EXPLORANDO LA FASCINANTE REGION ADONDE NO VALEN LAS HIPOTESES DE LA MECANICA ESTADISTICA DE BOLTZMANN-GIBBS

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Valencia, Enero 2012

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

> *In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92)* [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

TYPICAL SIMPLE SYSTEMS:

e.g., $W(N) \propto \mu^N \quad (\mu > 1)$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Euclidean geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

e.g., $W(N) \propto N^\rho \quad (\rho > 0)$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ **Entropy *Sq* (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**

POSTULATE FOR THE ENTROPIC FUNCTIONAL

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q=1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy S_q (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

- Concave
- Extensive
- Lesche-stable
- Finite entropy production per unit time
- Pesin-like identity (with largest entropy production)
- Composable
- Topsoe-factorizable

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

$DEFINITIONS:$ $q-logarithm:$ $$\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$$

$q-exponential:$ $$e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k\sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k\sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

- **Additive *versus* Extensive**

- Central Limit Theorem

- Predictions, verifications and applications

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)\, S_q(A)\, S_q(B) \;,$$

S_{BG} and $S_q^{Renyi}\ (\forall q)$ are additive, and $S_q\ (\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + ... + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \;,\; i.e.,\;\; S(N) \propto N \;\; (N \to \infty)$$

TYPICAL SIMPLE SYSTEMS:

$$W(N) \propto \mu^N \quad (\mu > 1)$$

$$\Rightarrow S_{BG}(N) = k_B \ln W(N) \propto N \quad (EXTENSIVE!)$$

TYPICAL COMPLEX SYSTEMS:

$$W(N) \propto N^{\rho} \quad (\rho > 0)$$

$$\Rightarrow S_q(N) = k_B \ln_q W(N) = k_B \frac{[W(N)]^{1-q} - 1}{1-q}$$

$$\propto N^{\rho(1-q)} \left[\text{if } q = 1 - \frac{1}{\rho}\right] \propto N \quad (EXTENSIVE!)$$

C. T., in eds. M. Gell-Mann and C. T. (Oxford University Press, 2004)
T., M. Gell-Mann and Y. Sato, Proc. Natl. Acad. Sc. USA **102**, 15377 (2005)

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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(Received 16 March 2008; revised manuscript received 16 May 2008; published 5 August 2008)

The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow \quad$ *Ising ferromagnet*

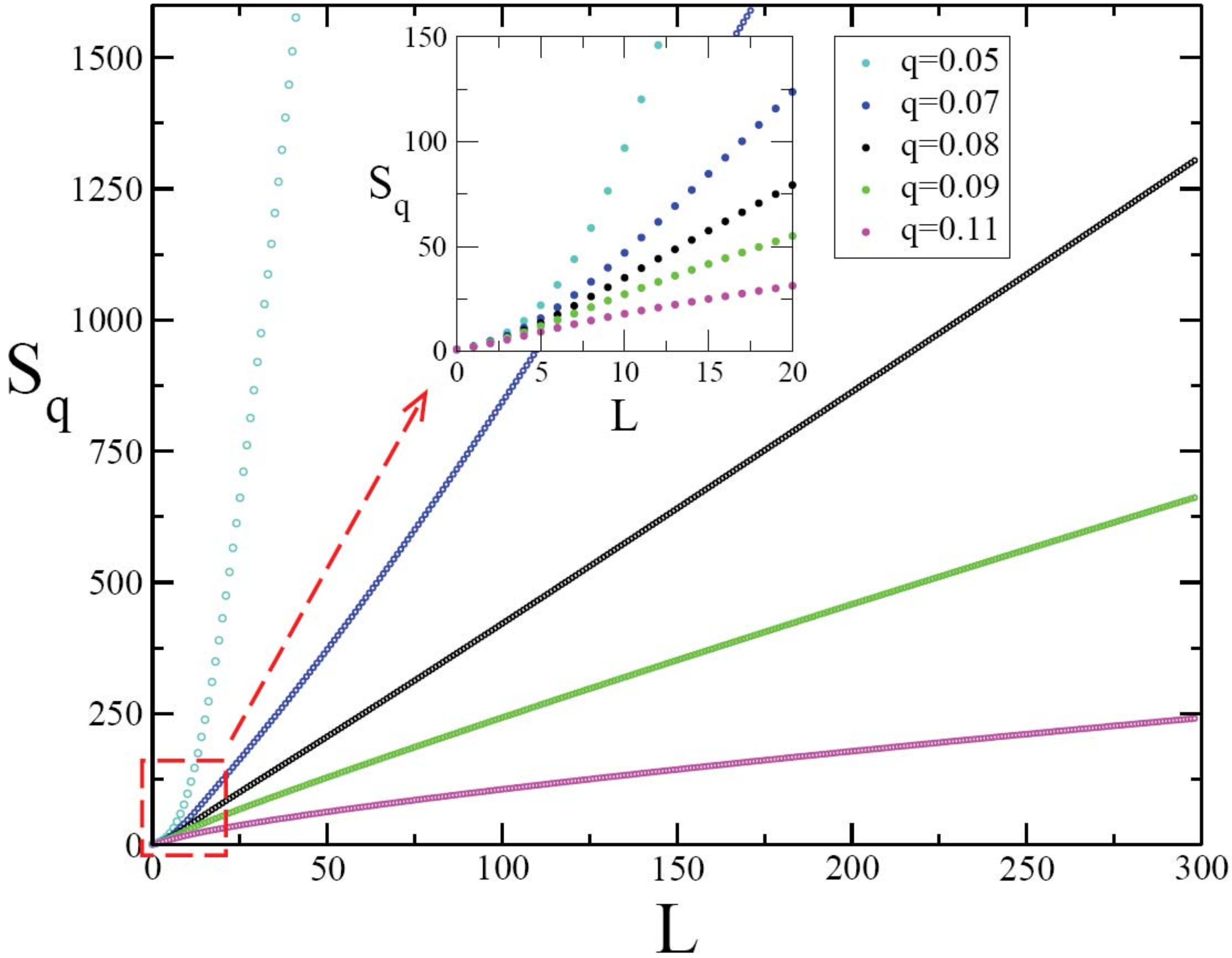
$0 < |\gamma| < 1 \quad \rightarrow \quad$ *anisotropic XY ferromagnet*

$\gamma = 0 \quad \rightarrow \quad$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

in P. Calabrese and J. Cardy, JSTAT P06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

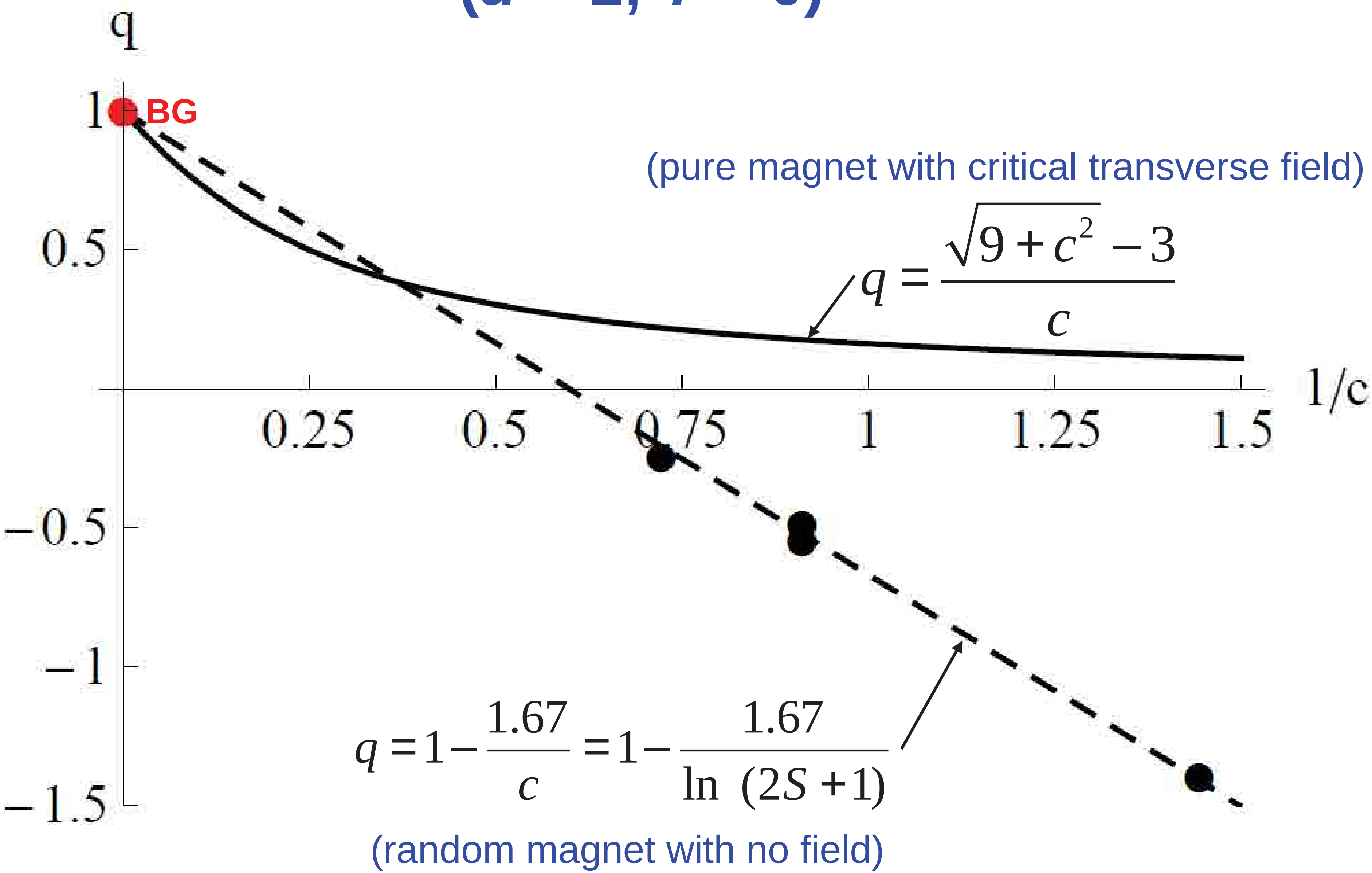
Ising and anisotropic XY ferromagnets $\Rightarrow \; c = \frac{1}{2} \;\Rightarrow\; q_{ent} = \sqrt{37}-6 \;\approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow \; c = 1 \;\Rightarrow\; q_{ent} = \sqrt{10}-3 \;\approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

(d = 1; T = 0)



A Saguia and MS Sarandy, Phys Lett A **374**, 3384 (2010)

Summarizing, for a wide class of quantum systems or subsystems with N elements, we know that

$$S_{BG}(N) \propto \ln L \propto \ln N \neq N \quad \text{for } d=1 \text{ quantum chains}$$

$$\propto L \propto \sqrt{N} \neq N \quad \text{for } d=2 \text{ bosonic systems}$$

$$\propto L^2 \propto N^{2/3} \neq N \quad \text{for } d=3 \quad \text{(black hole)}$$

$$\propto L^{d-1} \propto N^{(d-1)/d} \neq N \quad \text{for } d\text{-dimensional bosonic systems}$$

$$(d>1; \text{ area law})$$

$$\propto \frac{L^{d-1}-1}{d-1} \equiv \ln_{2-d} L \neq L^d \propto N \quad (d \geq 1) \qquad \text{(NONEXTENSIVE!)}$$

For the same class of quantum systems, we expect

$$S_{q_{ent}}(N) \propto L^d \propto N \quad (d \geq 1;\ q_{ent} \neq 1) \qquad \text{(EXTENSIVE!)}$$

(analytically and/or computationally shown for $d=1,2$)

F. Caruso and C. T., Phys Rev E **78,** 021101 (2008)

SYSTEMS	ENTROPY S_{BG} (additive)	ENTROPY S_q (q<1) (nonadditive)
Short-range interactions, weakly entangled blocks, etc	EXTENSIVE	NONEXTENSIVE
Long-range interactions (QSS), strongly entangled blocks, etc	NONEXTENSIVE	EXTENSIVE

↑

quarks-gluons, plasma, curved space ...?

- Additive *versus* Extensive

- **Central Limit Theorem**

- Predictions, verifications and applications

Milan j. math. 76 (2008), 307–328
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DOI 10.1007/s00032-008-0087-y
Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for q>1

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q - GENERALIZED CENTRAL LIMIT THEOREM:

S. Umarov, C.T. and S. Steinberg, Milan J Math **76,** 307 (2008)

q-Fourier transform:

$$F_q[f](\xi) \equiv \int_{-\infty}^{\infty} e_q^{ix\xi} \otimes_q f(x)\,dx = \int_{-\infty}^{\infty} e_q^{ix\xi[f(x)]^{q-1}} f(x)\,dx$$

$(q \geq 1)$

(nonlinear!)

