Chapter 1

Theoretical motivation

In the first section of this chapter an overview of the most important theoretical and phenomenological aspects related to the Standard Model (SM) are given with the aim of explaining the spontaneous symmetry breaking. The Higgs mechanism and *the defects of the Standard Model are also examined.* The second section includes an introduction to the supersymmetry and explains its motivations by the gauge hierarchy problem. Section three is devoted to the supersymmetric extension of the SM (the Minimal Supersymmetric Standard Model).

1.1 The Standard Model

The Standard Model is the most successful yet developed model to explain the physics of the fundamental particles and their interactions. The aim of this model is to reveal the relationship between matter and forces in terms of particles and underlying symmetries [1].

The current Standard Model is built on the successes of three previous theories. The first is Quantum Electrodynamics (QED), the theory which describes the electromagnetic (EM) interaction in terms of underlying U(1) symmetry. The success of QED has led to its use as a model for the theories that have followed. The next theory in the Standard Model progression is the electroweak interaction theory, guided by $SU(2) \times U(1)$ symmetry, first proposed by Glashow, Weinberg and Salam in the late 1960's and referred to as the Glashow-Weinberg-Salam model [2] [3] [4]. This theory incorporates the successful QED model and provides a description of the weak force in terms of the exchange of massive vector bosons [5]. The fact that these vector boson masses were successfully predicted from the theory before the particles were discovered is considered one of the major triumphs of the model. In the Standard Model, the masses of these bosons are generated by the Higgs mechanism. The third theory which makes up the Standard Model is Quantum Chromodynamics (QCD), guided by SU(3) symmetry. This is the quantum field theory describing the interactions of quarks through the strong "colour" field [5].

The fact that the Standard Model is made up from three gauge theories means that it is built up from three gauge symmetries. These symmetries correspond to the three relevant fundamental forces in particle physics, namely the strong, the weak, and the electromagnetic force. The Standard Model gauge group is the product group $SU(3) \times SU(2) \times U(1)$, with *SU(3), SU(2)* and *U(1)* component groups being associated with the colour, weak and hypercharge symmetries. The corresponding gauge bosons are the massless gluons of QCD, the massive W^+, W^- and Z of the weak interaction, and the massless photon of electromagnetism.

The spin $\pm$ 1/2, or fermion content of the model is divided into two parts, both of which appear to come in three families. The two parts which make up the fermion content of the model are known as the quarks and the leptons. Leptons are fermions that interact only under electroweak interactions, while the quarks are fermions that also interact strongly [6]. The observed fermion content of the model (leptons and quarks) is listed below in Table 1.1.

Table 1.1: The fermions of the Standard Model.

	MASS (GeV/c^2)	Q	Family	Hypercharge
QUARKS				
d	0.008	-1/3	1	1/3
u	0.004	+2/3	1	1/3
s	0.15	-1/3	2	1/3
c	1.2	+2/3	2	1/3
b	4.7	-1/3	3	1/3
t	175	+2/3	3	1/3
LEPTONS				
e	0.0005	-1	1	0
ν_e	< 0.000015, CL 90%	0	1	0
μ	0.105	-1	2	0
ν_μ	< 0.00017, CL 90%	0	2	0
τ	1.8	-1	3	0
ν_τ	< 0.0182, CL 95%	0	3	0

The predictions of the Standard Model have been tested and verified with unprecedented precision by experiments at the **L**arge **E**lectron **P**ositron Collider (LEP) [8], SLAC and many other laboratories. However, there are several incomplete aspects of the Standard Model which lack yet understanding, such as: the unverified origin of the mass, the interactions are not unified, an explanation for why there are only three families of particles does not exist; there are many (19) free parameters in the model [9]; gravitation is not included in the model; the mass scale is not "natural" (ie. radiative corrections to the Higgs mass suffer from large divergences requiring two quantities of the order $\sim 10^{32}$ which almost cancel to produce a result of 10^4, this fortuitous cancellation is considered "unnatural" [9] [10].); and it provides no dark matter candidate. These incomplete aspects of the model imply that the Standard Model can be considered only as an effective theory. That is, the Standard Model is a low energy approximation of a more general underlying theory.

One of the most important unanswered questions of the model is the origin of the masses. It is widely believed that this question can be answered by invoking the Higgs mechanism, which requires the onset of spontaneous symmetry breaking of the local gauge symmetry and provides a mass generation mechanism for both of the SU(2) weak gauge bosons and the observed massive quarks and leptons. It also predicts the existence of a massive scalar particle known as the Higgs boson. The best experimental verification of the existence of the Higgs Mechanism would be the discovery of its physically detectable manifestation, i.e. the existence of the Higgs boson.

1.1.1 Spontaneous Symmetry Breaking

Spontaneous symmetry breaking occurs when the Lagrangian of a system retains its invariance under the symmetry group, but the vacuum state (ground state of the system) does not exhibit the same invariance. The vacuum state breaks the symmetry [6]. To explain how this is achieved, we consider the case of a single complex scalar field, $\phi(x)$, and the *U(1)* invariant Lagrangian under phase rotations given by equation 1.1,

$$L(\phi, \phi^\dagger, \partial_\mu \phi, \partial_\mu \phi^\dagger) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi^\dagger \phi) \tag{1.1}$$

with $V(\phi^\dagger\phi) = \mu^2|\phi|^2 + \Lambda|\phi|^4$. For this system, the Euler-Lagrange equation implies that the vacuum state must satisfy

$$\phi_0(\mu^2 + 2\Lambda|\phi_0|^2) = 0. \tag{1.2}$$

This condition offers two possibilities: either both μ^2 ($mass^2$) and Λ (coupling strength) are positive, or μ^2 is negative while Λ remains positive (Λ cannot be negative as the potential must has a lower bound). The former case implies that there is only one solution, that of conventional vacuum with $\phi_0 = 0$, while the latter case permits a second solution:

$$|\phi_0^2| = \frac{-\mu^2}{2\Lambda} \equiv \frac{v^2}{2} \tag{1.3}$$

$$\phi_0 \equiv \langle\phi\rangle \frac{1}{\sqrt{2}} v e^{i\zeta}. \tag{1.4}$$

The second solution implies that the true vacuum corresponds to a non-zero vacuum expectation value (VEV) for ϕ. The spontaneous symmetry breaking potential is the famous "Mexican Hat" potential, with an infinite number of VEV degenetared states of minimum energy ($\langle\phi\rangle$), laying in a circle of $\frac{v}{\sqrt{2}}$ radius and centred at the origin. Figure 1.1 shows a 2-D slice of the potential $V(\phi^\dagger\phi)$ for both solutions to the spontaneous symmetry breaking condition.

