \begin{pmatrix} \nu_L(x) \\ e_L(x) \end{pmatrix} e^{i\alpha(x)}$$

$$\Rightarrow \quad i\partial_\mu \to i\partial_\mu - g\, W_\mu^a \frac{\sigma_a}{2} - g'\, y\, B_\mu$$

Invariance of Dirac equation under local SU(2) and U(1) produces weak and electric interaction through exchange of W^+, W^-, γ and Z^0.

However, invariances require,

$$m(\gamma) = m(W) = m(Z) = 0$$

and, because of $e = e_R + e_L$, $m\,\bar{e}e \not\mapsto m\,\bar{e}e$ in $\boldsymbol{L}_{\text{Dirac}}$:

$$m(e) = 0.$$

Higgs 1964: Introduction of a spin-0-doublet Φ and spontanuous symmetry breaking

$$L \to L - c\left[\bar{e}_R \begin{pmatrix}\Phi_1^* & \Phi_2^*\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix} + \begin{pmatrix}\bar{\nu}_L & \bar{e}_L\end{pmatrix}\begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix} e_R\right]$$

$$\begin{pmatrix}\Phi_1(x) \\ \Phi_2(x)\end{pmatrix} \to \begin{pmatrix}0 \\ \mathrm{v} + h(x)\end{pmatrix} \Rightarrow L \to L - c\,\mathrm{v}\left(\bar{e}_R e_L + \bar{e}_L e_R\right) + L_{\mathrm{int}}(h,e)$$

$$c\,\mathrm{v}\left(\bar{e}_R e_L + \bar{e}_L e_R\right) = m\,\bar{e}e$$

Higgs returns Dirac's electron mass which Glashow removed.

Glashow, Weinberg, Salam 1968:

Same Higgs mechanism produces masses for μ lepton and quarks u d c s, and gives quark mixing in W exchange.

Kobayashi, Maskawa 1973: Extension from 2 to 3 families

(discovery of τ-lepton 1975 , c-quark 1974, b 1977)

$$L = L_{Dirac,kin}(3l,3\nu,3u,3d) + L_{U1\otimes SU2\otimes SU3} + \ldots$$

$$-\left[C^{(l)}_{\alpha\beta}\, \bar{l}_{R\alpha}\, \Phi^{+} \begin{pmatrix} \nu_{L\beta} \\ l_{L\beta} \end{pmatrix} + C^{(d)}_{\alpha\beta}\, \bar{d}_{R\alpha}\, \Phi^{+} \begin{pmatrix} u_{L\beta} \\ d_{L\beta} \end{pmatrix} + C^{(u)}_{\alpha\beta}\, \bar{u}_{R\alpha}\, \Phi^{T} \varepsilon \begin{pmatrix} u_{L\beta} \\ d_{L\beta} \end{pmatrix} + h.c. \right]$$

Gauge invariance allows 3 arbitrary complex matrices C.

$C^{(u)}$ can be diagonalized by rotating $u_{L\alpha}$ and $u_{R\alpha}$:

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \rightarrow \begin{pmatrix} u \\ c \\ t \end{pmatrix}; \quad C^{(u)}_{\alpha\beta} \cdot v \rightarrow \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}; \quad \text{Doublets} \rightarrow \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \begin{pmatrix} t_L \\ b'_L \end{pmatrix}$$

Doublet partners d‘ s‘ b‘ of u c t are not mass eigenstates.

$C^{(d)}$ can be diagonalized by rotating $d_{L\alpha}$ and $d_{R\alpha}$:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \to \begin{pmatrix} d \\ s \\ b \end{pmatrix}; \quad C^{(d)}_{\alpha\beta} \cdot \mathrm{v} \to \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}; \quad \textit{Doublets} \to \begin{pmatrix} u'_L \\ d_L \end{pmatrix} \begin{pmatrix} c'_L \\ s_L \end{pmatrix} \begin{pmatrix} t'_L \\ b_L \end{pmatrix}$$

Dublet partners u‘ c‘ t‘ of d s b are not mass eigenstates.

W exchange $u_L \leftrightarrow W^+ d'_L$, $c_L \leftrightarrow W^+ s'_L$, $t_L \leftrightarrow W^+ b'_L$, $u'_L \leftrightarrow W^+ d_L$ ...

Difference between the 2 diagonalizing rotations is V_{CKM} :

$$\begin{pmatrix} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \\ \begin{pmatrix} c_L \\ s'_L \end{pmatrix} \\ \begin{pmatrix} t_L \\ b'_L \end{pmatrix} \end{pmatrix} = V \begin{pmatrix} \begin{pmatrix} u'_L \\ d_L \end{pmatrix} \\ \begin{pmatrix} c'_L \\ s_L \end{pmatrix} \\ \begin{pmatrix} t'_L \\ b_L \end{pmatrix} \end{pmatrix}; \qquad V = V_{\mathrm{CKM}}$$

$$\text{with} \quad V\,V^+ = 1$$

Introduced 1973 by M. Kobayashi and T. Maskawa, two-family mixing already 1963 by N. Cabibbo, named V_{CKM} since 1987.

Properties of V_{CKM}:

$$\begin{aligned} d' &= V_{ud} \cdot d + V_{us} \cdot s + V_{ub} \cdot b \\ s' &= V_{cd} \cdot d + V_{cs} \cdot s + V_{cb} \cdot b \\ b' &= V_{td} \cdot d + V_{ts} \cdot s + V_{tb} \cdot b \end{aligned}$$

V_{ik} is complex.

CPT theorem:

$$\begin{aligned} \bar{d}' &= V_{ud}^* \cdot \bar{d} + V_{us}^* \cdot \bar{s} + V_{ub}^* \cdot \bar{b} \\ \bar{s}' &= V_{cd}^* \cdot \bar{d} + V_{cs}^* \cdot \bar{s} + V_{cb}^* \cdot \bar{b} \\ \bar{b}' &= V_{td}^* \cdot \bar{d} + V_{ts}^* \cdot \bar{s} + V_{tb}^* \cdot \bar{b} \end{aligned}$$

Higgs mixes antiquarks in a different way than quarks.

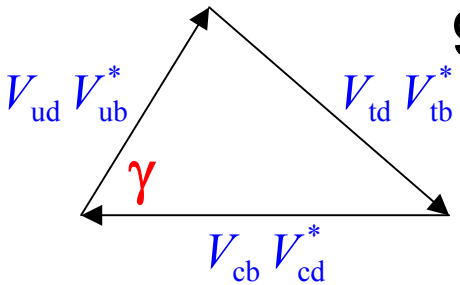
Origin of CP violation in the Standard Model, if $V_{ik} \neq V_{ik}^*$.

Phases of udcstb are unobservable (gauge priciple) $\Rightarrow$

$V_{ij} \to V_{ij}\, e^{i(\varphi_i - \varphi_j)}$ are unobservable. Invariants are:

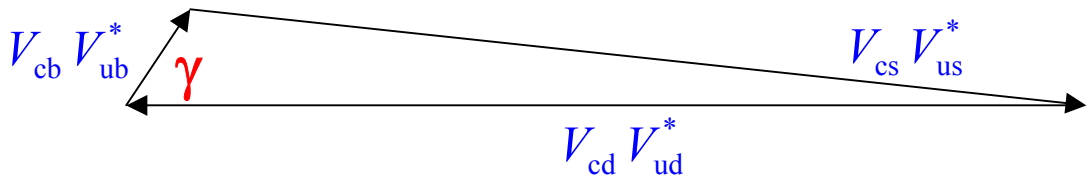
1) $|V_{ud}|^2$ 2) $V_{td} V_{tb}^* V_{cb} V_{cd}^*$ 3) $V_{td} V_{tb}^* V_{cb} V_{cs}^* V_{us} V_{ud}^*$ ...

Moduli, Re & Im of quartets, sextets ... [sin2β = Im(12-plet)]



9 moduli, 9 quartets, ... , 6 unitarity triangles,

18 angles, only 9 of them different.



$\gamma = \phi\left(-V_{ub}^* V_{ud} V_{cd}^* V_{cb}\right)$

V has $2*9 - 9_{VV^+=1} - 5_{\varphi_i - \varphi_j} = 4$ observable parameters

and an infinity of choices for these 4. Wolfenstein 1983:

V_{ud}, V_{us}, V_{cs}, V_{cb}, V_{tb} real and positive, parameters λ, A, ρ, η,

$|V_{us}| = \lambda$, $|V_{cb}| = A\lambda^2$, $|V_{ub}|\cos\gamma = A\lambda^3\rho$, $|V_{ub}|\sin\gamma = A\lambda^3\eta$.

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}; \qquad \begin{aligned} \bar{\rho} &= (1-\lambda^2/2)\rho, \\ \bar{\eta} &= (1-\lambda^2/2)\eta, \\ J &\approx A^2\lambda^6\,\bar{\eta}\,. \end{aligned}$$

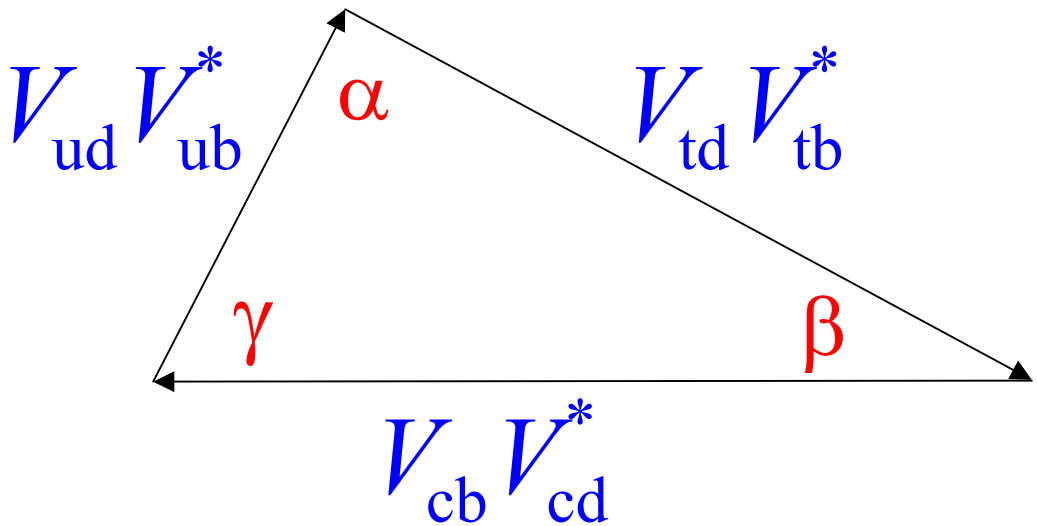
All 9 quartets have same imaginary part ***J***. $J \neq 0 \Leftrightarrow$ CPV.

Wolfenstein's λ, A, ρ, η have been very useful for 20 years.

Today we know $A\lambda^2$ better than A, and bands in the $\bar{\rho},\bar{\eta}$ plane start to be wider than in $A\lambda^3\,\bar{\rho}$, $A\lambda^3\,\bar{\eta}$. I advocate:

C, K, M, γ with $|V_{us}|=C$, $|V_{cb}|=K$, $|V_{ub}|=M$, $\gamma = \phi\left(-V_{ub}^* V_{ud} V_{cd}^* V_{cb}\right)$

$$V \approx \begin{pmatrix} 1-C^2/2 & C & M\cdot e^{-i\gamma} \\ -C & 1-C^2/2 & K \\ CK - \bar{M}\cdot e^{i\gamma} & -K & 1 \end{pmatrix}; \qquad \begin{aligned} \bar{M} &= (1-C^2/2)M, \\ J &\approx C\,K\,\bar{M}\sin\gamma. \end{aligned}$$


$V_{ud}V_{ub}^*$
α
$V_{td}V_{tb}^*$
γ
β
$V_{cb}V_{cd}^*$

Measurements of CKM Matrix Elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Three sources:

Super-allowed nuclear β^+-decays

β^- decays of polarized neutrons

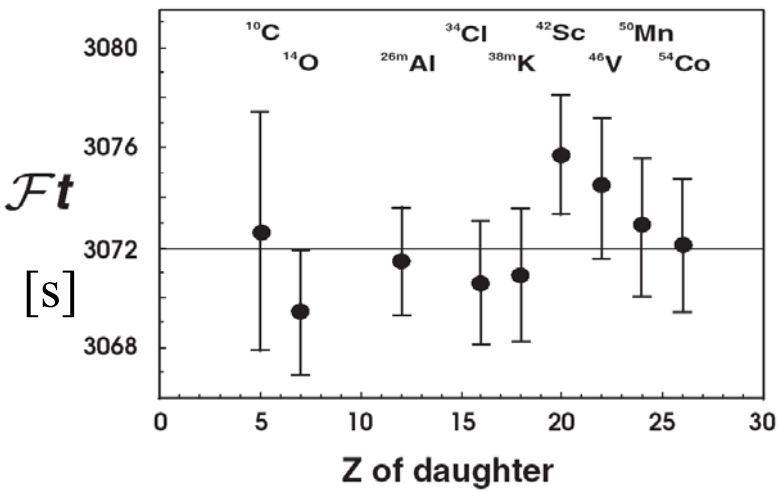
β^+ decay of the π^+

Super-allowed nuclear β^+-decays:

$0^+ \rightarrow 0^+$ nuclear transitions within same isospin multiplet, pure V.

I. S. Towner and J. C. Hardy 2003: $Ft = (3072.2 \pm 0.9 \pm 1.1)\ s$

fit error and estimated error on δ_C



$$|V_{ud}|^2 = \frac{2\,\pi^3\,\ln 2}{m_e^{\,5}} \cdot \frac{1}{2\,G_F^2\,(1+\Delta_{RV})\,Ft},$$

G_F from μ decay, Δ_{RV} = (2.40 ± 0.08)%

$|V_{ud}| = 0.9740 \pm 0.0001_{ft,exp} \pm 0.0004_{\Delta} \pm 0.0003_{Ft/ft}$

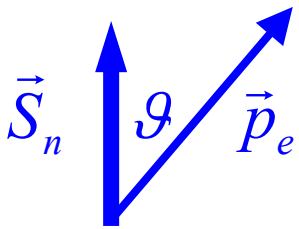
$$Ft = f \cdot t_{1/2} \cdot (1+\delta_R') \cdot (1+\delta_{NS} - \delta_C)$$

$|V_{ud}|_{Nucl} = 0.9740 \pm 0.0005$

β^- decays of polarized neutrons:

V and A transition because of $1/2^+ \rightarrow 1/2^+$. $G_V = G_F |V_{ud}|$,

but j_A only partially conserved. Determine $G_A/G_V = \lambda$ experimentally!



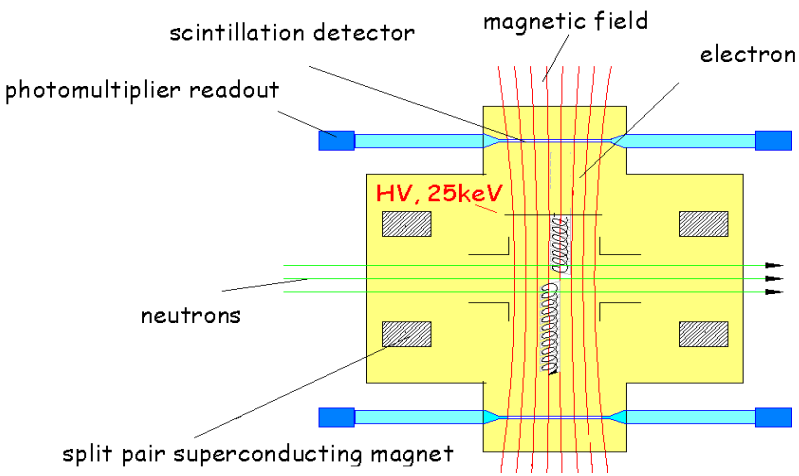
$$W(\vartheta) = 1 + \frac{v}{c} \cdot P \cdot A \cdot \cos\vartheta\,, \quad A = \frac{-2\lambda(\lambda+1)}{1+3\lambda^2} + o(1\%)$$

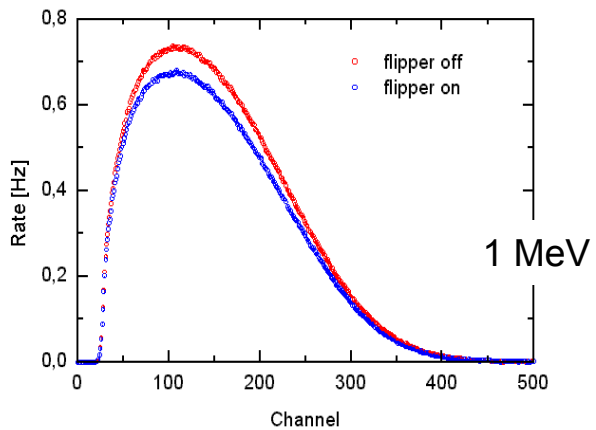
$$\frac{1}{\tau_n} = \frac{m_e^{\,5} \cdot G_F^2 \cdot |V_{ud}|^2}{2\,\pi^3} \cdot (1+3\lambda^2) \cdot f\,(1+\delta_R) \cdot (1+\Delta_{RV})\,.$$

Most recent experiment is PERKEO-II at ILL Grenoble;

neutrons of 25 K with P = $(98.9 \pm 0.3)\%$,

periodical spin-flip. e-detector:





$|V_{ud}|_n = 0.9717 \pm 0.0013$

β decay of π^+:

Recent experiment PIBETA at PSI; stopped π^+,

$\pi^+ \rightarrow \pi^0 e^+ \nu_e$, normalisation with $e^+ \nu_e$. Preliminary result:

$BF(\pi^+ \rightarrow \pi^0 e^+ \nu_e) = (1.044 \pm 0.007 \pm 0.009)\ 10^{-8}$. $|V_{ud}|_{\pi} = 0.9765 \pm 0.0056$

Summary of V_{ud}:

$|V_{ud}|_{Nucl} = 0.9740 \pm 0.0005$ (1)

$|V_{ud}|_{n} = 0.9741 \pm 0.0020$ S = 2.0 (2)

PERKEO-II alone: $|V_{ud}|_{n} = 0.9717 \pm 0.0013$ (3)

$|V_{ud}|_{\pi} = 0.9765 \pm 0.0056$ (4)

Average of (1) + (3) + (4) with S =1.3 : $|V_{ud}| = 0.9737 \pm 0.0007$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Sources: Old K_{l3} decays, new K_{l3} decays, hyperon β-decays, τ decays

K_{l3} decays: $K^+ \to \pi^0 l^+ \nu_e$ and $K^0 \to \pi^- l^+ \nu_e$, $0^- \to 0^-$, pure V

$$\Gamma_i = \frac{m_K^5 \cdot G_F^2 \cdot |V_{us}|^2}{192\,\pi^3} \cdot C_i^2 \cdot |f_+(0)|^2 \cdot I_i(\lambda_+, \lambda_0) \cdot (1+\delta_R), \quad f_{+,0}(q^2) = f_+(0) \cdot \left(1 + \lambda_{+,0} \frac{q^2}{m_\pi^2} + \ldots\right)$$

$$C_i = 1/\sqrt{2} \text{ for } K^+ \text{ and } K_L^0$$

Mode	BR (%)	$10^3\lambda_+$	$10^3\lambda_0$
K^+_{e3}	4.87 ± 0.06	27.8 ± 1.9	
K^0_{e3}	38.79 ± 0.27	29.1 ± 1.8	
$K^+_{\mu 3}$	3.27 ± 0.06	33 ± 10	4 ± 9
$K^0_{\mu 3}$	27.18 ± 0.25	33 ± 5	27 ± 6

$$\tau_{K^\pm} = (1.2384 \pm 0.0024) \times 10^{-8}\, s$$
$$\tau_{K_L} = (5.17 \pm 0.04) \times 10^{-8}\, s$$

⇐ Exp. input since ~1980.

Leutwyler and Roos 1984:

$$f_+^{K^0}(0) = 0.961\,, \; f_+^{K^+}(0) = 0.982\,.$$

Cirigliano 2003:

$|V_{us}|$ old Kl3 = 0.2201 ± 0.0024

New K_{l3} results:

Final K^+_{e3} result of BNL-E865, hep-ex/0305042

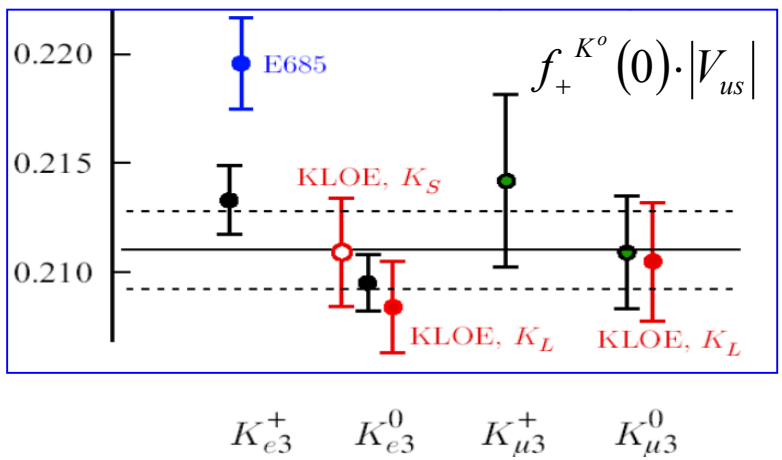
Preliminary KLOE results on three K^0_{l3} rates

BNL-E865: Aim $K^+ \to \pi^+\mu^+e^-$. Dedicated run for $K^+ \to \pi^0e^+\nu_e$ during one week 1998, $\pi^0 \to e^+e^-\gamma$. Normalised to $K^+ \to \pi^+\pi^0$, $\pi^+\pi^0\pi^0$. Result:

$$BF(K^+ \to \pi^0e^+\nu_e) = (5.13 \pm 0.02 \pm 0.09 \pm 0.04)\%$$

$$= (1.053 \pm 0.024) \cdot BF(\text{old}),\ 2.2\sigma$$

KLOE: $e^+e^- \to \phi(1020) \to K_SK_L$. hep-ex/0307016 gives preliminary K^0_{l3} results from 78 / pb. Values only given in this plot :



Tau decays: $\Gamma(\tau \to K\ n\pi\ \nu) / \Gamma(\tau \to \text{had}\ \nu)$ is sensitive to $|V_{us}|$.

Using ALEPH data and $m_s(2\ \text{GeV}) = (105 \pm 20)$ MeV, Gamiz et al find

$$|V_{us}|_\tau = 0.2179 \pm 0.0044_{exp} \pm 0.0009_{th}$$

Dominant error is experimental. Better measurements of rates and mass moments would give both m_s and $|V_{us}|$.

Summary of V_{us}: from old K_{l3}, E865, and τ decays:

$$|V_{us}| = 0.2210 \pm 0.0023$$

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Four Lectures on „Quark Mixing and CP Violation“

Second Lecture

Measurements of CKM Matrix Elements

Quarks: Higgs creates mass eigenstates d s b u c t

with Glahow partners u‘ c‘ t‘ d‘ s‘ b‘

Local SU(2) invariance creates g(u W d‘) with

$d' = V_{ud}\, d + V_{us}\, s + V_{ub}\, b$

Coherence length is very short, *o*(hadron sizes)

We observe d s b with probabilities $|V_{ud}|^2$ $|V_{us}|^2$ $|V_{ub}|^2$

Leptons: Higgs (?) creates mass eigenstates e μ τ ν_1 ν_2 ν_3

with Glahow partners ν_e ν_μ ν_τ e‘ μ‘ τ‘

Local SU(2) invariance creates g(e W ν_e) with

$\nu_e = U_{e1}\, \nu_1 + U_{e2}\, \nu_2 + U_{e3}\, \nu_3$

Coherence length is very long, *o*(galactic sizes)

We do not observe ν_1 ν_2 ν_3 , we detect λ = e μ τ

with probabilities $|\Sigma\, U_{e\alpha}\, U_{\lambda\alpha}{}^{*} \exp\,[i\, m(\nu_\alpha)^2\, L/E]\,|^2$

Quark oscillations?

Yes, but only at hadron distances, $d\bar{s} \rightarrow s\bar{d}$, $d\bar{b} \rightarrow b\bar{d}$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Old source: Production of dimuon pairs by neutrinos on nuclei

$$\nu_\mu\, d \to \mu^-\, c\,, \quad c \to s\, \mu^+\, \nu_\mu$$
$$\bar{\nu}_\mu\, \bar{d} \to \mu^+\, \bar{c}\,, \quad \bar{c} \to \bar{s}\, \mu^-\, \bar{\nu}_\mu$$

with $\boldsymbol{B} = \boldsymbol{BF}(c \to \mu^+ X)$:

$$\frac{\sigma(\nu \to \mu^+\mu^-) - \sigma(\bar{\nu} \to \mu^+\mu^-)}{\sigma(\nu \to \mu^-) - \sigma(\bar{\nu} \to \mu^+)} = \frac{3}{2} \cdot B \cdot |V_{cd}|^2$$

= (0.41 ± 0.07) % CDHS 1982

$= (0.534 \pm 0.021\,{}^{+0.025}_{-0.051})\%$ CCFR 1995

$\boldsymbol{B} = 0.099 \pm 0.012 \Rightarrow$ $|V_{cd}| = 0.224 \pm 0.016$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Sources: (1) β decays of D mesons and
(2) decays of real W ±

$D^+ \to \bar{K}^0 e^+ \nu$ and $D^0 \to K^- e^+ \nu$: $\Gamma = \frac{B}{\tau} = \frac{G_F^2 \cdot |V_{cs}|^2}{192\pi^3} \cdot \Phi \cdot |f(0)|^2 \cdot (1 + \delta_R)$

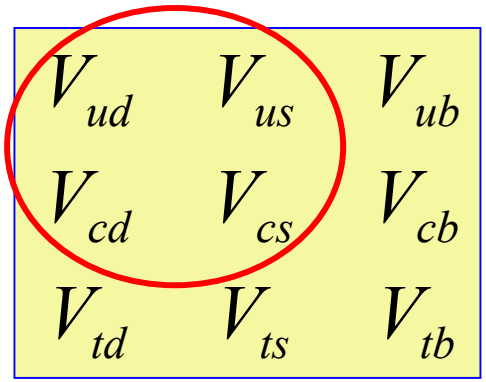
Constituent quark models and QCD sumrules:

$f(0) = 0.7 \pm 0.1$

$\Rightarrow$ $|V_{cs}| = 1.04 \pm 0.16$

2nd method, much more precise, needs 3rd quark family, will be discussed later.

Unitarity check of the udsc matrix:



Inputs: $|V_{ud}| = 0.9737 \pm 0.0007$

$|V_{us}| = 0.2210 \pm 0.0023$

$|V_{cd}| = 0.224 \pm 0.016$

$|V_{cs}| = 1.04 \pm 0.16$

$\|V_{ud}\|^2 + \|V_{us}\|^2$	$= 0.9969 \pm 0.0017$	$-1.8\ \sigma$
$\|V_{cd}\|^2 + \|V_{cs}\|^2$	$= 1.13 \pm 0.33$	$+0.4\ \sigma$
$\|V_{ud}\|^2 + \|V_{cd}\|^2$	$= 0.9983 \pm 0.0073$	$-0.2\ \sigma$
$\|V_{us}\|^2 + \|V_{cs}\|^2$	$= 1.13 \pm 0.33$	$+0.4\ \sigma$
$\|V_{ud}V_{cd}\| - \|V_{us}V_{cs}\|$	$= -0.012 \pm 0.039$	$0.3\ \sigma$
$\|V_{ud}V_{us}\| - \|V_{cd}V_{cs}\|$	$= -0.018 \pm 0.040$	$0.4\ \sigma$

From this unitarity check we cannot predict a 3rd family. Fit:

$C = \lambda = 0.2235 \pm 0.0033$

(S = 1.8)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Source: Inclusive and exclusive semileptonic B-meson decays

Inclusive decays:

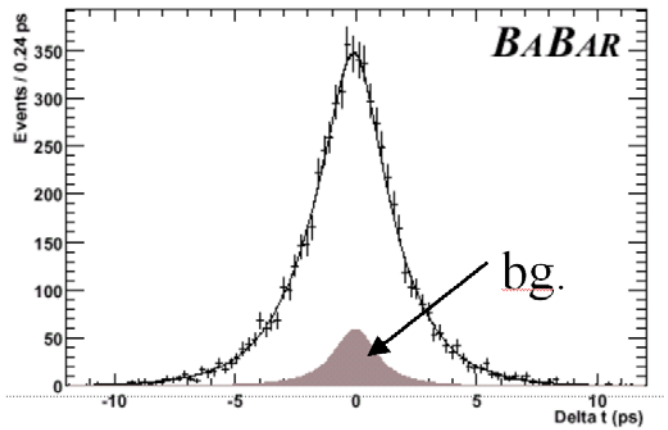
$$\Gamma(B \to l\nu\, X) = \Gamma(b \to l\nu\, c) + \Gamma(b \to l\nu\, u) + o\left(\Lambda_{QCD}^2 / m_b^2\right)$$

Primary information:

$\tau(B^0)$	1.534 +- 0.013 ps
$\tau(B^+)$	1.653 +- 0.014 ps

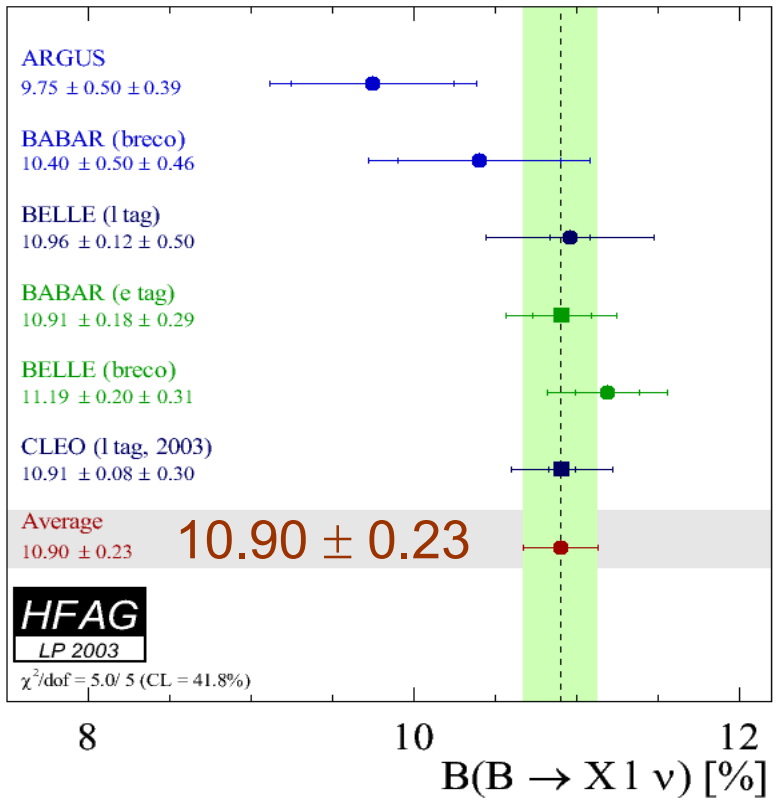
HFAG 07/03,

data from ALEPH, BABAR, BELLE, CDF, DELPHI, L3, OPAL, SLD

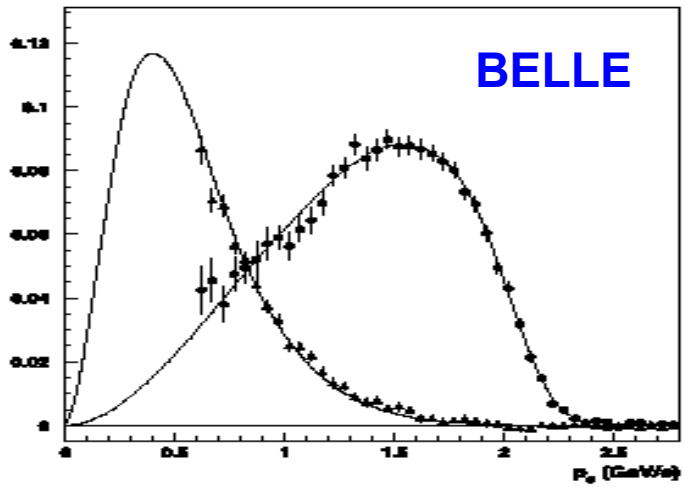


$$\Gamma(b \to e\nu\, X) = \frac{BF(b \to e\nu\, X)}{\tau(b)} = \frac{G_F^2 \cdot m_b^5}{192\,\pi^3} \cdot \left(\Phi\left(\frac{m_u}{m_b}\right) \cdot |V_{ub}|^2 + \Phi\left(\frac{m_c}{m_b}\right) \cdot |V_{cb}|^2 \right) + c(\alpha, \alpha_s)$$

Next information $\boldsymbol{BF}$(B → lν X):



Spectrum of signal leptons:



Integral over spectrum $\Rightarrow$ ***BF***,

98% b $\rightarrow$ c, 2% b $\rightarrow$ u,

σ (V_{cb}) from BF = 1.1%, σ (V_{cb}) from τ = 0.4%.

OPE of heavy quark effective QCD gives:

$$\frac{d^3\Gamma(B \to l\nu\, X_c)}{dm_X^2\, dE_l\, dq^2} = f\left(m_X^2, E_l, q^2 \middle| m_B, |V_{cb}|, \alpha_s, \overline{\Lambda}, \lambda_1, \lambda_2, \ldots\right)$$

Of special interest are the moments M_{ijk} of f. M_{000}:

$$\Gamma(B \to l\nu\, X_c) = \frac{G_F^2 |V_{cb}|^2}{192\,\pi^3} \cdot 0.369 \cdot m_B^5 \cdot \left[1 - 1.54\frac{\alpha_s}{\pi} - 1.65\frac{\overline{\Lambda}}{m_B}\left(1 - 0.87\frac{\alpha_s}{\pi}\right) - 0.95\frac{\overline{\Lambda}^2}{m_B^2} - 3.18\frac{\lambda_1}{m_B^2} + 0.02\frac{\lambda_2}{m_B^2} + o\left(\frac{\Lambda_{QCD}}{m_B}\right)^3\right]$$

„Heavy Quark Expansions“ (HQE) predict results for more moments, e. g. $\langle m_X^2 \rangle = M_{100}/M_{000}$, $\langle (m_X^2)^2 \rangle = M_{200}/M_{000}$, $\langle E_l \rangle = M_{010}/M_{000}$

Comparing measurements and calculations of „low“ M_{ijk} tests goodness of local duality and HQE, and determines λ_1, $\overline{\Lambda}$, $|V_{cb}|$..

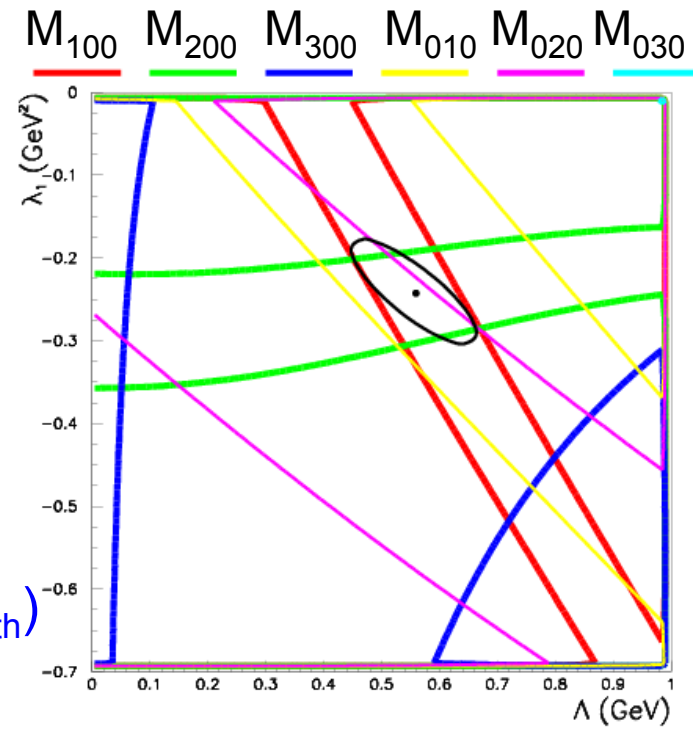
Moment measurements

DELPHI 2003 determines M_{i00} by reconstructing $B \to D^{**} l\nu$ events from 34 M Z at LEP. $\langle m_X^2 - \overline{m}_D^2 \rangle = (0.647 \pm 0.046 \pm 0.093)\ \mathrm{GeV}^2$, $\overline{m}_D = \left(m_D + 3 m_{D^*}\right)/4$. Also results for M_{200}, M_{300}

2002 they obtained $\langle E_l \rangle = (1.383 \pm 0.012 \pm 0.009)$ GeV, also M_{020}, M_{030}.

They fit in two mass schemes, with $\overline{\mathrm{MS}}$ scheme:

Fit Parameter	Fit Values	Fit Uncertainty	Syst. moments	Syst. theory
$\bar{\Lambda}$ (GeV)	0.542	± 0.065	± 0.087	± 0.04
λ_1 (GeV^2)	-0.238	± 0.055	± 0.028	± 0.06



With kinetic scheme:

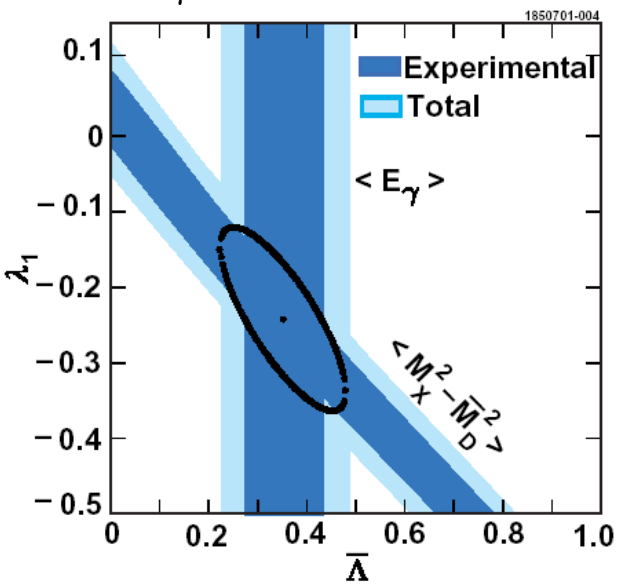
$|V_{cb}| = 0.0429\ (1 \pm 0.012_{\Gamma sl} \pm 0.019_{fit} \pm 0.010_{th})$

$\boldsymbol{BF}(B \to l\nu X)$ readjusted to 10.9%

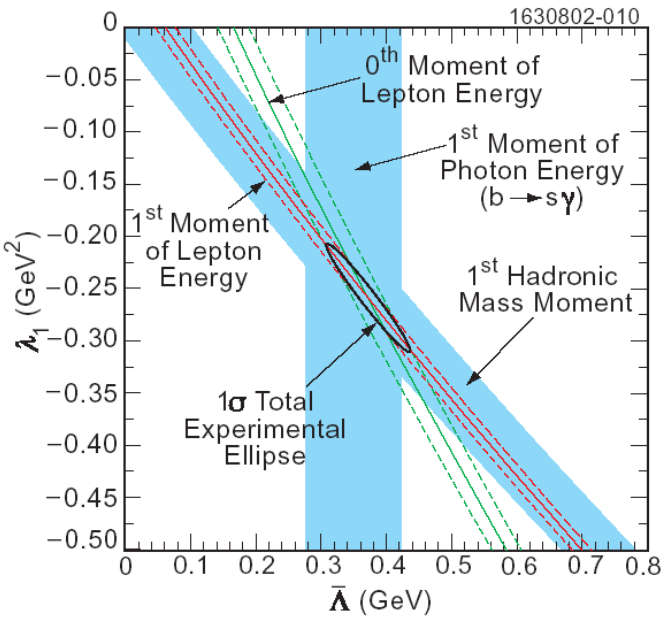
CLEO 2001 results:

from E_l spectrum,

$\langle m_X^2 - \bar{m}_D^2 \rangle$ (E_l >1.5 GeV)
$=(0.251 \pm 0.023 \pm 0.062)\ \mathrm{GeV}^2$
and $\langle E_\gamma \rangle$ from $b \to s\gamma$:



M_{010} (E_l >1.5 GeV) and
M_{000} (>1.7 GeV) / M_{000} (>1.5 GeV)



$|V_{cb}|^{(*)} = 0.0414\ (1 \pm 0.012_{\Gamma sl} \pm 0.022_{fit} \pm 0.020_{th})$

(*) $\boldsymbol{BF}(B \to l\nu X)$ re-adjusted to 10.9%

$|V_{cb}|^{(*)} = 0.0418\ (1 \pm 0.012_{\Gamma sl} \pm 0.012_{fit} \pm 0.022_{th})$

No combined fit performed.

BABAR 2003 results:

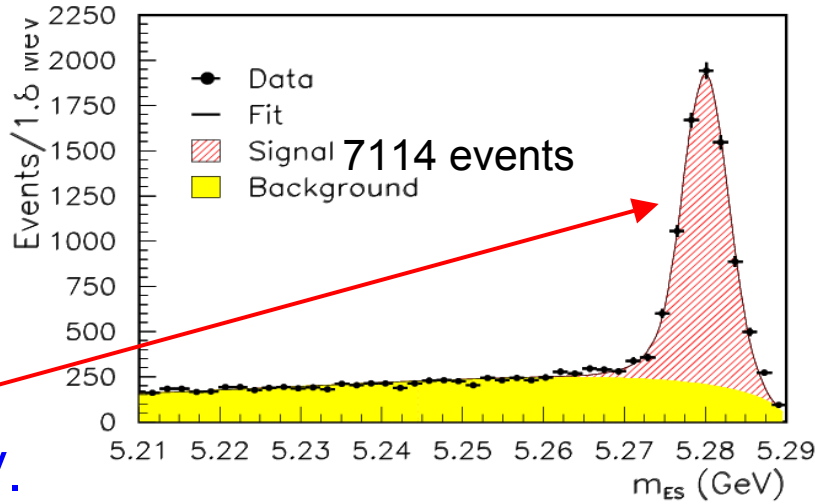
Moments M_{100} with

$E_l > 0.9 \ldots 1.6$ GeV,

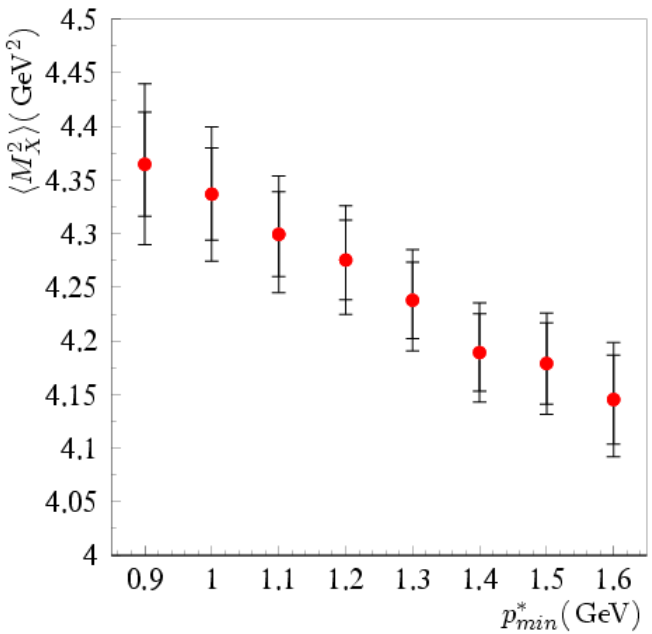
obtained from 89 M $B\bar{B}$

with B_{tag} fully reconstructed,

e and μ in B_{signal}, $E_l > 0.9$ GeV.

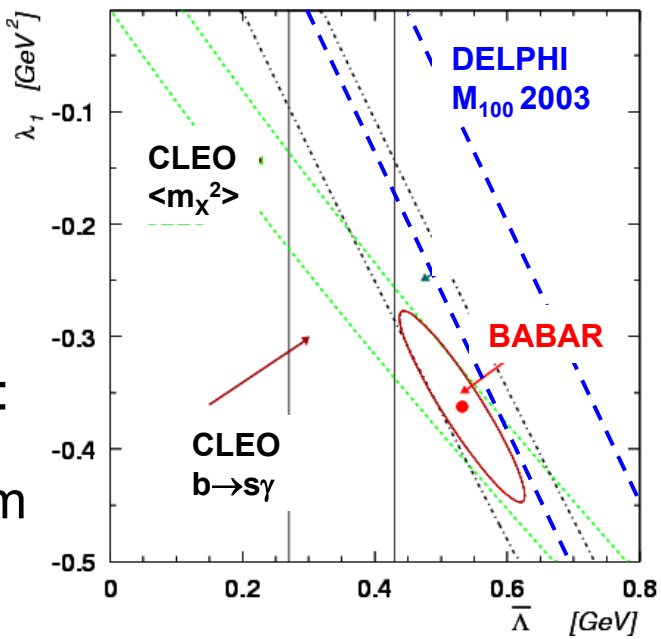


Obtained moments M_{100} ↓



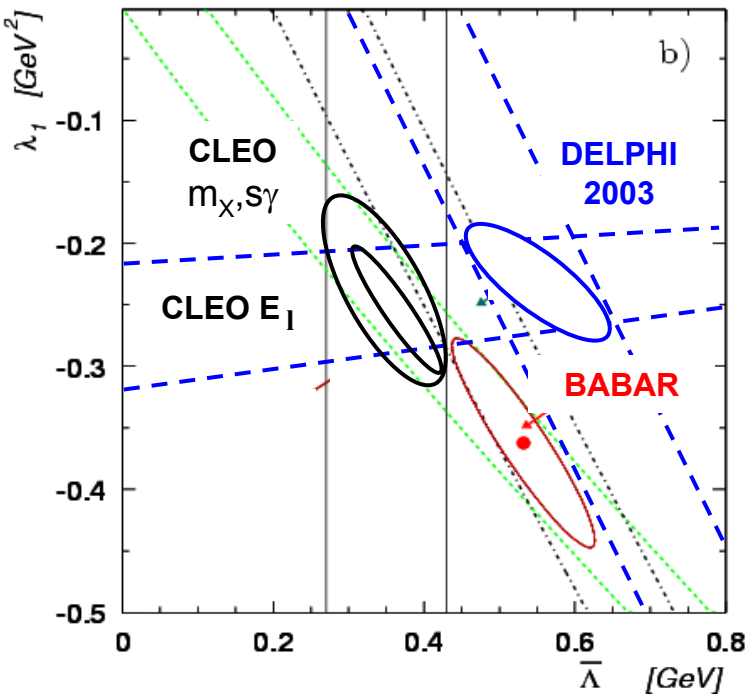
BABAR fits HQET parameters in 3 schemes, here pole mass:

quotes $|V_{cb}|$ from 1S scheme:



$0.0421(1 \pm 0.025_{exp} \pm 0.017_{th})$

Summary V_{cb} inclusive:



Moment measurements of $\frac{d^3\Gamma(B \to l\nu\, X_c)}{dm_X^2\, dE_l\, dq^2}$

and HQE / OPE offer the potential to determine $|V_{cb}|$ with $\sigma < 2\%$.