For q<1 see K.P. Nelson and S. Umarov, Physica A **389**, 2157 (2010)

Hilhorst function:

[H.J. Hilhorst, JSTAT P 10023 (2010)]

$$f_A(x) = \begin{cases} \dfrac{\left[\ |x|^{(q-2)/(q-1)} - A\right]^{1/(q-2)}}{C_q\ |x|^{1/(q-1)}\ \left\{1 + (q-1)\left[\ |x|^{(q-2)/(q-1)} - A\right]^{2(q-1)/(q-2)}\right\}^{1/(q-1)}} & \text{if } 0 \le A < |x|^{(q-2)/(q-1)} \\ 0 & \text{if } 0 \le |x|^{(q-2)/(q-1)} \le A \\ \text{with} \quad \int_{-\infty}^{\infty} dx\, f_A(x) = 1 & \end{cases}$$

Particular case: $A = 0$

$$f_0(x) = \frac{1}{C_q\ [1 + (q-1)x^2]^{1/(q-1)}}$$

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q-Generalization of the inverse Fourier transform

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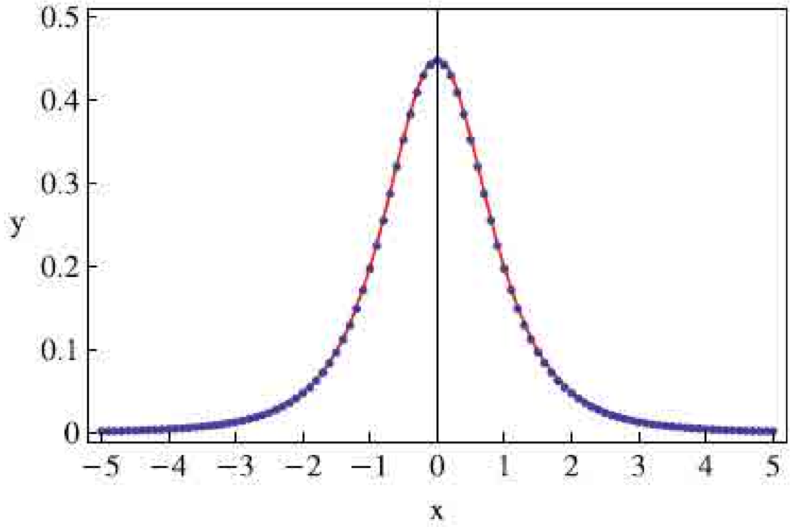
q-GENERALIZED INVERSE FOURIER TRANSFORM:

$$f(y) = \left[\frac{2-q}{2\pi} \int_{-\infty}^{+\infty} F_q[f(x+y)](\xi, y)\, d\xi \right]^{1/(2-q)} \qquad (1 \le q < 2)$$

Particular case $q = 1$:

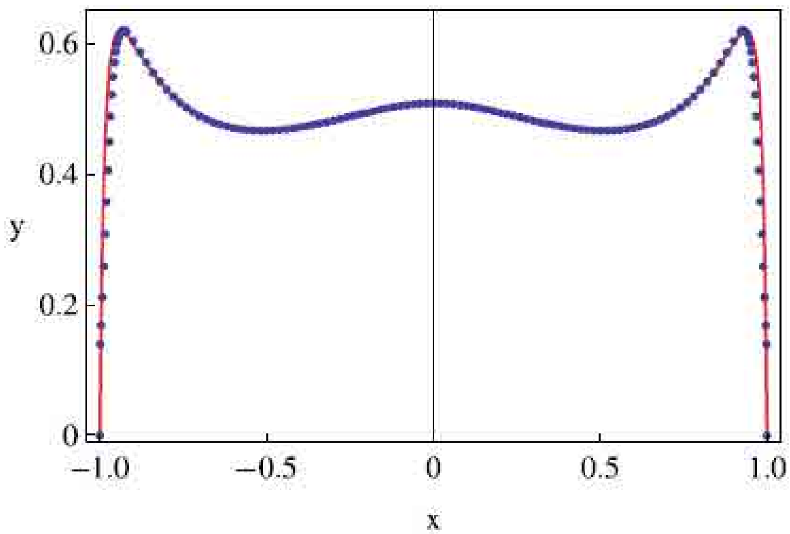
$$f(y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F[f(x+y)](\xi, y)\, d\xi = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F[f(x)](\xi)\, e^{-i\xi y}\, d\xi$$

M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)



← ***q* - Gaussian**

Fig. 1. Representation of $G_{3/2,1}(x)$. The continuous line corresponds to the analytical expression of the function; the dots were obtained by handling numerically Eqs. (1) and (6).



← **Hilhorst function**

Fig. 2. Representation of $f_{1,5/4}(x)$. The continuous line corresponds to the analytical expression of the function; the dots were obtained by handling numerically Eqs. (1) and (6). For all values of $x \in (-1, 1)$ we have used $\gamma = 2$ in Eq. (6), whereas for $x = \pm 1$ we have used $\gamma = 1$.

M. Jauregui and C. T.
Phys Lett A **375**, 2085 (2011)

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

q-moments remove the degeneracy associated with the inversion of the q-Fourier transform

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J. Stat. Mech. (2

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ *-scaled attractor* $\mathrm{F}(x)$ *when summing* $N\to\infty$ q*-independent identical random variables with symmetric distribution* $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q\equiv 2q-1,\; q_1 = \frac{1+q}{3-q}\right)$

	$q=1$ [*independent*]	$q\neq 1$ (*i.e.*, $Q\equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q<\infty$ $(\alpha=2)$	$\mathrm{F}(x)=$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathrm{F}(x)=G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x\rvert \ll x_c(q,2) \\ f(x)\sim C_q/\lvert x\rvert^{2/(q-1)} & \text{if } \lvert x\rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q\to 1} x_c(q,2)=\infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q\to\infty$ $(0<\alpha<2)$	$\mathrm{F}(x)=$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x\rvert \to\infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x\rvert \ll x_c(1,\alpha) \\ f(x)\sim C_\alpha/\lvert x\rvert^{1+\alpha} \\ \quad \text{if } \lvert x\rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha\to 2} x_c(1,\alpha)=\infty$ Levy-Gnedenko CLT	$\mathrm{F}(x)=L_{q,\alpha}$, *with same* $\lvert x\rvert\to\infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\,\alpha}(x) \sim C^*_{q,\alpha}/\lvert x\rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \text{(intermediate regime)} \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\,2}(x) \sim C^L_{q,\alpha}/\lvert x\rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \text{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

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A comprehensive classification of complex statistical systems and an axiomatic derivation of their entropy and distribution functions

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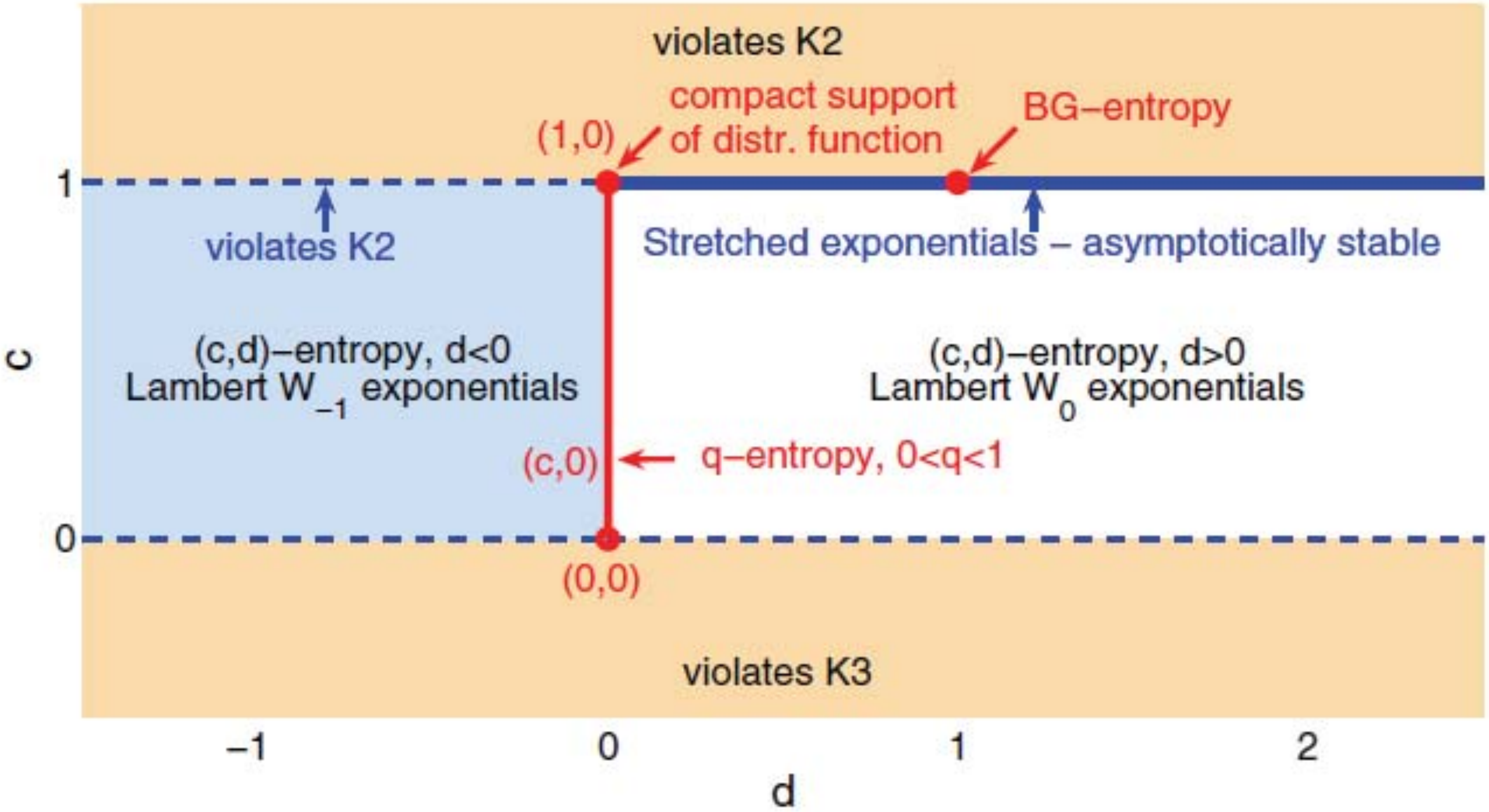

PHYSICAL REVIEW E **84**, 021121 (2011)

Group entropies, correlation laws, and zeta functions

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The notion of group entropy is proposed. It enables the unification and generaliztion of many different definitions of entropy known in the literature, such as those of Boltzmann-Gibbs, Tsallis, Abe, and Kaniadakis. Other entropic functionals are introduced, related to nontrivial correlation laws characterizing universality classes of systems out of equilibrium when the dynamics is weakly chaotic. The associated thermostatistics are discussed. The mathematical structure underlying our construction is that of formal group theory, which provides the general structure of the correlations among particles and dictates the associated entropic functionals. As an example of application, the role of group entropies in information theory is illustrated and generalizations of the Kullback-Leibler divergence are proposed. A new connection between statistical mechanics and zeta functions is established. In particular, Tsallis entropy is related to the classical Riemann zeta function.

$$S_q \leftrightarrow \varsigma(s) \equiv \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

$$= \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdot \frac{1}{1-7^{-s}} \cdot \frac{1}{1-11^{-s}} \mathrm{L}$$

- Additive *versus* Extensive

- Central Limit Theorem

- **Predictions, verifications and applications**

Thermostatistics of Overdamped Motion of Interacting Particles

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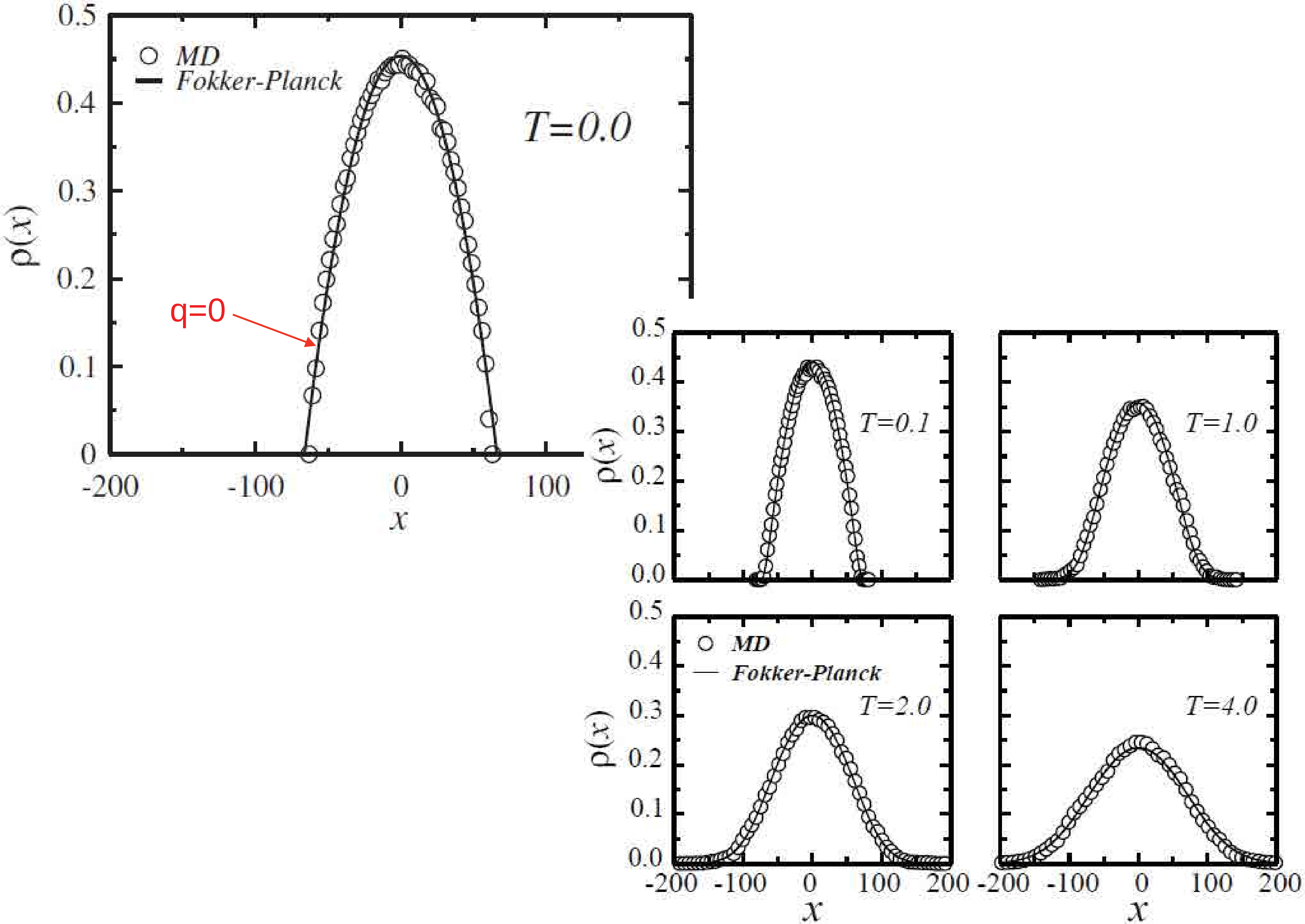
(Received 8 August 2010; published 22 December 2010)

We show through a nonlinear Fokker-Planck formalism, and confirm by molecular dynamics simulations, that the overdamped motion of interacting particles at $T = 0$, where T is the temperature of a thermal bath connected to the system, can be directly associated with Tsallis thermostatistics. For sufficiently high values of T, the distribution of particles becomes Gaussian, so that the classical Boltzmann-Gibbs behavior is recovered. For intermediate temperatures of the thermal bath, the system displays a mixed behavior that follows a novel type of thermostatistics, where the entropy is given by a linear combination of Tsallis and Boltzmann-Gibbs entropies.



