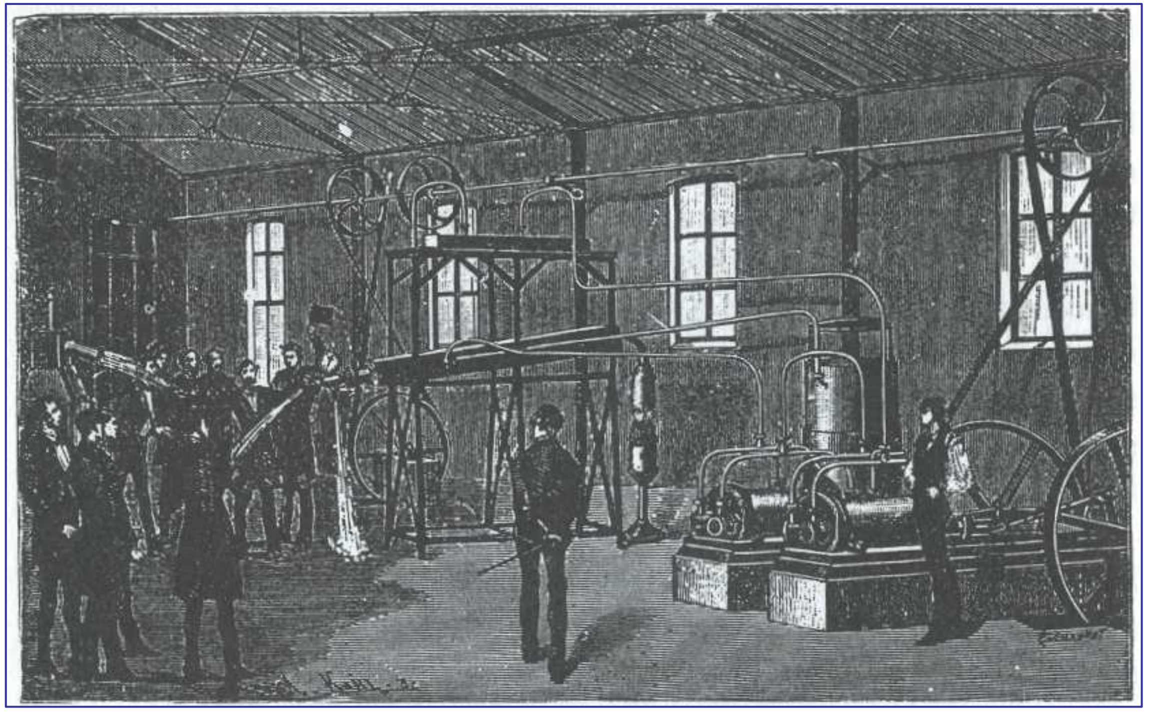
Valencia, 25 April 2007

# Ultracold atoms: overview and perspectives

*Franco Dalfovo*

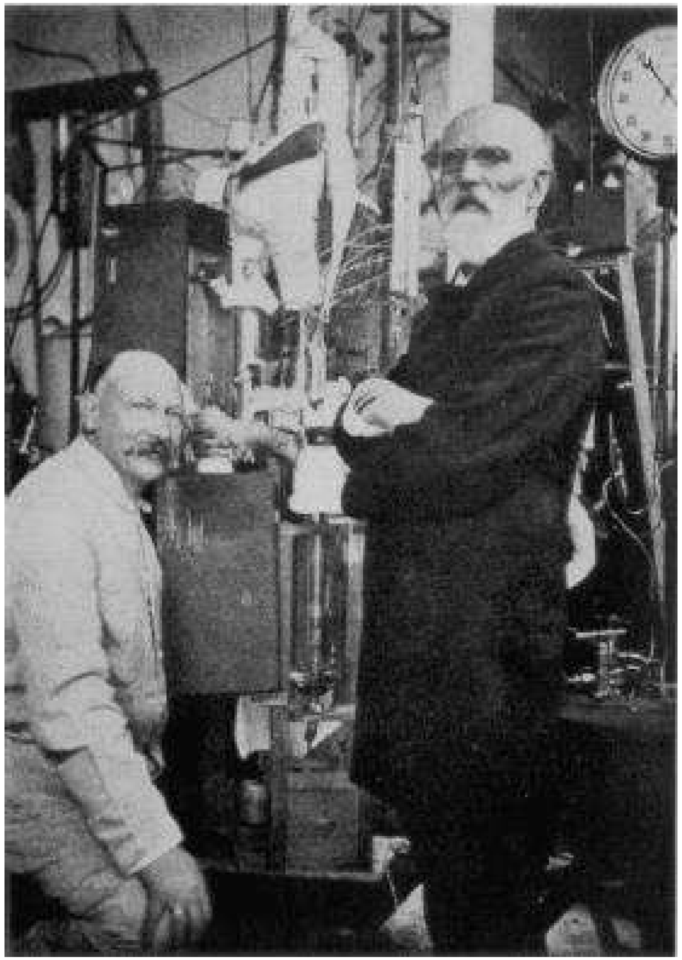
*INFM-BEC and*
*Dipartimento di fisica,*
*Università degli Studi di Trento*

# Hot is much simpler than cold !



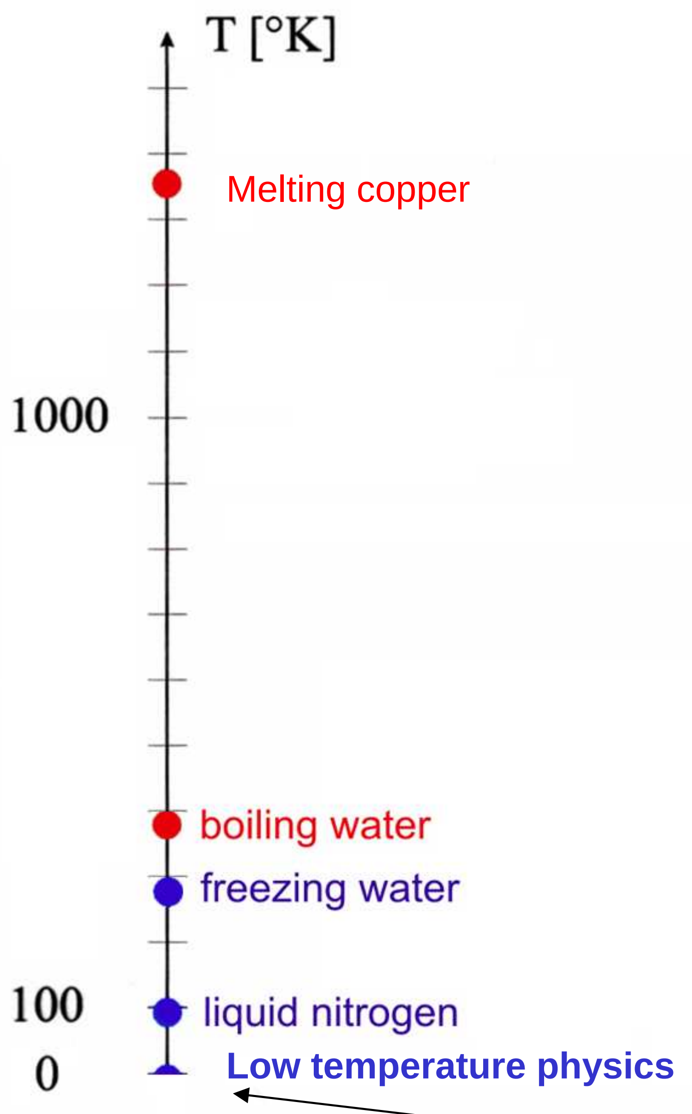
Production of liquid oxigen by Raoul Pictet (1877)

# Liquid helium observed at Leiden (1907)



Kamerlingh Onnes (left) and Van der Waals (right) with their refrigerator.

# Hot and cold




Just a boring little corner?

# Superfluid helium

**anomaly in the specific heat**
(Leiden, 1927)

**superfluidity**
(Allen and Misener, Kapitza,1938)

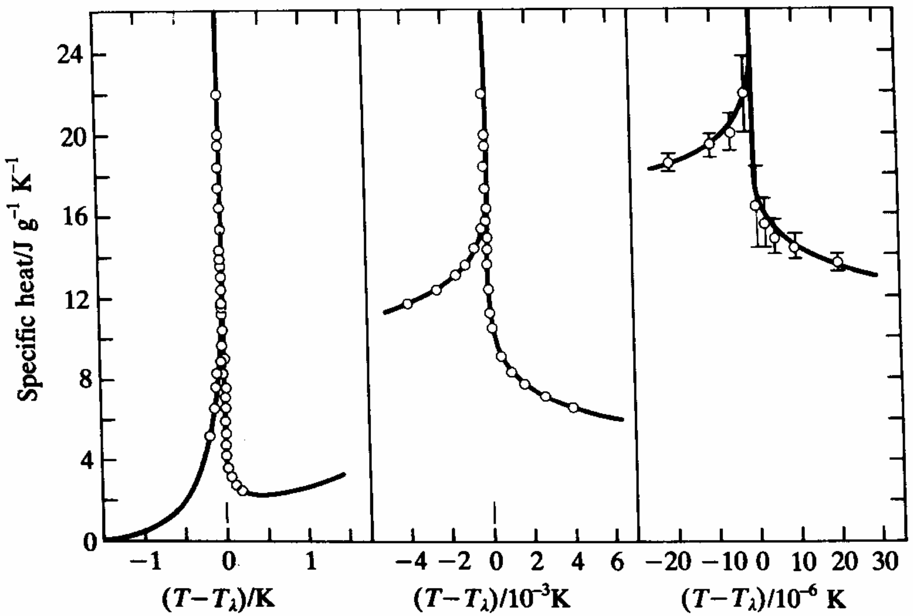
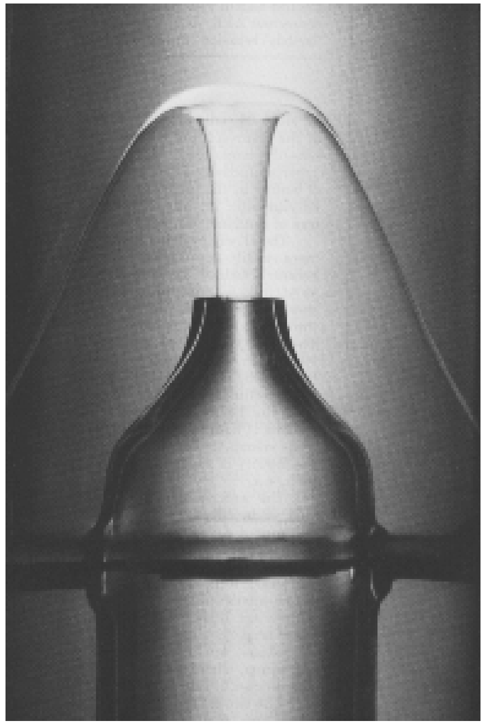



**Fig. 15.3.** The specific heat of liquid $^4$He under the saturated vapour pressure as a function of $T - T_\lambda$. The width of the small vertical line just above the origin indicates the portion of the diagram shown expanded (in width) in the curve directly to the right (after Buckingham and Fairbank [193].)



Fritz London intuition:

is superfluidity a manifestation of Bose-Einstein condensation ?



the effective mass $m^*$ being of the order of magnitude of the mass of the atoms. But in the present case we are obliged to apply Bose-Einstein statistics instead of Fermi statistics.

(3) In his well-known papers, Einstein has already discussed a peculiar condensation phenomenon of the 'Bose-Einstein' gas; but in the course of time the degeneracy of the Bose-Einstein gas has rather got the reputation of having only a purely imaginary existence. Thus it is perhaps not generally known that this condensation phenomenon actually represents a discontinuity of the derivative of the specific heat (phase transition of third order). In the accompanying figure the specific heat ($C_v$) of an *ideal* Bose-Einstein gas is represented as a function of $T/T_0$ where

$$T_0 = \frac{h^2}{2\pi m^* k}\left(\frac{n}{2,615}\right)^{2/3}.$$

With $m^*$ = the mass of a He atom and with the mol. volume $\frac{N_l}{n}$ = 27·6 cm.³ one obtains $T_0 = 3{\cdot}09°$. For $T < T_0$ the specific heat is given by

expected to furnish quantitative insight into the properties of liquid helium.

The conception here proposed might also throw a light on the peculiar transport phenomena observed with He II (enormous conductivity of heat[5], extremely small viscosity[6] and also the strange fountain phenomenon recently discovered by Allen and Jones[2]).

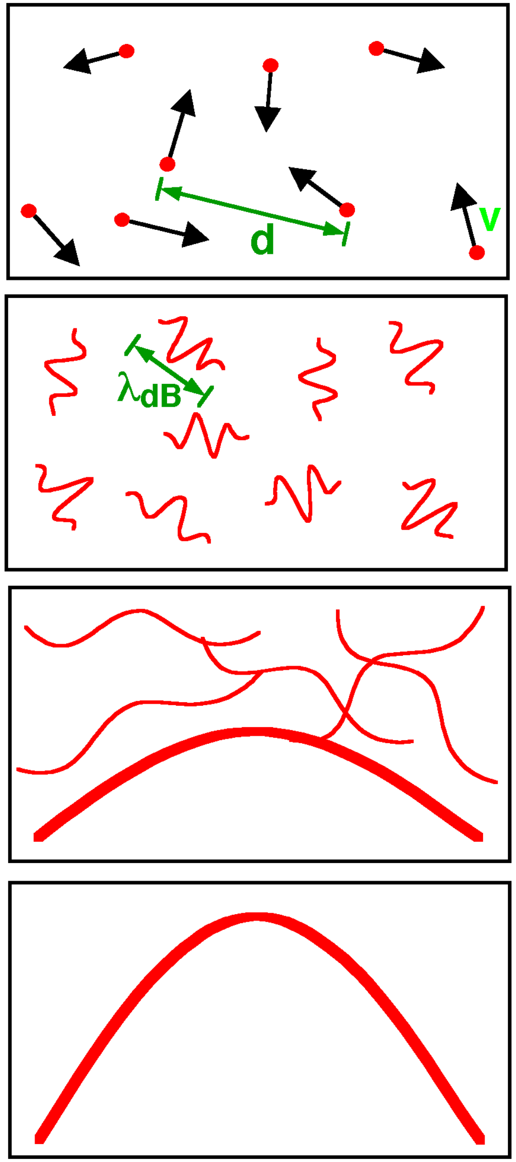
A detailed discussion of these questions will be published in the *Journal de Physique*.

F. LONDON.

Institut Henri Poincaré,
Paris.
March 5.

[1] Fröhlich, H., *Physica*, **4**, 639 (1937).
[2] Allen, J. F., and Jones, H., NATURE, **141**, 243 (1938).
[3] Simon, F., NATURE, **133**, 529 (1934).
[4] London, F., *Proc. Roy. Soc.*, A, **153**, 576 (1936).
[5] Rollin, *Physica*, **2**, 557 (1935); Keesom, W. H., and Keesom, H. P., *Physica*, **3**, 359 (1936); Allen, J. F., Peierls, R., and Zaki Uddin, M., NATURE, **140**, 62 (1937).
[6] Burton, E. F., NATURE, **135**, 265 (1935); Kapitza, P., NATURE, **141**, 74 (1938); Allen, J. F. and Misener, A. D., NATURE, **141**, 75 (1938).