<u>Duties for experiment:</u>

Identify inconsistency areas.

<u>for theory:</u> Identify best scheme (1S?), get more $1/m_b^3$ and α_s^2 terms

$|V_{cb}|_{incl} = 0.0429\ (1 \pm 0.012_{\Gamma sl} \pm 0.019_{fit} \pm 0.010_{th})$ DELPHI, kin

$0.0414\ (1 \pm 0.012_{\Gamma sl} \pm 0.022_{fit} \pm 0.020_{th})$ CLEO m, pole mass

$0.0418\ (1 \pm 0.012_{\Gamma sl} \pm 0.012_{fit} \pm 0.022_{th})$ CLEO E, pole mass

$0.0421\ (1 \pm 0.025_{exp} \pm 0.017_{th})$ BABAR, 1S

My average: $|V_{cb}|_{incl} = 0.0421 \pm 0.0013$ (3.0%)

Exclusive V_{cb} determination

$$\frac{d\Gamma\left(B^0 \to D^{*-}\ell\nu\right)}{dw}; \qquad w = \frac{m_{D^*}^2 + m_D^2 - q^2}{2 m_{D^*} m_D}$$

new results 2003 from DELPHI and BABAR

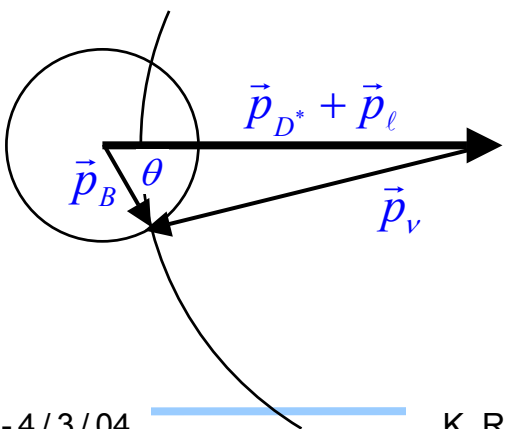
both with full reconstruction $D^{*+} \to D^0\pi^+ \,, D^0 \to K^-\pi^+, K^-\pi^+\pi^+\pi^-, K^-\pi^+\pi^0$

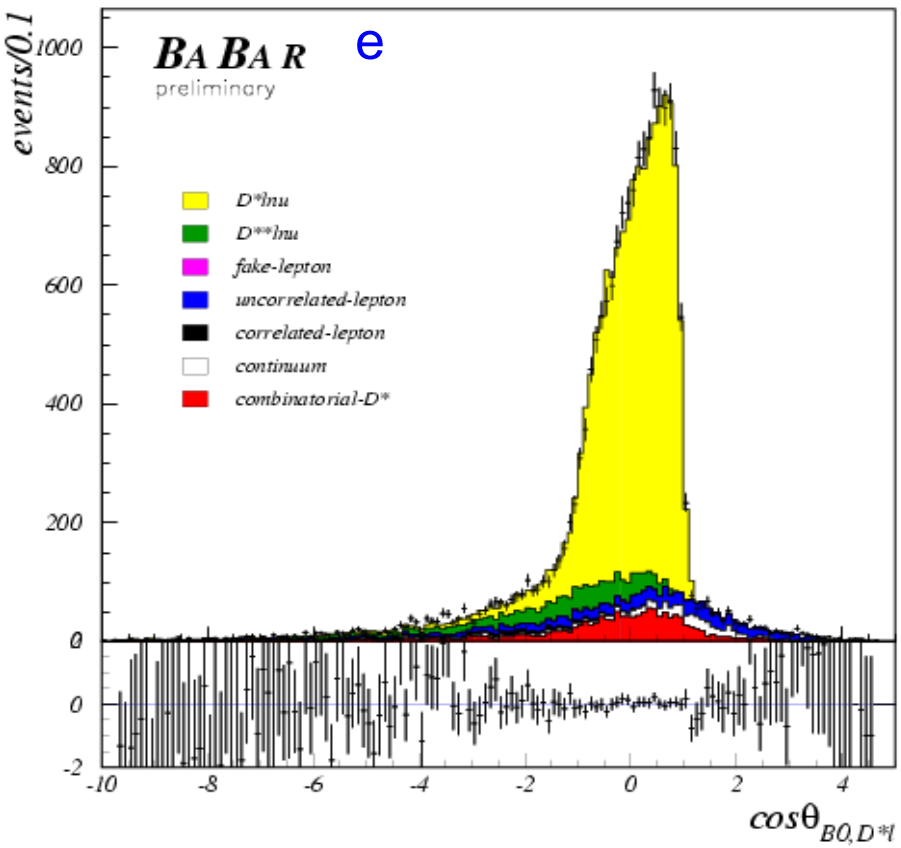
DELPHI 3.4 M Z, 1688 ev. (e+μ), BABAR 86 M ϒ(4S), 55700 (e+μ)

Delicate background from D**.

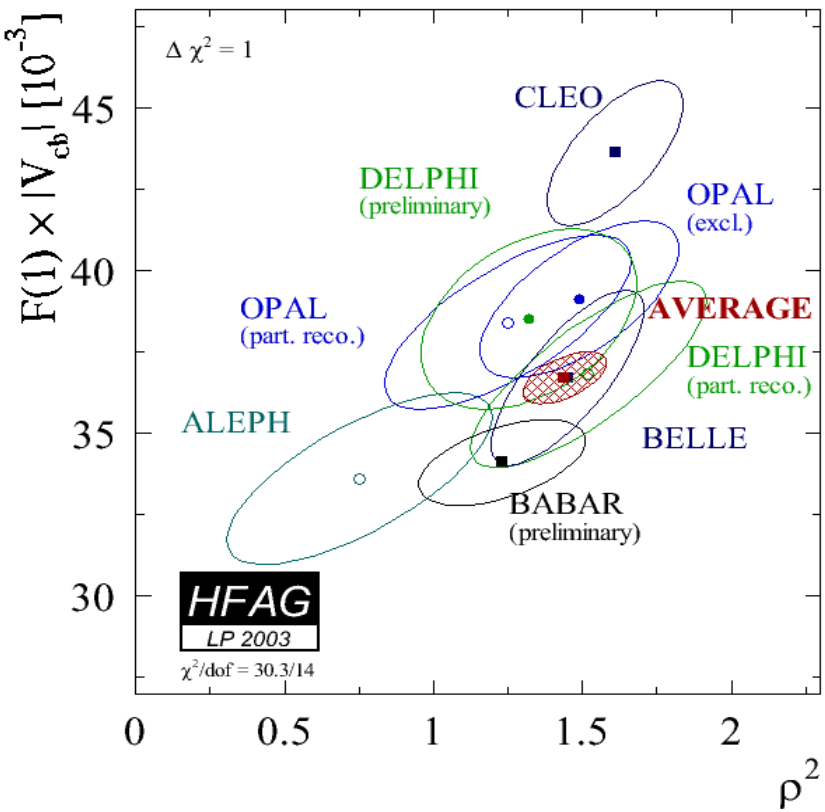
DELPHI uses hemisphere and vertex separation to identify extra tracks from B_{sig}

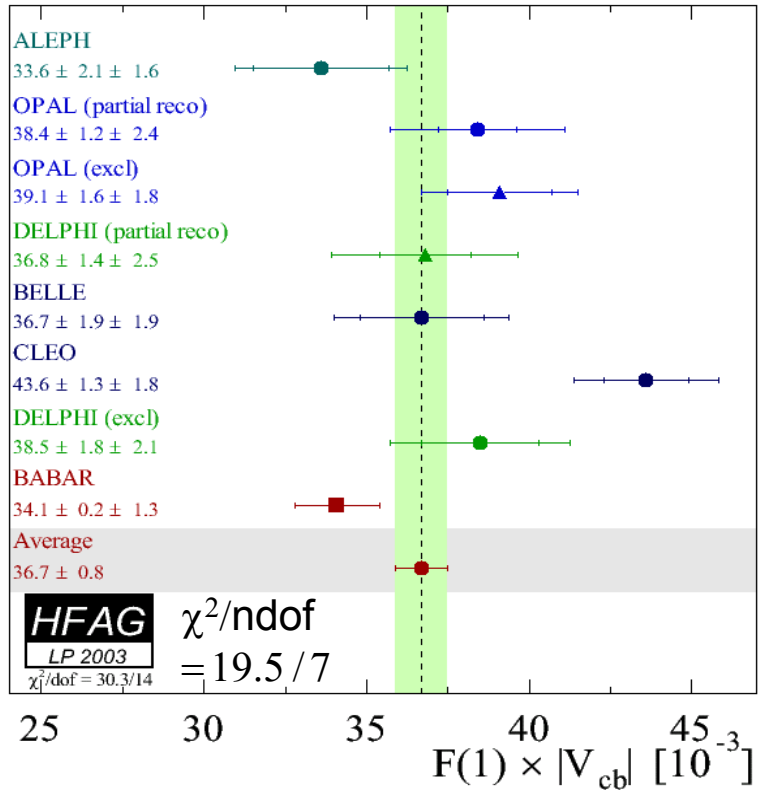
BABAR uses cos θ_{B,D^*l}





Summary for exclusive V_{cb}:





$|V_{cb}| \cdot F(1) = 0.0367 \pm 0.0013$ (S = 1.7)

$F(1) = 0.913\ ^{+0.030}_{-0.035}$ Lattice

$|V_{cb}|_{excl} = 0.0402 \pm 0.0020$

$|V_{cb}|_{incl} = 0.0421 \pm 0.0013$

$|V_{cb}| = 0.0415 \pm 0.0011$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$C = \lambda = 0.2235 \pm 0.0033$$

(±1.5 %)

$$K = A\,\lambda^2 = 0.0415 \pm 0.0011$$

(± 2.7 %) soon 2%

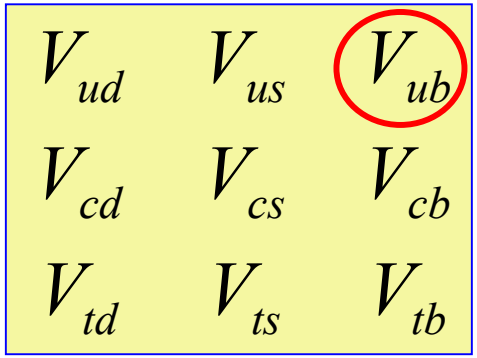
[We know $A\lambda^2$ better than A (± 4.0 %)]

$$CK = A\,\lambda^3 = 0.0093 \pm 0.0003$$

(±3.1 %)

$$V \approx \begin{pmatrix} 1 - C^2/2 & C & M \cdot e^{-i\gamma} \\ -C & 1 - C^2/2 & K \\ CK - \overline{M} \cdot e^{i\gamma} & -K & 1 \end{pmatrix}; \qquad \overline{M} = \left(1 - C^2/2\right) M,$$

$$J \approx C\,K\,\overline{M}\,\sin\gamma.$$



Source: Inclusive and exclusive semileptonic B-meson decays into charmless final states

Exclusive decays:

BELLE: BF for $B^0 \to \pi^- l\nu$, $B^+ \to \rho^0 l\nu$, $B^+ \to \omega l\nu$, no $|V_{ub}|$

BABAR: $|V_{ub}|$ from $B^0 \to \rho^- e^+\nu$, $B^+ \to \rho^0 e^+\nu$

with 55 M $\Upsilon(4S)$ and 5 form-factor calculations, „model dependent“

CLEO: $|V_{ub}|$ from $\pi^- l^+\nu$, $\pi^0 l^+\nu$, $\eta l^+\nu$, $\rho^- l^+\nu$, $\rho^0 l^+\nu$, $\omega l^+\nu$

with 10 M $\Upsilon(4S)$ and QCD form-factors only, „model independent“

Because of small statistics, both use isospin / quark-model constraints:

$$\Gamma(\rho^o \ell \nu) = \Gamma(\omega \ell \nu) = \Gamma(\rho^- \ell \nu)/2 \,,\; \Gamma(\pi^o \ell \nu) = \Gamma(\pi^- \ell \nu)/2$$

BABAR 2003:

$B^0 \to \rho^- e^+ \nu$

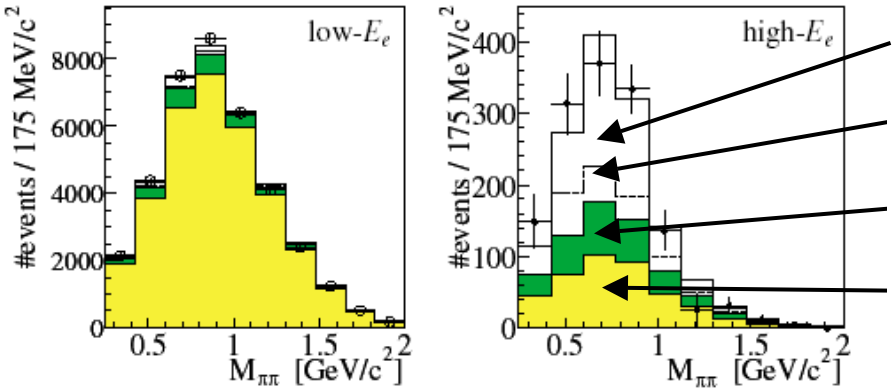
$B^+ \to \rho^0 e^+ \nu$

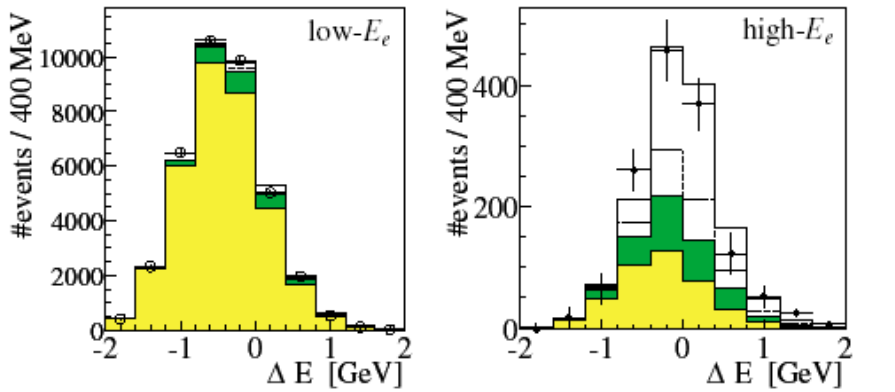
high - E_l: 2.3-2.7 GeV

low - E_l: 2.0-2.3 GeV

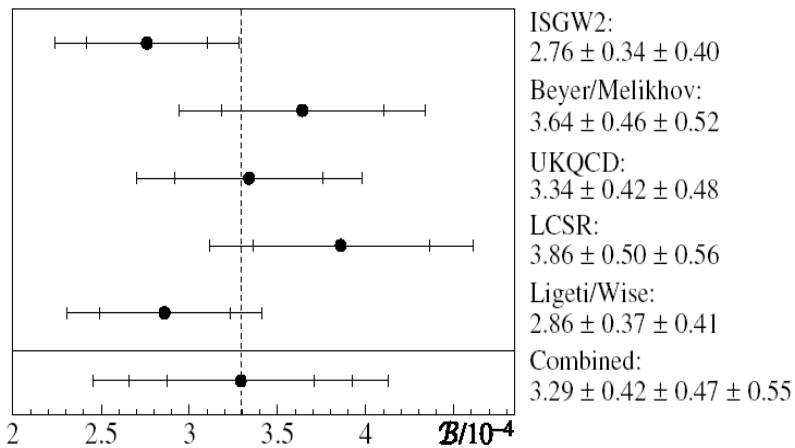
signals for ρ^- ⇒

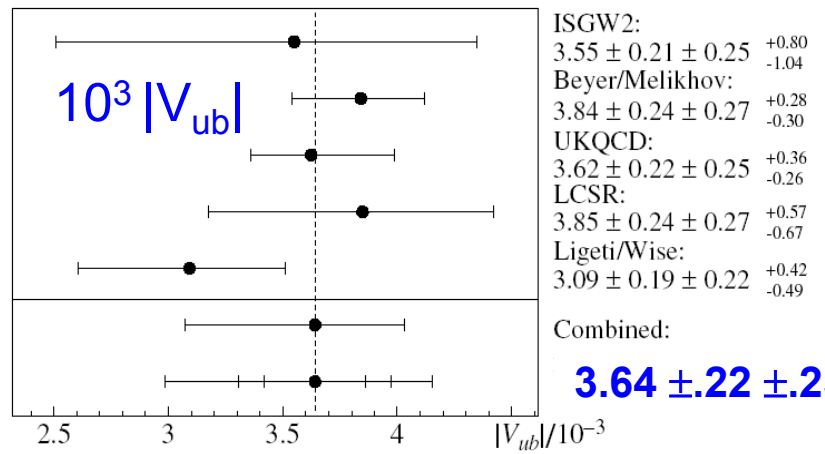
results from ρ^- & ρ^0 ⇓

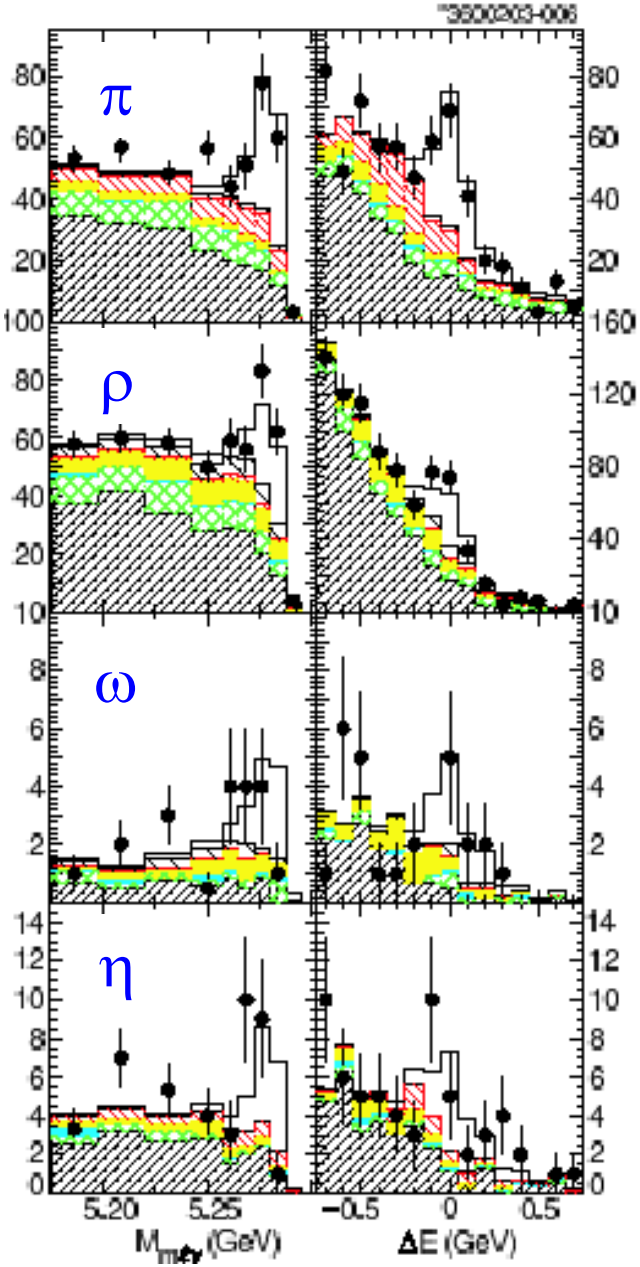




$10^4\ \boldsymbol{BF}(B^0 \to \rho^- e^+ \nu)$







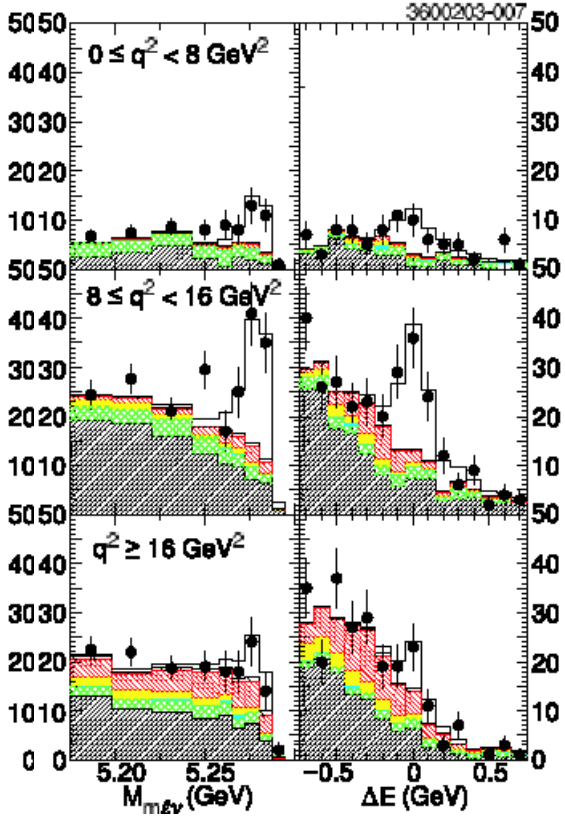
CLEO 2003:

Detector hermeticity ⇒ „ν reconstruction“

⇒ q^2 and E_l >1.0 GeV (π), >1.5 GeV (ρ)

⇐ Signals

in q^2 bins ⇓



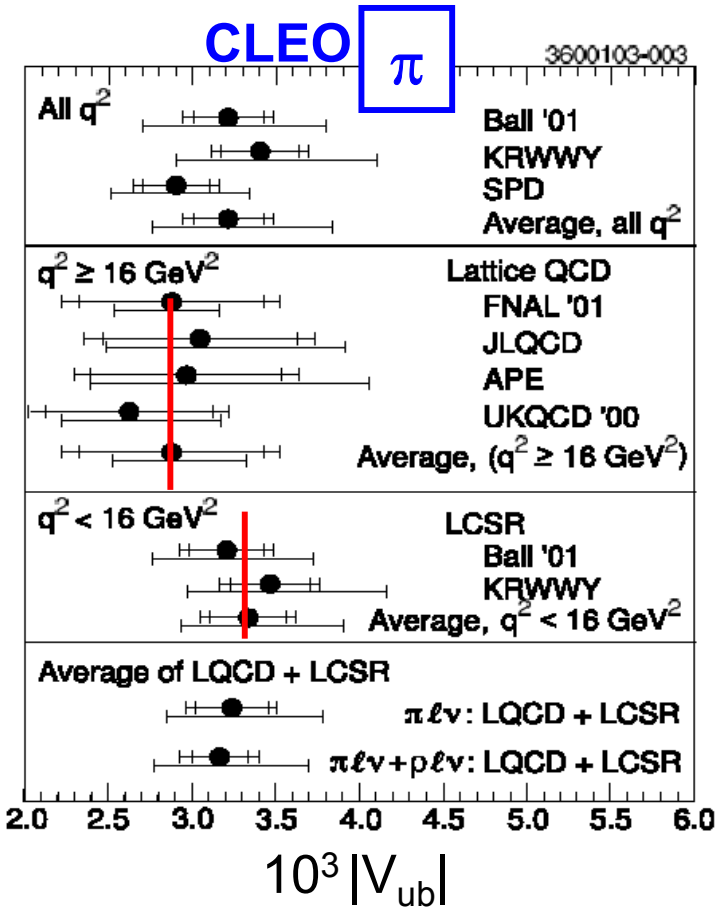
white = signal

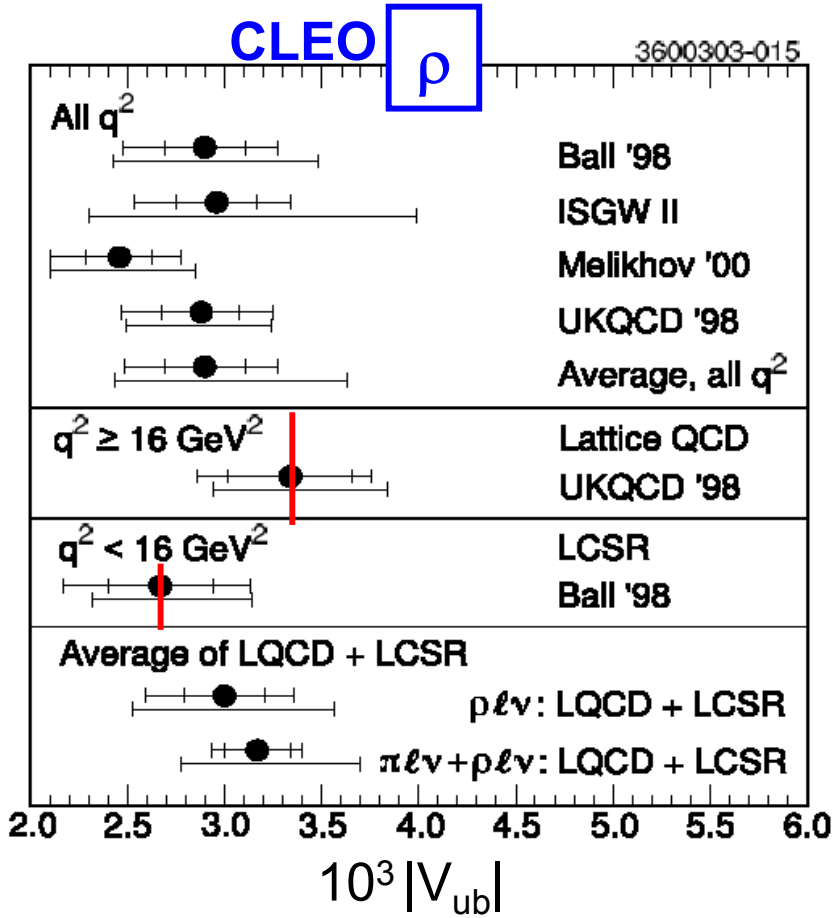
red = crossfeed

yellow = downfeed

green = continuum

black = b→clν



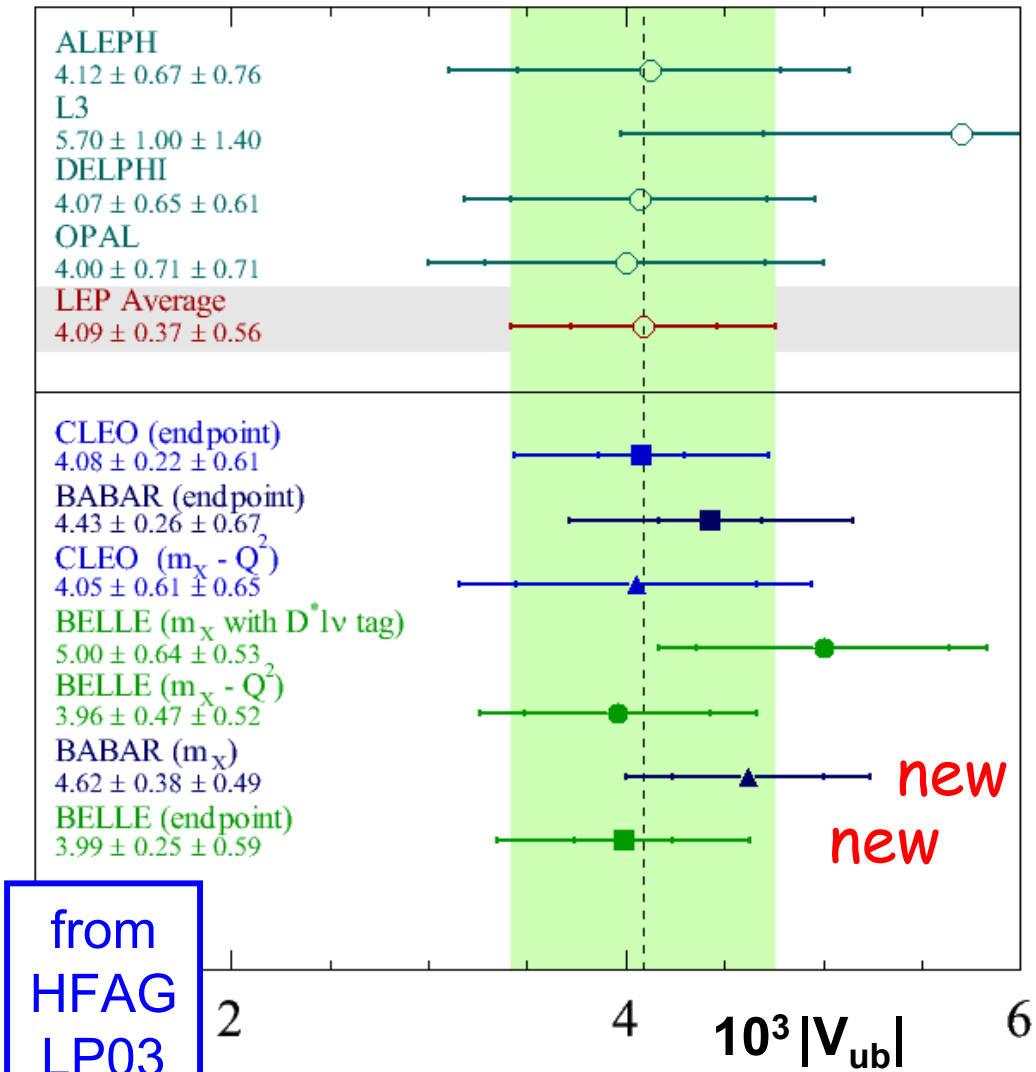


$$|V_{ub}|_{CLEO,\pi+\rho} = (3.17 \pm 0.17\ ^{+0.16\ +0.53}_{-0.17\ -0.39}) \times 10^{-3}$$

$$|V_{ub}|_{BABAR,\rho} = (3.64 \pm 0.22 \pm 0.25\ ^{+0.39}_{-0.56}) \times 10^{-3}$$

$$|V_{ub}|_{excl} = (3.40\ ^{+\ 0.24}_{-\ 0.33} \pm 0.40) \times 10^{-3}$$

Inclusive decays $b \to u l \nu$:



from HFAG LP03

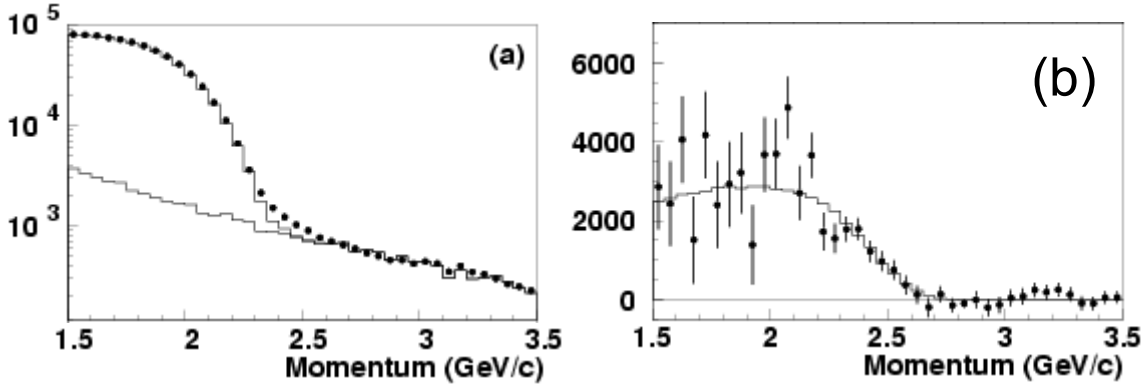
Three methods

1) $E_l > 2.3$ GeV, „endpoint“

2) $E_l > 1.0$ GeV
and $m_X < 1.5$ GeV,
requires tagged B,
less QCD-dependent.

3) $E_l > 1.0$ GeV
and $m_X < 1.5$ GeV
and $q^2 > 12$ GeV^2,
requires tagged B
and high statistics.
needs even less QCD.

BELLE:



29 MΥ(4S),

(a) B→eνX

(b) continuum

and b→c subtracted

$\boldsymbol{BF}$ (B →eνX$_u$) = $\Delta\boldsymbol{BF}$ (2.3 < E$_l$ < 2.6 GeV) / f$_u$(CLEO)

$|V_{ub}| = (3.99 \pm 0.17_{stat} \pm 0.16_{sys} \pm 0.59_{th})\,10^{-3}$

What is f$_u$? Already used in endpoint analyses of CLEO and BABAR.

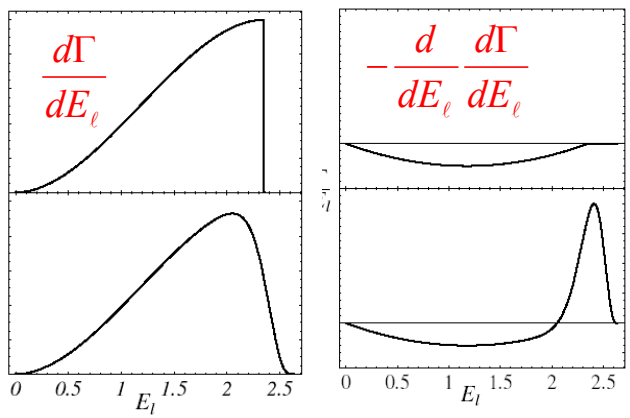
Determined by shape function, also concept of HQE,

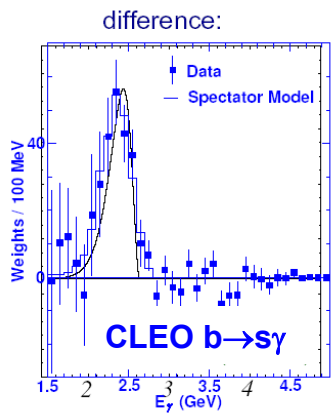
but beyond OPE, needs nonlocal operators („twists“).

From Z. Ligeti FPCP-03 Paris:

b-quark decay

B-meson decay



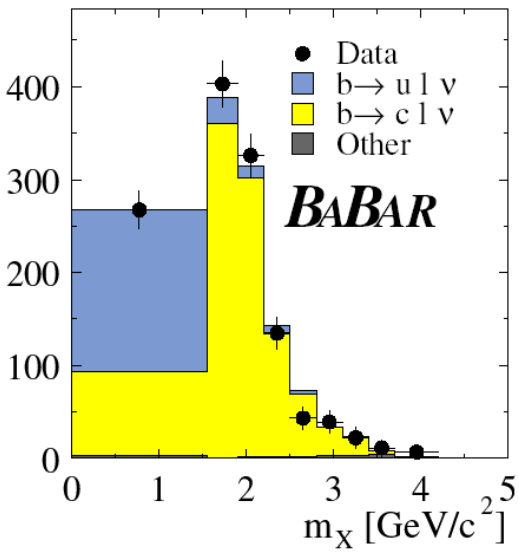


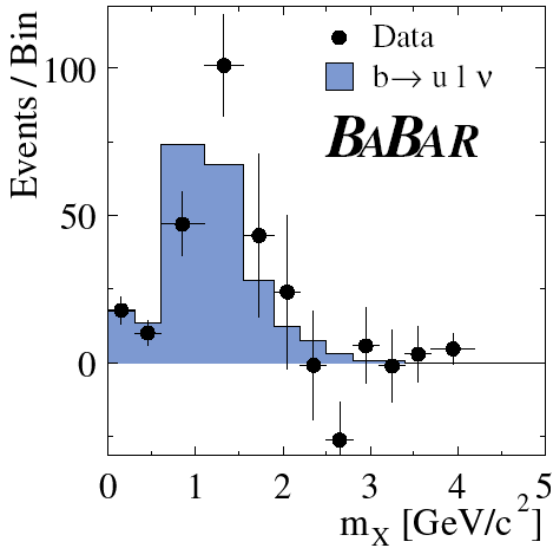
m_X Result of BABAR:

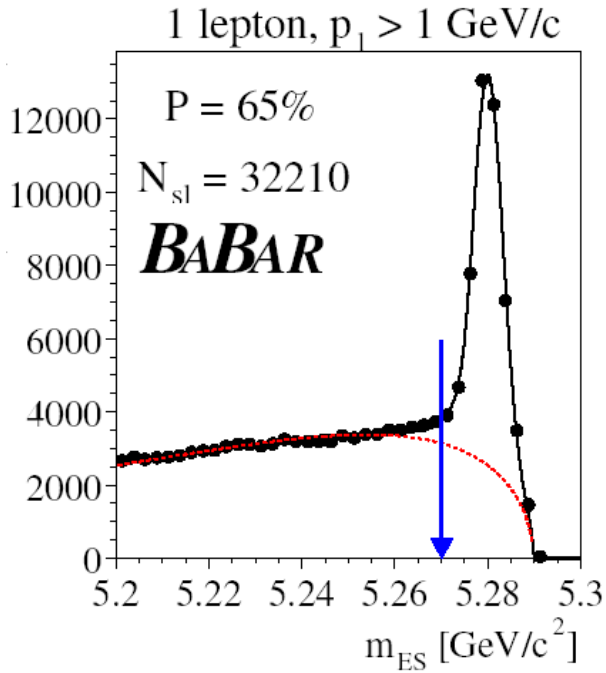
89 MΥ(4S), 32 k events with

$\Upsilon(4S) \rightarrow B_{reco}B_{sig}$, E_l >1 GeV, B_{reco} ⇒

reconstruct m_X in $B_{sig} \rightarrow l\nu X$ ⇓







Using the shape function ⇒

$$|V_{ub}| = (4.62 \pm 0.28_{stat} \pm 0.27_{sys} \pm 0.40_{shf} \pm 0.26_{\Gamma \rightarrow Vub}) \, 10^{-3}$$

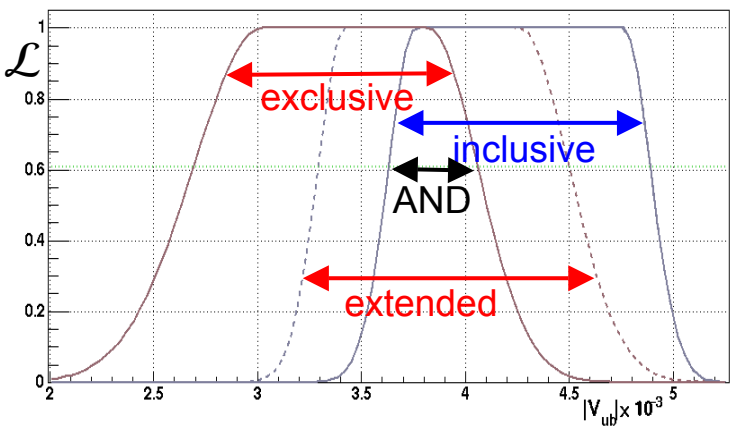
Summary for $V_{ub,incl}$:

No discussion of m_X-q^2 method here.
Best prospects in the future because of smallest dependence on HQE parameters. Take them from V_{cb} data?

HFAG does not yet present an average for $|V_{ub}|_{incl}$.