Andrade, Silva, Moreira, Nobre and Curado, Phys Rev Lett **105**, 260601 (2010)

COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

RAPID COMMU

PHYSICAL REVIEW A **67**, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

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(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A **245**, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a *q*-Gaussian;

(ii) $q = 1 + \dfrac{44 E_R}{U_0}$ where $E_R \equiv \text{recoil energy}$

$U_0 \equiv \text{potential depth}$

Tunable Tsallis Distributions in Dissipative Optical Lattices

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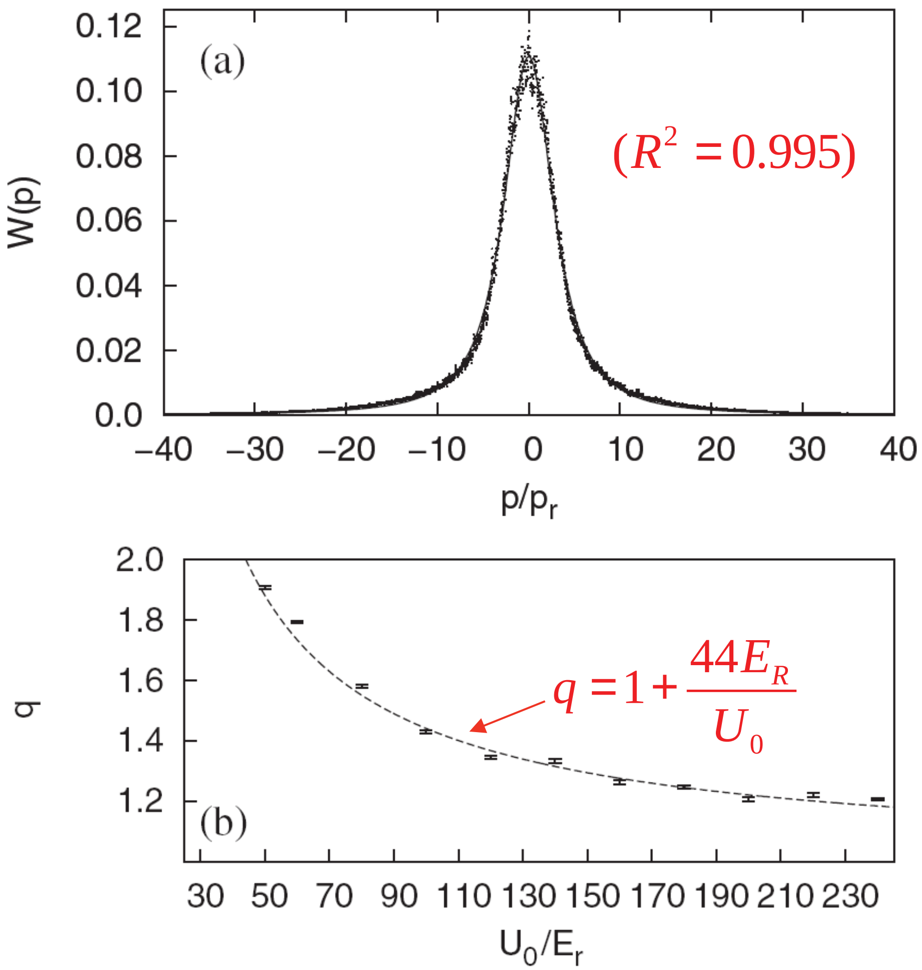
(Received 10 January 2006; published 24 March 2006)

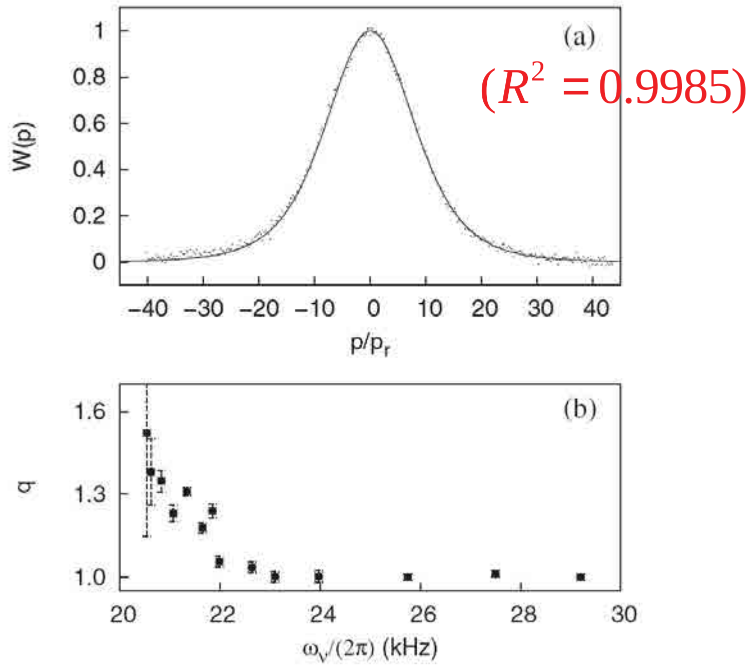
We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

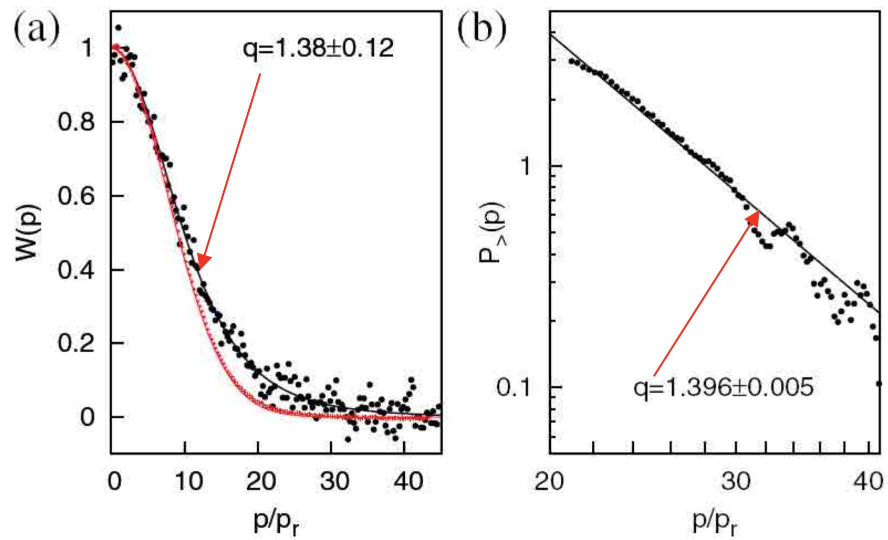
Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)



(Computational verification:
quantum Monte Carlo simulations)





(Experimental verification: Cs atoms)

LASER COOLING:

PRL 102, 063001 (2009) PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2009

Power-Law Distributions for a Trapped Ion Interacting with a Classical Buffer Gas

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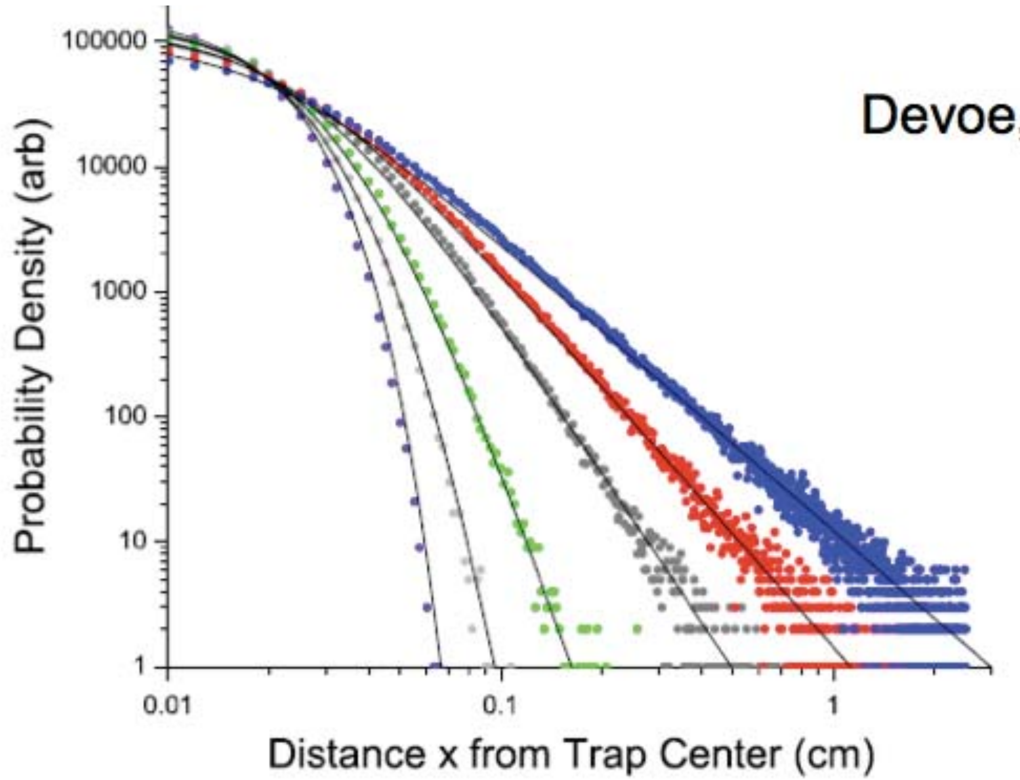
(Received 3 November 2008; published 10 February 2009)

Classical collisions with an ideal gas generate non-Maxwellian distribution functions for a single ion in a radio frequency ion trap. The distributions have power-law tails whose exponent depends on the ratio of buffer gas to ion mass. This provides a statistical explanation for the previously observed transition from cooling to heating. Monte Carlo results approximate a Tsallis distribution over a wide range of parameters and have *ab initio* agreement with experiment.



Devoe, Phys Rev Lett **102** (2009) 063001

$$T(x) = \frac{T(0)}{\left[1+(q-1)\left(\frac{x}{\sigma}\right)^2\right]^{\frac{1}{q-1}}}$$

FIG. 1 (color online). Monte Carlo distributions for a single $^{136}Ba^+$ ion cooled by six different buffer gases at 300 K ranging from $m_B = 4$ (left) to $m_B = 200$ (right). Note the evolution from Gaussian to power law (straight line) as the mass increases. The solid lines are Tsallis functions [Eq. (7)] with fixed $\sigma = 0.0185$ cm and the exponents of Table I.

TABLE I. Tsallis parameters n and q_T fit from Fig. 1.

Buffer gas	m_I/m_B	n	q_T
He	34.5	>60	1.03
Ar	3.40	8.2	1.12
Kr	1.70	3.8	1.26
Xe	1.0	1.98	1.51
170	0.80	1.50	1.80
200	0.68	1.15	1.87

Tissue Radiation Response with Maximum Tsallis Entropy

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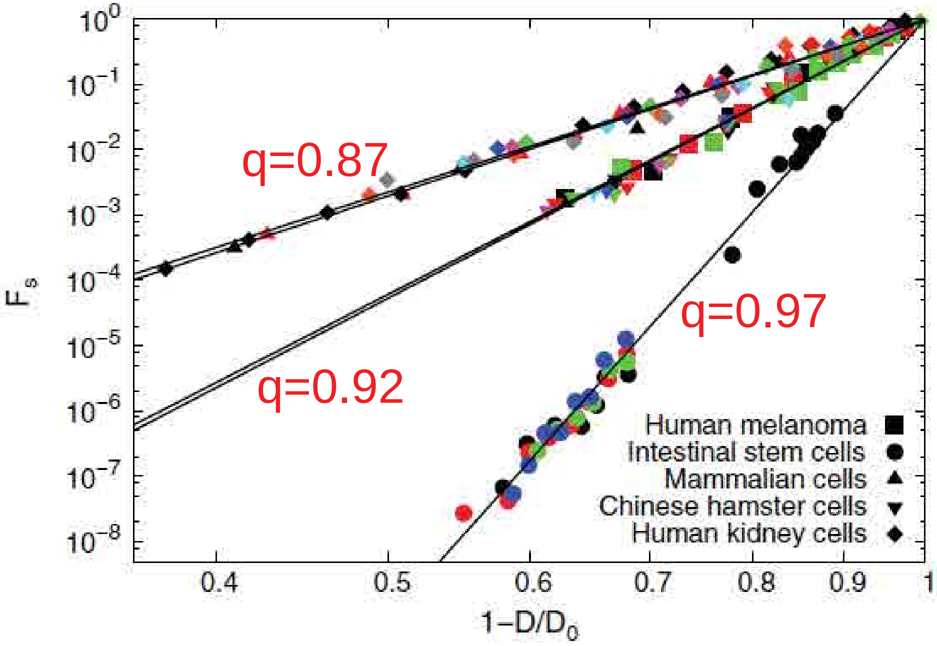
(Received 22 June 2010; published 7 October 2010)

The expression of survival factors for radiation damaged cells is currently based on probabilistic assumptions and experimentally fitted for each tumor, radiation, and conditions. Here, we show how the simplest of these radiobiological models can be derived from the maximum entropy principle of the classical Boltzmann-Gibbs expression. We extend this derivation using the Tsallis entropy and a cutoff hypothesis, motivated by clinical observations. The obtained expression shows a remarkable agreement with the experimental data found in the literature.



$$\gamma = \frac{2-q}{1-q}$$

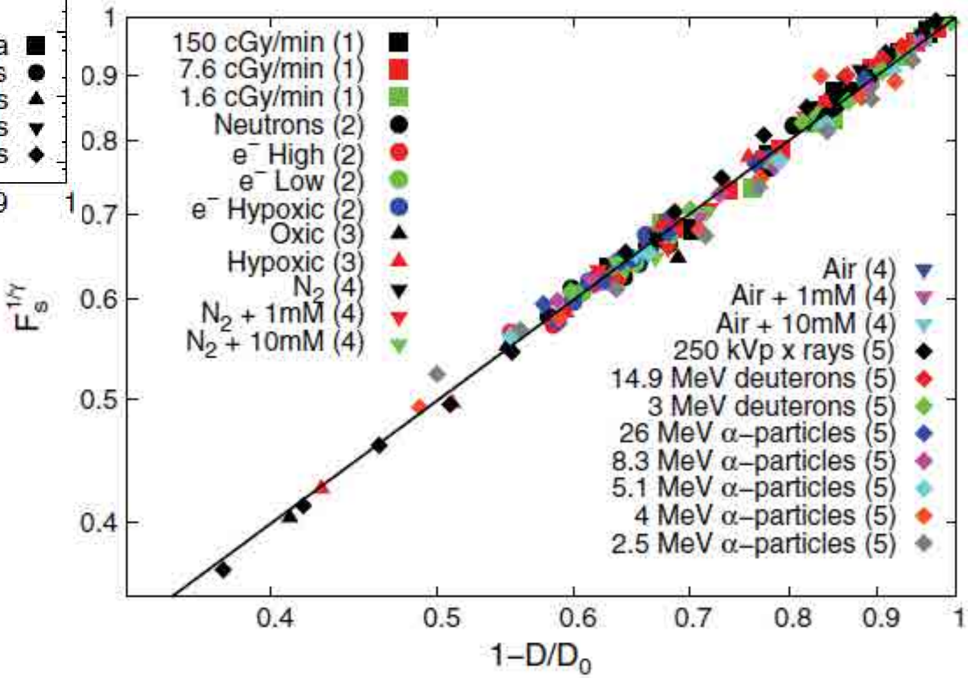


FIG. 2 (color online). Normalized survival fractions $(F_s)^{1/\gamma}$ as a function of the rescaled radiation dose, $1 - D/D_0$ for different tissues: intestinal stem cells (■), chinese hamster cells (●), human melanoma (▲), human kidney cells (▼), and cultured mammalian cells (◆) under different irradiation conditions detailed in [17–21] and grouped in [23]. The straight line shown is $y = x$.