# What is Bose-Einstein condensation (BEC)?




**High Temperature T:**

thermal velocity v

density $d^{-3}$

"Billiard balls"

**Low Temperature T:**

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

"Wave packets"

**$T=T_{crit}$: Bose-Einstein Condensation**

$\lambda_{dB} \approx d$

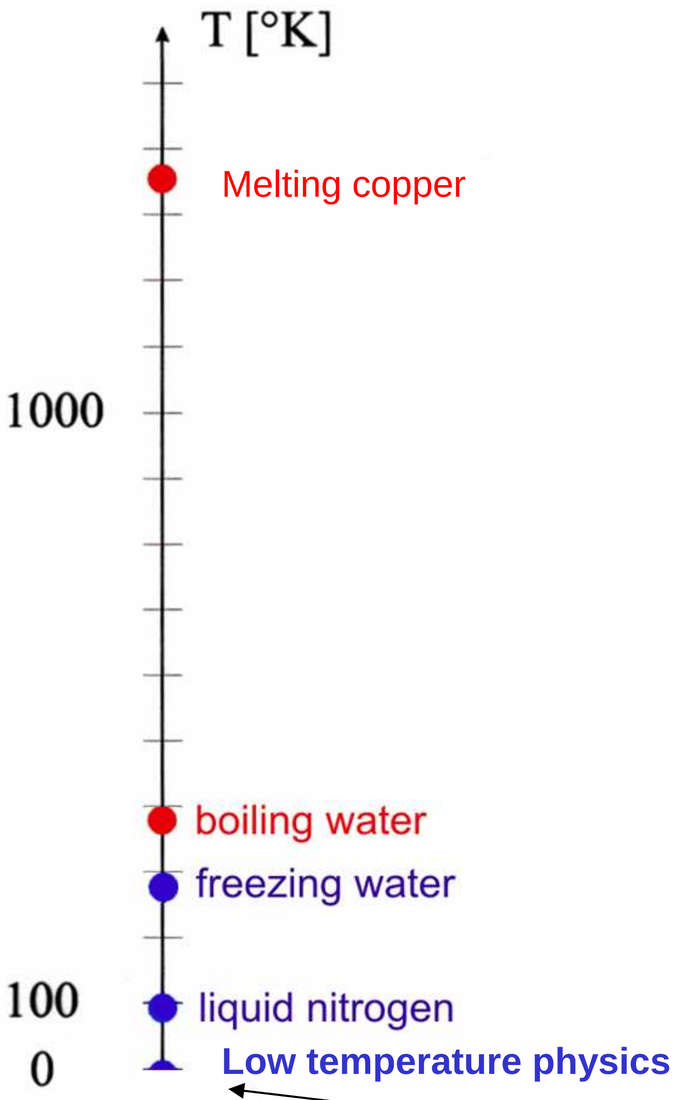
"Matter wave overlap"

**T=0: Pure Bose condensate**

"Giant matter wave"

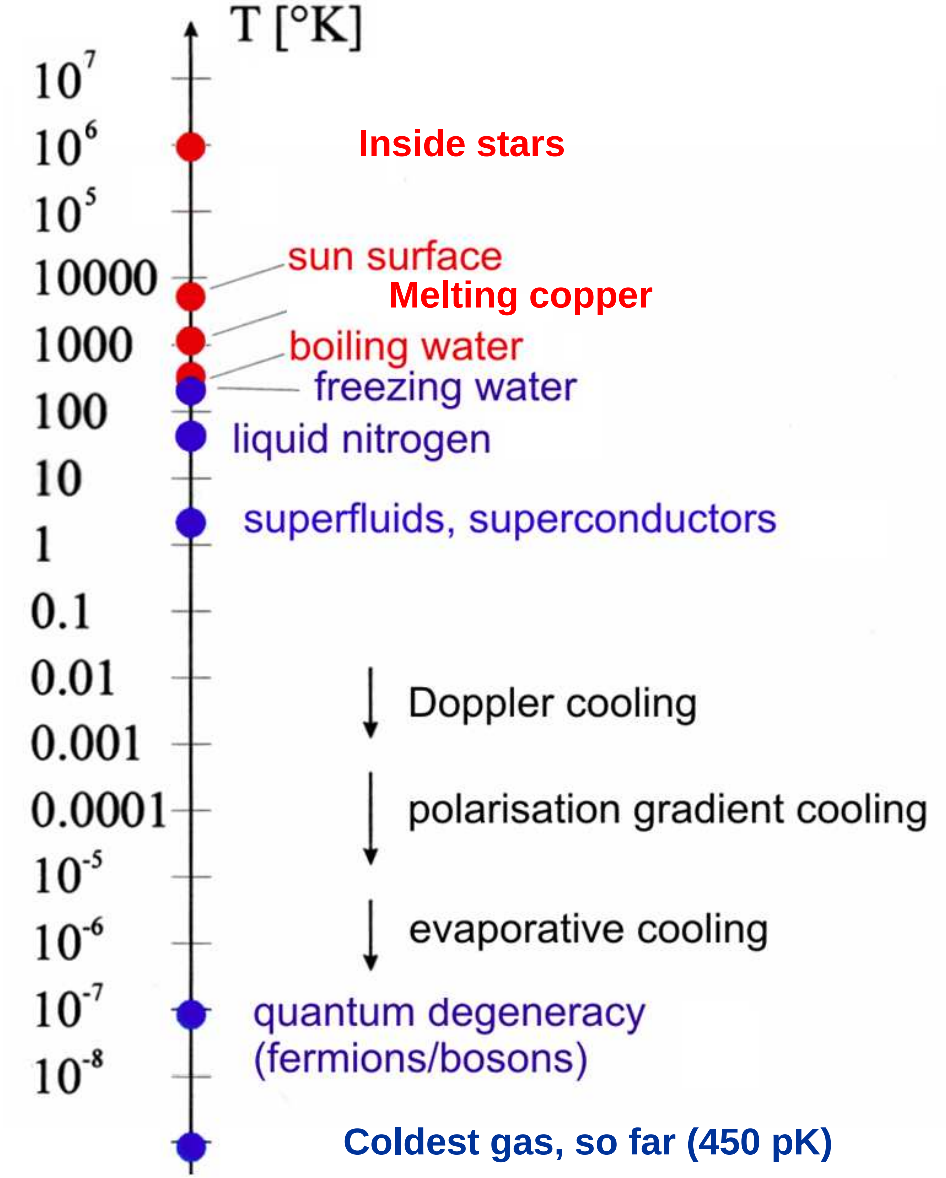
(Taken from W. Ketterle)

# Hot and cold




Just a boring little corner?

# Cold, very cold, ultracold

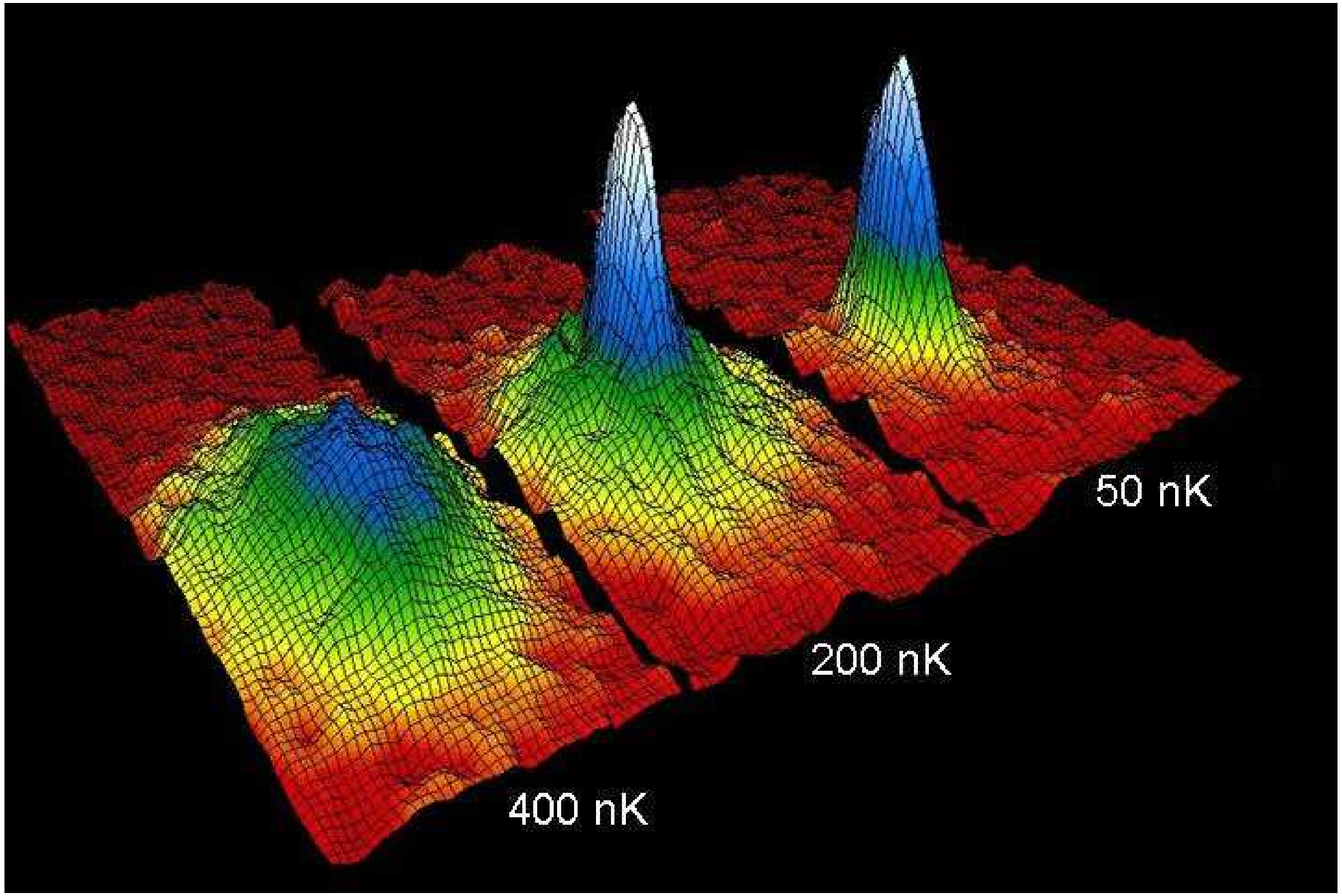



~~Just a boring little corner?~~

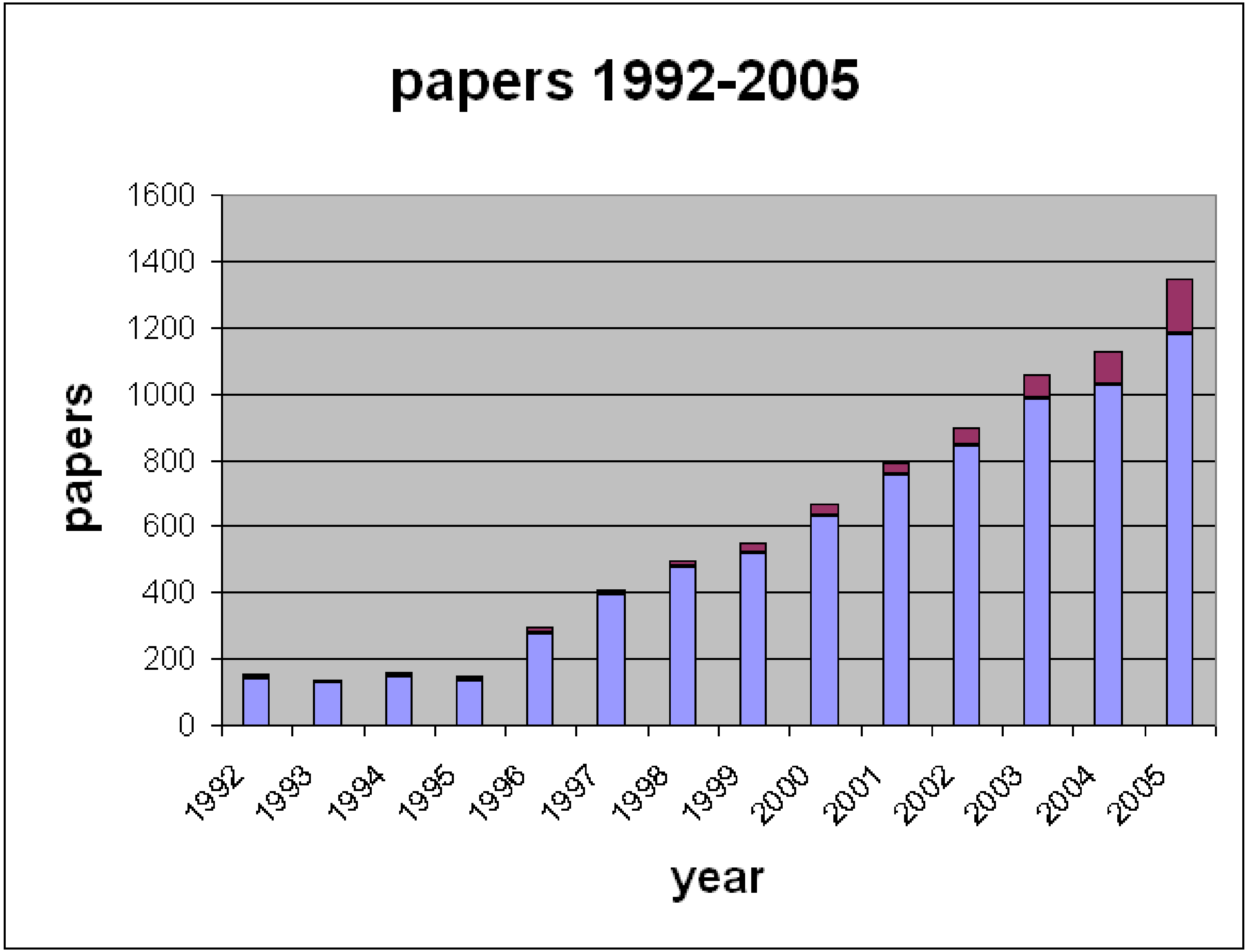
An amazingly interesting new frontier !!

The kingdom of Quantum Mechanics

# First images of BEC with Rb atoms (JILA 1995)




**At present: many condensates with more atomic species (Rb, Na, Li, H, He*, K, Cs, Yb), including condensates of molecules ($Rb_2$, $Na_2$, $Cs_2$).**




Source: ISI – Web of Science

Keywords: bos* AND condens* (blue); cold OR ultracold AND fermi* (red)

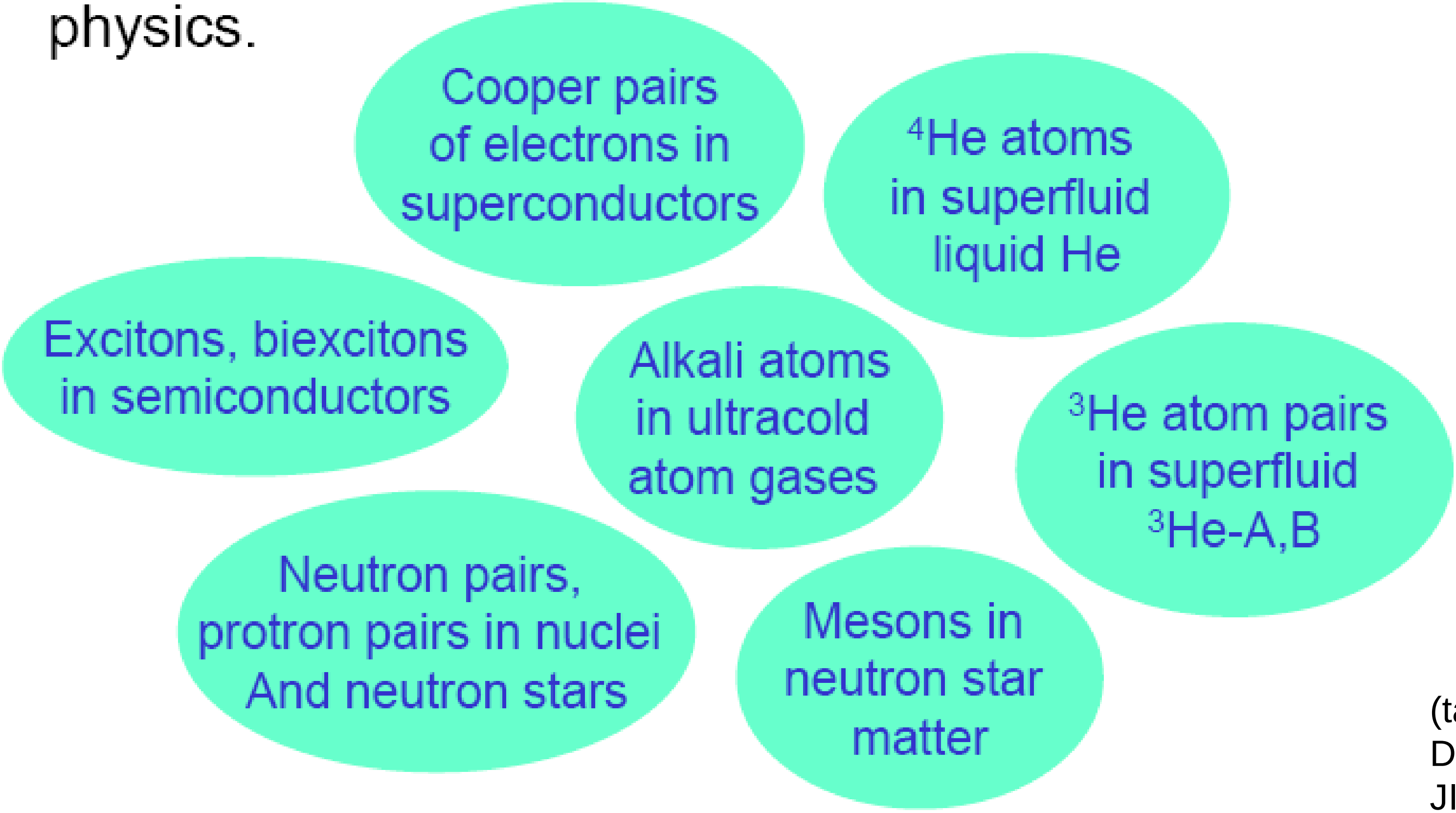
# Why BEC is important ?

Paradigm of statistical mechanics  (phase transition in the absence of interactions).

Exact description of the effects of interactions for dilute gases.

Fundamental concepts (long range order; spontaneous symmetry breaking; etc.) which play an important role in many areas of physics.

BEC shows up in condensed matter, nuclear physics, elementary particle physics, astrophysics, and atomic physics.




(taken from Debbie Jin, JILA)

Most recent: **polariton gas in 2D cavities**

# Why BEC is important ?

Paradigm of statistical mechanics (phase transition in the absence of interactions).

Exact description of the effects of interactions for dilute gases.

Fundamental concepts (long range order; spontaneous symmetry breaking; etc.) which play an important role in many areas of physics.

# Why BEC is important ?

Paradigm of statistical mechanics  (phase transition in the absence of interactions).

Exact description of the effects of interactions for dilute gases.

Fundamental concepts (long range order; spontaneous symmetry breaking; etc.) which play an important role in many areas of physics.

Tuning the interaction and/or  the external confining potentials:

from single-particle "textbook" physics → to correlated many-body physics

Most of the properties of dilute BECs are well described by the Gross-Pitaevskii equation (1961).

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[-\frac{\hbar^2}{2M}\nabla^2+V_{ext}(\mathbf{r})+g\,|\Psi(\mathbf{r},t)|^2\right]\Psi(\mathbf{r},t)$$

Complex function with modulus and phase!