My weighted inclusive mean:

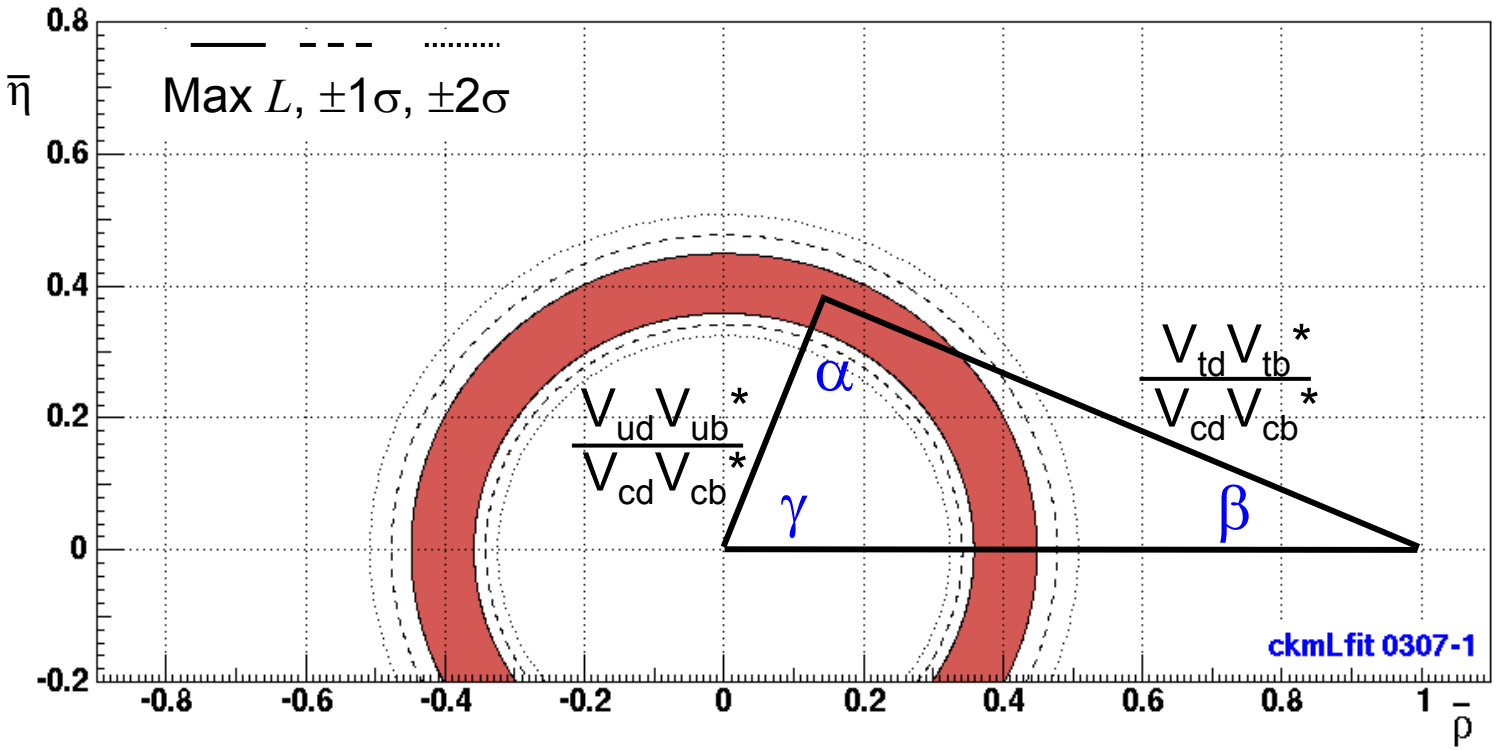
$$|V_{ub}|_{incl} = (4.26 \pm 0.13 \pm 0.50)\ 10^{-3}$$



$$|V_{ub}|_{excl} = (3.40\ ^{+\ 0.24}_{-\ 0.33} \pm 0.40)\ 10^{-3}$$

$$|V_{ub}| = (3.80\ ^{+\ 0.24}_{-\ 0.13} \pm 0.45)\ 10^{-3}$$

$|V_{ub}|^2 = A\,\lambda^3\,(\rho^2+\eta^2)$, $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$:



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{tb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$ again

was 1.04 ± 0.16

using $\Gamma(D \to \bar{K} e \nu)$

Decays of real W-bosons at LEP-2 increase precision:

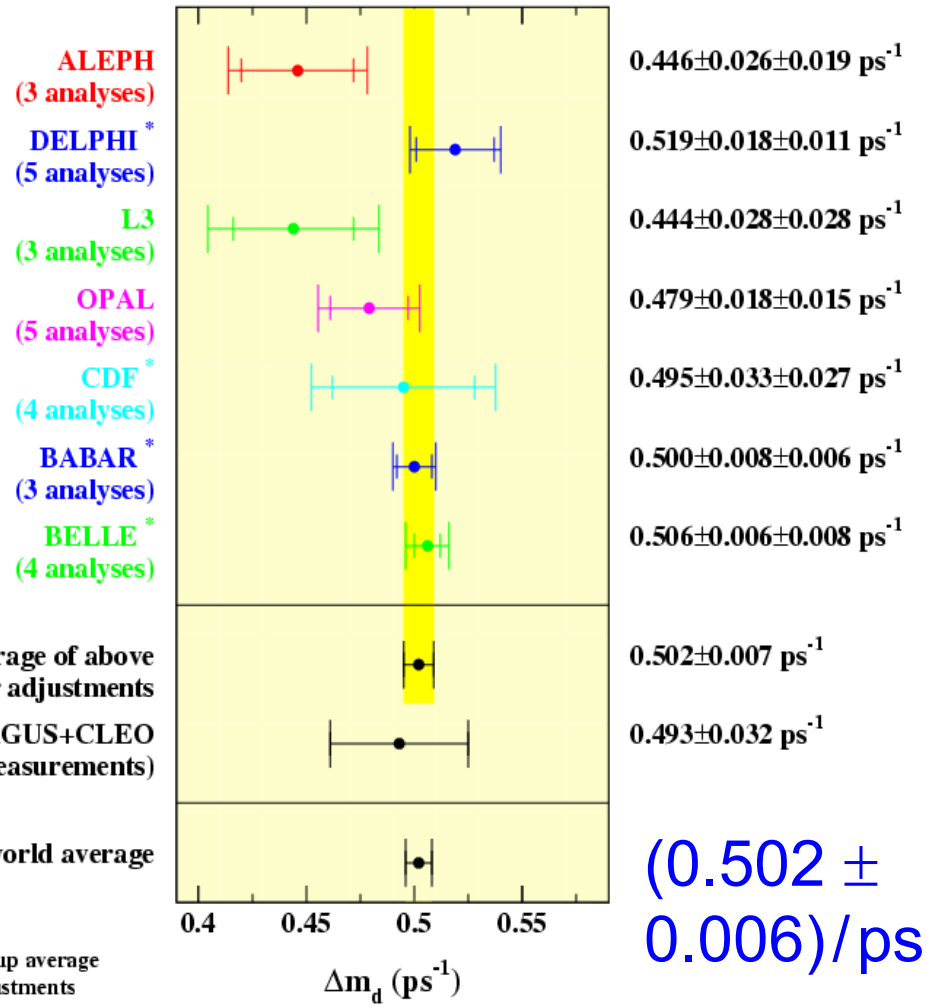
$$\frac{\Gamma(W \to hadrons)}{\Gamma(W \to e\nu)} \propto \frac{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}{1} = 2.039 \pm 0.026$$

only using that there are five quarks with m(q) < m(W) $\Rightarrow$

$$|V_{cs}| = 0.995 \pm 0.014$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

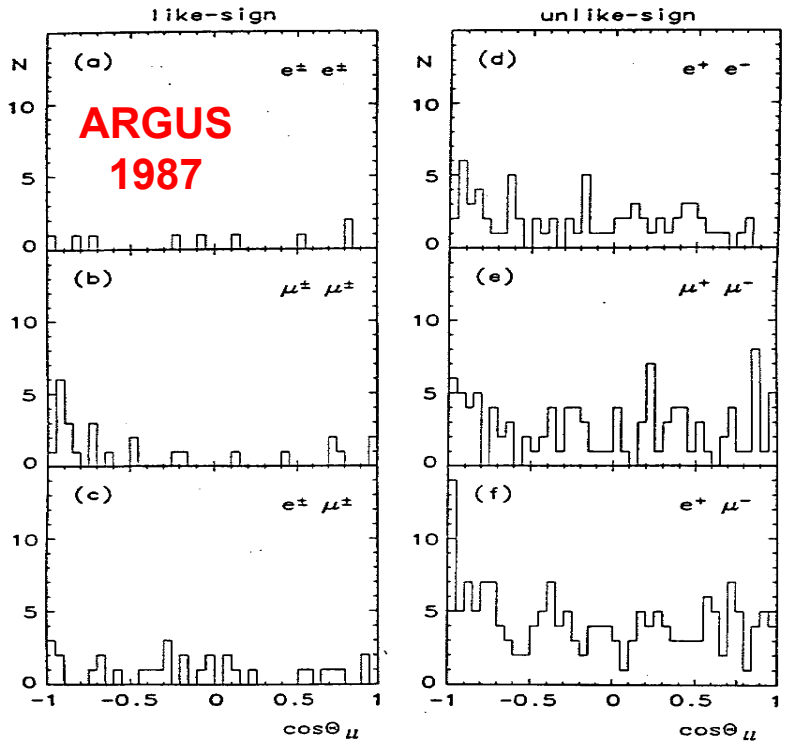
Source: $B^0\bar{B}^0$ Oscillations

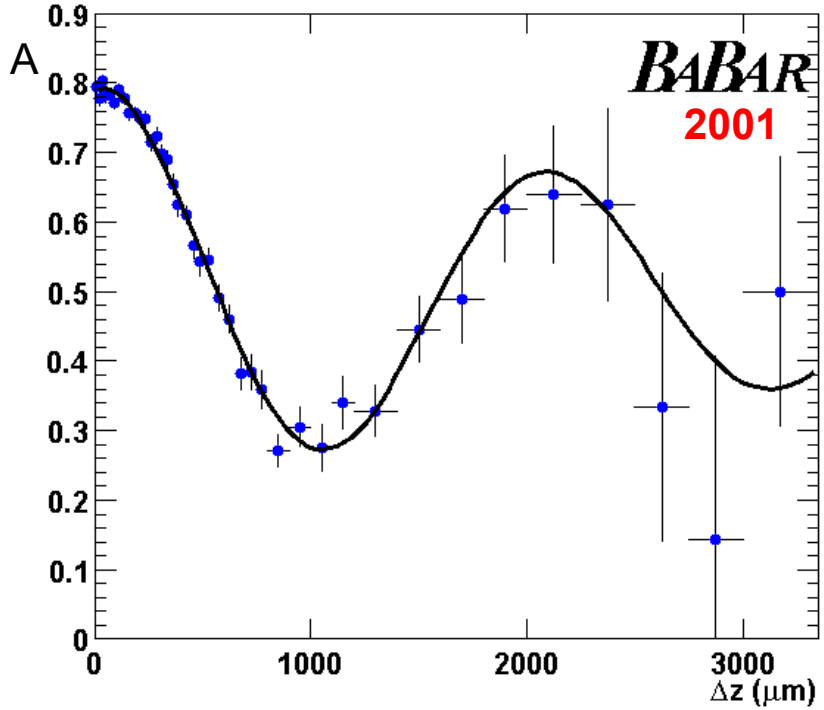


from HFAG LP03

* working group average without adjustments

$(0.502 \pm 0.006)/\text{ps}$





$$\Delta m_d = \frac{G_F^2 m_{B_d} f_{B_d}^2 B_{B_d} \eta_B}{6\pi^2} \cdot |V_{td}|^2 \cdot |V_{tb}|^2 \cdot f\left(m_t^2, m_W^2\right) = (.502 \pm .006)/\text{ps}$$

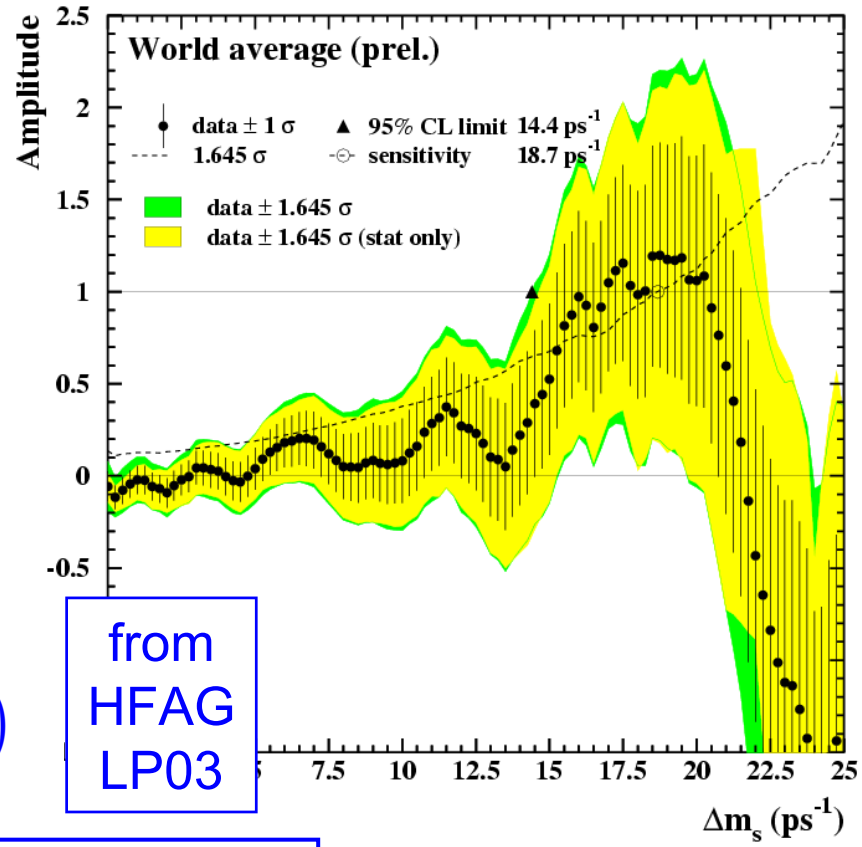
since ~ 2 y: $f_B^2 B_B = (223 \pm 33 \pm 12)^2\ \text{MeV}^2$ from lattice QCD

$$|V_{td}| \cdot |V_{tb}| = (9.2 \pm 1.4 \pm 0.5)\ 10^{-3}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

a) from $B_s\bar{B}_s$ oscillations

$\Delta m_s > 14.4/ps$ (95% CL)



from HFAG LP03

$$\Delta m_s = \frac{G_F^2 m_{B_s} f_{B_s}^2 B_{B_s} \eta_{B_s}}{6\pi^2} \cdot |V_{ts}|^2 \cdot |V_{tb}|^2 \cdot f\left(m_t^2, m_W^2\right)$$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.24 \pm 0.04 \pm 0.06$$

$|V_{ts}|\cdot|V_{tb}| > 0.033$

b) from b → sγ penguins

$BF(B \to X_s\gamma)$ [CLEO, ALEPH, BABAR, BELLE, Ali,Misiak]

$|V_{ts}|\cdot|V_{tb}| = 0.047 \pm 0.008$

Unitarity check of the full CKM matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \end{pmatrix}$$

$|V_{ud}|=0.9737\pm0.0007$ $|V_{ub}|=0.0038\pm0.0005$

$|V_{us}|=0.2210\pm0.0023$ $|V_{cb}|=0.0415\pm0.0011$

$|V_{cd}|= 0.224\pm0.016$

$|V_{cs}|= 0.995\pm0.014$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9969 \pm 0.0017 \quad -1.8\ \sigma$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.042 \pm 0.029 \quad +1.5\ \sigma$$

$$|V_{ud}V_{cd}| - |V_{us}V_{cs}| \pm |V_{ub}V_{cb}| = -0.002 \pm 0.016 \quad 0.1\ \sigma$$

The observed CKM matrix magnitudes fulfill unitarity reasonably well $\Leftrightarrow$ All processes which we call „weak“ can be described by the Standard weak interaction in which the CKM matrix is necessarily unitary.

Assuming Unitarity :

$|V_{tb}|^2 = 1 - |V_{cb}|^2 - |V_{ub}|^2$,

$|V_{tb}| = 0.99913 \pm 0.00009$

$|V_{td}| = (9.2 \pm 1.4 \pm 0.5)\ 10^{-3}$

$|V_{ts}|^2 = |V_{cb}|^2 + |V_{ub}|^2 - |V_{td}|^2$

$|V_{ts}| = 0.0406 \pm 0.0023$

(3 x better than from $B \rightarrow X_s\gamma$)

This helps to better measure $|V_{td}|$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \cdot \frac{|V_{td}|^2}{|V_{ts}|^2} \cdot \xi^2 \qquad \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.24 \pm 0.04 \pm 0.06$$

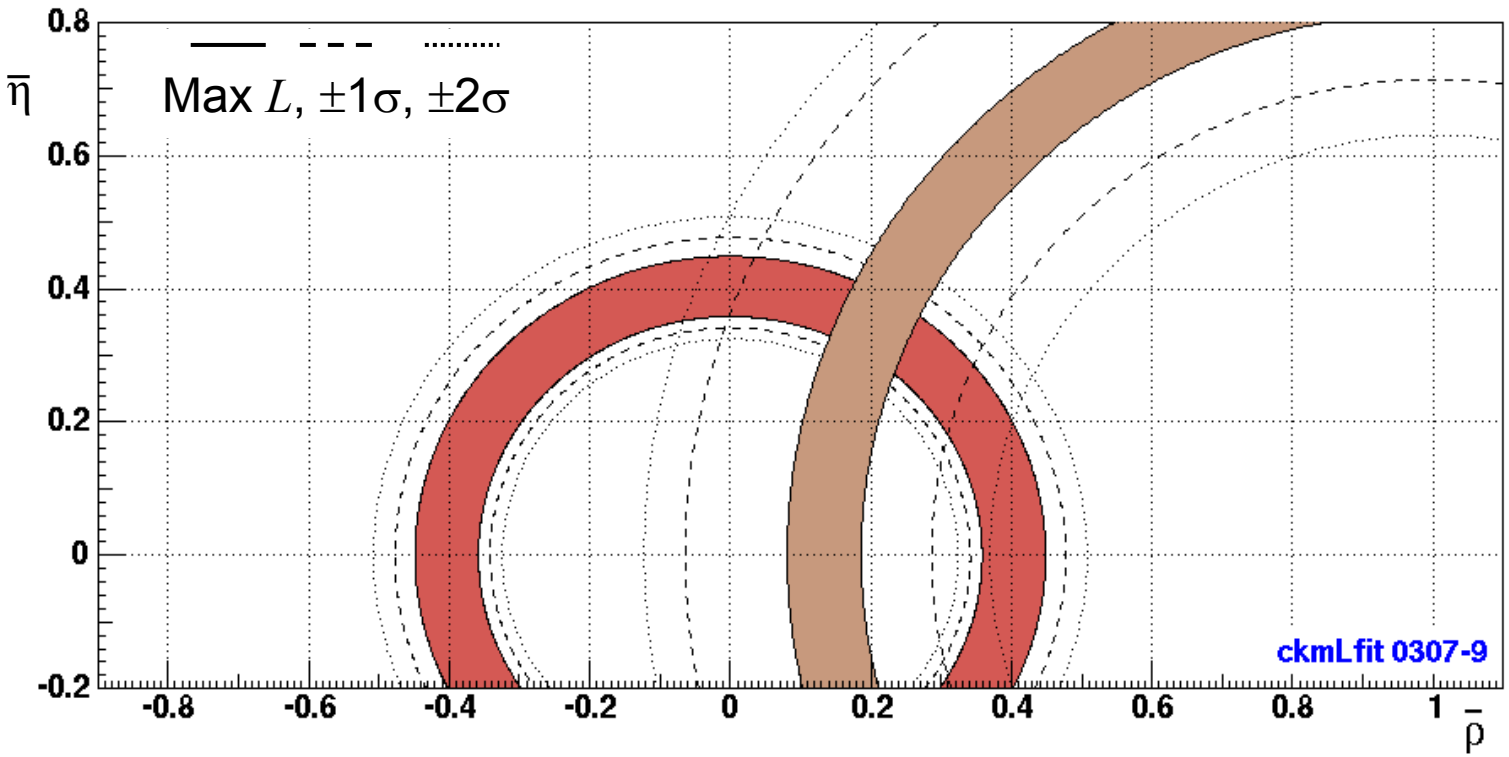
(from Lattice QCD)

$\Delta m_s > 14.4/\text{ps}$ (95% CL) gives

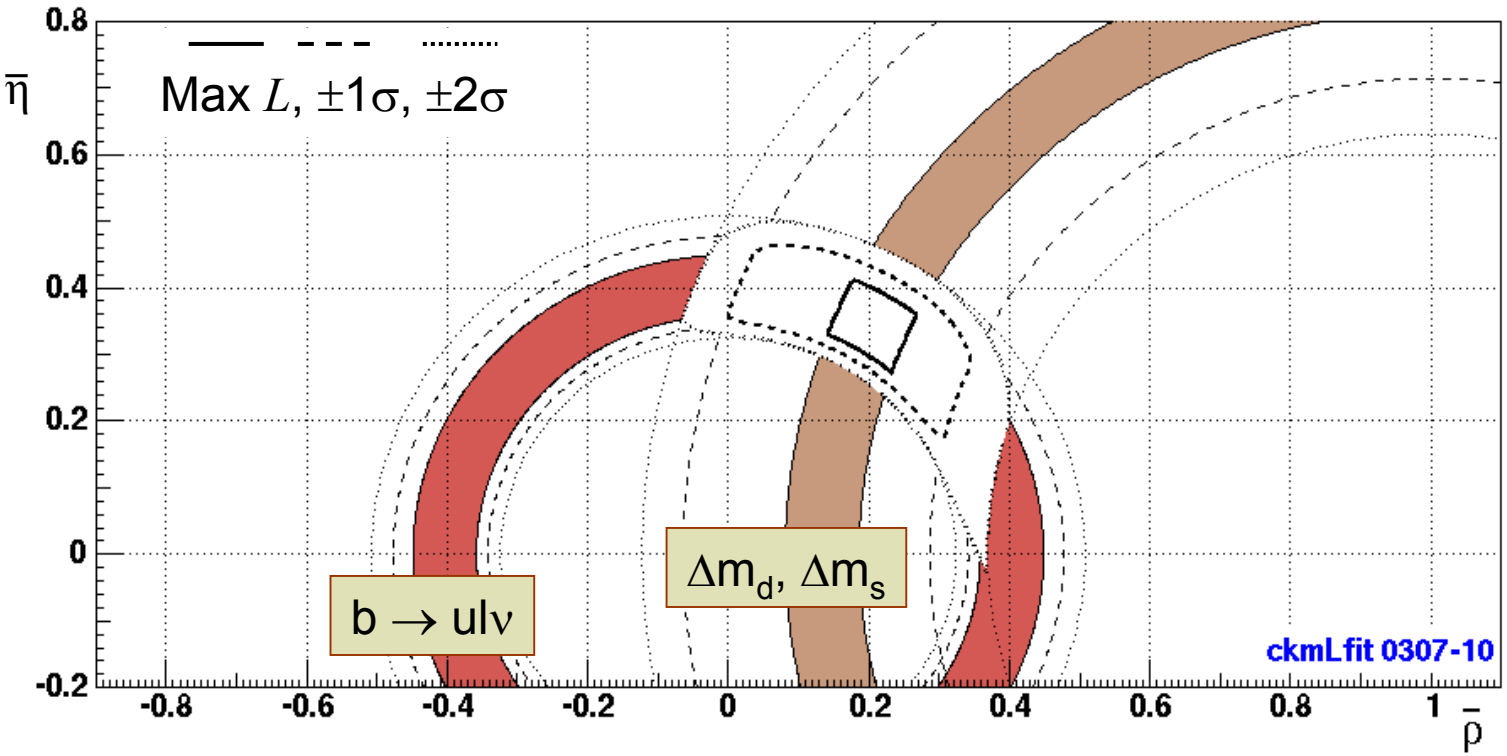
$|V_{td}| < 10.9 \cdot 10^{-3}\ (1\sigma)$

$|V_{td}| < 11.4 \cdot 10^{-3}\ (2\sigma)$

ckm Lfit


Max L, ±1σ, ±2σ
$\bar{\eta}$
$\bar{\rho}$
0.8
0.6
0.4
0.2
0
-0.2
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
ckmLfit 0307-9

ckm Lfit

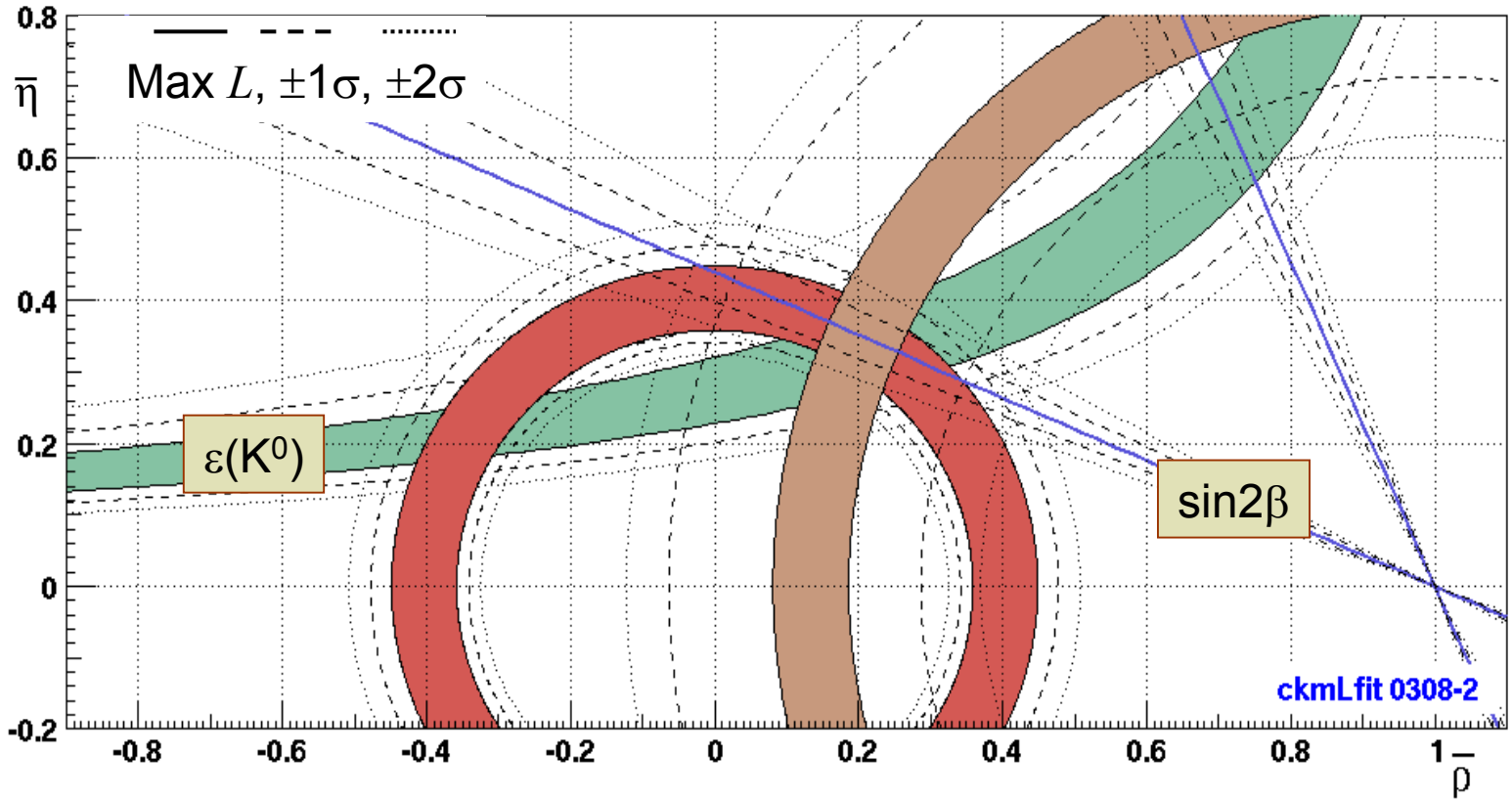


$\eta \neq 0 \Leftrightarrow J \neq 0 \Leftrightarrow$ CPV

Established with ≈ 2σ from CP-conserving observations only, thanks to the unitarity of V.

ckm Lfit

Two more entries from CP Violation:



- to be discussed next

K. R. Schubert, TU Dresden

IMFP04 Alicante, 1-5 March 2004

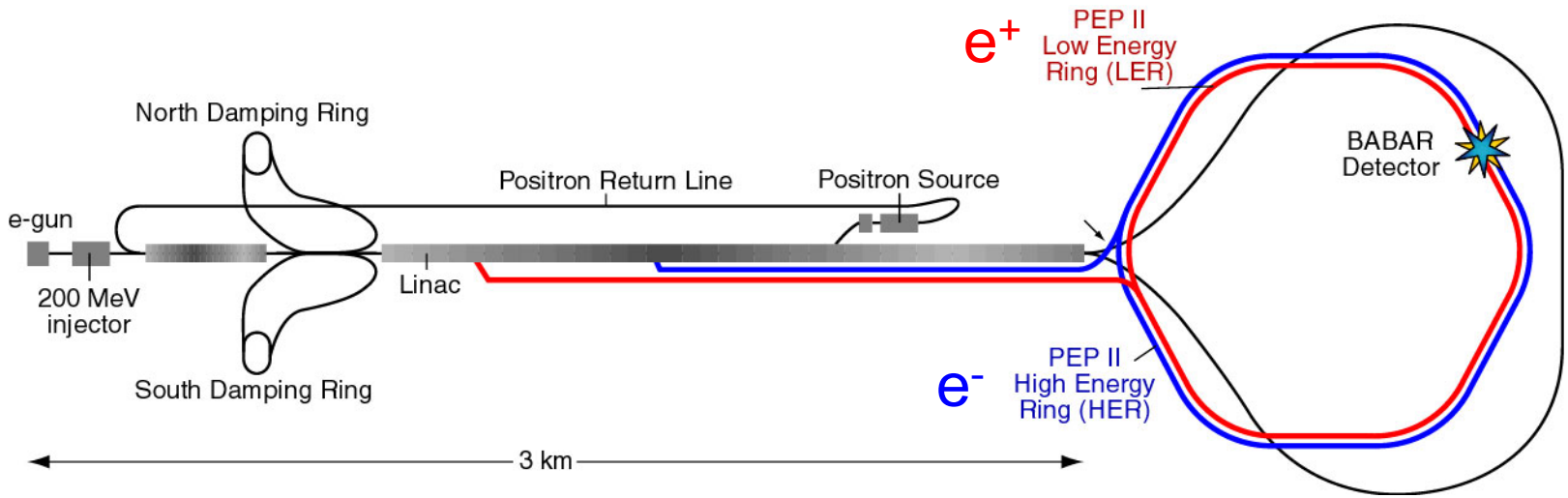
„Quark Mixing and CP Violation“

Third Lecture:
Four Types of CP Violation

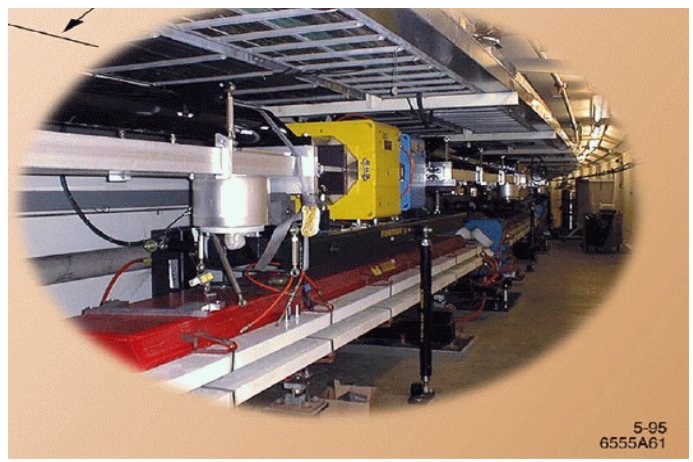
Short History of CPV (B):

1977	Discovery of b in $\Upsilon(9.46) = 1^3S_1\ b\bar{b}$ at FNAL
1978	Formation of $\Upsilon(9.46)$ and $\Upsilon(10.01)$ at DESY
1980	First B mesons $1^1S_0\ b\bar{q}$ at Cornell
1986-89	„B-Meson Factory“ plans at PSI
1987	ARGUS discovery of B^oB^o oscillations
1988	Start of PEP-II studies at SLAC
1993	Decisions for PEP-II and KEK-B
1995	BABAR „TDR“ & approval
7/98	First e^+e^- collisions in PEP-II
5/99	First e^+e^- events in BABAR
7/00	First BABAR & BELLE results at Osaka
10/00	PEP-II reaches design luminosity of $3 \cdot 10^{33}/cm^2/s$
3/01	BABAR and BELLE publish $\sin 2\beta$ in PRL
7/03	$\sin 2\beta$ contributes significantly to CKM matrix fit

The B-Meson Factory PEP-II:



	Design	Reached
E [GeV] e^- / e^+	9.0 / 3.1	yes
I [A] e^- / e^+	0.6 / 2.1	1.35 / 2.15
L [cm^{-2} s^{-1}]	3×10^{33}	7.9×10^{33}
L_{int} [pb^{-1}/day]	135	552 (13-02-04)



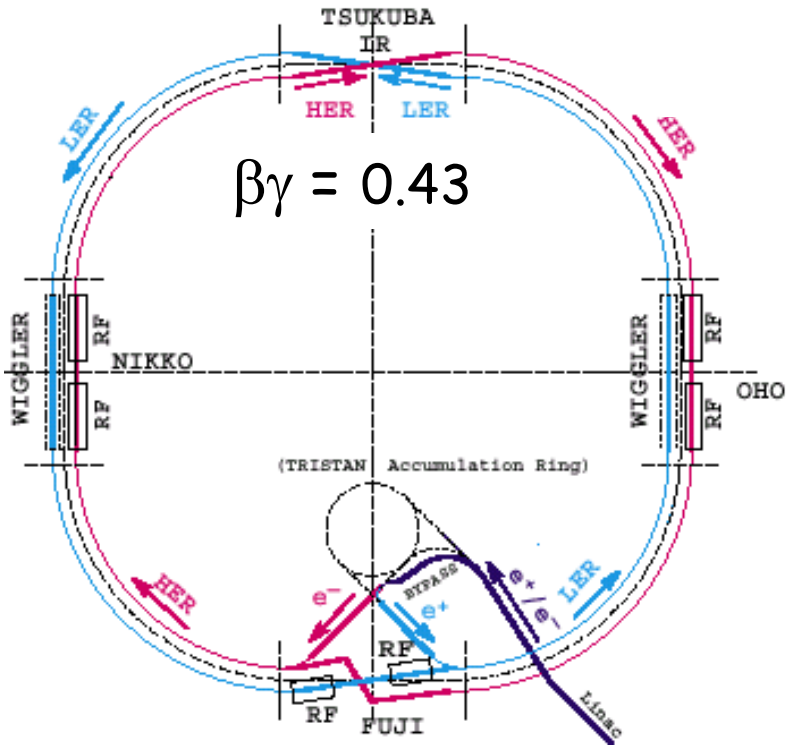
← Linac

← Fixed Target Experiments

← BABAR

← SLD

KEKB



asymmetric e^+e^- collider

- Two separate rings
  - e^+ (LER) : 3.5 GeV
  - e^- (HER) : 8.0 GeV
- E_{CM} : 10.58 GeV at $\Upsilon(4S)$
- Luminosity
  - target: 10^{34} /cm^2/s

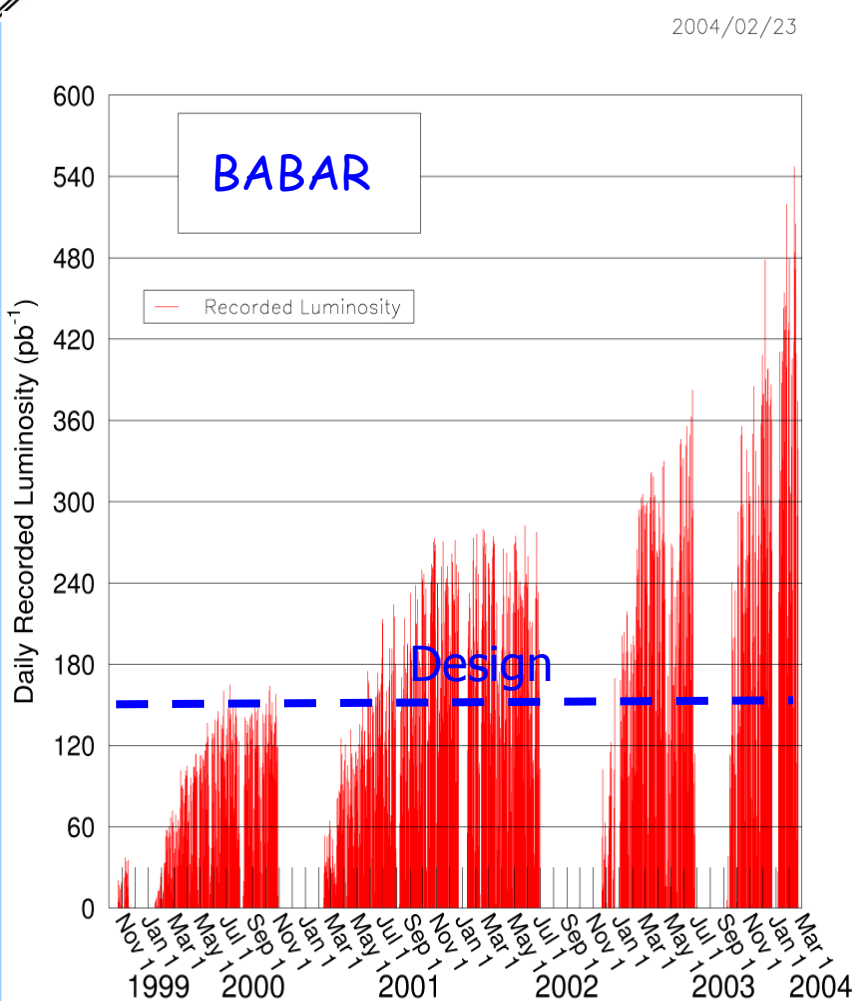
  achieved: 1.16×10^{34} /cm^2/s
- Small beam sizes:
  - $\sigma_y \approx 3$ μm; $\sigma_x \approx 100$ μm
- ±11 mrad crossing angle

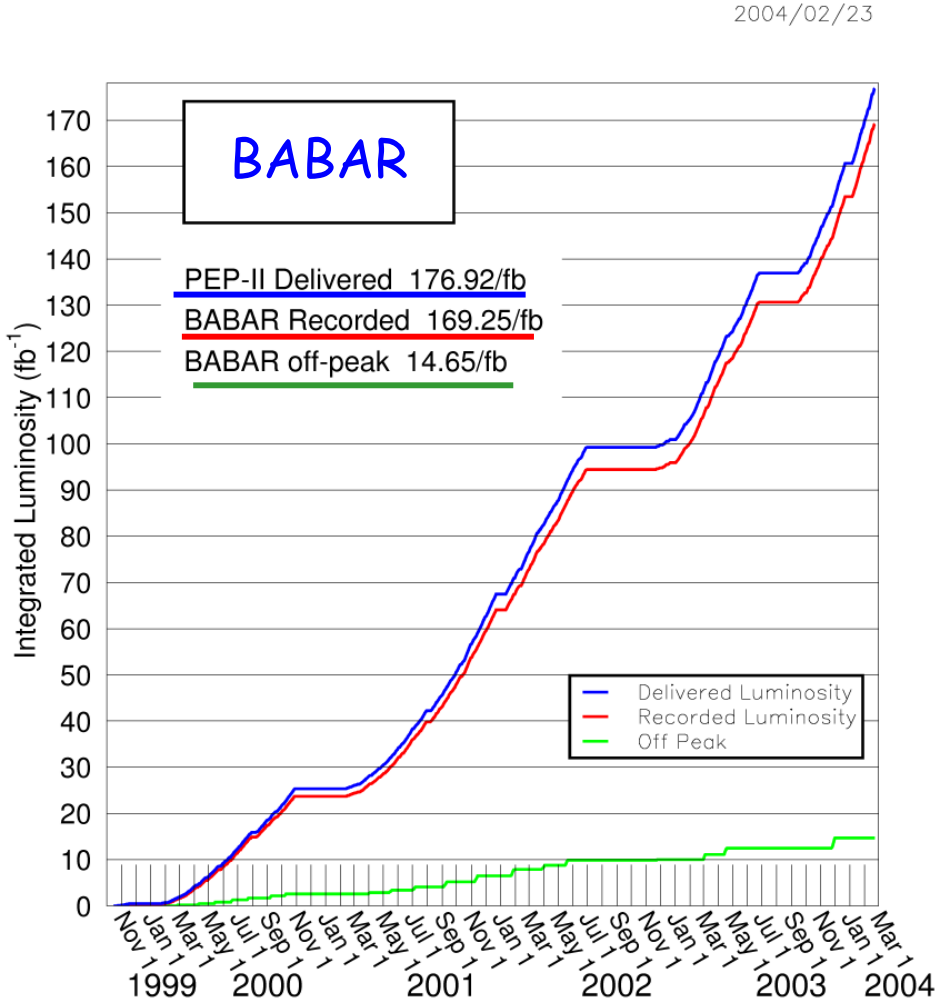
KEKB in the TRISTAN tunnel, L = 3.0 km

PEP-II: L = 2.2 km

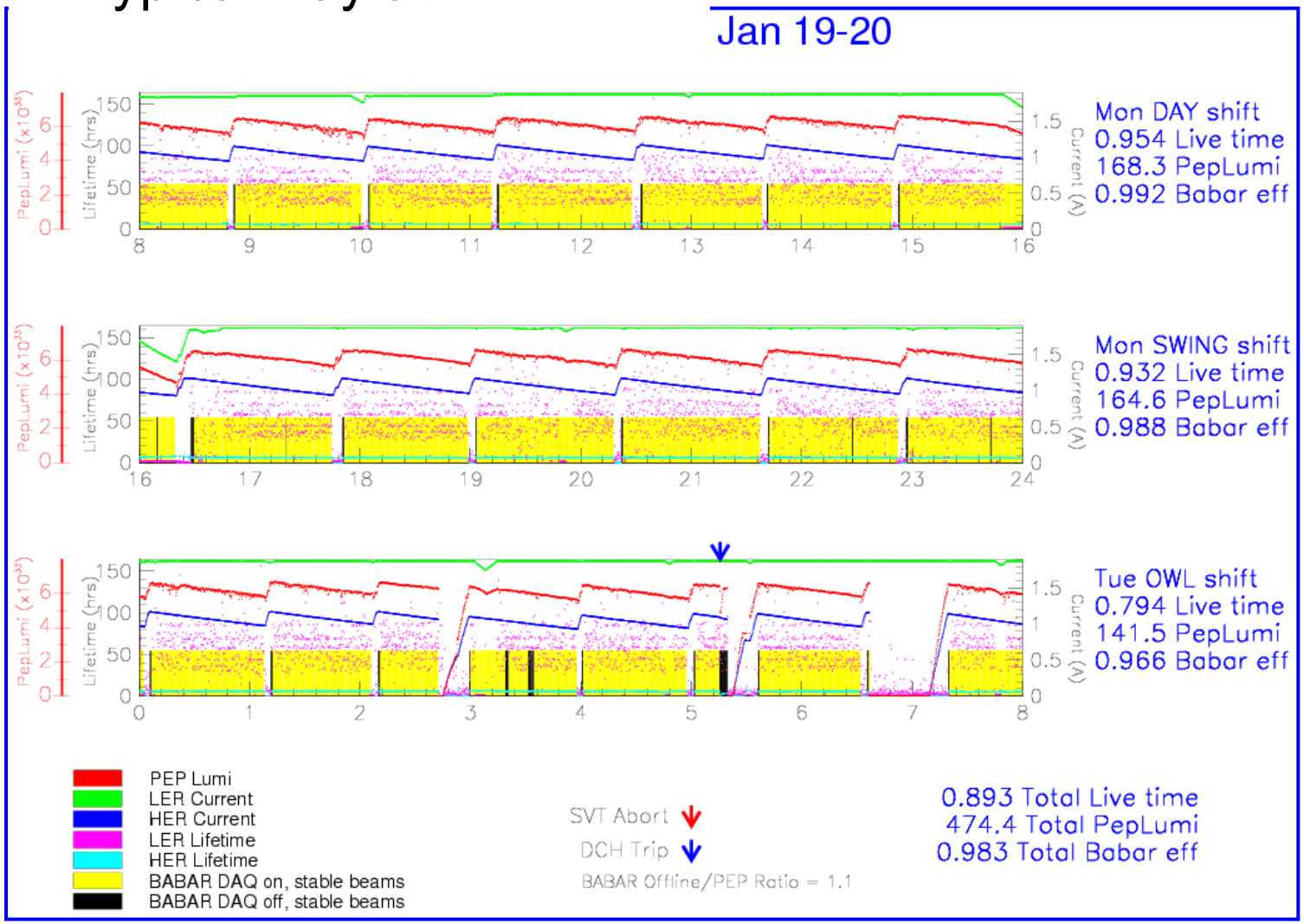
Daily and Integrated Luminosity at PEP-II,

October 1999 to February 2004



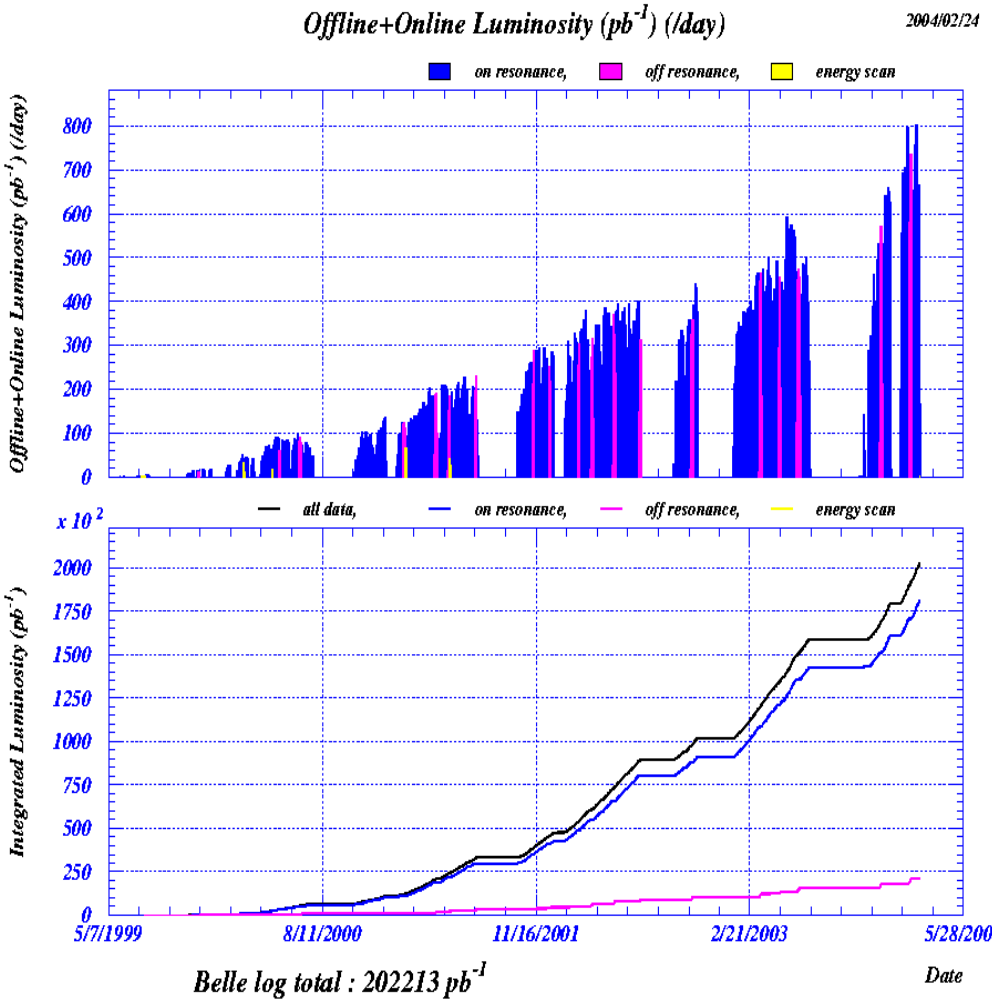


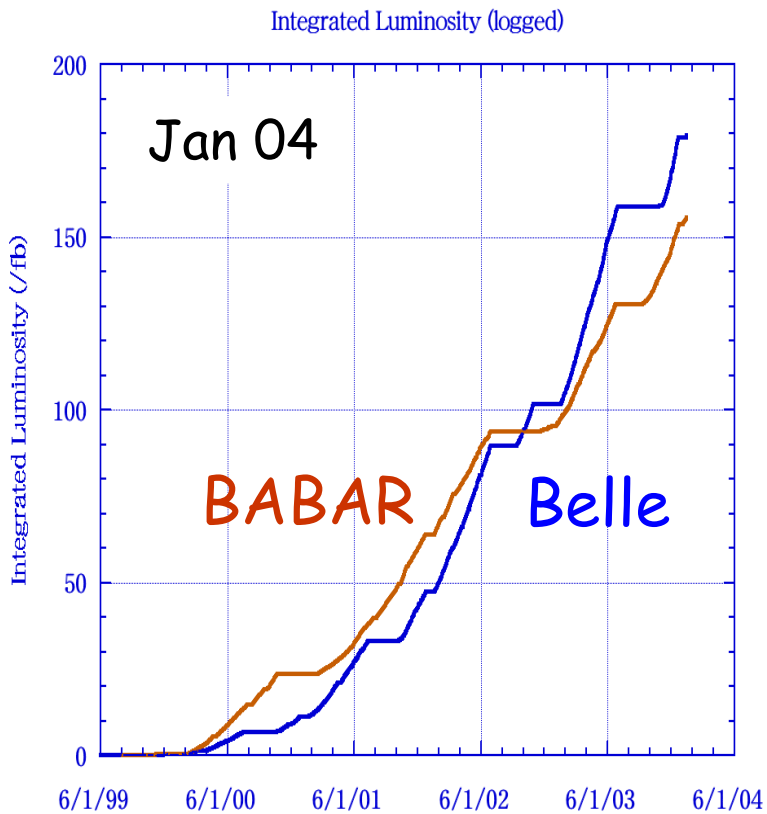
A Typical Day at PEP-II:



