Back To 1905

Einstein's solution of the Ultraviolet Problem for e.m radiation in a box in thermodinamic equilibrium.

$\boxed{\text{The Black Body Problem.}}$

Einstein's solution leads to:

1) The photon
2) wave - particle Duality
3) Quantum Mechanics.

Let us recall the physics argument used by Einstein in 1905.

The Problem:

Let us denote $\rho_\nu d\nu$ Energy/Volume of

e.m radiation with frequency between ν and $\nu + d\nu$

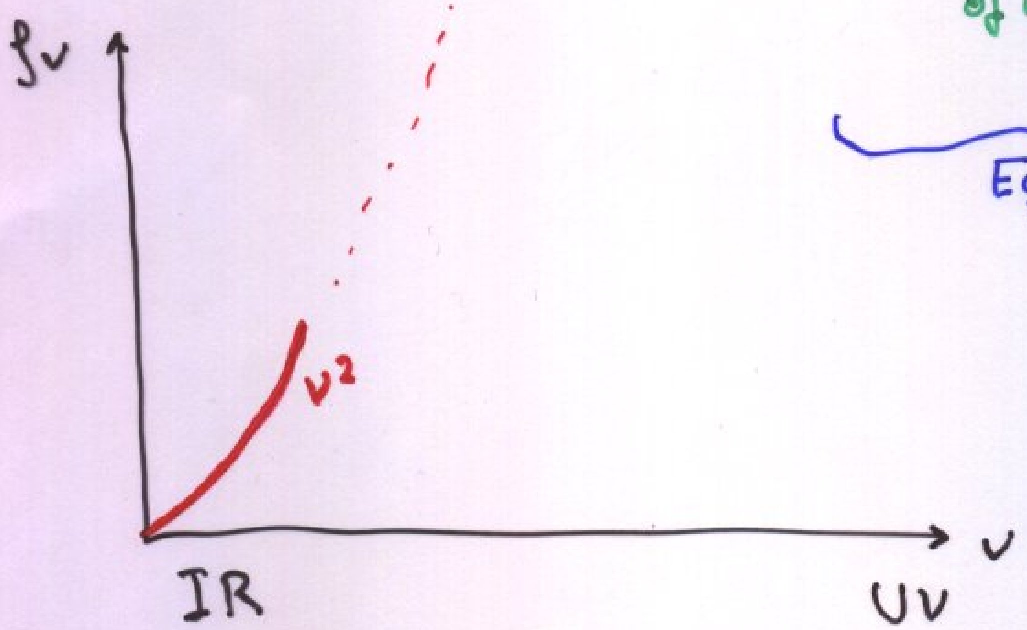
Using Maxwell's theory and

Equipartition theorem:

$$\boxed{\rho_\nu = K \frac{8\pi\nu^2}{c^3} T} = KT \underbrace{\left(\frac{8\pi\nu^2}{c^3}\right)}$$

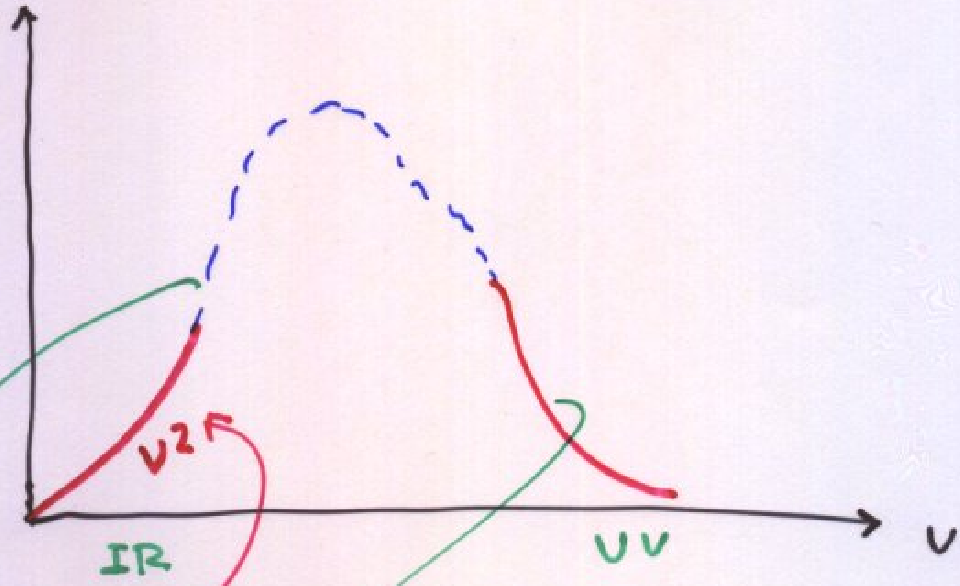
# of osilations with absulute value of wave vector equal ν/c

Equipartition.



UV problem: $E_{tot} = \int \rho_\nu d\nu \to \infty$!!

Experimental input.



Wien distribution:

$$\boxed{\rho_\nu = \alpha\, \nu^3 e^{-\beta \nu / T}}$$

experimental fit

α, β phonomenological constants.

UV/ IR interpolating formula.

Planck distribution:

IR.

UV

$$\boxed{\rho_\nu = \frac{\alpha\, \nu^3}{e^{\beta \nu / T} - 1}}$$

Einstein's Question:

What is the physical meaning of Wien distribution?