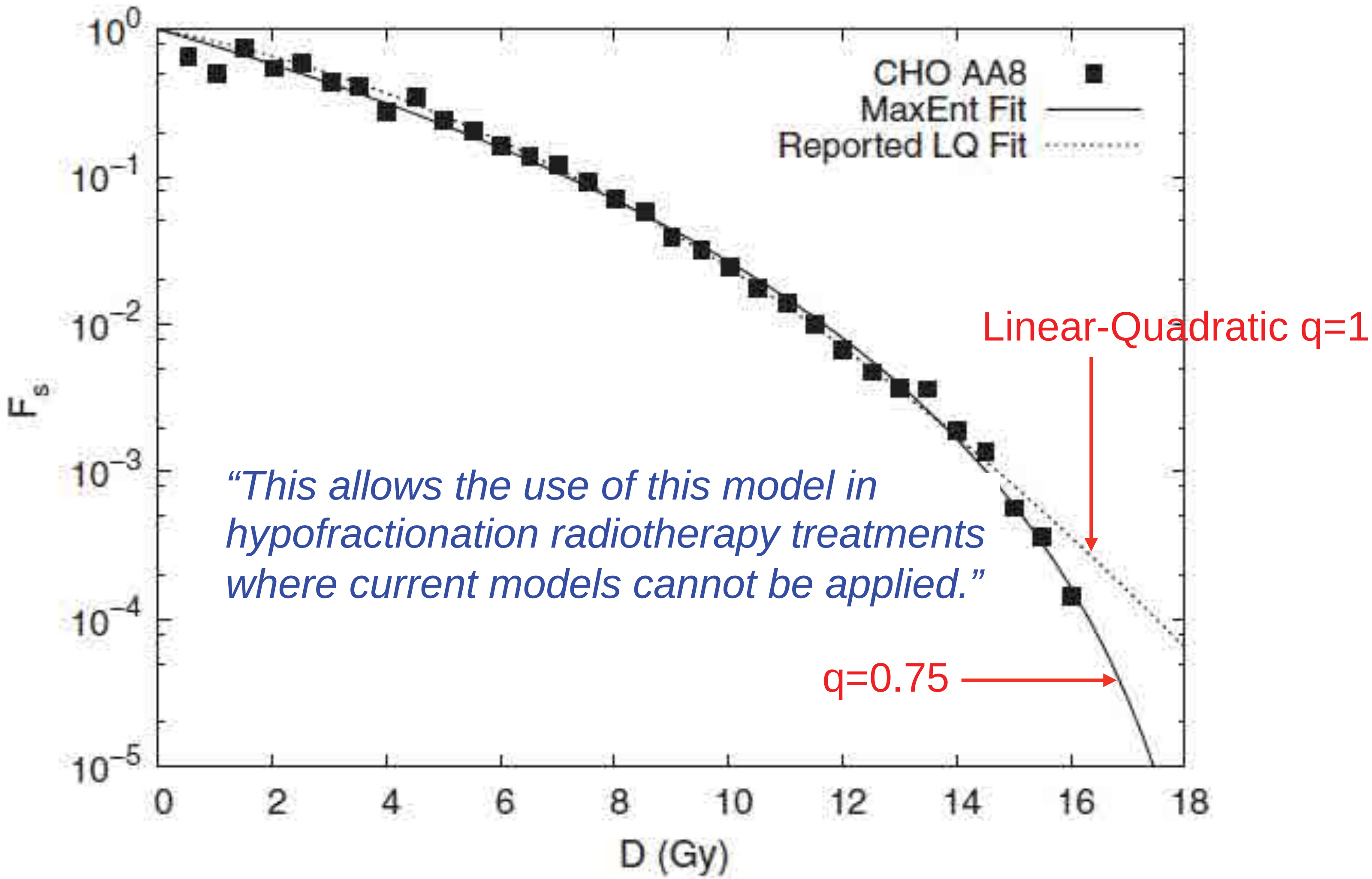


FIG. 3. Comparison between the LQ model best fit ($\alpha = 0.167 \pm 0.015$ Gy^{-1} and $\beta = 0.0205 \pm 0.0015$ Gy^{-2}) reported in [24] and our model fitted to $\gamma = 5.0 \pm 0.4$ and $D_0 = 19.4 \pm 0.4$ Gy for the cell line CHO AA8 under 250 k-Vp x rays.

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A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

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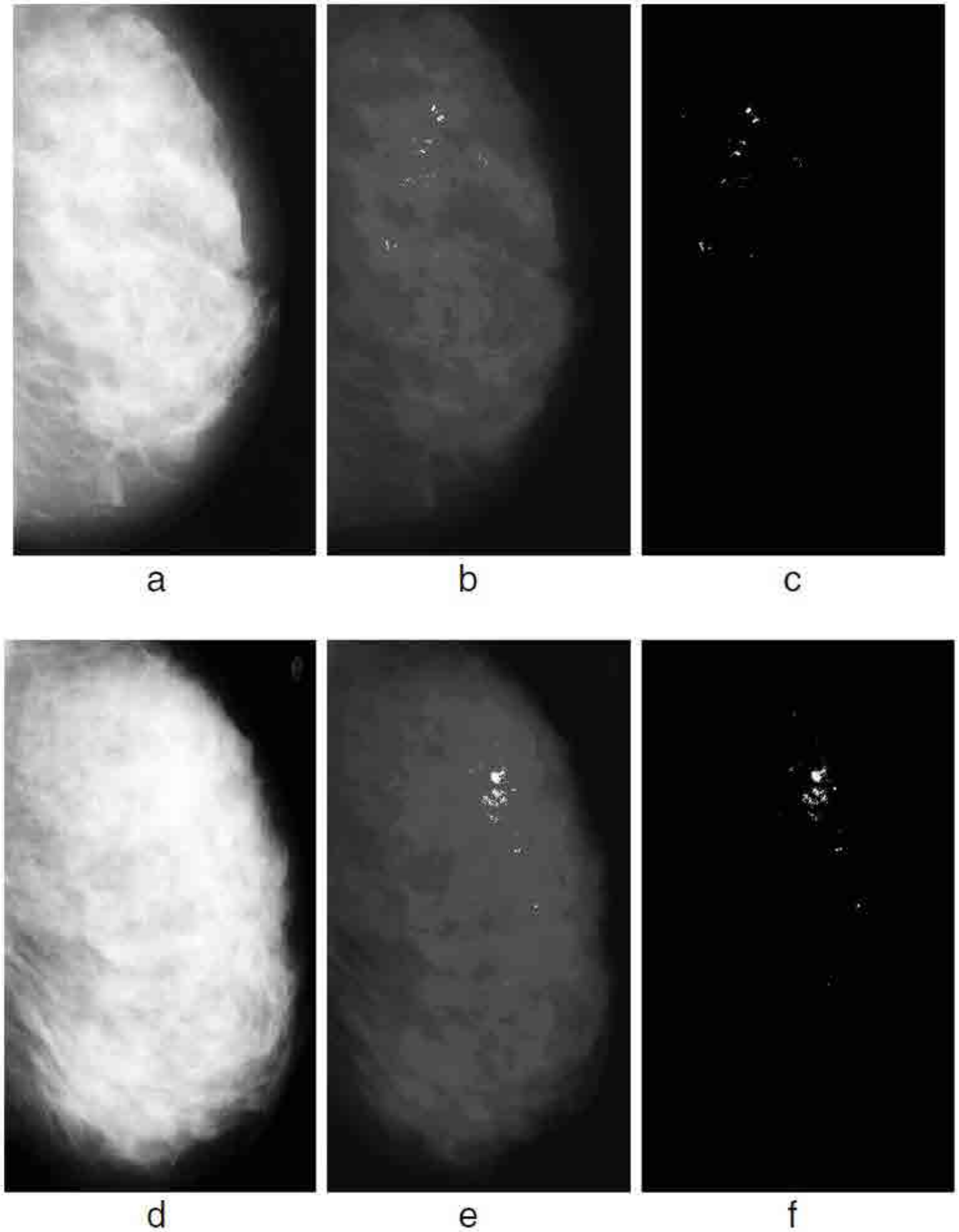
ABSTRACT

This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter 'q', which depends on the non-extensiveness of a mammogram. In previous studies, 'q' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of 'q'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.