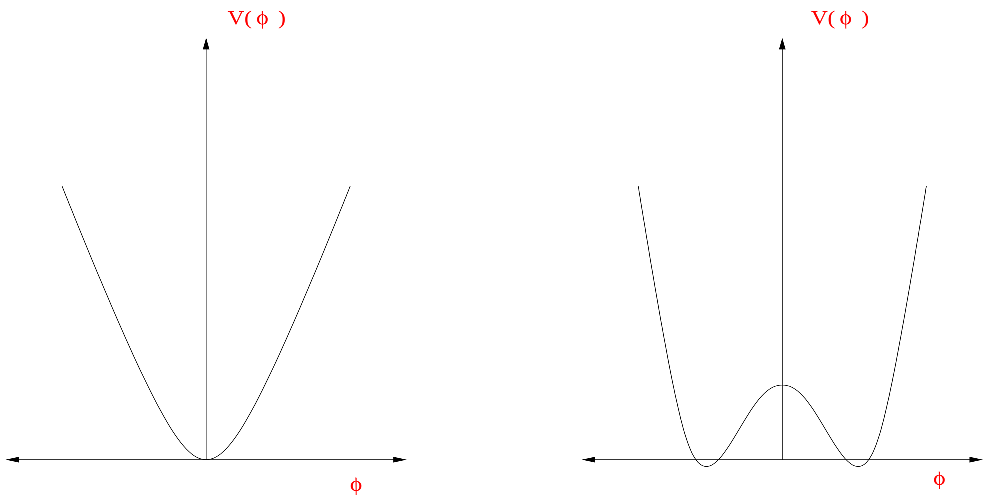

Figure 1.1: Two version of the simple U(1) invariant Higgs potentials: (a) The conventional vacuum Higgs potential with $\langle\phi\rangle = 0$, (b) a 2-d slice of the spontaneous symmetry breaking Higgs potential with $\langle\phi\rangle \neq 0$.

There is, however, a problem with the second solution of equation 1.1. The problem is that the scalar field has an unphysical negative solution for the mass. However, this situation can be remedied by means of a field redefinition of ϕ, where ϕ is expanded around its VEV. The form of this expansion is

$$\phi(x) = \frac{1}{\sqrt{2}}(\rho(x) + v)e^{i[g+\frac{\xi}{v}]} \tag{1.5}$$

with $\rho(x)$ and $\xi(x)$ being real fields. In this expansion, the introduction of $\rho(x)$ takes into account oscillations around the Mexican Hat minimum at $x = v/\sqrt{2}$, while $\xi(x)$ parametrises the motion around the minimum (ie. the minimum is really a flat circle). The effect on the Lagrangian of this field redefiniton is to cause an alteration of its form:

$$L = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{1}{2}\frac{(\rho+v)^2}{v^2}\partial_\mu\xi\partial^\mu\xi + \mu^2\rho^2 - \Lambda v\rho^3 - \frac{\Lambda}{4}\rho^4 - \frac{\mu^2 v^2}{4}. \tag{1.6}$$

The motivation for the field redefinition becomes now apparent. Equation 1.6 is the Lagrangian for two real scalar fields, one of which ($\rho(x)$) has a positive mass while the other ($\xi(x)$) is massless. Both are phisically acceptable fields. Further, this Lagrangian does not explicitly exhibit the original invarance under phase rotations (due to the expansion of ϕ around its VEV), nevertheless, by its pure construction, the Lagrangian still possesses the underlying internal symmetry of the gauge group. Because of this, the symmetry has been spontaneously broken, and is resulting in the appearance of a massles field (Nambu-Goldtone boson) which is free to move along the minima of the "Mexican Hat" potential.

1.1.2 The Higgs Mechanism

The exact mechanism by which the masses of the Standard Model vector bosons and fermions are generated remains unverified [10]. However, it is widely believed that the Higgs Mechanism is the key to explain this mass generation.

The Higgs mechanism is the application of the spontaneous symmetry breaking to a local gauge group. Again one considers the complex scalar field, but this time requires it to be invariant under local gauge transformation, as well as having the "Mexican Hat" potential. The Lagrangian of Equation 1.1 becomes

$$L = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger D^\mu\phi - \mu^2|\phi|^2 - \Lambda|\phi|^4, \tag{1.7}$$

where $F_{\mu\nu}$ is the field strength and D_μ the covariant derivative, D_μ given by

$$D_\mu \phi_i(x) = [\partial_\mu + igQ_i A_\mu(x)]\phi_i(x). \tag{1.8}$$

As before, we can develop ϕ around its vacuum expectation value in order to recover the physical scalar fields, but the expansion is a little more complicated due to the local nature of the required gauge invariance. In this case, the second term of the Lagragian written in Equation 1.7 becomes:

$$(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho + \frac{1}{2}\frac{(\rho+v)^2}{v^2}(\partial_\mu \xi)^\dagger \partial^\mu \xi \tag{1.9}$$

$$-g\frac{(\rho+v)^2}{v}A^\mu \partial_\mu \xi + \frac{1}{2}g^2(\rho+v)^2 A^\mu A_\mu.$$

Again the field redefinition generates masses, but this time for a vector gauge field. An additional term, which does not correspond to a standard interaction, is also generated. However, this can be solved by means of a redefinition of the gauge field, allowing the introduction of a massive field $B_\mu(x)$ defined by

$$B_\mu(x) = A_\mu(x) + \frac{1}{gv}\partial_\mu \xi(x) \tag{1.10}$$

which restores the Lagrangian to the form:

$$L = \frac{-1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}m_B^2 B^\mu B_\mu + \frac{\lambda^2}{2}(\rho^2 + 2\rho v)B^\mu B_\mu \tag{1.11}$$

$$+\frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \frac{1}{2}m_\rho^2 \rho^2 - \frac{\Lambda}{4}\rho^4 - \Lambda v \rho^3.$$

One of the physical fields introduced in the symmetry breaking process ($\xi(x)$) has disappeared. Once all traces of the Nambu-Goldstone boson field $\xi(x)$ have been removed, instead of a massless gauge field the theory contains a massive vector gauge field $B_\mu(x)$ with mass $m_B = gv$, as well as a massive scalar, $\rho(x)$ of mass $m_\rho = \sqrt{-2\mu^s}$. What has happened is that the spontaneous symmetry breaking originates the absorption of the Nambu-Golstone boson $\xi(x)$ by the original massless gauge field. Colloquially, the massless vector boson has "eaten" the scalar field $\xi(x)$, and by doing so has became the massive vector gauge field. This allows the calculation of the W and Z vector boson masses, given by the following formulae:

$$m_{W^{\pm}}^2 = \frac{g_2^2 v^2}{4},\ m_Z^0 = (g_1^2 + g_2^2)\frac{v^2}{4} = \frac{m_W^2}{cos^2\theta_W} \tag{1.12}$$

where g_1 and g_2 are coupling constants and θ_W is the Weinberg angle or weak mixing angle.