Einstein's Strategy:

Step 1 Since ρ_ν is equilibrium distribution we can find the ENTROPY such that

$$\boxed{\frac{\partial S}{\partial E} = \frac{1}{T} = -\frac{1}{\beta\nu}\ln\frac{\rho}{\alpha\nu^3}}$$

$\rightarrow \rho_\nu = \alpha\nu^3 e^{-\beta\nu/T}$

$$S(\rho) = -\frac{\rho}{\beta\nu}\left\{\ln\frac{\rho}{\alpha\nu^3} - 1\right\} \quad \Rightarrow$$

$$S = V S(\rho)d\nu = -\frac{E}{\beta\nu}\left\{\ln\frac{E}{V\alpha\nu^3 d\nu} - 1\right\}$$

Entropy of radiation with frequency between ν and $\nu + d\nu$

$\rho(\nu)d\nu = \frac{E}{V}$

Step 2 Use Boltzmann probabilistic interpretation of entropy.

$$\boxed{S = \kappa \ln [W]} \quad (1)$$

$$S - S_0 = \kappa \ln [W]$$

prob to find a system in volume V_0 concentrated in a smaller volume V

Step 3. Compute $S - S_0$ and compare with (1)

$$\boxed{S - S_0 = \frac{E}{\beta \nu} \ln \left(\frac{V}{V_0} \right)}$$

Wien's distribution

$$\frac{E}{\beta \nu} \equiv \kappa n \qquad \kappa \ln \underbrace{\left(\frac{V}{V_0} \right)^n}$$

prob of n-particles that live in V_0 to be in $V < V_0$.

⇒ Wien's distribution (U.V)

⇔ equilibrium distribution of a gas o n particles.

Photon

Moreover since:

$$n = \frac{E}{K\beta\nu}$$

$$\Rightarrow \boxed{E = K\beta\nu}$$

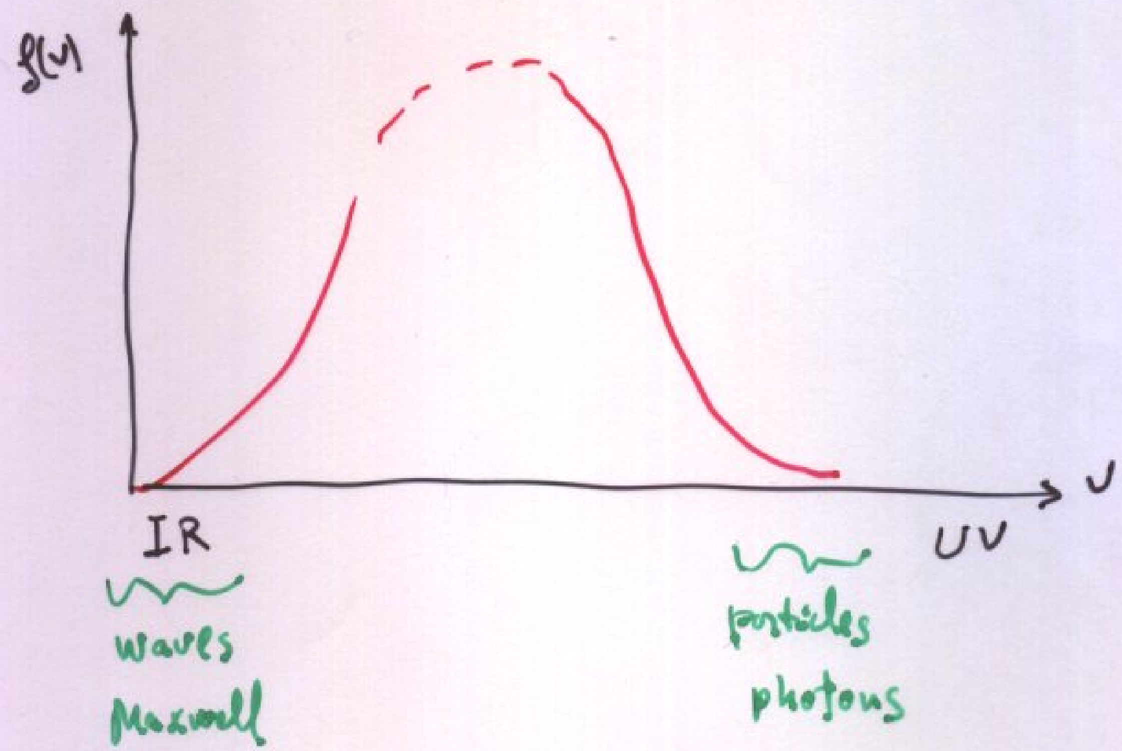
quantum of energy

Meaning of phenomenological constant β

$$\boxed{K\beta = h}$$

Planck constant.

<u>Einstein's conclusion:</u>



Wave - particle Duality

$\Rightarrow$ <u>Quantum Mechanics</u>

From Einstein 1905 to Maldacena 1997

Einstein (after Planck) — (Linear) Black Body Problem.

⬇

Quantum Mechanics

Maldacena (after Hawking t'Hooft) — (Non Linear) Black Body Problem.