The GP equation gives the behavior, in space and time, of the macroscopic matter wave *(see the analogy with Maxwell equations describing electromagnetic waves!).*

The GP equation accounts for the interatomic forces through a mean-field term, so that it takes the form of a nonlinear Schrödinger equation *(many analogies with nonlinear optics).*

# Some relevant properties of BEC

**Those due to interaction:**
- **sound propagation and collective oscillations**
- **solitary waves**

**Those due to phase coherence:**
- **interference**
- **atom laser**

**Superfluid properties (interaction + coherence):**
- **viscousless motion**
- **quantized vortices**
- **Josephson effect**

**Quantum phase transitions:**
- **Superfluid - Mott insulator**
- **Kosterlitz-Thouless**

## LINK BETWEEN BEC AND SUPERFLUIDITY

### ORDER PARAMETER

$$\Psi = n^{1/2}\, e^{iS}$$

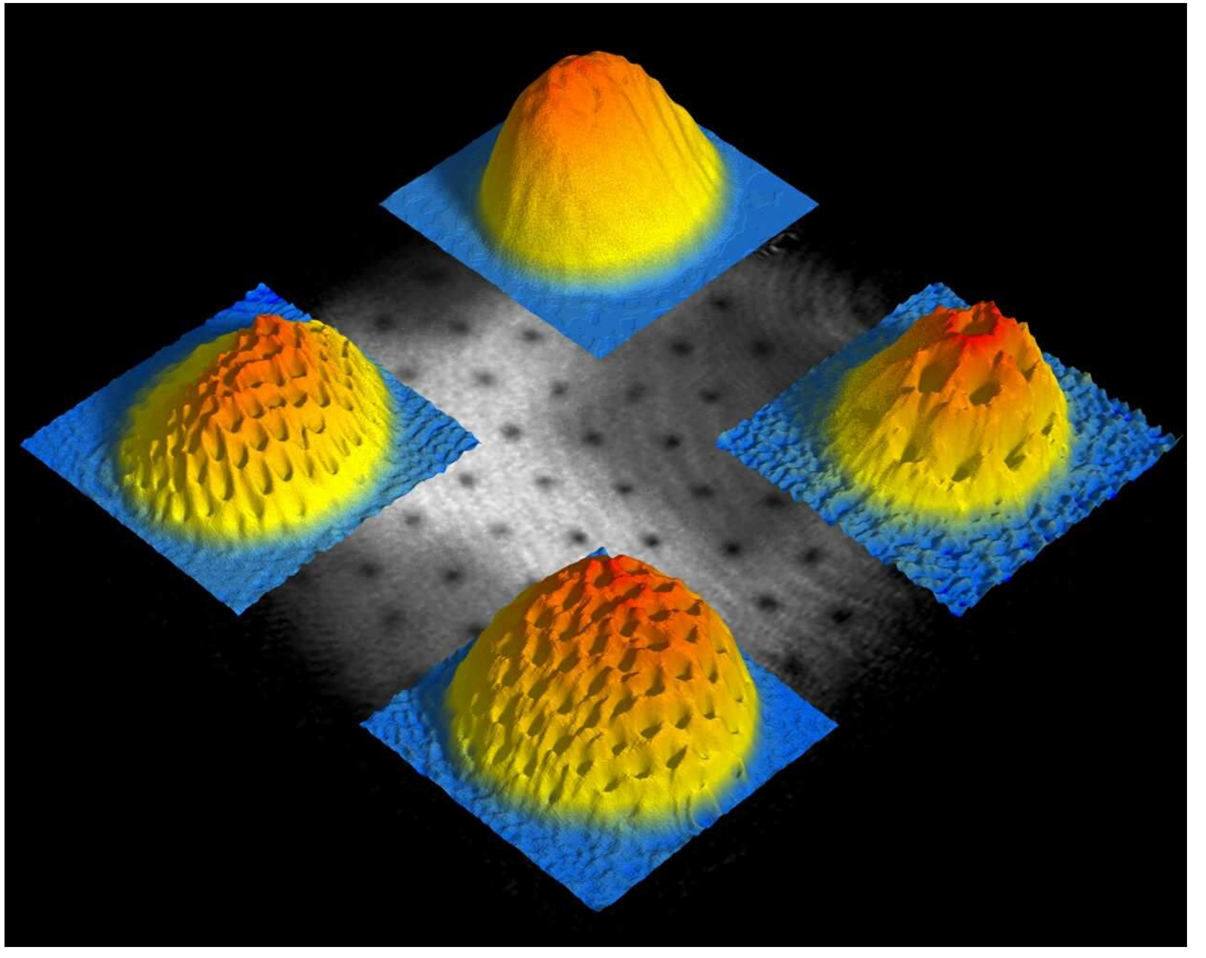
$n$ = condensate density

$S$ = phase

$\mathbf{v} = (\hbar/m)\,\nabla S$ = superfluid velocity

**IRROTATIONAL !**

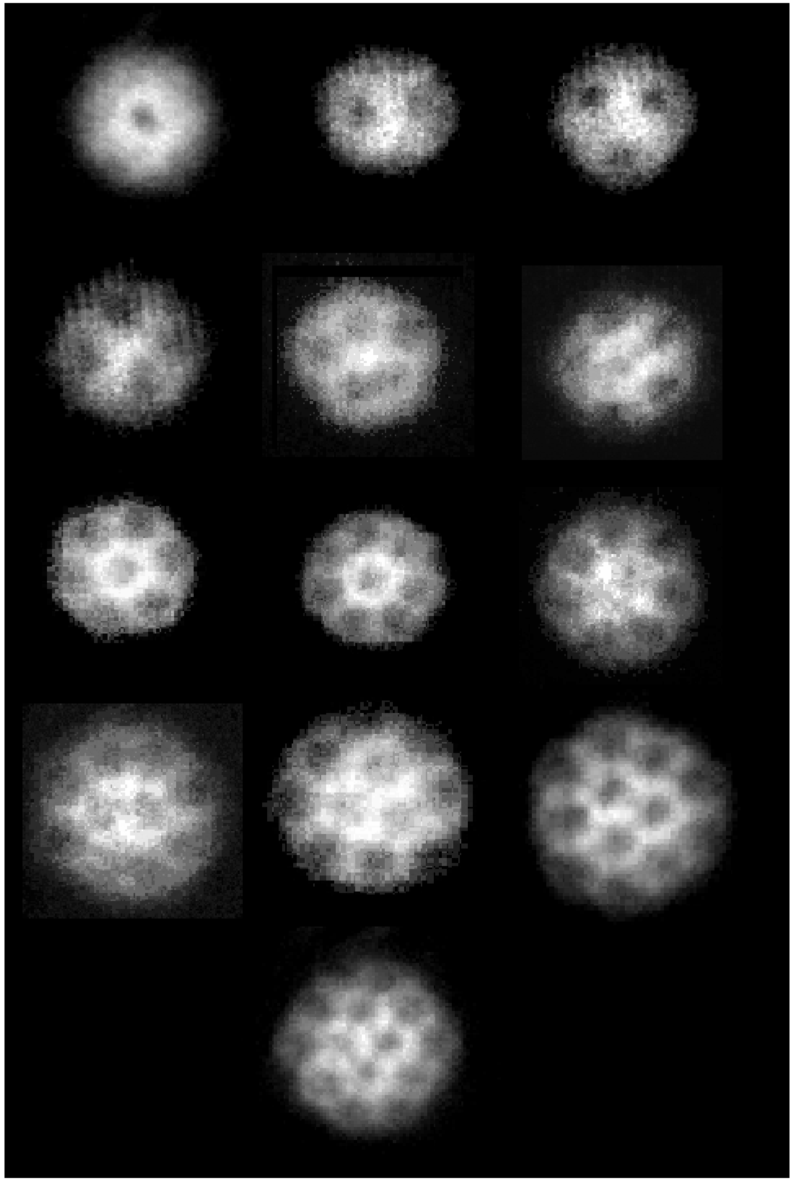
# Consequence of irrotationality: quantized vortices



(Ketterle et al., MIT, 2001)

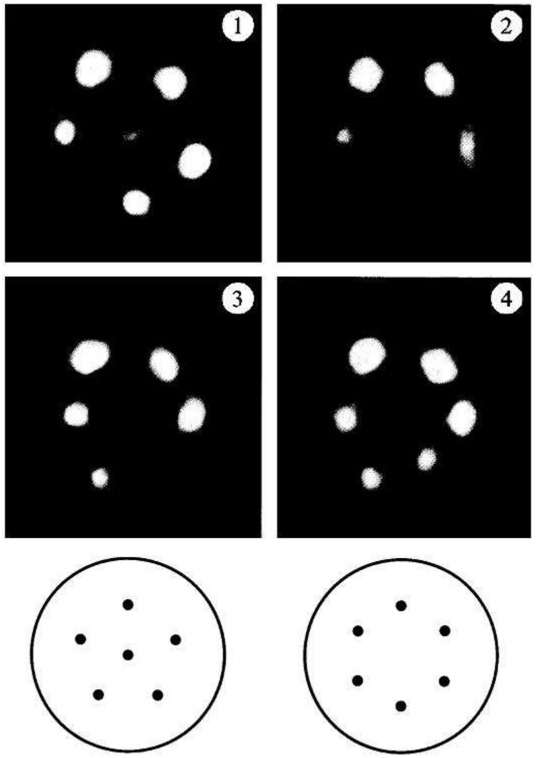
**Quantized vortices in BEC**

(Dalibard et al., ENS-Paris, 2000)

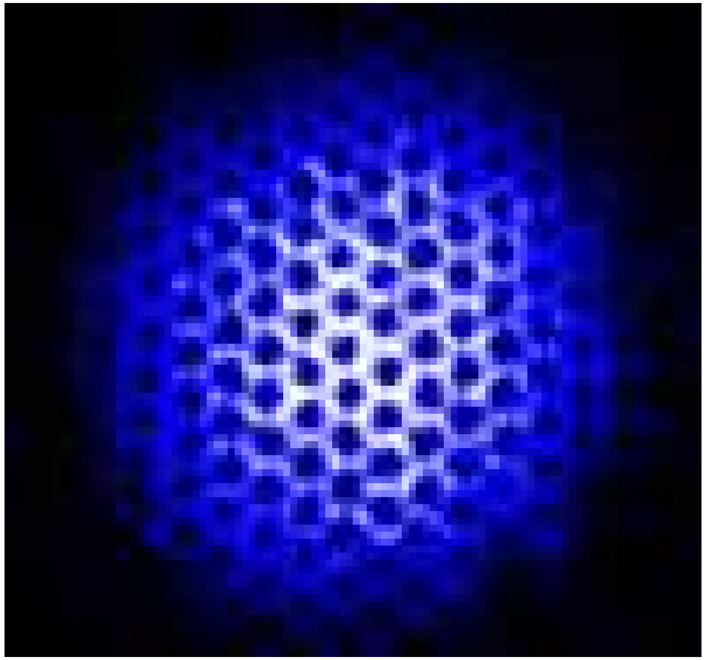


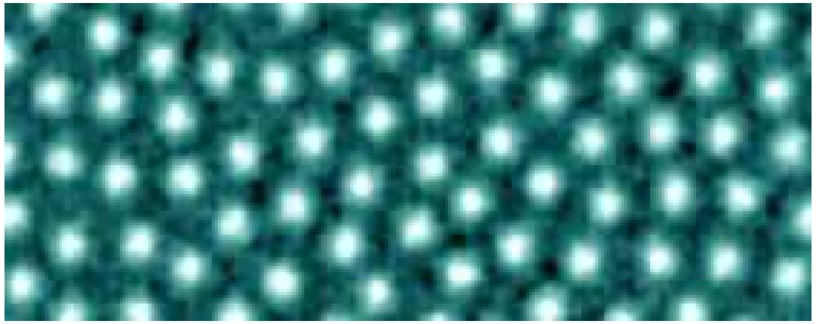
**Quantized vortices in superfluid helium**

(Packard et al., Berkeley, 1974)



# Vortex lattice





**Abrikosov lattice in type II superconductors**

**BEC**
(Cornell et al. JILA, 2002)

Another consequence of the phase of the order parameter:

# INTERFERENCE BETWEEN TWO CONDENSATES

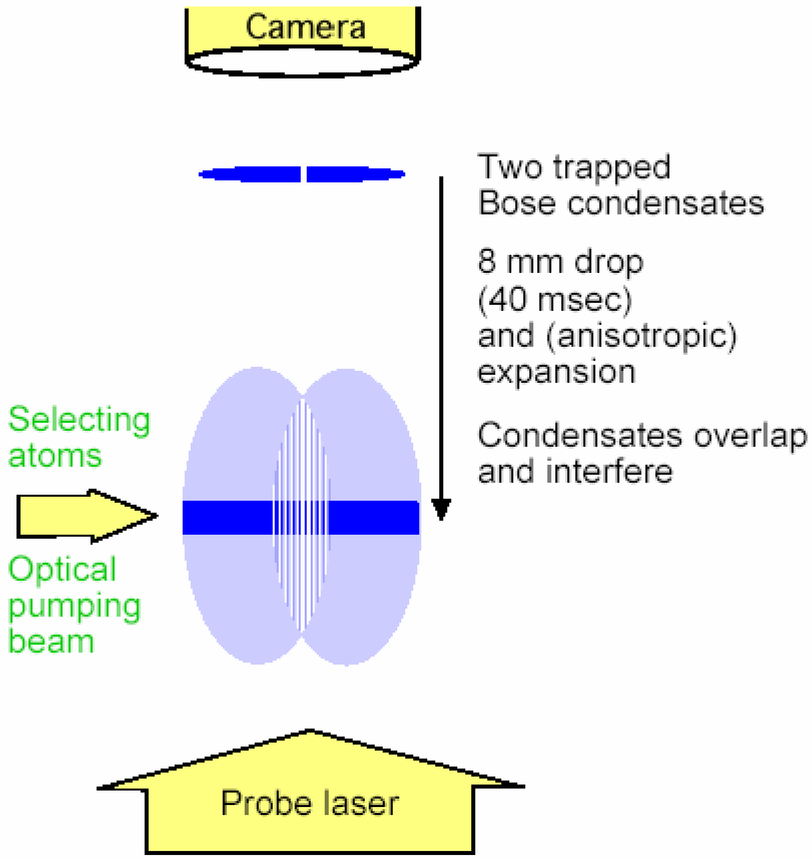



Fig. 13. Schematic setup for the observation of the interference of two Bose condensates, created in a double well potential. The two condensates were separated by a laser beam which exerted a repulsive force on the atoms. After switching off the trap, the condensates were accelerated by gravity, expanded ballistically, and overlapped. In the overlap region, a high-contrast interference pattern was observed by using absorption imaging. An additional laser beam selected absorbing atoms in a thin layer by optical pumping. This tomographic method prevented blurring of the interference pattern due to integration along the probe laser beam.

Another consequence of the phase of the order parameter:

# INTERFERENCE BETWEEN TWO CONDENSATES

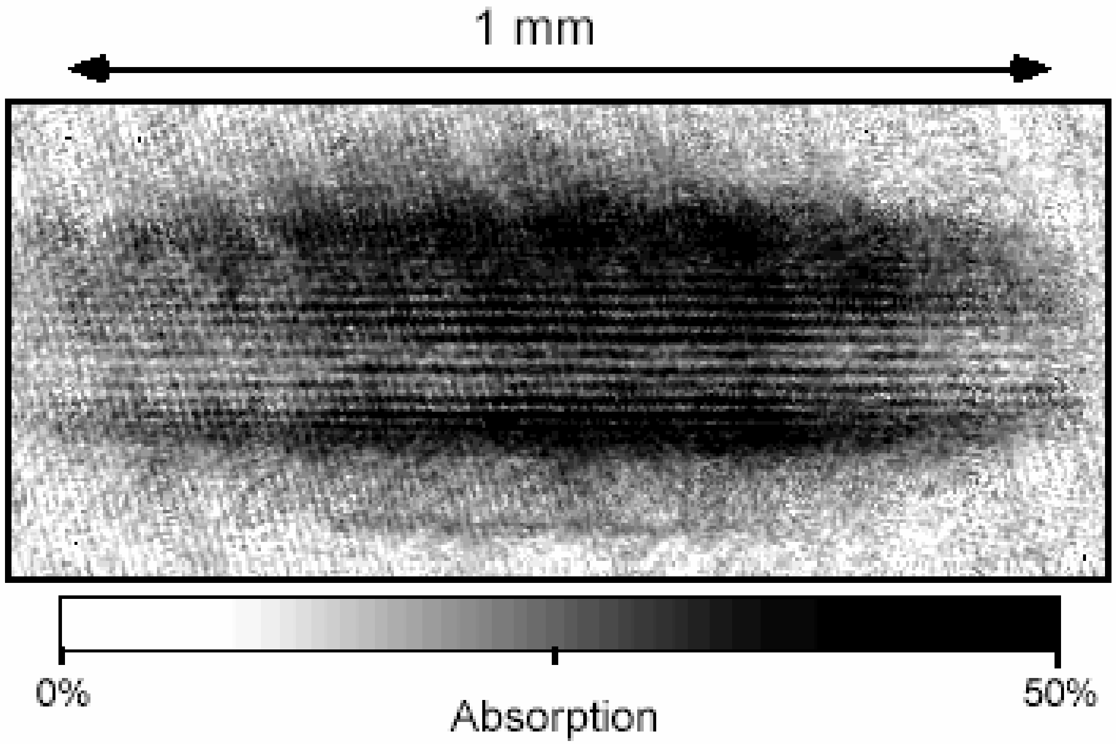
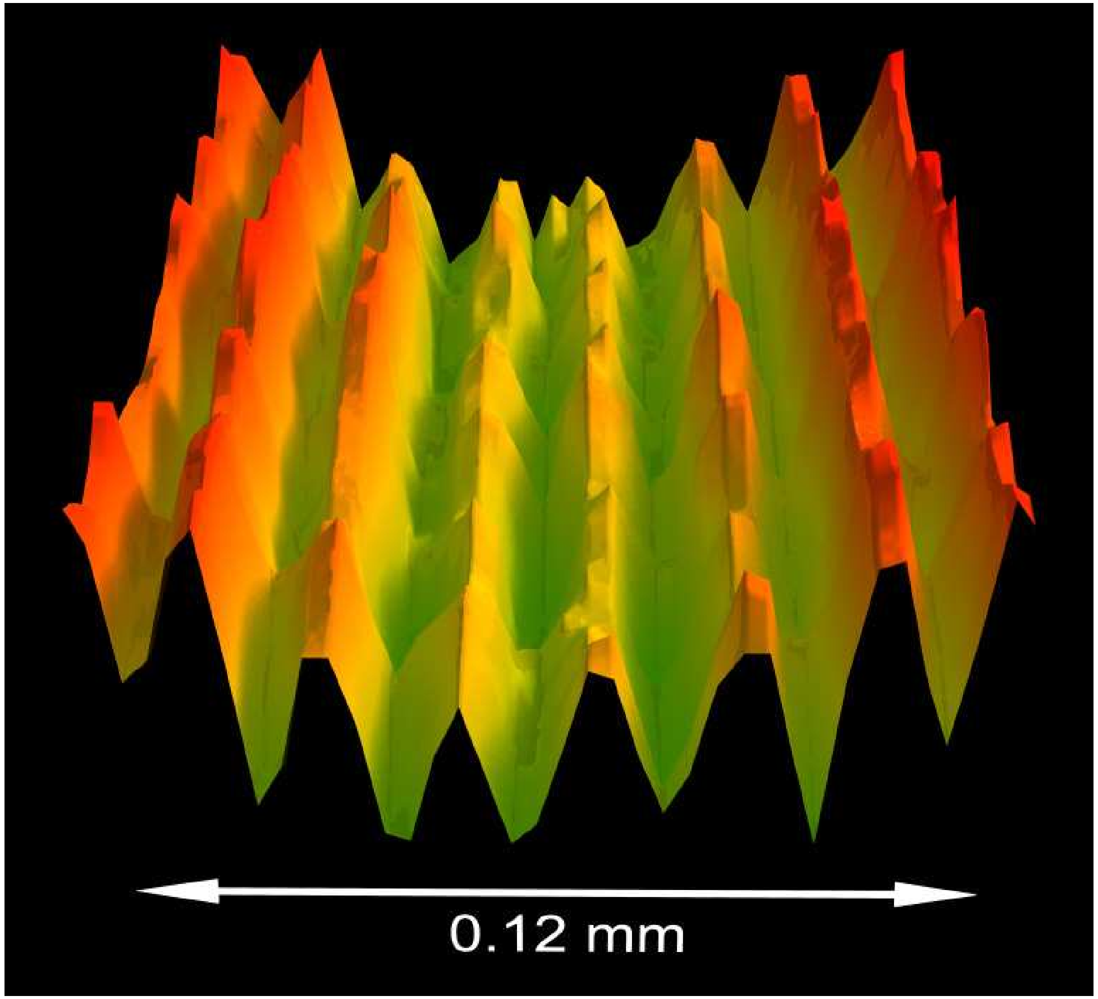