So, by means of spontaneous breaking of a local gauge symmetry (the Higgs mechanism), gauge bosons obtain their masses. In addition to give masses to the gauge bosons, the Higgs mechanism is remarkable because it can be simultaneously used to generate the fermions masses in the theory [10]. The introduction of this SU(2) Higgs doublet allows to write Yukawa-like terms of the form

$$\sqrt{\frac{1}{2}} g_f v(\overline{f_L} f_R + \overline{f_R} f_L) \tag{1.13}$$

with the mass of the fermion being given by $g_f v/\sqrt{2}$.

With this mechanism, the Standard Model predicts the existence of a Higgs boson (the physical manifestation of the Higgs mechanism) but does not predict its mass, because very little of what it has been measured on the Standard Model parameters depends directly of this mass. However, by imposing to the theory certain internal consistency requirements the range of possible values may be narrowed [6]. Two crucial requirements are triviality (a trivial theory would be non-interacting at low energies) and unitarity (the probability of scattering from a particular state to another one grows with energy and can be close to one) [10]. The mass upper bound obtained by imposing these constraints is of the order of 1 TeV/c^2.

Current experimental limits exclude the existence of the Higgs boson below $\sim$ 95 GeV (LEP II [7]). By the end of LEP II this limit could be extended to 110 GeV. Therefore it is crucial to have the facility of an accelerator for searching the Higgs up to at least 1 TeV/c^2. Either the Higgs will be produced or some other exotic particles signifying new physics will be discovered. The LHC will be the first accelerator in the world to explore this mass regime.

1.1.3 The Standard Model defects

It has been already said that there is not confirmed experimental evidence contradicting the Standard Model. Nevertheless, it seems obvious that the Standard Model is not a complete theory. Even if one accepts the rather bizarre set of group representations and hypercharges that it requires, the Standard Model contains at least 19 parameters: 3 gauge couplings $g_{1,2,3}$ and 1 CP-violating non-perturbative vacuum angle θ_3, 6 quarks and 3 charged-lepton masses with 3 charged weak mixing angles and 1 CP-violating phase δ, and 2 parameters: (μ, λ) or (m_H, m_W) to characterize the Higgs sector. Moreover, many more parameters are required if one wishes to accommodate observations from non-accelerator experiments. For example, neutrino masses and mixing introduce at least 7 parameters: 3 masses, 3 mixing angles and 1 CP-violating phase, cosmological inflation introduces at least 1 new mass scale of the order 10^{16} GeV, the cosmological baryon asymmetry is not explicable within the Standard Model, so one or more additional parameters are needed, and the cosmological constant may be non-zero.

It is convenient to organize the questions raised by the Standard Model into three broad categories. One is the **Problem of Mass**: do particle masses really originate from a Higgs boson, and, if so, why are these masses not much closer to the Planck mass $m_P \sim 10^{19}$ GeV? Another question is the **Problem of Unification**: can all the particle interactions be unified in a simple gauge group, and, if so, does it predict new observable phenomena such as baryon decay and/or neutrino masses, and does it predict relations between parameters of the Standard Model such as gauge coupling or fermion masses? Also there is the **Problem of Flavour**: what is the origin of the six flavours of quarks and leptons, and what explains their weak charged mixing current and CP violation? Finally, one of the most important problems that the Standard Model cannot solve is the "hierarchy problem" [11]. This does not mean really a difficulty with the Standard Model itself, but rather a disturbing sensitivity of the Higgs potential to the new physics in almost any imaginable extension of the SM.

1.1.3.1 The hierarchy problem

From measurements of the properties of the weak interactions it is known that the squared Higgs mass is of the order $m_H^2 \sim -(100\ GeV)^2$. However, m_H^2 is affected by enormous corrections from the virtual effects of every particle which couples, directly or indirectly, to the Higgs field.

For example in Figure 1.2a we show a correction to m_H^2 from a loop containing a Dirac fermion f with mass m_f. If the Higgs field couples to f with a term in the Lagrangian $-\lambda_f H\overline{f}f$, then the Feyman diagram in Figure 1.2a yields a correction

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2}[-2\Lambda_{UV}^2 + 6m_f^2 ln(\Lambda_{UV}/m_f) + ...]. \qquad (1.14)$$

Here Λ_{UV} is an ultraviolet momentum cutoff used to regulate the loop integral; it should be interpreted as the energy scale at which new physics enters to alter the high-energy behaviour of the theory. The largest corrections come when f is the top quark with $\lambda \approx 1$. In this case if Λ_{UV} is of the order of M_P, say, then this quantum correction to m_H^2 is 30 orders of magnitude larger than the aimed-for value of $m_H^2 \sim -(100GeV)^2$.