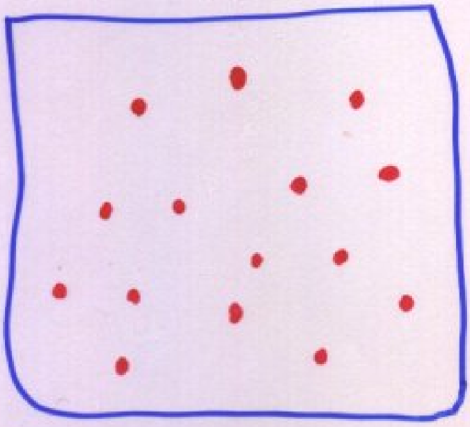
⬇

Quantum Gravity

Holography

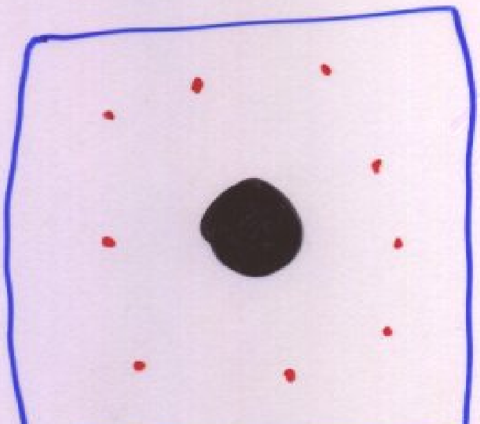
What is the (Non Linear) Black Body Problem?

Thermal radiation in a box but including gravity interaction.



maximal entropy corresponding to thermal radiation.

E, V



maximal entropy corresponding to black hole in equilibrium with Thermal radiation.

(new physics in the U.V)

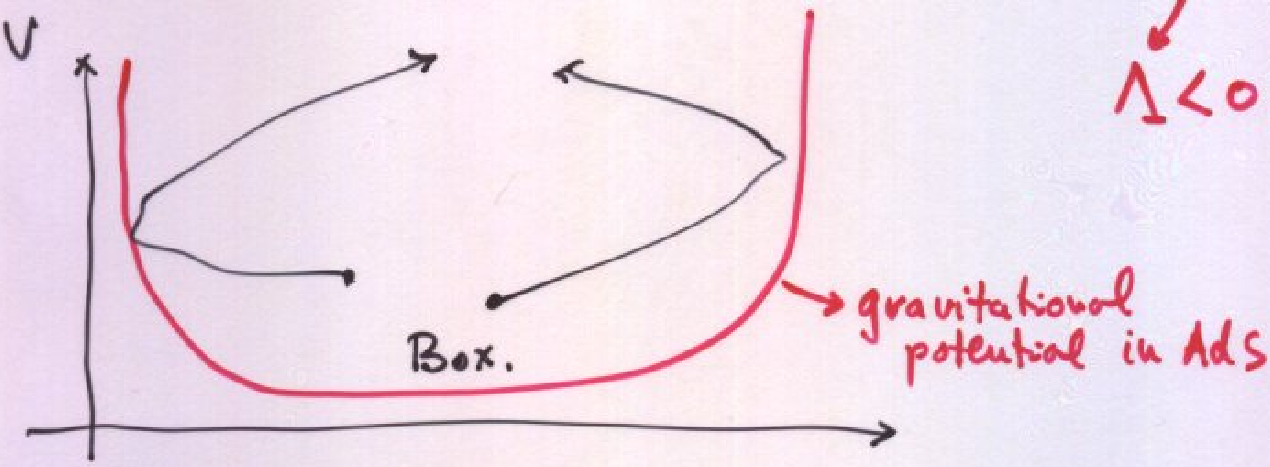
E', V

$E' > E$

• After General Relativity we can simulate a box as AdS space time.

$R = \Lambda$

$\Lambda < 0$



In these conditions we can use microcanonical ensemble to describe thermodinamics in AdS.

$$\underbrace{Z(\beta)}_{\text{partition function}} = \int dE \, \underbrace{N(E)}_{\text{density of states}} \, e^{-\beta E}$$

Laplace transform:

$$N(E) = \int d\beta \, e^{\beta E} Z(\beta)$$

$N(E)$ in the I.R : Thermal radiation

$d = 4$

$$Z \sim \exp\left(\frac{\pi^4}{90}\, g\, b^3\, \beta^{-3}\right)$$

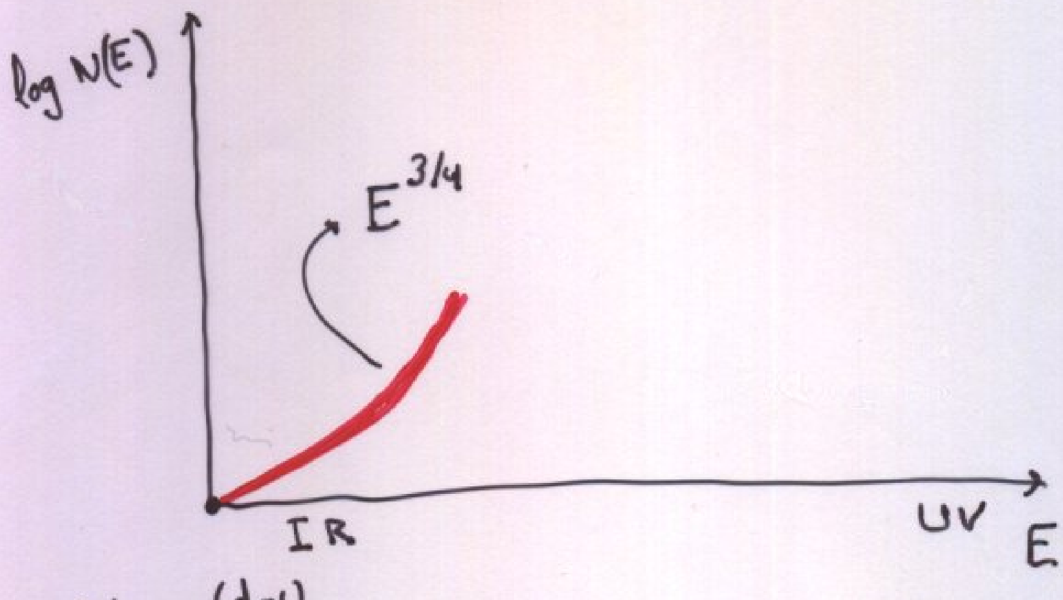
b → Size of the "box" (cosmological constant)

g → # of spin degrees of freedom.

Saddle point in Laplace transform.

$$\beta \sim \left(\frac{\pi^4}{30}\, g\, b^3\, E^{-1}\right)^{1/4}$$

$$\boxed{N(E) = \exp\left[\frac{4\pi}{3}\left(\frac{g}{30}\right)^{1/4}\left(b E\right)^{3/4}\right]}$$



$$3/4 = \frac{(d-1)}{d}$$

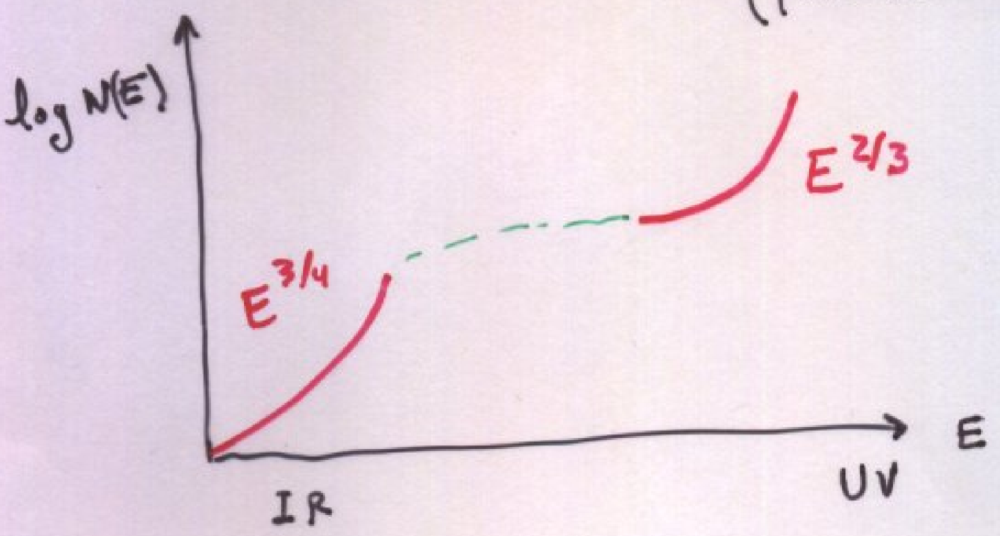
Let us now consider the U.V region.

By analogy with the black body problem let us use B.H information as phenomenological input.

In the U.V

$$\text{UV} \qquad \boxed{N(E) \sim \exp\left[\pi \left(2 m_{pl}\, b\right)^{2/3} \left(b E\right)^{2/3}\right]} \qquad (1)$$

m_{pl} → Newton constant.
(phenomenological constant)



Comparing (1) and

$$\text{IR} \qquad \boxed{N(E) = \exp\left[\frac{4\pi}{3}\left(\frac{g}{30}\right)^{1/4} \left(b E\right)^{3/4}\right]}$$

Notice: $(bE)^{3/4} \longrightarrow (bE)^{2/3}$

change in the exponent.

$(g/30)^{1/4} \longrightarrow (2 m_{pl}\, b)^{2/3}$

degrees of freedom → Newton constant.

As Einstein did in 1905 we can ask ourselves how to interpret the entropy in the UV region

$$\boxed{S \sim \log N(E) \sim (m_{pe}\, b)^{2/3} \, (b E)^{2/3}}$$

This is the entropy of a C.F.T in dimension $d = 3$

$$S \sim \sqrt{N} \, (b E)^{d-1/d}$$

b → size of box

$\sqrt{N}$ → # of degrees of freedom.