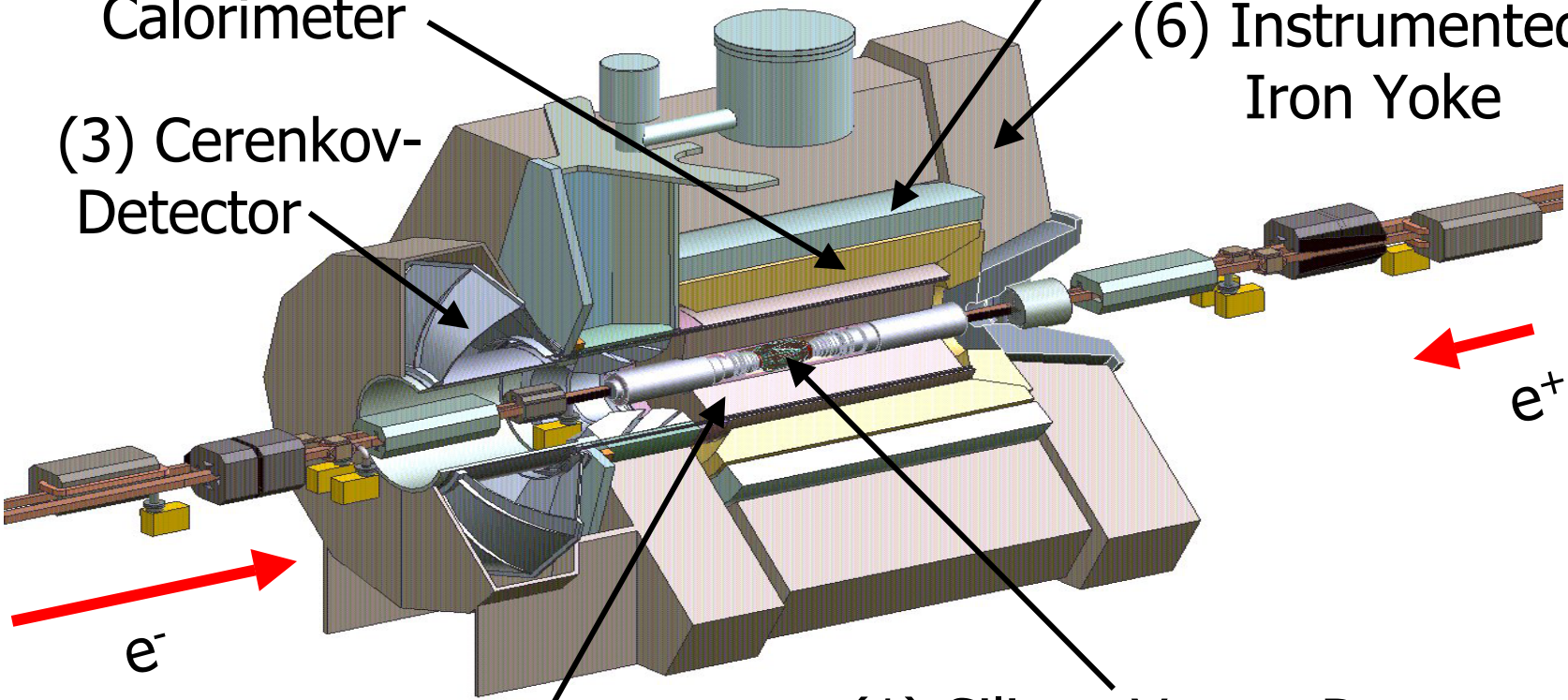
KEKB (23 Feb 04):

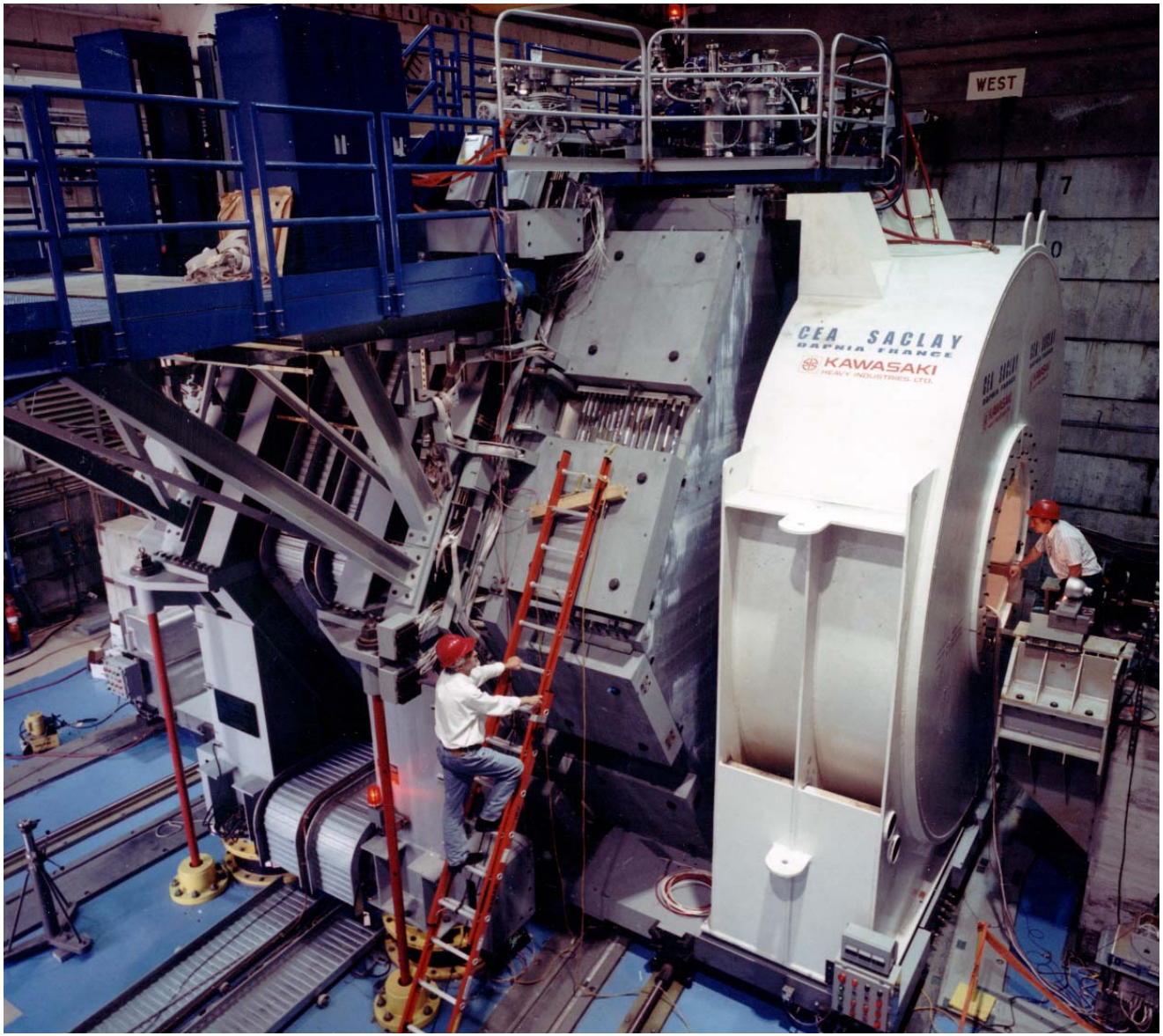
	PEP–II	KEKB	
$\dot{L}_{max}$	$7.9 \cdot 10^{33}$	$11.6 \cdot 10^{33}$	/cm^2/s
$\dot{L}_{bestday}$	552	818	/pb/d
$L_{recorded}$	169	202	/fb





The BABAR Detector:




WEST
CEA SACLAY
KAWASAKI

BABAR Collaboration: 10 Countries, 75 Institutes

China
Inst. of High Energy Physics, Beijing

Germany
RU Bochum
TU Dresden
U Heidelberg
U Rostock

France
LAPP Annecy
LAL Orsay
U Paris 6 et 7
Ecole Polytechnique
CEA Saclay

United Kingdom
U of Birmingham
U of Bristol
Brunel University
U of Edinburgh
U of Liverpool
Imperial College
Queen Mary & Westfield College
Royal Holloway, University of London
U of Manchester
Rutherford Appleton Laboratory

Italy
INFN Bari
INFN Ferrara
INFN Frascati
INFN Genova
INFN Milano
INFN Napoli
INFN Padova
INFN Pavia
INFN Pisa
INFN Roma
INFN Torino
INFN Trieste

Canada
U of British Columbia
McGill U
U de Montréal
U of Victoria

Netherlands
NIKHEF Amsterdam

Norway
U of Bergen

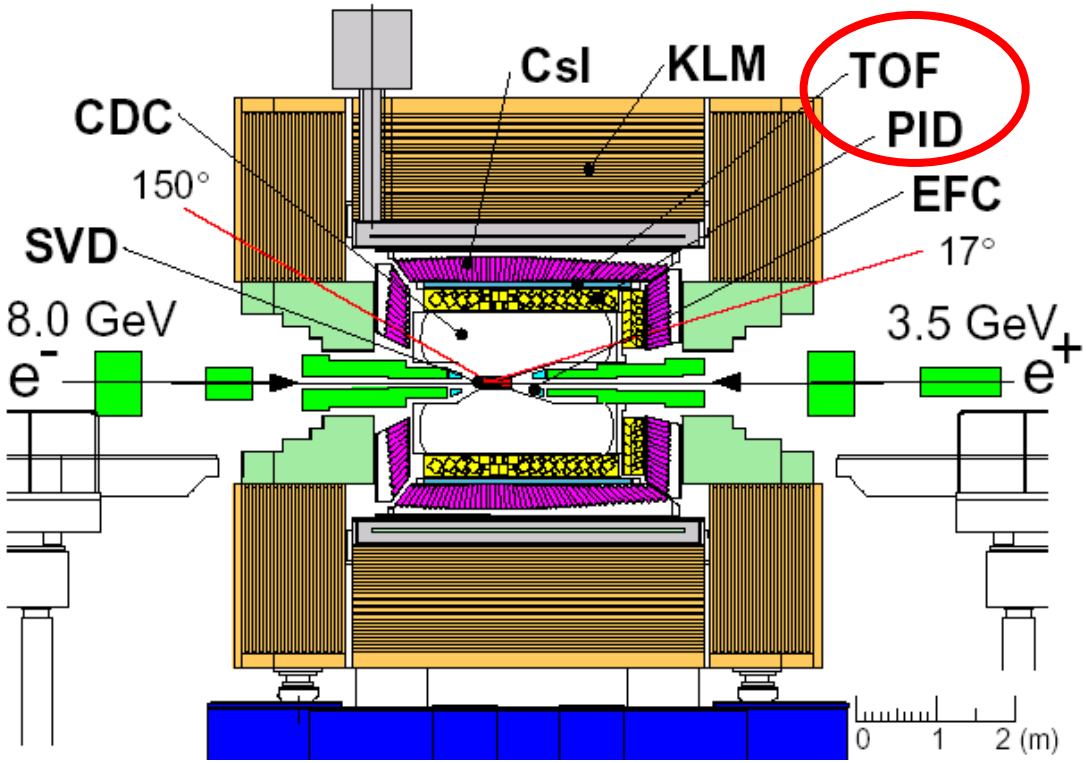
Russia
Budker Inst., Novosibirsk

San Francisco Peninsula Area Map

USA
Caltech Pasadena
UC, Irvine
UC, Los Angeles
UC, San Diego
UC, Santa Barbara
UC, Santa Cruz
U of Cincinnati
U of Colorado
Colorado State
Florida A&M
U of Iowa
Iowa State U
LBNL Berkeley
LLNL Livermore
U of Louisville
U of Maryland
U of Massachusets
MIT
U of Mississippi
Mount Holyoke College
Northern Kentucky U
U of Notre Dame
ORNL
U of Oregon
U of Pennsylvania
Prairie View A&M
Princeton U
SLAC
U of South Carolina
Stanford U
U of Tennessee
U of Texas at Dallas
Vanderbilt U
U of Wisconsin
Yale U

579 physicists

- SVD $\sigma \sim 55\mu m$ for 1GeV/c @ 90°
- CDC $\sigma_p/p \sim 0.35\%$ @ 1GeV/c; $\sigma_\pi(dE/dx) \sim 7\%$
- $K^\pm$ id: TOF ($\sigma \sim 100$ ps); Aerogel ($n = 1.01 \sim 1.03$)
- CsI $\sigma_E/E_\gamma \sim 1.5\%$ @ 1GeV
- KLM (RPCs) $\mu^\pm$: effic. > 90% ; ~2% fakes

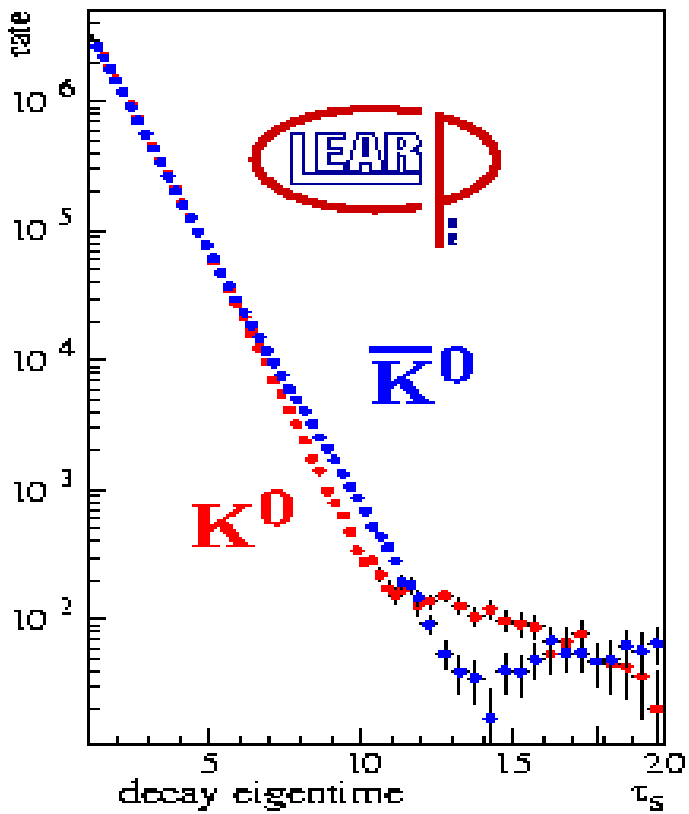


≈ 300 physicists, 49 institutes, 14 countries: Australia, Austria, China, Germany, India, Korea, Japan, Philippines, Poland, Russia, Slovenia, Switzerland, Taiwan, USA

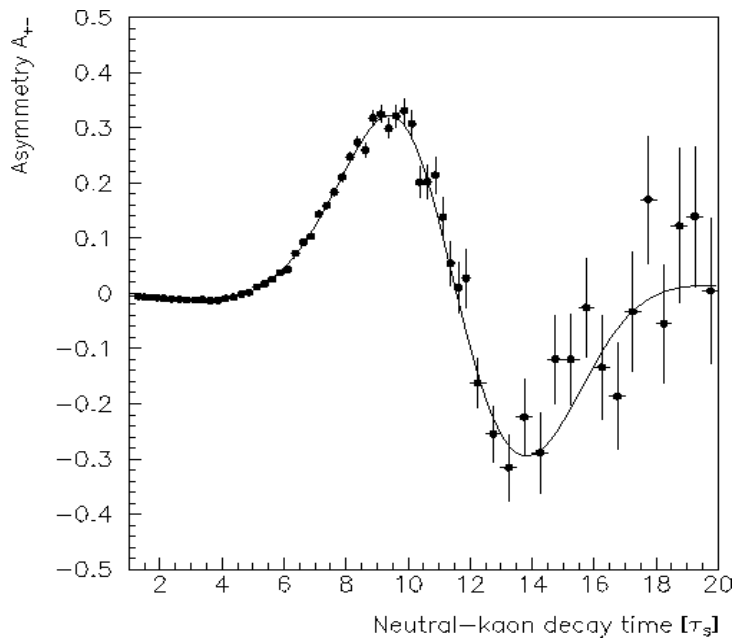
Introduction to CP Violation:

CPV (K) in CPLEAR 1999, $p\bar{p} \rightarrow K^+ \pi^- \bar{K}^0$ and $K^- \pi^+ K^0$;

K^0 and $\bar{K}^0 \rightarrow \pi^+ \pi^-$, Asymmetry between $\pi^+\pi^-$ decays of K^0 and $\bar{K}^0$ as function of time between production and decay:



$$a_{\pi\pi}(t) = \frac{N(\bar{K}^0 \rightarrow \pi^+\pi^-) - N(K^0 \rightarrow \pi^+\pi^-)}{N(\bar{K}^0 \rightarrow \pi^+\pi^-) + N(K^0 \rightarrow \pi^+\pi^-)}$$



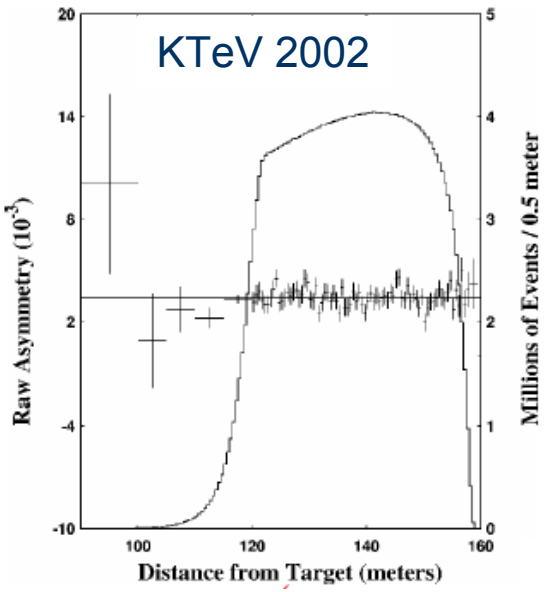
Second Example: CPV (K) in the Experiment KTeV 2002,

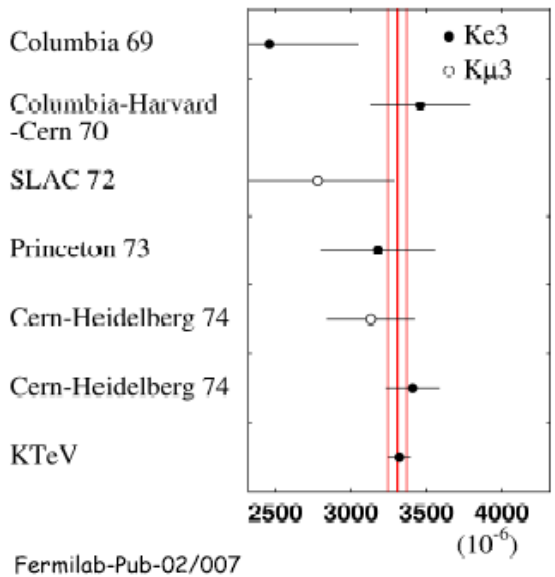
$K^0 \to \pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$, $\bar{K}^0 \to \pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$

$t \to \infty$ $t \to \infty$

Asymmetry between e^+ und e^- Rates,

$$a_{\pi e \nu} = \frac{N(\pi^- e^+ \nu) - N(\pi^+ e^- \bar{\nu})}{N(\pi^- e^+ \nu) + N(\pi^+ e^- \bar{\nu})}$$





$a_{\pi l \nu} = (3.27 \pm 0.12) \cdot 10^{-3}$ [PDG 2003]

$= (3.32 \pm 0.08) \cdot 10^{-3}$ [KTeV 2002 pelim.]

Four Types of CP Violation:

I CPV in Oscillations,
$\mathrm{Re}(\varepsilon_K)$, $\mathrm{Re}(\varepsilon_B)$,

II CPV in the Interference between Oscillations and Decay, $\mathrm{Im}(\varepsilon_0)$, $\sin 2\beta$

III Direct CPV in Decays into one final state,
$\mathrm{Re}(\varepsilon')$

IV Direct CPV in Decays into two final states,
$\mathrm{Im}(\varepsilon')$

Type-I CPV: CPV in Quark-Flavour Oscillations

Nature has four systems for this phenomenon:

$$\bar{s}d \leftrightarrow s\bar{d}\,, \quad c\bar{u} \leftrightarrow \bar{c}u\,, \quad \bar{b}d \leftrightarrow b\bar{d}\,, \quad \bar{b}s \leftrightarrow b\bar{s}\,.$$

$$K^0 \leftrightarrow \overline{K}^0\,, \quad D^0 \leftrightarrow \overline{D}^0\,, \quad B^0 \leftrightarrow \overline{B}^0\,, \quad B_s \leftrightarrow \overline{B}_s\,,$$

$$B_d^0 \leftrightarrow \overline{B}_d^0\,, \quad B_s^0 \leftrightarrow \overline{B}_s^0\,.$$

Time dependence of these transitions given by Schrödinger equation, e.g. for $\psi = \psi_1 K^0 + \psi_2 \overline{K}^0$:

For a stable particle we have $\mathrm{i}\dfrac{\partial}{\partial t}\psi = m\psi, \quad \psi(t) = \psi(0)\cdot e^{-\mathrm{i}mt}\,.$

for an unstable particle

$$\mathrm{i}\frac{\partial}{\partial t}\psi = \mu\psi, \quad \psi(t) = \psi(0)\cdot e^{-\mathrm{i}\mu t} = \psi(0)\cdot e^{-\mathrm{i}mt}\cdot e^{-\Gamma t/2}\,,$$

$$|\psi(t)|^2 = |\psi(0)|^2\cdot e^{-\Gamma t}\,, \quad m = (\mu+\mu^*)/2\,, \quad \Gamma/2 = (\mu-\mu^*)/2\mathrm{i}\,.$$

and for an unstable two-state system

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\mu_{11} & \mu_{12}\\ \mu_{21} & \mu_{22}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \qquad \begin{aligned} m_{ik} &= (\mu_{ik}+\mu_{ik}^+)/2\\ \Gamma_{ik}/2 &= (\mu_{ik}-\mu_{ik}^+)/2\mathrm{i}\end{aligned}$$

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\mu_{ik}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\left[\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

μ_{ik} has 8 real parameters. m_{ik} and Γ_{ik} are hermitian: $m_{21}=m_{12}^*, \quad \Gamma_{21}=\Gamma_{12}^*.$

CPT requires equal mass and equal decay rate for K^0 and $\bar{K}^0$,

$$m_{11}=m_{22}=m, \quad \Gamma_{11}=\Gamma_{22}=\Gamma. \quad 8 \rightarrow 6 \text{ parameters.}$$

m_{12} and Γ_{12} describe transitions from $\bar{K}^0$ to K^0

(Gell-Mann & Pais 1955), m_{21} and Γ_{21} from K^0 to $\bar{K}^0$.

The 6 parameters are: m_{11}, Γ_{11}, Re m_{12}, Im m_{12}, Re Γ_{12}, Im Γ_{12} .

Only 5 are observables because of arbitrary phases.

$K^0 \to K^0 e^{i\Phi}$ and $\bar{K}^0 \to \bar{K}^0 e^{-i\Phi}$ leads to $m_{jj} \to m_{jj}$, $\Gamma_{jj} \to \Gamma_{jj}$, but to

$m_{12} \to m_{12} e^{2i\Phi}$, $\Gamma_{12} \to \Gamma_{12} e^{2i\Phi}$ with the same phase Φ.

The 5 observables are: m, Γ, $|m_{12}|$, $\mathrm{Re}(\Gamma_{12}/m_{12})$, $\mathrm{Im}(\Gamma_{12}/m_{12})$.

$i\partial\psi_i / \partial t = \mu_{ik}\psi_k$ has well-defined solution $\psi(t)$ for each $\psi(0)$.

There are 2 and only 2 states with t-independent flavour composition.

With $|p|^2 + |q|^2 = 1$:

$$K_S^0(t) = \left(pK^0 + q\bar{K}^0\right) \cdot e^{-\gamma_S t}, \quad \gamma_S = i\,m_S + \Gamma_S/2,$$
$$K_L^0(t) = \left(pK^0 - q\bar{K}^0\right) \cdot e^{-\gamma_L t}, \quad \gamma_L = i\,m_L + \Gamma_L/2.$$

q/p is not an observable, only $|q/p|$. The 5 observables

m_S, m_L, Γ_S, Γ_L, $|q/p|$ are given by the 5 parameters of μ_{ik}

Explicit formulae in the literature, e.g.
Branco-Lavoura-Silva, „CP Violation“
an example:

$$\frac{q}{p} = \frac{-2m_{12}^* + i\Gamma_{12}^*}{2|m_{12}| + i|\Gamma_{12}|}$$

Evolution of the 2 states $\psi_K(0) = K^0$ and $\psi_{\overline{K}}(0) = \overline{K}^0$

$$\psi_K(t) = \frac{e^{-\gamma_S t} + e^{-\gamma_L t}}{2} K^0 + \frac{q}{p} \frac{e^{-\gamma_S t} - e^{-\gamma_L t}}{2} \overline{K}^0, \quad \psi_{\overline{K}}(t) = \frac{e^{-\gamma_S t} + e^{-\gamma_L t}}{2} \overline{K}^0 + \frac{p}{q} \frac{e^{-\gamma_S t} - e^{-\gamma_L t}}{2} K^0.$$

With $\Delta m = m_L - m_S$:

$$P(K^0 \to K^0) = P(\overline{K}^0 \to \overline{K}^0) = \left| \frac{e^{-\gamma_S t} + e^{-\gamma_L t}}{2} \right|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right],$$

equal because of CPT.

$$P(K^0 \to \overline{K}^0) = \frac{1}{4}\left| \frac{q}{p} \right|^2 \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right],$$

$$P(\overline{K}^0 \to K^0) = \frac{1}{4}\left| \frac{p}{q} \right|^2 \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos \Delta m t \right].$$

CP symmetric if and only if $\left| p/q \right| = 1 \quad \Leftrightarrow \quad \mathrm{Im}\left(\Gamma_{12} / m_{12} \right) = 0$

$$\varepsilon_K = \frac{p - q}{p + q}, \quad p = \frac{1 + \varepsilon_K}{\sqrt{2 + 2\left|\varepsilon_K\right|^2}}, \quad q = \frac{1 - \varepsilon_K}{\sqrt{2 + 2\left|\varepsilon_K\right|^2}}, \quad \frac{p}{q} = \frac{1 + \varepsilon_K}{1 - \varepsilon_K}.$$

We know (since 1964) that |p/q| ≠ 1, but very close to 1,

$$\left|\frac{p}{q}\right| = 1+\frac{2\,\mathrm{Re}(\varepsilon_K)}{1+|\varepsilon_K|^2} = 1+2\,\mathrm{Re}(\varepsilon_K), \quad \mathrm{Re}(\varepsilon_K)<<1.$$

Im ε_K is unobservable. If chosen small for simplicity, then

Re ε_K <<1 means small phase between Γ_{12} and m_{12}.

Re(Γ_{12}/m_{12}) is negative: $\Gamma_{12}/m_{12} = -|\Gamma_{12}/m_{12}|\, e^{-i\Phi_m} \quad \Rightarrow \quad \mathrm{Re}(\varepsilon_K) \approx \Phi_m/4$

Experimental Results: see page 66, based on $K^0 \to \ell^+$, $\overline{K}^0 \to \ell^-$:

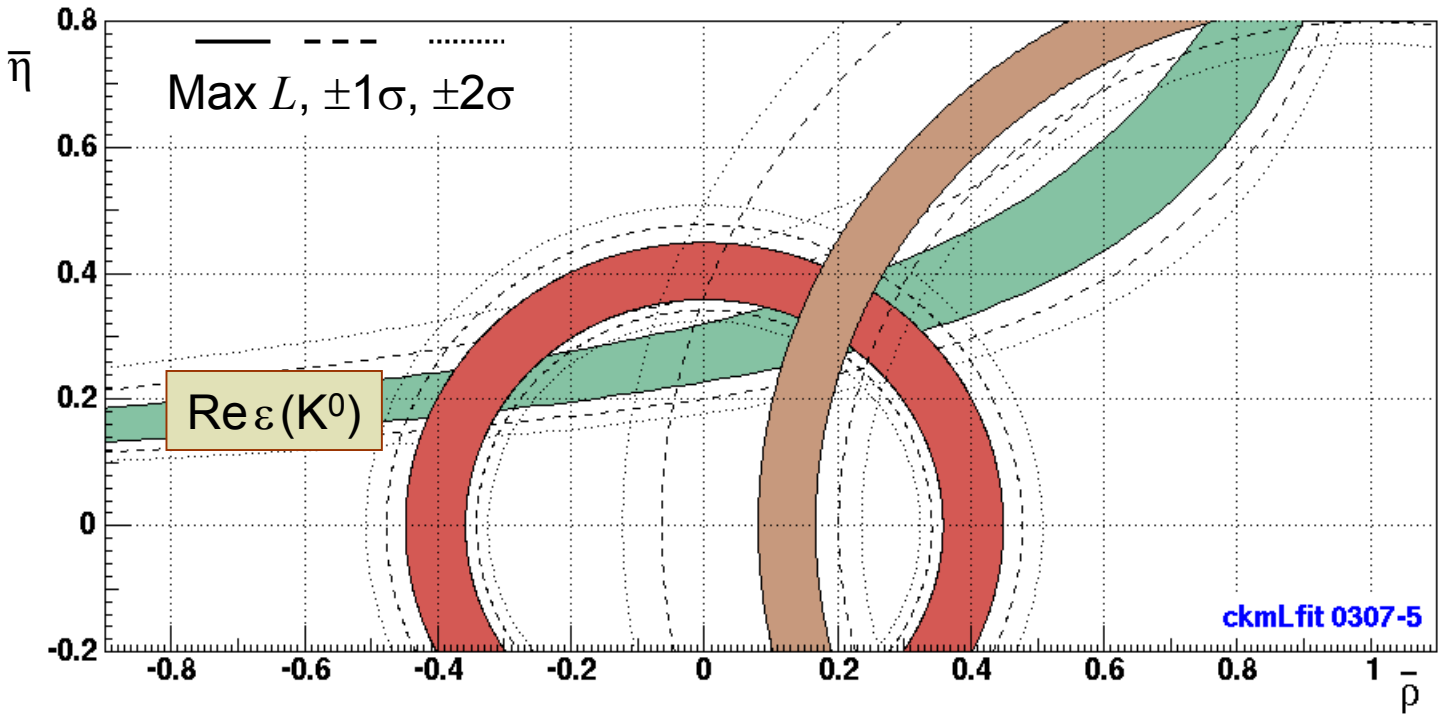
$$a_{\pi\ell\nu} = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)} = \frac{|p|^2-|q|^2}{|p|^2+|q|^2} = (0.327 \pm 0.012)\%.$$

$$\left|\frac{p}{q}\right| = 1.00327 \pm 0.00012\,, \quad \mathrm{Re}(\varepsilon_K) = (1.64 \pm 0.06)\cdot 10^{-3}, \quad \Phi_m = 0.0065 \approx 0.37^0.$$

For B^0 Mesons: $\mathrm{Re}(\varepsilon_B) = (0.7 \pm 3.2)\cdot 10^{-3}$. [PDG 2003]

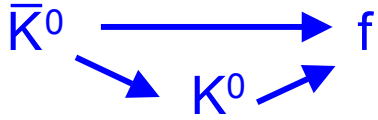
ckm Lfit

Standard Model calculation of Re ε leads to:



This exactly matches one of the two solutions of (ρ, η) from measurements of $|V_{ub}|$ and $|V_{td}|$.

Type-II CPV: Interference between Oscillations and Decay



In this type, f is a CP eigenstate, **CP** f = η_f f.

First example: f = $\pi\pi$, I = 0, η_f = +1.

$$< 2\pi, I=0 | T | K^0 > = A_0 \,, \quad < 2\pi, I=0 | T | \overline{K}^0 > = \overline{A}_0 \,.$$

[up to a common normalisation factor]

Decay rates N of states ψ_K and $\overline{N}$ of $\psi_{\overline{K}}$:

$$N(t) = \left| \frac{e^{-\gamma_S t} + e^{-\gamma_L t}}{2} A_0 + \frac{q}{p} \frac{e^{-\gamma_S t} - e^{-\gamma_L t}}{2} \overline{A}_0 \right|^2 ,$$

$$\overline{N}(t) = \left| \frac{e^{-\gamma_S t} + e^{-\gamma_L t}}{2} \overline{A}_0 + \frac{p}{q} \frac{e^{-\gamma_S t} - e^{-\gamma_L t}}{2} A_0 \right|^2 .$$

Def.: $\lambda = \dfrac{\overline{\quad}}{\quad 0 \quad} \Big/ \quad 0$

In contrast to q/p and $\overline{A}_0/A_0$, λ_0 is observable in magnitude and phase. Using λ_0:

$$N(t)=\frac{|A_0|^2|1+\lambda_0|^2}{4}\cdot\left|e^{-\gamma_S t}+\frac{1-\lambda_0}{1+\lambda_0}e^{-\gamma_L t}\right|^2,\quad \overline{N}(t)=\frac{|A_0|^2|1+\lambda_0|^2}{4}\cdot\left|\frac{p}{q}\right|^2\cdot\left|e^{-\gamma_S t}-\frac{1-\lambda_0}{1+\lambda_0}e^{-\gamma_L t}\right|^2.$$

With $\frac{1-\lambda_0}{1+\lambda_0}=\varepsilon_0 \quad\Leftrightarrow\quad \frac{1-\varepsilon_0}{1+\varepsilon_0}=\lambda_0$ we obtain for $a_{\pi\pi}=(\overline{N}-N)/(\overline{N}+N)$:

$$a_{\pi\pi}(t)=\frac{2\,\mathrm{Re}(\varepsilon_0)}{1+|\varepsilon_0|^2}-\frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cdot\left[\mathrm{Re}(\varepsilon_0)\cos\Delta m t+\mathrm{Im}(\varepsilon_0)\sin\Delta m t\right]}{e^{-\Gamma_S t}+|\varepsilon_0|^2 e^{-\Gamma_L t}}\qquad \varepsilon_0=\frac{\langle 2\pi, I=0|T|K_L\rangle}{\langle 2\pi, I=0|T|K_S\rangle}$$

$a_{\pi\pi}(t)=0$ for $\lambda_0=+1$ and $\lambda_0=-1$. CPV $\Leftrightarrow \lambda_0 \neq \pm 1$. Three possibilities:

$$\lambda_0=\frac{q\overline{A}_0}{pA_0}=\left|\frac{q}{p}\right|\cdot\left|\frac{\overline{A}_0}{A_0}\right|\cdot e^{-2i\phi_0}$$

(1) $|q/p|\neq 1$, CPV in oscillations,

(2) $|\bar{A}_0/A_0|\neq 1$, CPV in the decay, here forbidden,

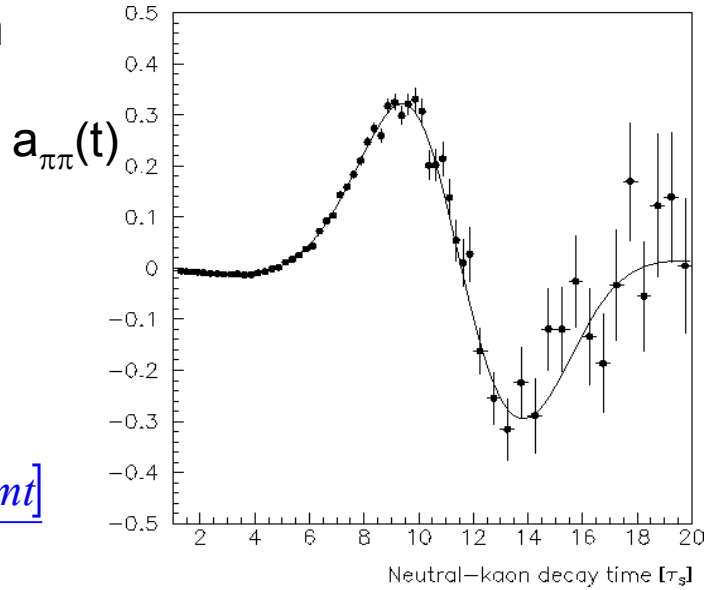
(3) $\Phi_0 \neq n\pi/2$, CPV in interference oscillation/decay.

$$\lambda_0=\left|\frac{q}{p}\right|e^{-2i\phi_0}=\left[1-2\,\mathrm{Re}(\varepsilon_K)\right]\left[1-2i\phi_0\right],\quad \varepsilon_0=\frac{1-\lambda_0}{1+\lambda_0}=\mathrm{Re}(\varepsilon_K)+i\phi_0\,,$$

$$\mathrm{Re}(\varepsilon_0)=\mathrm{Re}(\varepsilon_K),\quad \mathrm{Im}(\varepsilon_0)=\phi_0\,.$$

The CPLEAR result in the approximation

$\varepsilon_0 \approx \eta_{+-} = \langle \pi^+ \pi^- | T | K_L \rangle / \langle \pi^+ \pi^- | T | K_S \rangle$:



$$a_{\pi\pi}(t) = \frac{N(\bar{K}^0 \to \pi^+\pi^-) - N(K^0 \to \pi^+\pi^-)}{N(\bar{K}^0 \to \pi^+\pi^-) + N(K^0 \to \pi^+\pi^-)}$$

$$= \frac{2\mathrm{Re}(\varepsilon_0)}{1+|\varepsilon_0|^2} - \frac{2e^{-(\Gamma_S+\Gamma_L)t/2} \cdot [\mathrm{Re}(\varepsilon_0)\cos\Delta mt + \mathrm{Im}(\varepsilon_0)\sin\Delta mt]}{e^{-\Gamma_S t} + |\varepsilon_0|^2 e^{-\Gamma_L t}}$$

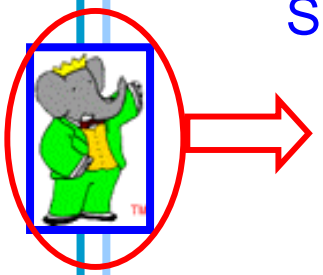
$$\varepsilon_0 = (2.27 \pm 0.02) \cdot 10^{-3} \cdot e^{i(43.3 \pm 0.5)^0}$$

Two types of CPV observed,

I: $\mathrm{Re}(\varepsilon_0) = \mathrm{Re}(\varepsilon_K) = 1{,}6 \cdot 10^{-3}$ = ¼ of the phase between m_{12} and Γ_{12}.

II: $\mathrm{Im}(\varepsilon_0) = \Phi_0 \quad = 1{,}5 \cdot 10^{-3}$ = ½ of the phase between q/p and $\bar{A}_0/A_0$.

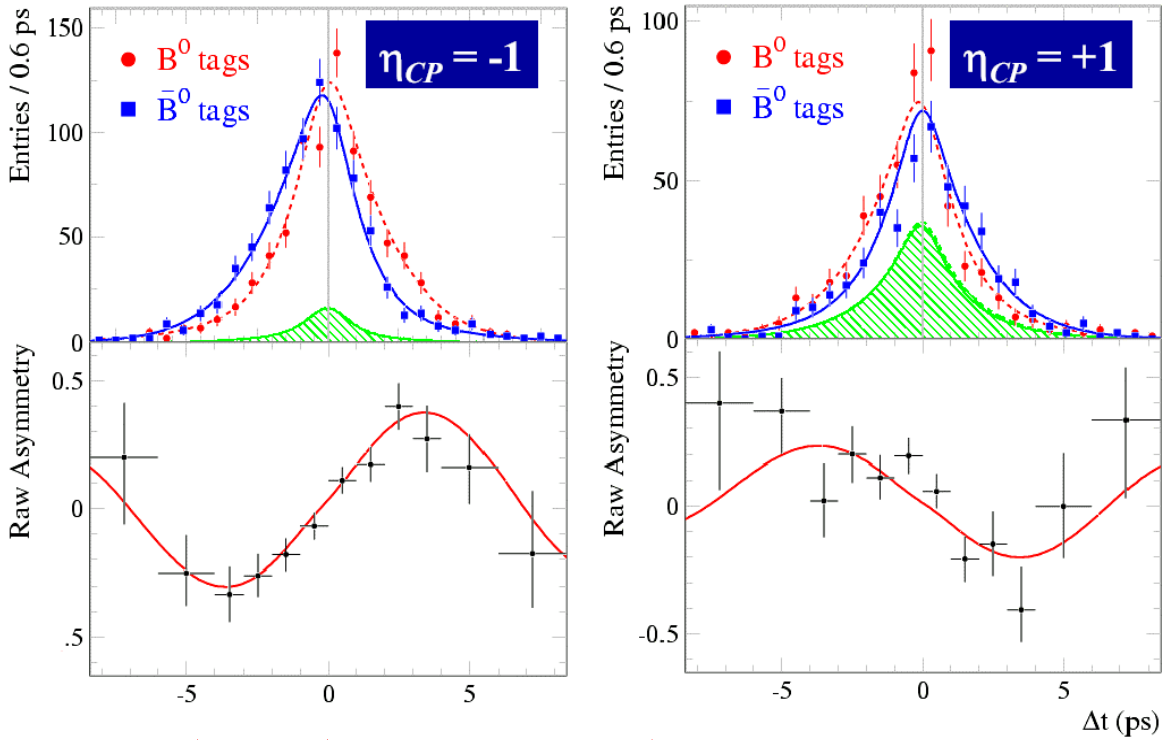
Second example: $B^0, \bar{B}^0 \to c\bar{c}\,K$



BABAR,

July 2002,

81/fb



$$a_{\psi K} = \frac{\Gamma\left(\overline{B}^0 \to J/\psi\, K_S^0\right) - \Gamma\left(B^0 \to J/\psi\, K_S^0\right)}{\Gamma\left(\overline{B}^0 \to J/\psi\, K_S^0\right) + \Gamma\left(B^0 \to J/\psi\, K_S^0\right)} = \sin 2\beta \cdot \sin\left(\Delta m_B t\right);$$

BABAR July 2002: $\sin 2\beta = 0.741 \pm 0.067 \pm 0.033$

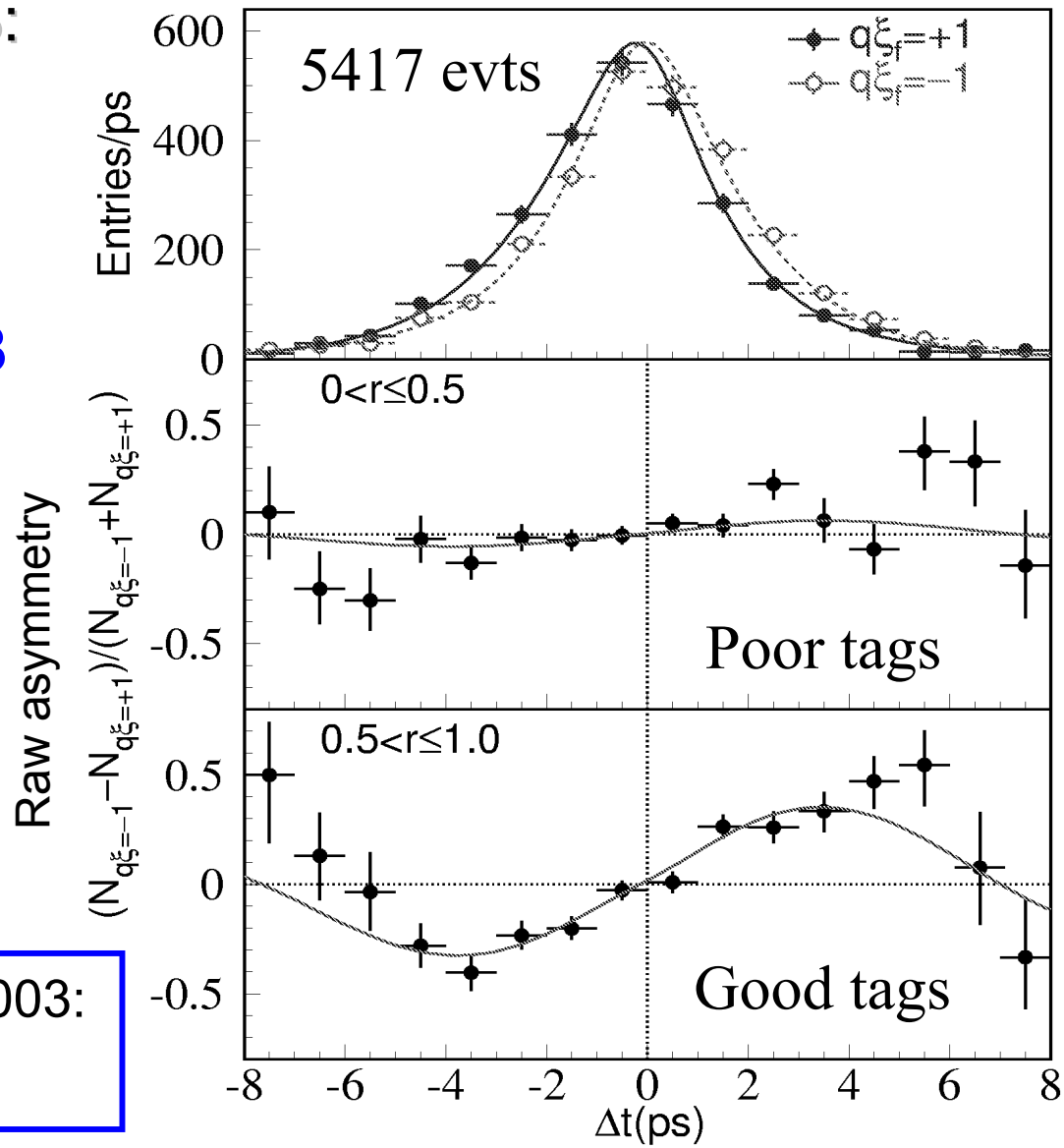
Belle August 2003:

140 fb^{-1}

$\sin 2\phi_1 =$

$0.733 \pm 0.057 \pm 0.028$

$(\phi_1 = \beta)$



World average Aug. 2003:

$\sin 2\beta = 0.737 \pm 0.048$.

$$\lambda_{\psi K} = \left(\frac{q}{p}\right)_B \frac{\overline{A}_{\psi K}}{A_{\psi K}}$$

with |q/p| = 1 and |$\bar{A}$/A| = 1, but with a relative phase 2β between q/p and $\bar{A}$/A .