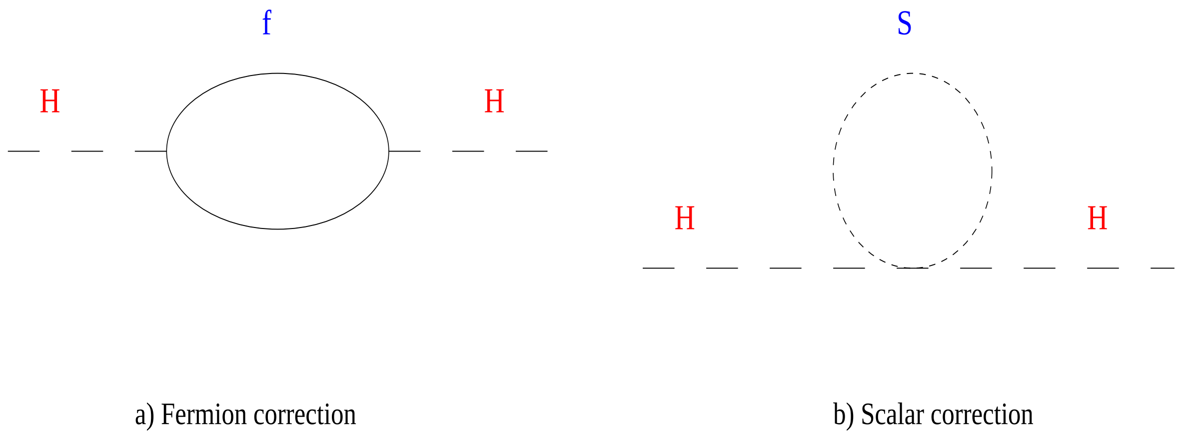

Figure 1.2: Correction to m_H^2 from: a) loop containing a Dirac fermion f with mass m_f, b) loop containing a heavy complex scalar particle S with mass m_S.

If the Higgs boson is a fundamental particle, either the assumption that it does not exist *any* heavy particle which couples (even indirectly or extremely weakly) to the Higgs scalar field has to be made, or some rather striking cancellation is needed between the various contributions to Δm_H^2. Fortunately, cancellation of all such contributions to scalar masses are not only possible, but are actually unavoidable once we merely assume that a symmetry relating fermions and bosons, called a *supersymmetry*, should exist.

1.2 Supersymmetry

Suppose there exists a heavy complex scalar particle S with mass m_S which couples to the Higgs with a lagrangian term $-\lambda_S|H|^2|S|^2$. Then the Feynman diagram in figure 1.2b gives a correction:

$$\Delta m_H^2 = \frac{|\lambda_S|^2}{16\pi^2}[\Lambda_{UV}^2 + 6m_S^2 ln(\Lambda_{UV}/m_S) + ...]. \tag{1.15}$$

The systematic cancellation of the dangerous contributions to Δm_H^2 can only be brought by the type of conspiracy which is better known to physicists as a symmetry. It is evident from comparing eqs. (1.14), (1.15) that the new symmetry ought to relate fermions and bosons, because of the relative minus sign between fermion loop and boson loop contributions to Δm_H^2. If each of the quarks and leptons of the Standard Model is accompanied by two complex scalars with $\lambda_S = |\lambda_f|^2$, then the Λ_{UV}^2 contributions of figures 1.2a and 1.2b will be neatly canceled (to explain this would go beyond the aims of this chapter, which is to give an introduction to supersymmetry).

A supersymmetry transformation turns a bosonic state into a fermionic state, and viceversa. The Q operator generating such transformations must be an anticommuting spinor, with

$$Q|Boson\rangle = |Fermion\rangle, \quad Q|Fermion\rangle = |Boson\rangle. \tag{1.16}$$

Spinors are intrinsically complex objects, so $Q^\dagger$ (the hermitian conjugate of Q) is also a symmetry generator. Because Q and $Q^\dagger$ are fermionic operators, they carry angular momentum of spin 1/2, so it is clear that sypersymmetry [12] must be a spacetime symmetry.

The single-particle states of a sypersymmetric theory fall naturally into irreducible representations of the supersymmetry algebra which are called *supermultiplets*. Each supermultiplet contains both fermion and boson states, which are commonly known as *superpartners* of each other. The $(mass)^2$ operator $-P^2$ commutes with the Q, $Q^\dagger$ operators, and with all spacetime rotation and translation operators, so it follows immediately that particles which inhabit the same irreducible supermultiplet must have equal eigenvalues of $-P^2$, and therefore equal masses.

The supersymmetry generators also commute with the generators of the gauge transformations. Therefore particles in the same supermultiplet must also be in the same representation of the gauge group, and so must have the same electric charges, weak isospin and color degrees of freedom.

Each supermultiplet contains an equal number of fermionic and bosonic degrees of freedom:

$$n_B = n_F. \tag{1.17}$$

The simplest possibility for a supermultiplet which is consistent with eq. (1.17) has:

- a single Weyl fermion (with two helicity states, so $n_F = 2$) and

• two real scalars (each with $n_B = 1$).

This combination of a two-componet Weyl fermion and a complex scalar field is called a *chiral or matter supermultiplet.*

The next simplest possibility for a supermultiplet contains:

• a spin-1 vector boson. A massless spin-1 boson has two helicity states, so the number of bosonic degrees of freedom is $n_B = 2$. Its superpartner is therefore a massless spin-1/2 Weyl fermion, again with two helicity states, so $n_F = 2$. Gauge bosons must transform as the adjoint representation of the gauge group, so their fermionic partners, called *gauginos*, must also.

Such a combination of spin-1/2 gauginos and spin-1 gauge bosons is called a *gauge or vector supermultiplet.*

The ordinary, non-extended, phenomenologically-viable type of supersymmetric model is sometimes called $N = 1$ supersymmetry, with N referring to the number of supersymmetries (the number of distinct copies of Q, $Q^\dagger$).

The names for the spin-0 partners of the quarks and leptons are constructed by prepending an "s", which is a short for scalar. Thus generically they are called *squarks* and *sleptons* (short for "scalar quark" and "scalar lepton"). The symbols for the squarks and sleptons are the same as for the corresponding fermions, but with a tilde used to denote the superpartner of a Standard Model particle.

The Higgs supersymmetry sector requires two Higgs doublets H_u, H_d with opposite hypercharges in order to give masses to all the matter fermions and to avoid gauge anomalies originated by the spin-1/2 higgsinos (this will be explain in the next section). H_u and H_d are the fields that appear in the Lagrangian, but the physical states of the Higgs boson, after electroweak symmetry breaking, are two charged ($H^\pm$) and three neutral (h, H, A) physical states. The h and H are the CP-odd scalars and A is the CP-even scalar.

1.3 The Minimal Supersymmetric Standard Model (MSSM)

This section explains the supersymmetric extension of the Standard Model (SM). Subsection 1.3.1 presents the SM particles and their superpartners. Subsection 1.3.2 discusses the superpotential and supersymmetric interactions of the MSSM. Subsection

1.3.3 presents the R-parity (also known as matter parity) and its consequences. Subsection 1.3.4 describes the soft supersymmetry breaking in the MSSM. Finally, the last subsection, 1.3.5, introduces the electroweak symmetry breaking and the Higgs bosons in the MSSM.