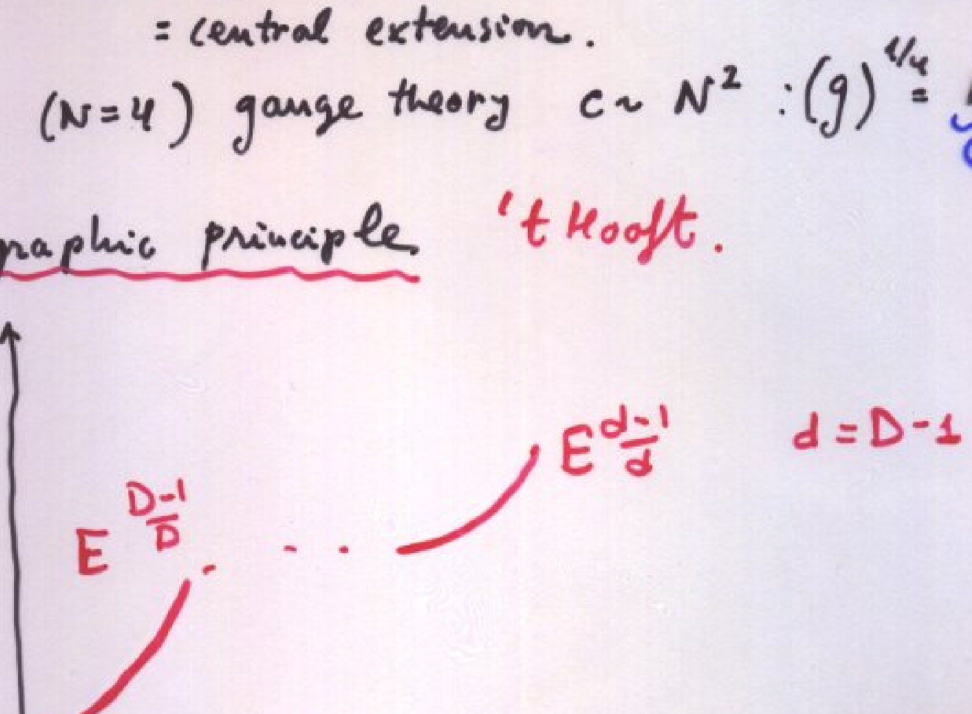
$\circledast$

(Recall: Comparing with IR $(g)^{1/4}$ — # of degrees of freedom.

For a CFT: # degrees of freedom = = central extension.

For YM ($N=4$) gauge theory $c \sim N^2$: $(g)^{1/4} = N^{1/2}$ $\circledast$)

<u>Holographic principle</u> 't Hooft.



gravity in D ⟷ CFT in $D-1 = d$

$\boxed{\text{Holographic Duality}}$

C.F.T degrees of freedom / gravity

correspondence.

$$\boxed{\sqrt{N} = \left(m_{pe}^{D}\, b\right)^{d-1/d}}$$

Newton cte in D-dimensions

$d = 4 \quad D = 5$

$$m_{pe}^{3} = \frac{b^{5}}{l_s^{8} g_s^{2}} \rightarrow \left(m_{pe}\, b\right)^{3/4} = \left[\frac{b^{8}}{l_s^{8} g_s^{2}}\right]^{1/4}$$

$$\left(g_s N\right)^{1/2} = \frac{b^{2}}{l_s^{2}}$$

Maldacena's AdS / CFT correspondence !!

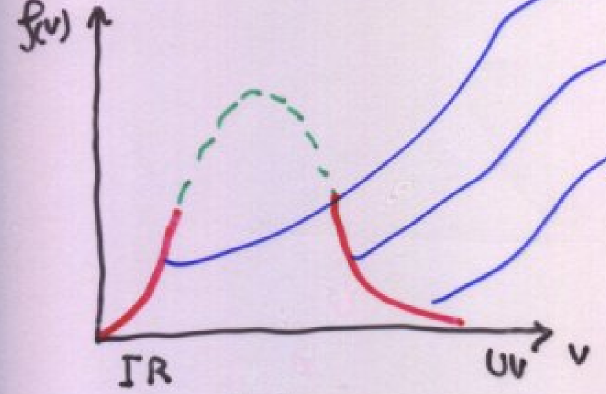
Here we have used string notation g_s, l_s

$g_s N = g_{YM}^{2} N =$ 't Hooft coupling.

(to be explained in a moment)

Summarizing the argument until this point.

1) (L) B.B problem.



Waves / equipartition

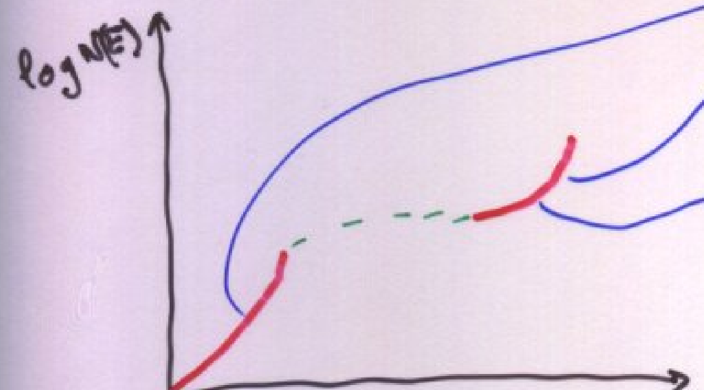
Wien phenomenological fit.

$S = \log\left(\frac{V}{V_0}\right)^n$ $n = E/\beta\nu$

particle interpretation $E = h\nu$

phenomenological cte

2) (NL) B.B problem.



Thermal radiation in dimension D
gas of gravitons

BH "phenomenological fit"

$S \sim \sqrt{N}\,(E\, b)^{d-1/d}$ $\sqrt{N} \sim (m_{pe}\, b)^{d-1/d}$

CFT in dimension $d = D-1$
Holographic interpretation

AdS/CFT correspondence.