Fig. 15. Interference pattern of two expanding condensates observed after 40 msec time of flight. The width of the absorption image is 1.1 mm. The interference fringes have a spacing of 15 μm and are strong evidence for the long-range coherence of Bose-Einstein condensates.

(Ketterle et al. MIT, 1996)

Another consequence of the phase of the order parameter:

# INTERFERENCE BETWEEN TWO CONDENSATES




**MATTER WAVES**

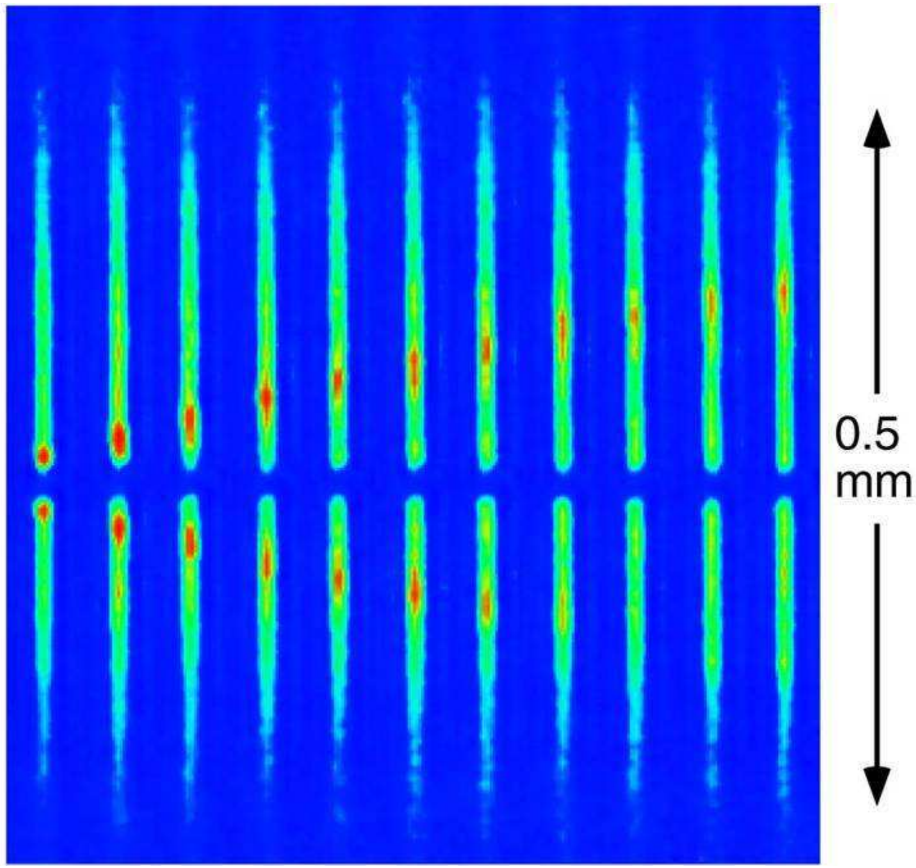
# Weakly interacting Bose gas: Bogoliubov sound

$$b_q = u_q a_q + v_q a^{\dagger}_{-q}$$

$$b^{\dagger}_q = u_q a^{\dagger}_q + v_q a_{-q}$$

**These transformations allow one to diagonalize the many-body Hamiltonian of a weakly interacting Bose gas**

**Interacting particles ➔ Free quasiparticles**




(Ketterle et al. MIT, 1997)

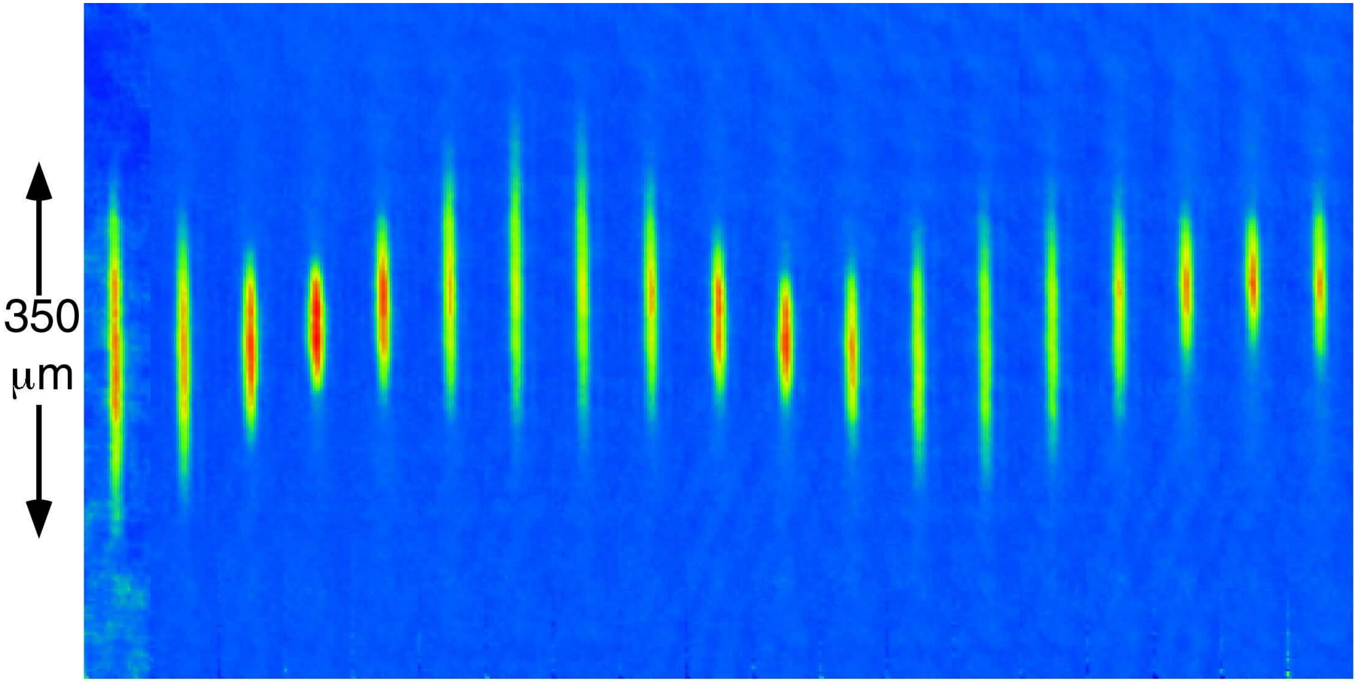
**Quasiparticle: one of the most important concepts in condensed matter physics** *!*

# Finite size: Collective oscillations

expt (MIT 97): $\omega = 1.57\omega_z$

theory (HD): $\omega = 1.58\,\omega_z$

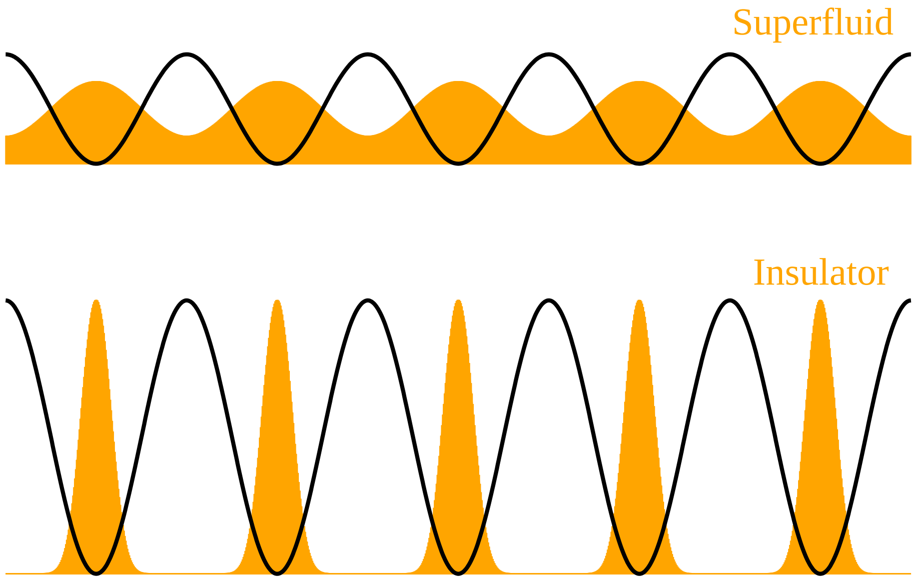
ideal gas: $\omega = 2\,\omega_z$



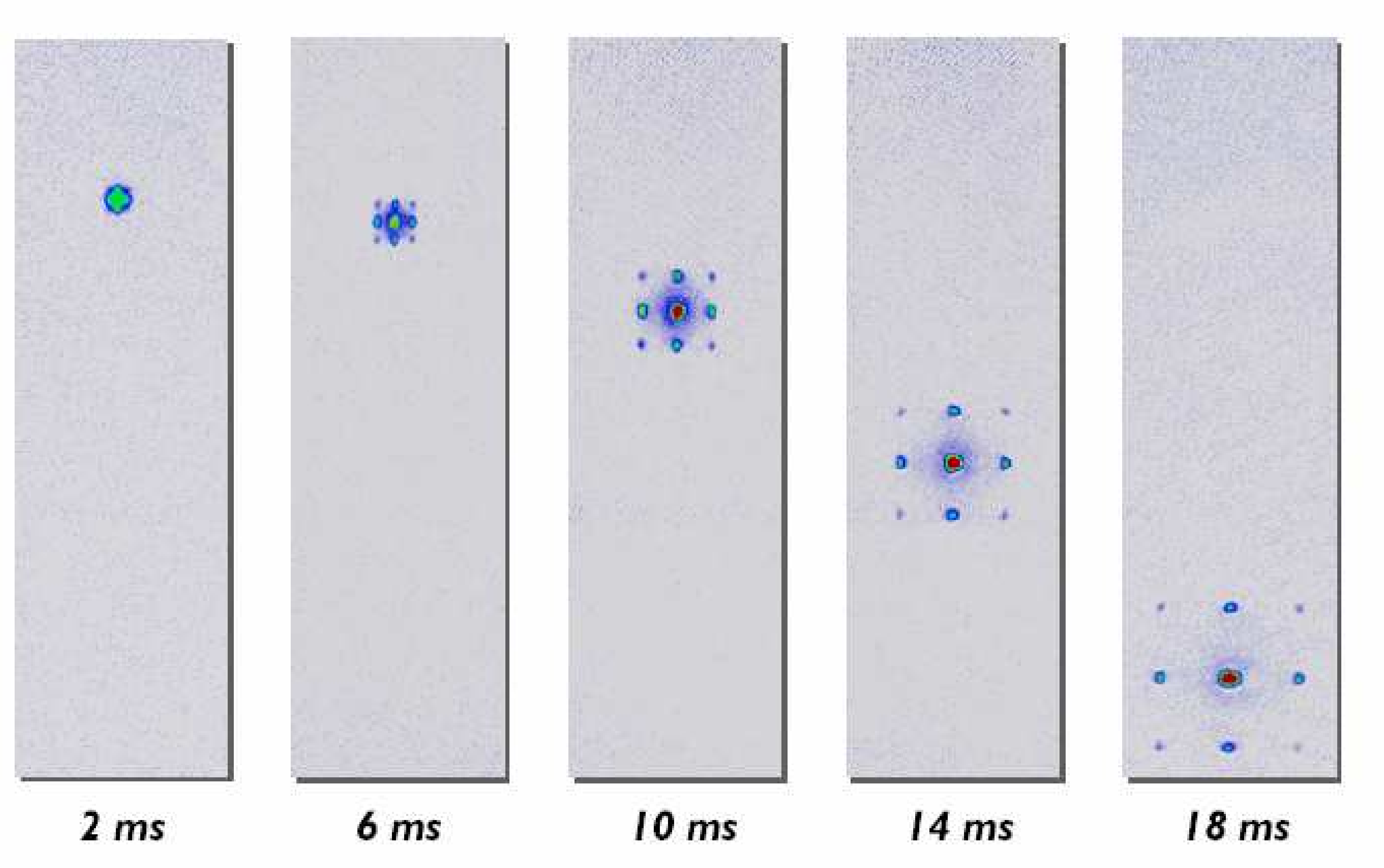

5 milliseconds per frame

(Ketterle et al. MIT, 1996)

# Atoms in optical lattices

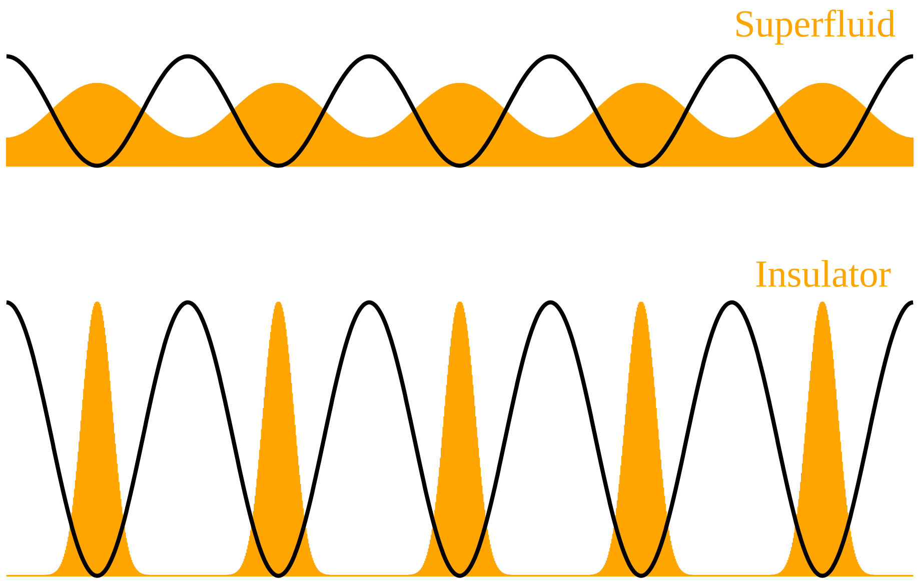


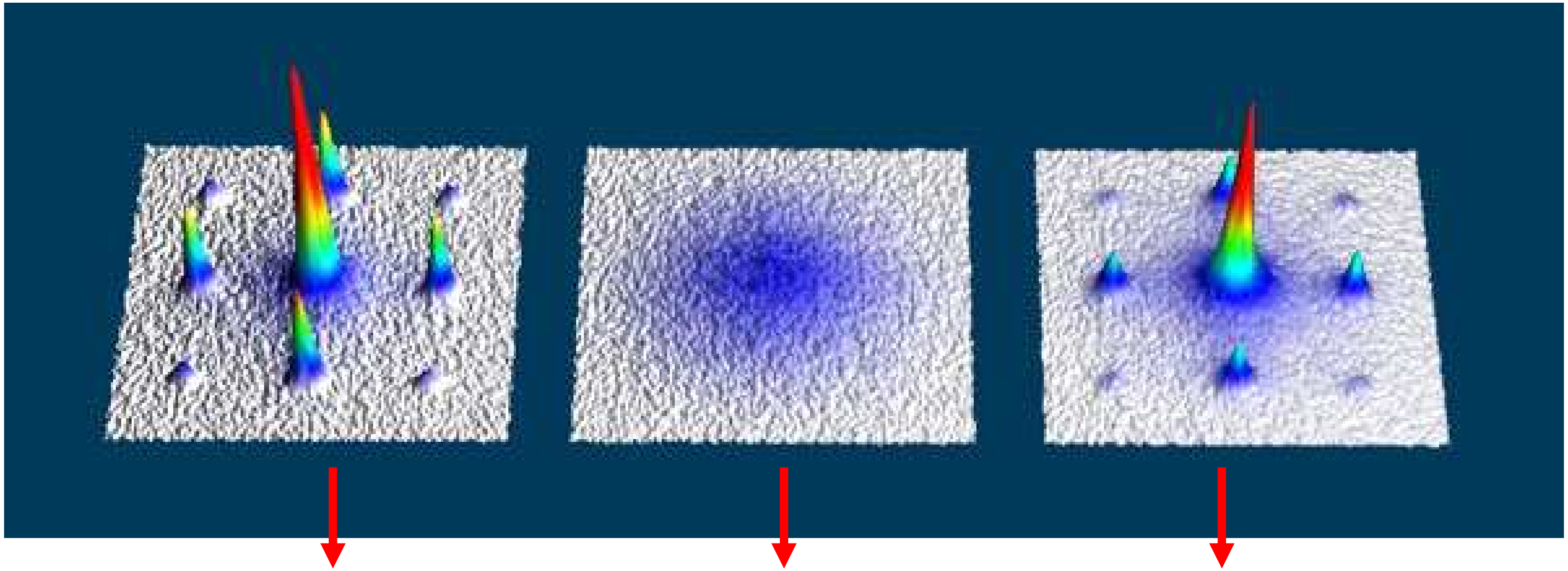
# Expanding a superfluid out of a lattice