$\lambda_{\psi K} = - e^{-2 i \beta}$.

$$\varepsilon_{\psi K} = \frac{1-\lambda_{\psi K}}{1+\lambda_{\psi K}} = \frac{1+e^{-2i\beta}}{1-e^{-2i\beta}} = \frac{1}{i\cdot\tan\beta}, \quad \mathrm{Re}(\varepsilon_{\psi K}) = 0, \quad \mathrm{Im}(\varepsilon_{\psi K}) = -\cot\beta .$$

$$a_{\psi K}(t) = \frac{-2e^{-(\Gamma_S+\Gamma_L)t/2}\cdot\left[\mathrm{Re}(\varepsilon_{\psi K})\cos\Delta mt + \mathrm{Im}(\varepsilon_{\psi K})\sin\Delta mt\right]}{e^{-\Gamma_S t} + \left|\varepsilon_{\psi K}\right|^2 e^{-\Gamma_L t}}$$

$$\Delta m \to \Delta m_B, \quad \Gamma_L \to \Gamma_H, \quad \Gamma_S \to \Gamma_L, \quad \Gamma_L = \Gamma_H \quad \Rightarrow \quad a_{\psi K}(t) = \frac{+2\cot\beta\cdot\sin\Delta m_B t}{1+\cot^2\beta} = \sin 2\beta\cdot\sin\Delta m_B t.$$

This asymmetry contains only Type II,

not types I and II as in K $\to\pi\pi$.

Standard Model derivation of $r(B^0 \to J/\psi K^0, K^0 \to \pi^+ \pi^-)$:

This is a cascade decay and requires both $B\bar{B}$ and $K\bar{K}$ oscillations.

The derivation here is only valid for early $K \to \pi\pi$ lifetimes

before $K\bar{K}$ mixing starts. $m_{12}(B^0)$ from box graph:

$$m_{12}\left(B^0\right) = -k \cdot \left(V_{tb} V_{td}^*\right)^2 , \quad \left|\Gamma_{12}\right| << \left|m_{12}\right| \quad \Rightarrow$$

$$\left(\frac{q}{p}\right)_B = \frac{-2m_{12}^* + i\Gamma_{12}^*}{2\left|m_{12}\right| + i\left|\Gamma_{12}\right|} \approx \frac{-2m_{12}^*}{2\left|m_{12}\right|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$

not a phase invariant quartet

$$f = J/\psi\left(\pi^+\pi^-\right)_K , \quad \text{CP } f = -f , \quad \eta_f = -1$$

phase-invariant 12-plet

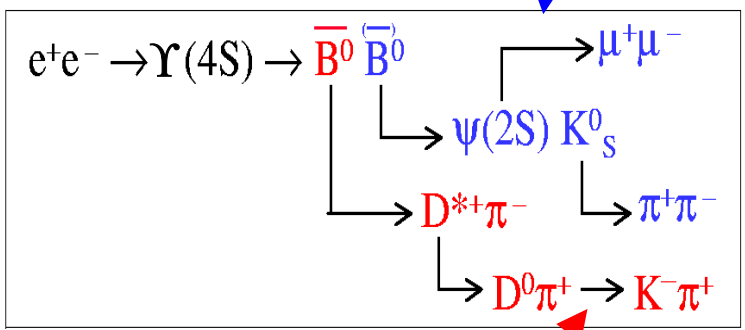
$$\lambda = \left(\frac{q}{p}\right)_B \frac{\bar{A}_f}{A_f} = \left(\frac{q}{p}\right)_B \eta_f \frac{\bar{A}_{\bar{f}}}{A_f} = (-1) \cdot \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \cdot \frac{V_{us} V_{ud}^*}{V_{us}^* V_{ud}} = -e^{-2i\beta}$$

not a phase-invariant octet

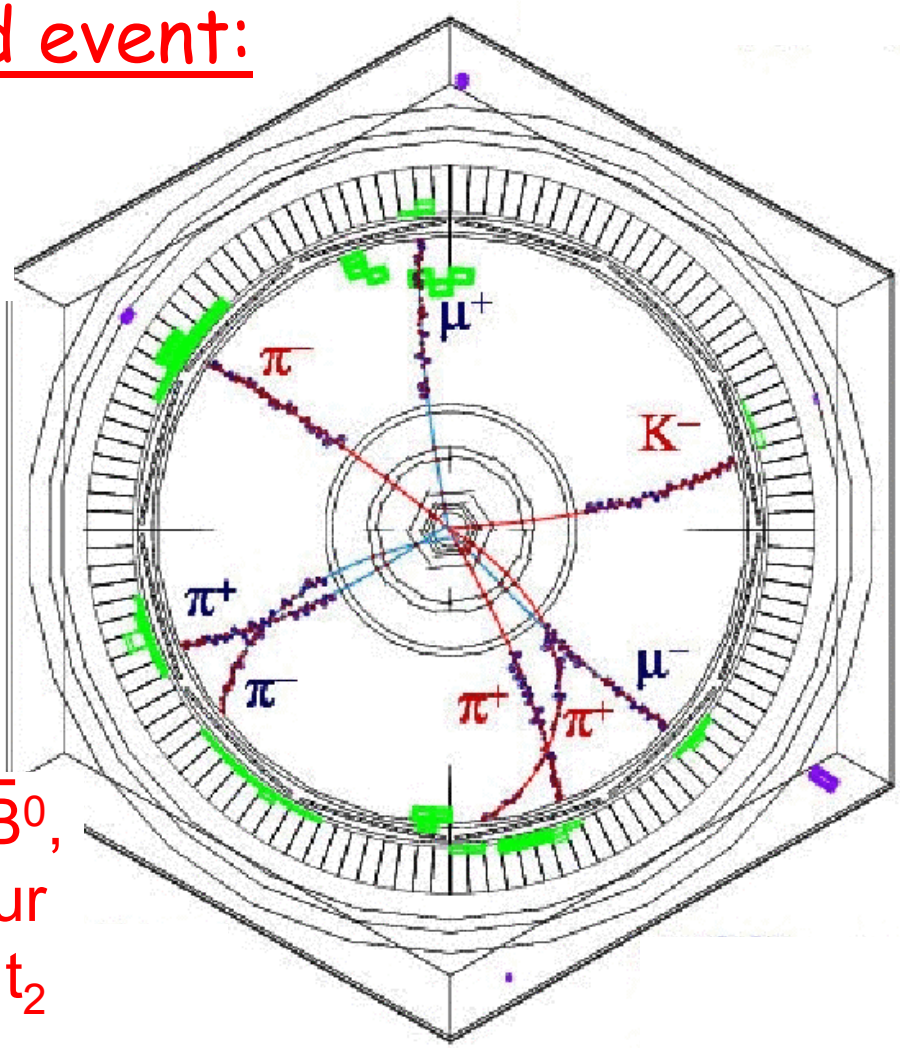
$B \to c\bar{c}K$ $\quad$ $K \to \pi\pi$

A fully reconstructed event:

CP eigenstate, decay at t_1, either from $\bar{B}^0$ or from B^0



flavour eigenstate,here from $\bar{B}^0$, decay at t_2, „tags“ the flavour of the other B at time t_2

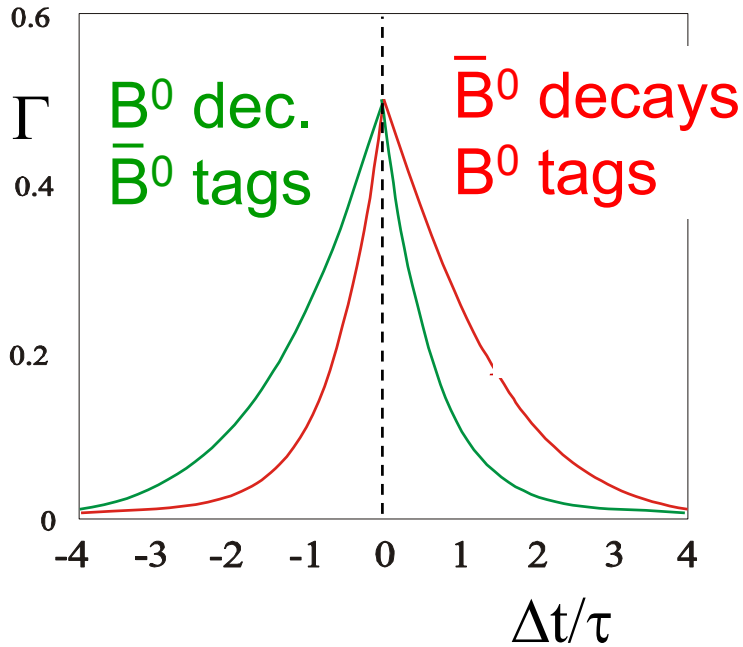


Such events are very rare, o(10^{-6}), therefore BABAR and Belle use inclusive tagging for CP asymmetry measurement.

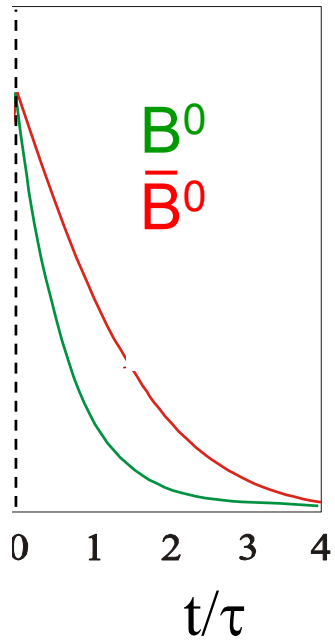
Searched CP-Asymmetry:

$$A = \frac{\Gamma\left(\overline{B}^0 \to J/\psi\, K_S^0\right) - \Gamma\left(B^0 \to J/\psi\, K_S^0\right)}{\Gamma\left(\overline{B}^0 \to J/\psi\, K_S^0\right) + \Gamma\left(B^0 \to J/\psi\, K_S^0\right)} = \sin 2\beta \cdot \sin\left[\Delta m\left(t_{decay} - t_{tag}\right)\right]$$

Because of coherence:



Incoherent case:

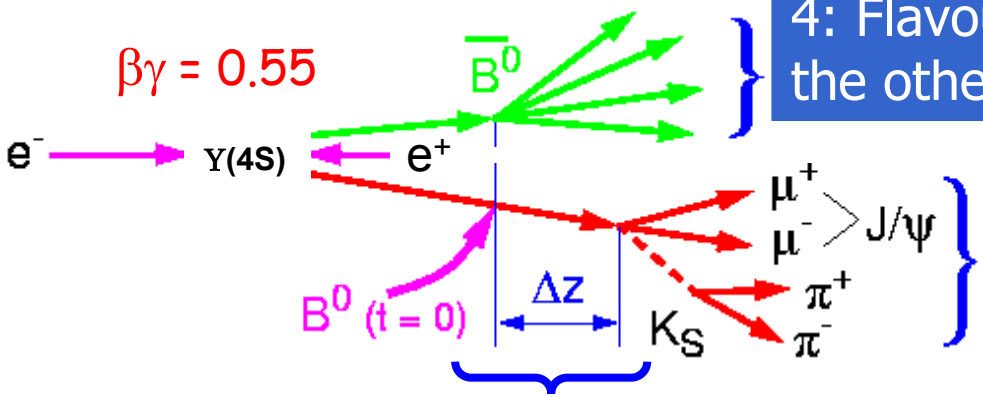


Integral of *A* over all Δt is zero.

Therefore it is necessary to measure the Δt dependence (and - for technical reasons - to boost the ϒ4S.)

Measurement of time-dependent CP-Asymmetry:

$$\tilde{A}(\Delta\tilde{t}) = \frac{\Gamma(\bar{B}^0 \to J/\Psi K_S^0) - \Gamma(B^0 \to J/\Psi K_S^0)}{\Gamma(\bar{B}^0 \to J/\Psi K_S^0) + \Gamma(B^0 \to J/\Psi K_S^0)} = D \cdot \sin 2\beta \cdot \int \sin(\Delta m\, \Delta t) \cdot r(\Delta\tilde{t} - \Delta t)\, d\Delta t$$



4: Flavour Determination of the other B Meson ("tag")

1, 2, 3: B Reconstruction into CP Eigenstate

6: Determination of $\Delta t = \Delta z/\beta\gamma c$

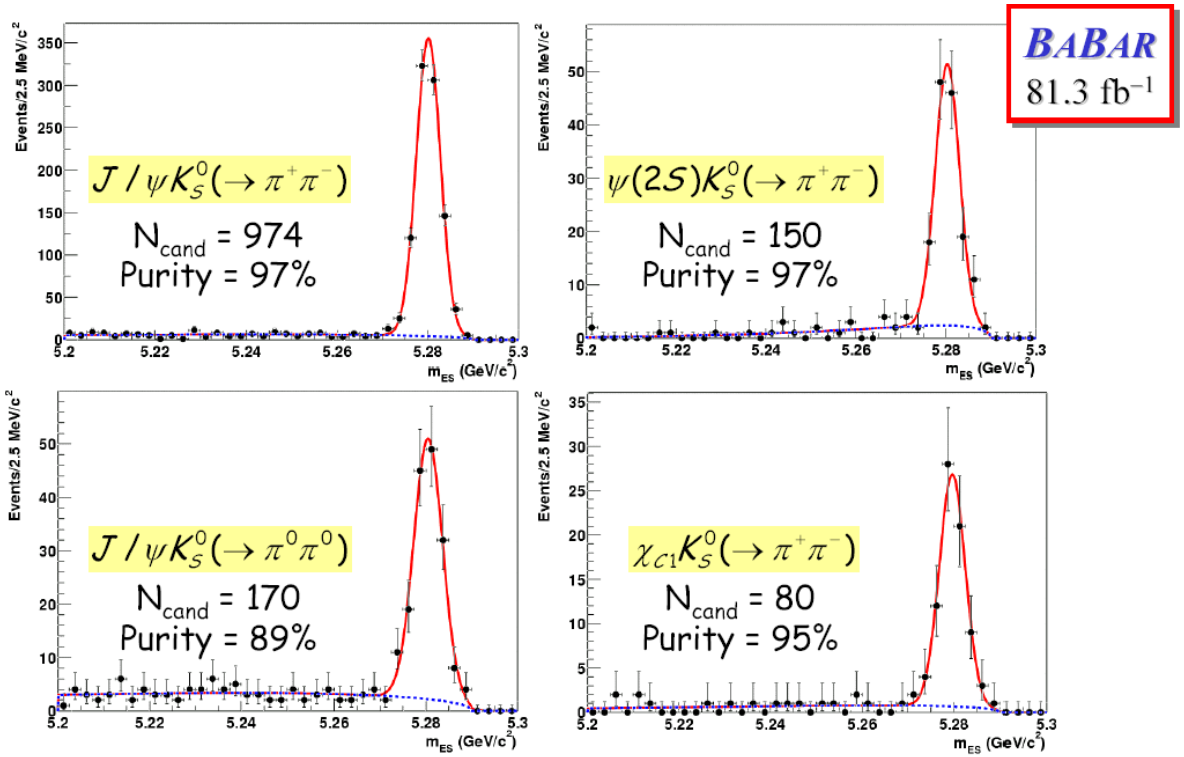
7: Determination of the Δz resolution function

5: Determination of the fraction *w* of mistags

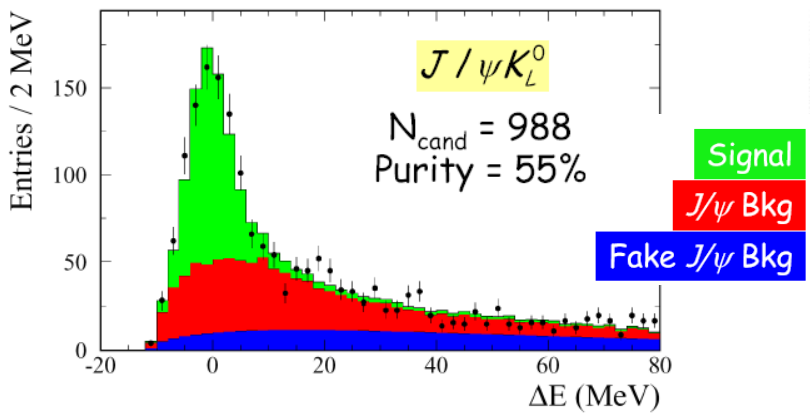
Dilution D = (1-2w) reduces observed asymmetry.

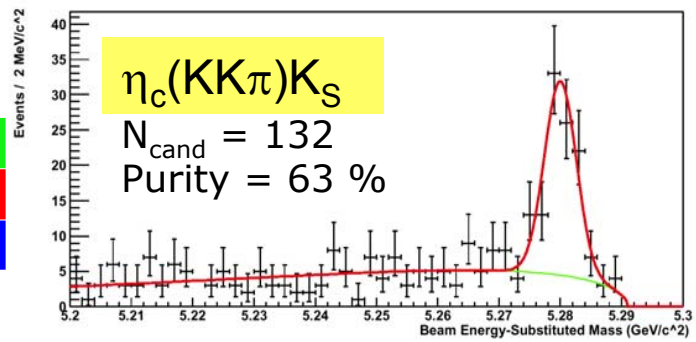
8: sin2β Fit to both Time Distributions

$c\bar{c}K_S$ modes:

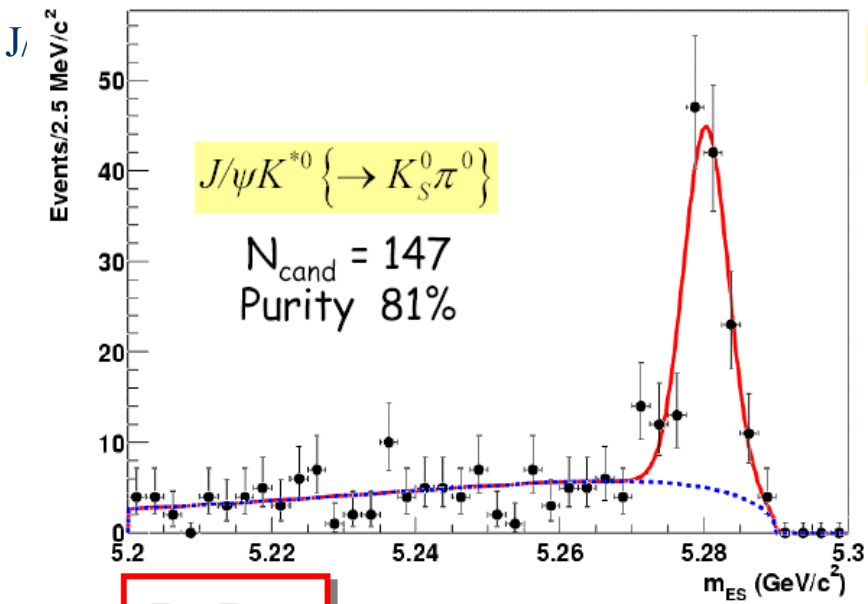


$J/\psi K_L$:





One more mode:

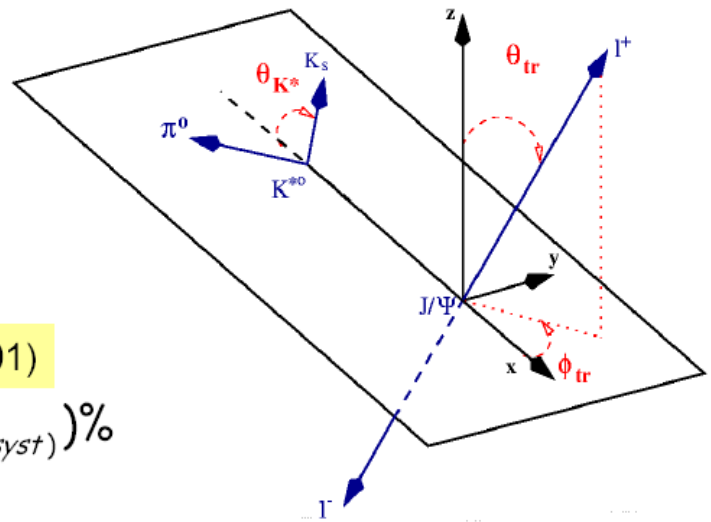


BABAR
81.3 fb⁻¹

$P \to VV$: mixture of *CP* states

$D_\perp = 1 - 2 \cdot R_\perp$,

$R_\perp \equiv |A_\perp|^2$ fraction of *CP*-odd



BABAR PRL **87**, 241801 (2001)

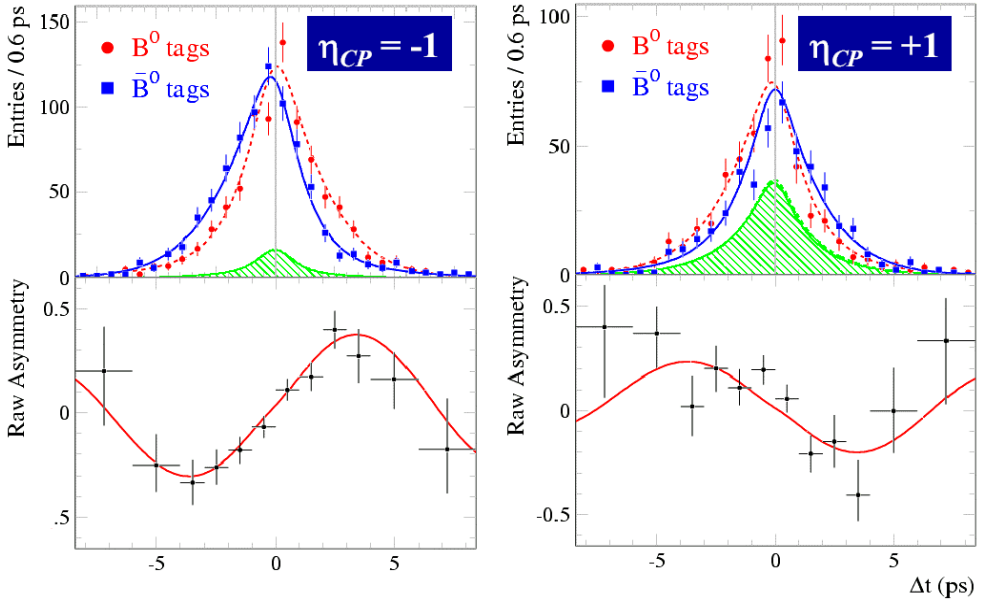
$R_\perp = (16.0 \pm 3.2_{(stat)} \pm 1.4_{(syst)})\%$

BELLE hep-ex/0205021

$R_\perp = (19.1 \pm 2.3_{(stat)} \pm 2.6_{(syst)})\%$

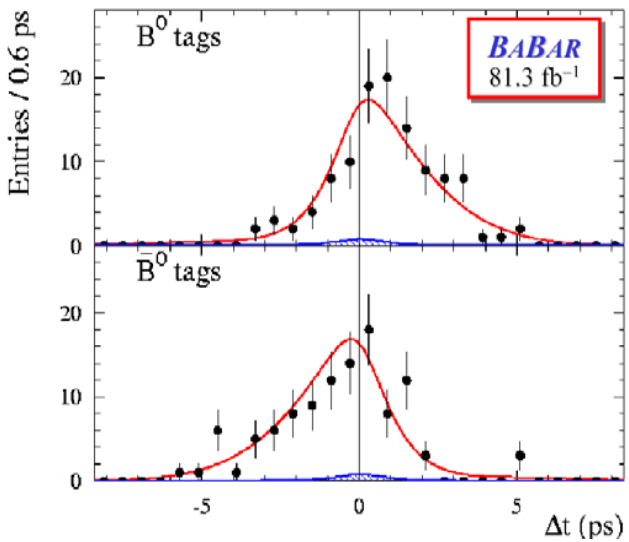
Conclude: J/ψ mostly *CP* even

$D = 0.68 \pm 0.07$

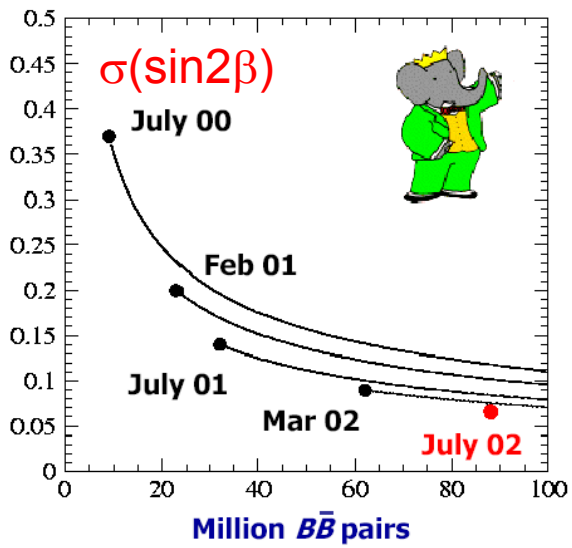


220 lepton-tagged
η_f = -1 events

98% purity
3.3% mistag rate
20% better Δt
resolution

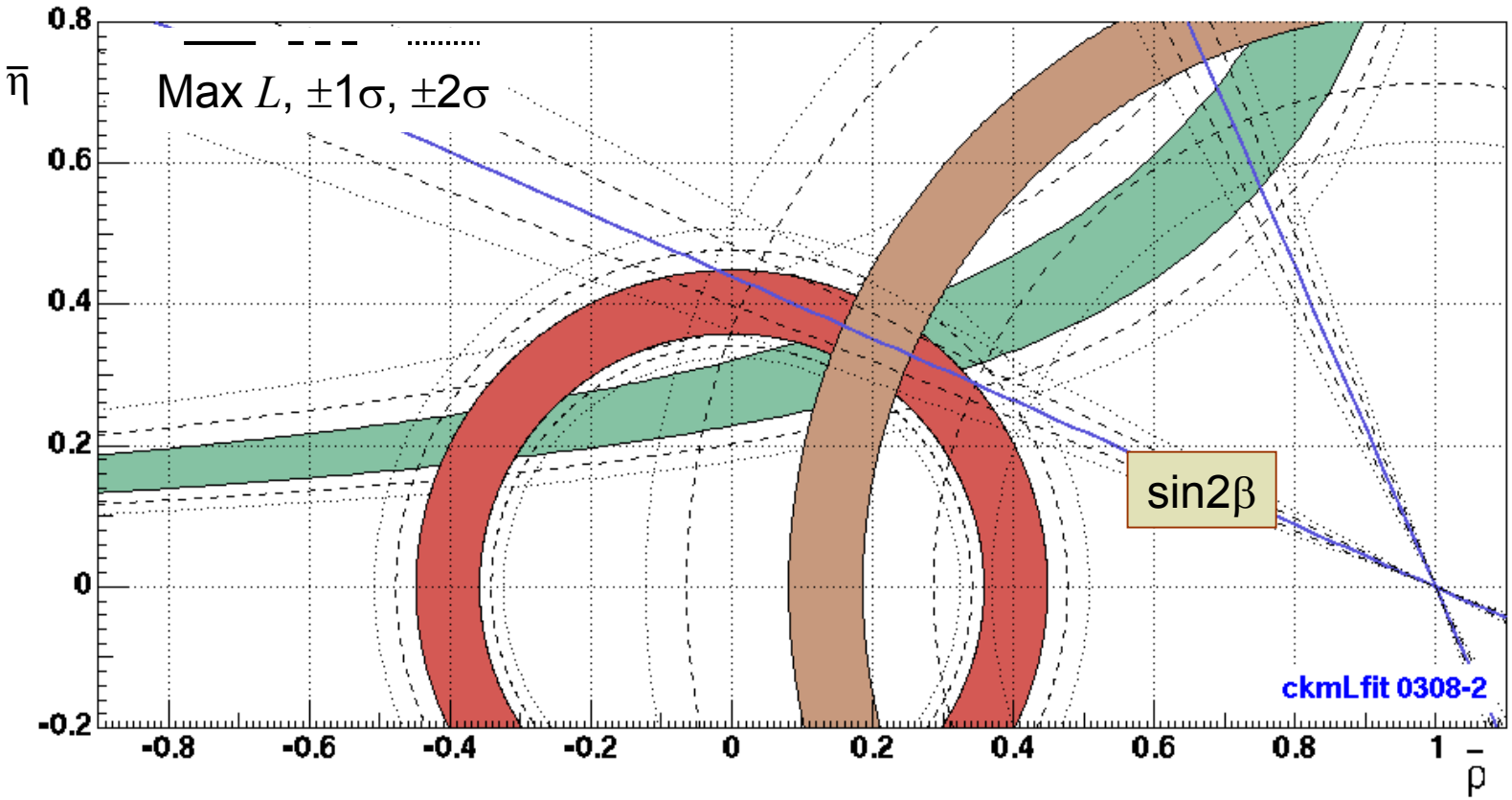


Results have been improving by more than just luminosity gain

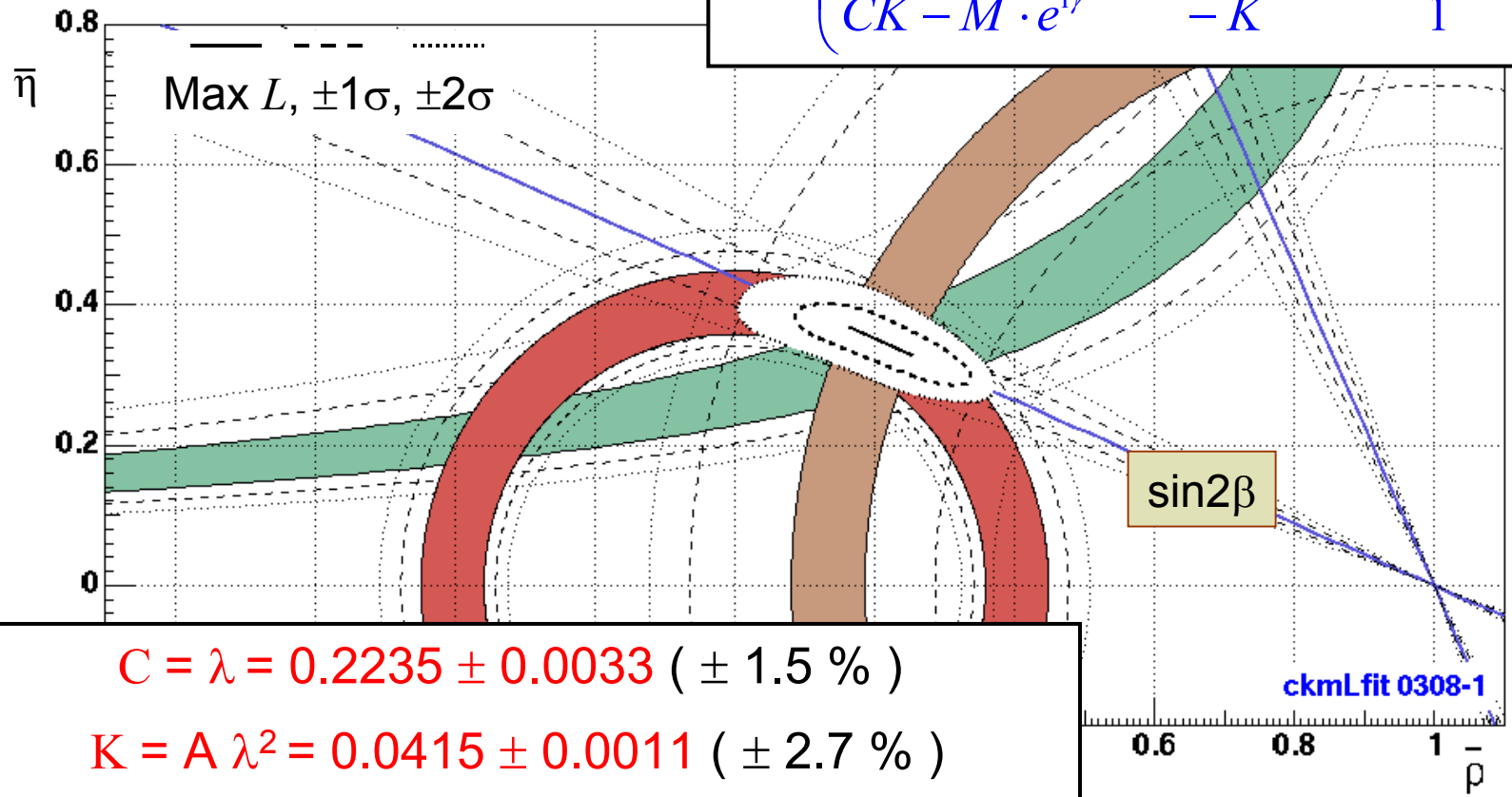


ckm Lfit

With world average 08/2003, sin2β = 0.737 ± 0.048 :



$$V \approx \begin{pmatrix} 1-C^2/2 & C & M\cdot e^{-i\gamma} \\ -C & 1-C^2/2 & K \\ CK-\overline{M}\cdot e^{i\gamma} & -K & 1 \end{pmatrix}$$



$C = \lambda = 0.2235 \pm 0.0033$ (± 1.5 %)

$K = A\,\lambda^2 = 0.0415 \pm 0.0011$ (± 2.7 %)

$M = A\,\lambda^3 \sqrt{\rho^2 + \eta^2} = 0.00385 \pm 0.00033$ (± 9 %)

$\gamma = \text{atan}\,(\eta/\rho) = (58 \pm 19)^o$ (± 5 % of 360^o)

Type-III CPV: Direct CPV in decays into one final state

$A = \langle f | T | B^0 \rangle, \; \overline{A} = \langle \bar{f} | T | \overline{B}^0 \rangle, \quad CPV \Leftrightarrow |\overline{A} / A| \neq 1$

CPT Symmetry requires :

$A = a \cdot e^{i\varphi} \cdot e^{i\delta} \quad \Rightarrow \quad \overline{A} = a \cdot e^{-i\varphi} \cdot e^{i\delta} \quad \Rightarrow \quad |\overline{A} / A| = 1$

Way out : two amplitudes contribute to the same final state, $A = A_1 + A_2$,

$A_j = a_j \cdot e^{i\varphi_j} \cdot e^{i\delta_j}$ with weak phases φ_j and strong phases δ_j

$\overline{A} = \overline{A}_1 + \overline{A}_2 \quad \Rightarrow$ Rates :

$$|A|^2 = a_1^2 + a_2^2 + 2 \cdot a_1 a_2 \cdot \cos(\varphi_1 - \varphi_2 + \delta_1 - \delta_2)$$

$$|\overline{A}|^2 = a_1^2 + a_2^2 + 2 \cdot a_1 a_2 \cdot \cos(-\varphi_1 + \varphi_2 + \delta_1 - \delta_2)$$

$$|\overline{A}|^2 - |A|^2 = 4 \cdot a_1 a_2 \cdot \sin(\varphi_1 - \varphi_2) \cdot \sin(\delta_1 - \delta_2)$$

CPV violation requires $\varphi_1 - \varphi_2 \neq 0$ and $\delta_1 - \delta_2 \neq 0$.

This type has not yet been firmly observed in B-meson decays, but in K-meson decays!

ε'

Example for Type III: The real part of ε'

$$\left|\pi^+\pi^-\right\rangle = \sqrt{2/3}\cdot\left|\pi\pi, I=0\right\rangle + \sqrt{1/3}\cdot\left|\pi\pi, I=2\right\rangle$$

$$A_{+-} = \left\langle \pi^+\pi^- \left| T \right| K^0 \right\rangle = \sqrt{2/3}\cdot a_0\, e^{i\delta_0} e^{i\varphi_0} + \sqrt{1/3}\cdot a_2\, e^{i\delta_2} e^{i\varphi_2}$$

$$\overline{A}_{+-} = \left\langle \pi^+\pi^- \left| T \right| \overline{K}^0 \right\rangle = \sqrt{2/3}\cdot a_0\, e^{-i\delta_0} e^{i\varphi_0} + \sqrt{1/3}\cdot a_2\, e^{-i\delta_2} e^{i\varphi_2}$$

[all on this page in the approximation $a_2 \ll a_0$]

$$a_{+-} = \frac{N\left(\overline{K}^0 \to \pi^+\pi^-\right) - N\left(K^0 \to \pi^+\pi^-\right)}{N\left(\overline{K}^0 \to \pi^+\pi^-\right) + N\left(K^0 \to \pi^+\pi^-\right)} = \frac{\left|\overline{A}_{+-}\right|^2 - \left|A_{+-}\right|^2}{\left|\overline{A}_{+-}\right|^2 + \left|A_{+-}\right|^2} = \sqrt{2}\,\frac{a_2}{a_0}\sin\left(\delta_2-\delta_0\right)\sin\left(\varphi_2-\varphi_0\right)$$

Definition of K-meson physics for ε' : $\eta_{+-} = \dfrac{\left\langle \pi^+\pi^- \left| T \right| K_L^0 \right\rangle}{\left\langle \pi^+\pi^- \left| T \right| K_S^0 \right\rangle} = \varepsilon_0 + \varepsilon'$

$$\Leftrightarrow \quad \lambda_{+-} = \frac{q\,\overline{A}_{+-}}{p\,A_{+-}} = \frac{1-\eta_{+-}}{1+\eta_{+-}} = 1 - 2\varepsilon_0 - 2\varepsilon' \quad \Rightarrow \quad \left|\lambda_{+-}\right| = 1 - 2\,\mathrm{Re}\left(\varepsilon_0\right) - 2\,\mathrm{Re}\left(\varepsilon'\right) = \left|\frac{q}{p}\right|\cdot\left|\frac{\overline{A}_{+-}}{A_{+-}}\right|$$

$$\left|\frac{q}{p}\right| = 1 - 2\,\mathrm{Re}\left(\varepsilon_K\right) = 1 - 2\,\mathrm{Re}\left(\varepsilon_0\right) \quad \Rightarrow \quad \left|\frac{\overline{A}_{+-}}{A_{+-}}\right| = 1 - 2\,\mathrm{Re}\left(\varepsilon'\right); \qquad \boxed{a_{+-} = -2\,\mathrm{Re}\left(\varepsilon'\right)}$$

$$\mathrm{Re}\left(\varepsilon'\right) = -\frac{a_2}{\sqrt{2}\,a_0}\,\sin\left(\delta_2-\delta_0\right)\,\sin\left(\varphi_2-\varphi_0\right)$$

Type-IV CPV: Direct CPV in decays into two final states

Type III was $a_{III} = \frac{N(\overline{K}^0 \to f) - N(K^0 \to f)}{N(\overline{K}^0 \to f) + N(K^0 \to f)} \neq 0 \quad \Leftrightarrow \quad R_{III} = \left| \frac{\langle f \mid T \mid \overline{K}^0 \rangle}{\langle f \mid T \mid K^0 \rangle} \right|^2 \neq 1$

with CP eigenstate f, **CP** f = η_f f.

Now two CP eigenstates f und g with CP eigenvalues η_f and η_g .

CP conservation $\Leftrightarrow R_{IV}(f,g) = \frac{\langle f \mid T \mid \overline{K}^0 \rangle \langle g \mid T \mid K^0 \rangle}{\langle f \mid T \mid K^0 \rangle \langle g \mid T \mid \overline{K}^0 \rangle} = \eta_f \cdot \eta_g = +1 \text{ oder } -1$

In contrast to R_{III} , R_{IV} is observable in magnitude and phase.

CPV of Type IV: $R_{IV}(f,g) = \frac{\langle f \mid T \mid \overline{K}^0 \rangle \langle g \mid T \mid K^0 \rangle}{\langle f \mid T \mid K^0 \rangle \langle g \mid T \mid \overline{K}^0 \rangle} \neq \eta_f \cdot \eta_g$

If $|R_{IV}| \neq 1$, then we must have at least one CPV of Type III.

If $R_{IV} = e^{i\phi} \neq \eta_f \cdot \eta_g = +1$ or -1, then we have CPV of Type IV.

Observed example: Im ε'.

The imaginary part of ε‘

$$\left|\pi^+\pi^-\right\rangle = \sqrt{2/3}\cdot\left|\pi\pi, I=0\right\rangle + \sqrt{1/3}\cdot\left|\pi\pi, I=2\right\rangle$$
$$\left|\pi^0\pi^0\right\rangle = -\sqrt{1/3}\cdot\left|\pi\pi, I=0\right\rangle + \sqrt{2/3}\cdot\left|\pi\pi, I=2\right\rangle$$

$$\eta_{+-} = \frac{\left\langle\pi^+\pi^-\,|\,T\,|\,K_L^0\right\rangle}{\left\langle\pi^+\pi^-\,|\,T\,|\,K_S^0\right\rangle} = \varepsilon_0 + \varepsilon'$$
$$\eta_{00} = \frac{\left\langle\pi^0\pi^0\,|\,T\,|\,K_L^0\right\rangle}{\left\langle\pi^0\pi^0\,|\,T\,|\,K_S^0\right\rangle} = \varepsilon_0 - 2\varepsilon'$$

in the language of K physics; now change into B-physics language:

A_{+-} , $\bar{A}_{+-}$, r_{+-} defined on page 17; in the same way for $\pi^0\pi^0$:

$$A_{00} = \left\langle\pi^0\pi^0\left|T\right|K^0\right\rangle, \quad \bar{A}_{00} = \left\langle\pi^0\pi^0\left|T\right|\bar{K}^0\right\rangle, \quad \lambda_{00} = \frac{q\,\bar{A}_{00}}{p\,A_{00}}$$

$$\lambda_{+-} = \frac{1-\eta_{+-}}{1+\eta_{+-}} = 1-2\varepsilon_0-2\varepsilon', \quad \lambda_{00} = \frac{1-\eta_{00}}{1+\eta_{00}} = 1-2\varepsilon_0+4\varepsilon'$$

$$R_{IV}\left(\pi^0\pi^0,\pi^+\pi^-\right) = \frac{\left\langle\pi^0\pi^0\,|\,T\,|\,\bar{K}^0\right\rangle\left\langle\pi^+\pi^-\,|\,T\,|\,K^0\right\rangle}{\left\langle\pi^0\pi^0\,|\,T\,|\,K^0\right\rangle\left\langle\pi^+\pi^-\,|\,T\,|\,\bar{K}^0\right\rangle} = \frac{\bar{A}_{00}\,A_{+-}}{A_{00}\,\bar{A}_{+-}} = \frac{\lambda_{00}}{\lambda_{+-}} = 1+6\varepsilon'$$
$$= \left(1+6\,\mathrm{Re}\,\varepsilon'\right)\left(1+6\,i\,\mathrm{Im}\,\varepsilon'\right) = \left(1+6\,\mathrm{Re}\,\varepsilon'\right)\cdot e^{6i\,\mathrm{Im}\,\varepsilon'}$$

With Re ε‘ = 0 and Im ε‘ ≠ 0, $R_{IV} = e^{i\phi}$, Type-IV CPV.

Determining ε‘

$$A_0 = \langle \pi\pi, I=0 | T | K^0 \rangle, \quad A_2 = \langle \pi\pi, I=2 | T | K^0 \rangle, \quad A_n = a_n\, e^{i\varphi_n}\, e^{i\delta_n}$$

$$\overline{A}_0 = \langle \pi\pi, I=0 | T | \overline{K}^0 \rangle, \quad \overline{A}_2 = \langle \pi\pi, I=2 | T | \overline{K}^0 \rangle, \quad \overline{A}_n = a_n\, e^{-i\varphi_n}\, e^{i\delta_n}$$

$$R_{IV} = 1 + 6\varepsilon' = \frac{\lambda_{00}}{\lambda_{+-}} = \frac{\left(-\overline{A}_0 + \sqrt{2}\,\overline{A}_2\right)\left(\sqrt{2}\,A_0 + A_2\right)}{\left(-A_0 + \sqrt{2}\,A_2\right)\left(\sqrt{2}\,\overline{A}_0 + \overline{A}_2\right)} = 1 + \frac{6\,i\,a_2}{\sqrt{2}\,a_0}\, e^{i(\delta_2-\delta_0)} \sin(\varphi_2 - \varphi_0)$$

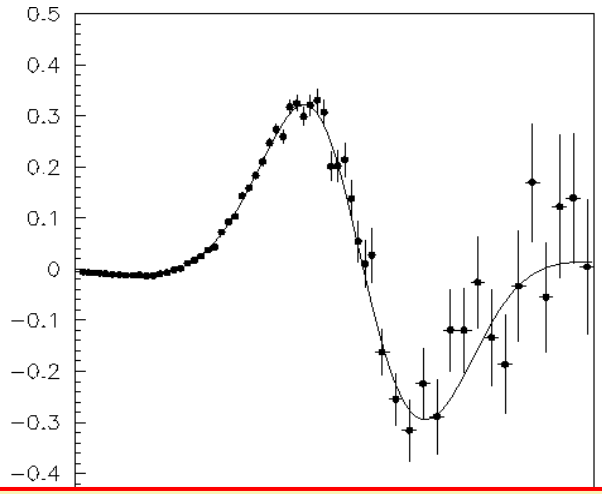
$$\mathrm{Re}(\varepsilon') = \frac{-a_2}{\sqrt{2}\,a_0} \sin(\delta_2 - \delta_0) \sin(\varphi_2 - \varphi_0), \quad \mathrm{Im}(\varepsilon') = \frac{a_2}{\sqrt{2}\,a_0} \cos(\delta_2 - \delta_0) \sin(\varphi_2 - \varphi_0)$$

CPV of Type III: Weak and strong phases of the two amplitudes in the final state f must be different.