"phenomenological constant".

$$\boxed{\begin{array}{lll} \text{Gauge} & \text{– Gravity} & \text{Correspondence} \\ d=4 & d=5 & \end{array}}$$

or

What is String Theory.

Back to 1980's Polyakov

Gauge theory Observables: Wilson Loops

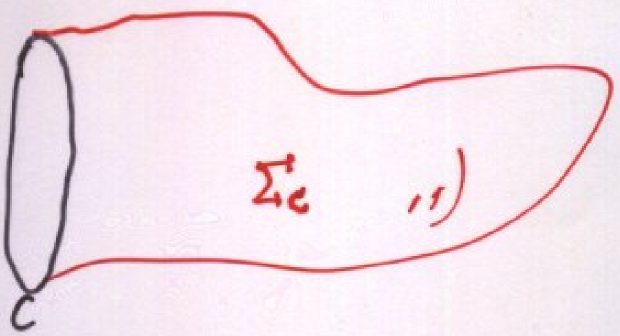
$$A(C) = \exp \oint_C A(\xi)\, d\xi$$

String Theory is just a representation of $\langle A(C) \rangle$ as sum over random surfaces Σ_C with boundary C.

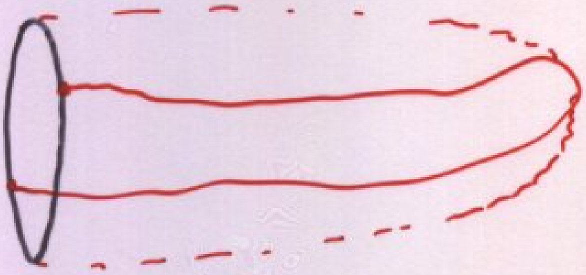
$$\boxed{\langle A(C) \rangle = \sum_{\Sigma_C} \exp S(\Sigma_C)}$$

gauge theory

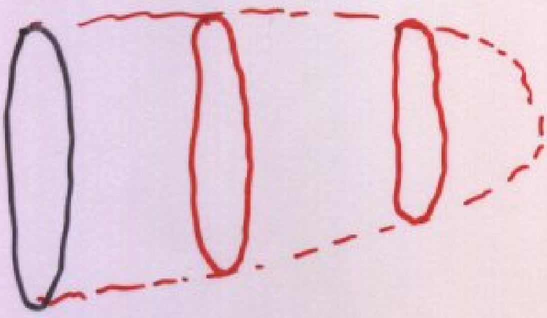
String theory



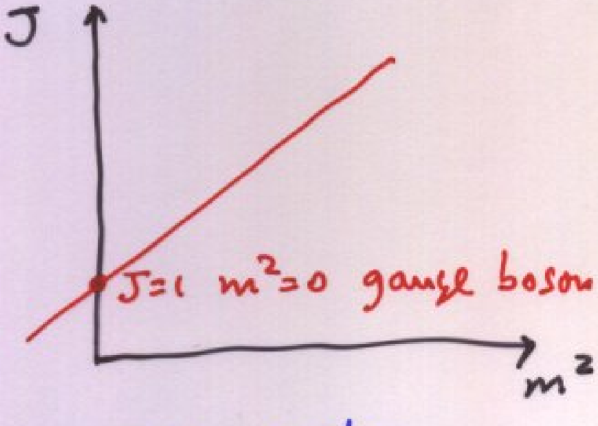
Dual Interpretation



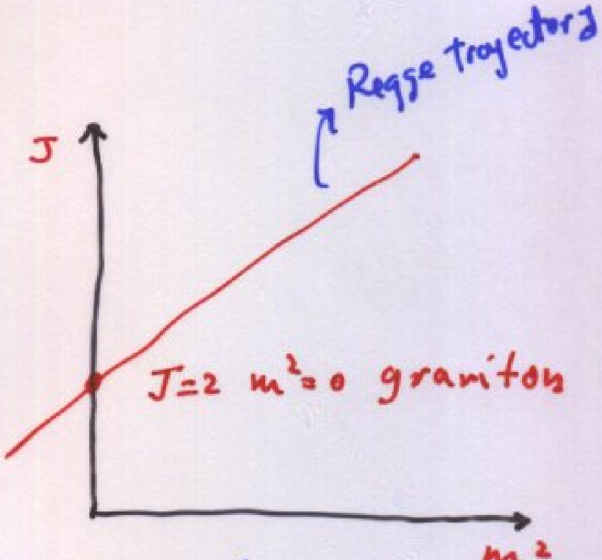
open



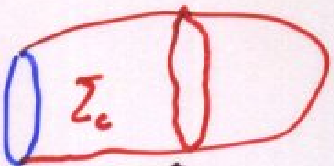
closed



open spectrum



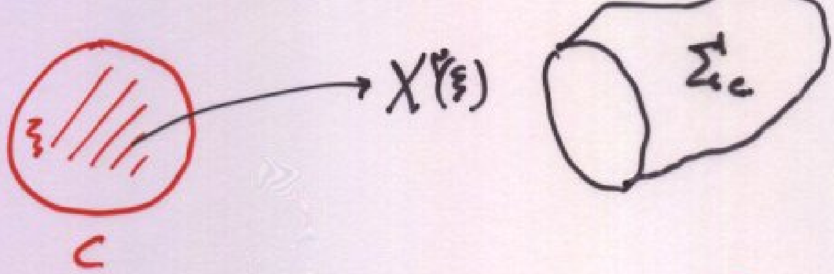
closed spectrum.

$$\langle A(C) \rangle = \sum_{\Sigma_c}$$ 

gauge — gravity correspondence.