Bloch et al. Munich, 2002

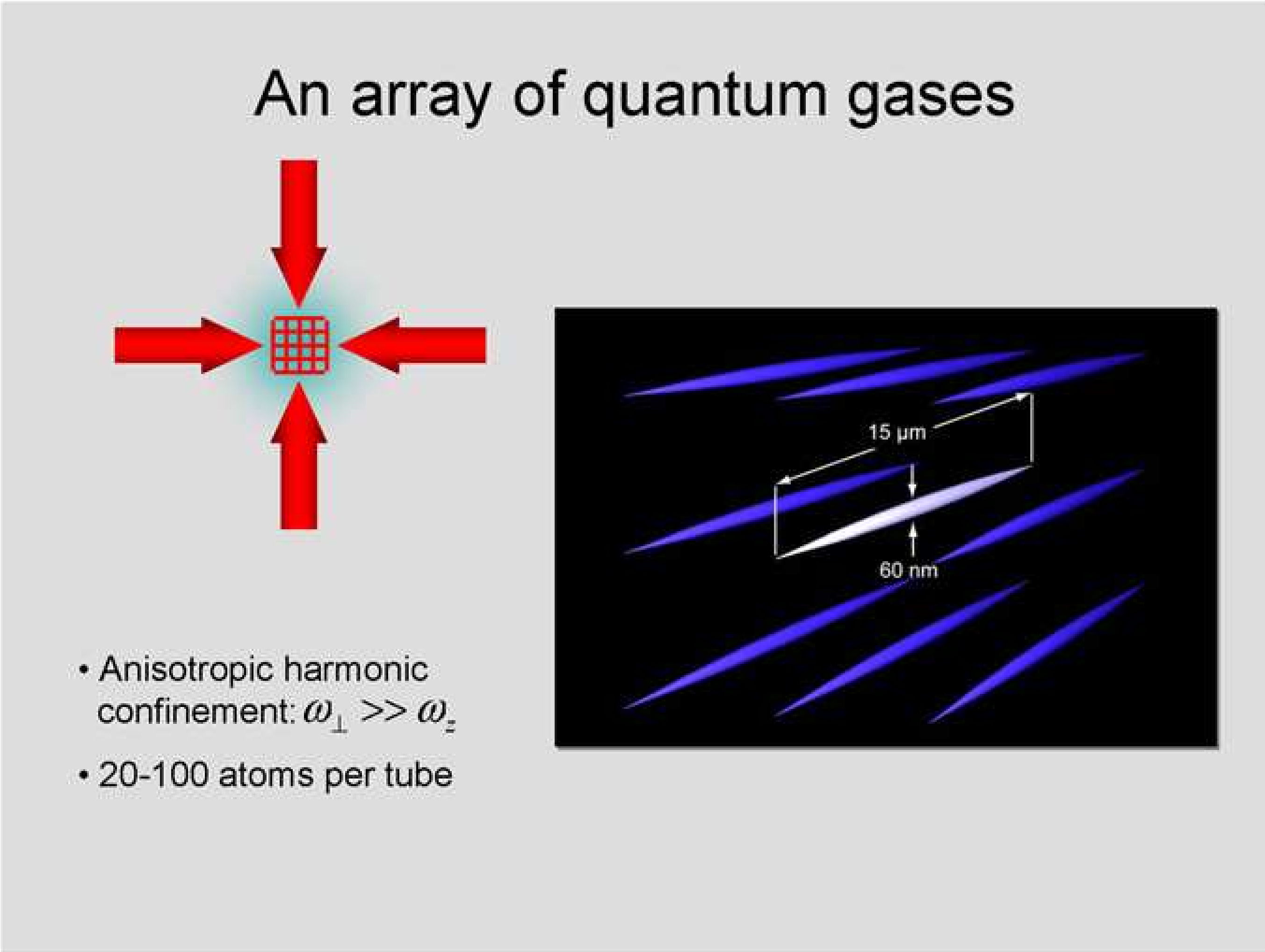
Superfluid to Mott insulator quantum phase transition



superfluid Mott insulator superfluid

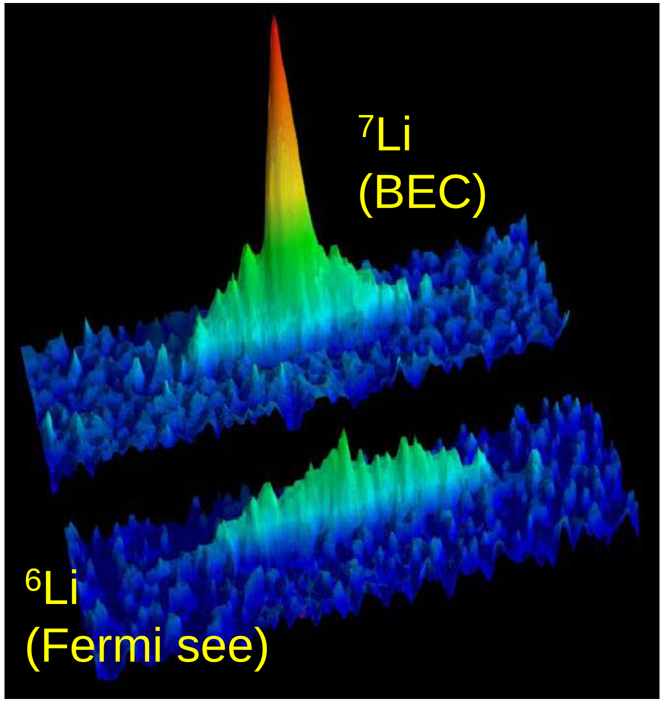
# Physics at low D



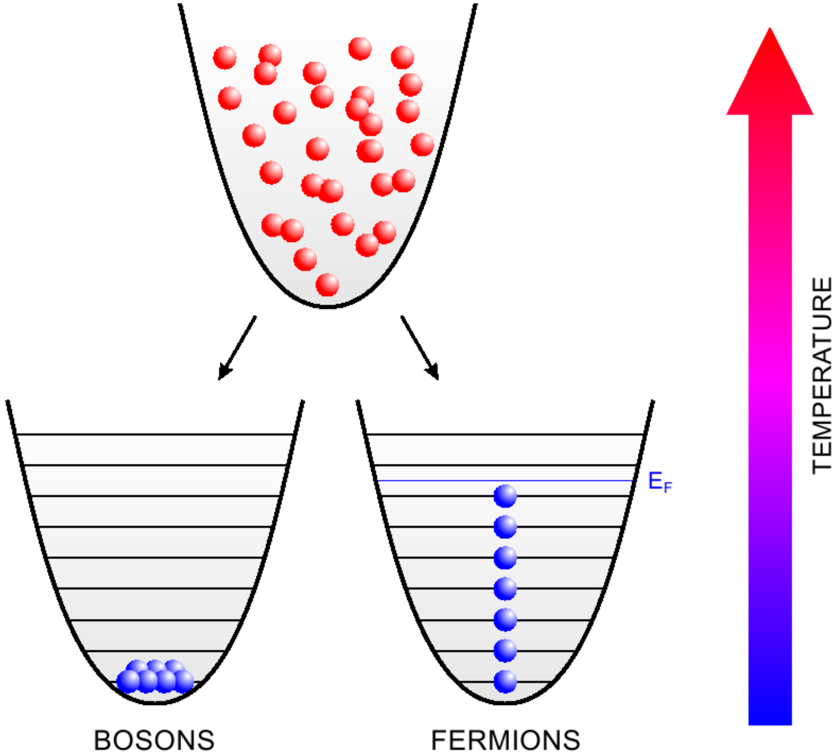
# What about fermions?

A simple argument:

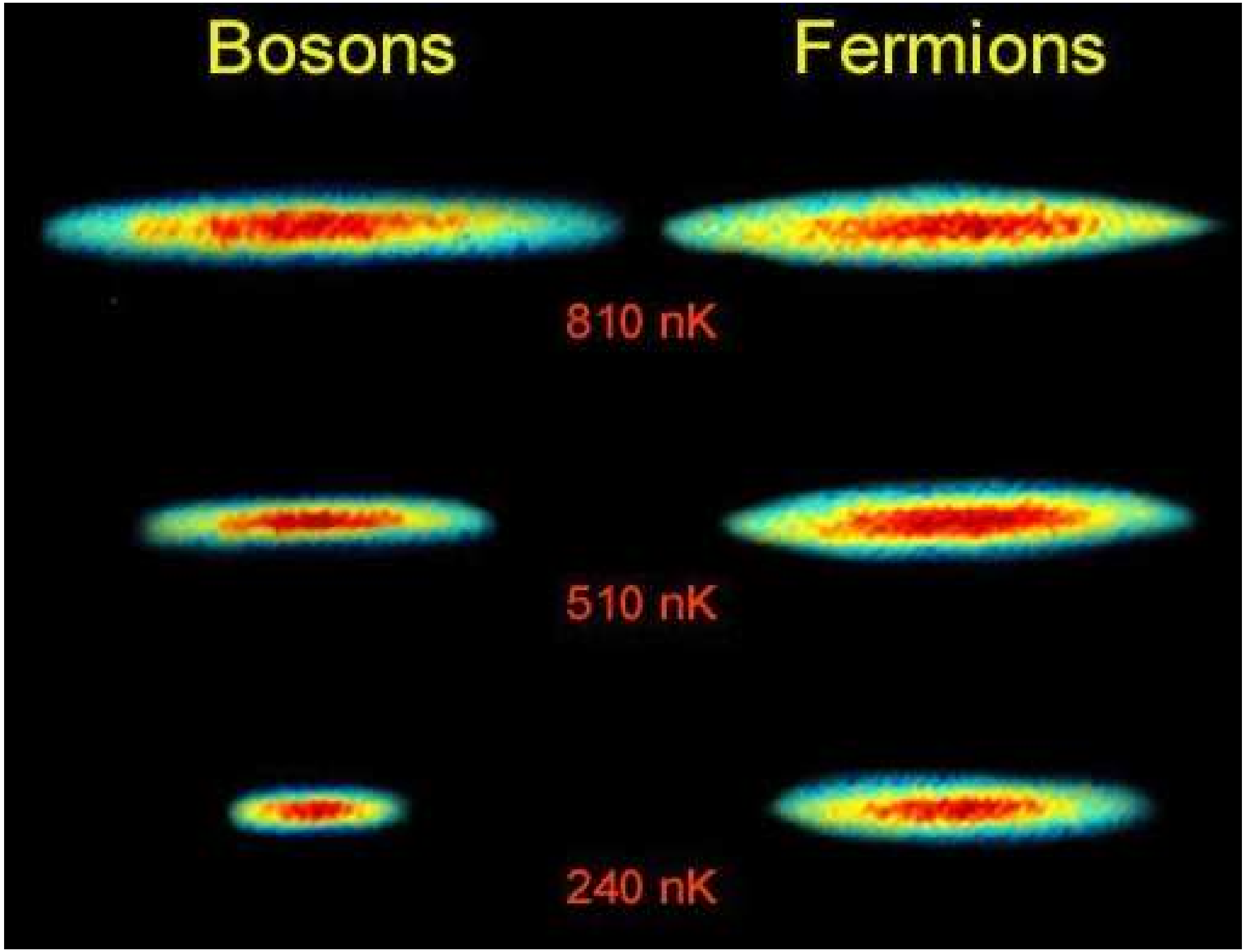
- Condensation is only possible for BOSONS.
- FERMIONS behave differently, due to Pauli.




(Salomon, ENS, 2001)

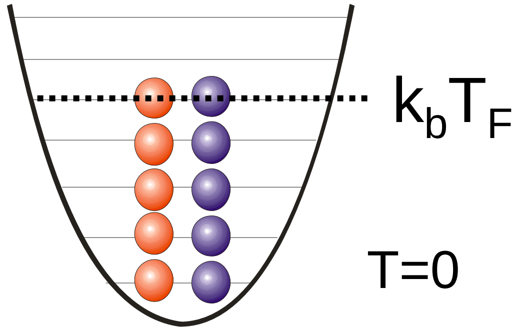


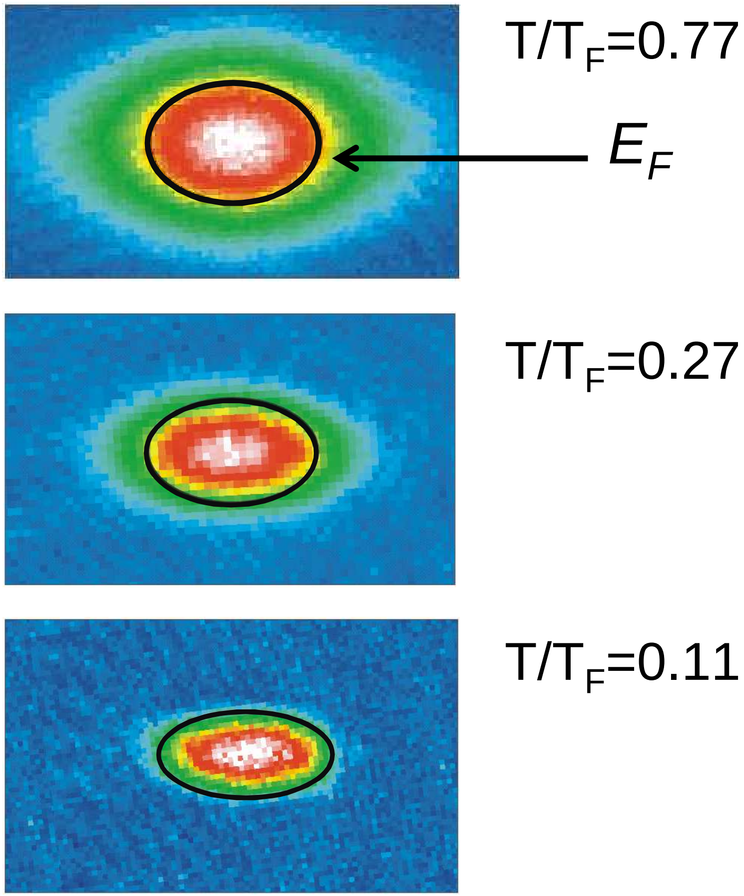
# Observing quantum statistics



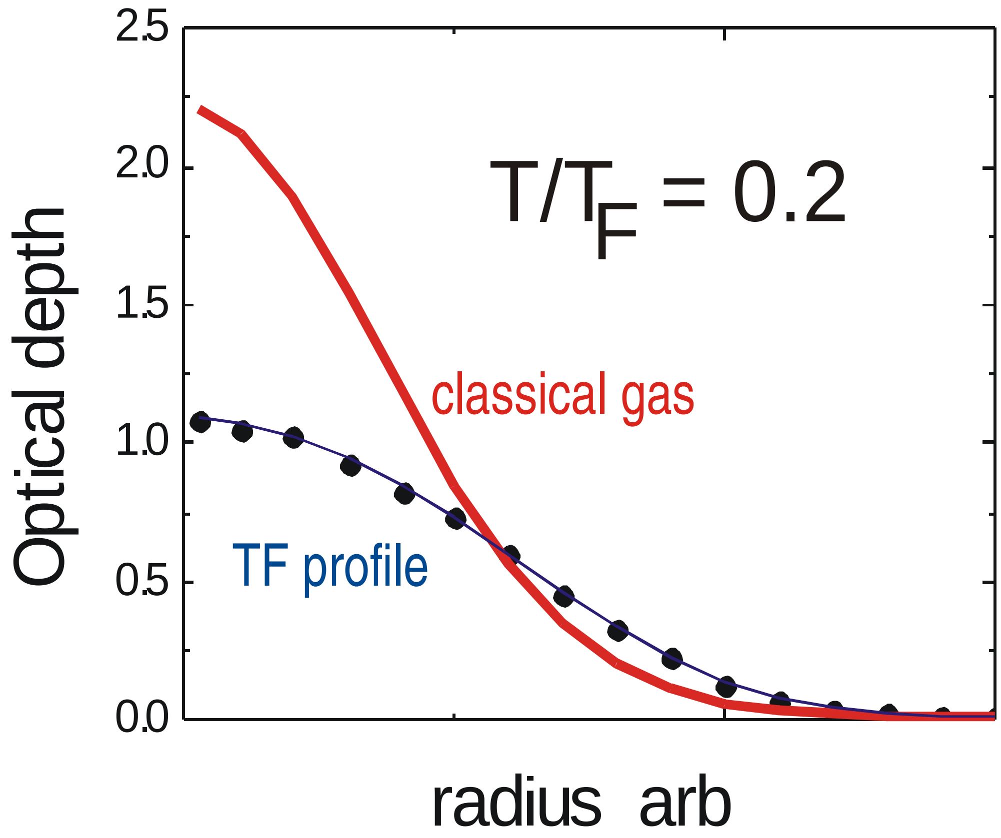

(Rice, 2001)

# Fermi degeneracy




(Regal et al., JILA)



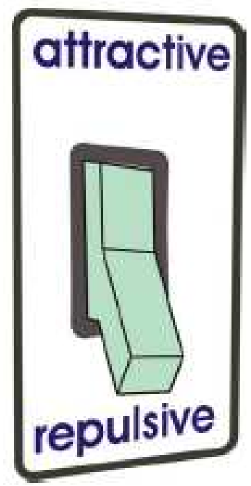
# BUT... Things are even more interesting

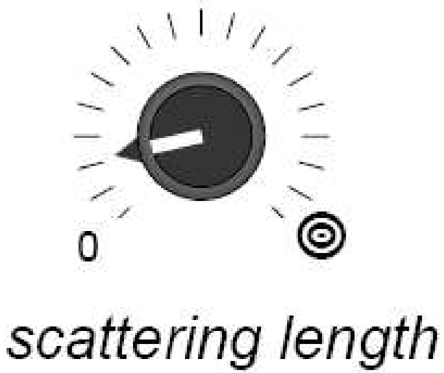
Fermionic atoms in the same spin state do not interact in s-wave (Pauli principle) !!