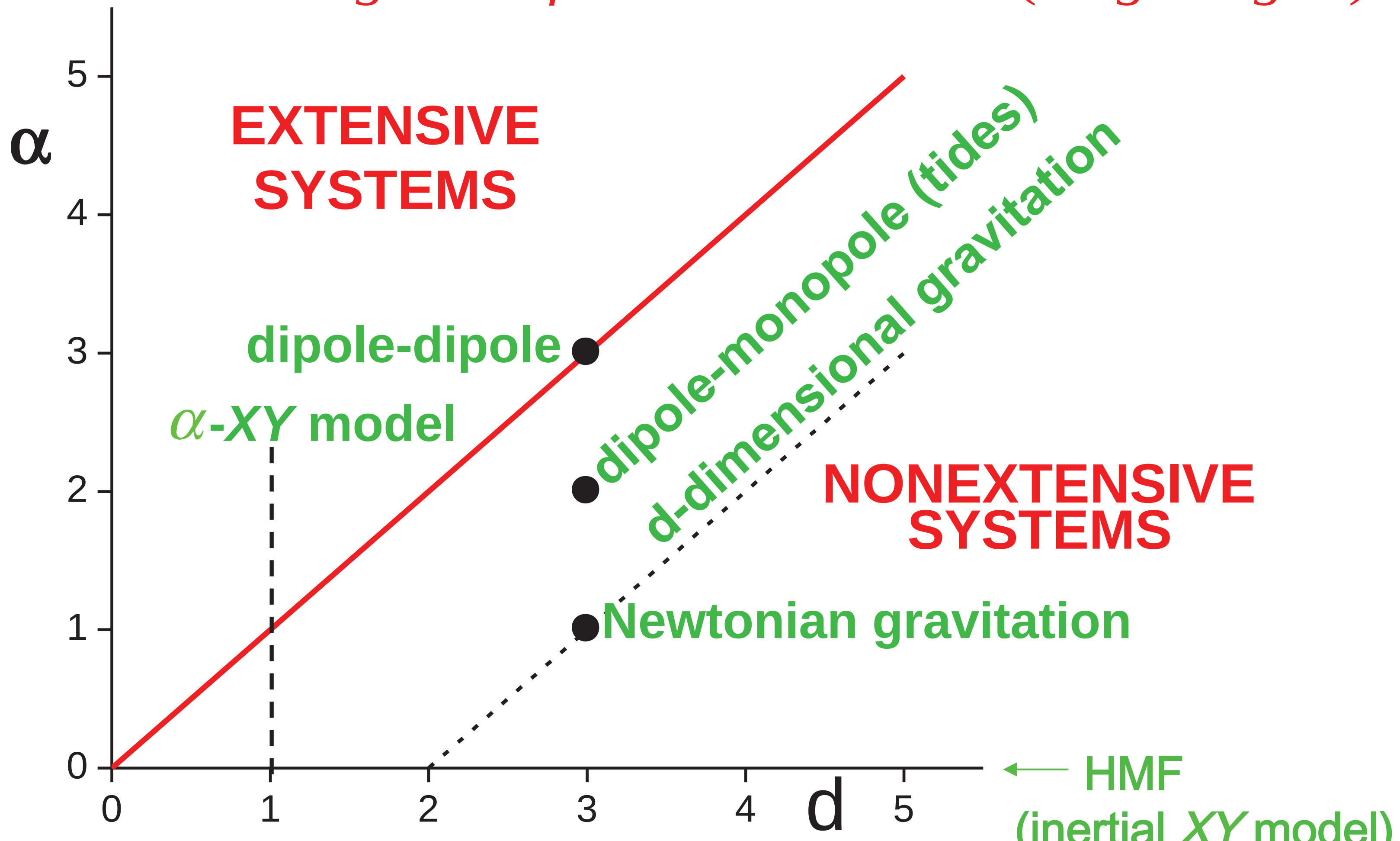

a
b
c
d
e
f

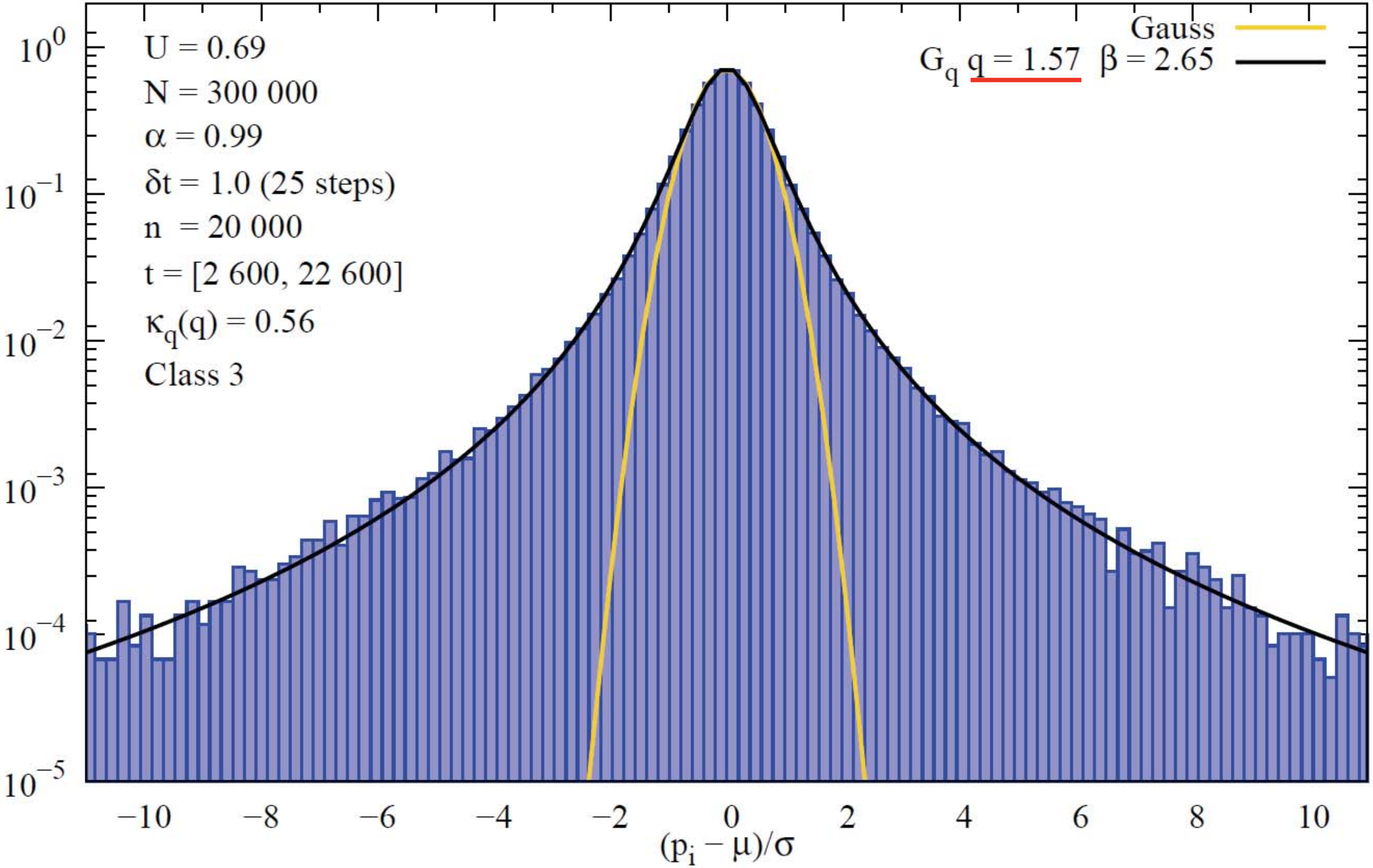
CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A>0,\ \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

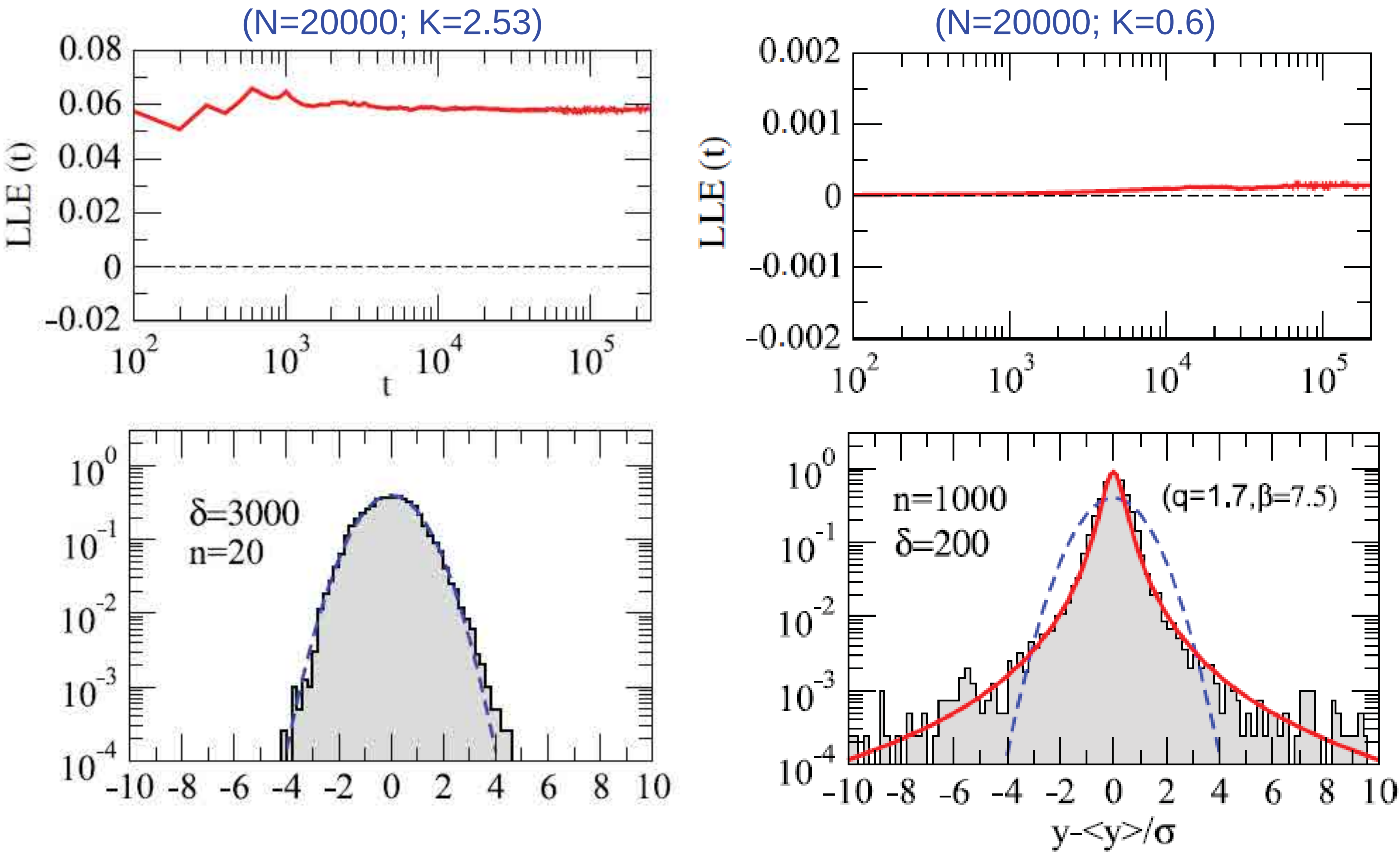
non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*





L.J.L. Cirto, V.R.V. Assis and C. T. (2011)

KURAMOTO MODEL: (N nonlinearly coupled oscillators)



G. Miritello, A. Pluchino and A. Rapisarda, Physica A **388**, 4818 (2009)

CONSERVATIVE MC MILLAN MAP:

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1 + y_n^2} + \varepsilon y_n$$

$$\mu \neq 0 \Leftrightarrow \text{nonlinear dynamics}$$

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2011), in press

$(\mu, \varepsilon) = (1.6, 1.2)$ $(\lambda_{max} \approx 0.05)$

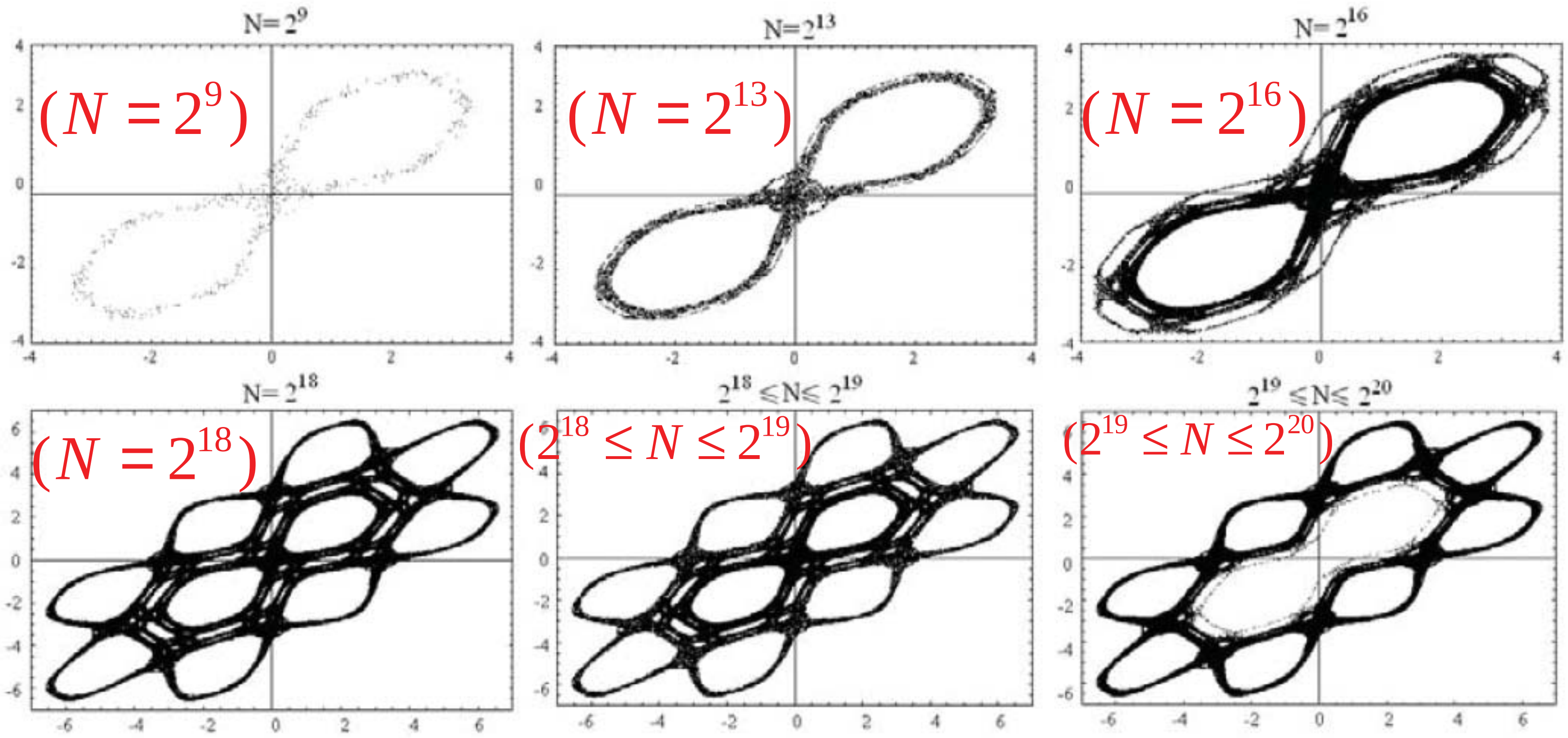
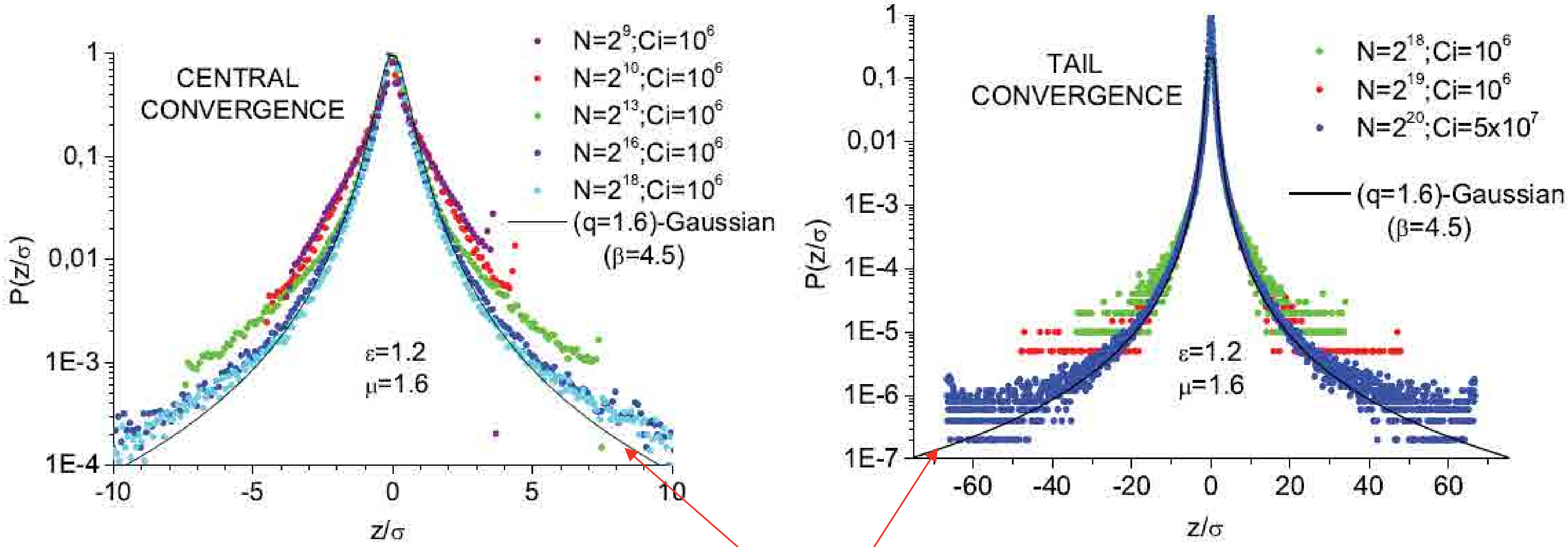