String Action:

$$S(\Sigma_c) = \frac{1}{\alpha'} \int \partial X^\mu \partial X^\nu G_{\mu\nu}(X)\, d^2\xi$$



The simplest possibility $G_{\mu\nu}$ = Minkowski
$d = 4$
(where the gauge theory is living)

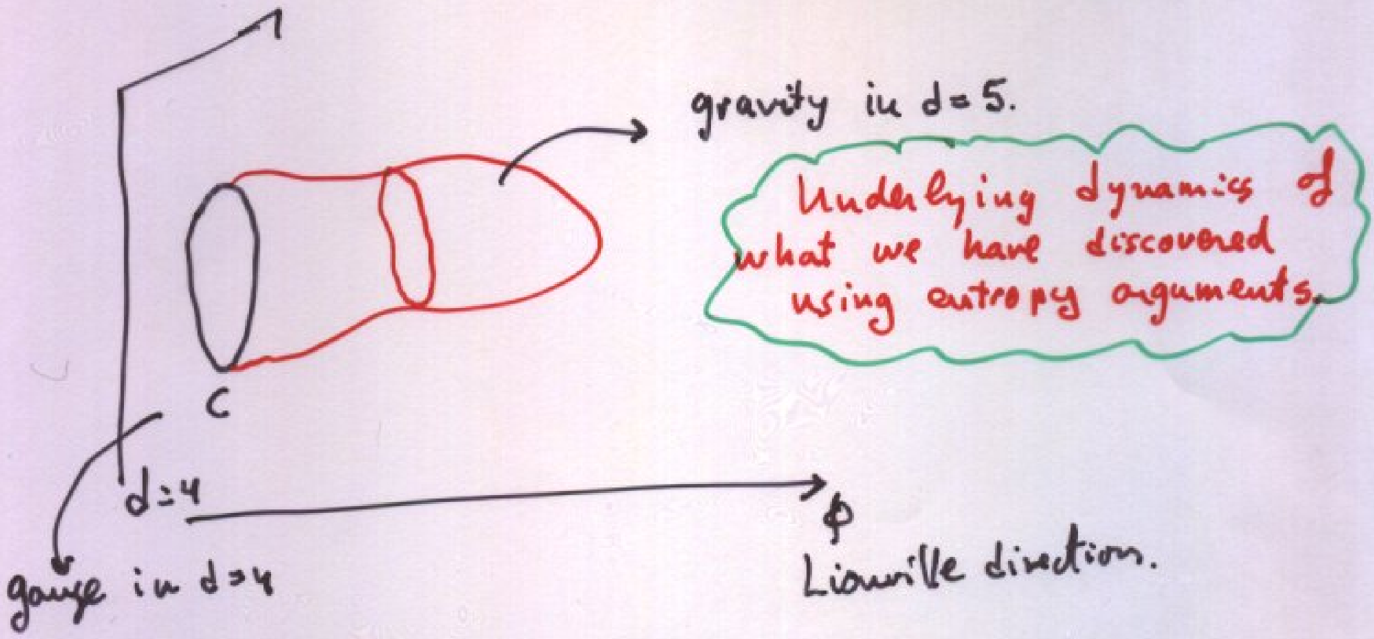
However:

Consistency of String Theory (QM + special relativity)

➡ Dynamical Generation of extra dimension (Liouville mode).

$$ds^2 = g(\phi)\, dx^2 + d\phi^2$$

$g(\phi) \to$ Liouville gravity potential.



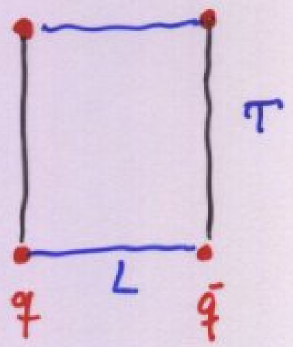
Let us come back to our box with gravity.

Once we have discovered that in the UV very complicated gravity effects are nicely described by a CFT (gauge) in 4D we can ask ourselves what is the gauge description of the IR region where from the gravity 5D point of view we just have thermal radiation.

Since we are breaking SUSY and we are working with a Non Abelian gauge theory we expect confinement in the IR.

This is in fact the qualitative result.

Confinement order parameter.