Mixtures of fermionic atoms in different spin states can interact in s-wave.

Three interesting cases:

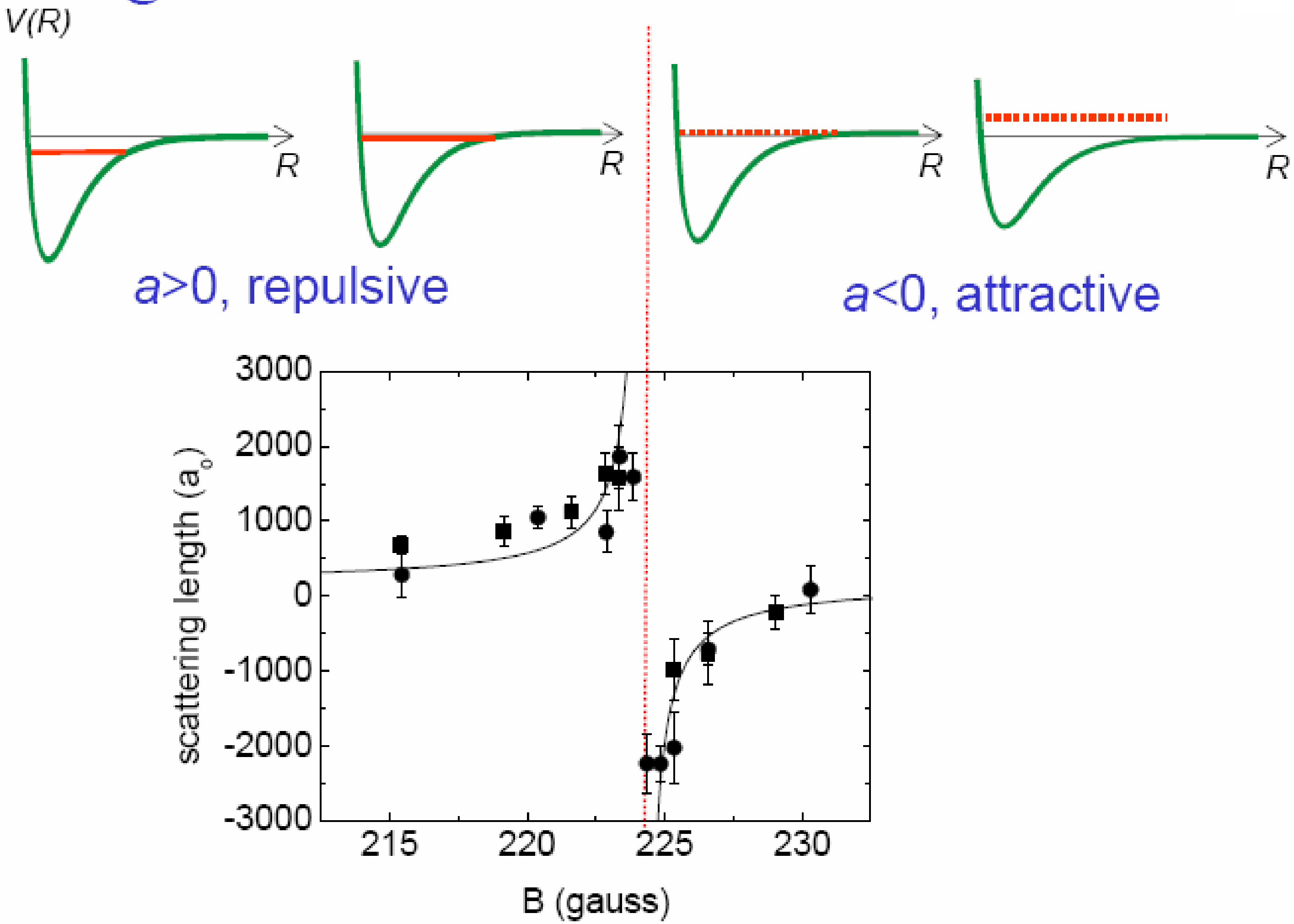
- weak attractive interaction → BCS superfluid phase
- weak repulsive interaction and molecular bound state → BEC of molecules
- infinite s-wave scattering length → unitary regime



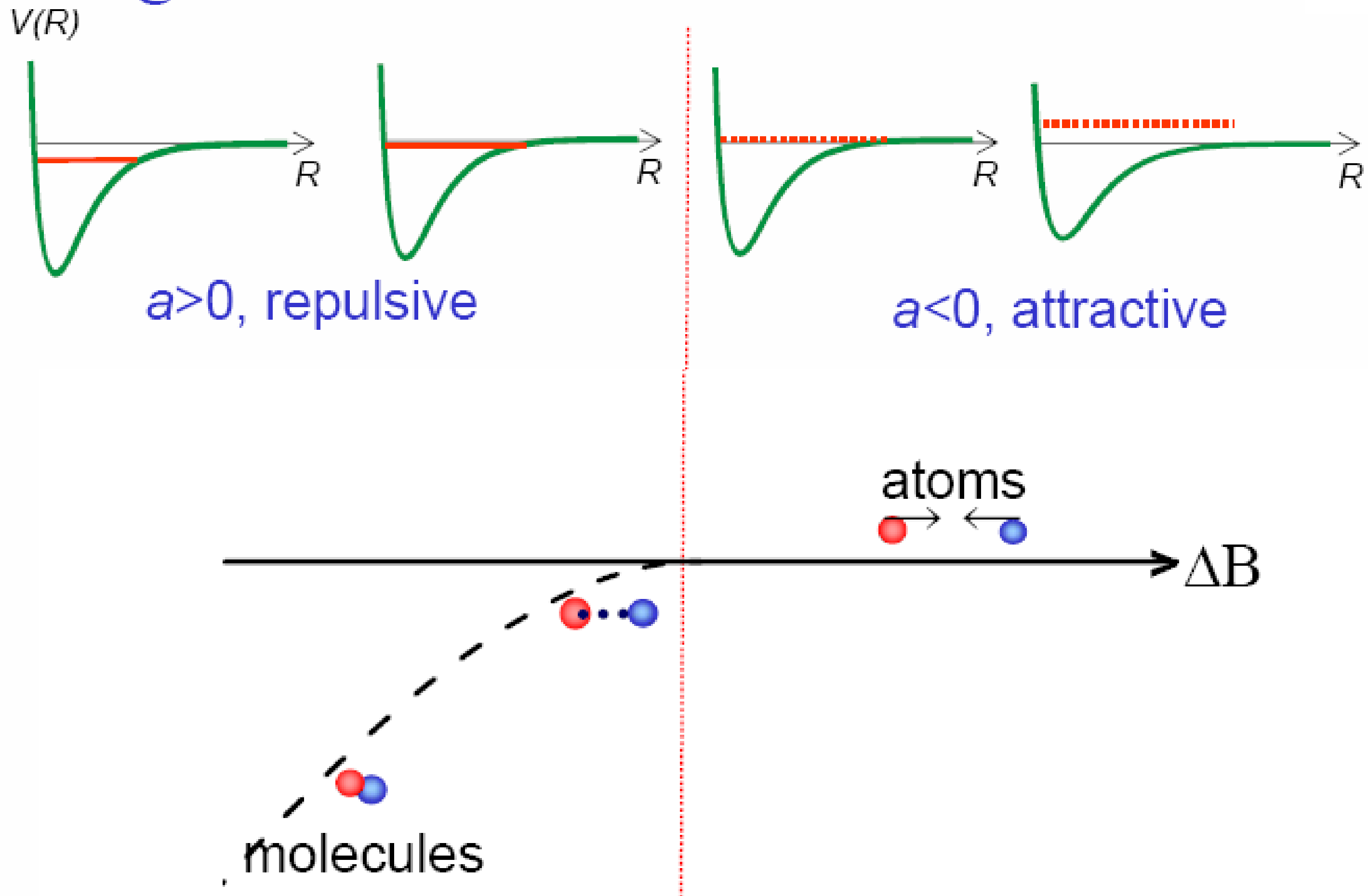
# Magnetic-field Feshbach resonance

(taken from Debbie Jin, JILA)



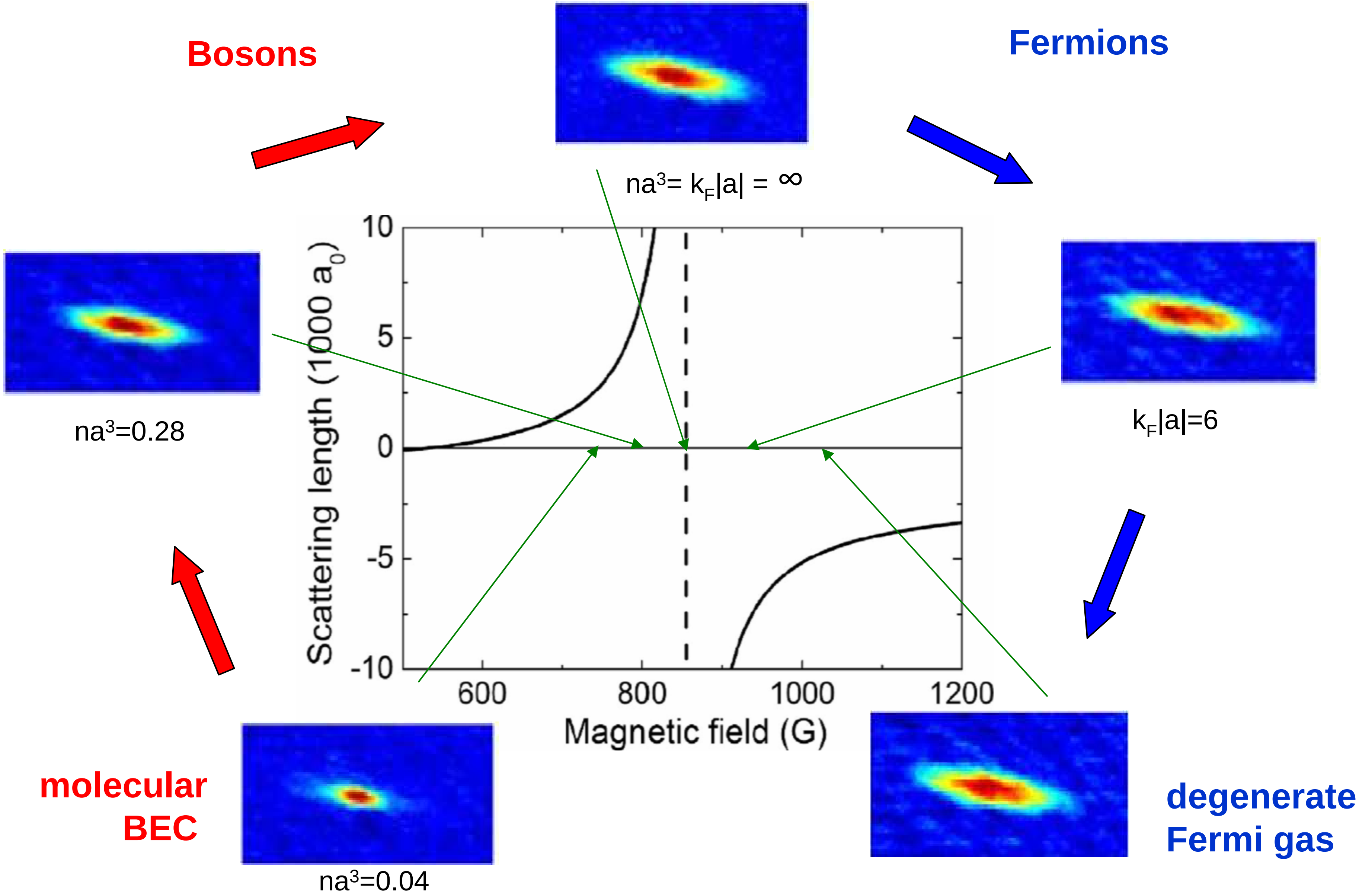
# Magnetic-field Feshbach resonance

(taken from Debbie Jin, JILA)

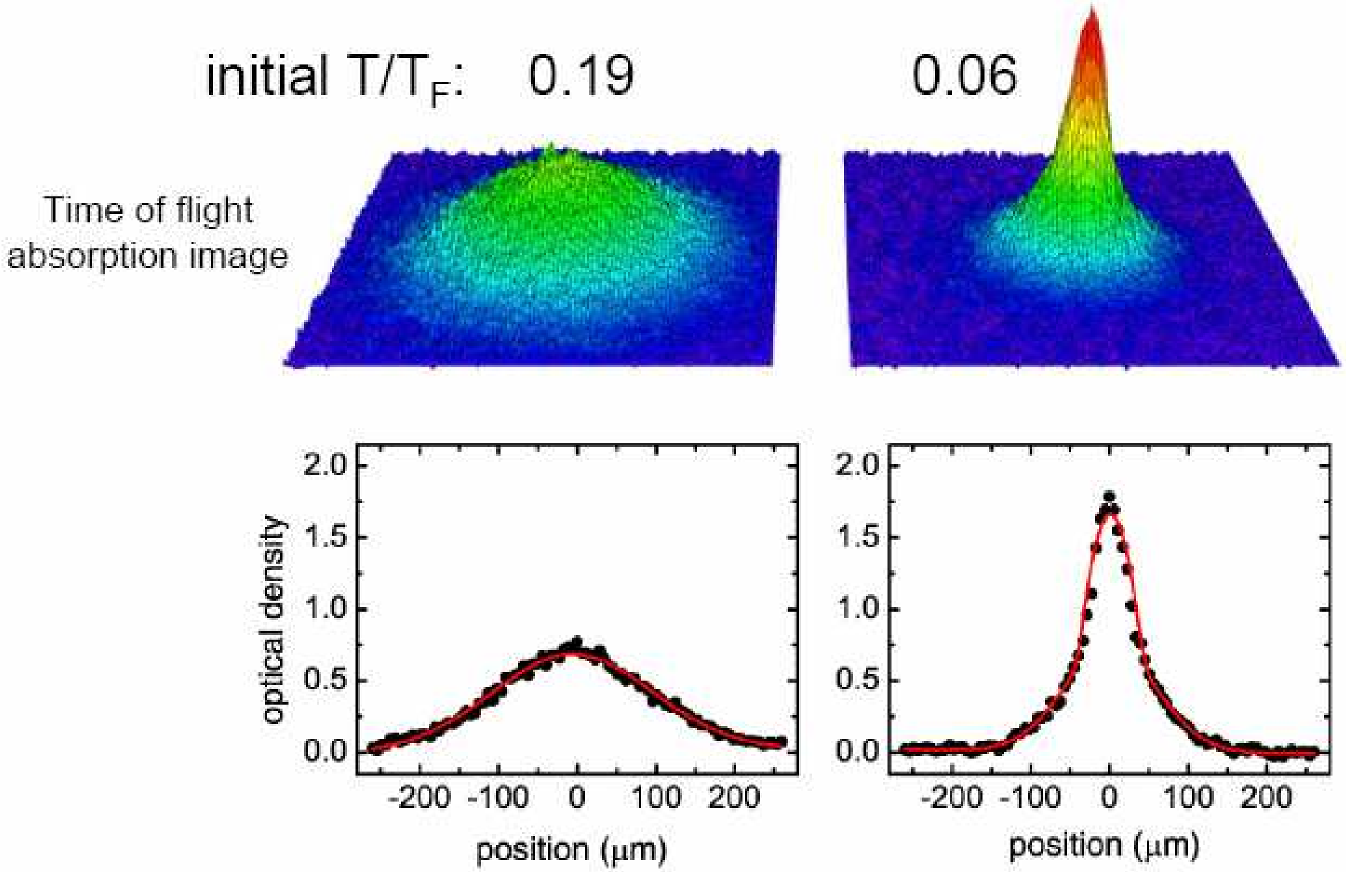


# BCS-BEC crossover

$^6$Li atoms @ Innsbruck



# Molecular Condensate




M. Greiner, C.A. Regal, and D.S. Jin, Nature **426**, 537 (2003).

Experiments with ultracold fermions:

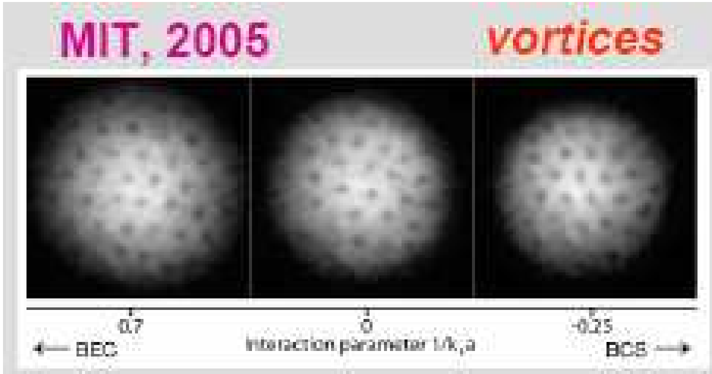
$^{40}K$: JILA (1999)
Florence (2002)
Zurich (2004)
Hamburg (2004)
Toronto (2005)

$^{6}Li$: Rice (2001)
ENS (2001)
Duke (2001)
MIT (2002)
Innsbruck (2003)

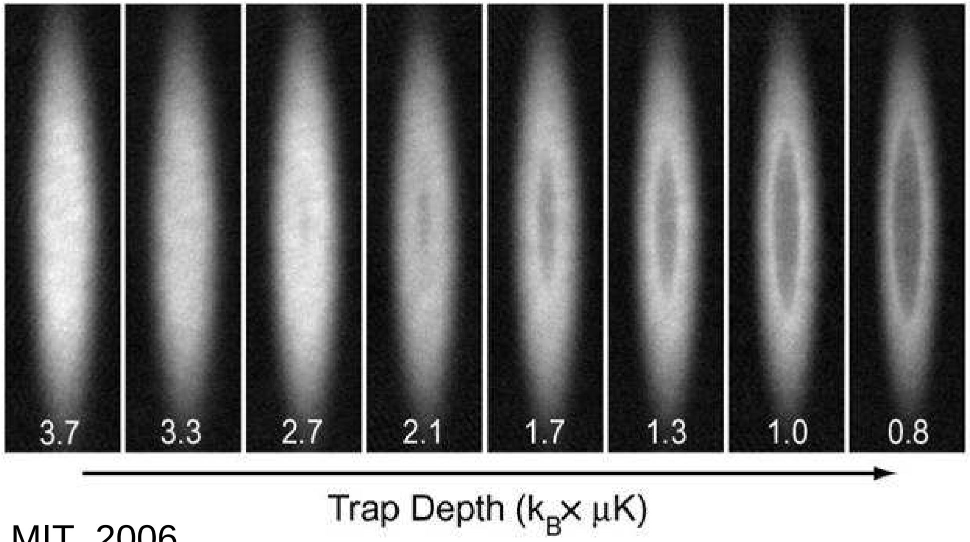
## Are Fermi gases superfluid?

Superfluid hydrodynamics in the expansion (Duke, 2002)
Pair condensation (JILA, MIT, 2004)
Collective modes (Innsbruck, Duke, 2004)
Measuring the BCS pairing gap (Innsbruck, 2004)
Heat capacity (Duke, 2005)
Quantized vortices (MIT, 2005)

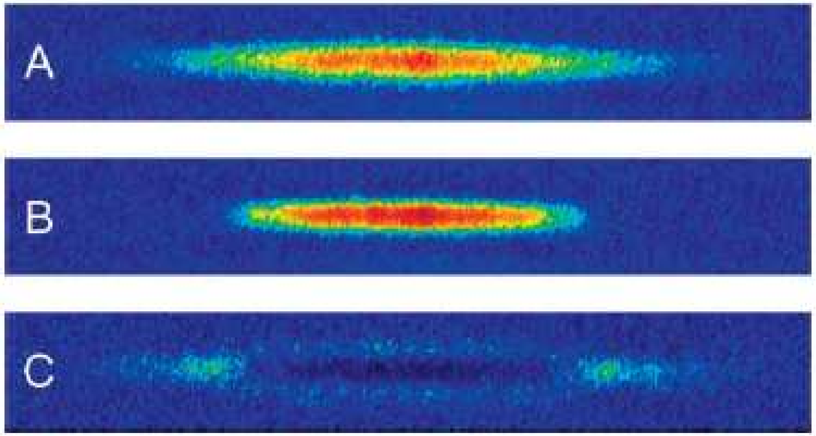
**Yes**



# Hot topic: Fermi superfluids with imbalanced populations



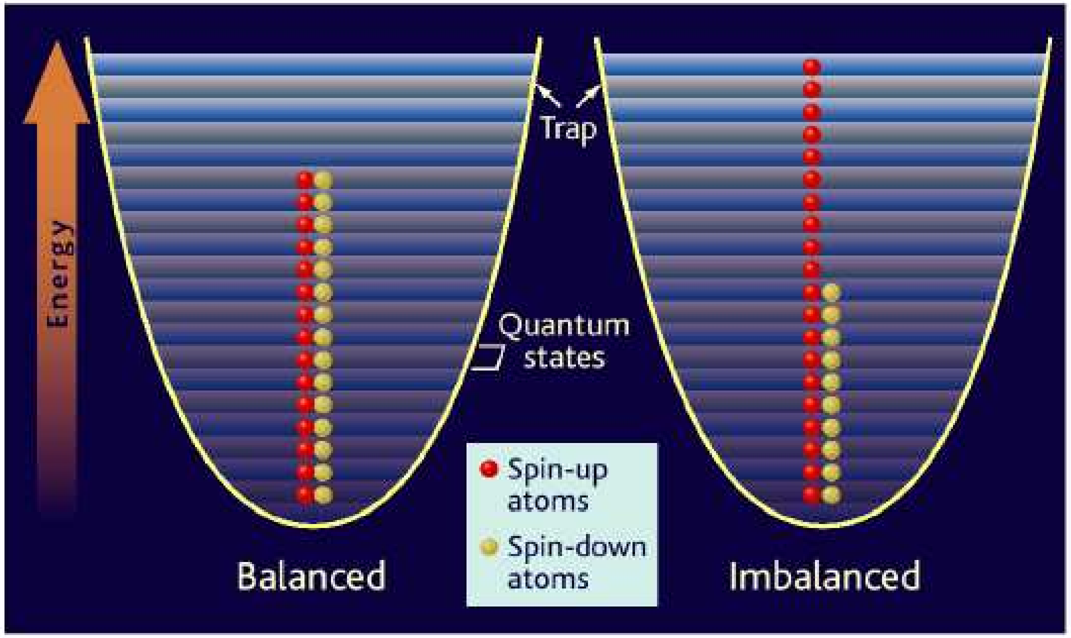

MIT, 2006




Rice, 2006

**A lot of theory papers in the last year.**

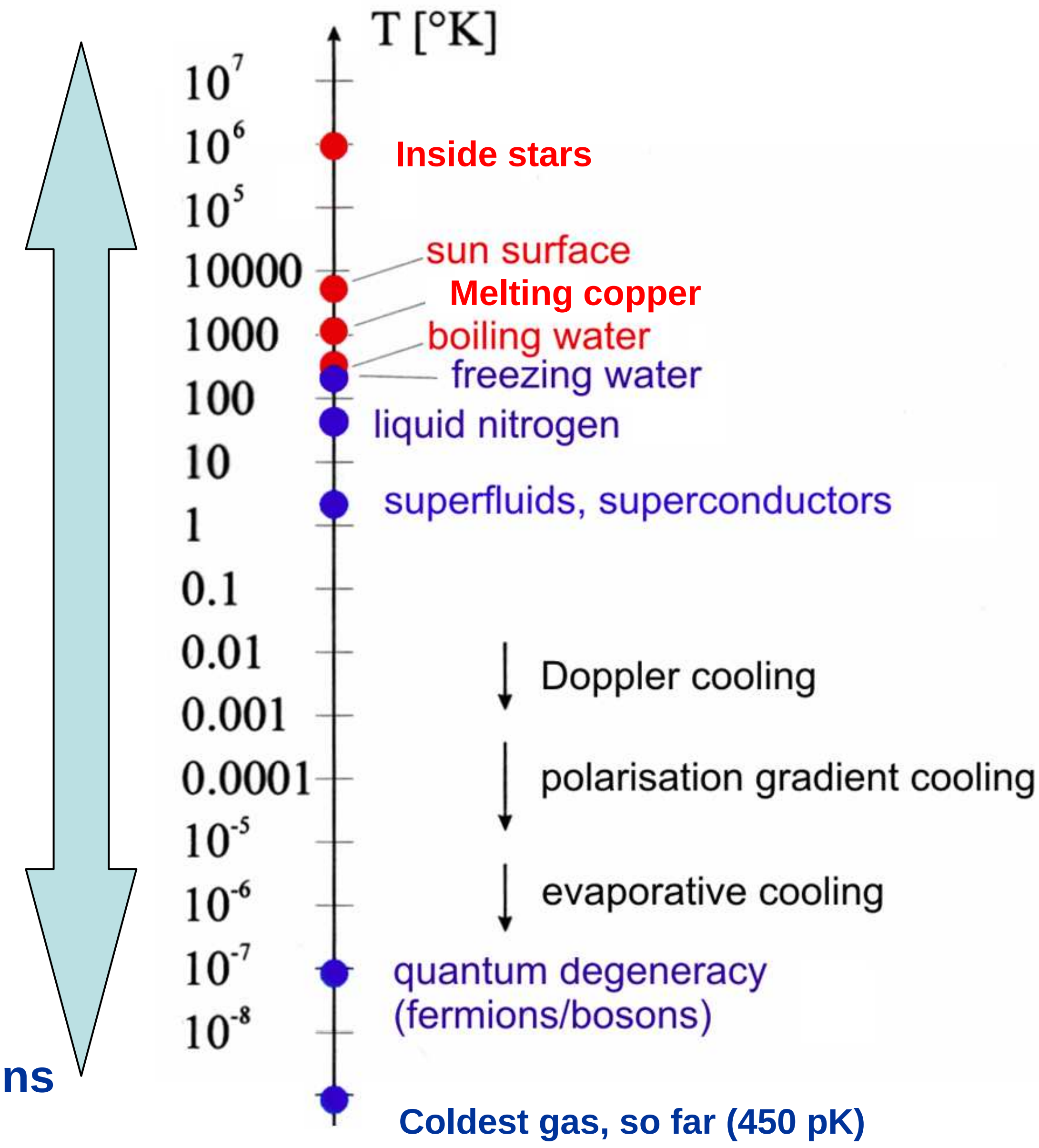
**Interesting physics in common with quark-gluon plasma and color superconductivity. In this case: balance of strange (s) and light (u,d) quarks.  Possible implications in the physics of neutron stars.**



**Color superconductivity**

A lot of physics
in common

**Ultracold superfluid fermions**



# Perspectives

New states:
- Entanglement and Schroedinger cats
- Superfluid-Insulator transition
- Fermi-Bose mixtures
- BEC of dipolar molecules
- Tonks-Girardeau gas
- BCS-BEC crossover
- Fermi gas at unitarity

New settings:
- Low Dimension
- Fast rotation
- Optical lattices
- Ring traps
- Atoms on chips
- Interferometers

New Probes
- RF Spectroscopy
- Noise Correlations

# What I am working on…

**Parametric resonances in BEC**
(collaboration with M. Kraemer, C. Tozzo and M. Mogugno)

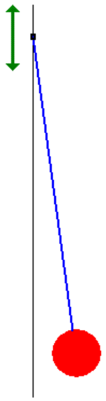
**Solitons in 2D BEC**
(collaboration with S. Tsuchiya and L. Pitaevskii)

**Bogoliubov – de Gennes equations for fermions**
(collaboration with E. Furlan, M. Antezza, S. Stringari and L. Pitaevskii)

just two words about parametric resonances…

# Parametric resonance

Classical example: the **vertically driven pendulum.**
Stationary solutions: $\varphi = 0$ and $\varphi = 180°$. In the undriven case, these solutions are always stable and unstable, respectively. But vertical driving can change stability into instability and vice versa.
The dynamics is governed by the Mathieu equation:



$$\frac{d^2\varphi}{dt^2} + \gamma\frac{d\varphi}{dt} + (\omega_0^2 + a\cos 2\pi f t)\,\varphi = 0$$

$$f_0 \equiv \frac{\omega_0}{2\pi} = \frac{f}{2}n$$

$\varphi(t) = c(t)\exp(\lambda t)$, where $c(t+1/f) = c(t)$.

$\lambda$ Floquet exponent. If is is real and positive, then the oscillator is parametrically unstable.

# Parametric resonance

Very general phenomenon (classical oscillators, nonlinear optics, systems governed by a Non-Linear Schroedinger Equation, Hamiltonian chaotic systems, etc.)

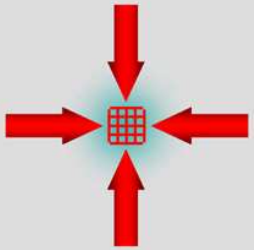
Previously discussed in the context of BEC by several authors (Castin and Dum, Kagan and Maksimov, Kevrekidis et al., Garcia-Ripoll et al., Staliunas, Longhi and De Valcarcel, Salasnich et al., Salmond et al., Haroutyunyan and Nienhuis, Rapti et al).

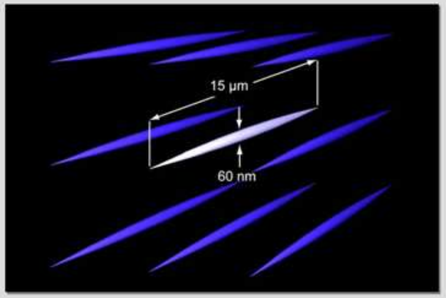
Our contribution:

Parametric amplification of Bogoliubov phonons in BEC in modulated 1D optical lattices

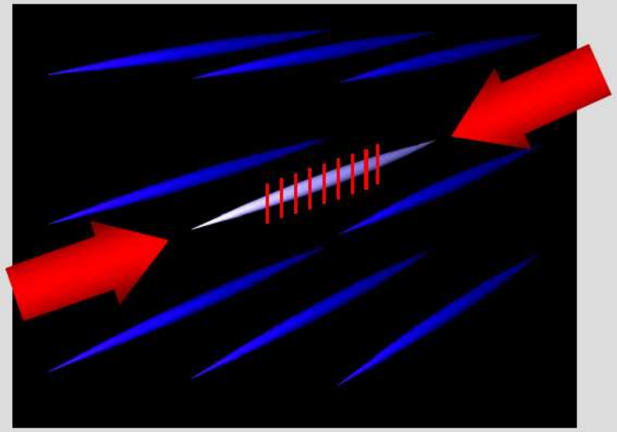
Faraday pattern in toroidal condensates

## An array of quantum gases



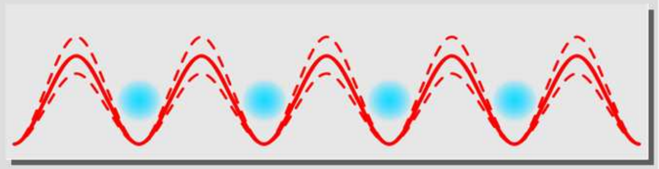



- Anisotropic harmonic confinement: $\omega_\perp >> \omega_z$
- 20-100 atoms per tube