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2011), in press



$$p \propto e_q^{-\beta (z/\sigma)^2}$$

with $(q, \beta) = (1.6, 4.5)$

G. Ruiz, T. Bountis and C. T.
Int J Bifurcat Chaos (2011), in press

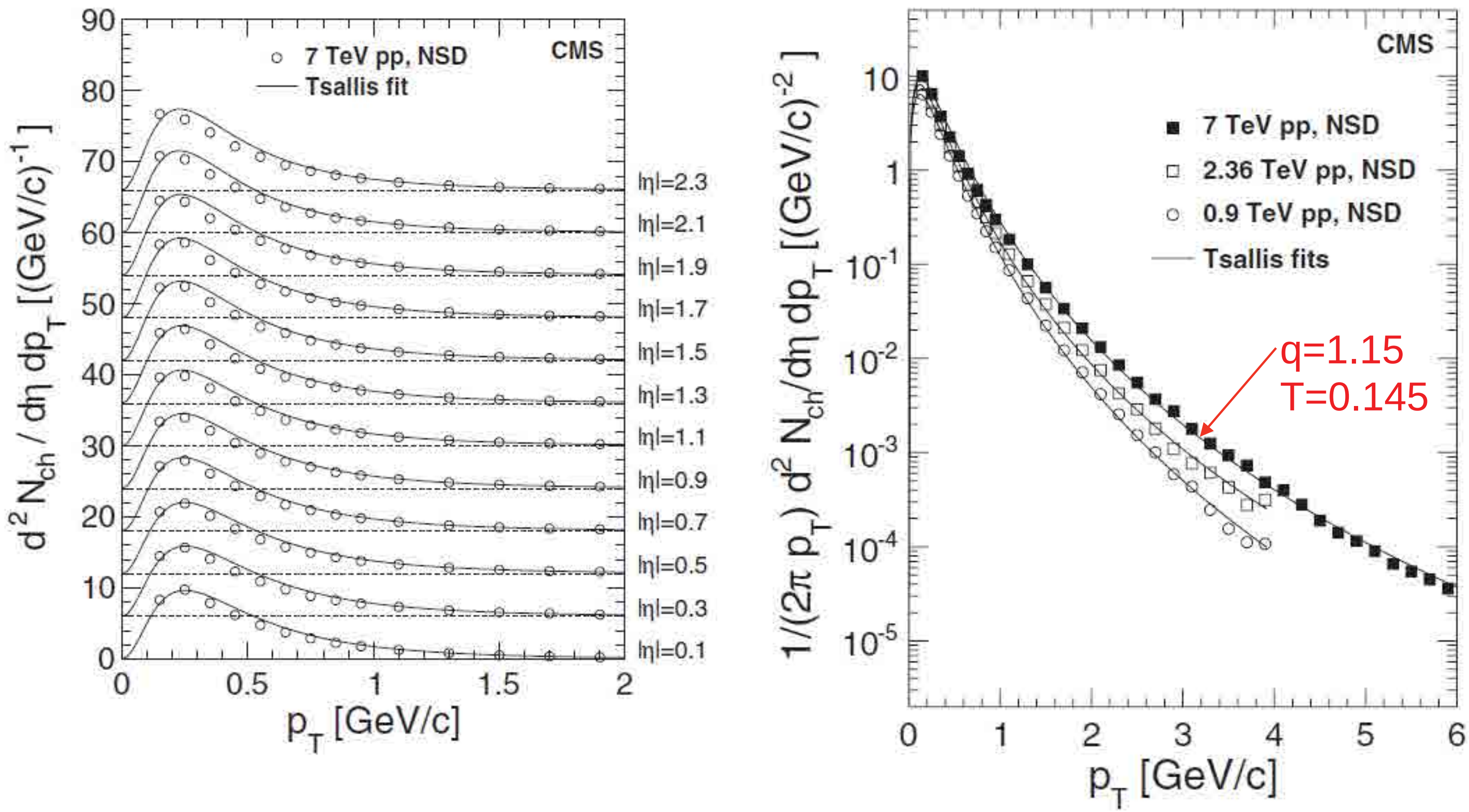
LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

~ 2500 scientists/engineers from 183 institutions of 38 countries

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.*[*]
(CMS Collaboration)
(Received 18 May 2010; published 6 July 2010)



Eur. Phys. J. C (2011) 71:1655
DOI 10.1140/epjc/s10052-011-1655-9

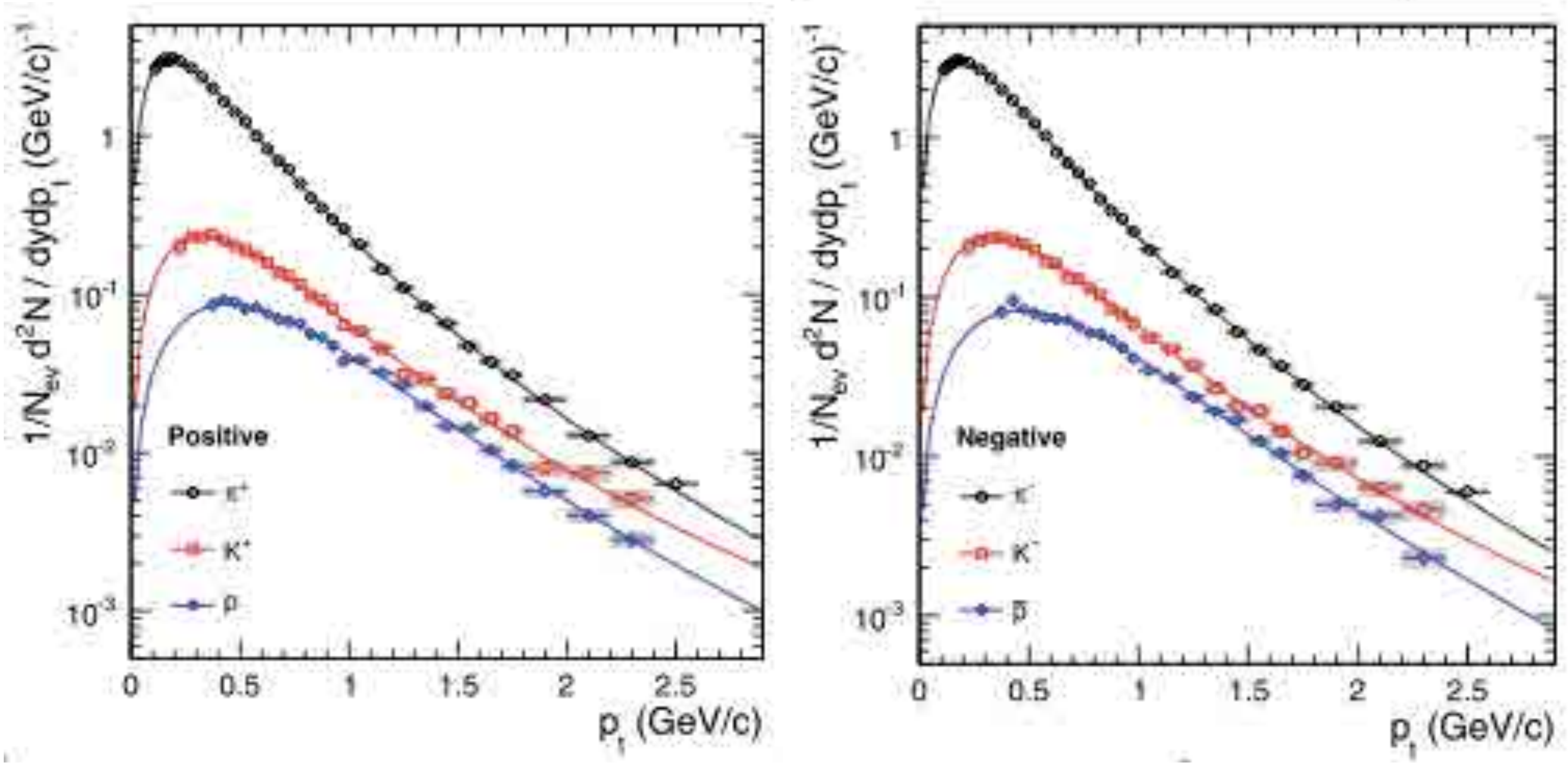
THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Experimental Physics

Production of pions, kaons and protons in pp collisions at $\sqrt{s} = 900$ GeV with ALICE at the LHC

The ALICE Collaboration

K. Aamodt[77], N. Abel[43], U. Abeysekara[75], A. Abrahantes Quintana[42], A. Abramyan[112], D. Adamová[85], M.M. Aggarwal[25], G. Aglieri Rinella[40], A.G. Agocs[18], S. Aguilar Salazar[63], Z. Ahammed[53], A. Ahmad[2], N. Ahmad[2], S.U. Ahn[38,b], R. Akimoto[99], A. Akindinov[46], D. Aleksandrov[68], B. Alessandro[104], R. Alfaro Molina[63], A. Alici[13], E. Almaráz Aviña[63], J. Alme[8], T. Alt[43,c], V. Altini[5], S. Altinpinar[31], C. Andrei[17], A. Andronic[31], G. Anelli[40], V. Angelov[43,c], C. Anson[27], T. Antičić[113], F. Antinori[40,d], S. Antinori[13], K. Antipin[36], D. Antończyk[36], P. Antonioli[14], A. Anzo[63], L. Aphecetche[71], H. Appelshäuser[36], S. Arcelli[13], R. Arceo[63], A. Arend[36], N. Armesto[91], R. Arnaldi[104], T. Aronsson[72], I.C. Arsene[77,e], A. Asryan[97], A. Augustinus[40], R. Averbeck[31], T.C. Awes[74], J. Äystö[49], M.D. Azmi[2], S. Bablok[8], M. Bach[35], A. Badalà[24], Y.W. Baek[38,b], S. Bagnasco[104], R. Bailhache[31,f], R. Bala[105], A. Baldisseri[88], A. Baldit[26], J. Bán[56], R. Barbera[23], G.G. Barnaföldi[18], L.S. Barnby[12], V. Barret[26], J. Bartke[29], F. Barile[5], M. Basile[13], V. Basmanov[93], N. Bastid[26], B. Bathen[70], G. Batigne[71], B. Batyunya[34], C. Baumann[70,f], I.G. Bearden[28], B. Becker[20,g], I. Belikov[98], R. Bellwied[33], E. Belmont-Moreno[63], A. Belogianni[4], L. Benhabib[71], S. Beole[103], I. Berceanu[17], A. Bercuci[31,h], E. Berdermann[31], Y. Berdnikov[39], L. Betev[40], A. Bhasin[48], A.K. Bhati[25], L. Bianchi[103], N. Bianchi[37], C. Bianchin[78], J. Bielčík[80], J. Bielčíková[85], A. Bilandzic[3], L. Bimbot[76], E. Biolcati[103], A. Blanc[26], F. Blanco[23,i], F. Blanco[61], D. Blau[68], C. Blume[36], M. Boccioli[40], N. Bock[27], A. Bogdanov[67], H. Bøggild[28], M. Bogolyubsky[82], J. Bohm[95], L. Boldizsár[18], M. Bombara[55], C. Bombonati[78,k], M. Bondila[49], H. Borel[88], A. Borisov[50], C. Bortolin[78,m], S. Bose[52], L. Bosisio[100], F. Bossú[103], M. Botje[3], S. Böttger[43], G. Bourdaud[71], B. Boyer[76], M. Braun[97], P. Braun-Munzinger[31,32,c], L. Bravina[77], M. Bregant[100,l], T. Breitner[43], G. Bruckner[40], R. Brun[40], E. Bruna[72], G.E. Bruno[5], D. Budnikov[93], H. Buesching[36], P. Buncic[40], O. Busch[44], Z. Buthelezi[22],

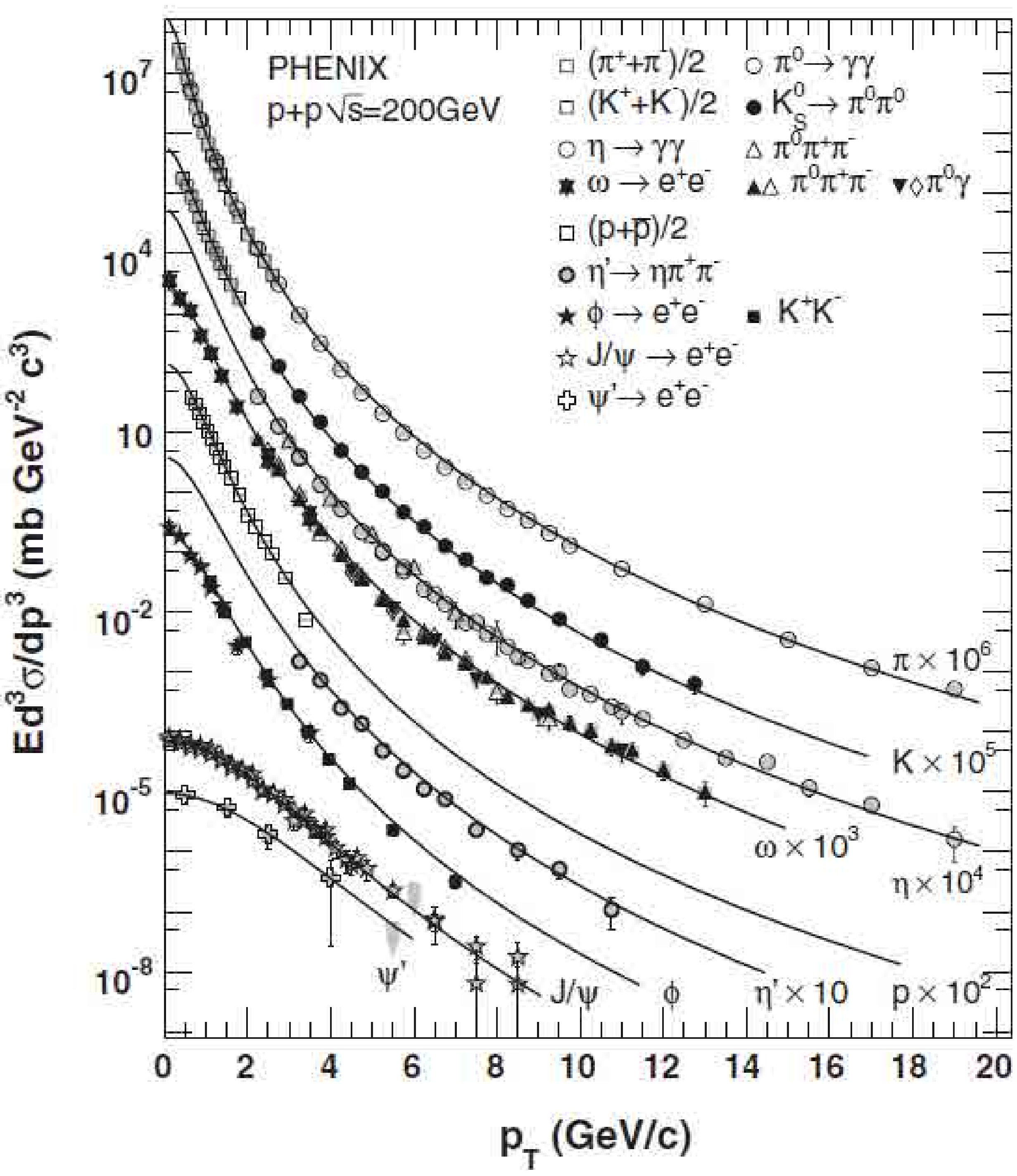

1/N_ev d²N / dydp_t (GeV/c)⁻¹
p_t (GeV/c)
Positive
π⁺
K⁺
p
Negative
π⁻
K⁻
p̄

PHYSICAL REVIEW D **83,** 052004 (2011)