Type IV: Only the weak phases of the two amplitudes, which contribute to the two states f and g, must be different.

Measured in $\pi\pi \to \pi\pi$: $\delta_2 - \delta_0 = -42^o \pm 4^o \Rightarrow \phi(\varepsilon') = 48^o \pm 4^o$

Measuring ε‘



The CPLEAR result in the approximation

$\varepsilon_0 \approx \eta_{+-} = \langle\pi^+\pi^-|T|K_L\rangle / \langle\pi^+\pi^-|T|K_S\rangle$:

$$\varepsilon_0 = (2.27 \pm 0.02)\cdot 10^{-3} \cdot e^{i(43.3\pm 0.5)^0}$$

$$a_{\pi\pi}(t) = \frac{N(\bar{K}^0 \to \pi^+\pi^-) - N(K^0 \to \pi^+\pi^-)}{N(\bar{K}^0 \to \pi^+\pi^-) + N(K^0 \to \pi^+\pi^-)}$$

$$= \frac{2\,\mathrm{Re}(\varepsilon_0)}{1+|\varepsilon_0|^2} - \frac{2e^{-(\Gamma_S+\Gamma_L)t/2}\cdot[\mathrm{Re}(\varepsilon_0)\cos\Delta m t + \mathrm{Im}(\varepsilon_0)\sin\Delta m t]}{e^{-\Gamma_S t} + |\varepsilon_0|^2 e^{-\Gamma_L t}} \qquad = 0 \quad \text{for} \quad t = 0$$

With $\eta_{+-} = \varepsilon_0 + \varepsilon'$ we get $a_{\pi\pi}(t=0) = -2\,\mathrm{Re}\,\varepsilon' \neq 0$,

but too small to be measured. $\varepsilon_0 = o(10^{-3})$, $\varepsilon' = o(10^{-6})$.

$$\frac{\langle \pi^+\pi^- | T | K_L \rangle}{\langle \pi^+\pi^- | T | K_S \rangle} = \varepsilon_0 + \varepsilon' , \quad \frac{\langle \pi^0\pi^0 | T | K_L \rangle}{\langle \pi^0\pi^0 | T | K_S \rangle} = \varepsilon_0 - 2\varepsilon' ,$$

$$\frac{\left|\langle \pi^+\pi^- | T | K_L \rangle\right|^2 / \left|\langle \pi^+\pi^- | T | K_S \rangle\right|^2}{\left|\langle \pi^0\pi^0 | T | K_L \rangle\right|^2 / \left|\langle \pi^0\pi^0 | T | K_S \rangle\right|^2} = 1 + 6 \cdot \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon_0}\right).$$

NA48 (CERN), 2002: $\mathrm{Re}(\varepsilon'/\varepsilon_0) = (14.7 \pm 2.2)\cdot 10^{-4}$

KTeV (FNAL), 2003: $\mathrm{Re}(\varepsilon'/\varepsilon_0) = (20.7 \pm 2.8)\cdot 10^{-4}$

PDG average: $\mathrm{Re}(\varepsilon'/\varepsilon_0) = (16.7 \pm 2.3)\cdot 10^{-4}$

Recall: $\varepsilon_0 = (2.27 \pm 0.02)\cdot 10^{-3} \cdot e^{i(43.3\pm 0.5)^0}$

Since the phases of ε_0 and ε' are known,

we can transform this result

into Re ε' and Im ε' :

$$\mathrm{Re}\,\varepsilon' = (2.6 \pm 0.4)\cdot 10^{-6}$$
$$\mathrm{Im}\,\varepsilon' = (2.9 \pm 0.4)\cdot 10^{-6}$$

Conclusion on CP Violation so far:

In laboratory experiments with K and B mesons we have found CP-violating effects with four different origins:

in K-$\bar{K}$ oscillations, Re(ε_K), origin is phase $\Delta\phi$ between m_{12} and Γ_{12} ,

in the interference between oscillation and dacay, Im(ε_0), sin2β,

origin is $\Delta\phi$ between q/p and $\bar{A}/A$,

in decays into one final state, Re(ε'),

origin $\Delta\phi$ and $\Delta\delta$ between A_1 and A_2 ,

in decays into two final states, Im(ε'),

origin $\Delta\phi$ between A_1 and A_2 (no $\Delta\delta$) .

This classification does not mean that there are only these 4 CPV types.

Standard Model contains and describes observations of all 4 types.

End of Lecture III. Next a few more recent measurements:

K. R. Schubert, TU Dresden
IMFP04 Alicante, 1-5 March 2004

„Quark Mixing and CP Violation“

Fourth Lecture: Tests of CPT and New CP Violation Measurements

This Conference celebrates, as many other events this year,

the 50th Birthday of CERN

The celebration tomorrow covers mainly the 2nd 25 years,

for good reasons:

- the most important results are from this 2nd half,
- we all are more familiar with the 2nd half, and
- the 1st half is aready well covered by a Festschrift:

„Highlights of 25 years of Physics at CERN“,

L. van Hove and M. Jacob, Physics Reports 62 (1980) 1

- Nuclear Physics
- Hadron Spectroscopy
- Hadron Collisions at ISR Energies
- QED
- Weak Interactions

The „Weak Interactions“ chapter

covers the following „highlights“ :

- $\pi^+ \rightarrow e^+\nu$ (1958)
- $\pi^+ \rightarrow \pi^0 e^+\nu$ (1962)
- the phase ϕ_{00} of η_{00} (1970)
- T is violated, T invariance with CPT violation is ruled out (1970)
- Precision measurements of $\Delta m\,(K^0)$, $|\eta_{+-}|$, and ϕ_{+-} (1974)
- Discovery of Neutral Currents with Gargamelle (1977)
- Measurements of $\sin^2\theta_W$ (CDHS, BEBC)

I will not review this CPT test here,

but only quote the main results:

[Phys Lett 31 B (1970) 662]

$$K_S = \frac{1}{\sqrt{2}}\left[(1+\varepsilon+\delta)K + (1-\varepsilon-\delta)\overline{K}\right]$$

$$K_L = \frac{1}{\sqrt{2}}\left[(1+\varepsilon-\delta)K + (1-\varepsilon+\delta)\overline{K}\right]$$

CPT Invariance: $\delta = 0$

T Invariance: $\varepsilon = 0$

$$\mathrm{Re}(\varepsilon) = \frac{1}{4}\cdot \mathrm{Im}\left(\frac{\Gamma_{12}}{m_{12}}\right), \quad \delta = \frac{1}{2}\cdot\frac{(m_{22}-m_{11}) - \mathrm{i}(\Gamma_{22}-\Gamma_{11})/2}{(m_L - m_S) - \mathrm{i}(\Gamma_L - \Gamma_S)/2}$$

Using the Bell-Steinberger unitarity relation and all available K_S and K_L decay rates:

$$\mathrm{Re}(\varepsilon) = (1.68 \pm 0.30)\cdot 10^{-3}, \quad \mathrm{Re}(\delta) = (0.07 \pm 0.43)\cdot 10^{-3}, \quad \mathrm{Im}(\delta) = (-0.30 \pm 0.45)\cdot 10^{-3}$$

More recent data have improved the errors on δ.

CPLEAR finds $\mathrm{Re}(\delta) = (0 \pm 0.3)\cdot 10^{-3}, \quad \mathrm{Im}(\delta) = (0 \pm 0.05)\cdot 10^{-3}$

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SLAC-PUB-10247
hep-ex/0311037

November 2003

Limits on the Decay-Rate Difference of Neutral B Mesons and on CP, T, and CPT Violation in $B^0\overline{B}^0$ Oscillations

B. Aubert,[1] R. Barate,[1] D. Boutigny,[1] J.-M. Gaillard,[1] A. Hicheur,[1] Y. Karyotakis,[1] J. P. Lees,[1] P. Robbe,[1] V. Tisserand,[1] A. Zghiche,[1] A. Palano,[2] A. Pompili,[2] J. C. Chen,[3] N. D. Qi,[3] G. Rong,[3] P. Wang,[3] Y. S. Zhu,[3] G. Eigen,[4] I. Ofte,[4] B. Stugu,[4] G. S. Abrams,[5] A. W. Borgland,[5] A. B. Breon,[5] D. N. Brown,[5] J. Button-Shafer,[5] R. N. Cahn,[5] E. Charles,[5] C. T. Day,[5] M. S. Gill,[5] A. V. Gritsan,[5] Y. Groysman,[5] R. G. Jacobsen,[5] R. W. Kadel,[5] J. Kadyk,[5] L. T. Kerth,[5] Yu. G. Kolomensky,[5] J. F. Kral,[5] G. Kukartsev,[5] C. LeClerc,[5] M. E. Levi,[5] G. Lynch,[5] L. M. Mir,[5] P. J. Oddone,[5] T. J. Orimoto,[5] M. Pripstein,[5] N. A. Roe,[5] A. Romosan,[5] M. T. Ronan,[5] V. G. Shelkov,[5] A. V. Telnov,[5] W. A. Wenzel,[5] K. Ford,[6] T. J. Harrison,[6] C. M. Hawkes,[6] D. J. Knowles,[6] S. E. Morgan,[6] R. C. Penny,[6] A. T. Watson,[6] N. K. Watson,[6] T. Deppermann,[7] K. Goetzen,[7] H. Koch,[7] B. Lewandowski,[7] M. Pelizaeus,[7] K. Peters,[7] H. Schmuecker,[7] M. Steinke,[7] N. R. Barlow,[8] J. T. Boyd,[8] N. Chevalier,[8] W. N. Cottingham,[8] M. P. Kelly,[8] T. E. Latham,[8] C. Mackay,[8] F. F. Wilson,[8] K. Abe,[9] T. Cuhadar-Donszelmann,[9] C. Hearty,[9] T. S. Mattison,[9] J. A. McKenna,[9] D. Thiessen,[9] P. Kyberd,[10] A. K. McKemey,[10] V. E. Blinov,[11] A. D. Bukin,[11] V. B. Golubev,[11] V. N. Ivanchenko,[11] E. A. Kravchenko,[11] A. P. Onuchin,[11] S. I. Serednyakov,[11] Yu. I. Skovpen,[11] E. P. Solodov,[11] A. N. Yushkov,[11] D. Best,[12] M. Bruinsma,[12] M. Chao,[12] D. Kirkby,[12]

C. Angelini, G. Batignani, S. Bettarini, M. Bondioli, F. Bucci, G. Calderini, M. Carpinelli, V. Del Gamba, F. Forti, M. A. Giorgi, A. Lusiani, G. Marchiori, F. Martinez-Vidal, M. Morganti, N. Neri, E. Paoloni, M. Rama, G. Rizzo, F. Sandrelli, and J. Walsh

...

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\mu_{ik}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\left[\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

If CPT, $m_{11}=m_{22}$, $\Gamma_{11}=\Gamma_{22}$, and

$$B_L^0(t)=\left(pB^0+q\overline{B}^0\right)\cdot e^{-\gamma_L t},\quad \gamma_L=i\,m_L+\Gamma_L/2,$$
$$B_H^0(t)=\left(pB^0-q\overline{B}^0\right)\cdot e^{-\gamma_H t},\quad \gamma_H=i\,m_H+\Gamma_H/2.$$

If CPT violated, also two eigenstates B_L, B_H with

$$\Delta m=m_H-m_L\approx 2\left|m_{12}\right|,\quad \Delta\Gamma=\Gamma_H-\Gamma_L\approx 2\left|m_{12}\right|\mathrm{Re}\left(\Gamma_{12}/m_{12}\right),$$
$$\left|\frac{q}{p}\right|\approx 1-\frac{1}{2}\mathrm{Im}\left(\Gamma_{12}/m_{12}\right),$$
$$\delta m=\left(m_{11}-m_{22}\right)/2,\quad \delta\Gamma=\left(\Gamma_{11}-\Gamma_{22}\right)/2,$$
$$z=2\frac{\delta m-\mathrm{i}\delta\Gamma/2}{\Delta m-\mathrm{i}\Delta\Gamma/2},\quad \mathrm{Re}\,z\approx 2\delta m/\Delta m,\quad \mathrm{Im}\,z\approx-\delta\Gamma/\Delta m.$$

Def. of |q/p| as with CPT, but B_L, B_H given by p, q, and z.

{ explicit expressions:

$$B_L^0(t) = \left[p \cdot \sqrt{1-z} \cdot B^0 + q \cdot \sqrt{1+z} \cdot \overline{B}^0 \right] \cdot e^{-\gamma_L t}$$

$$B_H^0(t) = \left[p \cdot \sqrt{1+z} \cdot B^0 - q \cdot \sqrt{1-z} \cdot \overline{B}^0 \right] \cdot e^{-\gamma_H t}$$

z = - 2 δ from p. 100

q/p same as in CPT-conserving case }

For pairs B_{tag} B_{rec} from the Y(4S) we have with $\Delta t = t_{rec} - t_{tag}$:

$$\frac{dN}{dt} \propto e^{-\Gamma|\Delta t|}\left[\frac{c_+}{2}\cdot\cosh\left(\frac{\Delta\Gamma}{2}\Delta t\right)+\frac{c_-}{2}\cdot\cos(\Delta m\Delta t)-\mathrm{Re}s\cdot\sinh\left(\frac{\Delta\Gamma}{2}\Delta t\right)+\mathrm{Im}s\cdot\sin(\Delta m\Delta t)\right]$$

Coefficients c_+, c_-, s given as functions of $|q/p|$, λ_{CP}, $(\lambda_{tag}, \lambda_{fl})$, and z :

$B_{tag}B_{rec}$	c_+
$B^0\ B^0$	$\lvert A_{Bt}\rvert^2\lvert A_{Bf}\rvert^2\ \lvert p/q\rvert^2$
$B^0\ \bar{B}^0$	$\lvert A_{Bt}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2$
$\bar{B}^0\ B^0$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{Bf}\rvert^2$
$\bar{B}^0\ \bar{B}^0$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2\ \lvert q/p\rvert^2$
$B^0\ B_{CP}$	$\lvert A_{Bt}\rvert^2\lvert A_{CP}\rvert^2\ \lvert p/q\rvert^2[1+\lvert\lambda_{CP}\rvert^2-4\,\mathrm{Re}\,\lambda_{CP}\,\mathrm{Re}\,\lambda_{Bt}+2\,\mathrm{Re}\,z\,\mathrm{Re}\,\lambda_{CP}-2\,\mathrm{Im}\,z\,\mathrm{Im}\,\lambda_{CP}]$
$\bar{B}^0\ B_{CP}$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{CP}\rvert^2\ [1+\lvert\lambda_{CP}\rvert^2-4\,\mathrm{Re}\,\lambda_{CP}\,\mathrm{Re}\,\bar{\lambda}_{\bar{B}t}-2\,\mathrm{Re}\,z\,\mathrm{Re}\,\lambda_{CP}-2\,\mathrm{Im}\,z\,\mathrm{Im}\,\lambda_{CP}]$

c_+

$B_{tag}B_{rec}$	c_-
$B^0\ B^0$	$-\lvert A_{Bt}\rvert^2\lvert A_{Bf}\rvert^2\ \lvert p/q\rvert^2$
$B^0\ \bar{B}^0$	$\lvert A_{Bt}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2$
$\bar{B}^0\ B^0$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{Bf}\rvert^2$
$\bar{B}^0\ \bar{B}^0$	$-\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2\ \lvert q/p\rvert^2$
$B^0\ B_{CP}$	$\lvert A_{Bt}\rvert^2\lvert A_{CP}\rvert^2\ \lvert p/q\rvert^2[-1+\lvert\lambda_{CP}\rvert^2-4\,\mathrm{Im}\,\lambda_{CP}\,\mathrm{Im}\,\lambda_{Bt}-2\,\mathrm{Re}\,z\,\mathrm{Re}\,\lambda_{CP}+2\,\mathrm{Im}\,z\,\mathrm{Im}\,\lambda_{CP}]$
$\bar{B}^0\ B_{CP}$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{CP}\rvert^2\ [1-\lvert\lambda_{CP}\rvert^2+4\,\mathrm{Im}\,\lambda_{CP}\,\mathrm{Im}\,\bar{\lambda}_{\bar{B}t}+2\,\mathrm{Re}\,z\,\mathrm{Re}\,\lambda_{CP}+2\,\mathrm{Im}\,z\,\mathrm{Im}\,\lambda_{CP}]$

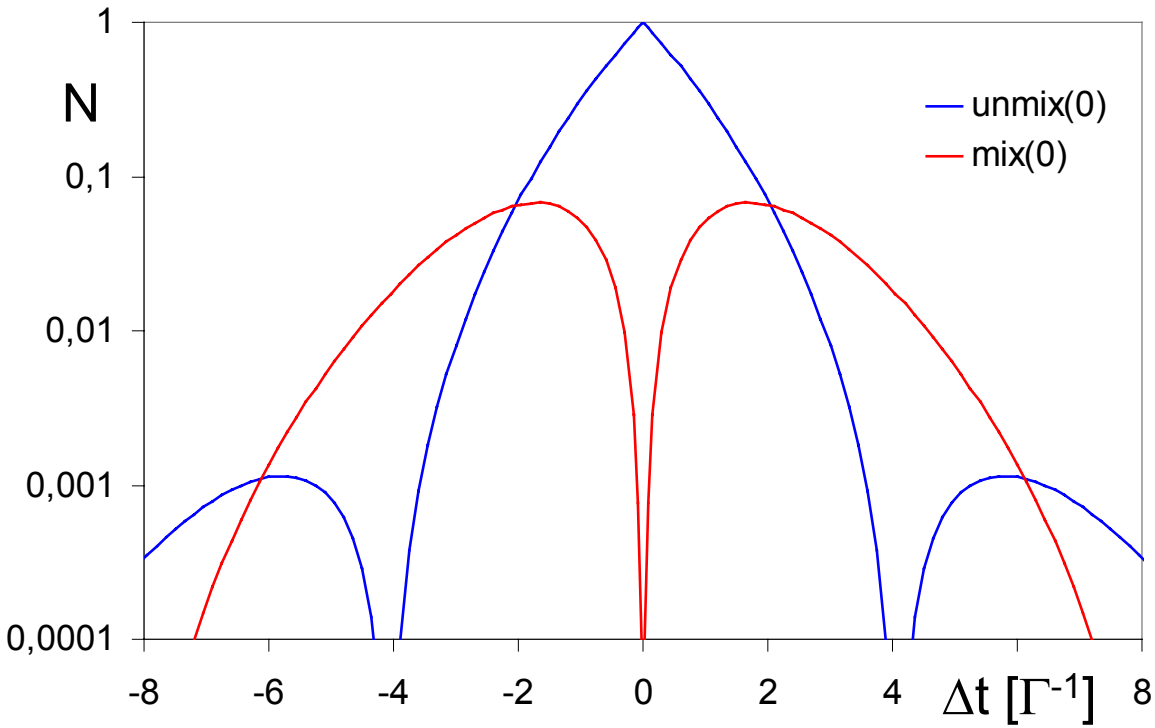
c_-

$B_{tag}B_{rec}$	s
$B^0\ B^0$	$\lvert A_{Bt}\rvert^2\lvert A_{Bf}\rvert^2\ \lvert p/q\rvert^2[-\lambda^*_{Bf}+\lambda^*_{Bt}]$
$B^0\ \bar{B}^0$	$\lvert A_{Bt}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2[\lambda_{Bt}-\bar{\lambda}_{\bar{B}f}-z]$
$\bar{B}^0\ B^0$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{Bf}\rvert^2[\bar{\lambda}_{\bar{B}t}-\lambda_{Bf}+z]$
$\bar{B}^0\ \bar{B}^0$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert \bar{A}_{\bar{B}f}\rvert^2\ \lvert q/p\rvert^2[\bar{\lambda}^*_{\bar{B}t}-\bar{\lambda}^*_{\bar{B}f}]$
$B^0\ B_{CP}$	$\lvert A_{Bt}\rvert^2\lvert A_{CP}\rvert^2\ \lvert p/q\rvert^2[\lambda^*_{Bt}-\lambda^*_{CP}+\lvert\lambda_{CP}\rvert^2(-z+\lambda_{Bt})]$
$\bar{B}^0\ B_{CP}$	$\lvert \bar{A}_{\bar{B}t}\rvert^2\lvert A_{CP}\rvert^2\ [\bar{\lambda}_{\bar{B}t}-\lambda_{CP}+\lvert\lambda_{CP}\rvert^2\bar{\lambda}^*_{\bar{B}t}+z]$

s

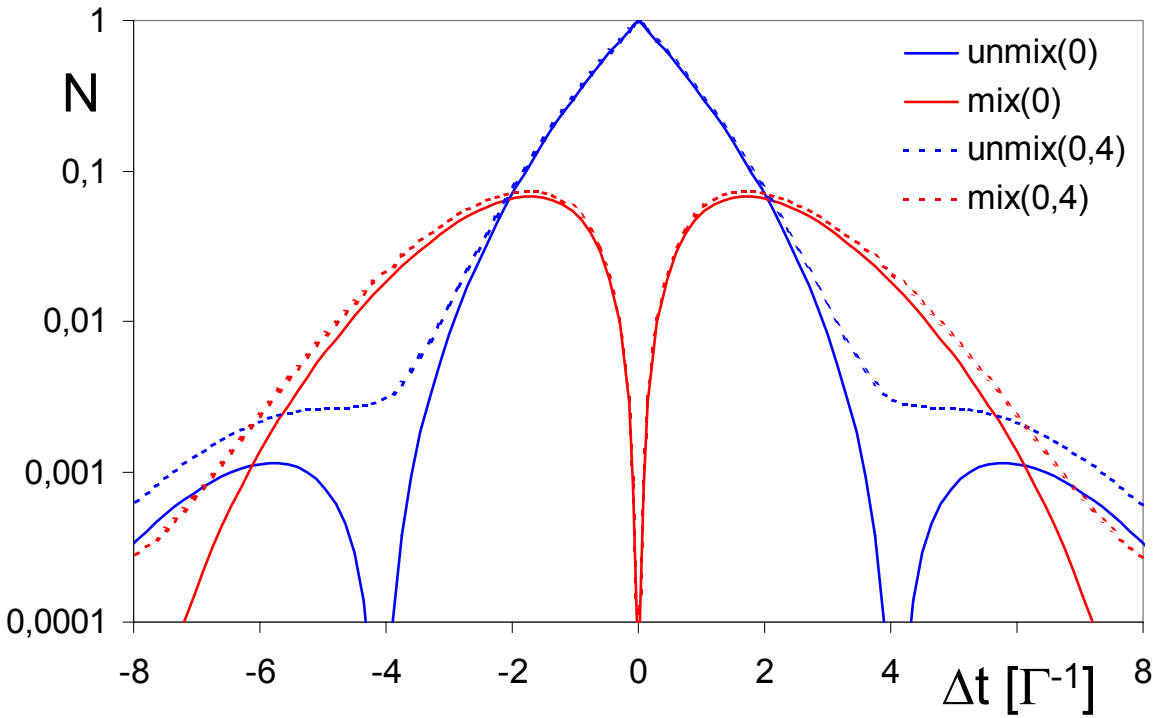
Flavour-tag and Flavour-rec

$\Delta\Gamma/\Gamma = 0$



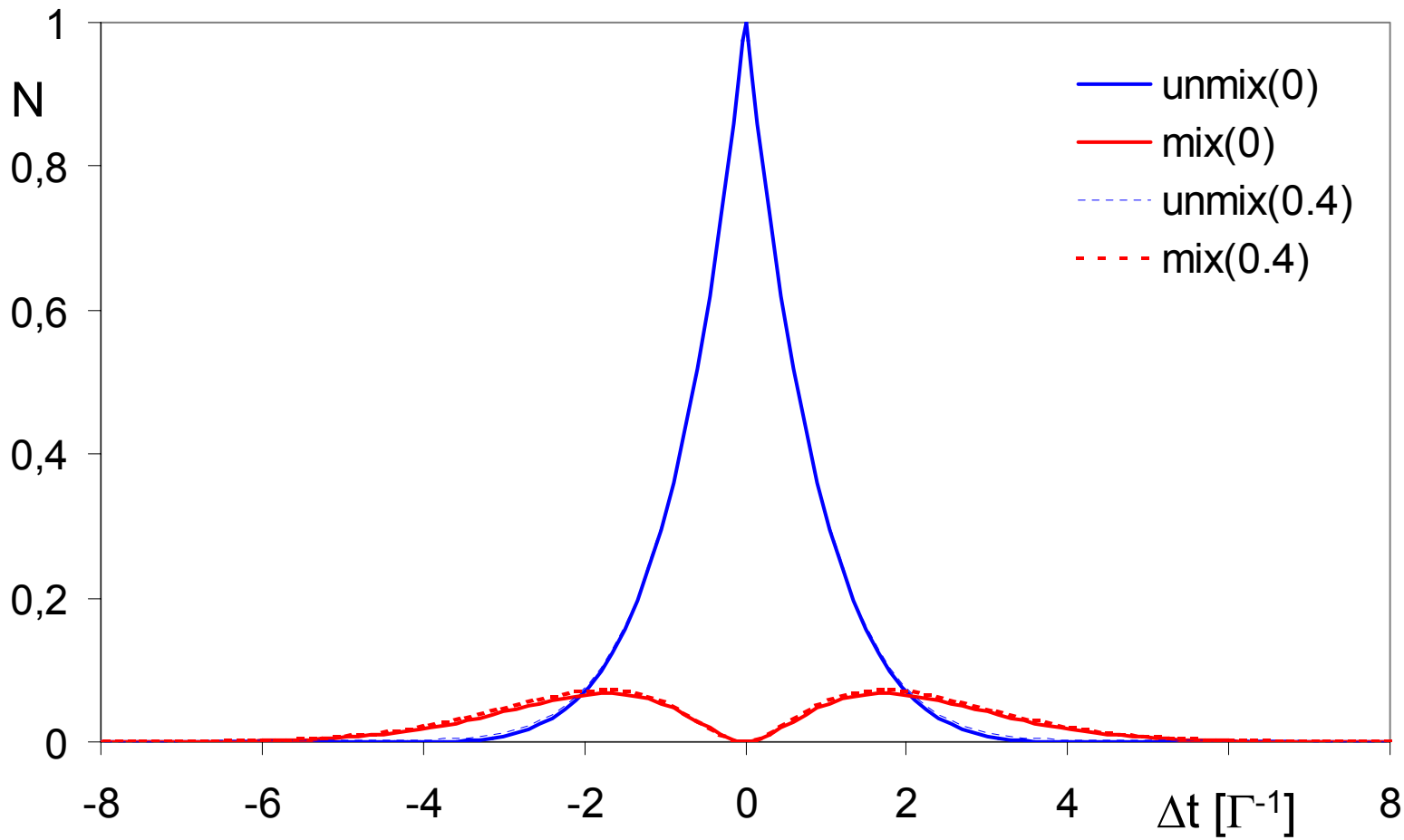
Flavour-tag and Flavour-rec

$\Delta\Gamma/\Gamma = 0$ and $\Delta\Gamma/\Gamma = 0.4$



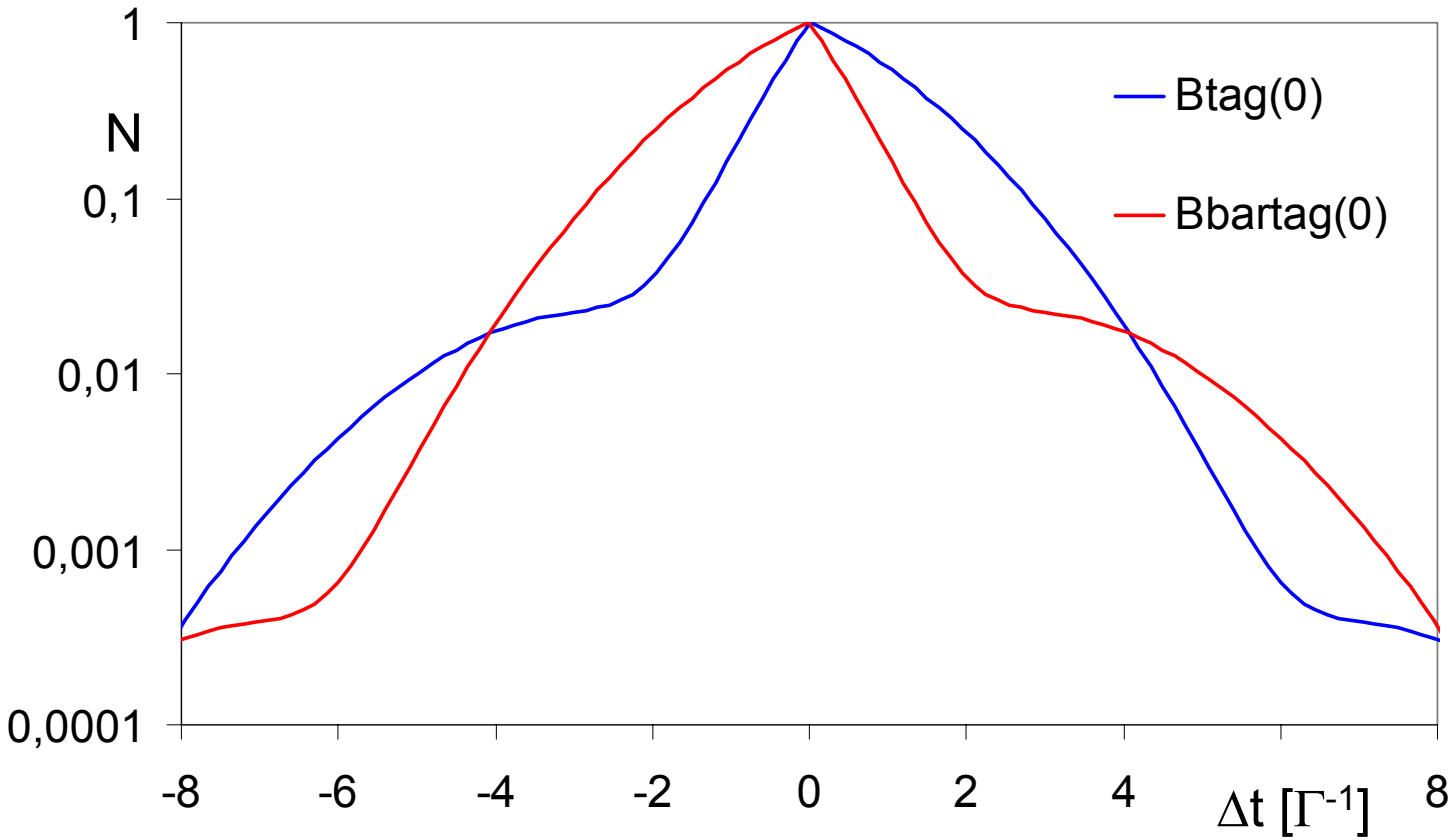
Flavour-tag and Flavour-rec

$\Delta\Gamma/\Gamma = 0$ and $\Delta\Gamma/\Gamma = 0.4$



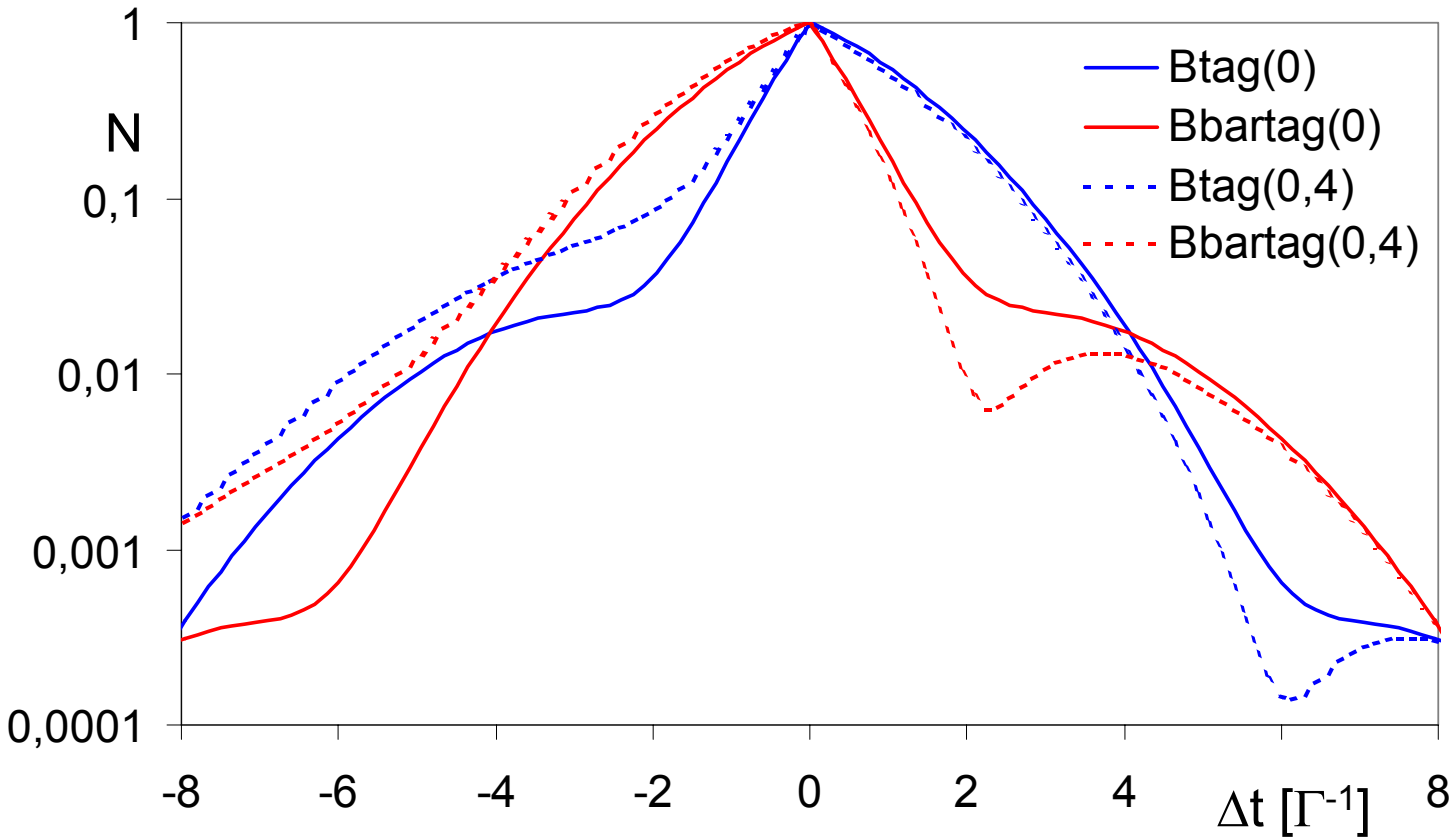
Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma = 0$



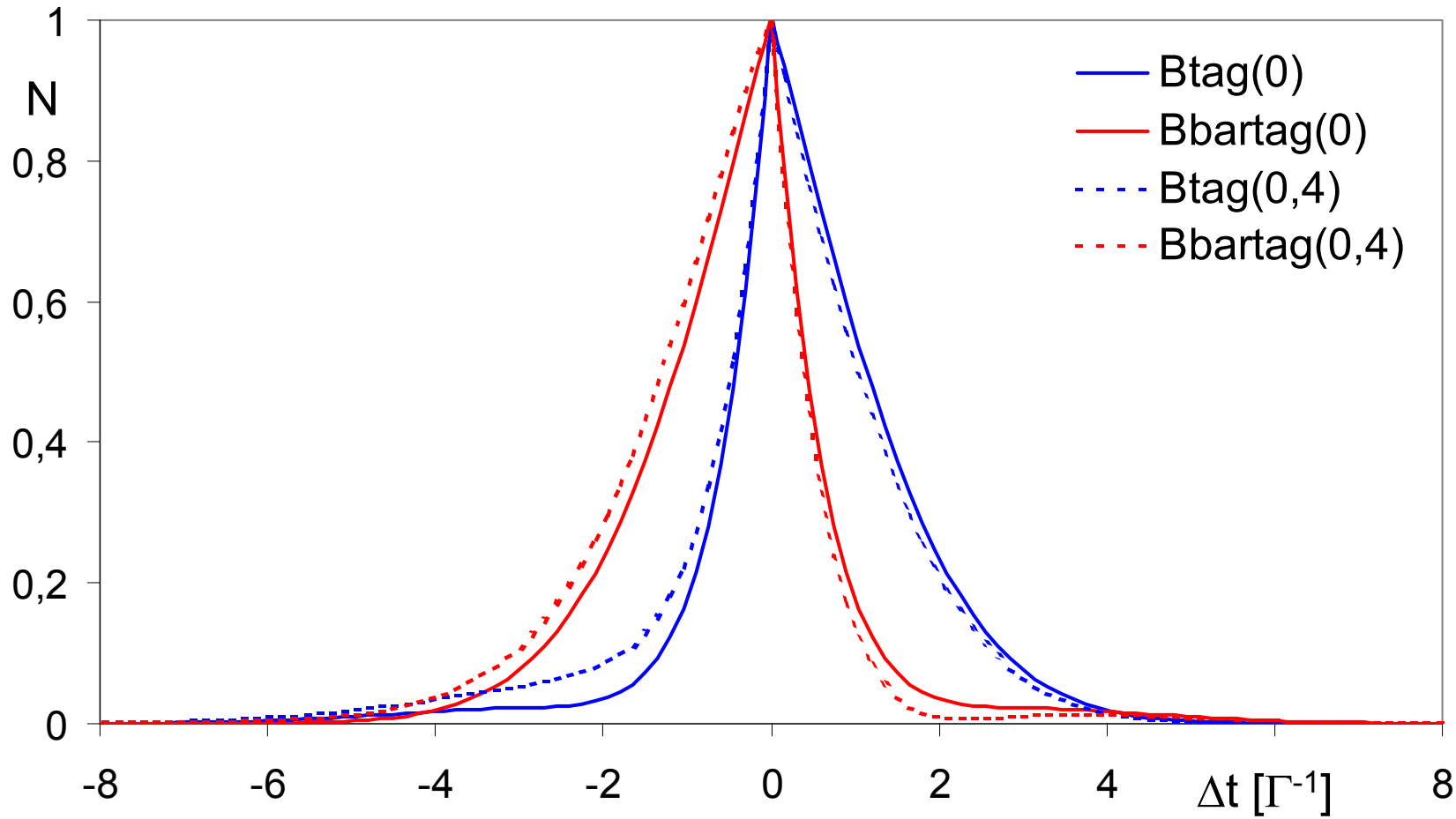
Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma = 0$ and $\Delta\Gamma/\Gamma = 0.4$



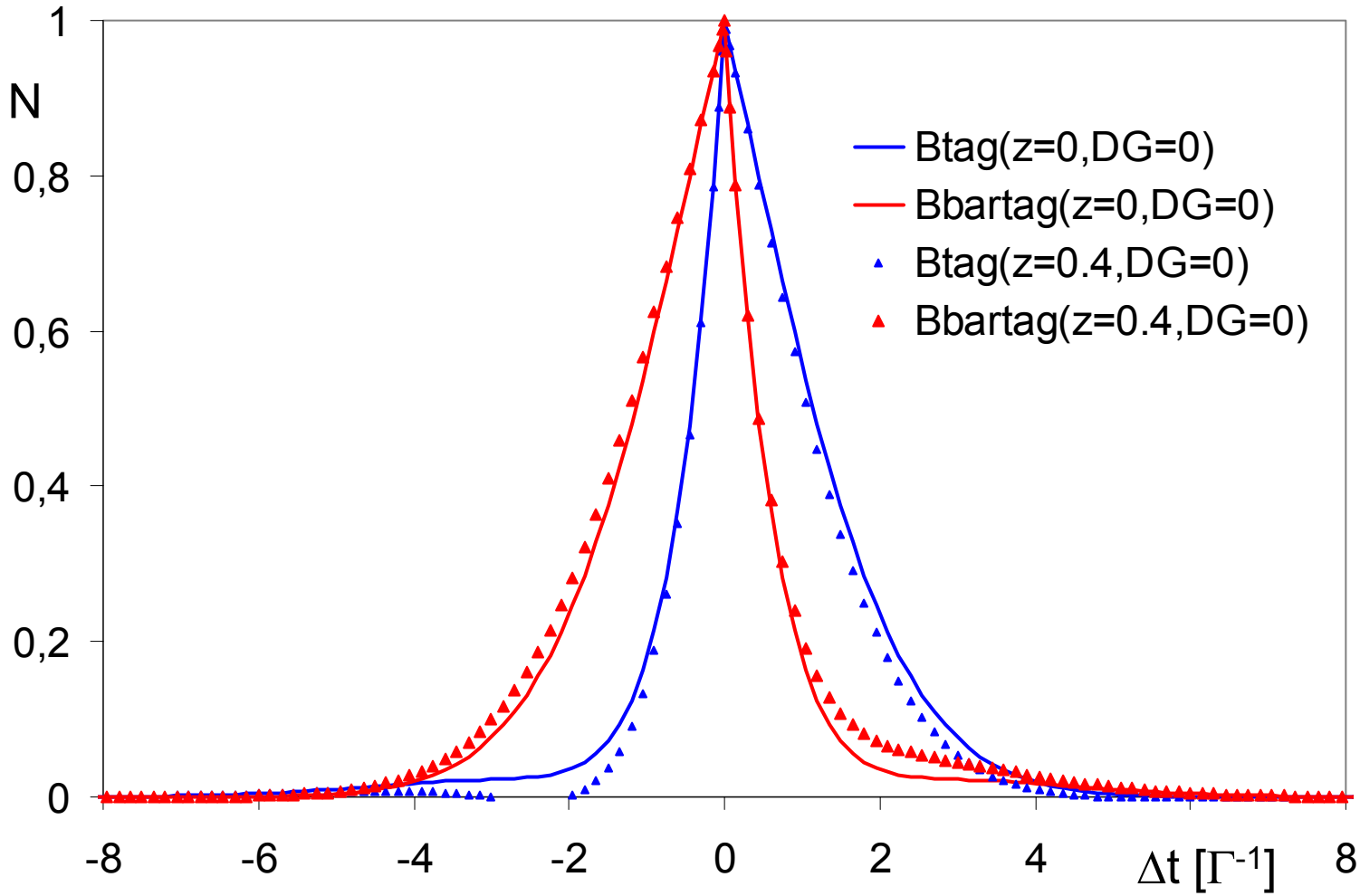
Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma = 0$ and $\Delta\Gamma/\Gamma = 0.4$



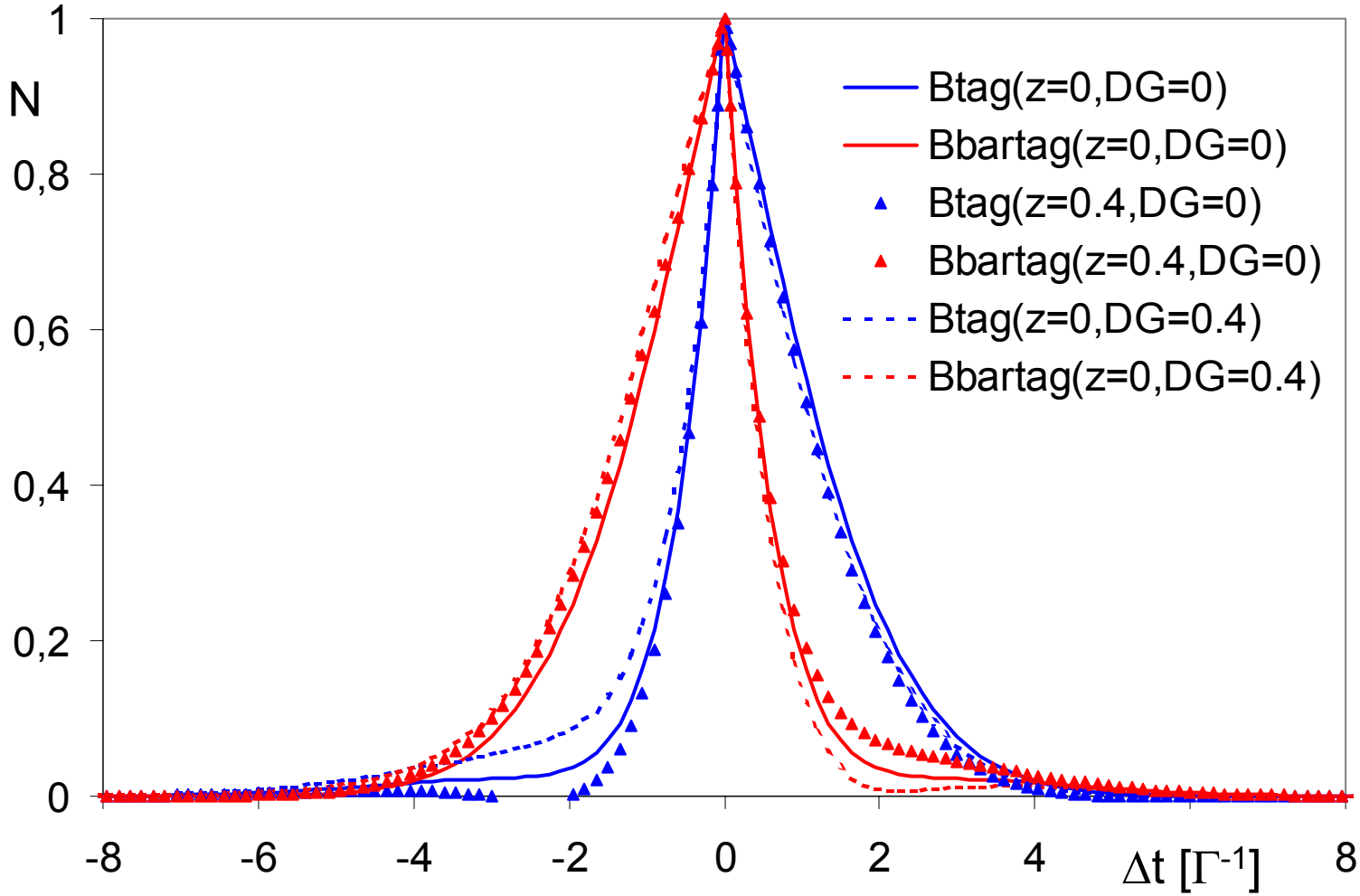
Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma = 0$; z = 0 and z = 0.4



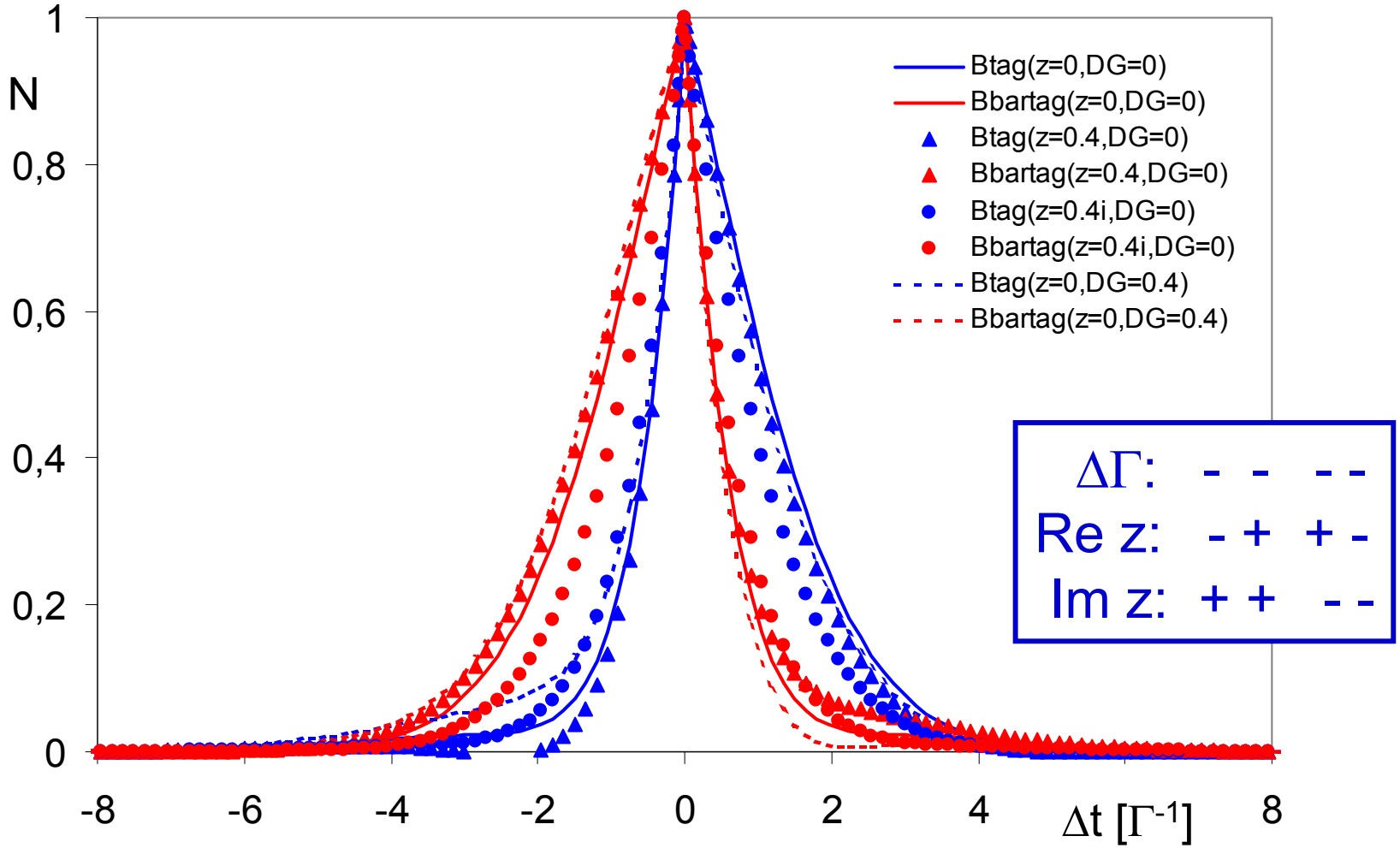
Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma$ = 0 and 0.4; z = 0 and z = 0.4



Flavour-tag and CP-rec

$\Delta\Gamma/\Gamma$ = 0 and 0.4; z = 0, z = 0.4, and z = 0.4 i



Δm, $|q/p|$,and Im z

dominantly from the sample tag + flavour-rec,

$\Delta\Gamma/\Gamma$, Im λ(sin2β), and Re z

from the sample tag + CP-rec.