Experiment by Esslinger et al., Zurich

## Bragg Spectroscopy in an Optical Lattice



$$V_{ax}(y,t) = V_{ax,0}(1 + A_{mod}\sin(2\pi\nu_{mod}t))\sin^2(ky)$$

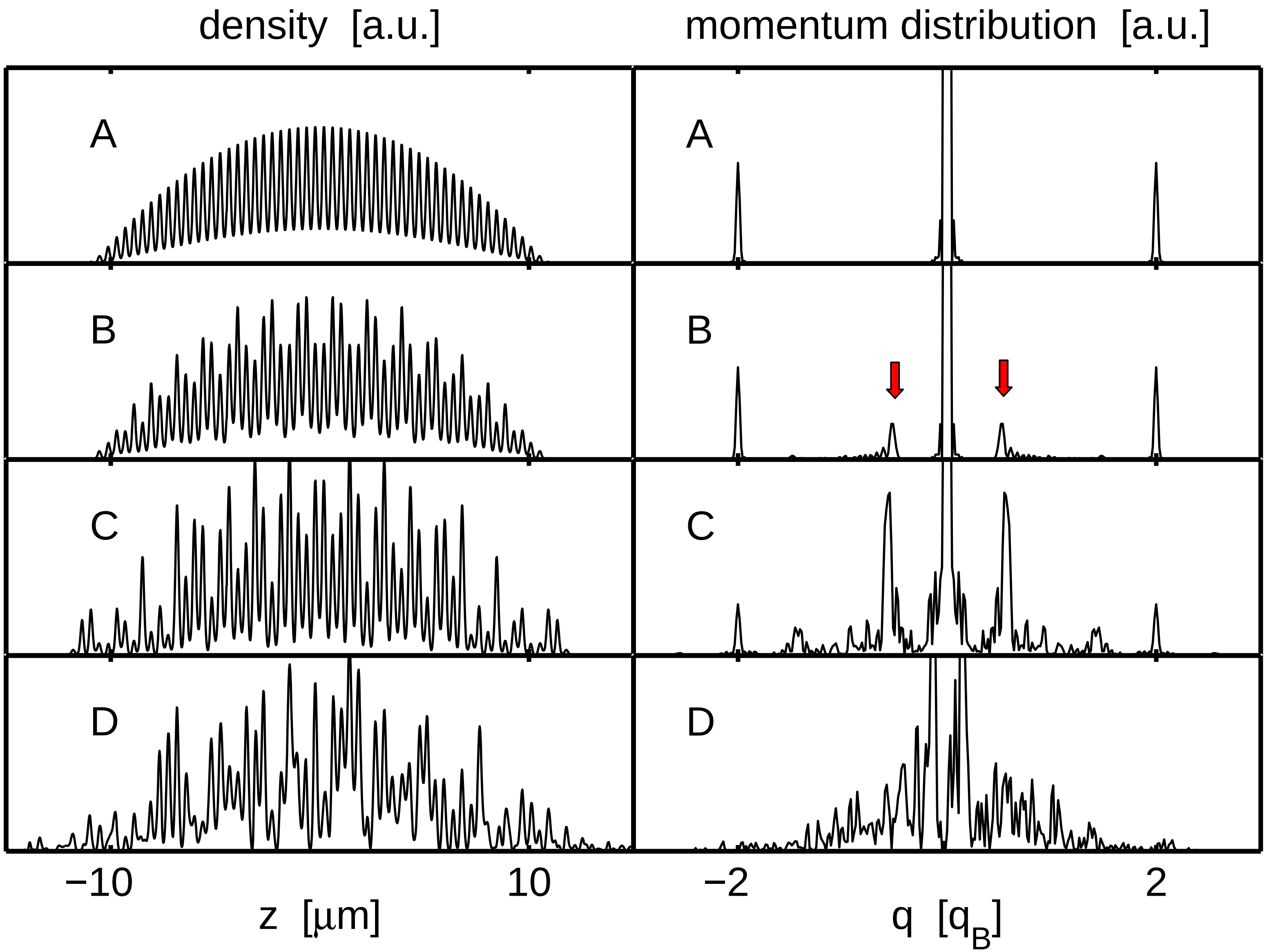
Frequency sidebands at $\pm\nu_{mod}$

Energy transfer: $\Delta E = \hbar(2\pi\nu_{mod})$

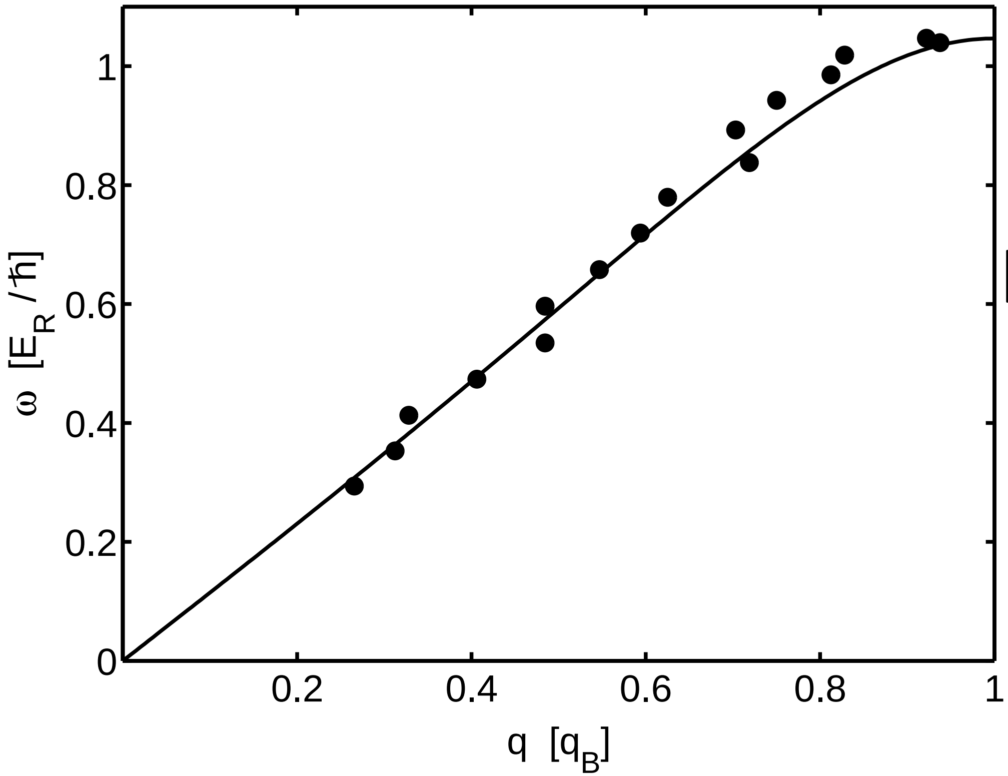
Momentum transfer: $\Delta \boldsymbol{p} = 2\hbar \boldsymbol{k}$ = reciprocal lattice vector

## Gross-Pitaevskii simulations

C. Tozzo, M. Krämer, and F. Dalfovo, PRA **71**, 061602(R) (2005)
and PRA **72**, 023613 (2005)



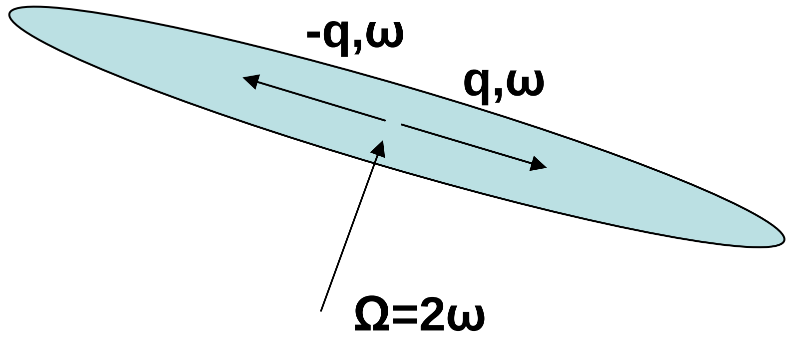
Recipe: for a given Ω find the resonant q, then plot Ω/2 vs. q and compare with the frequency of axial Bogoliubov phonons in the lattice $\omega_q$




$$\omega_q = \Omega / 2$$

**resonance condition for parametric amplification**

the main mechanism of parametric amplifications is a coupling between pairs of counter-propagating Bogoliubov excitations. This coupling is caused by the modulation of the background in which the excitations live.




**Important remark:** in order to be parametrically amplified, the “resonant” mode must be present at t=0 (seed excitation). The parametric amplification is sensitive to the initial quantum and/or thermal fluctuations.

## Remarks on thermal and quantum seed

Two limiting cases:

$k_B T \gg \hbar\omega_{jk}$

Thermal fluctuations.
Possible measurement of T, even when the thermal cloud is not visible (selective amplification of thermally excited modes).

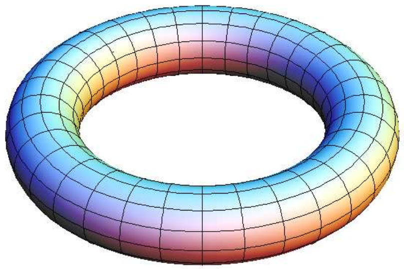
$k_B T \ll \hbar\omega_{jk}$

Amplification of quantum fluctuations.
Analogous to parametric down-conversion in quantum optics.
Source of entangled counter-propagating quasiparticles.

example: $|\Psi\rangle \propto |j, k\rangle |j', -k\rangle + |j, -k\rangle |j', k\rangle$

Dynamic Casimir effect: the environment in which quasiparticles live is periodically modulated in time and this modulation transforms virtual quasiparticles into real quasiparticles (as photons in oscillating cavities).

# Toroidal condensates



Procedure:

i) The condensate is initially prepared in a torus.
ii) The transverse harmonic potential is periodically modulated in time.
iii) Both the trap and the modulation are switched off and the condensate expands.

We solve numerically the time dependent GP equation

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2M}\nabla^2 + V_{\rm ext} + g|\psi|^2\right]\psi$$

We use the Wigner representation for fluctuations at equilibrium at step (i).

$$\psi = \psi_0 + \sum_i [c_i u_i + c_i^* v_i^*]$$

# Faraday pattern

**no modulation** | **modulation**

**in trap**

**after expansion**

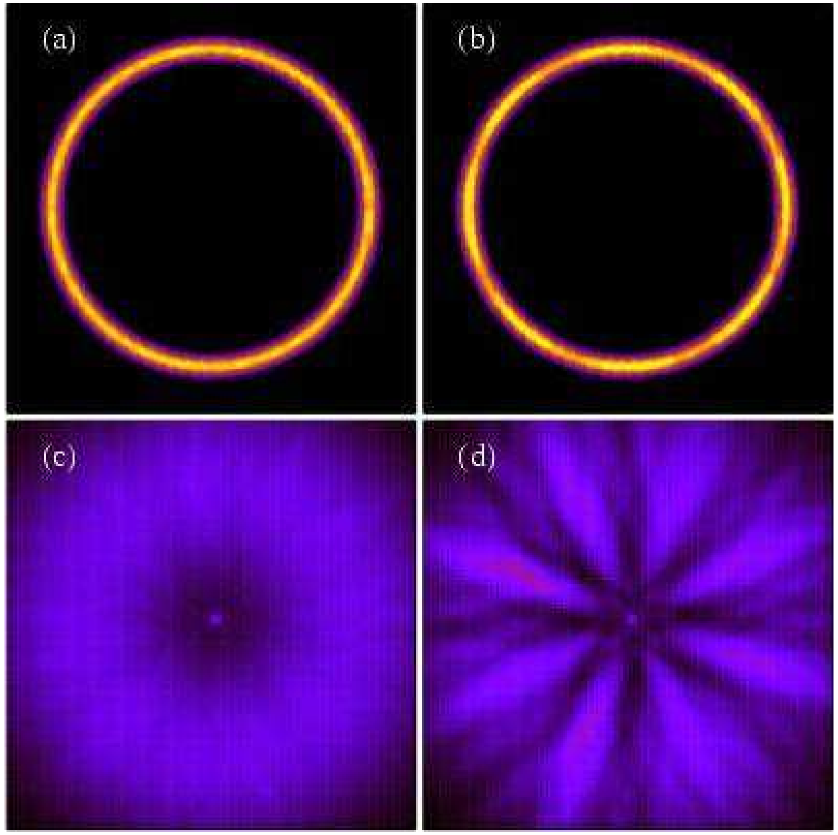
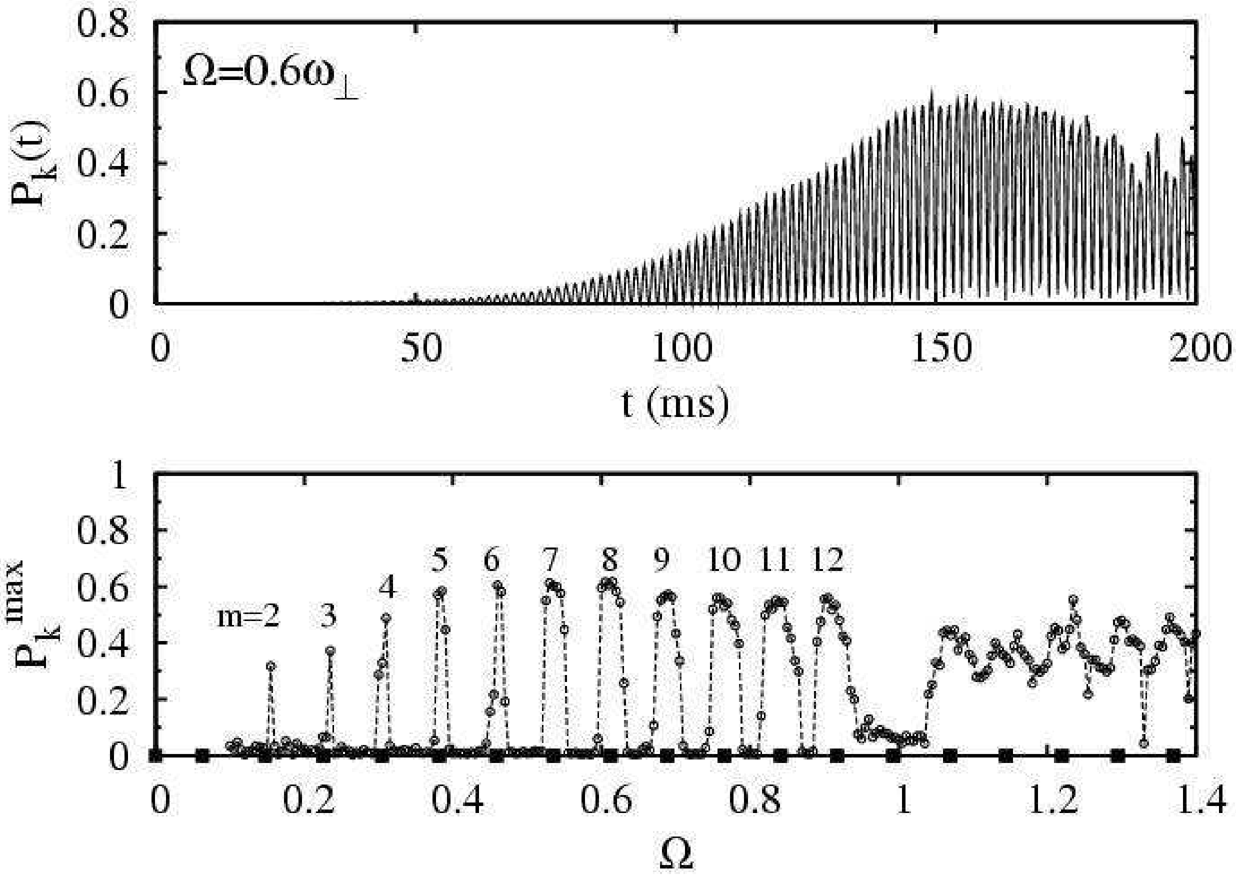



FIG. 1: Top: a) density distribution of a condensate at equilibrium in the toroidal trap; b) same condensate after a transverse modulation of frequency $\Omega = 0.6\omega_{\perp}$, duration $t_{mod} = 130.15$ ms and amplitude $A = 0.1$. Bottom: c) the condensate in (a) freely expanded for a time $t_{exp} = 7$ ms; d) same expansion but for the condensate in (b). The quantity in all plots is the column density, i.e., the density integrated along the $z$-axis perpendicular to the torus.

M. Modugno, C. Tozzo, F. Dalfovo,
Phys. Rev. A **74**, 061601(R) (2006)

# Pattern visibility (in trap)




$$P_k = 2\pi \int dr_\perp \, r_\perp |\tilde{\psi}(k, r_\perp)|^2$$

FIG. 2: Top: Amplitude of the $\pm k$ components of the Fourier transform of the order parameter along the torus, where the wavevector $k$ satisfies the resonance condition $\omega(k) = \Omega/2$. The amplitude is plotted as a function of the modulation time for $\Omega = 0.6\omega_\perp$ and $A = 0.1$. Bottom: maximum value of $P_k$ in the interval $0 < t_{mod} < 200$ ms obtained in simulations at different frequency $\Omega$ (small circles with dashed line). Solid squares on the horizontal axis correspond to twice the eigenfrequencies $\omega(k_m)$ of the Bogoliubov equations, with $k = 2\pi m/L$ and $m$ integer.

PRL **98**, 095301 (2007) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2007

# Observation of Faraday Waves in a Bose-Einstein Condensate

P. Engels* and C. Atherton
*Washington State University, Department of Physics and Astronomy, Pullman, Washington 99164, USA*

M. A. Hoefer
*National Institute of Standards and Technology, Boulder, Colorado 80305, USA*

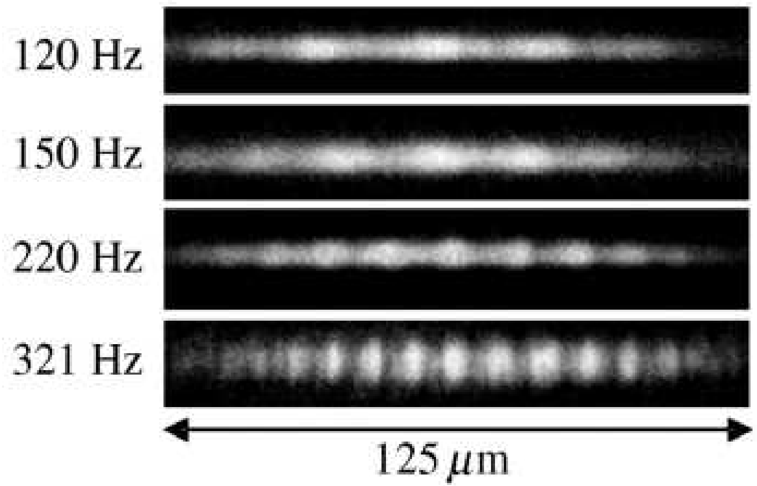



FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

# Yet a lot of activities with ultracold atoms worldwide…

## part of them in Trento, at


BOSE EINSTEIN CONDENSATION

INFM

UNITN: **Stringari, Pitaevskii, Dalfovo, Giorgini.**
INFM researchers: **Smerzi, Carusotto, Menotti, Calarco, Recati.**
Postdocs: **Lobo, De Chiara, Tsuchiya, Antezza, Pezzè.**
Students: **Pilati, Bariani, Bausmerth, Furlan, Piazza**.
Former members: Minniti, Guilleumas, Vichi, Brunello, Falco, Zambelli, Viverit, Weidong Li, Giorgetti, Poulsen, Pedri, Kraemer, Ianeselli, Trefzger, Jackson, Cozzini, Tozzo, Astrakharchik, Wouters, Idziaszek, Orso.

http://bec.science.unitn.it