Measurement of neutral mesons in $p + p$ collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production

A. Adare,[11] S. Afanasiev,[25] C. Aidala,[12,36] N. N. Ajitanand,[53] Y. Akiba,[47,48] H. Al-Bataineh,[42] J. Alexander,[53] K. Aoki,[30,47] L. Aphecetche,[55] R. Armendariz,[42] S. H. Aronson,[6] J. Asai,[47,48] E. T. Atomssa,[31] R. Averbeck,[54] T. C. Awes,[43] B. Azmoun,[6] V. Babintsev,[21] M. Bai,[5] G. Baksay,[17] L. Baksay,[17] A. Baldisseri,[14] K. N. Barish,[7] P. D. Barnes,[33] B. Bassalleck,[41] A. T. Basye,[1] S. Bathe,[7] S. Batsouli,[43] V. Baublis,[46] C. Baumann,[37] A. Bazilevsky,[6] S. Belikov,[6,*] R. Bennett,[54] A. Berdnikov,[50] Y. Berdnikov,[50] A. A. Bickley,[11] J. G. Boissevain,[33] H. Borel,[14] K. Boyle,[54] M. L. Brooks,[33] H. Buesching,[6] V. Bumazhnov,[21] G. Bunce,[6,48] S. Butsyk,[33,54] C. M. Camacho,[33] S. Campbell,[54] B. S. Chang,[62] W. C. Chang,[2] J.-L. Charvet,[14] S. Chernichenko,[21] J. Chiba,[26] C. Y. Chi,[12] M. Chiu,[22] I. J. Choi,[62] R. K. Choudhury,[4] T. Chujo,[58,59] P. Chung,[53] A. Churyn,[21] V. Cianciolo,[43] Z. Citron,[54] C. R. Cleven,[19] B. A. Cole,[12] M. P. Comets,[44] P. Constantin,[33] M. Csanád,[16] T. Csörgő,[27] T. Dahms,[54] S. Dairaku,[30,47] K. Das,[18] G. David,[6] M. B. Deaton,[1] K. Dehmelt,[17] H. Delagrange,[55] A. Denisov,[21] D. d'Enterria,[12,31] A. Deshpande,[48,54] E. J. Desmond,[6] O. Dietzsch,[51] A. Dion,[54] M. Donadelli,[51] O. Drapier,[31] A. Drees,[54] K. A. Drees,[5] A. K. Dubey,[61] A. Durum,[21] D. Dutta,[4] V. Dzhordzhadze,[7] Y. V. Efremenko,[43] J. Egdemir,[54] F. Ellinghaus,[11] W. S. Emam,[7] T. Engelmore,[12] A. Enokizono,[32] H. En'yo,[47,48] S. Esumi,[58] K. O. Eyser,[7] B. Fadem,[38] D. E. Fields,[41,48] M. Finger, Jr.,[8,25] M. Finger,[8,25] F. Fleuret,[31] S. L. Fokin,[29] Z. Fraenkel,[61,*] J. E. Frantz,[54] A. Franz,[6] A. D. Frawley,[18] K. Fujiwara,[47] Y. Fukao,[30,47] T. Fusayasu,[40] S. Gadrat,[34] I. Garishvili,[56] A. Glenn,[11] H. Gong,[54] M. Gonin,[31] J. Gosset,[14] Y. Goto,[47,48] R. Granier de Cassagnac,[31] N. Grau,[12,24] S. V. Greene,[59] M. Grosse Perdekamp,[22,48] T. Gunji,[10] H.-Å. Gustafsson,[35,*] T. Hachiya,[20] A. Hadj Henni,[55] C. Haegemann,[41] J. S. Haggerty,[6] H. Hamagaki,[10] R. Han,[45] H. Harada,[20] E. P. Hartouni,[32] K. Haruna,[20] E. Haslum,[35] R. Hayano,[10] M. Heffner,[32] T. K. Hemmick,[54] T. Hester,[7] X. He,[19] H. Hiejima,[22] J. C. Hill,[24] R. Hobbs,[41] M. Hohlmann,[17] W. Holzmann,[53] K. Homma,[20] B. Hong,[28] T. Horaguchi,[10,47,57] D. Hornback,[56] S. Huang,[59] T. Ichihara,[47,48] R. Ichimiya,[47] H. Iinuma,[30,47] Y. Ikeda,[58] K. Imai,[30,47] J. Imrek,[15] M. Inaba,[58] Y. Inoue,[49,47] D. Isenhower,[1] L. Isenhower,[1] M. Ishihara,[47] T. Isobe,[10] M. Issah,[53] A. Isupov,[25] D. Ivanischev,[46] B. V. Jacak,[54,†] J. Jia,[12] J. Jin,[12] O. Jinnouchi,[48] B. M. Johnson,[6] K. S. Joo,[39] D. Jouan,[44] F. Kajihara,[10] S. Kametani,[10,47,60] N. Kamihara,[47,48] J. Kamin,[54] M. Kaneta,[48] J. H. Kang,[62] H. Kanou,[47,57] J. Kapustinsky,[33] D. Kawall,[36,48] A. V. Kazantsev,[29] T. Kempel,[24] A. Khanzadeev,[46] K. M. Kijima,[20] J. Kikuchi,[60] B. I. Kim,[28] D. H. Kim,[39] D. J. Kim,[62] E. Kim,[52] S. H. Kim,[62] E. Kinney,[11] K. Kiriluk,[11] Á. Kiss,[16] E. Kistenev,[6] A. Kiyomichi,[47] J. Klay,[32] C. Klein-Boesing,[37] L. Kochenda,[46] V. Kochetkov,[21] B. Komkov,[46] M. Konno,[58] J. Koster,[22] D. Kotchetkov,[7] A. Kozlov,[61] A. Král,[13] A. Kravitz,[12] J. Kubart,[8,23] G. J. Kunde,[33] N. Kurihara,[10]

PHENIX @ RHIC


PHENIX
p+p √s=200GeV
(π⁺+π⁻)/2
π⁰→ γγ
(K⁺+K⁻)/2
K⁰S→ π⁰π⁰
η → γγ
π⁰π⁺π⁻
ω → e⁺e⁻
π⁰π⁺π⁻
π⁰γ
(p+p̄)/2
η'→ ηπ⁺π⁻
φ → e⁺e⁻
K⁺K⁻
J/ψ → e⁺e⁻
ψ'→ e⁺e⁻
π × 10⁶
K × 10⁵
ω × 10³
η × 10⁴
ψ'
J/ψ
φ
η' × 10
p × 10²
10⁷
10⁴
10
10⁻²
10⁻⁵
10⁻⁸
0
2
4
6
8
10
12
14
16
18
20
Ed³σ/dp³ (mb GeV⁻² c³)
pT (GeV/c)

$q \simeq 1.10$

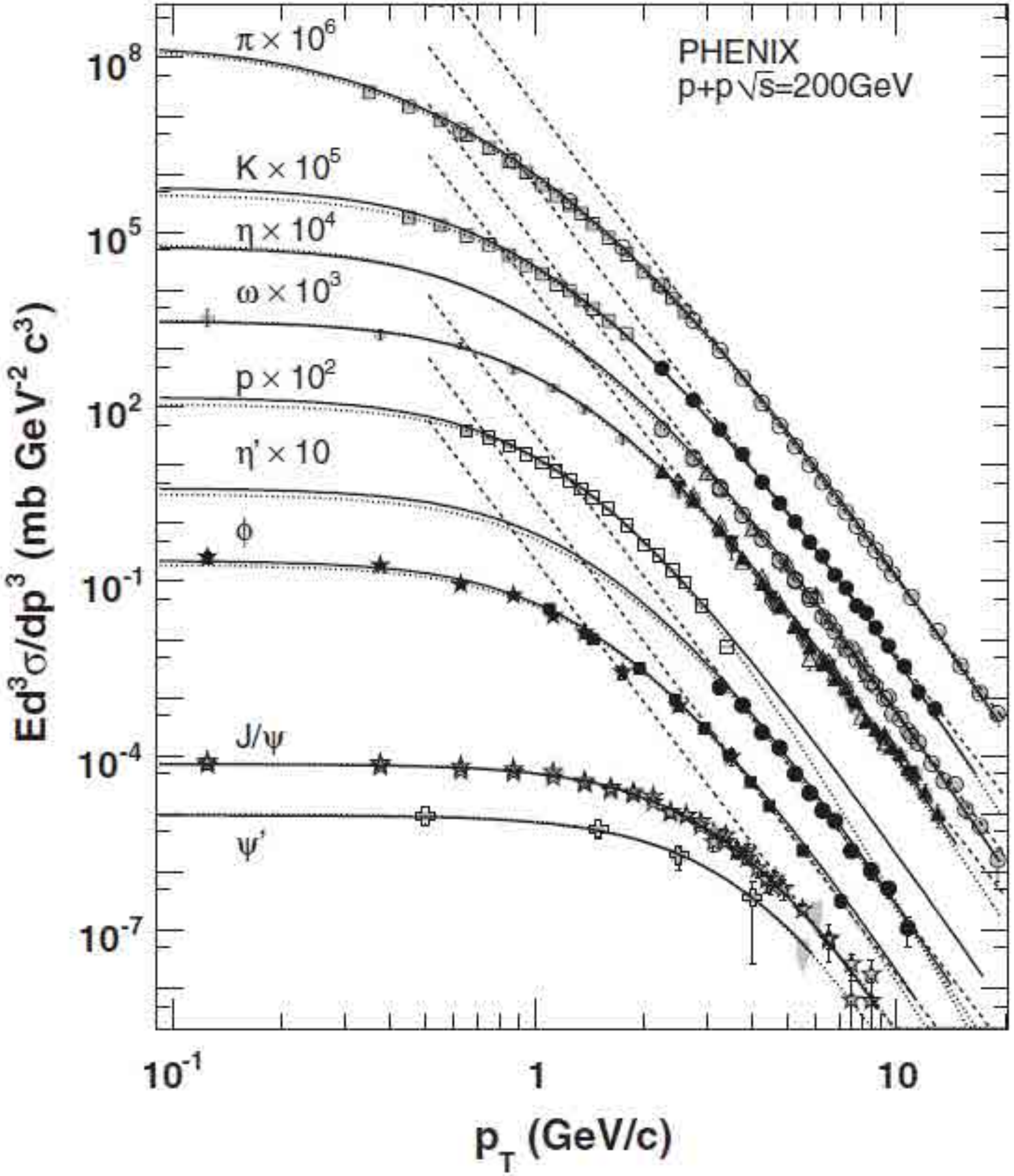


FIG. 13. The p_T spectra of various hadrons measured by PHENIX fitted to the power law fit (dashed lines) and Tsallis fit (solid lines). See text for more details.