1.3.1 MSSM spectrum of particles

The chiral supermultiplets of a minimal phenomenologically viable extension of the Standard Model are summarized in Table 1.2, classified according to their transformation properties under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which combines ($u_L$, d_L) and (ν, e_L) degrees of freedom into $SU(2)_L$ doublets. Here we have followed the standard convention that all chiral supermultiplets are defined in terms of left-handed Weyl spinors, so that the *conjugates* of the right-handed quarks and leptons (and their superpartners) appear in Table 1.2. Q stands for the $SU(2)_L$-doublet chiral supermultiplet containing $\widetilde{u}_L$, u_L (with weak isospin component $L_3 = 1/2$) and $\widetilde{d}_L$, d_L (with $L_3 = -1/2$), while $\overline{u}$ stands for the $SU(2)_L$-singlet supermultiplet containing $\widetilde{u}^*_R$, $u^\dagger_R$. There are three families for each of the quark and lepton supermultiplets, but we have used first-family representatives only. The superpartners of the left- and right-handed parts of the electron Dirac field are called left- and right-handed selectrons and they are denoted by $\widetilde{e}_L$ and $\widetilde{e}_R$. It is important to keep in mind that the "handedness" here does not refer to the helicity of the selectrons, since they are spin 0 particles, but to that of their superpartners. In the SM the neutrinos are always left-handed, so the sneutrinos are denoted generically by $\widetilde{\nu}$ with a subscript to indicate which lepton flavour they carry.

The two Higgs doublets (H_u, H_d) have hypercharge Y= +1/2 and -1/2 respectively. This is due to the structure of the supersymmetric theories: only a $Y = 1/2$ Higgs chiral supermultiplet can have the necessary Yukawa couplings to give masses to up-type quarks with charge +2/3, and only a Y = -1/2 Higgs can give masses to down-type quarks with charge -1/3 and to charged leptons. The neutral scalar that corresponds to the physical SM Higgs boson is a linear combination of H^0_u and H^0_d. The fermionic partners of the Higgs scalars are called higgsinos. It is interesting to note that the Higgs chiral supermultiplet H_d has exactly the same Standard Model gauge quantum numbers as the left-handed sleptons and leptons L_i.

The electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ has associated with it the spin-1 gauge bosons W^+, W^0, W^- and B^0, as well as the spin-1/2 superpartners $\widetilde{W}^+, \widetilde{W}^0, \widetilde{W}^-$ and $\widetilde{B}^0$, called *winos* and *bino*. After electroweak symmetry breaking, the W^0, B^0 gauge eigenstates mix to give mass eigenstates Z^0 and γ. The corresponding gaugino mixtures of $\widetilde{W}^0$ and $\widetilde{B}^0$ are called zino ($\widetilde{Z}^0$) and photino ($\widetilde{\gamma}$). Table 1.3 summarizes the gauge supermultiplets of a minimal supersymmetric extension of the Standard Model.

Table 1.2: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The hypercharge is computed using the equation $Q_{em} = L_3 + Y$ where L_3 is the third isospin component, Q_{em} is the electromagnetic charge and Y is the hypercharge.

Names		spin 0	spin 1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
squarks, quarks (× 3 families)	Q	$(\widetilde{u}_L, \widetilde{d}_L)$	(u_L, d_L)	(3,2,1/6)
	$\overline{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	($\overline{3}$,1,-2/3)
	$\overline{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	($\overline{3}$,1,1/3)
sleptons, leptons (× 3 families)	L	$(\widetilde{\nu}, \widetilde{e}_L)$	(ν, e_L)	(1,2, -1/2)
	$\overline{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	(1,1,1)
Higgs, Higgsinos	H_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	(1,2,1/2)
	H_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	(1,2,-1/2)

Table 1.3: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C \times SU(2)_L \times U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	(8,1,0)
winos, W bosons	$\widetilde{W}^\pm$, $\widetilde{W}^0$	$W^\pm$, W^0	(1,3,0)
bino, B boson	$\widetilde{B}^0$	B^0	(1,1,0)

Higgsinos and electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking. The neutral higgsinos $\widetilde{H}_u^0$ and $\widetilde{H}_d^0$, and the neutral gauginos $\widetilde{B}$ and $\widetilde{W}^0$ combine to form four neutral mass eigenstates called *neutralinos*, $\widetilde{X}_{i=1,2,3,4}^0$. The charged higgsinos $\widetilde{H}_u^{\pm}$ and $\widetilde{H}_d^{\pm}$, and the winos $\widetilde{W}^{\pm}$, mix to form two mass eigenstates with charge ± 1 called *charginos* $\widetilde{X}_{i=1,2,3,4}^{\pm}$.

1.3.2 The superpotential and supersymmetric interactions

The superpotential for the MSSM is given by:

$$W_{MSSM} = \overline{u} y_u Q H_u - \overline{d} y_d Q H_d - \overline{e} y_e L H_d + \mu H_u H_d \tag{1.18}$$

where H_u, H_d, Q, L, $\overline{u}$, $\overline{d}$, $\overline{e}$ are the chiral superfields corresponding to the chiral supermultiplets given in Table 1.2. The dimensionless Yukawa coupling parameters: y_u, y_d, y_e are 3×3 matrices in the familie space. The Yukawa matrices determine the masses and CKM (Cabibbo-Kobayashi-Maskawa) mixing angles of the ordinary quarks and leptons after the neutral scalar components of H_u and H_d get VEVs (Vacuum Expectation Value). The μ term is the supersymmetric version of the Higgs boson mass in the Standard Model.

Since the Yukawa interactions y^{ijk} in a general supersymmetric theory must be completely symmetric under interchange of i, j, k, we know that y_u, y_d and y_e imply not only Higgs-quark-quark and Higgs-lepton-lepton coupling as in the Standard Model, but also squark-Higgsino-quark and slepton-Higgsino-lepton interactions. To illustrate this, some of the interactions which involve the top-quark Yukawa coupling are shown in figure 1.3.

Figure 1.3a is the Standard Model-like coupling of the top quark to the neutral complex scalar Higgs boson. In figure 1.3b, it is shown the coupling of left-handed top squark $\widetilde{t}_L$ to the neutral higgsino field H_u^0 and right-handed top quark, while in figure 1.3c the right-handed top-squark field couples to $\widetilde{H}_u^0$ and t_L. For each of the three interactions, there is a similar one by changing $H_u^0 \to H_u^+$ and $t_L \to -b_L$ (with tildes where appropriate). All of these interactions are required by the supersymmetry to have the same strength y_t. This is also an incontrovertible prediction of softly-broken supersymmetry at tree-level.