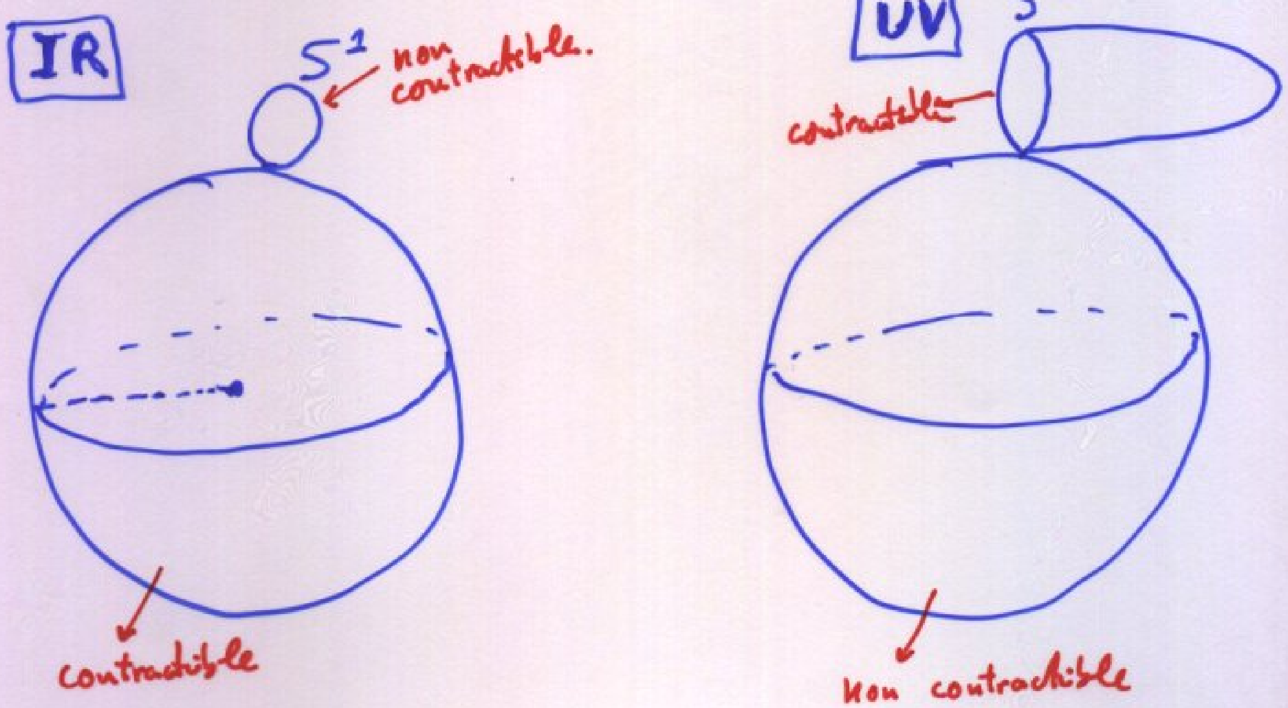


$$\sim \exp{-V(L)T} = \langle P \rangle$$

$\langle P \rangle \neq 0$ deconfinement

$\langle P \rangle = 0$ Confinement.

The box Geometry.



$$\langle P \rangle = \exp \oint_{S^1} A$$

If S^1 non contractible (IR) $\longrightarrow$ $Z(N)$ center of group $\xrightarrow{\text{after sum}}$ $\langle P \rangle = 0$ Confinement.

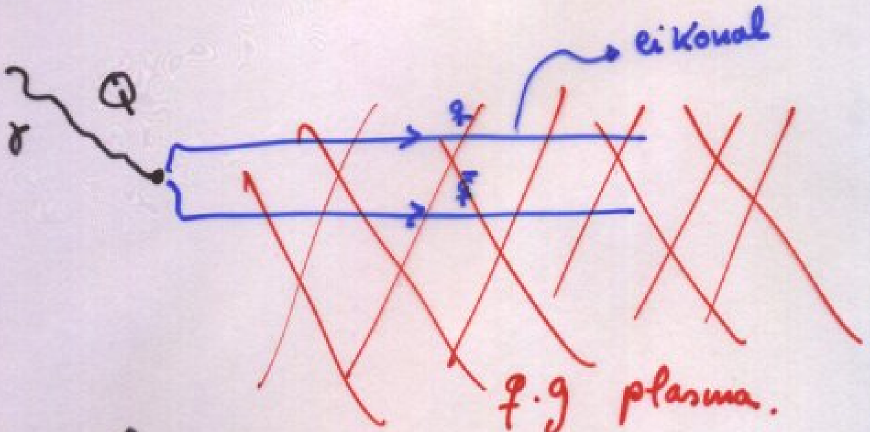
If S^1 contractible $\Rightarrow$ $\langle P \rangle \neq 0$

Gravity IR - box $\rightarrow$ Confinement

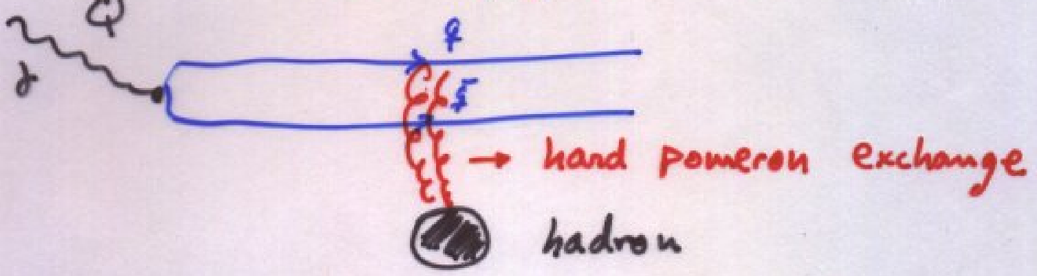
Gravity UV - box $\rightarrow$ Deconfinement $\Rightarrow$ Good model of quark-gluon plasma.

Some physics in the Box in the U.V

D.I.S on quark gluon plasma.



(1)



(1) ↔