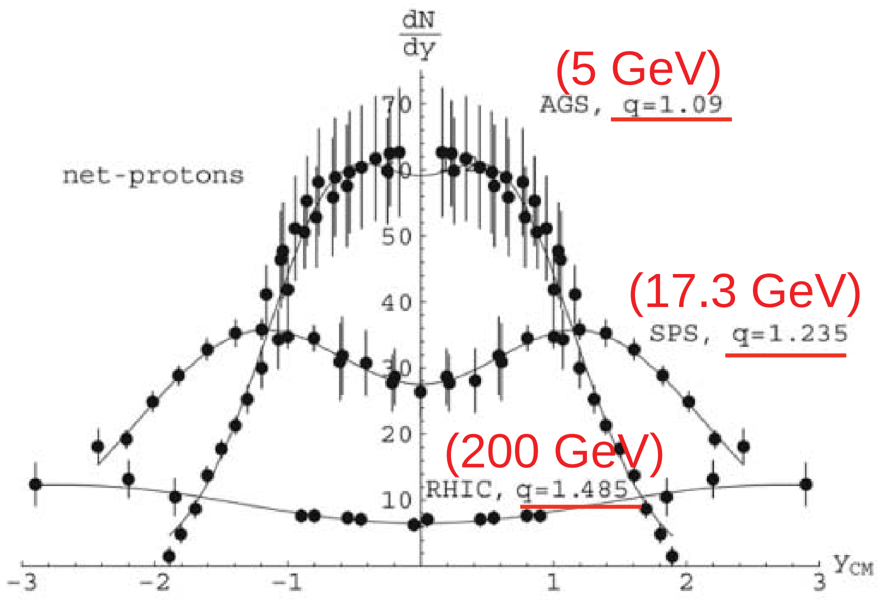
$q \simeq 1.10$

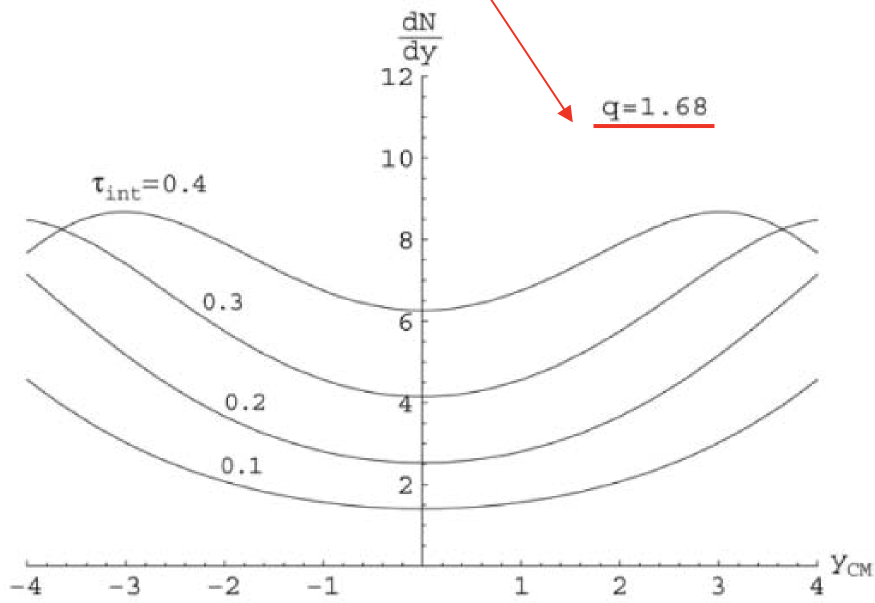
RAPIDITY DISTRIBUTION FOR PROTON PRODUCTION

IN HEAVY-NUCLEI COLLISIONS:

P. Czerski, Int J Mod Phys A **26**, 638 (2011)

prediction for LHC





q-PLANE WAVES:

1) New representation of Dirac delta:

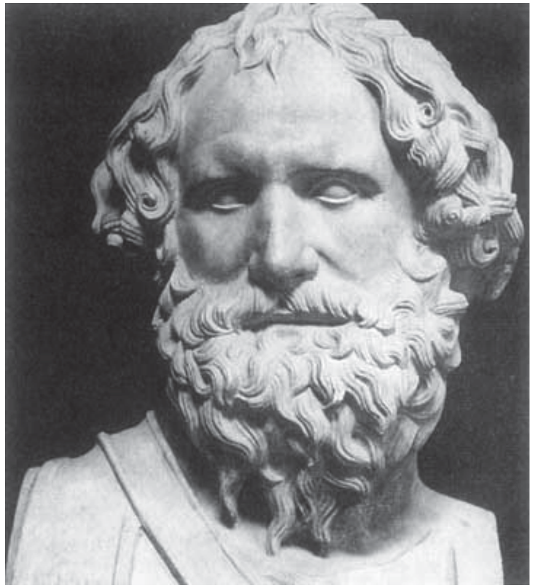
$$\delta(x) = \frac{2-q}{2\pi}\int_{-\infty}^{\infty} dk\ e_q^{-ikx} \quad (1 \leq q < 2)$$

i.e.,

$$\int_{-\infty}^{\infty} dx\ \delta(x - x_0)\, f(x) = f(x_0)$$

M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

2) New representation of π :



Archimedes

(c. 287 BC – c. 212 BC)

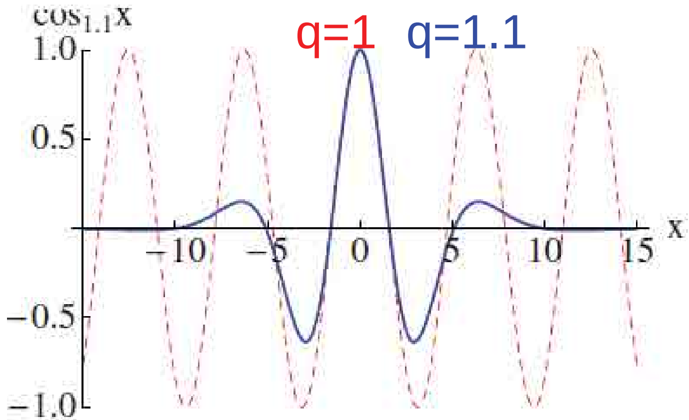
$$\pi = n \sum_{k=0}^{\lfloor \frac{n+1}{2} \rfloor - 1} (-1)^k \frac{\Gamma\left(n-k-\frac{1}{2}\right)\Gamma\left(k+\frac{1}{2}\right)}{\Gamma(2k+2)\Gamma(n-2k)}, \quad \forall n \in \mathbb{N}$$

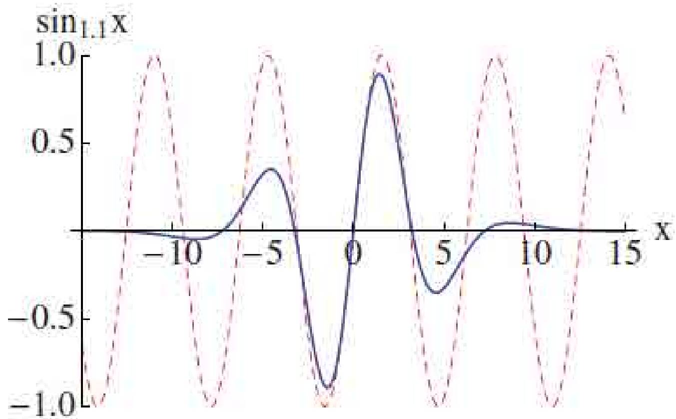
M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

3) q-plane waves are square integrable ($0 < q < 3$):

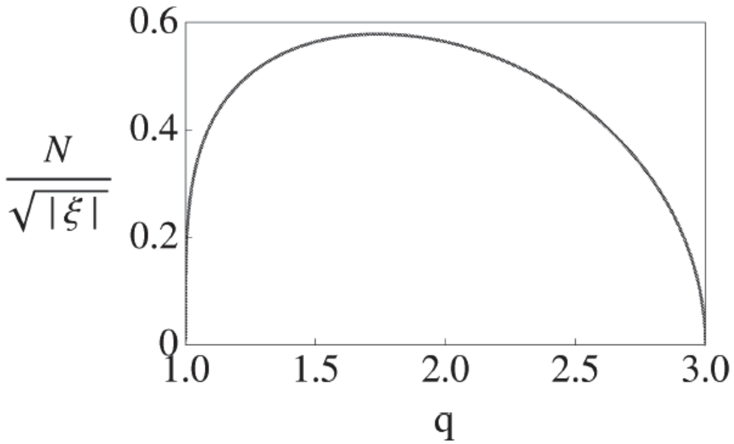
$$\psi(x,t) = e_q^{i(k\,x - \omega\,t)} \text{ satisfies } \frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2} \text{ with } \omega = ck$$

$$\Psi(x) \equiv N\, e_q^{i\xi x} = N(\cos_q \xi x + i \sin_q \xi x) \text{ with } \int_{-\infty}^{\infty} dx\, |\Psi(x)|^2 = 1$$





$$\frac{N}{\sqrt{|\xi|}} = \left[\frac{(q-1)\,\Gamma\left(\frac{1}{q-1}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{3-q}{2(q-1)}\right)} \right]^{1/2}$$



See also E.P. Borges, C. T., J.G.V. Miranda and R.F.S. Andrade, J Phys A **37**, 9125 (2004)

M. Jauregui and C. T., J Math Phys **51**, 063304 (2010)

PRL **106**, 140601 (2011) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2011

Nonlinear Relativistic and Quantum Equations with a Common Type of Solution

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(Received 25 October 2010; published 4 April 2011)

Generalizations of the three main equations of quantum physics, namely, the Schrödinger, Klein-Gordon, and Dirac equations, are proposed. Nonlinear terms, characterized by exponents depending on an index q, are considered in such a way that the standard linear equations are recovered in the limit $q \to 1$. Interestingly, these equations present a common, solitonlike, traveling solution, which is written in terms of the q-exponential function that naturally emerges within nonextensive statistical mechanics. In all cases, the well-known Einstein energy-momentum relation is preserved for arbitrary values of q.

4) $q-$generalized Schroedinger equation

(quantum non-relativistic spinless free particle)

$$i\hbar \frac{\partial}{\partial t}\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right] = -\frac{1}{2-q}\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right]^{2-q} \quad (q \in R)$$

Its exact solution is given by

$$\Phi(\vec{x},t) = \Phi_0\, e_q^{i(\vec{p}\,.\,\vec{x}-Et)/\hbar} = \Phi_0\, e_q^{i(\vec{k}\,.\,\vec{x}-\omega t)}$$

with

$$E = \frac{p^2}{2m} \quad \text{(Newtonian relation!)}$$

$$E = \hbar\omega$$

$$p = \hbar k$$

$\Big\}\ \forall q$

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

5) *q*-generalized Klein-Gordon equation:

(quantum relativistic spinless free particle: e.g., mesons π)

$$\nabla^2 \Phi(\vec{x},t) = \frac{1}{c^2}\frac{\partial^2 \Phi(\vec{x},t)}{\partial t^2} + q\,\frac{m^2c^2}{\hbar^2}\Phi(\vec{x},t)\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right]^{2(q-1)} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi(\vec{x},t) = \Phi_0\, e_q^{\,i(\vec{p}\,.\,\vec{x}-Et)/\hbar} = \Phi_0\, e_q^{\,i(\vec{k}\,.\,\vec{x}-\omega\, t)}$$

with

$$E^2 = p^2c^2 + m^2c^4 \qquad (\forall q) \qquad \text{(Einstein relation!)}$$

Particular case: $m = 0 \;\Rightarrow\;$ *q*-plane waves

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

6) *q*-generalized Dirac equation:

(quantum relativistic spin $1/2$ matter and anti-matter free particles:

e.g., electron and positron)

$$i\hbar \frac{\partial \Phi(\vec{x},t)}{\partial t} + i\hbar c\left(\vec{\alpha}.\vec{\nabla}\right)\Phi(\vec{x},t) = \beta m c^2 A^{(q)}(\vec{x},t)\,\Phi(\vec{x},t) \qquad (q \in R)$$

with

$$\vec{\alpha} \equiv \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}; \quad \beta \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (4 \times 4 \text{ matrices})$$

$$A^{(q)}_{i\,j}(\vec{x},t) \equiv \delta_{ij}\left[\frac{\Phi_j(\vec{x},t)}{a_j}\right]^{q-1} \qquad \left(A^{(1)}_{i\,j}(\vec{x},t) = \delta_{ij}\right) \qquad (4 \times 4 \text{ matrix})$$

where $\{a_j\}$ are complex constants.

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) \equiv \begin{pmatrix} \Phi_1\left(\vec{x},t\right) \\ \Phi_2\left(\vec{x},t\right) \\ \Phi_3\left(\vec{x},t\right) \\ \Phi_4\left(\vec{x},t\right) \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{\,i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{\,i\left(\vec{k}\,.\,\vec{x}-\omega\, t\right)}$$

with $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ being the same $\forall q$

hence

$$E^2 = p^2c^2 + m^2c^4 \quad (q \in R) \quad \text{(Einstein relation!)}$$

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BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

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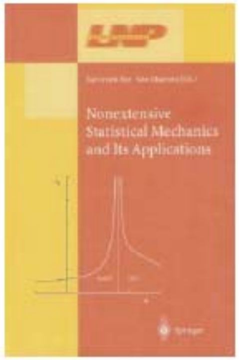

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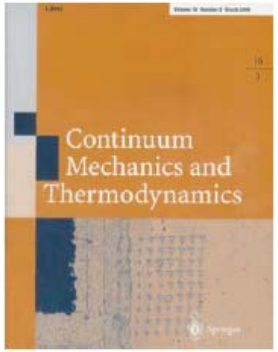

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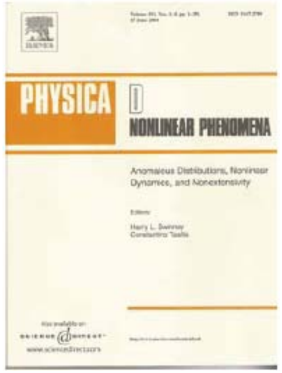

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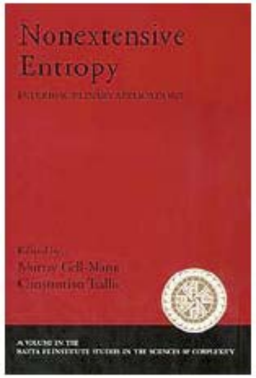

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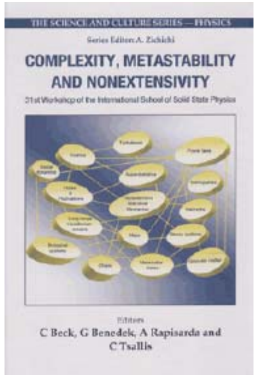

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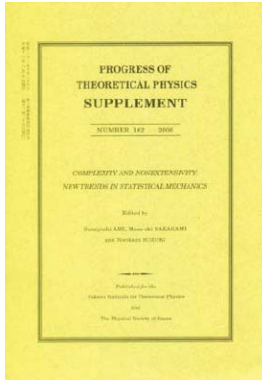

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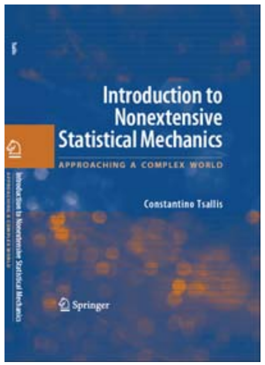

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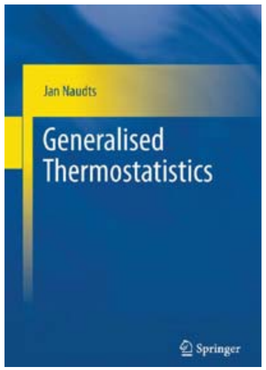

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Introduction to Nonextensive Statistical Mechanics

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