There exist also scalar quartic interactions with strength proportional to y_t^2, as can be seen in figure 1.4. Three of them are shown in the figure, but there are nine more, which can be obtained by replacing $\widetilde{t}_L \to \widetilde{b}_L$ and/or $H_u^0 \to H_u^+$ in each vertex. This

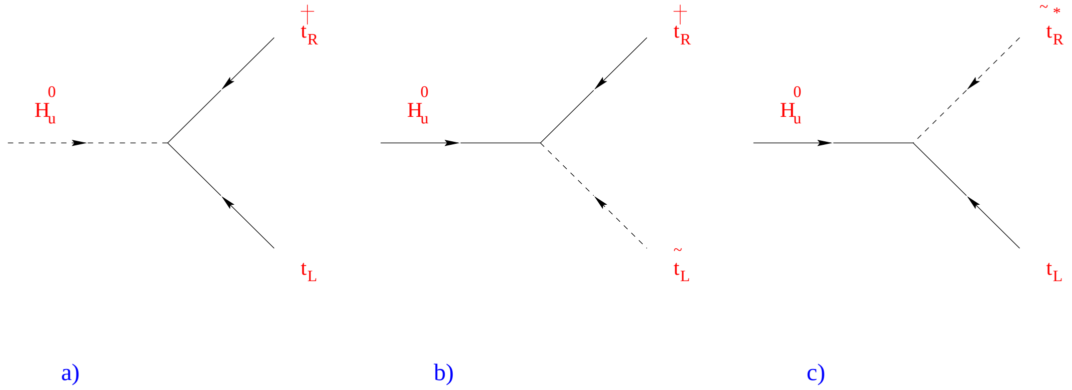

Figure 1.3: The top-quark Yukawa coupling (a) and its supersymmetrizations (b), (c), all of strength y_t.

illustrates the remarcable economy of supersymmetry; there are many interactions determined by only a single parameter! In a similar way, the existence of all other quark and lepton Yukawa couplings in the superpotential leads not only to the Higgs-quark-quark and Higgs-lepton-lepton in the lagrangian terms as in the ordinary Standard Model, but also to squark-higgsino-quark and slepton-higgsino-lepton terms, and scalar quartic coupling terms $[(squark)^4$, $(slepton)^4$, $(squark)^2(slepton)^2$, $(squark)^2(Higgs)^2$, and $(slepton)^2(Higgs)^2]$.

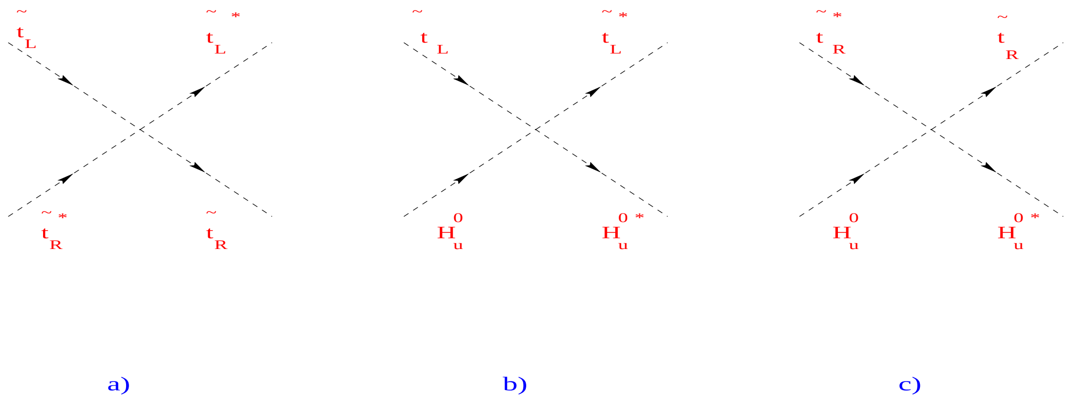

Figure 1.4: Some of the quartic interactions with strength proportional to y_t^2.

Decay and especially production processes for superpartners in the MSSM are typically dominated by the supersymmetric interactions of the gauge-coupling strength. The couplings of the Standard Model gauge bosons (photon, $W^{\pm}$, Z^0) to the MSSM particles are determined completely by the gauge invariance of the kinetic terms in the lagrangian. The gauginos also couple to (squark, quark), (slepton, lepton) and (Higgs, higgsino) pairs. The Feynman diagram for this interaction is shown in figure 1.5a.

Figure 1.5b,c shows in a similar way the coupling of (squark, quark), (lepton, slepton) and (Higgs, higgsino) pairs to the winos and bino, with strengths proportional to the electroweak gauge couplings g and g' respectiveley.

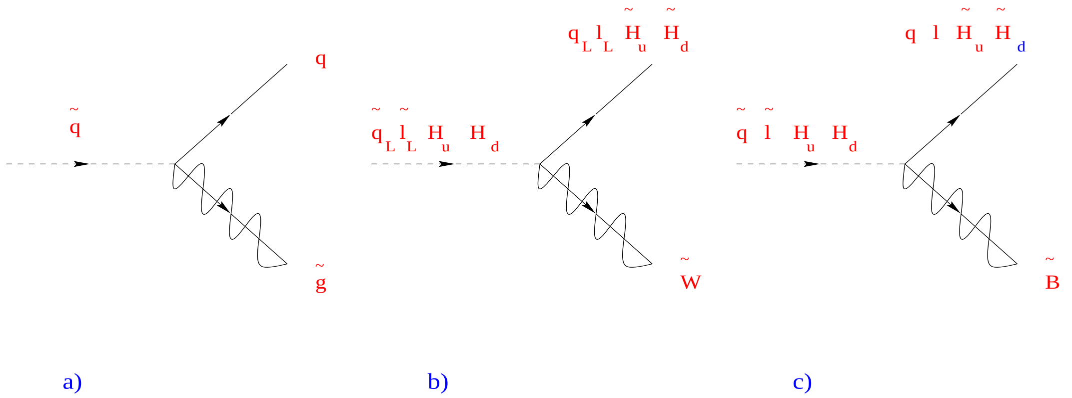

Figure 1.5: Coupling of the gluino, wino, and bino to MSSM (scalar, fermion) pairs.