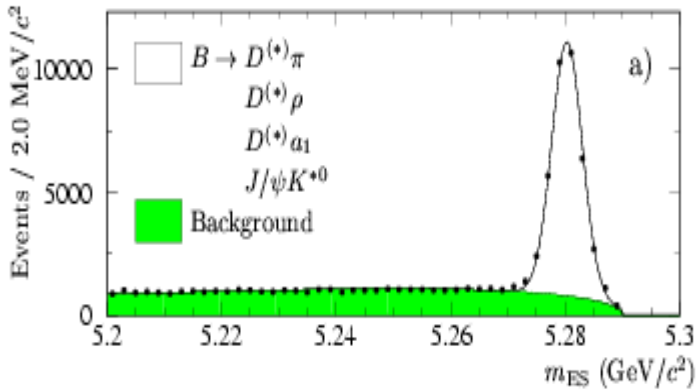
BABAR analysis with 88 M Y(4S):

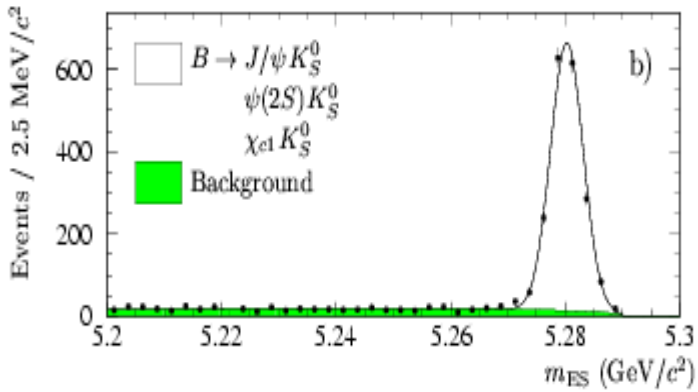
rec events, dN/dm_{ES}

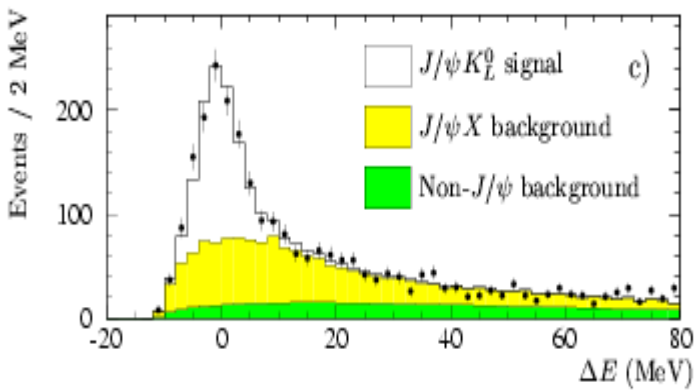
Flavour sample

CP sample with $\eta = -1$

CP sample with $\eta = +1$







after tagging and vertexing:

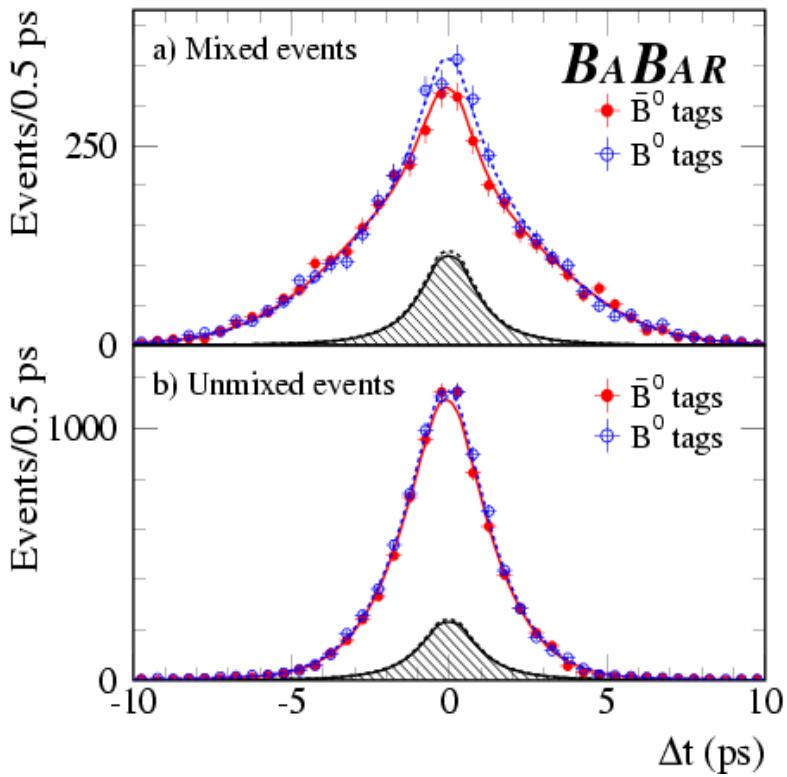
N = 20.6 K
82% purity

N = 1.2 K
94 % purity

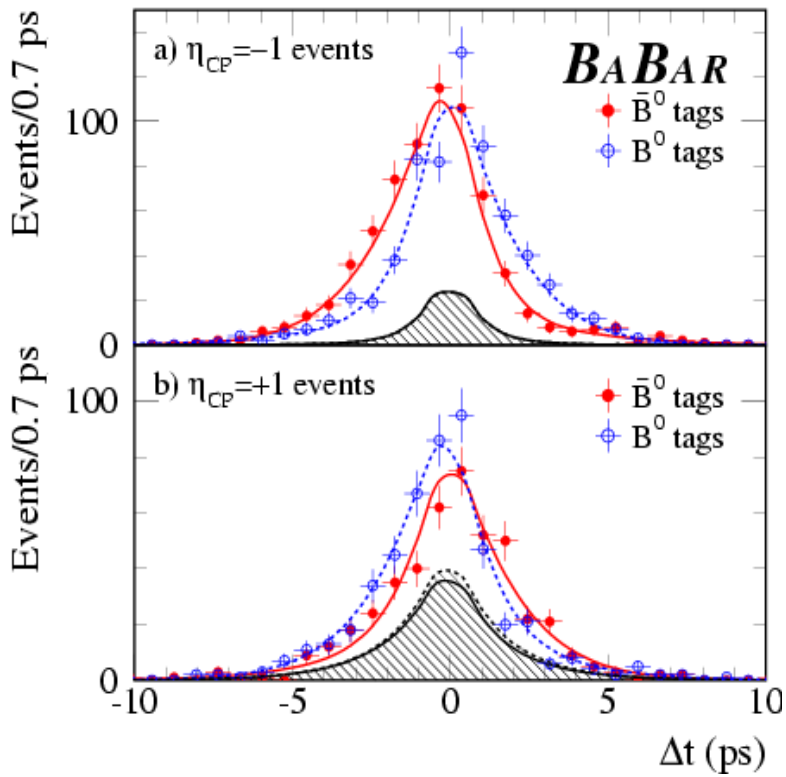
N = 0.5 K
55% purity

rec events, Δt distributions

flavour samples



CP samples



BABAR results:

|q/p| =

1.029±0.013 ±0.011

Dileptons:

0.998±0.006±0.007

Comb.: 1.005±0.008

sgn(Reλ)ΔΓ/Γ =

-0.008±0.037±0.018

sgn(Reλ)Rez =

0.021±0.053±0.051

Imz =

0.038±0.029±0.025

|z| vs |q/p|

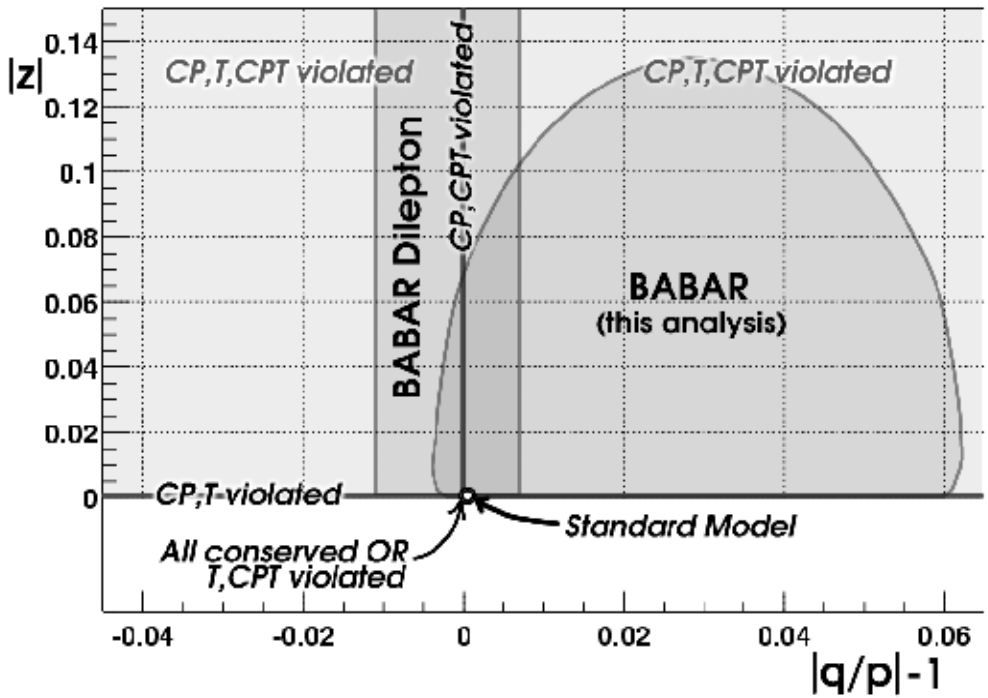


FIG. 4: Favored regions at 68% confidence level in the $(|q/p|\text{-}1, |z|)$ plane determined by this analysis and by the *BABAR* measurement of the dilepton asymmetry [11].

Summary of hep-ex/0311037:

Neither $\Delta\Gamma/\Gamma$ nor CPT violation influence BABAR's $\sin 2\beta$.

$\Delta\Gamma/\Gamma$ compatible with zero, St.Model expects $o(3 \cdot 10^{-3})$,

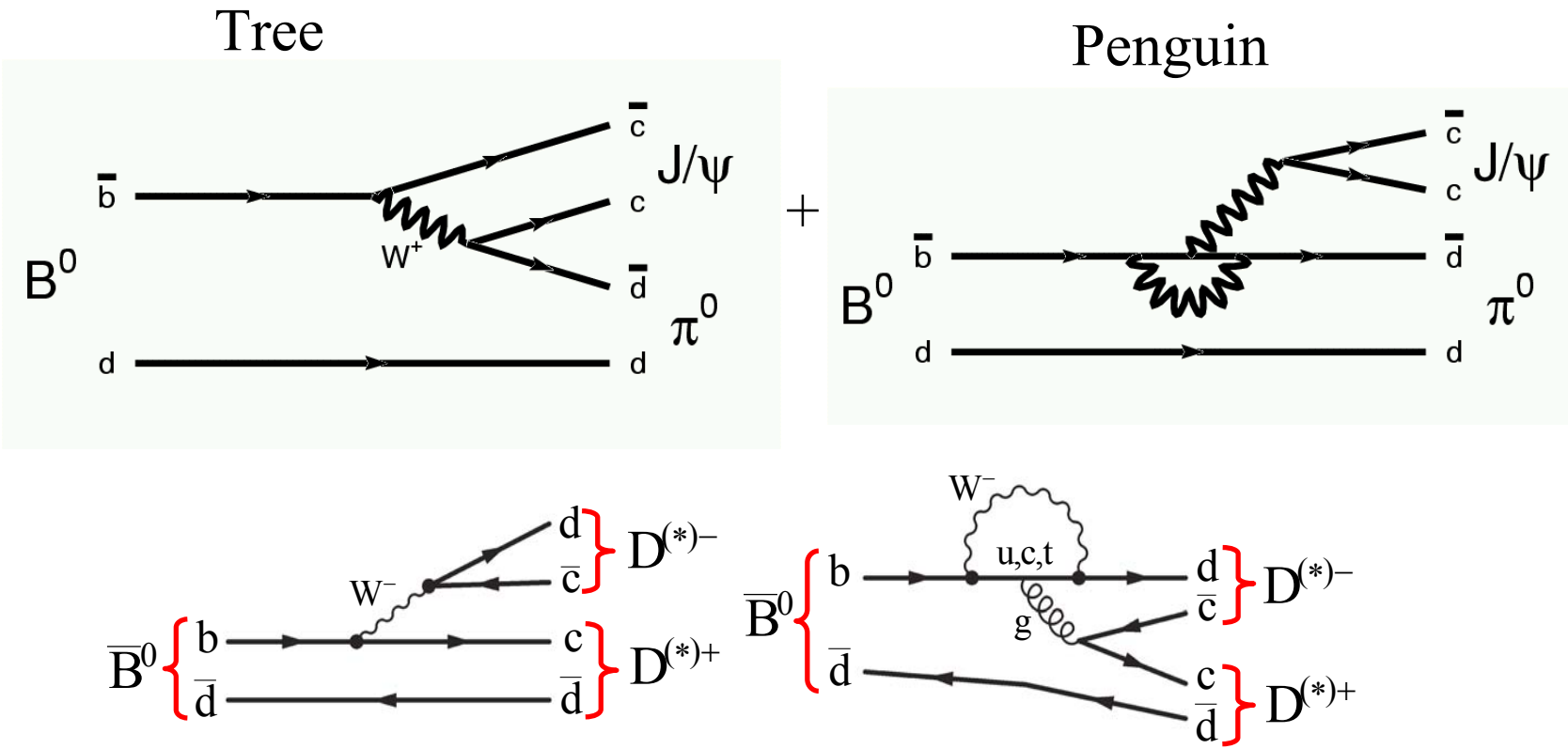
$|q/p|$ compatible with 1, less precise than in dileptons,

No deviation from CPT invariance seen.

BABAR B^0	K^0 System
$\|q/p\| = 1.005 \pm 0.008$	1.00327 ± 0.00012
$\|\Delta\Gamma/\Gamma\| < 0.08$ (90% CL)	-0.99827 ± 0.00001
$\Delta m/\Gamma = 0.770 \pm 0.011$ (WA)	0.4741 ± 0.0014
$\|2\delta m/\Gamma\| < 0.10$ (90% CL)	$< 1 \cdot 10^{-4}$
$2\delta\Gamma/\Gamma \in [-0.16, 0.04]$ (90% CL)	

Recent results
on CPV (B)

CPV in b→(c $\bar{c}$ d) Decays, J/ψ π^0 and $D^{(*)+}$ $D^{(*)-}$

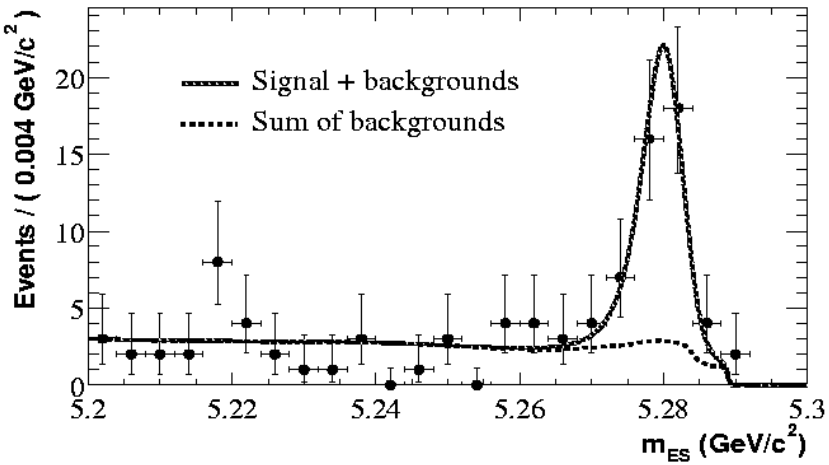


The same CPV phase as in B→ J/ψ K_S but may have penguin pollution.

$B \rightarrow \psi \pi^0$

BABAR:

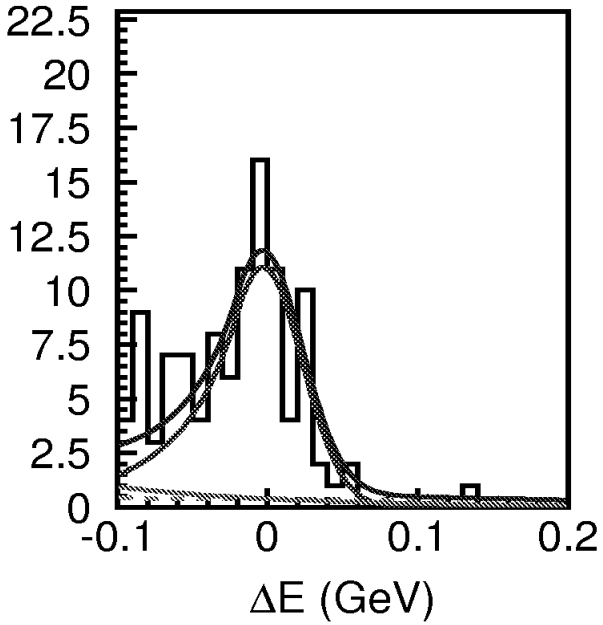
81 fb^{-1}

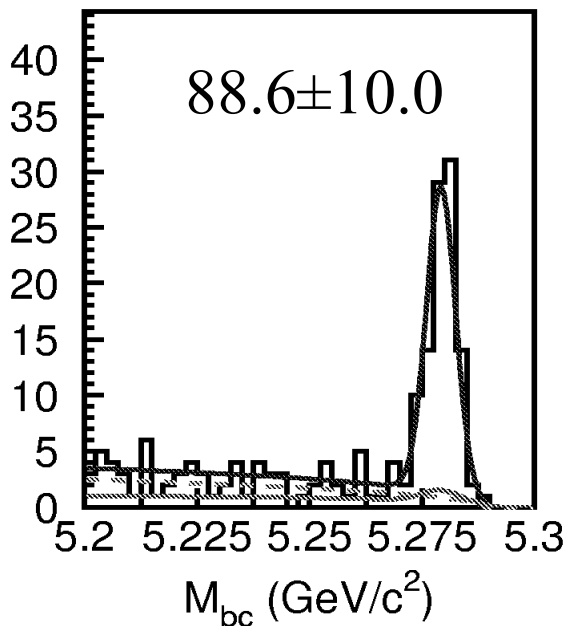


40±7

Belle:

140 fb^{-1}



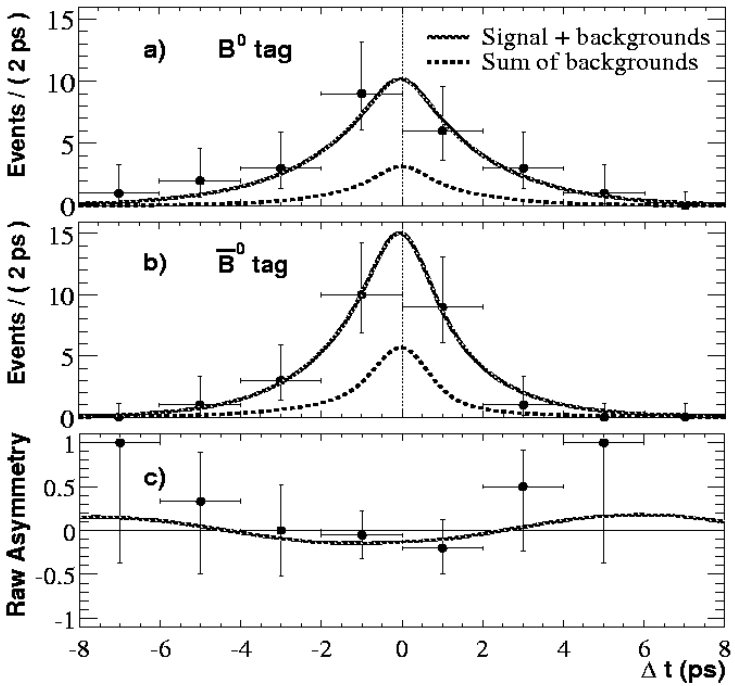


8.2 peaking bkg

from T. Browder, LP03

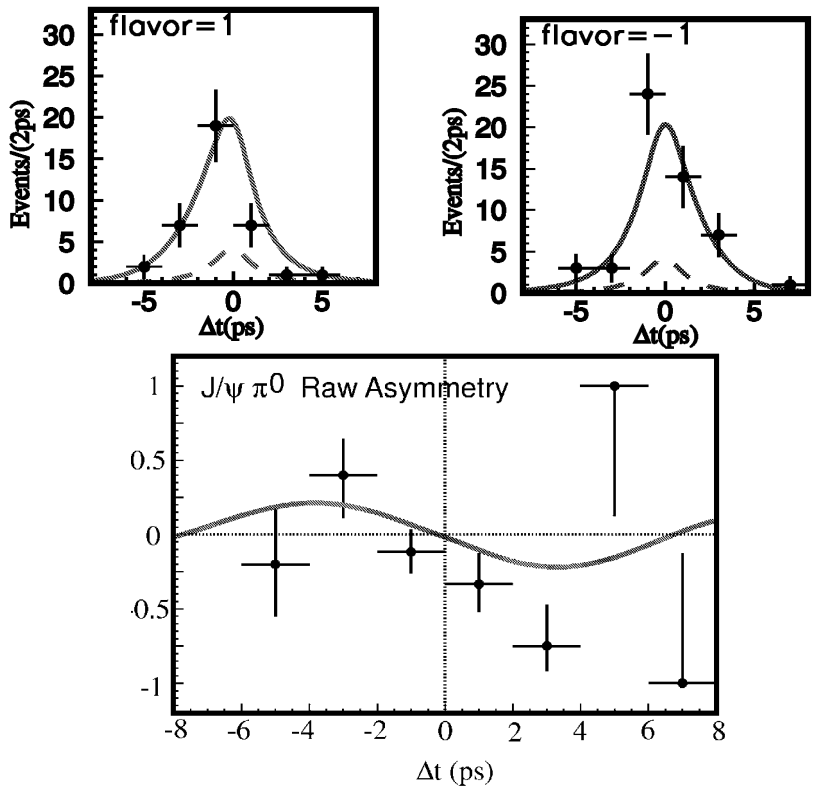
CPV in b→(c $\bar{c}$ d) decays: B→ψ π^0

BABAR 2003



$$\sin(2\varphi_{1\text{eff}}) = 0.05 \pm 0.49 \pm 0.16$$

Belle 2003

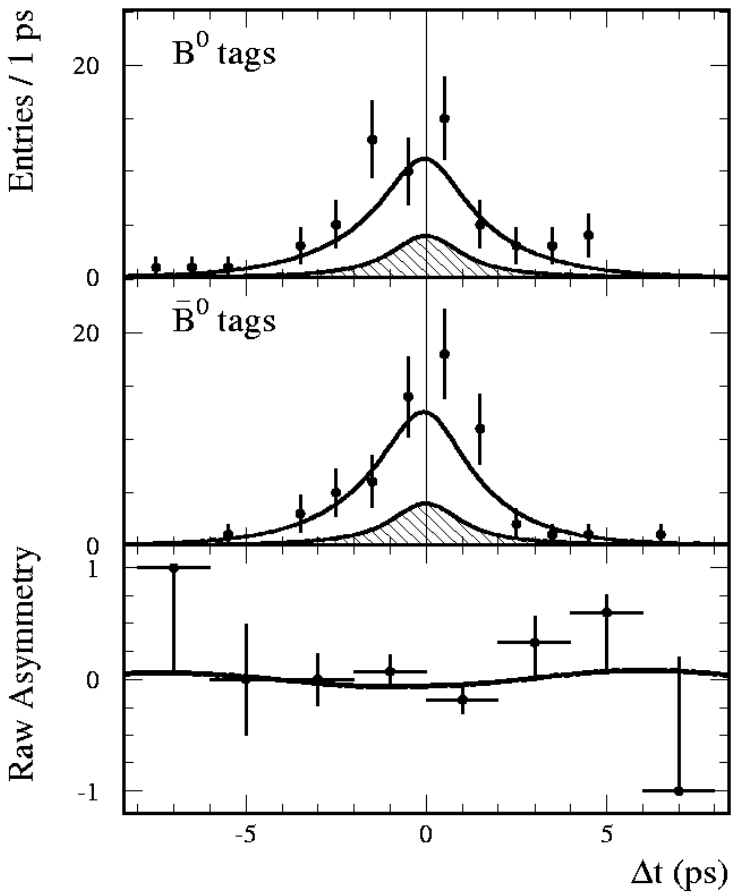


$$\sin(2\varphi_{1\text{eff}}) = 0.72 \,^{+0.37}_{-0.42} \pm 0.08$$

from T. Browder, LP03

CPV in b→(c $\bar{c}$ d) decays: B→D^{*+} D^{*-}

BABAR 2003



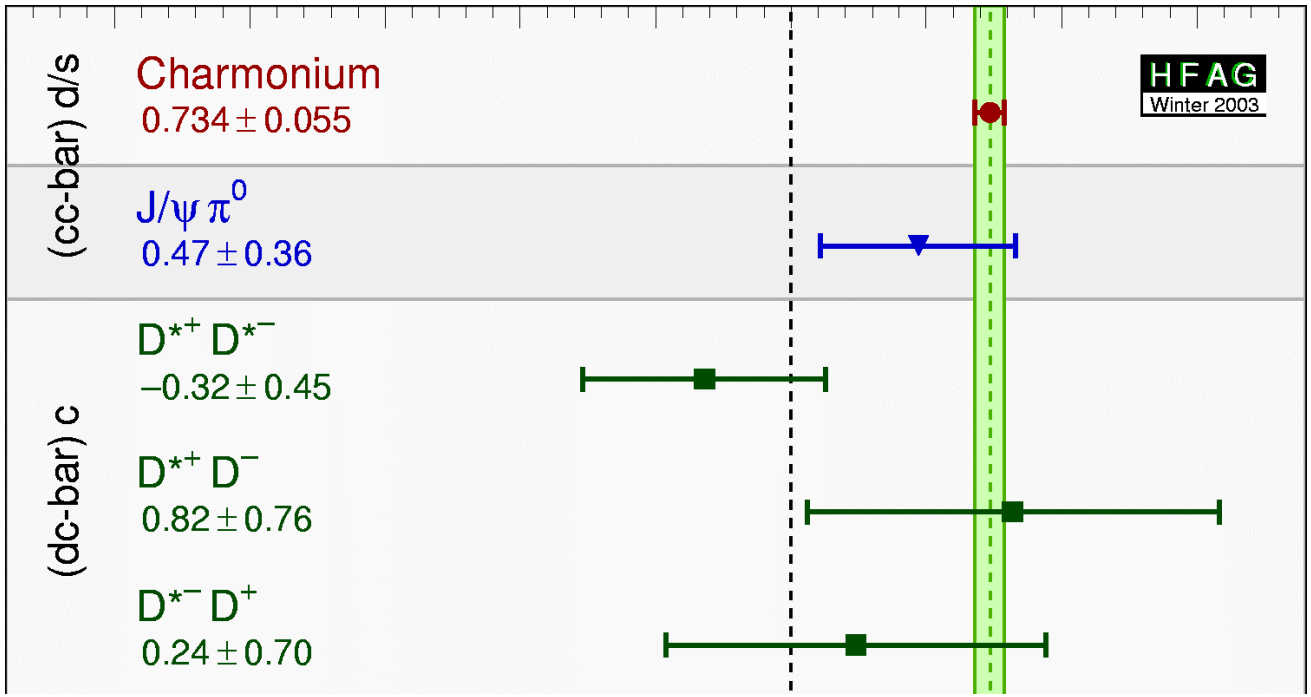
$\sin(2\varphi_{1\text{eff}})=-0.05\pm0.29\pm0.10$

2.5 σ from b→c $\bar{c}$ s, might indicate SM penguin contribution is present.

$|\lambda|=0.75\pm0.19\pm0.02$

Consistent with 1 or no direct CPV

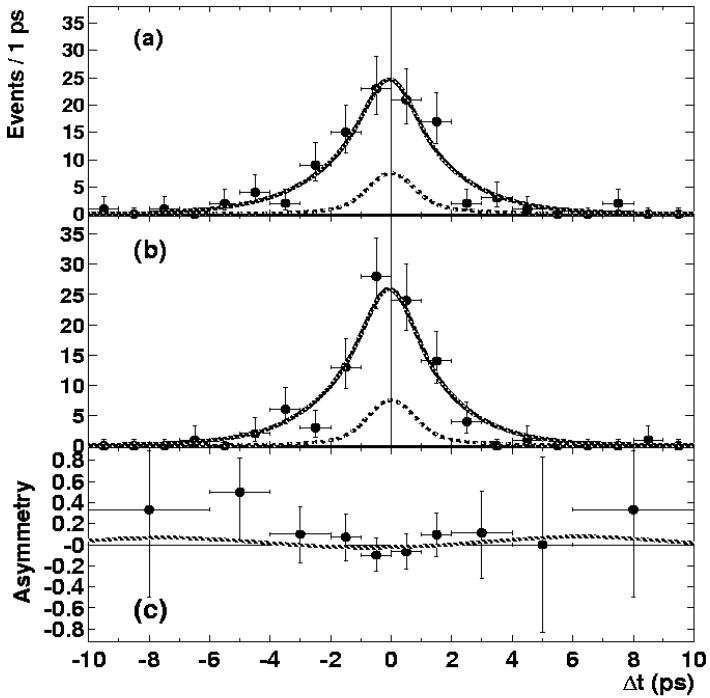
Summary of CPV in b→(c $\bar{c}$ d) decays:



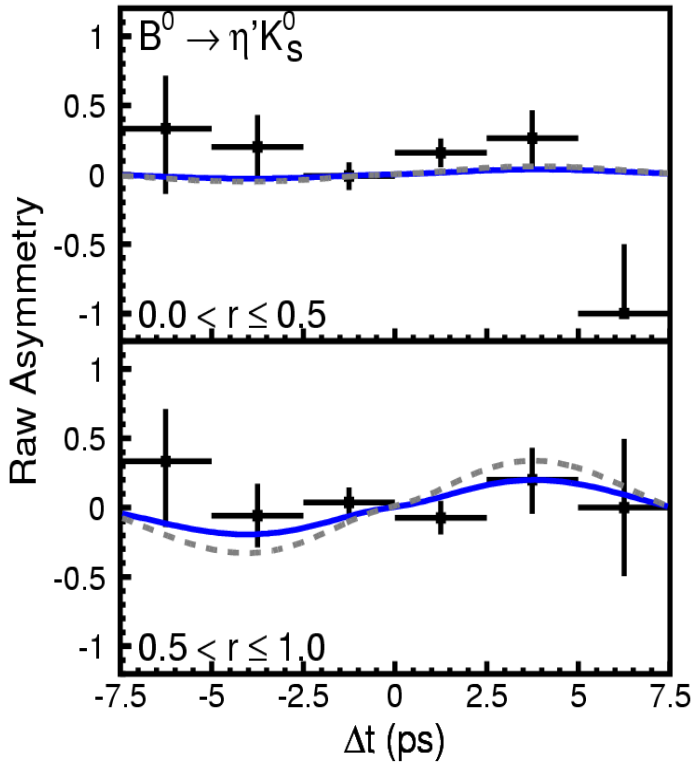
Errors are large for these modes so that it is difficult to verify whether there is large penguin pollution. There is a 2.5 σ "hint" for penguin pollution in Babar's result for B→D* D*

Search for New Physics in $B \rightarrow \eta' K_S$ (SM: only penguin)

BABAR 2003 [81 fb^{-1}]



Belle 2003 [140 fb^{-1}]

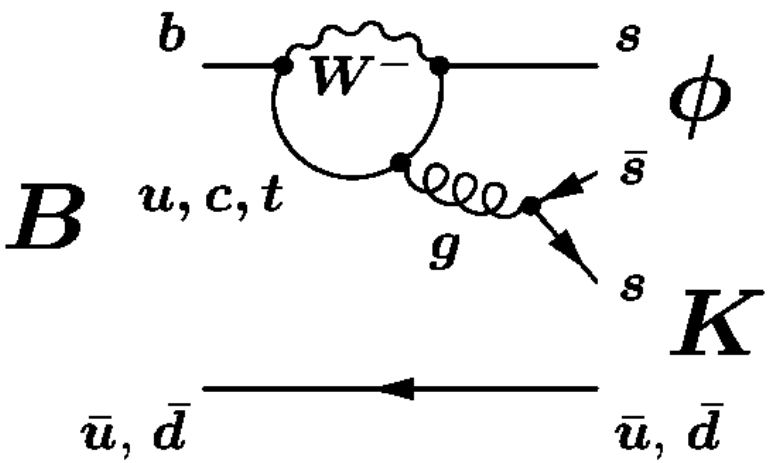


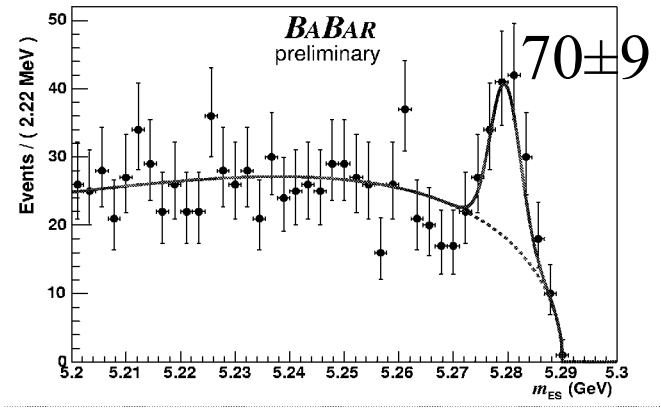
Babar: $S_{\eta' Ks} = 0.02 \pm 0.34 \pm 0.03$

Belle: $S_{\eta' Ks} = 0.43 \pm 0.27 \pm 0.05$

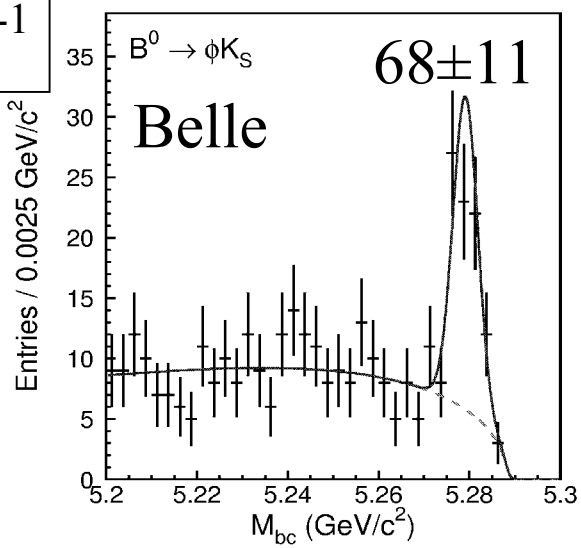
Search for New Physics in $B \rightarrow \phi K_S$ (SM: only penguin)

110 fb^{-1}





140 fb^{-1}

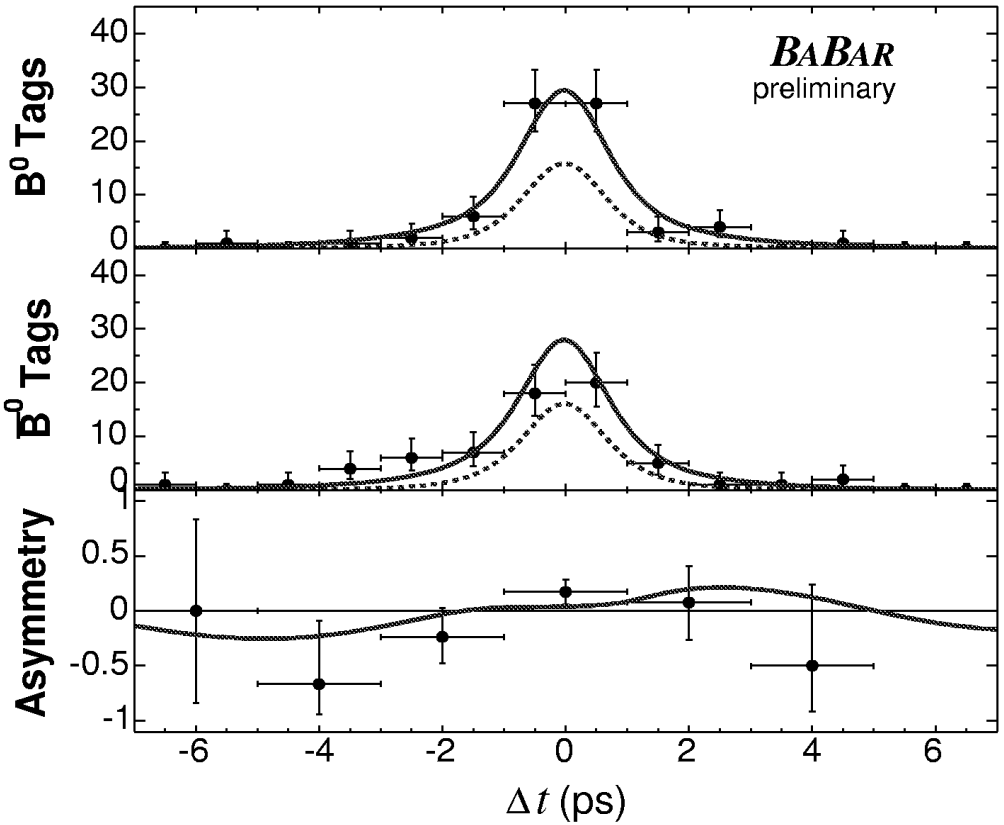


In the SM, $\sin 2\beta^{eff}$

$= \sin 2\beta$ ($B \rightarrow J/\psi\, K_S$)

BABAR 2003:

110 fb^{-1}

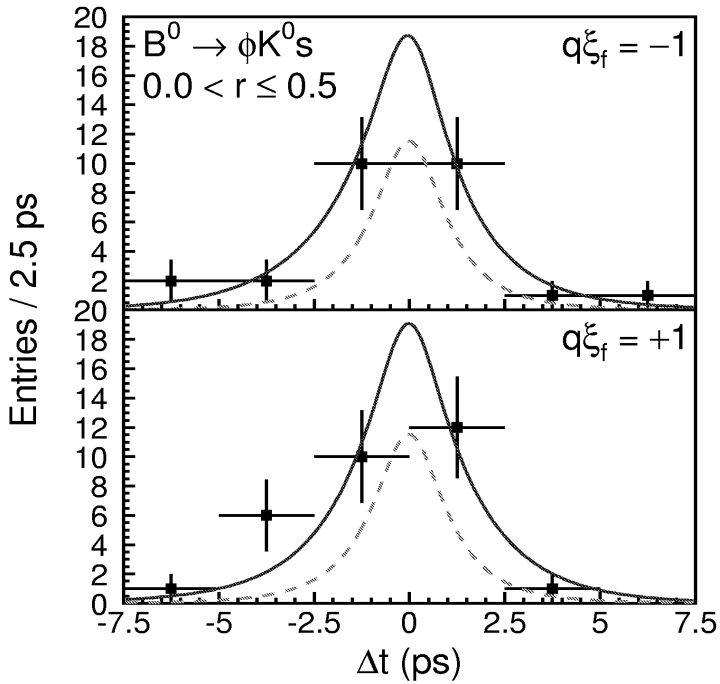


$$\sin 2\beta\,(\varphi\ K_S) = +0.45 \pm 0.43 \pm 0.07$$

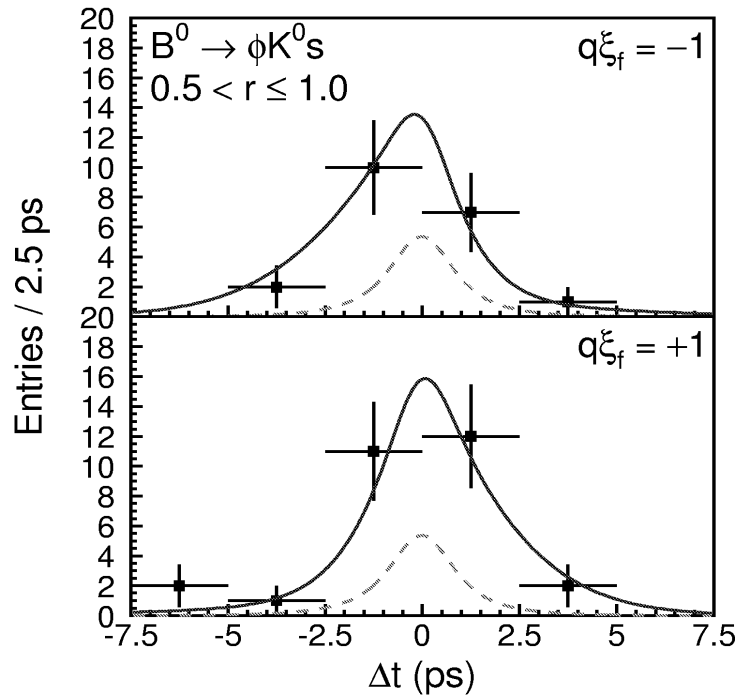
Belle 2003: CP Asymmetry in B→φ K_S

140 fb^{-1}

Poor tags



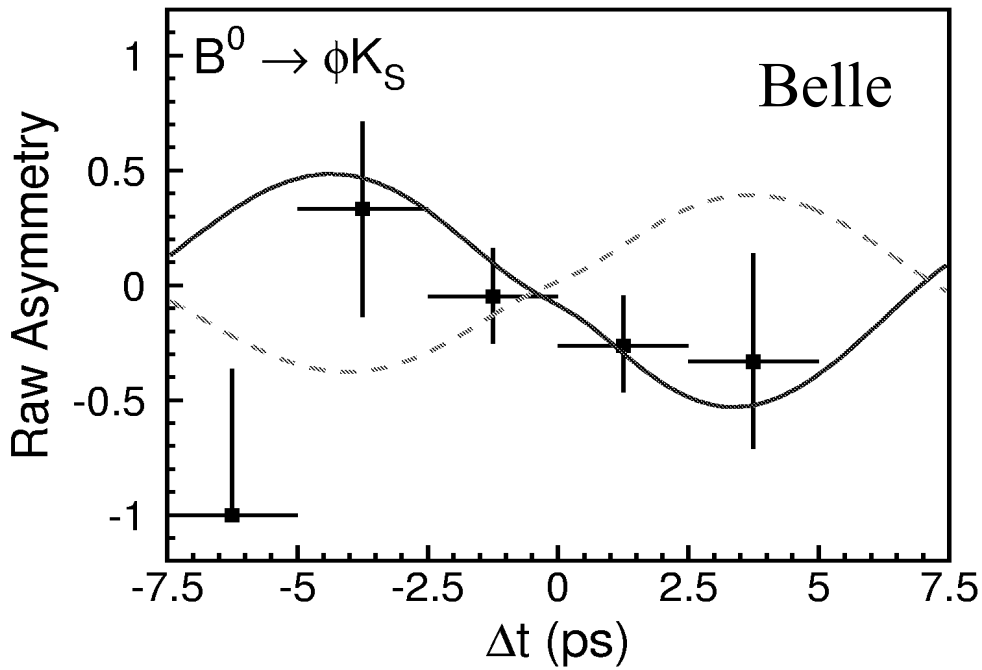
Good tags



from T. Browder, LP03

CP Asymmetry in B→φ K_S

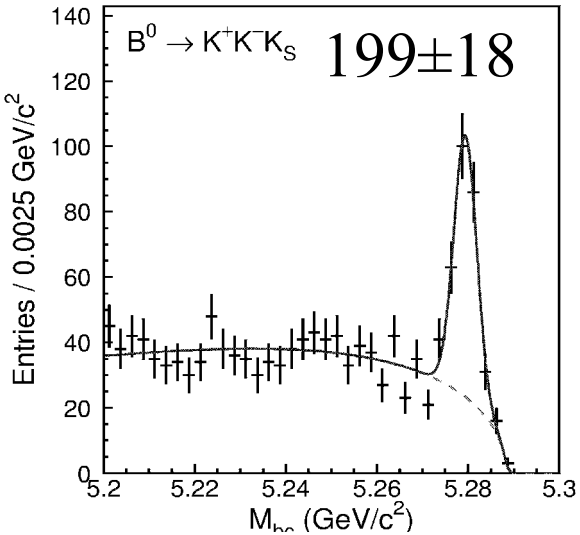
140 fb^{-1}

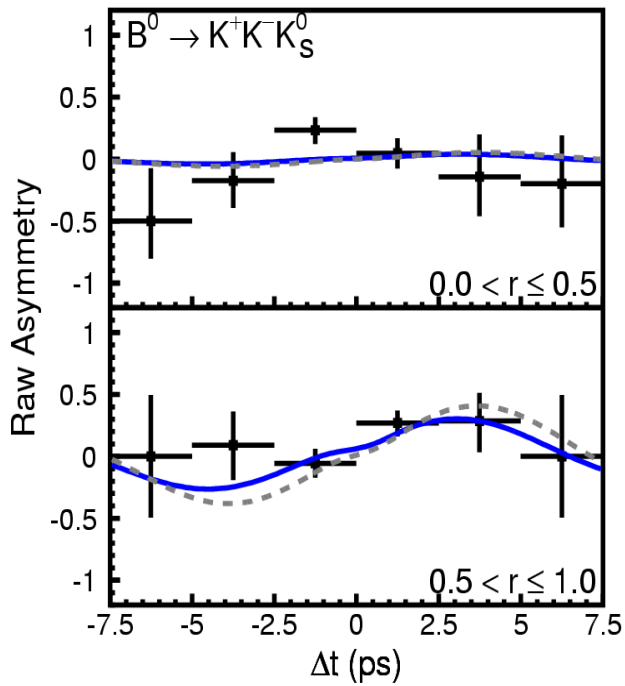


3.5σ off

Belle: $\sin 2\varphi_{1\text{eff}} = -0.96 \pm 0.50\ ^{+0.09}_{-0.11}$

BABAR: $\sin 2\beta_{\text{eff}} = +0.45 \pm 0.43 \pm 0.07$





CPV in B→ K^+ K^- K_S penguin decay. (without φ K_S)

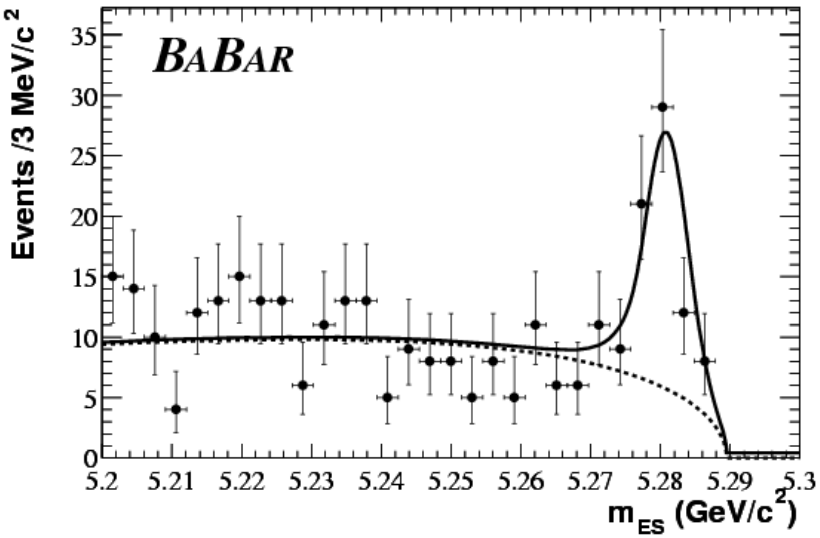
Belle 2003:[140 fb^{-1}]

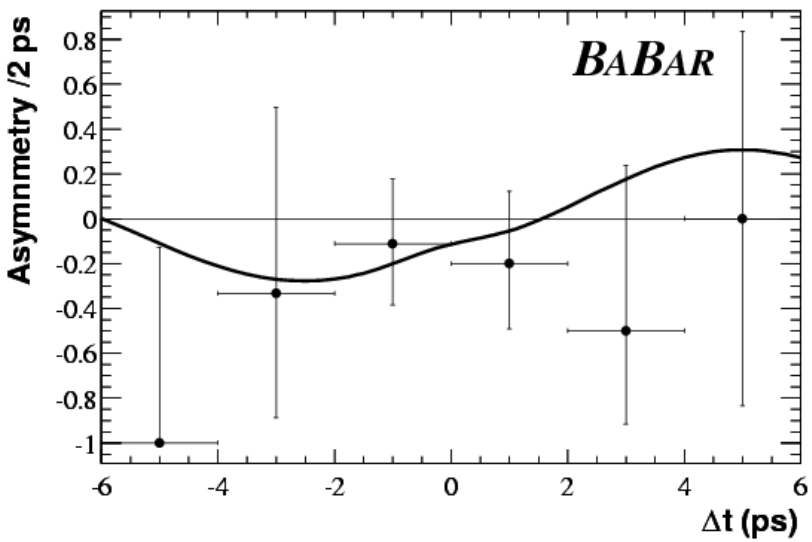
$$-S_{KKK_S} = 0.51 \pm 0.26 \,^{+0.18}_{-0.00}$$

The third error is due to uncertainty in the CP content.

{ I checked this sign in the Belle paper, since $\eta_{CP}(KKK_S) = +1$, $\lambda = + e^{-i2\beta}$ }

BABAR: CPV with $B^0 \rightarrow K_S \pi^0$ [b→s d $\bar{d}$]





N	=	123 ± 16
C	=	$0.40^{+0.27}_{-0.28} \pm 0.10$
S	=	$0.48^{+0.38}_{-0.47} \pm 0.11$
S (C=0)	=	$0.41^{+0.41}_{-0.48} \pm 0.11$

from T. Browder, LP03

Summary on β measurements:

$c\bar{c}s$ (tree dominated): $\sin 2\beta = 0.74 \pm 0.05$

in agreement with V_{CKM} and contributing to its precision

$c\bar{c}d$ (tree + penguin): $J/\psi\,\pi^0$ and $D^{(*)+}D^{(*)-}$

$\sin 2\beta_{eff} = \sin 2\beta$ except $D^{*+}D^{*-}$ (BABAR), 2.5 σ off,
could be SM penguins.

$s\bar{d}d$ (penguin dominated), $K_S\pi^0$: $\sin 2\beta_{eff} = \sin 2\beta$

$s\bar{s}d$ (penguin dominated), $\eta' K_S$, ϕK_S, $K^+K^-K_S$

$\sin 2\beta_{eff} = \sin 2\beta$ in $\eta' K_S$ (BABAR, Belle),

$= \sin 2\beta$ in ϕK_S (BABAR),

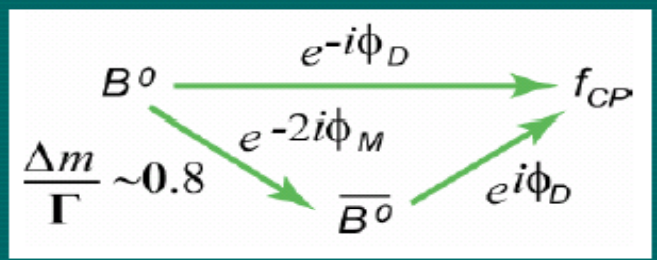
$= 3.5\ \sigma$ off in ϕK_S (Belle), NP ??

$= \sin 2\beta$ in $K^+K^-K_S$ (Belle).

$\alpha = \phi_2$

start with new result on $B \to \pi^+\pi^-$

CP violation in $B^0 \overline{B}^0$ system



$$A_{CP}(t) \equiv \frac{\Gamma(\overline{B}^0 \to f_{CP}; t) - \Gamma(B^0 \to f_{CP}; t)}{\Gamma(\overline{B}^0 \to f_{CP}; t) + \Gamma(B^0 \to f_{CP}; t)} = A_f \cos(\Delta m t) + S_f \sin(\Delta m t)$$

$$A_f = \frac{|\lambda_f|^2 - 1}{|\lambda_f|^2 + 1}$$

$$S_f = \frac{2Im(\lambda_f)}{|\lambda_f|^2 + 1}$$

$$\lambda_f \equiv e^{-2i\phi_M} \frac{A(\overline{B} \to f)}{A(B \to f)}$$

Standard model predictions

	$b \to c\bar{c}s$	$b \to c\bar{c}d$	$b \to s\bar{s}s$	$b \to u\bar{u}d$
example	$J/\psi K_S$	$J/\psi \pi^0$	ϕK_S	$\pi^+\pi^-$
A_f	0	0	small	$\neq 0$
S_f	$\sin 2\phi_1$	$\sin 2\phi_1$	$\sin 2\phi_1$	"$\sin 2\phi_2$"

Note: $A_f \neq 0 \Rightarrow \Gamma(B \to f) \neq \Gamma(\overline{B} \to \overline{f}) \Rightarrow$ direct CP violation.

from H. Sagawa, Belle, 19/01/04

$$S_{\pi\pi} = \sqrt{1 - A_{\pi\pi}^2}\,\sin 2\phi_{2eff}$$

$$= \sqrt{1 - A_{\pi\pi}^2}\,\sin 2(\phi_2 + \theta)$$

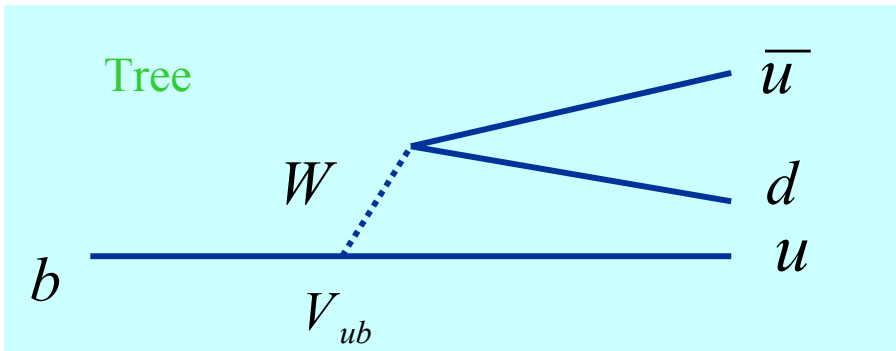
$A_{\pi\pi} = 0,\ \ S_{\pi\pi} = \sin 2\phi_2$

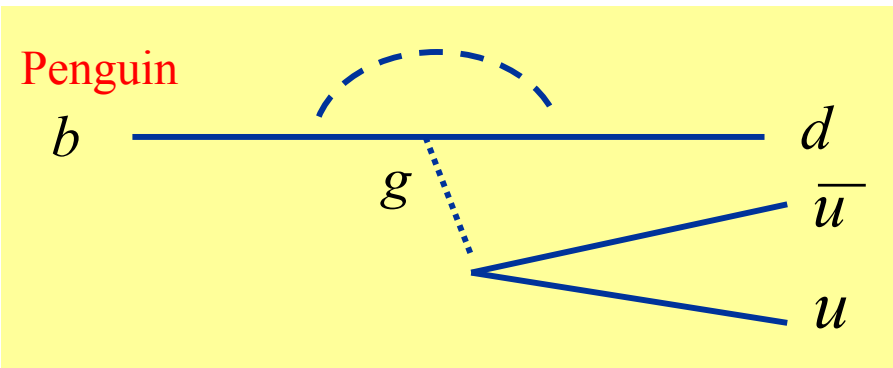
if "penguin" is negligible

Direct *CP* violation in $B^0 \to \pi^+\pi^-$

Penguin diagram is not expected to be small

theoretical preference: $|P/T| \sim 0.3$





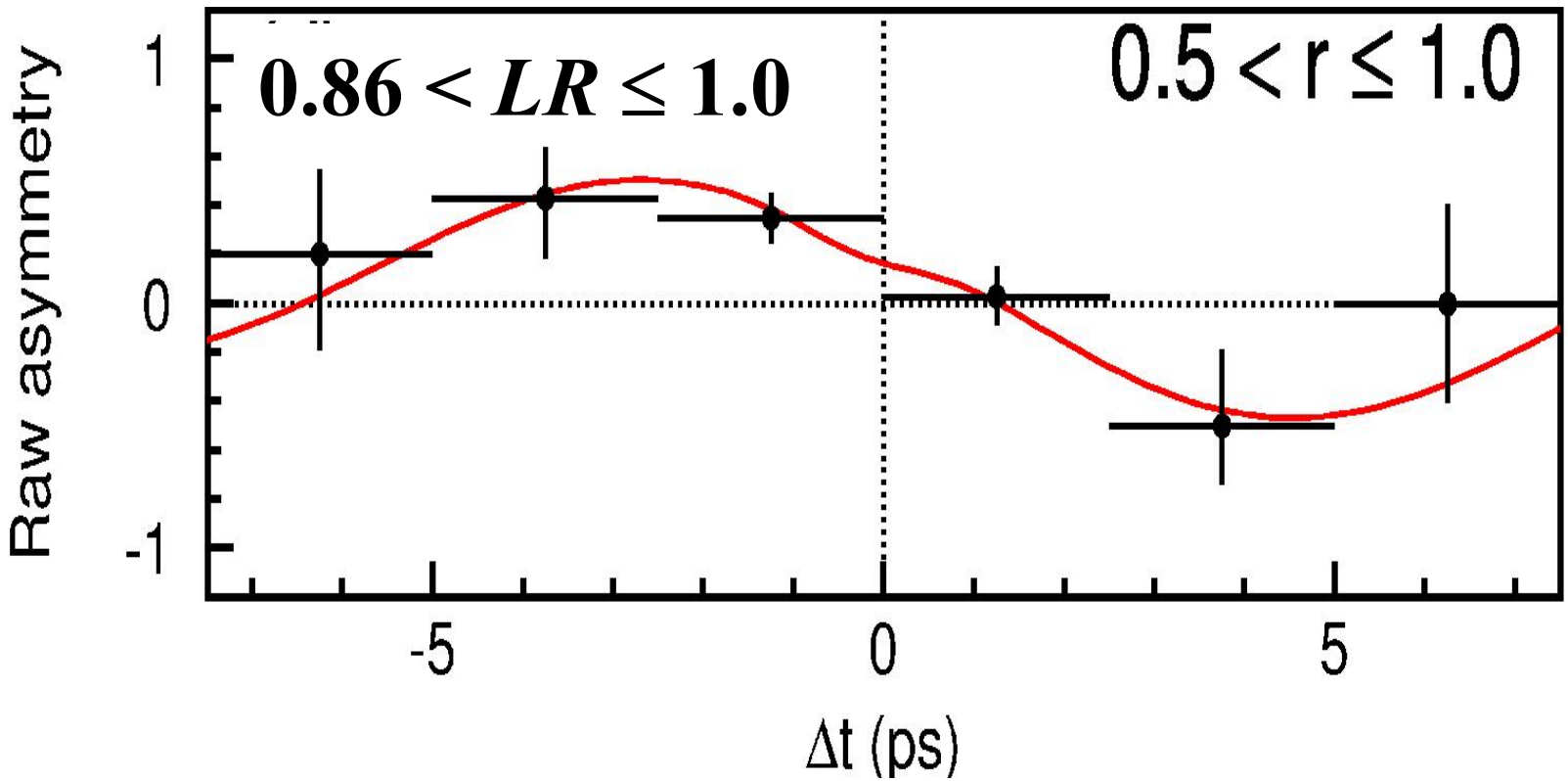
Nasty "pollution" for precise ϕ_2 measurements

Wonderful "contribution" for the first observation of direct *CP* violation in *B*-meson decays !

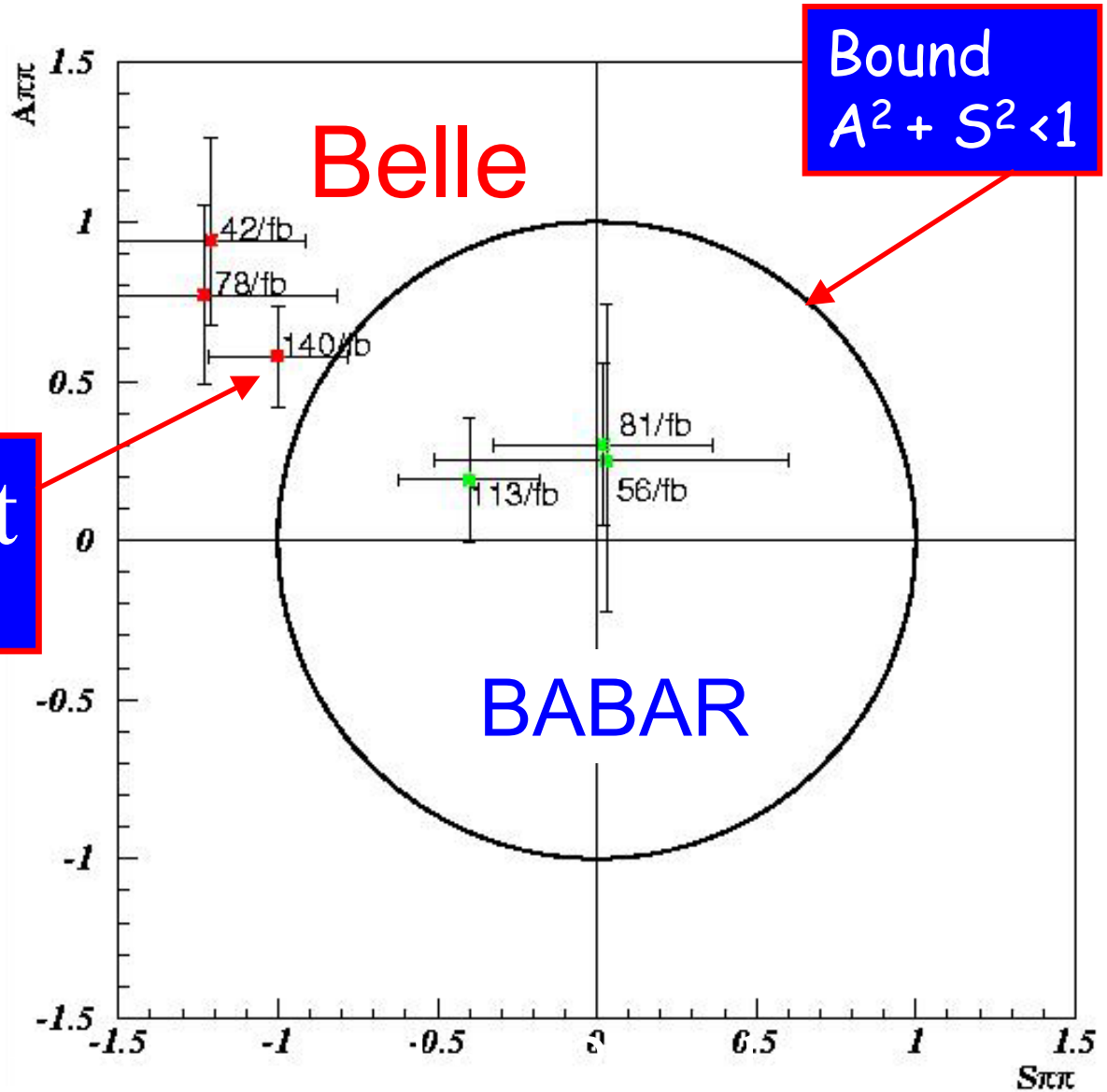
Belle results (140fb^{-1})

1529 ev. (all *LR-r* regions)

$$A_{\pi\pi} = +0.58 \pm 0.15(\text{stat}) \pm 0.07(\text{syst})$$
$$S_{\pi\pi} = -1.00 \pm 0.21(\text{stat}) \pm 0.07(\text{syst})$$



History of $A_{\pi\pi}$ and $S_{\pi\pi}$



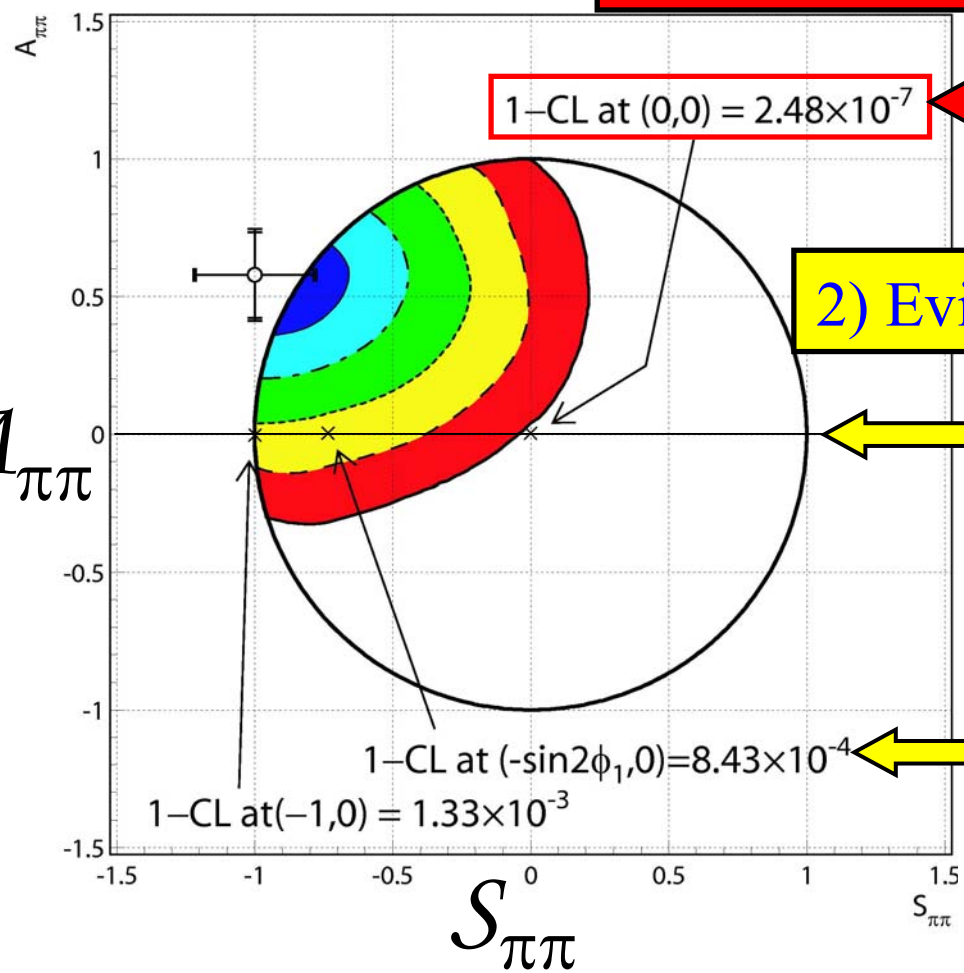
from H. Sagawa, Belle, 19/01/04

Significance

Feldman-Cousins Analysis

1) Observation of *CP* violation

(5.2σ)



2) Evidence for direct *CP* Violation

≥3.2σ

for $\mathcal{A}_{\pi\pi}$=**0** and any $S_{\pi\pi}$

3.3σ

for "**superweak**" case

Constraints on the CKM angle ϕ_2

$$A(B^0 \to \pi^+\pi^-) = -(|T|e^{i\delta_T}e^{i\phi_3}) + |P|e^{i\delta_P}),$$

$$A(\overline{B}^0 \to \pi^+\pi^-) = -(|T|e^{i\delta_T}e^{-i\phi_3}) + |P|e^{i\delta_P}),$$

$$\lambda_{\pi\pi} = e^{i\phi_2}\frac{1+|P/T|e^{i(\delta+\phi_3)}}{1+|P/T|e^{i(\delta-\phi_3)}}$$

4 parameters

$$S_{\pi\pi} = [\sin 2\phi_2 + 2|P/T|\sin(\phi_1 - \phi_2)\cos\delta - |P/T|^2 \sin 2\phi_1]/R_{\pi\pi},$$

$$A_{\pi\pi} = -[2|P/T|\sin(\phi_1+\phi_2)\sin\delta]/R_{\pi\pi},$$

$$R_{\pi\pi} = 1 - 2|P/T|\cos(\phi_1+\phi_2)\cos\delta + |P/T|^2$$

convention taken from
M.Gronau and J.L.Rosner
Phys. Rev. D65, 093012 (2002)

$\delta \equiv \delta_P - \delta_T$

Strong phase difference

$\|P/T\|$	0.15-0.45 (representative) Theory ~0.3
ϕ_1	23.7 deg (Belle & BABAR combined)

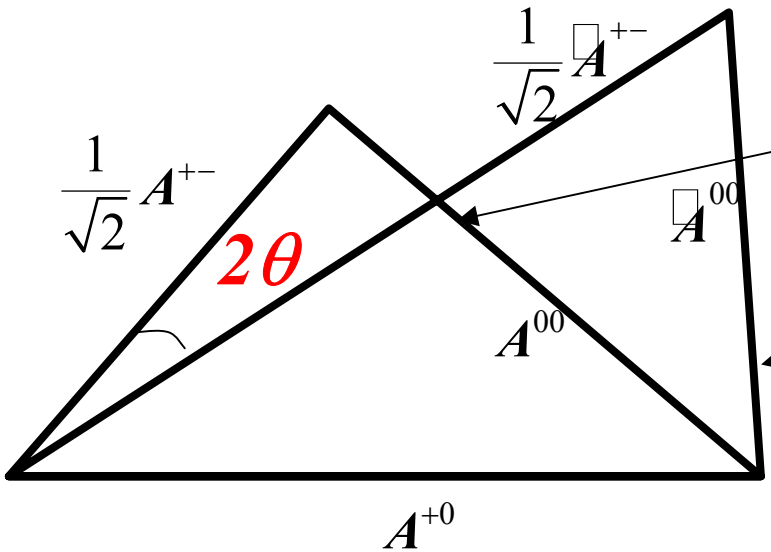
$\pi\pi$ isospin analysis

($\rightarrow \theta = \phi_2 - \phi_{2eff}$)

$$S_{\pi\pi} = \sqrt{1 - A_{\pi\pi}^2}\,\sin 2\phi_{2eff}$$
$$= \sqrt{1 - A_{\pi\pi}^2}\,\sin 2(\phi_2 + \theta)$$

Gronau-London

Isospin triangles



$$A \text{ amplitude}: \frac{1}{\sqrt{2}}A^{+-} + A^{oo} = A^{+0}$$

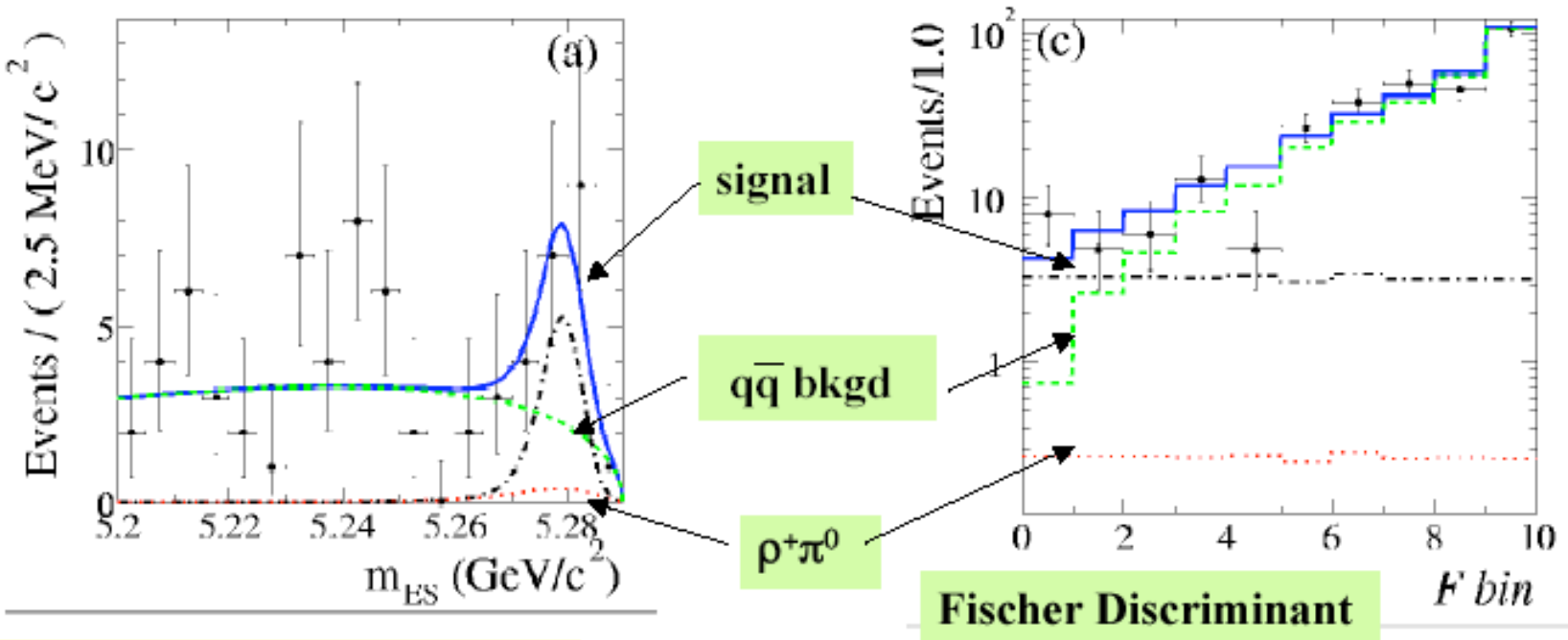
$$\bar{A} \text{ amplitude}: \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{oo} = \bar{A}^{+0}$$

Experimental inputs for B $\bar{\text{B}}$ average rates of $\pi^+\pi^-$ and $\pi^0\pi^0$ give bounds on θ

BaBar: The decay $B \to \pi^0 \pi^0$

- Used their full data set (113 fb^{-1})
- Measured Br($B \to \rho^+ \pi^0$) [Br($B \to \rho^+ \pi^0$) =(11±2.7)x10^{-6}]

Control over a major background source. Br about half of previous assumed value



$$N(\pi^0\pi^0) = 46^{+14+2}_{-13-3}$$

$$\overline{B}(B^0 \to \pi^0\pi^0) = 2.1 \pm 0.6 \pm 0.3 \times 10^{-6}$$

Significance: 4.2 σ ➔Observation

Table 1: The recent results of $BR(B \to \pi\pi)$ and $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ decays. The unit is 10^{-6}.

Mode	Belle	BaBar	CLEO	New Avg.
$\pi^+\pi^-$	$4.4 \pm 0.6 \pm 0.3$	$4.7 \pm 0.6 \pm 0.2$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	4.55 ± 0.44
$\pi^+\pi^0$	$5.3 \pm 1.3 \pm 0.5$	$5.5^{+1.0}_{-0.9} \pm 0.6$	$4.6^{+1.8+0.6}_{-1.6-0.7}$	5.27 ± 0.79
$\pi^0\pi^0$	$1.7 \pm 0.6 \pm 0.2$	$2.1 \pm 0.6 \pm 0.3$	< 4.4	1.90 ± 0.47
$\mathcal{A}_{\pi\pi}$	$+0.77 \pm 0.27 \pm 0.08$	$+0.19 \pm 0.19 \pm 0.05$	–	$+0.38 \pm 0.16$
$\mathcal{S}_{\pi\pi}$	$-1.23 \pm 0.41^{+0.08}_{-0.07}$	$-0.40 \pm 0.20 \pm 0.03$	–	-0.58 ± 0.20

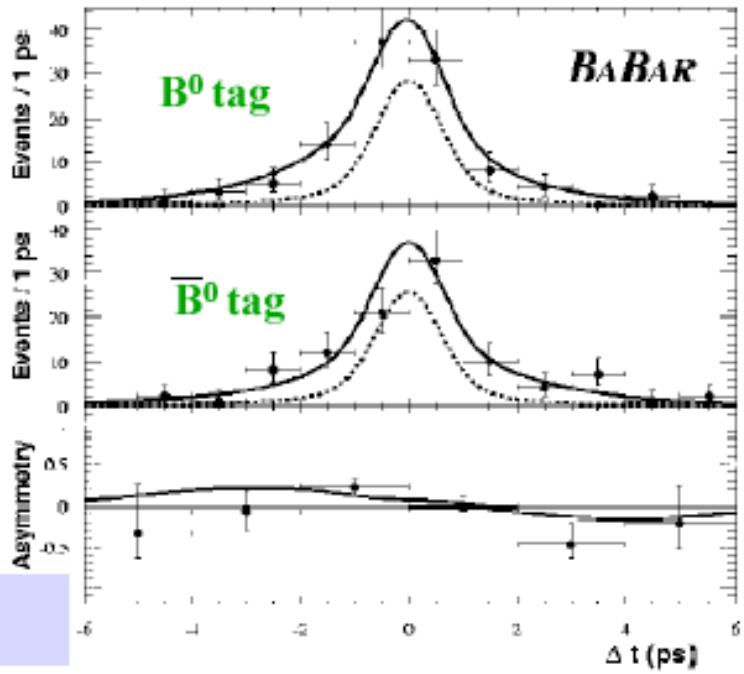
new

HFAG Summer 03

$\pi^+ \pi^-$ BABAR 2003:

- Reprocessed Run1+2 (82 fb^{-1})
- Added Run 3 ((31 fb^{-1})

$$N_{\pi\pi} = 265.9 \pm 24.0$$
$$N_{K\pi} = 873.3 \pm 37.5$$
$$N_{KK} = 12.5 \pm 10.4$$



CPV Results

Run1+2(reprocessed)

S=-0.252±0.27 C=-0.166 ±0.22

Run 3

S=-0.67±0.35 C=-0.33 ±0.34

$$S_{\pi\pi} = -0.40 \pm 0.22 \pm 0.03$$
$$C_{\pi\pi} = -0.19 \pm 0.19 \pm 0.05$$
$$A_{K\pi} = -0.107 \pm 0.041 \pm 0.013$$

$C_{\pi\pi} = - A_{\pi\pi}$

14#

$\pi^+ \pi^-$ Summary of HFAG Feb 2004:

Experiment	$S_{\pi\pi}$	$C_{\pi\pi} = -A_{\pi\pi}$
BABAR'03 N(BB)=123m	$-0.40 \pm 0.22 \pm 0.03$	$-0.19 \pm 0.19 \pm 0.05$
updated Winter'04: Belle'02 N(BB)=152m	$-1.00 \pm 0.21 \pm 0.07$	$-0.58 \pm 0.15 \pm 0.07$
Average	-0.74 ± 0.16	-0.46 ± 0.13

$1.9\ \sigma \Rightarrow$ $S = -0.74 \pm 0.30$

$1.5\ \sigma \Rightarrow$ $S = -0.46 \pm 0.20$

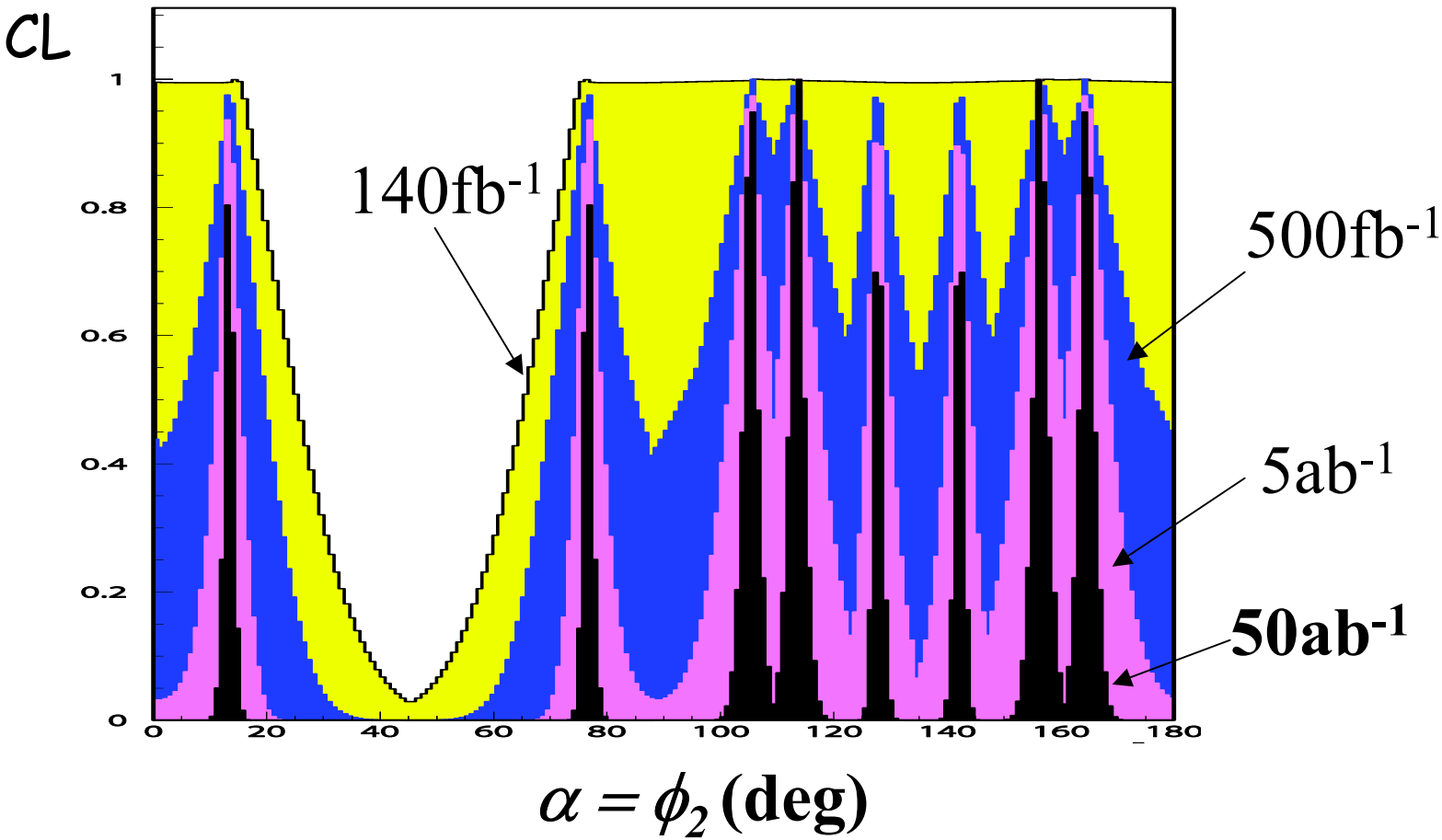
Even with scaled errors there is evidence for CP violation,

2.5 σ for type-II (mixing-decay interference)

and 2.3 σ for type-III (direct).

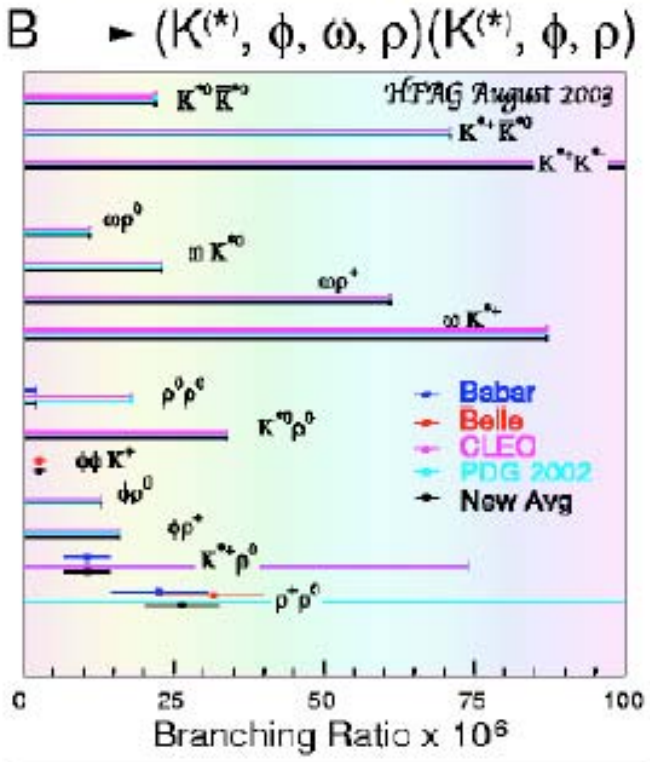
Evidence for two types of CPV, but nearly no constraint on α.

Prospects:



from H. Sagawa, Belle, 19/1/04

A promising channel:the B→ρρ system



The components of B→ρρ are identifeid: $B^0 \to \rho^+\rho^-$ (BaBar), $B^+ \to \rho^+\rho^0$ (Belle & BaBar)

Decays nealy 100% longitudinally polarized [CP-even]

Limit has been set on $B^0 \to \rho^0\rho^0$.

Quinn-Grossman bound applied to the longitudenal polarization component of the rates:

$$\sin^2(\alpha_{eff} - \alpha) \le (f_L^{\rho^0\rho^0} \times B^{\rho^0\rho^0})/(f_L^{\rho^+\rho^0} \times B^{\rho^+\rho^0}) < 0.10 \quad @90\ \%\ c.l.$$

$|\alpha_{eff} - \alpha| < 19^o$, ~ 2.5 times better than in $\pi\pi$.

from H. Jawahery, LP 03

Summary on α measurements:

In $\pi^+\pi^-$ decays of tagged B^0 and $\bar{B}^0$ decays, evidence for type-II CP violation, $2\,\mathrm{Im}\,\lambda / (1+|\lambda|^2) = -0.74 \pm 0.30$ and for type-III CP violation, $(1-|\lambda|^2) / (1+|\lambda|^2) = -0.46 \pm 0.20$
This is no evidence for new physics, but understandable in the St.Model as as tree-penguin interference.

In spite of measured $\pi^{\pm}\pi^0$ and untagged $\pi^0\pi^0$ rates, there is big uncertainty on $\alpha_{eff} - \alpha$. Much more (~10x) data needed.

Because of very small $\rho^0\rho^0$ rate, uncertainty on $\alpha_{eff} - \alpha$ is ~ 3 x smaller in B to $\rho\rho$. Data will be available soon.