The dimensionful terms in the supersymmetric part of the MSSM lagrangian are all dependent on μ. The μ-term and the Yukawa couplings in the superpotential, eq. (1.18), combine to yield $(scalar)^3$ of the form represented in figure 1.6 for the MSSM case. These couplings which are proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$, respectively, play an important role in determing the mixing of top squarks, bottom squarks, and tau sleptons.

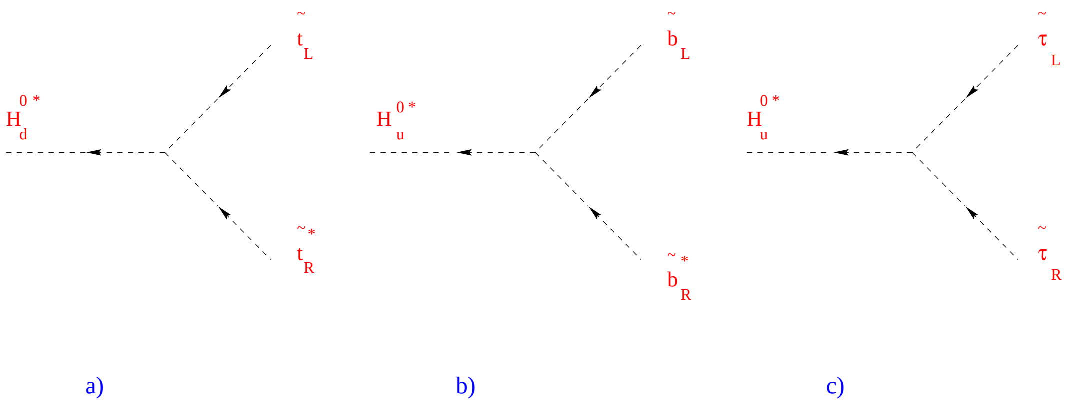

Figure 1.6: Some of the supersymmetric cubic couplings proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$.

1.3.3 R-parity and its consequences

In the MSSM one needs to add a new symmetry which has the effect of eliminating the possibility of baryon (B) and lepton (L) violating terms in the renormalizable superpotential. This new symmetry is called "R-parity" [13], also known as "matter parity", which is defined for each MSSM particle as:

$$P_R = (-1)^{3(B-L)+2s} \tag{1.19}$$

where B and L are the baryon and lepton quantum numbers and s is the spin of the particle.

The advantage of R-parity is that it can be in principle an *exact* and fundamental symmetry, while B and L themselves cannot, since they are known to be violated by non-perturbative electroweak effects. Even though with exact matter parity conservation in the MSSM, one expects that baryon number and total lepton number violation will occur in very tiny amounts, due to non-renormalizable terms in the Lagrangian.

The R-parity assignment is very useful for phenomenology because all of the Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$), while all of the squarks, sleptons, gauginos and higgsinos have odd R-parity ($P_R = -1$).

The odd R-parity particles are known as "supersymmetric particles" or "sparticles" for short, and they are distinguished by a tilde (see tables 1.2 and 1.3). If R-parity is exactly conserved, then there can be no mixing between the sparticles and the $P_R = +1$ particles. Furthermore, every interaction vertex in the theory contains an even number of $P_R = -1$ sparticles. This has three extremely important phenomenological consequences:

- The lightest sparticle with $P_R = -1$, called the "lightest supersymmetric particle" or LSP, must be absolutely stable. If the LSP is electrically neutral it interacts only weakly with ordinary matter, and therefore can be an attractive candidate [14] for the non-baryonic dark matter which seems to be required by cosmology.

- Each sparticle other than the LSP must eventually decay into a state which contains an odd number of LSPs (usually just one).

- In collider experiments, sparticles can only be produced in even numbers (usually two at a time).

1.3.4 Soft supersymmetry breaking in the MSSM

A realistic phenomenological model must contain supersymmetry breaking, because if supersymmetry were unbroken there would have to exist, for example, slectrons with the same mass as the electrons, and such particles would be very easy to detect, but they have not been discovered yet. From a theoretical perspective, we expect that supersymmetry, if it exists at all, should be an exact symmetry which is spontaneously broken. In other words, the ultimate model should have a lagrangian density which is invariant under supersymmetry, but a vacuum state which is not. Many models of the spontaneous symmetry breaking have indeed been proposed. They always involve the MSSM extending to include new particles and interactions at very high mass scales, and there is no consensus on exactly how this should be done. However, from a practical point of view, it is extremely useful to simply parametrize our ignorance of these issues by just introducing extra terms which break supersymmetry explicitly in the effective lagrangian. The extra supersymmetry-breaking coupling should be soft in order to be able to naturally maintain a hierarchy between the electroweak scale and the Planck mass scale. In the context of a general renormalizable theory, the possible soft supersymmetry-breaking terms in the lagrangian are:

$$
\begin{aligned}
L_{soft}^{MSSM} = & -\frac{1}{2}(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}) + c.c. \\
& -(\tilde{\overline{u}}\mathbf{a_u}\widetilde{Q}H_u - \tilde{\overline{d}}\mathbf{a_d}\widetilde{Q}H_d - \tilde{\overline{e}}\mathbf{a_e}\tilde{\overline{L}}H_d) + c.c \\
& -\widetilde{Q}^\dagger\mathbf{m_Q^2}\widetilde{Q} - \widetilde{L}^\dagger\mathbf{m_L^2}\widetilde{L} - \tilde{\overline{u}}\mathbf{m_{\overline{u}}^2}\tilde{\overline{u}}^\dagger - \tilde{\overline{d}}\mathbf{m_{\overline{d}}^2}\tilde{\overline{d}}^\dagger - \tilde{\overline{e}}\mathbf{m_{\overline{e}}^2}\tilde{\overline{e}}^\dagger \\
& -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + c.c.)
\end{aligned}
\tag{1.20}
$$

where M_3, M_2 and M_1 are the gluino, wino, and bino mass terms. The second line in the lagrangian contains the cubic scalar couplings. Each of a_u, a_d, a_e is a complex 3×3 matrix in a family space, with mass dimensions. They are in one-to-one correspondence with the Yukawa coupling matrices in the superpotential. The third line consists of squarks and slepton mass terms, where m_Q^2, m_L^2, $m_{\overline{u}}^2$, $m_{\overline{d}}^2$, $m_{\overline{e}}^2$ are 3×3 matrix. Finally, in the last line we have supersymmetry-breaking contributions to the Higgs potential, being $m_{H_u}^2$, $m_{H_d}^2$ and b $(mass)^2$ terms.

L_{soft}^{MSSM} introduces many new parameters which were not present in the ordinary Standard Model. A careful counting reveals that there are 105 parameters: masses, phases and mixing angles in the MSSM lagrangian which cannot be rotated away by redefining the phases and basics flavours for the quark and lepton supermultiplets, and which have no counterpart in the ordinary Standard Model. Thus, in principle, supersymmetry appears to introduce a tremendous arbitrariness in the lagrangian.

Fortunately, there is already good experimental evidence that some sort of powerful "organizing principle" must govern the soft terms [12]. This is because most of the new parameters in eq. (1.20) involve flavour mixing or CP violation of the type which is already severely restricted by the experiments [15].

1.3.4.1 Electroweak symmetry breaking and the Higgs bosons

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets: $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^+, H_d^0)$, rather than just one as in the ordinary Standard Model. The reason is the following: the features of the W superpotential (equation 1.18 in section 1.3.2), which determines the chiral interactions, do not allow to introduce complex conjugates. If complex conjugates are introduced in the superpotential, supersymmetry is automatically broken in an uncontrolled way. To avoid this problem, the complex conjugates are substituted by another Higgs doublet. As it has been explained in section 1.3.2, supersymmetry must be broken in a controlled way by introducing soft terms in the potential.

Having established the necessary conditions for H_u^0 and H_d^0 to get non-zero VEVs, we can now require that they are compatible with the observed phenomenology of electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. Being their VEVs: $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$, at the minimum of the potential. These VEVs can be connected to the known mass of the Z^0 boson and the electroweak gauge couplings:

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \sim (174\ GeV)^2. \tag{1.21}$$

The ratio of the two VEVs is traditionally written as:

$$tan\beta = v_u/v_d \tag{1.22}$$

The $tan\beta$ value is not fixed, but it depends on the MSSM lagrangian parameters in a calculable way. Since $v_u = vsin\beta$ and $v_d = vcos\beta$ were taken to be real and positive, we have a requirement that $0 < \beta < \pi/2$.

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublets, or eight real scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bonsons G^0 and $G^\pm$ which become the longitudinal modes of the Z^0 and $W^{\pm}$ massive vector bosons. The remaining five Higgs scalar mass eigenstates consist of one CP-odd neutral scalar A^0, a charge +1 scalar H^+ and its conjugate charge -1 scalar H^-, and two CP-even neutral scalars h^0 and H^0.

The masses of A^0, H^0 and $H^{\pm}$ can in principle be arbitrarily large, but the mass of h^0 is bounded from the requirements given above [12]:

$$m_{h^0} \leq 150\ GeV \tag{1.23}$$

that is assuming that none of the MSSM sparticles have masses exceeding 1 TeV and that all of the couplings in the theory remain perturbative up to the unification scale. The particles A^0, H^0 and $H^{\pm}$ are much heavier and nearly degenerate, forming an isospin doublet which decouples from sufficiently low-energy experiments.

In the MSSM the masses and CKM mixing angles of the quarks and leptons are determined by the Yukawa coupling of the superpotential and by the $tan\beta$ parameter. This is because the top, charm and up quarks get masses proportional to $v_u = vsin\beta$ and the bottom, strange, and down quarks as well as the charge leptons get masses proportional to $v_d = vcos\beta$.

There are good theoretical motivations to consider models with large $tan\beta$. For example: models based on the Grand Unification Theory (GUT) gauge group $SO(10)$ can unify the running top, bottom and tau Yukawa couplings at the unification scale; this requires $tan\beta$ to be roughly of the order of m_t/m_b [16].

Bibliography

[1] Hall, Nina, A Supercollider for Europe, New Scientist v.131, p. 35-9, July 27, 1991.

[2] Glashow, S.L., Nucl. Phys. 22, 579 (1961).

[3] Weinberg, S., Phys. Rev. Lett. 19, 1264 (1967).

[4] Salam, A., Elementary Particle Theory, N. Svartholm, ed.(Stockholm: Almquist and Wiksell, 1986).

[5] Coughlan, G.D. and Dodd, J.E., The Ideas of Particle Physics: An Introduction for Scientists, Cambridge University Press, Cambridge, 1991.

[6] Quigg, Chris., Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, Addison-Wesley, Reading, Massachusetts, 1993.

[7] Ellis, J., Beyond the Standard Model for Hillwalkers, CERN-TH/98-329, 1998.

[8] Jenni, P. and White, H., Physics and Experimental Challenge of Future Hadron Colliders, Proceeding of a lectures series given at Eigth Lake Louisse Winter Institute, Lake Louise, Alberta, Canada, February, 1993.

[9] Gelmini, G., Beyond the Standard Model, Proceedings of the Eighth Lake Louisse Winter Institute, Lake Louise, Alberta. Canada, February, 1993.

[10] Gunion, J. et al., The Higgs Hunter's Guide, Addison-Wesley, Redwood City, California, 1990.

[11] Weinberg, S., Phys. Rev. D 13, 974 (1976); Phys. Rev. D 19, 1277 (1979); Susskind, L., Phys. Rev. D 20, 2619 (1979); Hooft, G. 't., in Recent development in gauge theories, Proceedings of the NATO Advanced Summer Institute, Cargese 1979, ed. Hooft, G. 't. et al. (Plenum, New York 1980).

[12] Martin S. P., A Supersymmetry Primer, hep-ph/9709356.

[13] Farrar, G. R. and Fayet, P., Phys. Lett. B 76, 575 (1978).

[14] Goldber, H., Phys. Rev. Lett. 50, 1419 (1983); Ellis, J., Hagelin, J., Nanopoulos, D.V., Olive, K., and Srednicki, M., Nucl. Phys. B238, 453 (1984).

[15] Gabbiani, F., Gabrielli, E., Masiero, A., and Silvestrini, L., Nucl. Phys. B477, 321 (1996) and references therein.

[16] Lopez, C., Phys. lett B 126, 54 (1983); Arason, H., et al., Phys. Rev. Lett. 67, 2933(1991); Barger, V., Berger, M.S., and Ohmann, P., Phys. Rev. D 47, 1093 (1993); Anderson, G. et al, Phys. Rev. D 49, 3660 (1994).